Quantum-Correlated Measurements Related to the Determination of γ/ϕ_3

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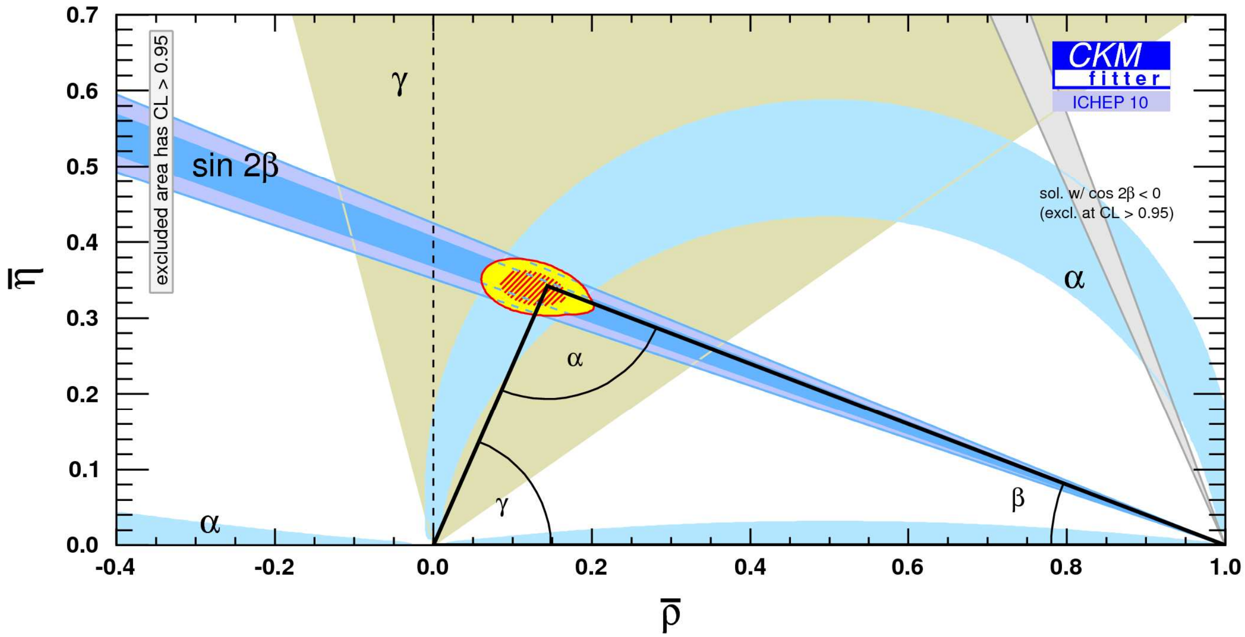
On behalf of the CLEO-c collaboration

Outline

- Determination of γ/ϕ_3 with $B\rightarrow DK$
- The role of quantum-correlated $\psi(3770)\rightarrow D^0\overline{D}^0$ decays
- Current quantum-correlated measurements
  - **$D^0\rightarrow K^0h^+h^-$ (h=π or K)**
    - Impact on γ/ϕ_3
  - $D^0\rightarrow K^-\pi^+$ [David Asner's talk coming up next]
  - **$D^0\rightarrow K^-\pi^+\pi^0$ and $D^0\rightarrow K^-\pi^+\pi^+\pi^-$**
    - Impact on γ/ϕ_3
- Conclusions and the future

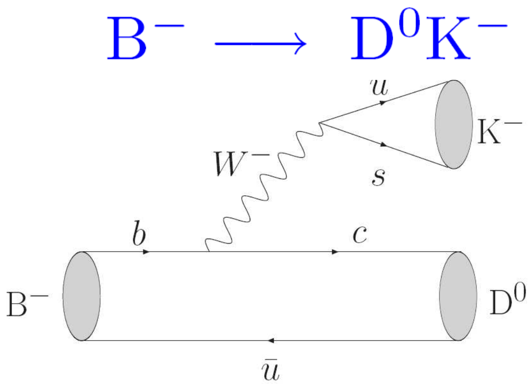
Status of direct determination of γ

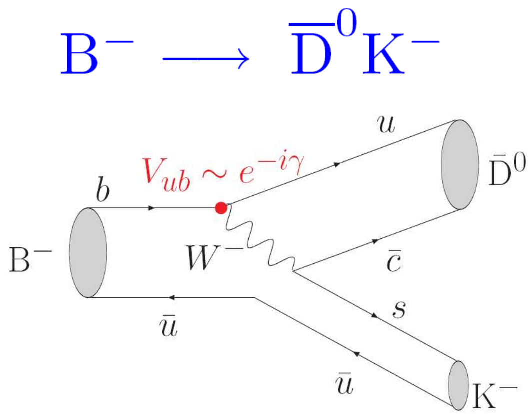


- γ is the least well determined angle of the unitarity triangle with an uncertainty of ~20° from direct measurements
  - $\sigma_\beta = 1°$

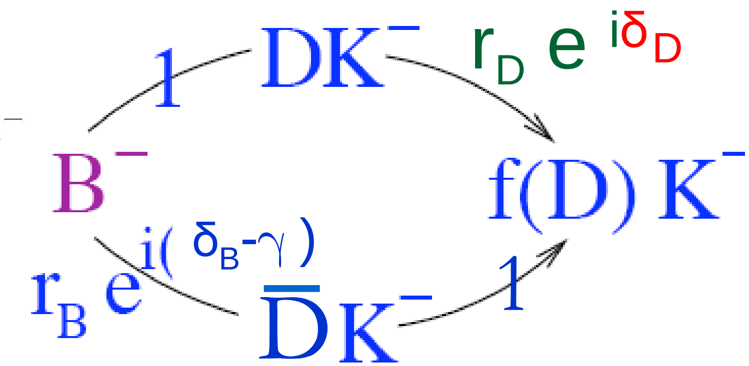
- Comparison of measurements of γ in tree and loop processes sensitive to new physics
  - **Side opposite - B-mixing measurements loop only**

γ from $B^\pm \rightarrow DK^\pm$





$$\frac{< B^- \longrightarrow \overline{D}^0 K^- >}{< B^- \longrightarrow D^0 K^- >} = r_B e^{i(\delta_B - \gamma)}$$



- Sensitivity through interference between $b \rightarrow u$ and $b \rightarrow c$ transitions
- Require D^0 and $\overline{D}^0$ decay to a common final state, f(D):

  $K^0_S hh$; $K\pi$; $K\pi\pi\pi$; $K\pi\pi^0$

- Comparison of B^- and B^+ rates allow γ to be extracted
- But other parameters to be considered
  - in particular δ_D – **accessed in quantum-correlated D-decays**

r_D & δ_D analogous to B-decay quantities. For multibody decays, these vary over Dalitz space

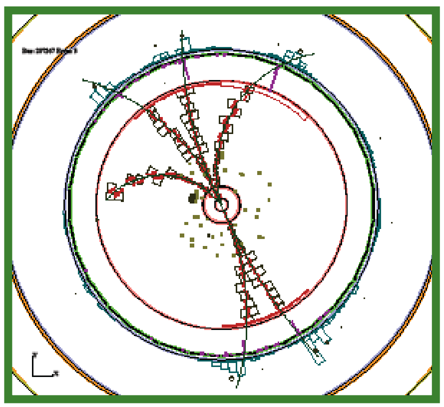
CP-tagging at the ψ(3770)

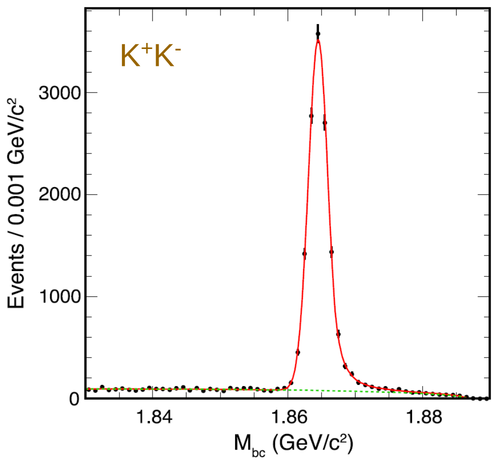
- Quantum correlations in process $e^+e^- \to \psi(3770) \to D^0\overline{D}^0$ allow for *CP-tagging*.
- Reconstruct one D in a mode of interest & other to a CP-eigenstate,
  - For example if tag is K^+K^- (CP+), given that the ψ(3770) is C=-1, signal decay is CP−

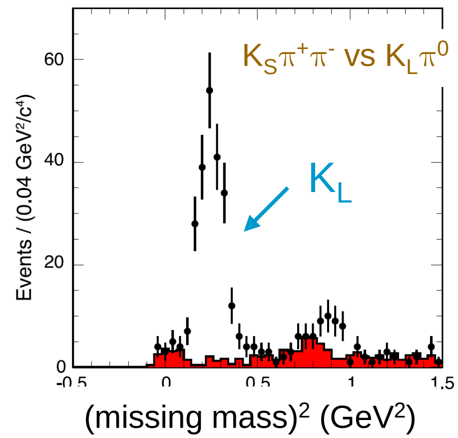
Threshold running has other practical advantages

Very clean – no fragmentation particles.

Unseen particle reconstruction through kinematic constraints







$K_S\pi^+\pi^-$ vs $K^+\pi^-$

CLEO-c accumulated 818 pb^{-1} at ψ(3770)

Hermetic detector with excellent EM calorimetry and hadron PID

CP-tagged D-decays: the essential idea

Dalitz plots of CP-tagged decays at the Ψ(3770) provide additional info to flavour tagged events

Sensitivity to the cosine of strong phase difference between the D^0 & $\bar{D}^0$ **(cosδ)**

$D^{*+} \rightarrow D^0 \pi^+$

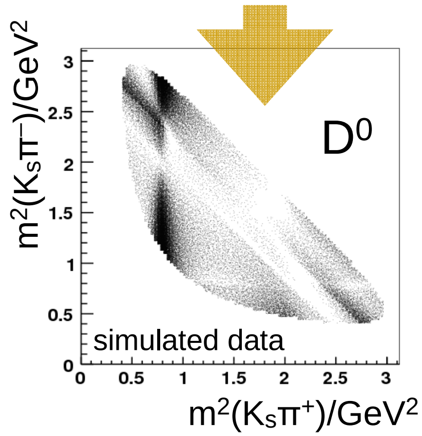
$D^0 \rightarrow K_S \pi^+ \pi^-$

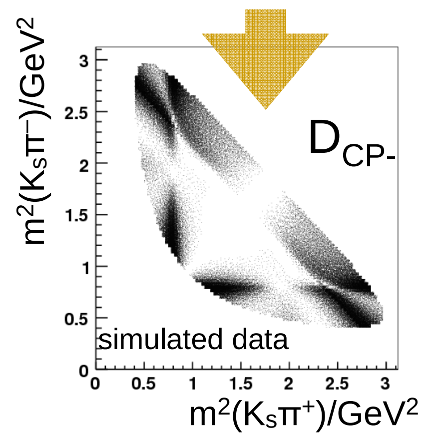
$\psi'' \rightarrow D_a D_b$

$D_a \rightarrow K^+ K^-$ eg. CP+

$D_b \rightarrow K_S \pi^+ \pi^-$

Flavour tagged distribution $\propto$ $|D^0|^2$ or $|\bar{D}^0|^2$





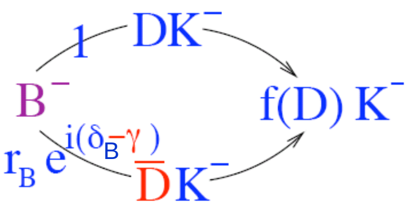
CP-tagged $\propto$ $|D^0|^2 + |\bar{D}^0|^2 \pm 2\,|D^0||\bar{D}^0|$ $\cos\delta$

In a Dalitz-plot bin combinations of flavour & CP-tagged data give access to cosδ

In addition, quantum-correlations allow *other* hadronic decays to be used

Study of $D \to K_S \pi^+ \pi^-$ and $D \to K_S K^+ K^-$ Dalitz Plots in Quantum-correlated Decays

B-factory $B \to D(K_S h^+ h^-)K$ Dalitz Plots for γ

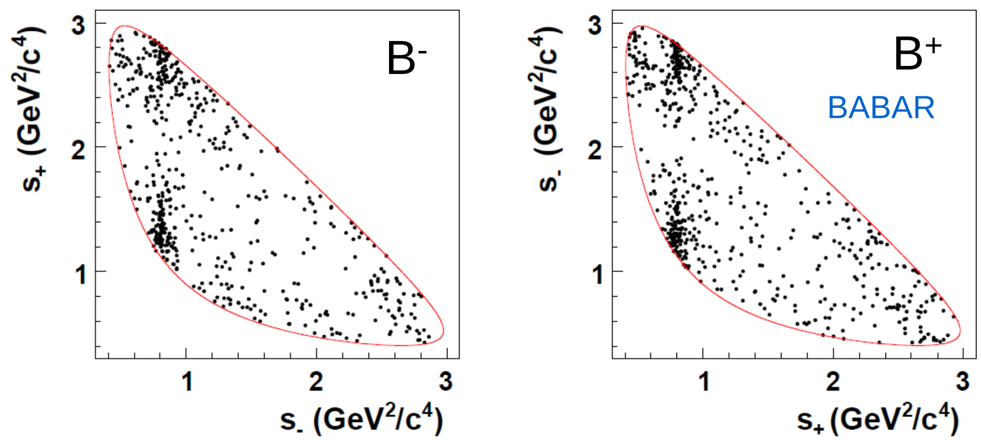


- A powerful choice of common state f(D) is $K_s h^+ h^-$
  - BABAR - PRL **105,** 121801 (2010)
  - Belle - PRD **81,** 112002 (2010)

Differences between B^- and B^+ Dalitz plots allow γ to be extracted in unbinned fit…

…need to understand different amplitudes from D^0 and $\overline{D}^0$ decay across Dalitz space, esp. variation in strong phase

$B^\pm \to (D \to K^0_S \pi^+ \pi^-)K^\pm$



Approach of B-factories: construct Dalitz plot model of $K_S\pi^+\pi^-$ with flavour-tagged decays – estimated model uncertainty of 3-9° which is << statistical error

But LHCb and future facilities will start to be limited by this model uncertainty **–**
Highly desirable to have high precision *model independent* approach

Binned Model-Independent Fit

Binned fit proposed by Giri *et al.* [PRD 68 (2003) 054018] and developed by Bondar & Poluektov [EPJ C 55 (2008) 51; EPJ C47 (2006) 347] removes model dependence by relating events in bin i of Dalitz plot to *experimental observables.*

$B^\pm$ events in bin i of Dalitz plot

Number of events for flavour-tagged D sample

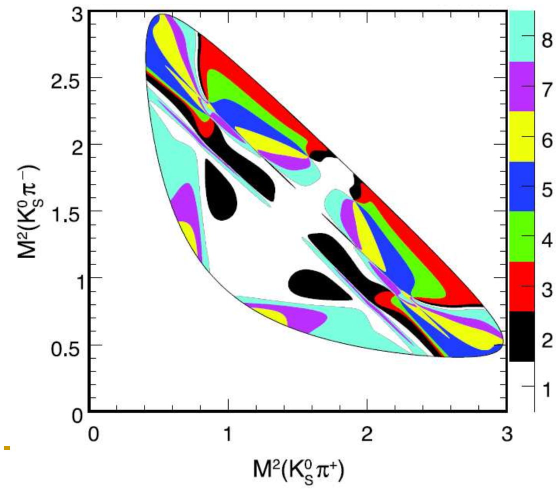
$x_\pm = r_B \cos(\delta_B \pm \gamma)$

$y_\pm = r_B \sin(\delta_B \pm \gamma)$

$$N_i^\pm = h(K_{\pm i} + r_B^2 K_{\mp i} + 2\sqrt{K_i K_{-i}}(x_\pm c_i \pm y_\pm s_i))$$

c_i, s_i : average in bin of cosine, sine of strong phase

Can be measured in quantum correlated decays at ψ(3770)!



Choosing bins of *expected* similar strong phase difference maximises statistical precision

Here take 8 bins of equal spacing in $\Delta\delta_D$ (using as reference model: BaBar, PRL 95 (2005) 121802)

Loss in statistical sensitivity w.r.t. unbinned result…(here ~20%) but no model error!

CLEO-c Quantum-Correlated $K_{S,L}\pi^+\pi^-$ Analysis

First measurements of strong-phase differences R. Briere *et al.*, PRD 80 (2009) 032002

Uses 818 pb^{-1} of $\psi(3770)$ data

- Flavour tags: ~20,000 double-tags
- CP-tags: ~1700 double-tags
- $K^0\pi^+\pi^-$ vs $K^0\pi^+\pi^-$ events: ~1700
- $K_L\pi^+\pi^-$ events are also used:
  CP-odd $K_S\pi^+\pi^- \approx$ CP-even $K_L\pi^+\pi^-$
  Introduces a limited model-dependence to correct for difference

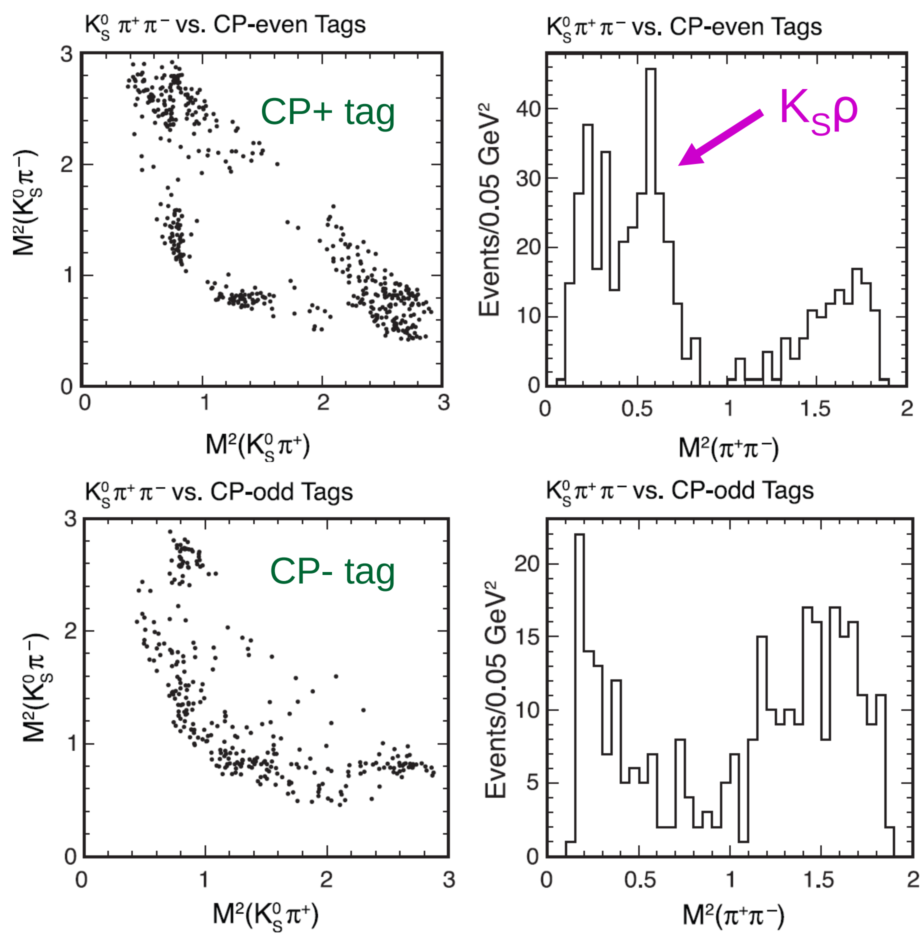
Signal to background 10-100 depending on tag mode

Tag		$K^0_S\pi^+\pi^-$	$K^0_L\pi^+\pi^-$
$K^-\pi^+$		1444	2857
$K^-\pi^+\pi^0$		2759	5133
$K^-\pi^+\pi^+\pi^-$		2240	4100
$K^-e^+\nu$		1191	
K^+K^-	(CP+)	124	357
$\pi^+\pi^-$	(CP+)	61	184
$K^0_S\pi^0\pi^0$	(CP+)	56	
$K^0_L\pi^0$	(CP+)	237	
$K^0_S\pi^0$	(CP−)	189	288
$K^0_S\eta$	(CP−)	39	43
$K^0_S\omega$	(CP−)	83	
$K^0_S\pi^+\pi^-$		473	1201

CP-tagged $K_s\pi^+\pi^-$ Dalitz plots

Clear differences seen between CP-odd and CP-even:

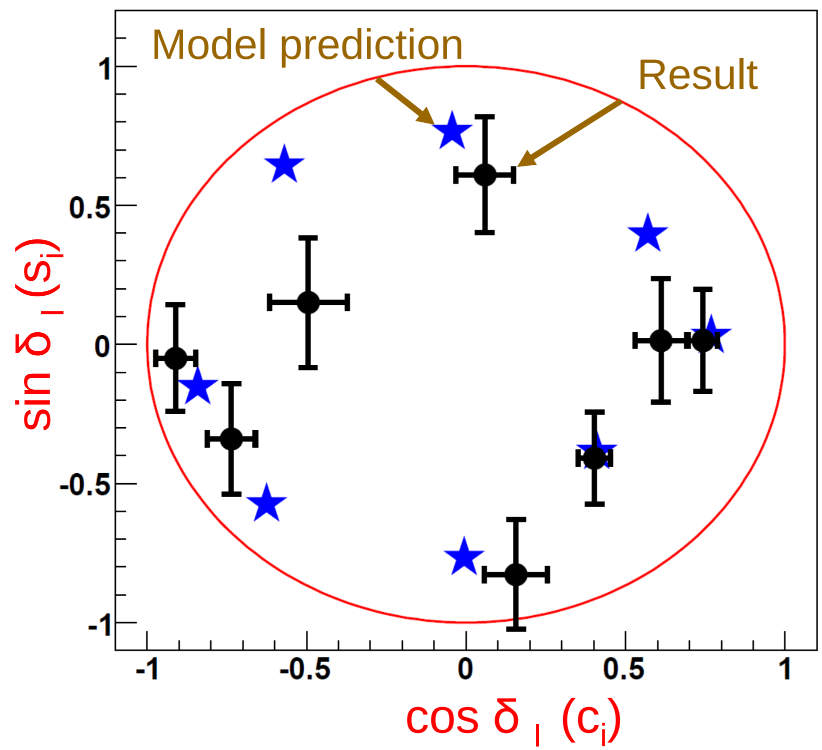
CLEO-c, PRD 80 (2009) 032002



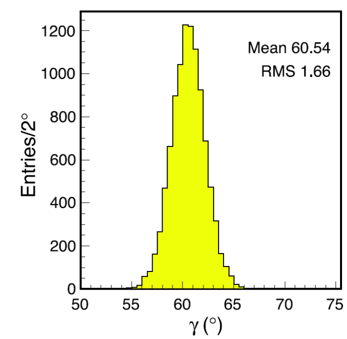
First CLEO-c results and γ/ϕ_3 impact

R. Briere *et al.*, PRD 80 (2009) 032002

(model = BABAR PRL 95 (2005) 121802)



Projected uncertainty on γ arising from uncertainty on c_i & s_i is 1.7^o :



- Smaller than model error
- Plus experimental in origin - dominated by finite CLEO-c statistics

Downside - binning leads to ~20% loss in σ_{stat} relative to unbinned approach

Recent developments (arXiv:1010.2817)

CLEO-c has re-performed $K_S\pi^+\pi^-$ c_i & s_i measurements with same data & approach (+ some improvements on systematics) but with alternative binnings. **Why?**

1. Better model → better chance bin choice will give expected statistical precision

   - Much improved BABAR model [PRD 78 (2008) 034023] . e.g. K-matrix for $\pi\pi$ S-wave & better description of $K\pi$ S-wave. Take as baseline.

   (Aside: even more recent BABAR model (PRL 105) *very* similar to this.)

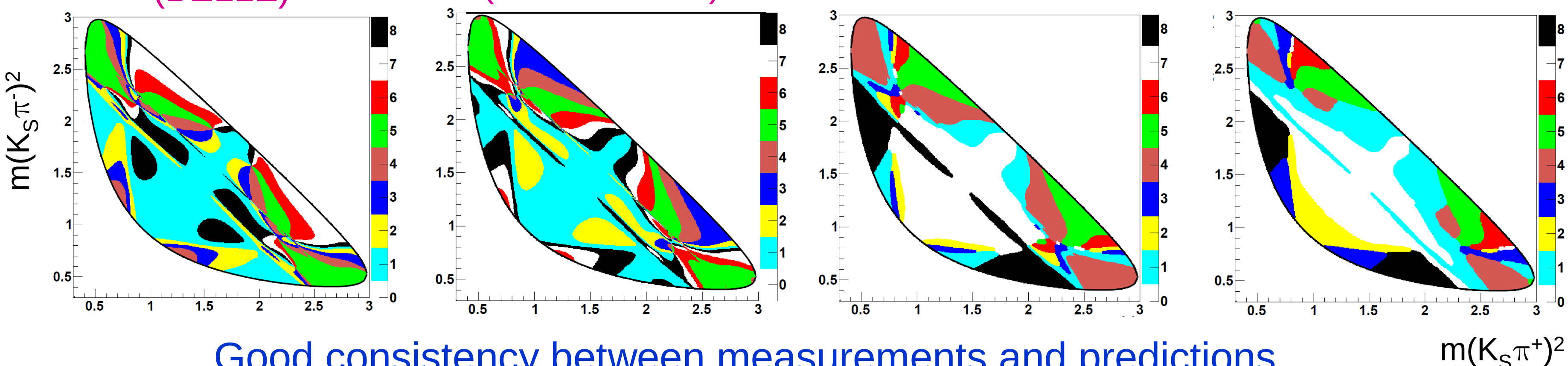
2. Within given model, possible to find binnings with better statistical precision than original equal $\Delta\delta_D$ choice.

   - 'optimal binning' which in low background environment gives ~10% improvement in statistical sensitivity w.r.t. equal $\Delta\delta_D$ choice
   - 'modified optimal binning' which does same as above, but for scenario where more background expected (use LHCb expectations)

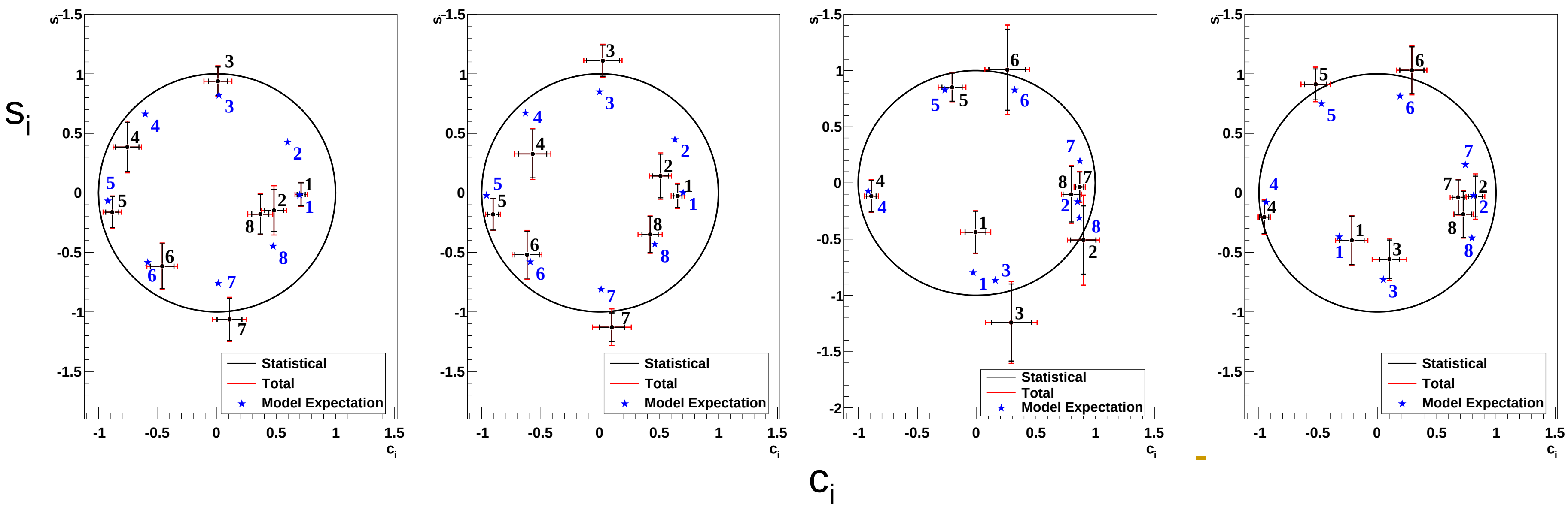
- More binnings give experiments opportunity for cross-checks
  - Produce equal $\Delta\delta_D$ binning results using Belle model [PRD 81 (2010) 112002]

New $K_S\pi^+\pi^-$ binnings – preliminary results

Equal $\Delta\delta_D$ (BELLE) | Equal $\Delta\delta_D$ (BABAR 2008) | Optimal binning (BABAR 2008) | Modified optimal (BABAR 2008)



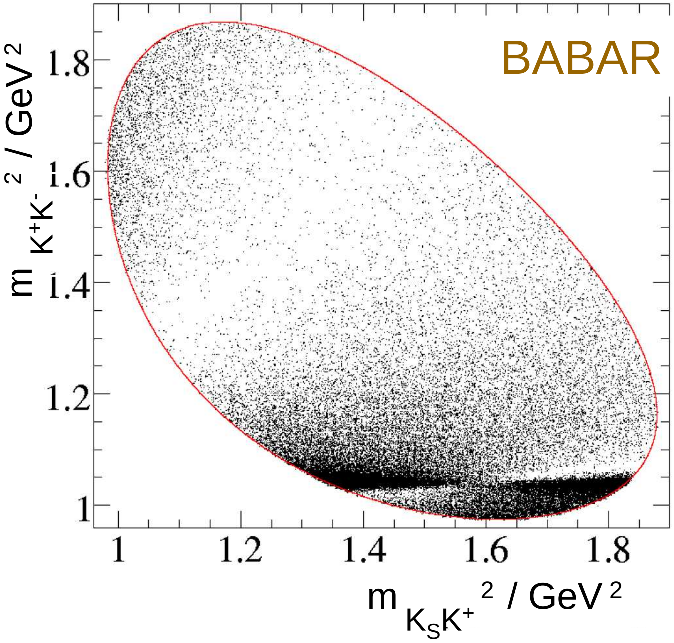
Good consistency between measurements and predictions



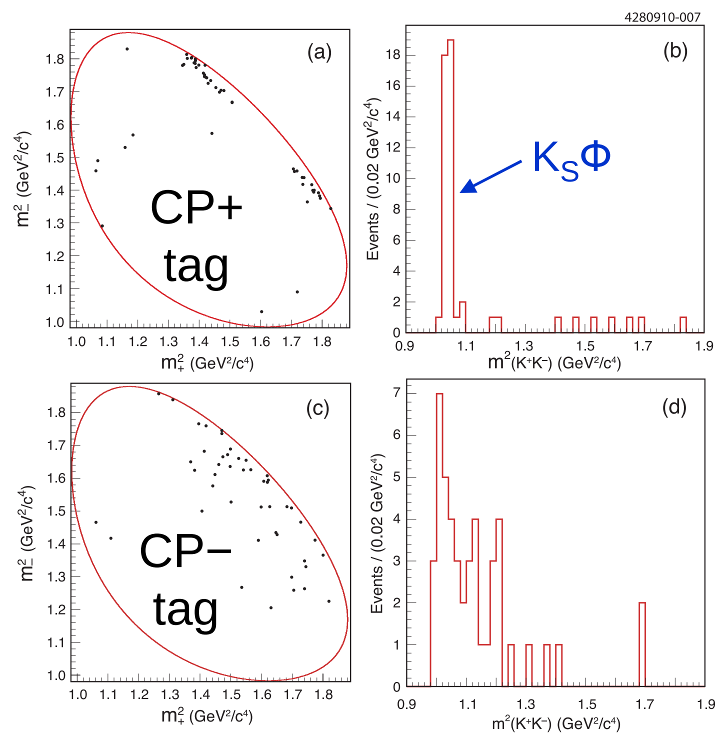
$D^0 \to K^0_S K^+ K^-$

Dalitz γ analysis has been extended to $B^- \to D(K^0_S K^+ K^-)K^-$. Pioneered by BABAR [PRD **78** 034023 (2008) & PRL **105**, 121801 (2010)] who have built an amplitude model with flavour tagged decays

Flavour tagged $K_S KK$ decays



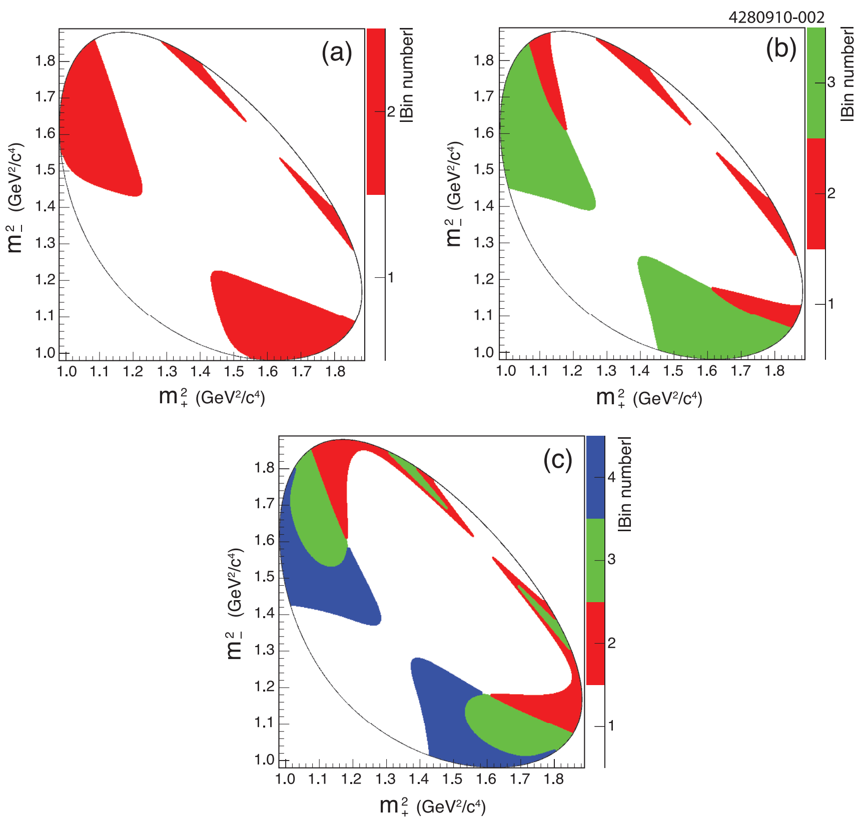
Measurement of c_i's and s_i's also performed at CLEO-c using ~550 quantum-correlated double-tags

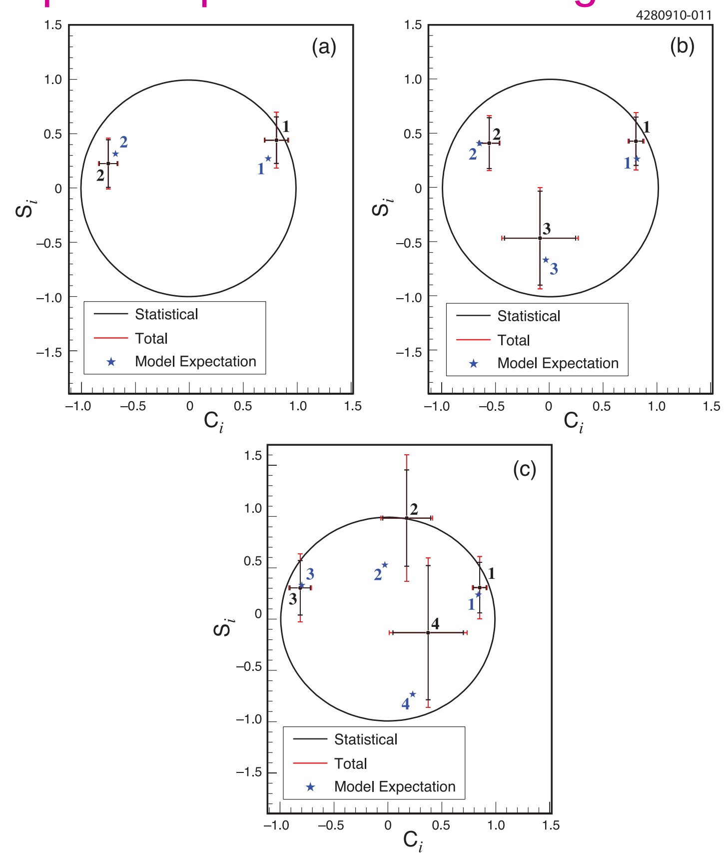


$K_SK^+K^-$ c_i, s_i analysis

c_i and s_i results calculated with equal $\Delta\delta_D$ binning for 2, 3 and 4 bins

Negligible improvement in sensitivity when attempts to optimise the binning are made





Above based on latest model from BABAR (PRL 105 121801 (2010)).

Impact on γ/ϕ_3 determination

- We have estimated the systematic error on γ/ϕ_3 resulting from the uncertainties on the strong-phase parameters for each mode:
  - **1.7° to 3.9° for $K^0_S\pi\pi$** (depending on binning)
  - **3.2° to 3.9° for $K^0_S KK$** (depending on binning)
- Same order or smaller than current model error (3°- 9°) incurred in the binned methods
- Limitation is statistical precision on s_i
  - BES-III can in principle reduce this by a factor of three or more
    - assuming a 10 fb^{-1} data and similar performance to CLEO-c
  - Leading to γ/ϕ_3 error due to strong-phase parameters of order 1°
  - This level of precision is suitable for the future e^+e^- facilities and the proposed LHCb upgrade
    - See excellent talk by A. Poluektov at CKM 2010

CLEO-c coherence factor analysis of $D \rightarrow K\pi\pi\pi$, $K\pi\pi0$

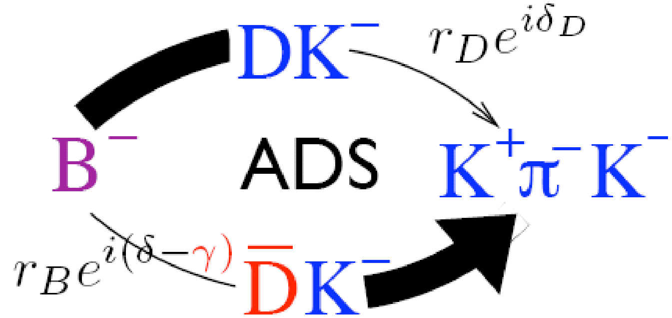
Atwood-Dunietz-Soni (ADS) Method

PRL 78, 3257 (1997)

f(D) = non-CP Eigenstate (e.g. $K^+\pi^-$)

$$\frac{< D^0 \longrightarrow K^+\pi^- >}{< \overline{D}^0 \longrightarrow K^+\pi^- >} = r_D e^{i\delta_D}$$

~0.06



$$\Gamma(B^- \rightarrow (K^-\pi^+)_D K^-) \propto 1 + (r_B r_D^{K\pi})^2 + 2 r_B r_D^{K\pi} \cdot \cos(\delta_B - \delta_D^{K\pi} - \gamma) \quad (1)$$

$$\Gamma(B^- \rightarrow (K^+\pi^-)_D K^-) \propto r_B^2 + (r_D^{K\pi})^2 + 2 r_B r_D^{K\pi} \cdot \cos(\delta_B + \delta_D^{K\pi} - \gamma) \quad (2)$$

$$\Gamma(B^+ \rightarrow (K^+\pi^-)_D K^+) \propto 1 + (r_B r_D^{K\pi})^2 + 2 r_B r_D^{K\pi} \cdot \cos(\delta_B - \delta_D^{K\pi} + \gamma) \quad (3)$$

$$\Gamma(B^+ \rightarrow (K^-\pi^+)_D K^+) \propto r_B^2 + (r_D^{K\pi})^2 + 2 r_B r_D^{K\pi} \cdot \cos(\delta_B + \delta_D^{K\pi} + \gamma) \quad (4)$$

- From counting these 4 rates, together with those from CP eigenstates (*KK*,*ππ*), a determination of γ can be made
- Can determine δ_D from rates but **external constraints extremely helpful**

Multi-body ADS

Mode	Branching Ratio
$K\pi$	3.89%
$K\pi\pi^0$	13.9%
$K3\pi$	8.1%

- $B \to D(K\pi\pi\pi)K$ and $B \to D(K\pi\pi^0)K$ can also be used for ADS analyses
  - Significantly larger branching fractions than $B \to D(K\pi)K$
- However, need to account for the resonant substructure
  - In principle each point in the phase space has a different strong phase associated with it
- Atwood and Soni [PRD **68** 033003 (2003)] showed how to modify the usual ADS equations for this case
  - Introduce **coherence parameter $R_{K3\pi}$** which dilutes interference term sensitive to γ

$$\Gamma(B^- \to (K^+\pi^-\pi^-\pi^+)_D K^-) \propto r_B^2 + (r_D^{K3\pi})^2 + 2 r_B r_D^{K3\pi} R_{K3\pi} \cos(\delta_B + \delta_D^{K3\pi} - \gamma)$$

- $R_{K3\pi}$ ranges from
  - 1=coherent (dominated by a single mode) to
  - 0=incoherent (several significant components)

CLEO-c $K\pi\pi\pi$ & $K\pi\pi^0$ QC Analysis

Sensitivity to the $K\pi\pi\pi$ coherence factor and average strong phase difference comes from counting the following classes of double-tagged events:

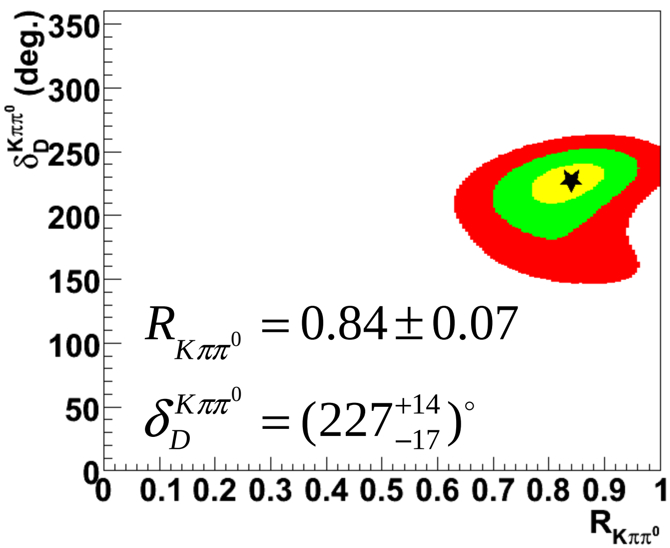
Double tag Rate	Sensitive to
$K^\pm\pi^\mp\pi^+\pi^-$ vs $K^\pm\pi^\mp\pi^+\pi^-$	$(R_{K3\pi})^2$
$K^\pm\pi^\mp\pi^0$ vs $K^\pm\pi^\mp\pi^0$	$(R_{K\pi\pi^0})^2$
$K^\pm\pi^\mp\pi^+\pi^-$ vs CP	$R_{K3\pi}\cos(\delta^{K3\pi})$
$K^\pm\pi^\mp\pi^0$ vs CP	$R_{K\pi\pi^0}\cos(\delta^{K\pi\pi^0})$
$K^\pm\pi^\mp\pi^+\pi^-$ vs $K^\pm\pi^\mp$	$R_{K3\pi}\cos(\delta^{K3\pi} - \delta^{K\pi})$
$K^\pm\pi^\mp\pi^0$ vs $K^\pm\pi^\mp$	$R_{K\pi\pi^0}\cos(\delta^{K\pi\pi^0} - \delta^{K\pi})$
$K^\pm\pi^\mp\pi^+\pi^-$ vs $K^\pm\pi^\mp\pi^0$	$R_{K3\pi}R_{K\pi\pi^0}\cos(\delta^{K3\pi} - \delta^{K\pi\pi^0})$

CLEO-c Coherence Factor Analysis

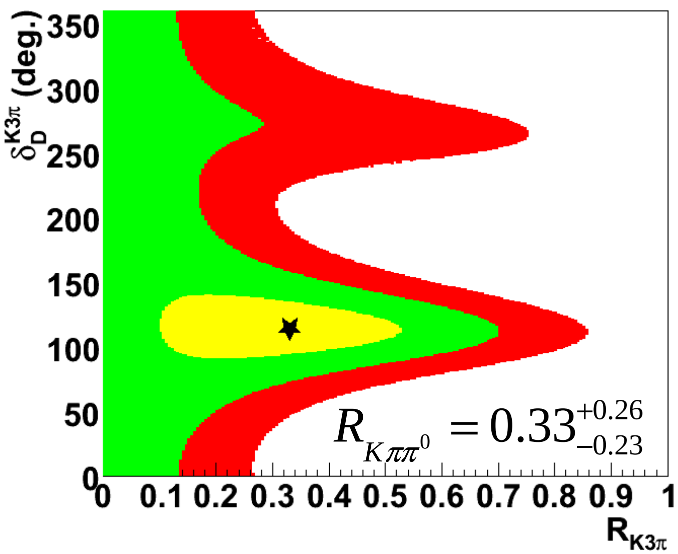
Double-tag technique can also be used to measure mean strong phase difference, δ, and ‘coherence factor’, R, for decays such as D^0, $D^0 \to K^-\pi^+\pi^0$ and $K^-\pi^+\pi^-\pi^+$

Coherence factor expresses decay to which intermediate resonances act in phase if final state is used in an inclusive manner in $B \to DK$ γ measurement.

$K\pi\pi^0$ – very coherent, acts similarly to two-body decay. High γ sensitivity !



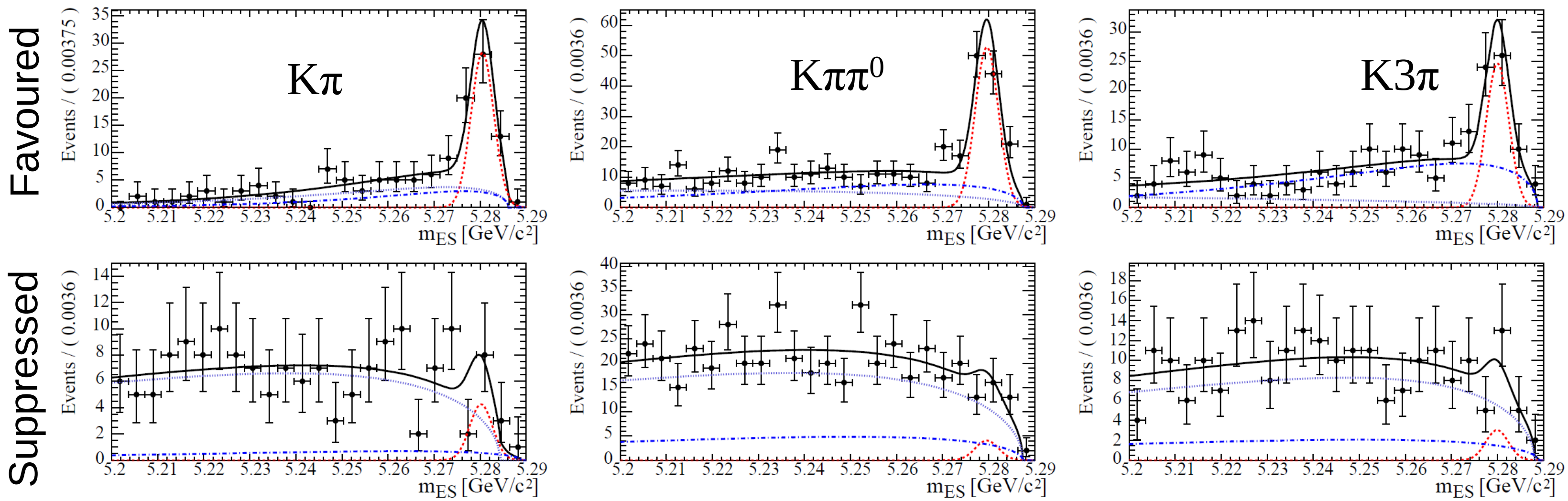
$K\pi\pi\pi$ – lower coherence favoured, so less sensitivity to γ (but helps fix r_B !)



N. Lowrey et al., PRD 80 (2009) 031105

Impact on γ/ϕ_3 – e+e−

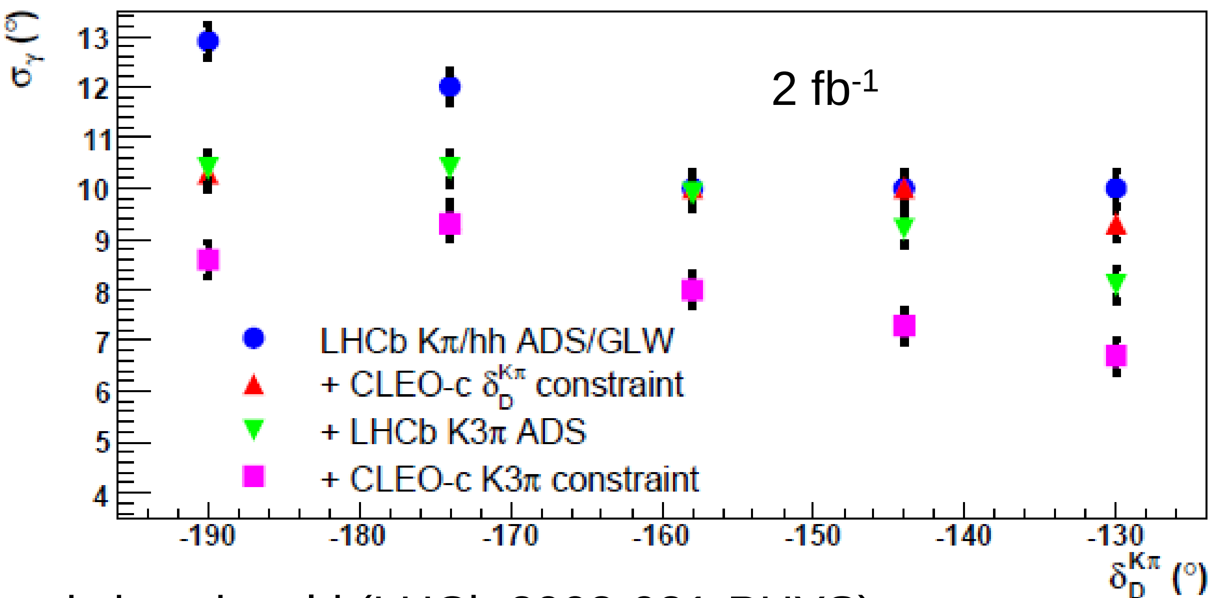
- Coherence factor results have been used in BABAR search for suppressed ADS decays in $B^0 \rightarrow D^0 K^{*0}$ [PRD 80 (2009) 031102]



- Clear gain in using these modes in terms of statistics and coherence factor allows external constraints on otherwise unknown parameters
- Result: best constraint on CKM suppressed to CKM favoured amplitude in $B^0 \rightarrow D^0 K^{*0}$
- Charged ADS will benefit from adding these modes as well

Impact on γ/ϕ_3 - LHCb

- LHCb have also studied the impact of constraints on coherence factor and strong phases (including that of Kπ) in terms of the addition of multibodv modes and CLEO-c inputs



However, study is quite old (LHCb 2008-031-PHYS)
Yields and some assumptions were updated in LHCb roadmap published at the end of last year (http://arxiv.org/abs/0912.4179) – an ‘independent’ update here focusing on the import of charm inputs and adding Kππ

Impact on γ/ϕ_3 - LHCb

- With 2 fb^{-1}
  - $B^+ \to DK^+$ ($D \to K\pi$, $K3\pi$, KK, $\pi\pi$) and $B^0 \to DK^{*0}$ ($D \to K\pi$, KK,$\pi\pi$)
  - $\sigma(\gamma) =$ 9.9° (Including $\delta_{K\pi}$ constraint from D-mixing)
  - **$\sigma(\gamma) =$ 8.5°** (Including CLEO-c results on $K3\pi$)
    - Remember this is just improving r_B
- Now add $D \to K\pi\pi^0$ (Assumed 1/2 $K3\pi$ yield same background)
  - $\sigma(\gamma) =$ 9.7° (Including $\delta_{K\pi}$ constraint from D-mixing)
  - **$\sigma(\gamma) =$ 7.5°** (Including CLEO-c results on $K3\pi$ and $K\pi\pi^0$)
    - Equivalent to ~70% more B data

Impact on γ/ϕ_3 - LHCb

- With 2 fb^{-1}
  - $B^+ \to DK^+$ ($D \to K\pi$, $K3\pi$, KK, $\pi\pi$) and $B^0 \to DK^{*0}$ ($D \to K\pi$, KK, $\pi\pi$)
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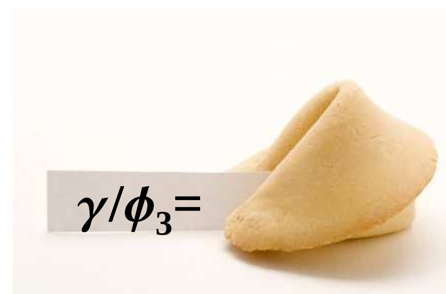
  - **$\sigma(\gamma) =$ 7.5°** (Including CLEO-c results on $K3\pi$ and $K\pi\pi^0$)
    - Equivalent to ~70% more B data
- Assume BES-III reduce uncertainty on coherence factors and phase by factor three
  - **$\sigma(\gamma) =$ 6.9°**
  - **Non-trivial improvement**



Conclusions

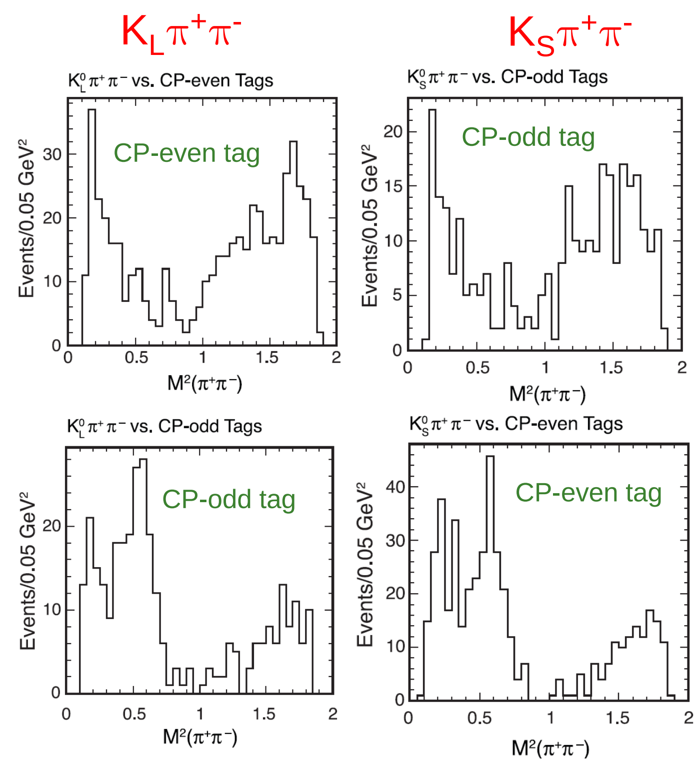
- Second generation quantum correlation measurements are being produced by CLEO-c →
  - Model-independent determination of γ from $B \rightarrow D(K^0_S hh)K$ with only 10% loss in statistical precision over model-dependent method with experimentally driven systematic <= that from model
  - Coherence factor analysis will improve the determination of γ in ADS decays of $B \rightarrow DK$
- Other modes where measurements of strong-phase parameters can aid γ
  - c_i and s_i for $K^0_S\pi\pi\pi^0$ and $\pi\pi\pi^0$
  - Coherence factor for $K^0_S K\pi$
  - Suppressed mode $KK\pi\pi$
  - Binned analysis of $K3\pi$ finding regions of higher coherence
    - Use K^0hh tag to improve determinations of parameters in $K3\pi$ and $K\pi\pi^0$
- **Most measurements statistically limited so significant improvements in all the above can be made by BES-III**
  - **Measurements ready for the next generation e^+e^- machines and an upgraded LHCb**

Backup

A Word on $K_L\pi^+\pi^-$ in CLEO-c Analysis

CP-odd $K_S\pi^+\pi^-$ ≈ CP-even $K_L\pi^+\pi^-$ & so latter can be used to increase statistics

This approximate equality is seen in data



There is however a correction term:

$$-A\left(D^0 \to K_L^0\pi^+\pi^-\right) =$$

$$\underbrace{A\left(D^0 \to K_S^0\pi^+\pi^-\right)}_{\text{CF+DCS}} - \underbrace{\sqrt{2}A\left(D^0 \to K^0_{\text{flavour}}\pi^+\pi^-\right)}_{\text{DCS}}$$

Correction order $\tan^2\theta_c$ – accounting for this introduces small model dependence

In analysis we measure separate c_i', s_i' for $K_L\pi^+\pi^-$, which differ from c_i, s_i by offsets which are floated in fit, but constrained with conservative uncertainties

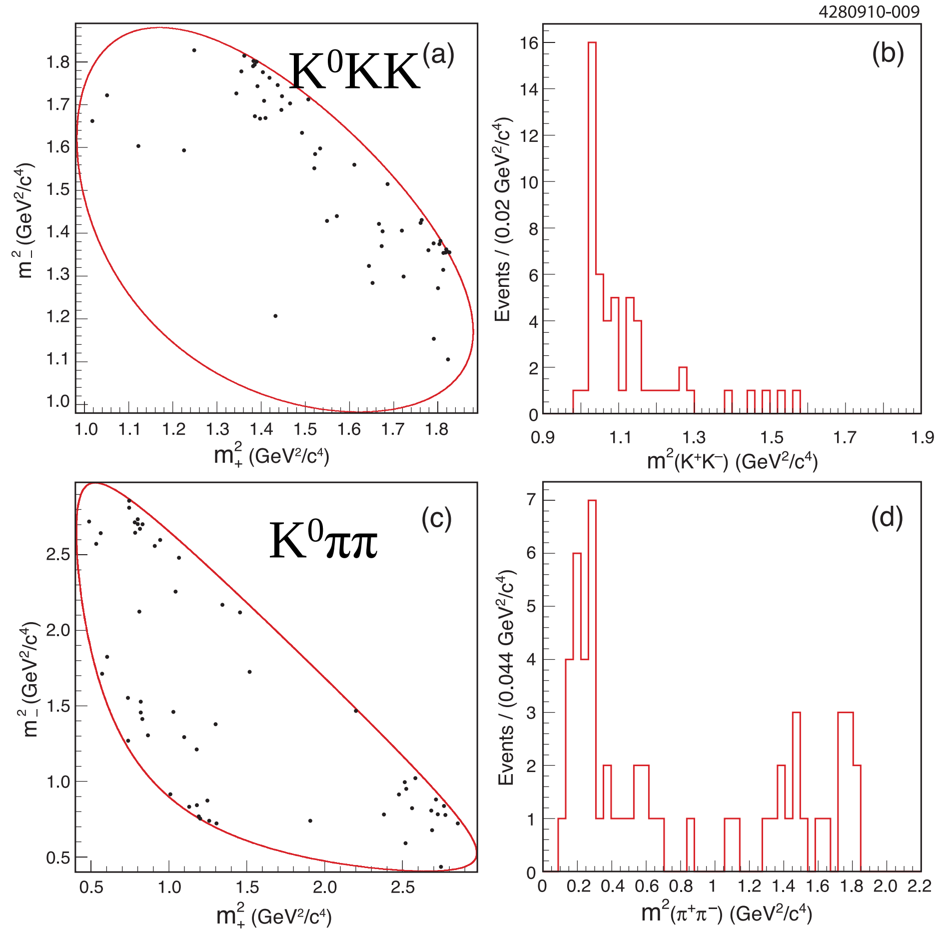
Systematic uncertainties

Uncertainty	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
(Pseudo-)flavor statistics	0.005	0.010	0.009	0.012	0.005	0.013	0.013	0.010
Momentum resolution	0.007	0.013	0.016	0.022	0.007	0.021	0.021	0.016
Mode-to-mode normalization	0.007	0.010	0.015	0.018	0.008	0.014	0.024	0.013
Multiple-candidate selection	0.014	0.014	0.024	0.022	0.008	0.014	0.032	0.019
DCS correction	0.001	0.002	0.001	0.002	0.002	0.004	0.003	0.003
Dalitz plot acceptance	0.004	0.005	0.009	0.008	0.006	0.009	0.011	0.006
Tag-side background	0.024	0.032	0.049	0.059	0.027	0.046	0.079	0.046
$K^0_S\pi^+\pi^-$ signal-side background	0.014	0.020	0.028	0.034	0.016	0.025	0.049	0.026
$K^0_L\pi^+\pi^-$ signal-side background	0.017	0.035	0.032	0.047	0.017	0.022	0.046	0.032
Continuum background	0.020	0.026	0.031	0.038	0.017	0.029	0.049	0.031
Total systematic	0.042	0.063	0.080	0.098	0.042	0.072	0.124	0.075
Statistical plus $K^0_L\pi^+\pi^-$ model	0.036	0.068	0.088	0.119	0.045	0.102	0.105	0.069
$K^0_L\pi^+\pi^-$ model alone	0.013	0.018	0.039	0.068	0.024	0.040	0.068	0.034
Total	0.056	0.093	0.119	0.154	0.062	0.125	0.163	0.102

Uncertainty	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
Total systematic	0.043	0.066	0.044	0.072	0.026	0.059	0.096	0.045
Statistical plus $K^0_L\pi^+\pi^-$ model	0.098	0.182	0.086	0.202	0.131	0.197	0.131	0.150
$K^0_L\pi^+\pi^-$ model alone	0.037	0.038	0.000	0.000	0.030	0.006	0.000	0.025
Total	0.106	0.193	0.097	0.214	0.133	0.206	0.162	0.157

Tagging with $K^0_S\pi\pi$

- Quantum correlations mean that one can improve the determination of K^0KK strong-phase parameters by tagging with the higher statistics $K^0\pi\pi$ mode and using the strong-phase parameters measured for that decay
  - 60% of the events used in the analysis are of this type
  - Use results for equal-strong phase binning based on the BABAR model



Coherence Factor Analysis Event Yields

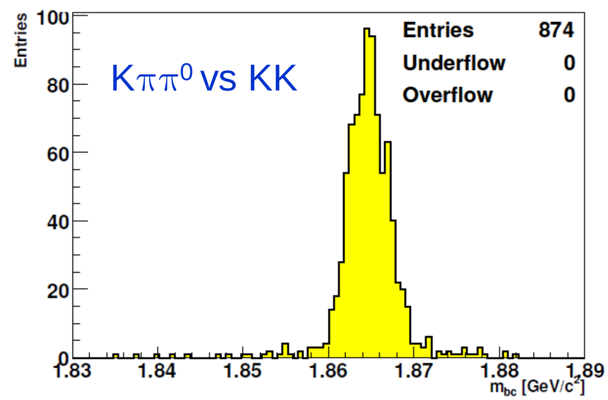
Analysis based on full 818 pb^{-1} $\psi(3770)$ CLEO-c dataset

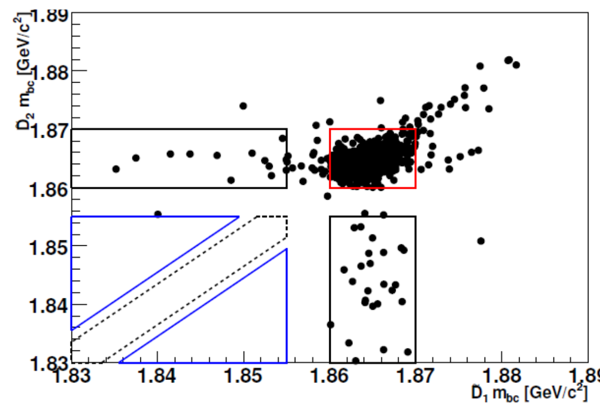
Use 10 separate CP-tags:

CP Tag	$K3\pi$ yield	$K\pi\pi^0$ yield
KK, $\pi\pi$	782	1100
$K_S\pi^0$	705	891
$K_S\omega(\pi^+\pi^-\pi^0)$	319	389
$K_S\pi^0\pi^0$	283	406
$K_S\phi(K^+K^-)$	53	91
$K_S\eta(\{\gamma\gamma, \pi^+\pi^-\pi^0\})$	164	153
$K_S\eta'(\pi^+\pi^-\eta)$	36	61
$K_L\pi^0$	695	1234
$K_L\omega(\pi^+\pi^-\pi^0)$	296	449
Total	**3465**	**4774**

CP = 1, *CP* = -1

Flat background assessed from m_{bc} space; peaking from MC



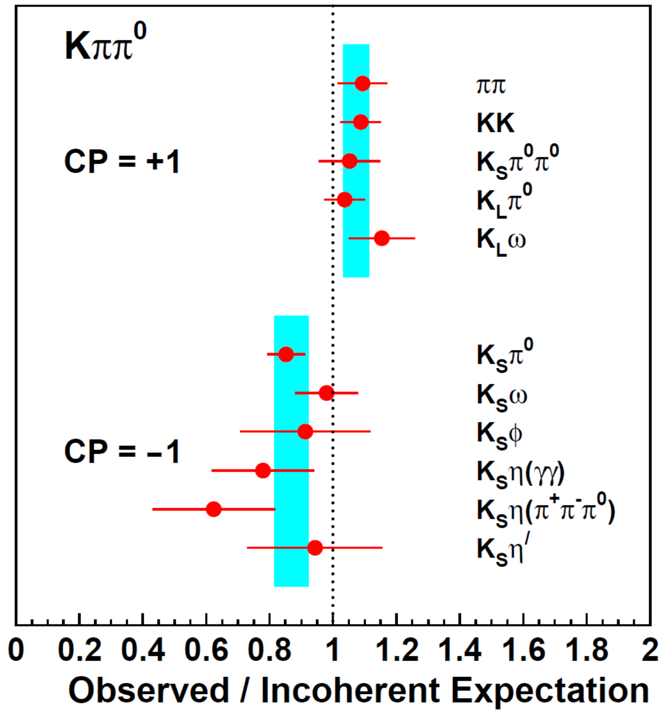


Other classes of double tags are suppressed (but generally very sensitive to physics parameters) so yields low: eg. 29 $K^{\pm}\pi\pi\pi$ vs $K^{\pm}\pi\pi\pi$ events

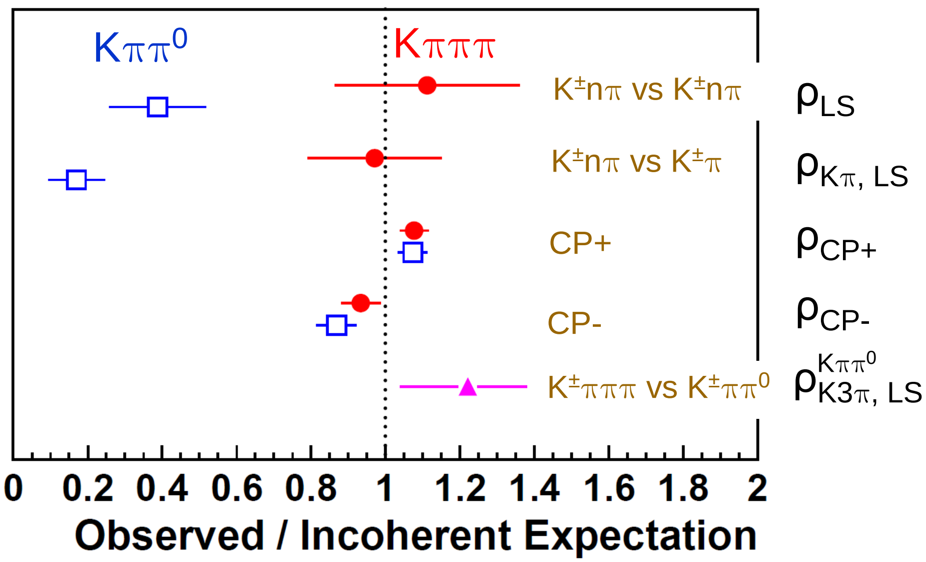
Results for observables

Calculate ratio of observed number of events, ρ, to expected number with zero coherence (≡ no quantum-correlations being present)

CP-tag results internally consistent



Results for all observables



$K\pi\pi^0$ looks very coherent; $K\pi\pi\pi$ does not (note that expected sign of shift for given parameter value varies between observables)

Results for Observables & Parameter Extraction

Observable	Value ± stat ± syst
$\rho^{K3\pi}_{CP+}$	1.077 ± 0.024 ± 0.029
$\rho^{K3\pi}_{CP-}$	0.933 ± 0.027 ± 0.046
$\rho^{K3\pi}_{LS}$	1.112 ± 0.226 ± 0.102
$\rho^{K3\pi}_{K\pi,LS}$	0.971 ± 0.169 ± 0.062
$\rho^{K\pi\pi^0}_{CP+}$	1.073 ± 0.020 ± 0.035
$\rho^{K\pi\pi^0}_{CP-}$	0.868 ± 0.023 ± 0.049
$\rho^{K\pi\pi^0}_{LS}$	0.388 ± 0.127 ± 0.026
$\rho^{K\pi\pi^0}_{K\pi,LS}$	0.170 ± 0.072 ± 0.027
$\rho^{K\pi\pi^0}_{K3\pi,LS}$	1.221 ± 0.169 ± 0.080

- Systematic for ρ_{CP} dominated by an internal uncertainty associated with normalisation, which is statistical in nature

- Systematics for other observables are small, and dominated by knowledge of BRs

Observables depend on R and δ, as well as ratio of DCS to CF amplitudes, r_D, and the D mixing parameters x and y.

$$\rho^{K3\pi}_{LS} \cong \frac{1 - R^2_{K3\pi}}{1 + \frac{x^2+y^2}{2\left(r_D^{K3\pi}\right)^2} - \frac{R_{K3\pi}}{r_D^{K3\pi}}\left(y\cos\delta_D^{K3\pi} - x\sin\delta_D^{K3\pi}\right)}$$

$$\rho^{K3\pi}_{K\pi,LS} \propto \frac{1 + \left(\frac{r_D^{K3\pi}}{r_D^{K\pi}}\right)^2 - 2\frac{r_D^{K3\pi}}{r_D^{K\pi}} R_{K3\pi}\cos\delta_D^{K3\pi}}{1 + \frac{x^2+y^2}{2\left(r_D^{K\pi}\right)^2} - \frac{1}{r_D^{K\pi}}\left(y\cos\delta_D^{K\pi} - x\sin\delta_D^{K\pi}\right)}$$

$$\rho^{K3\pi}_{CP\pm} \cong 1 \pm \Delta^{K3\pi}_{CP} \text{ where } \Delta^{K3\pi}_{CP} = y - r_D^{K3\pi} R_{K3\pi}\cos\delta_D^{K3\pi}$$

Perform fit to extract R and δ , using external constraints on other parameters