

# Recent progress on D semileptonic decays from Lattice QCD

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HPQCD collaboration

- Introduction: D semi-leptonic decay

- D to K | v

$$D^0 \rightarrow K^- l \nu, \quad D^+ \rightarrow \bar{K}^0 l \nu$$

- D to π | v

$$D^0 \rightarrow \pi^- l \nu, \quad D^+ \rightarrow \pi^0 l \nu$$

$$\boxed{\frac{d}{dq^2} \Gamma \propto |V_{cx}|^2 |f_+(q^2)|^2}$$

- Testing Lattice formalism

- $f_+(q^2=0)$  as well as the shape of  $f_+(q^2)$

- Direct determination of CKM matrix elements

- $|V_{cs}|$  and  $|V_{cd}|$

- Unitarity tests

- second row and column unitarity tests

- Testing the Standard Model!

- From experiment:  $f_+^K(0) |V_{cs}|$        $f_+^\pi(0) |V_{cd}|$

	D to K	D to $\pi$
CLEO-c (2009)	1.2 %	3 %
BaBar (2007+update)	1.4 %	
Belle (2006)	3.3 %	5.8 %

- From Lattice:  $f_+^K(0)$        $f_+^\pi(0)$

	D to K	D to $\pi$
HPQCD (2010)	2.5 %	
ETMC (2010, Preliminary)	5.2 % (stat.)	9 % (stat.)
Fermilab/MILC (2005)	10 %	10 %

- ongoing project: Fermilab/MILC and QCDSF

- Lattice calculations without using quenched approximations

	Configuration type	Light quark action	Heavy quark action	Lattice spacing (~fm)
HPQCD (2010, D to K)	MILC asqtad $N_f=2+1$	HISQ	HISQ	0.12, 0.09
ETMC (preliminary)	ETMC Twisted Mass $N_f=2$	Twisted Mass	Twisted Mass	0.102, 0.086, 0.068
Fermilab/MILC (preliminary)	MILC asqtad $N_f=2+1$	asqtad	Fermilab	0.12, 0.09
Fermilab/MILC (2005)	MILC asqtad $N_f=2+1$	asqtad	Fermilab	0.12

# Outline

- Charm quarks on the lattice
- Fermilab/MILC (2005)
  - The first unquenched lattice calculation
- HPQCD (2010, D to K)
  - Most recent report with a factor of 4 smaller errors
  - New methods: form factor from a scalar current  
simultaneous modified z-expansion fit
- ETMC (2010, preliminary)
- Fermilab/MILC (2010, preliminary)
- Result overview
- Summary

- Charm quarks on the lattice

- It is difficult to satisfy

$$am_c = m_c / \text{cutoff} \ll 1$$

- > NRQCD, HQET, or Heavy clover with Fermilab interpretation
- > Operator matching may be required.

- Charm quarks on the lattice

- It “was” difficult to satisfy

$$am_c = m_c / \text{cutoff} \ll 1$$

- > NRQCD, HQET, or Heavy clover with Fermilab interpretation
- > Operator matching may be required.
- Newly developed highly improved quark actions
  - HISQ (Highly Improved Staggered Quark) action
    - Leading error starts at  $O(\alpha_s (am_c)^2 v^2/c^2)$  and  $O((am_c)^4 v^2/c^2)$   
(If  $am_c=0.6$ , it is about 2%)
  - Twisted Mass or clover wilson action
    - set  $am_c < 1/2$  with  $O((am_c)^2)$  errors

- Fermilab/MILC (2005)

- First unquenched lattice calculation
- Using MILC asqtad 2+1 configurations
  - asqtad light quarks and Fermilab charm quark
- $a \sim 0.12$  fm “coarse” ensembles
  - $am_l$  down to  $am_s/8$   $\rightarrow$  easier chiral extrapolation
  - 400~500 confs
  - discretization effect considered by power counting
- Becirevic and Kaidalov (BK) parametrization
- Staggered ChPT

$$f_+^K(0) = 0.73(3)(7)$$

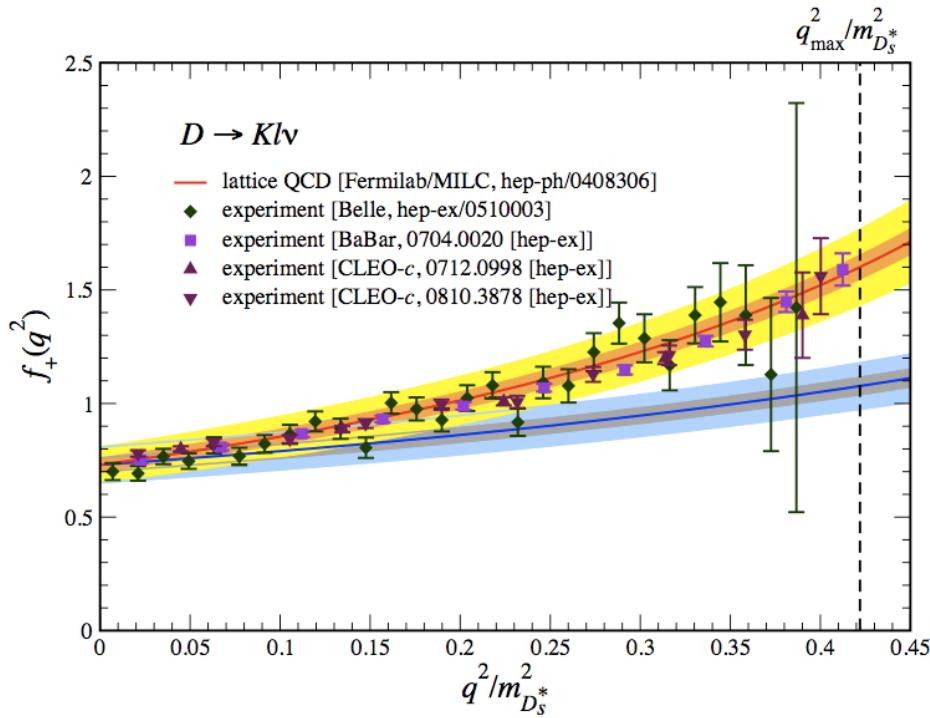
$$f_+^\pi(0) = 0.64(3)(6)$$

<b>fit</b>	<b>3 %</b>	<b>scale</b>	<b>1 %</b>
<b>Chiral fit</b>	<b>3 (2) %</b>	<b>Light q</b>	<b>2 %</b>
<b>BK</b>	<b>2 %</b>	<b>Ep</b>	<b>5 %</b>
<b><math>\rho V \mu</math></b>	<b>1 %</b>	<b>Charm q</b>	<b>7 %</b>

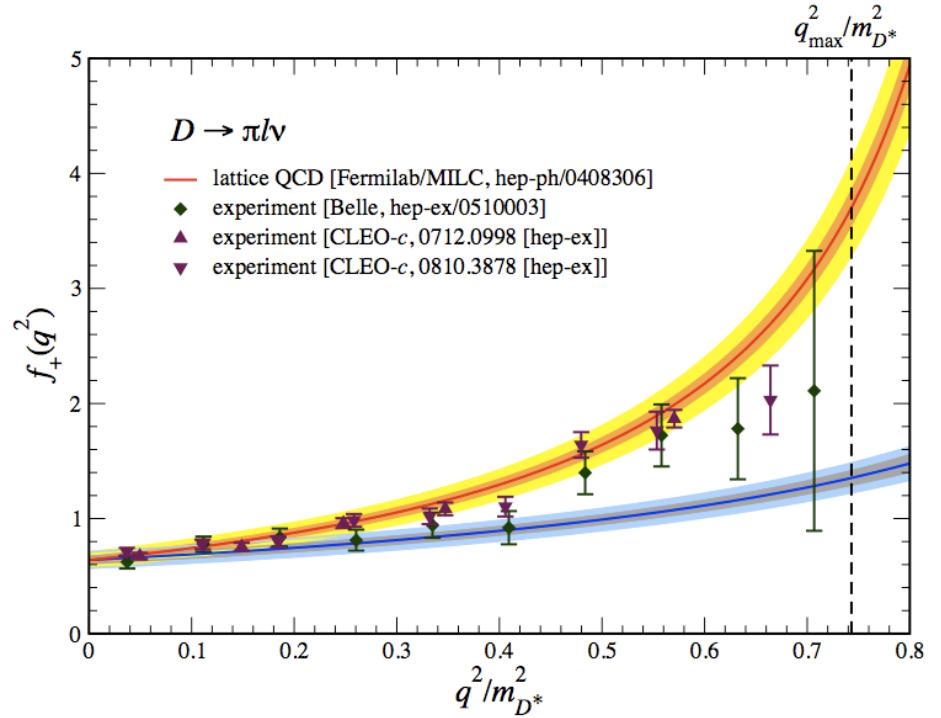
Total 10 % error

- Fermilab/MILC (2005)

**D to K**



**D to  $\pi$**



The lattice calculation successfully predicted the shape!

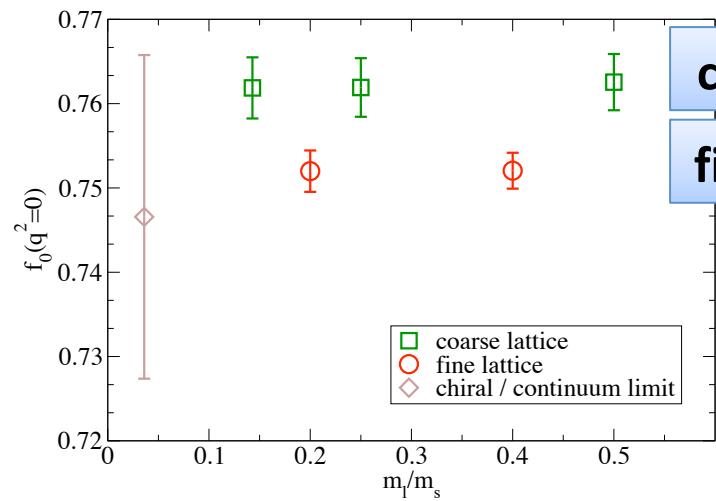
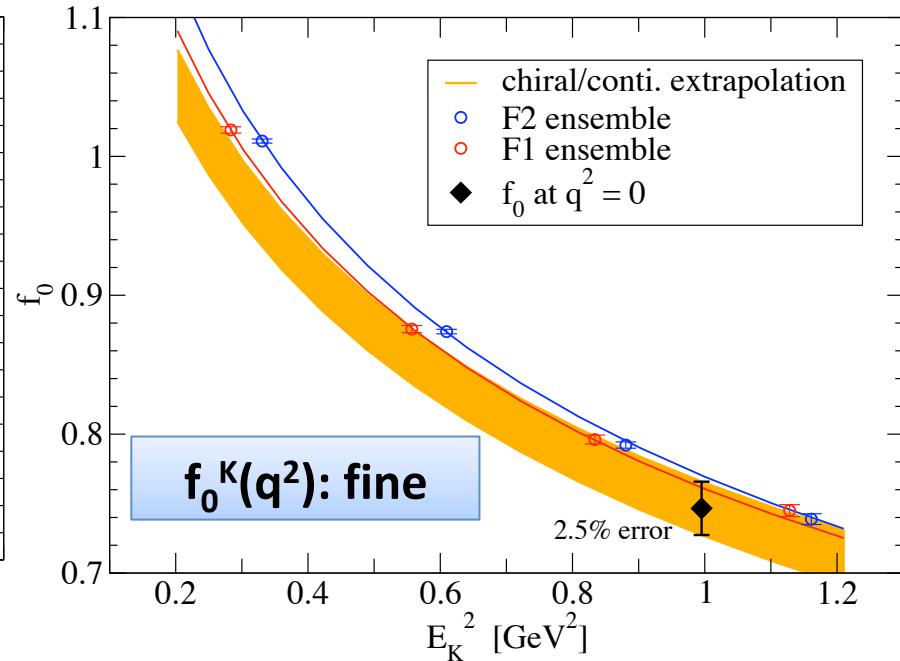
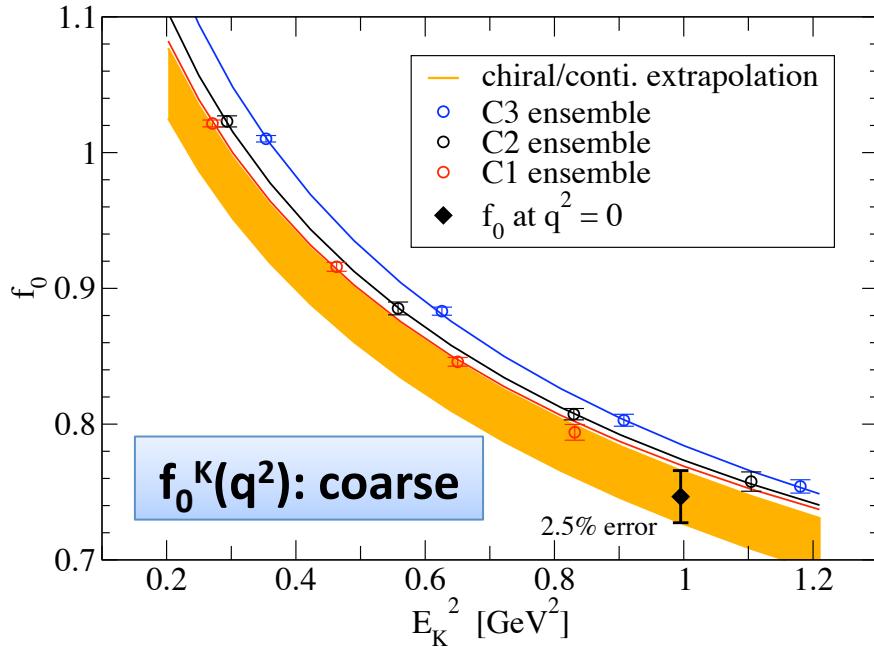
## • HPQCD (2010) D to K l v

\* HN et al. arXiv:1008.4562

- Using MILC asqtad 2+1 configurations
  - HISQ valence light and charm quarks
- $a \sim 0.12$  fm “coarse” and  $a \sim 0.09$  fm “fine” ensembles
  - 600 confs for each ensemble
  - random-wall source
- Total 2.5 % error on  $f_+(K)(0)$ 
  - Small discretization errors by using HISQ action (7 %  $\rightarrow$  2 %)
  - Using a scalar current  $\rightarrow$  no need for operator matching
  - Simultaneous modified z-expansion fit for chiral / continuum extrapolations
  - sophisticated simultaneous correlator fits
- Only for  $f_0(q^2)$ , but  $f_+(0) = f_0(0)$
- D to  $\pi$  l v is under way

## • HPQCD (2010) D to K Inv

- Chiral / continuum extrapolations using modified z-expansion



$f_0^K(q^2=0)$  at the physical limit  
and each ensemble

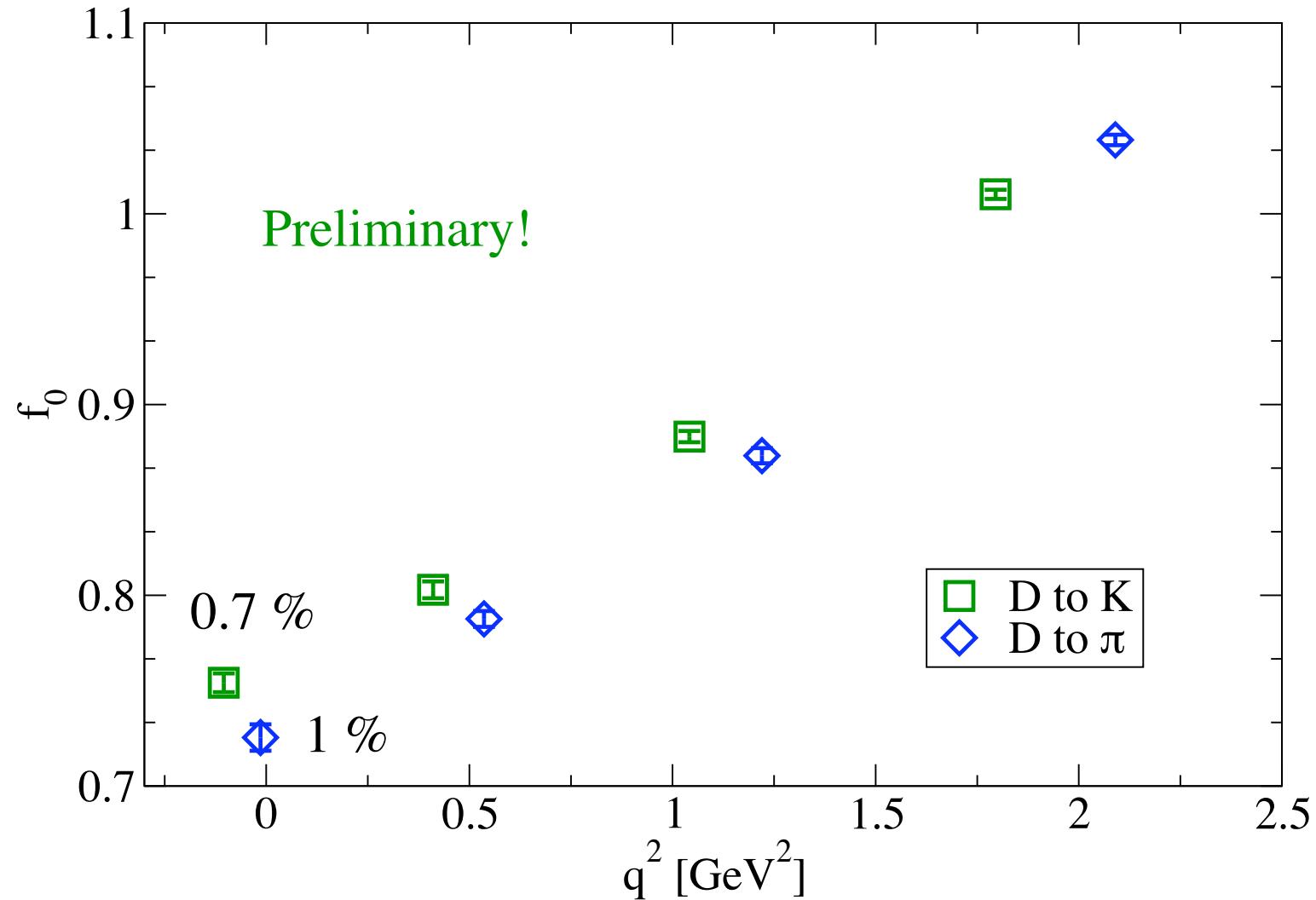
- HPQCD (2010) Error budget and results on  $f_+^K(0)$ .

<b>statistical</b>	<b>1.5 %</b>
<b>Lattice scale</b>	<b>0.2 %</b>
<b>Input meson mass</b>	<b>0.1 %</b>
<b>Light quark dependence</b>	<b>0.6 %</b>
<b>Strange quark dependence</b>	<b>0.7 %</b>
<b>Sea quark dependence</b>	<b>0.4 %</b>
<b><math>am_c</math> extrapolation</b>	<b>1.4 %</b>
<b><math>aE_K</math> extrapolation</b>	<b>1.0 %</b>
<b>Finite volume</b>	<b>0.01 %</b>
<b>Charm quark tuning</b>	<b>0.05 %</b>
<b>Total</b>	<b>2.5 %</b>

$$f_+^K(0) = 0.747 (11)(15)$$

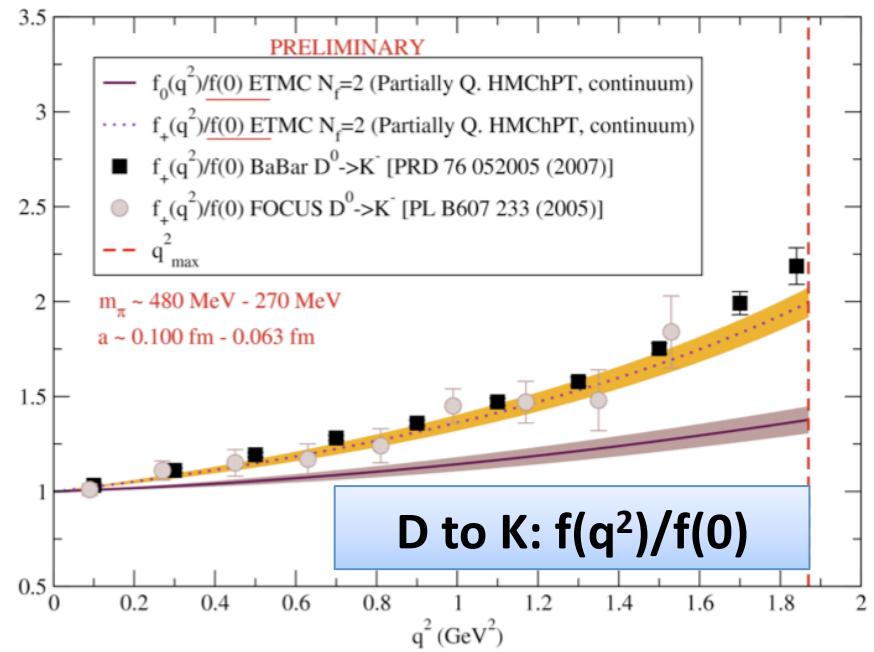
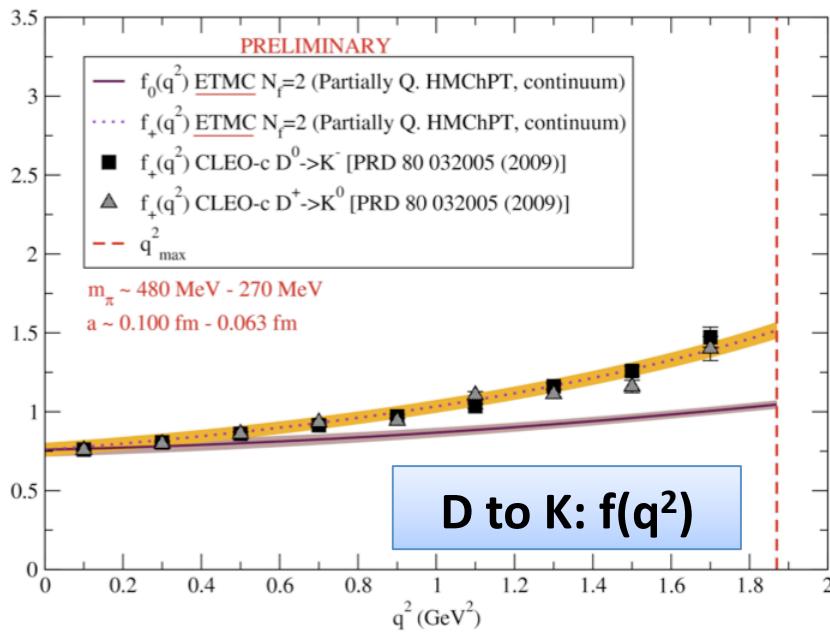
$$|V_{cs}| = 0.961 (11)(24)$$

- HPQCD D to  $\pi$  | v



- ETMC (preliminary)

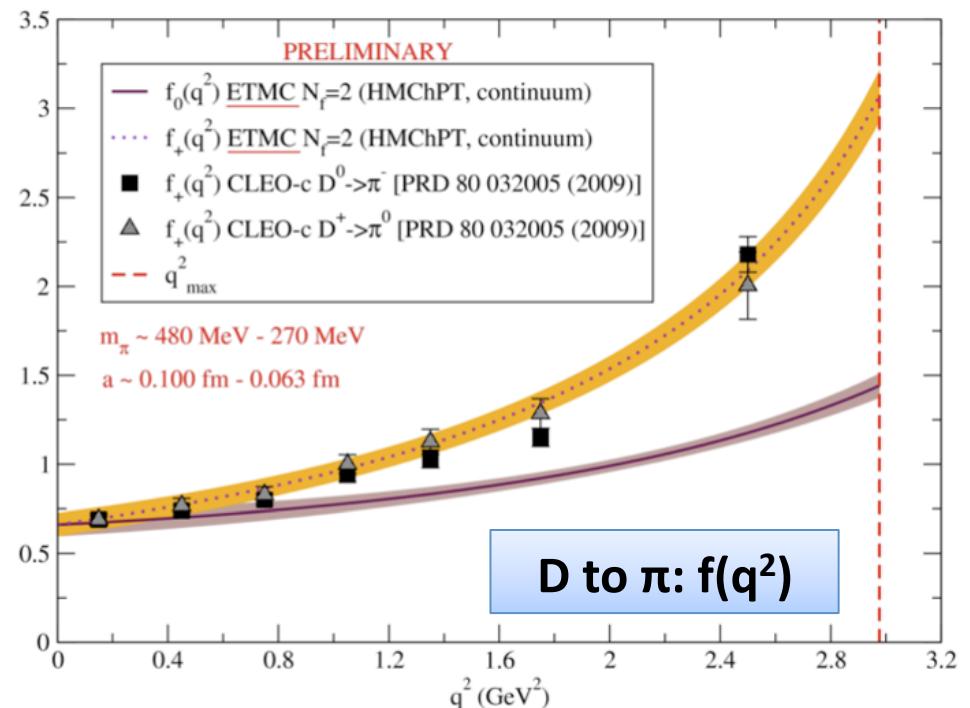
- Using ETMC  $N_f = 2$  configurations
- $a \sim 0.102, 0.086, \text{ and } 0.068 \text{ fm}$
- Twisted boundary conditions
- Using ratios and double ratios
  - reduce errors and cancel the renormalization factors
- HMChPT extrapolations
  - including  $O(a^2)$  terms



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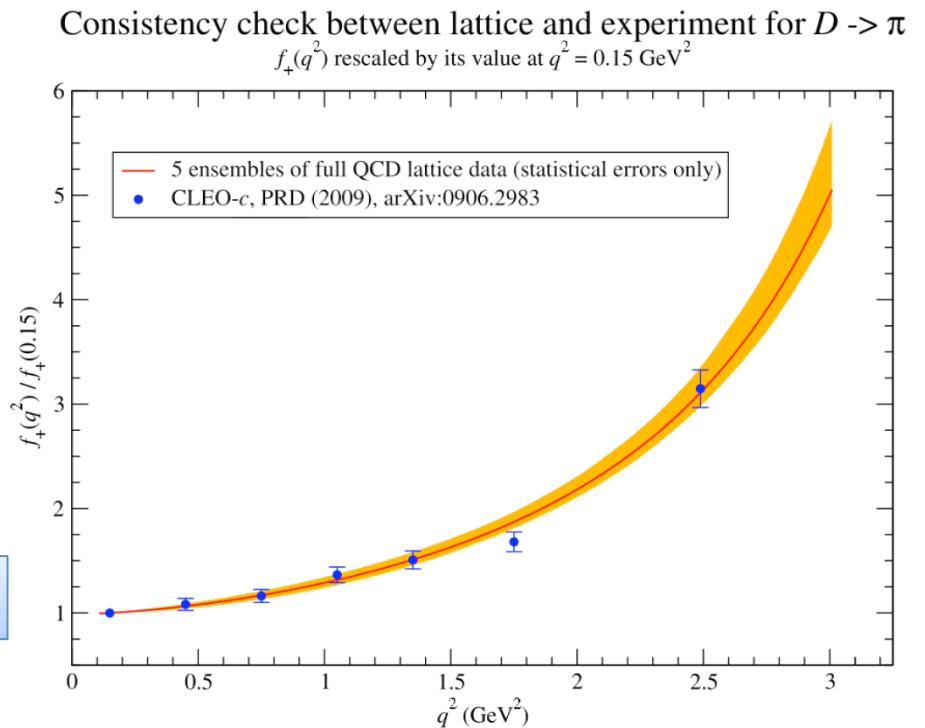
$f_+^\pi(0) = 0.76(4)_{\text{stat.}}$   
 $f_+^\pi(0) = 0.66(6)_{\text{stat.}}$



- Fermilab/MILC (preliminary)

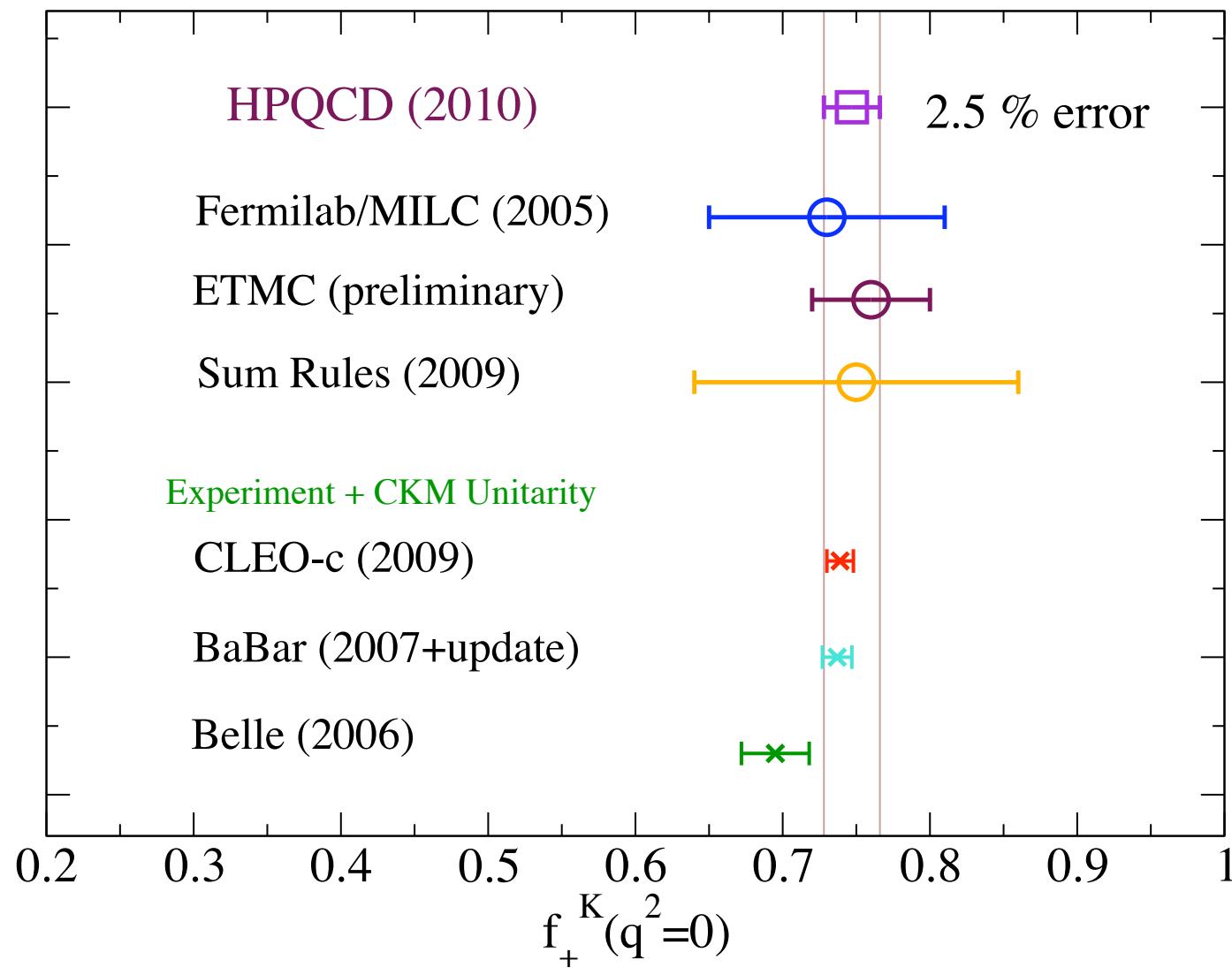
- Using MILC asqtad 2+1 configurations
  - asqtad light quark and Fermilab charm quark
- $a \sim 0.12$ , and  $0.09$  fm
  - $\sim 2000$  confs. high statistics.
- PQSHMChPT extrapolations
  - include NLO and analytic NNLO terms
  - Use only up to  $p = (1, 1, 0)$

D to  $\pi$ :  $f_+(q^2)/f_+(0.15 \text{ GeV}^2)$

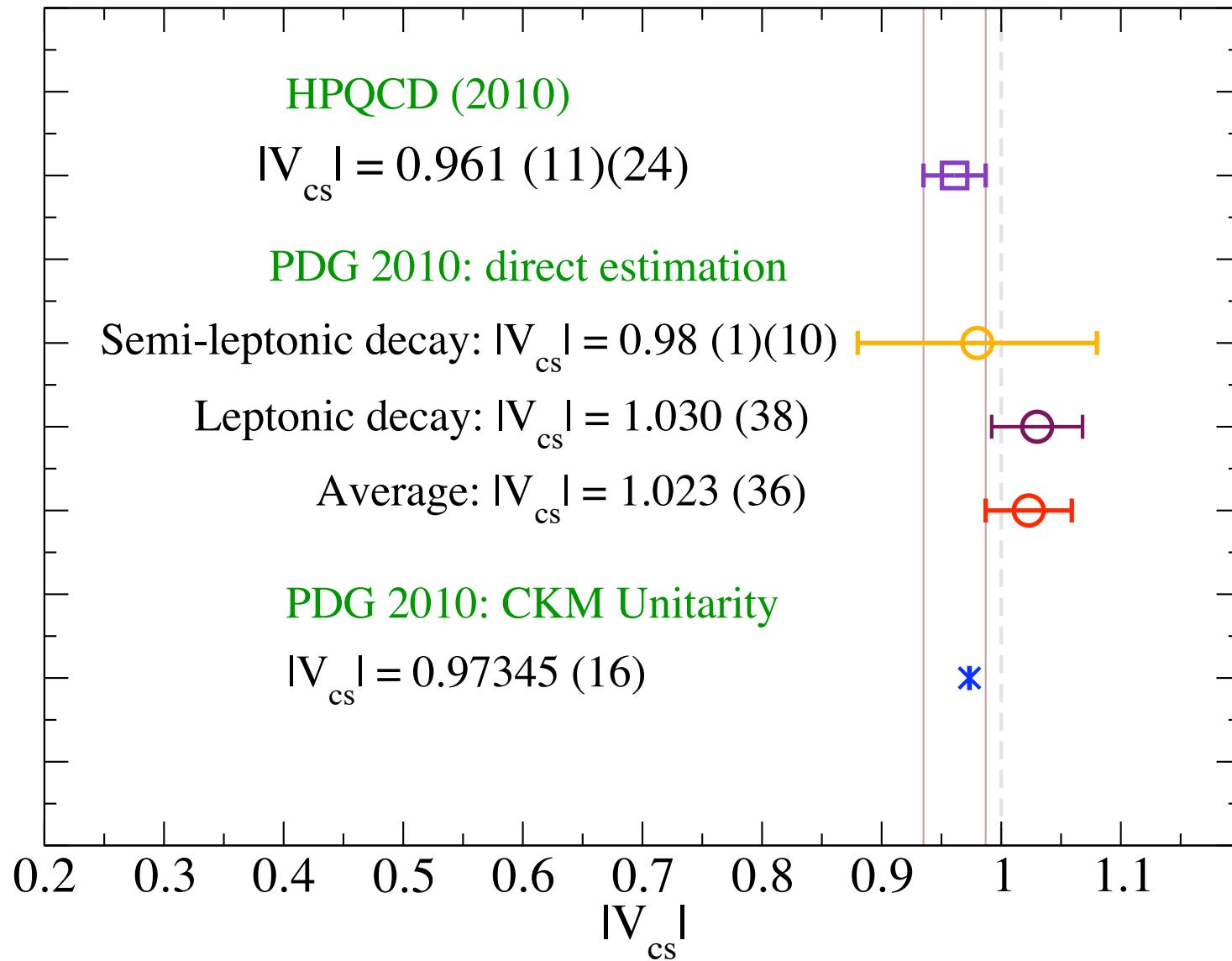


- Result overview D to K Inv

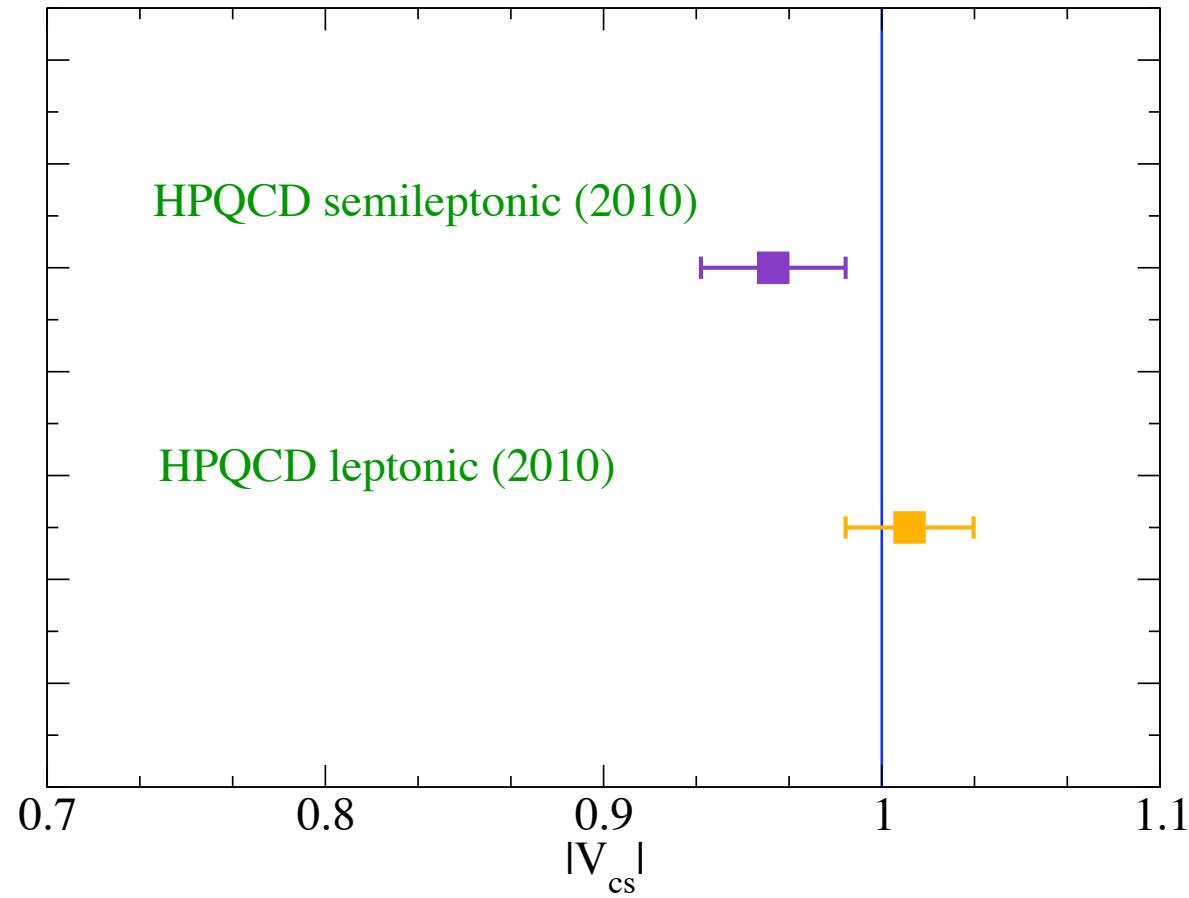
$f_+(0)$



- Result overview **D to K l ν** Direct determination of  $|V_{cs}|$



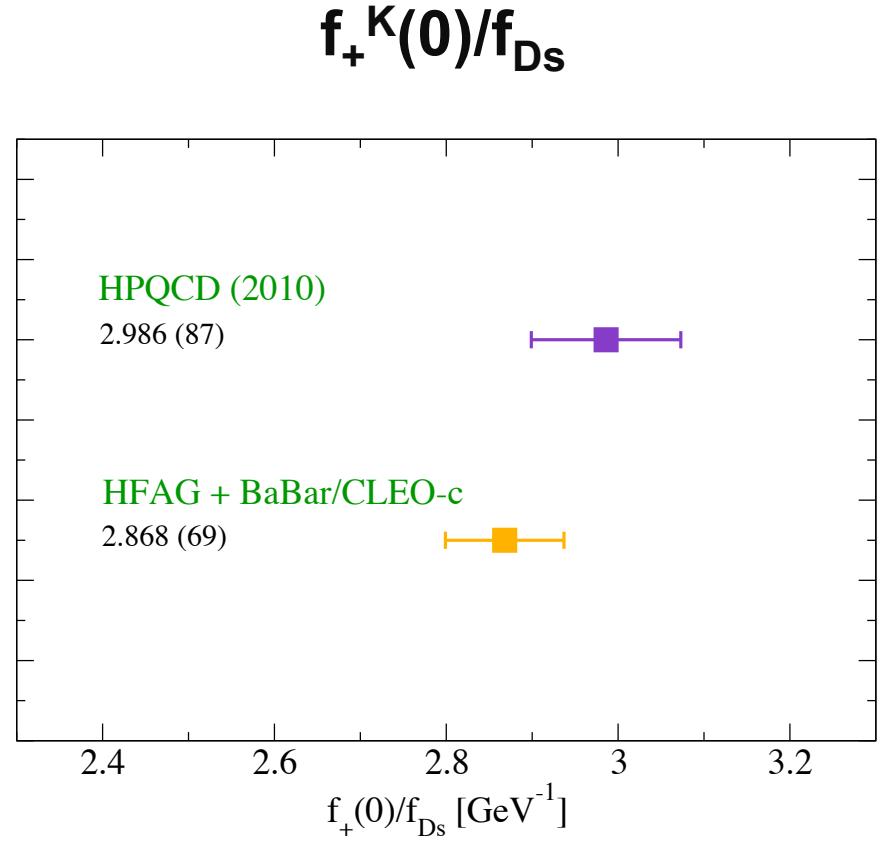
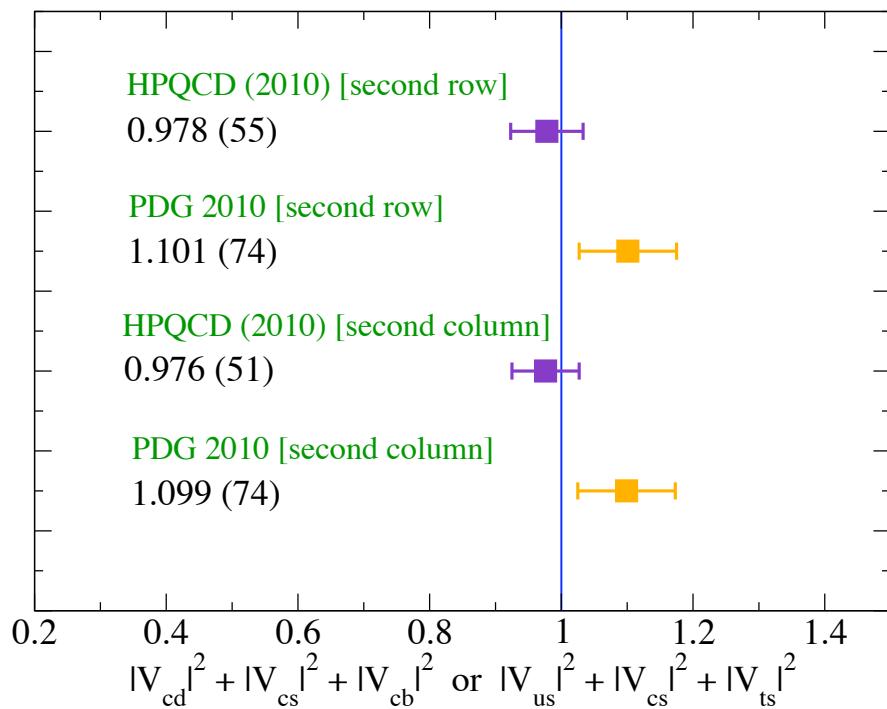
- $|V_{cs}|$  from D semileptonic and  $D_s$  leptonic decays



- Used new HFAG included new BaBar results for leptonic decays

- Result overview D to K l ν

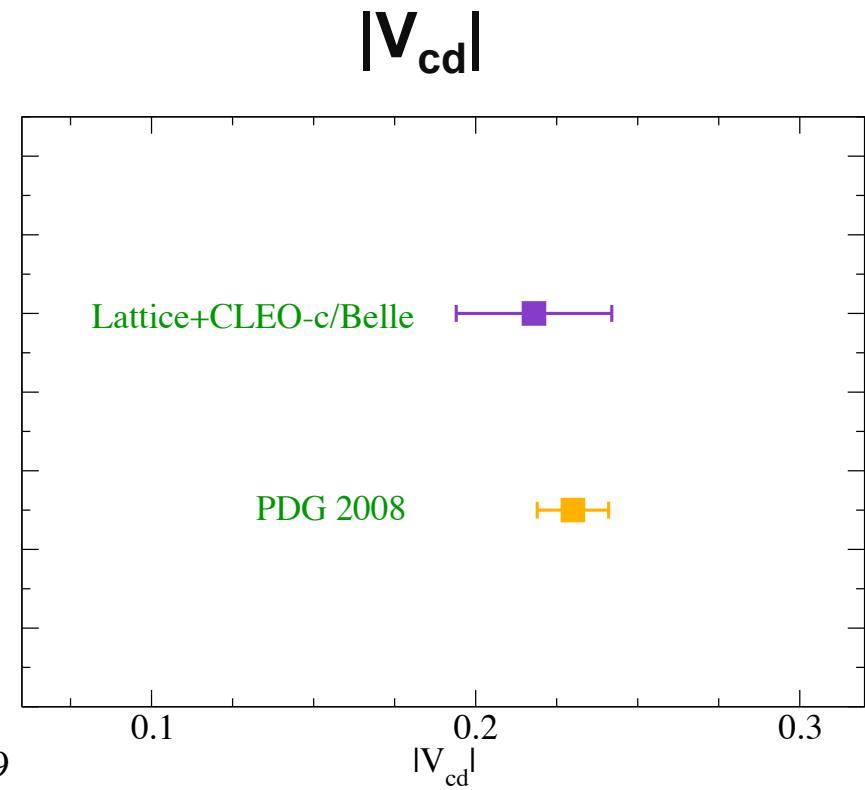
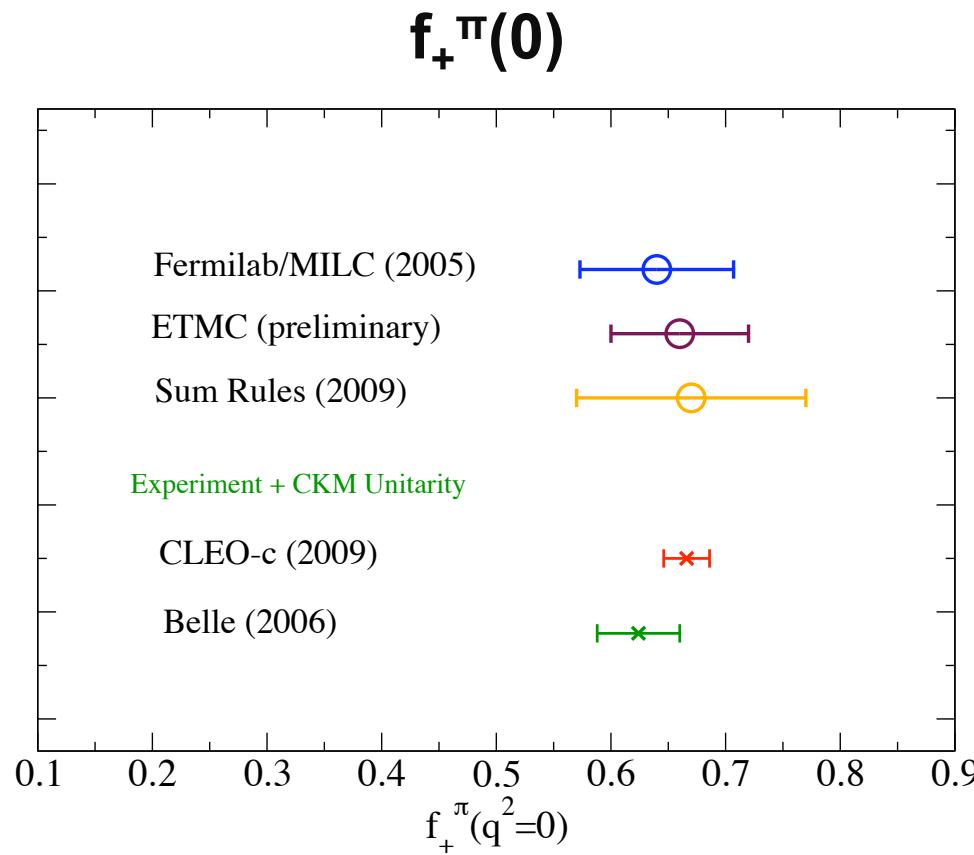
## Unitarity check



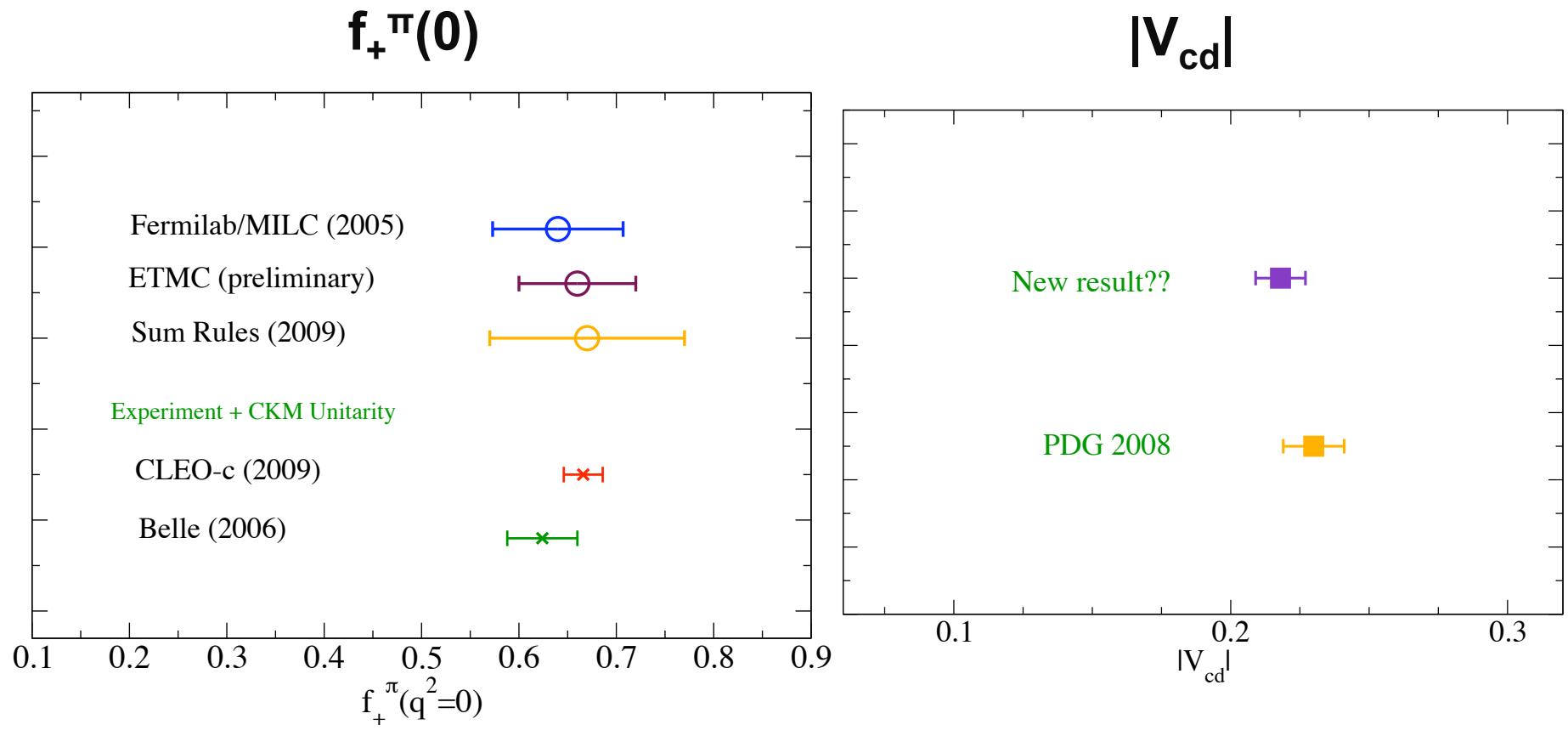
$$\boxed{|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2}$$

$$\boxed{|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2}$$

- Result overview D to  $\pi$  Inv



- Result overview D to  $\pi$  I v



- waiting for more results!

- Summary

- Studies for the exclusive D semileptonic decays have been very successful!
  - D to K: experiment 1% and lattice 3 % errors
  - D to  $\pi$ : experiment 3 % and lattice 10 % errors
- Many lattice projects on D semileptonic decays are ongoing
  - ETMC, Fermilab/MILC, HPQCD, and QCDSF
- Recently, HPQCD reports a new lattice calculation on D to K semileptonic decays with 3% errors.
  - Precise direct determination of  $|V_{cs}|$
  - Applying new methods
    - HISQ action (small discretization errors)
    - scalar matrix element (no need for operator matching)
    - modified z-expansion fit
  - Need for D to  $\pi$  analysis

# Backup slides

- HPQCD (2010) New Method 1.

- Using HISQ action for both charm and light quarks
- Form factors

$$\langle V^\mu \rangle = [(p_D^\mu + p_K^\mu) - \frac{m_D^2 - m_K^2}{q^2} q^\mu] f_+(q^2) + \frac{m_D^2 - m_K^2}{q^2} q^\mu f_0(q^2)$$

- PCVC and “fully nonperturbative matching”

$$q^\mu \langle V_\mu^{cont} \rangle = (m_c - m_q) \langle S^{cont} \rangle$$

$$(m_D - E_\pi) \langle V_0^{latt.} \rangle Z_t + \vec{p}_\pi \cdot \langle \vec{V}^{latt.} \rangle Z_S = (m_c - m_q) \langle S^{latt.} \rangle$$

- Furthermore,  $f_0(q^2)$  and  $f_+(0)$  can be determined just from  $\langle S \rangle$  with no need for operator matching!

$$f_0(q^2) = \frac{m_c - m_q}{m_D^2 - m_K^2} \langle S \rangle, \quad f_+(q^2 = 0) \equiv f_0(q^2 = 0)$$

- HPQCD (2010) New Method 2.

- z-expansion: model independent parameterization

$$f_0(q^2) = \frac{1}{P(q^2)\Phi_0(q^2, t_0)} \sum_{k=0}^{\infty} a_k(t_0) z(q^2, t_0)^k$$

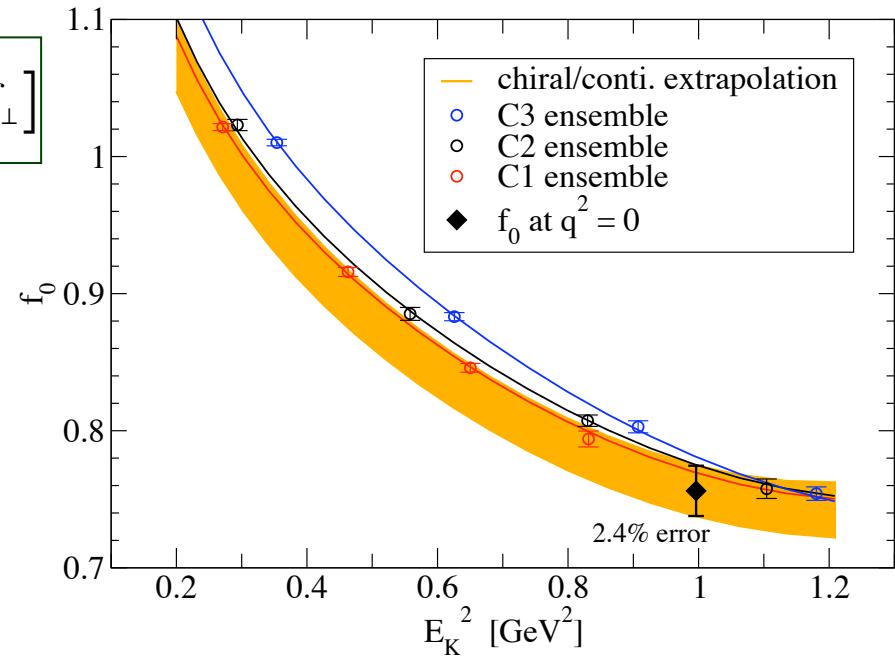
All kinematics are absorbed by P,  $\Phi$ , and z

Applicable to individual ensembles

- HMPQChPT

$$f_0(q^2) = \frac{\sqrt{2M_D}}{M_D^2 - M_K^2} [(M_D - M_K)f_{\parallel} + (E_K^2 - M_K^2)f_{\perp}]$$

Questionable at large  $E_K^2$



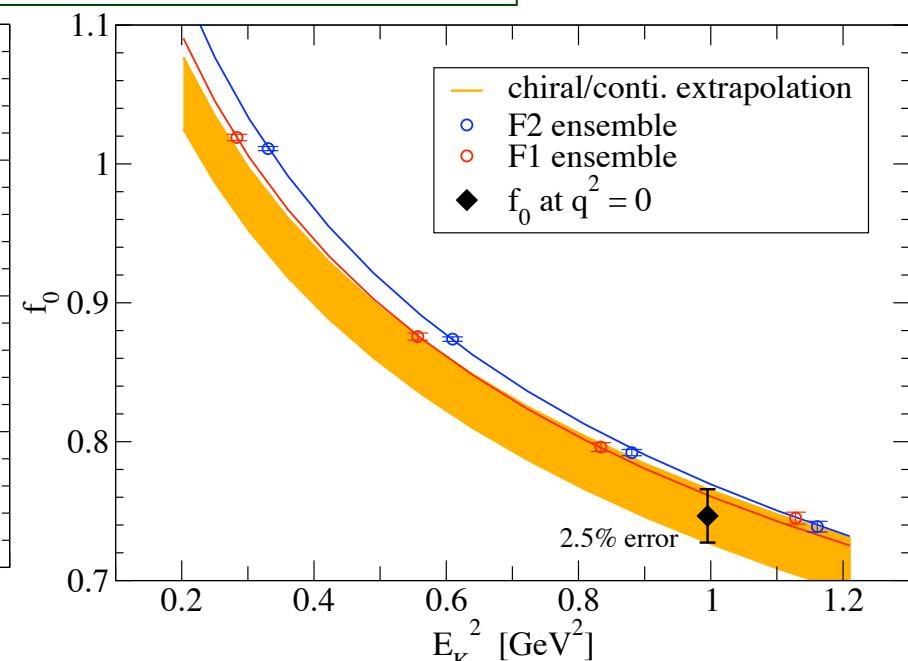
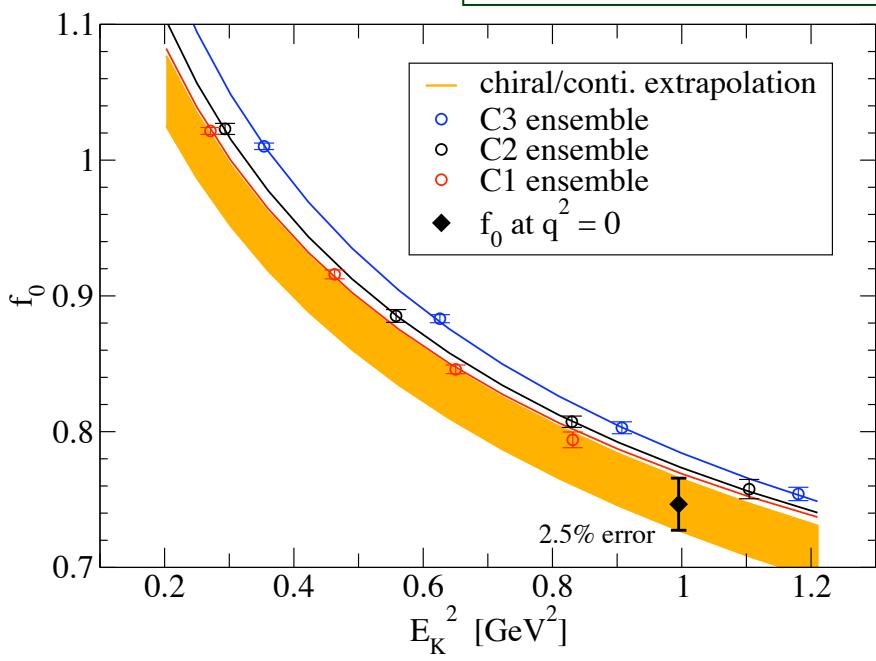
- HPQCD (2010) New Method 2.

- Simultaneous modified z-expansion fit

$$f_0(q^2) = \frac{1}{P \Phi_0} \left( a_0 D_0 + a_1 D_1 z + a_2 D_2 z^2 \right) \left( 1 + b_1 (a E_K)^2 + b_2 (a E_K)^4 \right)$$

$$D_i = 1 + c_1^i x_l + c_2^i \delta x_s + c_3^i x_l \log(x_l) + f_i (1/2 \delta M_\pi^2 + \delta M_K^2) \\ + d_i (am_c)^2 + e_i (am_c)^4$$

$$x_l = \frac{M_\pi^2}{(4\pi f_\pi)^2}, \quad \delta x_s = \frac{M_{\eta_s}^2 - M_{\eta_s^{phys}}^2}{(4\pi f_\pi)^2}$$



- HPQCD (2010) New Method 2.

- Simultaneous modified z-expansion fit

$$f_0(q^2) = \frac{1}{P \Phi_0} \left( a_0 D_0 + a_1 D_1 z + a_2 D_2 z^2 \right) \left( 1 + b_1 (a E_K)^2 + b_2 (a E_K)^4 \right)$$

$$\begin{aligned} D_i &= 1 + c_1^i x_l + c_2^i \delta x_s + c_3^i x_l \log(x_l) + f_i (1/2 \delta M_\pi^2 + \delta M_K^2) \\ &\quad + d_i (am_c)^2 + e_i (am_c)^4 \end{aligned}$$

$$x_l = \frac{M_\pi^2}{(4\pi f_\pi)^2}, \quad \delta x_s = \frac{M_{\eta_s}^2 - M_{\eta_s^{phys}}^2}{(4\pi f_\pi)^2}$$

