The Decay Constants f_{D_s} and f_{D^+} from Lattice QCD

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Review CHARM 2010

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Charm 2010
The 4th International Workshop on Charm Physics

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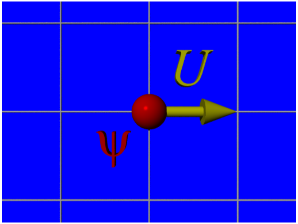
Lattice QCD is an *ab initio* method



$$S_{Dirac} = \bar{\Psi}(\not{D} + m)\Psi$$

Discretized derivative

$$D_\mu \Psi(x) = \frac{1}{2a}\left[U_\mu(x)\Psi(x+\hat{\mu}) - U^\dagger_\mu(x-\hat{\mu})\Psi(x-\hat{\mu})\right]$$

Procedure (executive summary):

- Choose the bare coupling constant, g_0, extract α_s at short distance e.g. from plaquette.
- Determine lattice spacing, a, from e.g. HQ potential.
- Quark masses m_u, m_d, m_s, m_c and m_b determined by reproducing masses of π, K, η_c and η_b mesons.
- QCD parameters now completely fixed in other computations.

“Gold-plated” quantities in LQCD

Lattice QCD technology for process is understood, not computationally expensive and statistcal signal to noise ratio is good.

- Stable particles not near threshold.
- At most one stable hadron in both inital and final states.
- Low to moderate momentum transfer in process.

Spectroscopy, leptonic decays, semileptonic decays and neutral meson mixing!

Lattice "Gold" Decays and the CKM Matrix

Leptonic and semileptonic decays plus mixing...

$$\begin{pmatrix} \begin{array}{c} |V_{ud}| \\ \pi \to \ell\bar{\nu} \\ \\ |V_{cd}| \\ D \to \ell\bar{\nu} \\ D \to \pi\ell\bar{\nu} \\ |V_{td}| \\ B\text{-}\bar{B} \text{ mixing:} \\ \hat{B}_{B_d} \text{ and } f_B \end{array} & \begin{array}{c} |V_{us}| \\ K \to \ell\bar{\nu} \\ K \to \pi\ell\bar{\nu} \\ |V_{cs}| \\ D_s \to \ell\bar{\nu} \\ D \to K\ell\bar{\nu} \\ |V_{ts}| \\ B_s\text{-}\bar{B}_s \text{ mixing:} \\ \hat{B}_{B_s} \text{ and } f_{B_s} \end{array} & \begin{array}{c} |V_{ub}| \\ B \to \pi\ell\bar{\nu} \\ B \to \rho\ell\bar{\nu} \\ |V_{cb}| \\ B \to D^*\ell\bar{\nu} \\ B \to D\ell\bar{\nu} \\ |V_{tb}| \approx 1 \\ \\ \\ \end{array} \end{pmatrix}$$

K-$\bar{K}$ mixing: $|\epsilon_K| \sim B_K \bar{\eta}(1 - \bar{\rho})$

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Charm systems and lattice QCD

Charmonium and D mesons are ideal systems for testing lattice QCD methods.

- Abundant and relatively precise experimental data.
- Test lattice technologies for both heavy and light quarks.
- Same techniques used for bottom: $m_c \to m_b$.
- CKM physics and possible new physics signals,
- e.g. nonstandard leptonic D_s decays, Dobrescu and Kronfeld, arχiv:0803.0512 – the "f_{D_s} puzzle" circa 2007.

Three talks at CHARM 2010: Charmonium (C. Detar), semileptonic decays (H. Na) and decay constants (this talk).

Decay constants

Experiment determines the product $f_{D_q}|V_{cq}|$,

$$\Gamma(D_q \to \ell\nu) = \frac{G_f^2}{8\pi} m_\ell^2 \left(1 - \frac{m_\ell^2}{M_{D_q}^2}\right)^2 M_{D_q} f_{D_q}^2 |V_{cq}|^2 \quad ,$$

while the lattice can compute f_{D_q} from first princples.

A complete lattice calculation of the f_{D_q} must address:

- dynamical (sea) quark effects,
- discretization effects and finding the continuum limit $a \to 0$,
- chiral extrapolation to the physical light quarks,
- tuning errors in determinations of "a" and quark masses.

Lattice studies of f_{D_s} and f_{D^+}

Focus on results from three collaborations with features:

	gauge sets			valence quarks	
collab.	**name**	$\mathbf{n_f}$	**sea quarks**	**light**	**charm**
HPQCD	MILC	$2+1$	asqtad	HISQ	HISQ
FNAL/MILC	MILC	$2+1$	asqtad	asqtad	FNAL clover
ETMC	ETMC	2	tw-mass	tw-mass	tw-mass

HPQCD: C.T.H. Davies, *et al.*, arχiv:1008.4018 and
E. Follana, *et al.*, arχiv:0706.1726

FNAL/MILC: C. Bernard, *et al.*, LATTICE 2010 and
C. Aubin, *et al.*, arχiv:hep-lat/0506030

ETMC: B. Blossier, *et al.*, arχiv:0904.0954

Dynamical sea quarks

Neglecting vaccuum polarization ($\mathbf{n_f = 0}$, quenched QCD) leads to 10-20% uncertainties.

Effects from a quenched strange quark, e.g. in the ETMC $n_f = 2$ study, are difficult to estimate *a priori*.

The heavier charm mass motivates a perturbative bound on effects from quenched charm. HPQCD esimates this error to be $\ll 0.01\%$ for f_{D_s}.

Note: MILC/FNAL, HPQCD and ETMC are now generating gauge configurations including dynamical charm ($n_f = 2 + 1 + 1$)

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MILC three flavor gauge sets

The MILC collaboration has made publicly available sets of gluon configurations having three flavors dynamical quarks [A. Bazavov, *et al.*, arχiv:0903.3598].

- **Quenching is no longer dominant systematic.**
- One flavor $m_h \approx m_s$, two flavors $m_s/10 \leq m_l \leq m_s$.
- Numerically less expensive than other methods.
- **Lighter quarks reduce "chiral" extrapolation systematics.**
- **Leading gluons errors $\mathcal{O}\left(\alpha_s^2 a^2\right)$ and "Asqtad" improved staggered quarks $\mathcal{O}\left(\alpha_s a^2\right)$.**
- Lattice spacings of 0.045, 0.06, 0.09, 0.12 and 0.15 fm.

Open science

- Sets (ensembles) of gauge configurations are expensive to generate, requiring large amounts of time on the fastest computers in the world.
- The MILC, ETMC and other sets of configurations are openly shared worldwide via the ILDG, the International Lattice Data Grid.
- A rich set of open source LQCD application codes are available in the MILC and Chroma codes which use the USQCD SciDAC portable LQCD libraries.

Valence quarks are improved

MILC/FNAL

light: Asqtad improved stag. leading errors $\mathcal{O}(\alpha_s a^2)$

charm: Clover in FNAL interpretation $\mathcal{O}(\alpha_s a^2 \Lambda^2, a^4 \Lambda^4)$

HPQCD

HISQ (Highly Improved Stag. Quark) for both light and charm.

light: $\mathcal{O}(\alpha_s a^2)$, though smaller than asqtad.

charm: leading error $\mathcal{O}(\alpha_s a^2 m_c^2)$

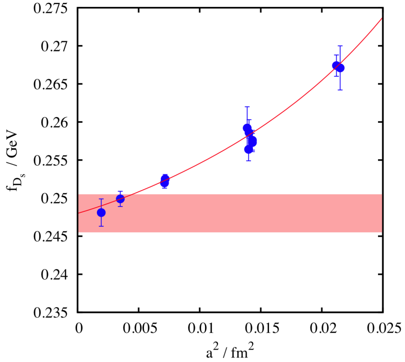
ETMC

Twisted-mass quarks for both light and charm.

$$\mathcal{L}_{doublet} = \bar{\chi} \left(\not{D} + m_q + i \mu_q \gamma_5 \tau^3 \right) \chi$$

At tuned twist, $\mathcal{O}(a^2 \mu_q^2)$ errors for $q =$ light and charm.

HPQCD $\mathbf{f_{D_s}}$ extrapolation



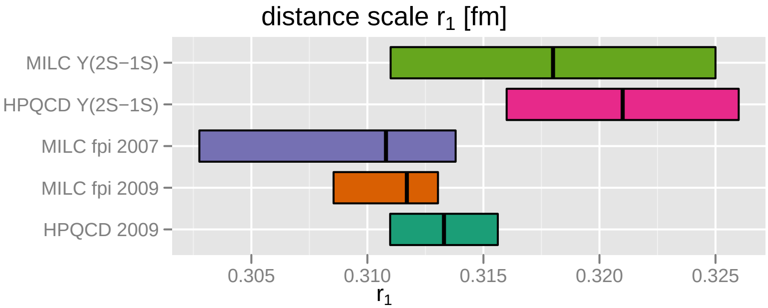
Extrapolation in a^2 setting quarks to their physical masses

- most precise lattice result
- new result for f_{D_s} only
- 2σ higher than 2007 result!
- now at five lattice spacings
- better tuning of quark masses and lattice spacing
- full f_{D^+} update to follow?
- significant lattice spacing dependence

a [fm]	error
0.15	7.8%
0.12	4.2%
0.09	1.7%

Lattice spacing recalibration on MILC lattices

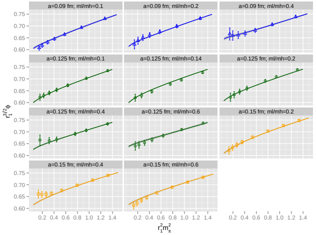
Distance r_1 defined by the HQ potential



- older $\Upsilon(2S\text{-}1S)$ gave a larger r_1 (top two values)
- recent MILC and HPQCD give a lower r_1 (bottom two values)
- net effect on f_{D_s} is smaller than naive rescaling since quark masses must be retuned

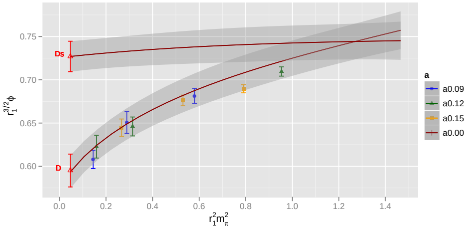
FNAL/MILC $\mathbf{f_{D_s}}$ and $\mathbf{f_{D^+}}$ extrapolation

Fit at finite *a* and simulated sea quarks



- $a = 0.09$, 0.12 and 0.15 fm
- eleven sets of gluon configurations
- χ logs not apparent at finite "a"

Extrap. $a \to 0$ and all $m_q \to$ physical



- PRELIMINARY result
- physical f_{D_q} indicated in red
- "full QCD" subset of data points overlay the extrapolation

Detailed error budgets

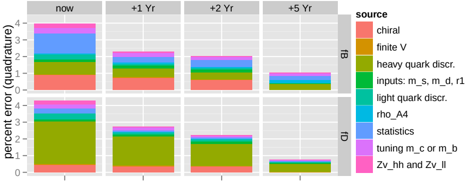
HPQCD

source	f_{D_s}
statistics /valence tuning	0.57
r_1 / a (lat. spacing)	0.15
r_1	0.57
a^2 extrap.	0.40
sea-quark extrap.	0.34
finite vol.	0.10
m_{η_s} (m_s tune)	0.13
QED in D_s	0.10
QED and annih. m_{η_c}	0.00
quenched charm	0.00
total	1.0

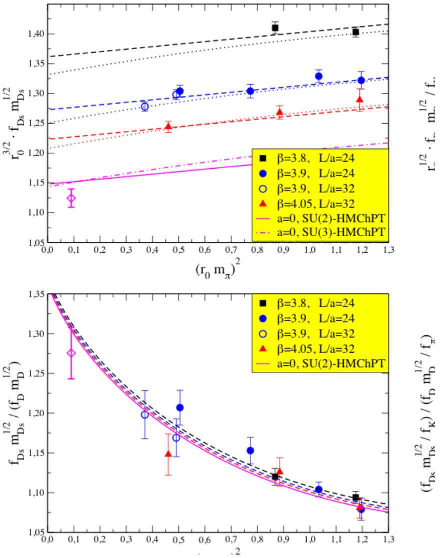
FNAL/MILC

source	f_{D_s}	f_{D^+}	f_{D_s}/f_{D^+}
stat. + disc. effects	2.9	3.6	1.1
chiral extrapolation	0.8	1.4	1.2
inputs r_1, m_s, m_d and m_u	0.7	0.8	0.1
input m_c or m_b	1.2	1.0	0.2
Z_V^{hh} and Z_V^{qq}	1.0	1.0	0
higher-order ρ_{A_4}	0.3	0.3	0.2
finite volume	0.2	0.4	0.4
total	3.5	4.2	1.7

Predicted improvements



ETMC $\mathbf{f_{D_s}}$ extrapolation ($\mathbf{n_f = 2}$)



- top: extrap. in both m_l^{sea} and a for $\phi_s = f_{D_s}\sqrt{m_{D_s}}$
- bottom: extrapolation of ratio ϕ_s/ϕ_d
- bulk of many syst. errors cancel in ratio
- both $SU(2)$ and $SU(3)$ chiral P.Th. fits shown
- lattice spacings $a = 0.065$, 0.085 and $0.10\ \mathrm{fm}$

Summary of lattice results

Three flavor f_{D_s} results differ at the 1.4σ level

Results in $2+1$ flavor QCD			
collaboration	f_{D_s} [MeV]	f_{D^+} [MeV]	f_{D_s}/f_{D^+}
HPQCD	248.0 ± 2.5	213 ± 4	1.164 ± 0.018
FNAL/MILC	261.4 ± 9.2	220.3 ± 9.3	1.19 ± 0.02

Results in 2 flavor QCD			
collaboration	f_{D_s} [MeV]	f_{D^+} [MeV]	f_{D_s}/f_{D^+}
ETMC	244 ± 8	197 ± 9	1.24 ± 0.03

HPQCD: f_{D^+} based on older ratio and updated f_{D_s}

FNAL/MILC: PRELIMINARY

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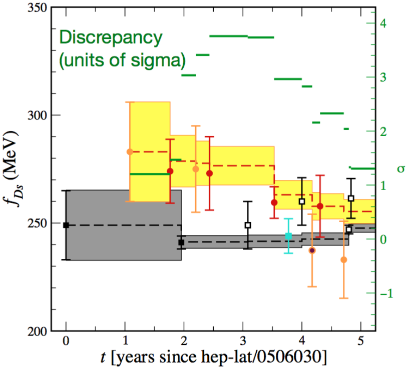
Brief history of f_{D_s}



Kronfeld, arχiv:0912.0543 + updates

- Gray bands lattice three-flavor avg.
- Yellow bands expt. avg.
- Leftmost ($t = 0$) result accompanied by successful prediction of f_{D^+} by FNAL/MILC.
- HPQCD 2007 ($t \approx 2$) result provoked the "f_{D_s} puzzle" (3.8σ discrepency).
- Lattice avg. has come up.
- Expt. has come down.

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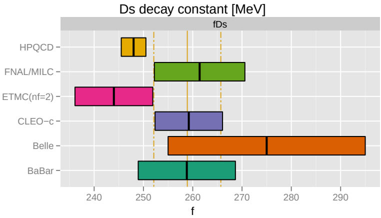
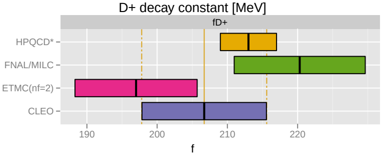
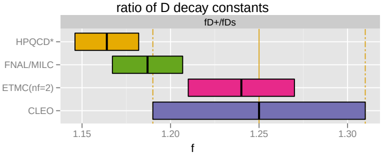
Comparisons of lattice to recent experiment







- Includes recent f_{D_s} update from BaBar.
- My unofficial expt. average pending HFAG f_{D_s} update.
- HPQCD and expt. f_{D_s} differ at about the 1.5σ level.

BaBar: P. del Amo Sanches, *et al.*, arχiv:1008.4080

Belle: K. Abe, *et al.*, arχiv:0709.1340

CLEO: D. Cassel, ICHEP 2010, Paris

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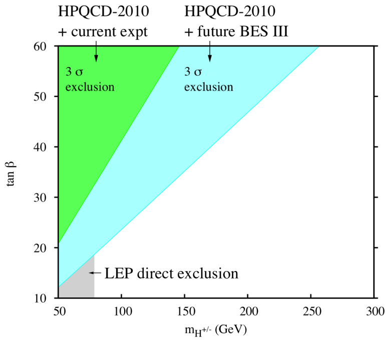
BES-III and future lattice

Bounds on 2-Higgs doublet (type-II)



from HPQCD using A.G. Akeroyd and F. Mahmoudi, arχiv:0902.2393

- 1-2% decay constant measurements by BES-III a welcome challenge for lattice!
- HPQCD: update to f_{D^+}?
- FNAL/MILC: extend asqtad to finer lattices and higer statistics.
- FNAL/MILC/HPQCD: HISQ valence+sea quarks with $n_f = 2 + 1 + 1$.
- ETMC: A four dynamical flavor prelim. f_{D_s} shown at LATTICE2010.