On the structure of X(3872) and Z(4430)

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1. Experimental Background

X(3872)

◇ Basic information

- Close to the threshold of $D^0\bar{D}^{*0}$:

  $m_X = (3871.2 \pm 0.5)$ MeV $[M(D^0\bar{D}^{*0}) = 3871.81 \pm 0.36$ MeV$]$

- Narrow: $\Gamma_X < 2.3$ MeV
- Quantum number: $J^{PC} = 1^{++}(2^{-+}?)$

◇ Decay modes

$\mathcal{B}(B^\pm \to K^\pm X) \times \mathcal{B}(X \to \rho(\pi^+\pi^-)J/\psi) = (1.14 \pm 0.20) \times 10^{-5}$

$R_{\rho/\gamma} = \frac{\Gamma_{\psi\rho}}{\Gamma_{\psi\gamma}} = 4\text{-}7$

$R_{\rho/\omega} = \frac{\Gamma_{\psi\rho}}{\Gamma_{\psi\omega}} = 1.0 \pm 0.5$

- $\omega \to \pi^+\pi^-\pi^0$ for $m_{3\pi} > 0.75$ MeV [Belle:hep-ex/0505037]
- Not confirmed by Babar for $m_{3\pi} > 0.7695$ MeV [Babar: arXiv:0711.2047]
- $R_{\rho/\omega} \simeq 1$ shows that there is large isospin violation in decays of X.

X(3872)

◇ Production

- B-production

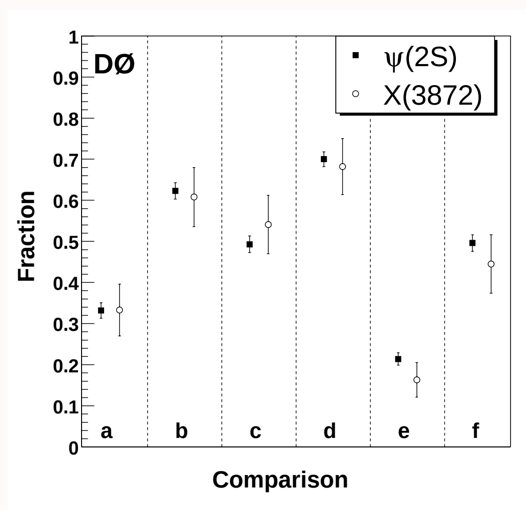
$\mathcal{B}^+ = \mathcal{B}(B^+ \to X(3872)K^+) < 3.2 \ \times 10^{-4}$ Babar: PRL 96(2006) 052002

$R_{n/c} = \frac{\mathcal{B}(B^0 \to X(3872)K^0)}{\mathcal{B}(B^+ \to X(3872)K^+)} = 0.50 \pm 0.30 \pm 0.05$ Babar: PRD 73(2006) 011101

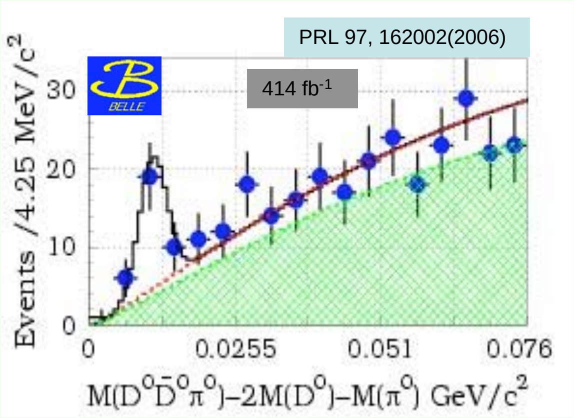
$R_{n/c} = 0.94 \pm 0.24 \pm 0.10$ K. Trabelsi [Belle Collaboration], This Workshop

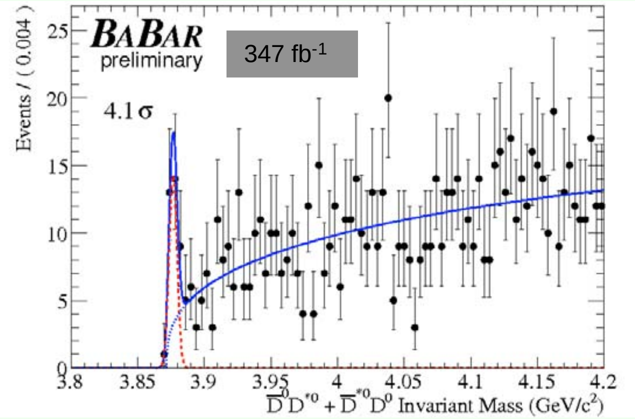
- Production at hadron collider
  - Comparison of event-yield fractions for X and ψ' in the following regions:

a. $p_T > 15$ GeV
b. $|y| < 1$
c. $\cos(\theta_\pi) < 0.4$
d. $dl < 0.01$
e. isolation = 1
f. $\cos(\theta_\mu) < 0.4$



X(3875)

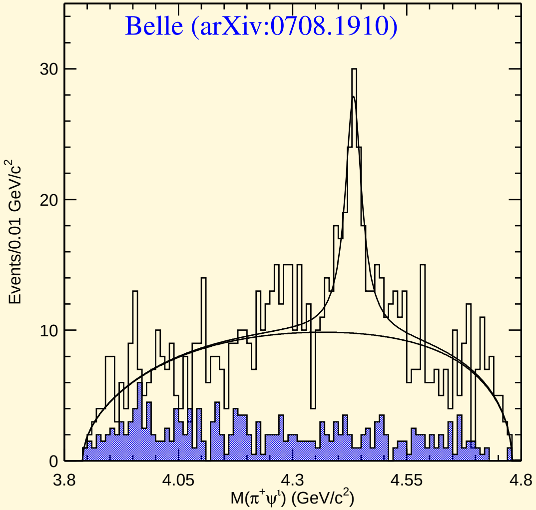




also in Babar: arXiv:0708.1565v2

$m_X = 3875.4 \pm 0.7^{+1.2}_{-2.0}$ MeV	$3875.1^{+0.7}_{-0.5} \pm 0.5$ MeV
$\mathcal{B}^+ \cdot \mathcal{B}(X \to D^0 \bar{D}^0 \pi^0) = (1.02 \pm 0.31^{+0.21}_{-0.29}) \cdot 10^{-4}$	$(1.67 \pm 0.36 \pm 0.47) \cdot 10^{-4}$
$R_{n/c} = \frac{\mathcal{B}(B^0 \to X K^0)}{\mathcal{B}(B^+ \to X K^+)} = 1.7 \pm 0.9$	$(1.33 \pm 0.69 \pm 0.43)$
$R_{\rho/DD\pi} = \frac{\Gamma_{\psi\rho}}{\Gamma_{DD\pi}} = 0.11 \pm 0.05$	$(0.07^{+0.3}_{-0.2})$

$Z^+(4430)$



$m_Z = 4433 \pm 4 \pm 1$ MeV $[M(D^*D_1) = 4430 \pm 4$ MeV, $M(D^*D_1') = 4435 \pm 26$ MeV$]$

$\Gamma_Z = 44^{+17+30}_{-13-11}$ MeV

$\mathcal{B}(B \to KZ)\mathcal{B}(Z \to \pi^+\psi') = (4.1 \pm 1.0 \pm 1.3) \times 10^{-5}$

$R_{\psi'/\psi} = \frac{\Gamma_{Z\to\psi'\pi}}{\Gamma_{Z\to J/\psi\pi}} \gg 1$ "J/ψ suppression problem"

2. $X(3872)$: Theory

2.1. Before 2007

- Highlight
  - Closeness to the threshold： $m_X - M(D^0 D^{*0}) = -0.6 \pm 0.6$ MeV
  - Isospin violation： $R_{\rho/\omega} \sim 1$
  - Small width & Large production rate
  - X(3872) v.s. X(3875)
- Molecule model
  - Mass， J^{PC}， $R_{\rho/\omega}$， ······
  - B-production： $\mathcal{B} < 1 \times 10^{-4}$， $R_{n/c} < 0.1$
  - Decay： $R_{\rho/DD\pi} > 10$
- Charmonium model： χ'_{c1}
  - Large production rate
  - mass： Potential models， Lattice， Coupled channel effects
  - Decays： Isospin violation？
- Other interpretations： 1^{++} cusp, hybrid charmonium, tetraquark state...

2.2. X(3872) as a charmonium state χ'_{c1}

◇ B-Production (C.M., Y.J. Gao and K.T. Chao, hep-ph/0506222)

	Experimental Data	Charmonium Model	Molecule Models
$10^4 \times \mathrm{Br}^+$	1.0-3.2	3-6	$0.07\text{-}1.0^d$
$R_{n/c}$	$0.5 \pm 0.3 \pm 0.05^a$ 1.7 ± 0.9^b $0.94 \pm 0.24 \pm 0.10^c$	0.7-0.9	$< 0.1^d$ $0.06\text{-}0.29^e$

a. BaBar, using $X \to J/\psi\pi^+\pi^-$, PRD 73 011101

b. Belle, using $X \to DD\pi$, PRL 97 162002

c. K. Trabelsi [Belle Collaboration], using $X \to J/\psi\pi^+\pi^-$, This Workshop

d. E. Braaten and M. Kusunoki, PRD 71 074005

e. E.S. Swanson, Phys. Rept. 429 243

- As a by-product: $\mathcal{B}(X \to J/\psi\pi^+\pi^-) = (2\text{-}4)\%$

◇ Large-P_T Production at Tevatron

- Relativistic expansion of Fock state

$$|\psi\rangle = \mathcal{O}(1)|c\bar{c}(^3S_1)\rangle^{\underline{1}} + \mathcal{O}(v)|c\bar{c}(^3P_J)\rangle^{\underline{8}} + \mathcal{O}(v^2)|c\bar{c}(^3S_1)\rangle^{\underline{8}} + \ldots$$

$$\sigma(P_T) \propto \qquad P_T^{-8} \qquad\qquad P_T^{-6} \qquad\qquad P_T^{-4}$$

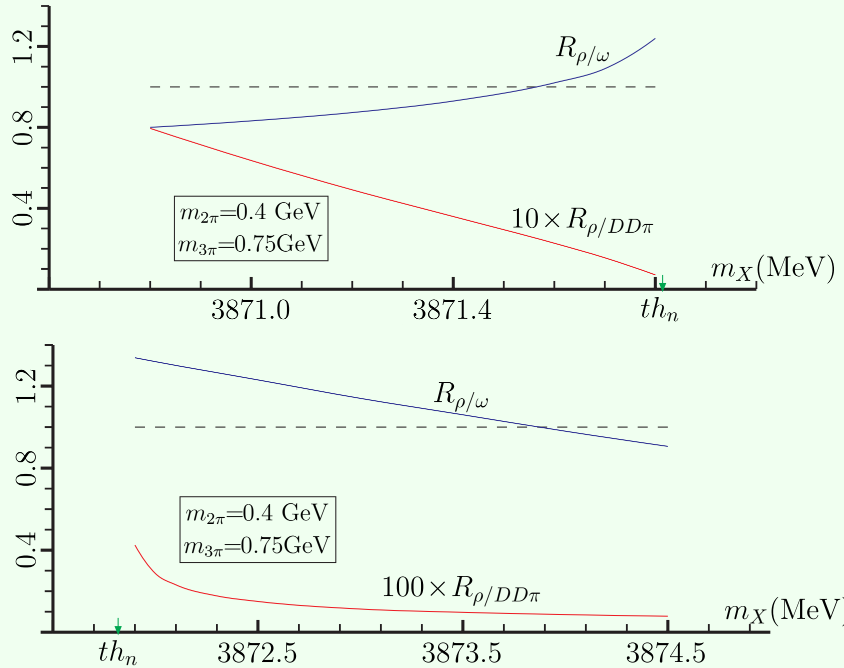
$$|\chi_{cJ}\rangle = \mathcal{O}(1)|c\bar{c}(^3P_J)\rangle^{\underline{1}} + \mathcal{O}(v)|c\bar{c}(^3S_1)\rangle^{\underline{8}} + \ldots$$

$$\sigma(P_T) \propto \qquad P_T^{-6} \qquad\qquad P_T^{-4}$$

- The large P_T production rate of ψ' and χ'_{c1} are both mainly from the matrix elements of operator $\mathcal{O}^{\underline{8}}(^3S_1)$, so that their kinematical distributions should be similar, as we have seen.
- Provided $\langle\mathcal{O}^{\underline{8}}(^3S_1)\rangle^{\psi'} \approx \langle\mathcal{O}^{\underline{8}}(^3S_1)\rangle^{\chi'_{c1}}$, the production rates will be equal. That can be used to deduce

$$\mathcal{B}(X \to J/\psi\pi\pi) \approx (4\text{-}6)\%$$

◇ Decays of X(3872) as charmonium (C.M. and K.T. Chao, PRD 75 114002)



- $R_{\rho/\omega} \sim 1$ is consistent with experimental data very well.
  - The large isospin violation is due to both the difference between $th_n = m_{D^0} + m_{D^{*0}}$ and $th_c = m_{D^+} + m_{D^{*-}}$ and the large difference between the phase spaces of $J/\psi\rho$ and $J/\psi\omega$.(M. Suzuki, PRD 72 114003)
- The prediction on $R_{\rho/DD\pi}$ is roughly consistent with experimental data when $m_X < th_n$, but about two order smaller when $m_X > th_n$.

♠ Experimental data favor charmonium model if $m_X < th_n$.

2.3. Coupled-channel effects

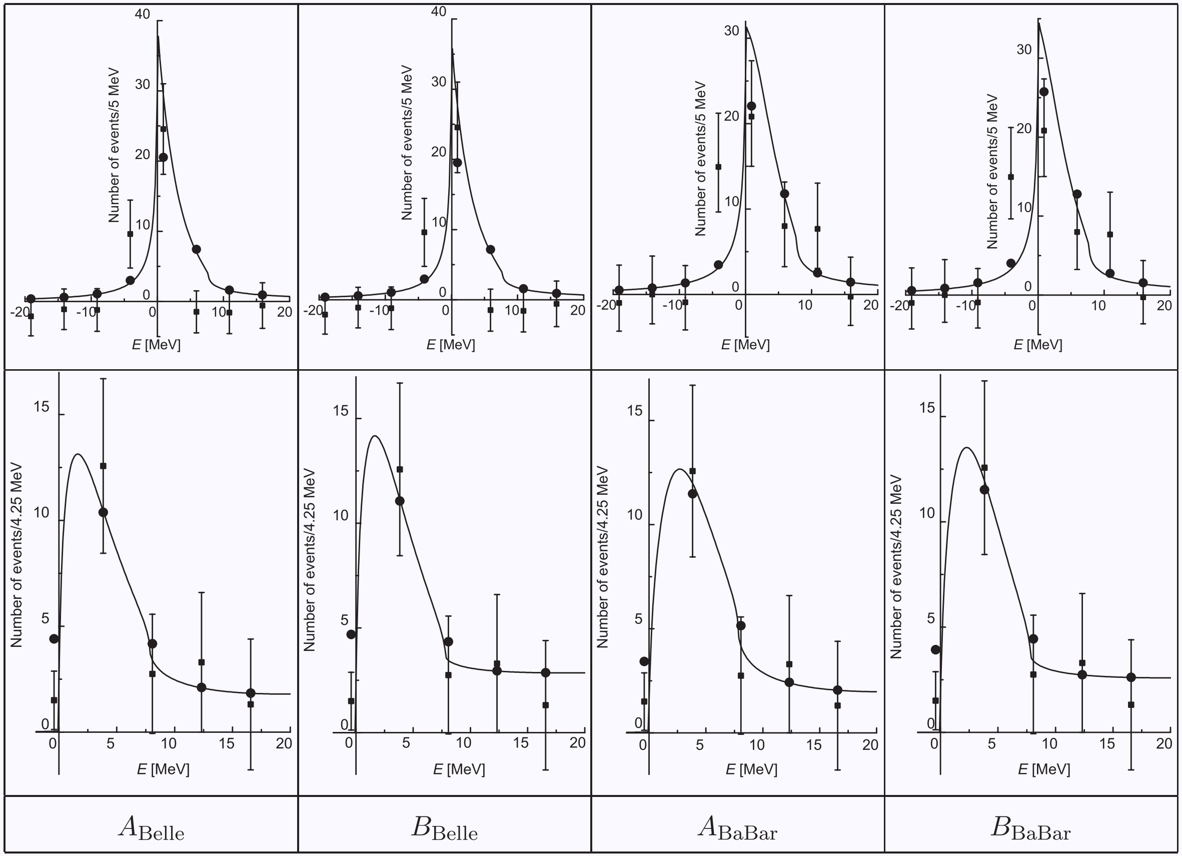
([1] Yu.S. Kalashnikova, PRD 72 034010; [2] B.Q. Li and K.T. Chao, in preparation)

- Summary of Ref. [2]:
  - The physical state with $J^{PC} = 1^{++}$ has a mass solution near the threshold within 10 MeV.
  - The mass spliting between 1^{++} state and 2^{++} state is about 60 MeV, which is consistent with the measurement of $m_{Z(3930)}$ if the $Z(3930)$ state can be identified with χ'_{c2}.
  - In the 1^{++} physical state, the probability of bare χ'_{c1} state $Z = (20\text{-}70)\%$, which is very sensitive to the exact value of mass.
- The influences on our naive charmonium model:
  - The predictions on the production rates and the partial widths should scale as Z.
  - The coupling constant g_{XDD^*} should scale as $\sqrt{Z}$.
  - The ratios $R_{\rho/\omega}$ and $R_{\rho/DD\pi}$ are insensitive to Z.
  - No evident contradictions between the coupled-channel improved charmonium model (CCICM) and the experimental data if $Z > 30\%$.

2.4. X(3872) as a virtual state

[C. Hanhart et al., PRD 76 034007]

◇ Fit by a virtual state



$$E = m_X - th_n$$

Scattering length: $a \approx -4$ fm

$$R_{\rho/DD\pi} = 0.1$$

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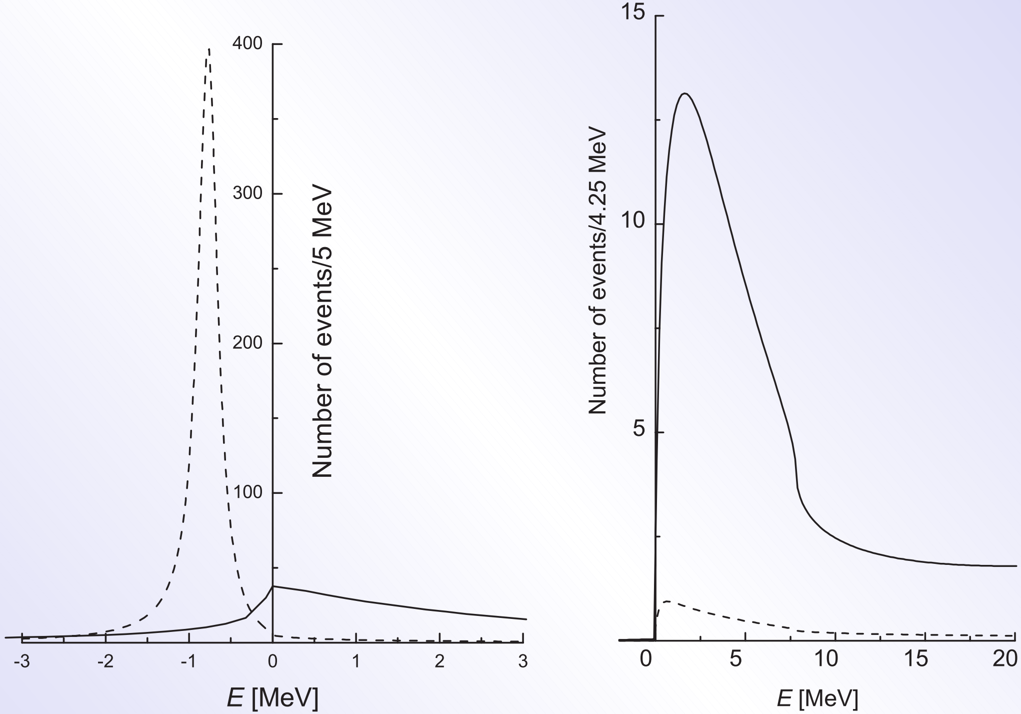
◇ Fit by a molecule state



$$a \approx 4 \text{ fm}$$

$$R_{\rho/DD\pi} = 1.7$$

2.5. Summary of X(3872)

- Summary:
  - X(3872) behaves like a charmonium state χ'_{c1} in production.
  - Provided X(3872) is a charmonium state, the isospin violation in its decay can be accounted for by the interferences between $D^0\bar{D}^{*0}$ and $D^{\pm}D^{*\mp}$ intermediate states.
  - The line shapes of X(3872) and X(3875) can be roughly consistent with each other if X(3872) is a virtual state in $D^0\bar{D}^{*0}$ channel.
  - X(3872) is mostly like a state induced by the coupled-channel effects between bare χ'_{c1} and $D\bar{D}^*$ channels dynamically. It is produced mainly through its short-sistance $c\bar{c}$ component, and behaves as a virtual state in long distance ($> m_{\pi}^{-1}$).
- Work needs to be done:
  - To replace the Flatte fit by the CCICM one.
  - To improve our calculations by considering the long-distant behaviors of X(3872).
  - To find $X \to \psi'\gamma$ for experimental physicists.

    $\Gamma(X \to \psi'\gamma) = (50\text{-}80)Z$ KeV, for CCICM

    $\Gamma(X \to \psi'\gamma) \approx 0.03$ KeV, for molecule model (E.S. Swanson, PLB 598 189)

3. $Z^+(4430)$: Theory

3.1. What we have in market by now

- S-wave threshold effect of $DD_1(2420)$: J.L. Rosner, 0708.3496
- S-wave threshold cusp effect of $DD_1(2420)$: D.V. Bugg, 0709.1254
- Tetraquark state: L. Maiani et al., 0708.3997; S.S. Gershtein, 0709.2058
  - First radial excitations of 1^+ state with flavor $[cu][\bar{c}\bar{d}]$.
  - Two-body decay modes should be dominant: $DD^*, D^*D^*, \psi^{(\prime)}\pi, \psi^{(\prime)}\rho$...
  - Bottom partner $Z_{b\bar{b}}$ with mass about 10.7 GeV: K.M. Cheung, 0709.1312
- Baryonium: C.F. Qiao, 0709.4066
  - Belong to the series of $Y(4260), Y(4361), Z^+(4430), Y(4664)$...

- Resonance of $\bar{D}^* D_1(2420)(\bar{D}^* D_1'(2430))$: CM and K.T. Chao, 0708.4222
  - Formed by some attractive interactions (say, π-exchange) between its components
  - Molecule or virtual state, depending on the strength of the attractive force.
  - S-wave coupling to $\bar{D}^* D_1(\bar{D}^* D_1')$: $J^P(Z) = 0^-$ or 1^-
  - Should be in an isospin-triplet: $Z^+ \approx \frac{1}{\sqrt{2}}(D^{*+}\bar{D}_1^{(\prime)0} - \bar{D}^{*0}D_1^{(\prime)+})$
  - Dominant decay mode: $Z \to D^* D^* \pi$ predicted
  - Dominant production mechanism: $B \to D^* D_s^{(*)} \to ZK$ to be checked

⋆ 0^- molecule in QCD sum rules: $m_Z = (4.40 \pm 0.10)$ GeV S.H. Lee et al., 0710.1029

⋆ 0^- molecule in EFT: $m_{Z_{b\bar{b}}} = 11.05$ GeV G.J. Ding, 0711.1485

⋆ Nonet partners and their b-production: Y. Li et al., 0711.0497

⋆ One-pion exchange between $D^* D_1^{(\prime)}$: NOT strong enough to form a molecule X. Liu et al., 0711.0494

3.2. Decays of $Z^+(4430)$ as a resonance

(C.M. and K.T. Chao, arXiv:0708.4222[hep-ph])

$\diamondsuit$ $Z \to D^* D^* \pi$

$$\Gamma(Z(0^-) \to D_1 D^* \to D^* D^* \pi) = 25 \text{ MeV}$$

$$\Gamma(Z(0^-) \to D_1' D^* \to D^* D^* \pi) = 37 \text{ MeV}$$

$$\Gamma(Z(1^-) \to D_1 D^* \to D^* D^* \pi) = 32 \text{ MeV}$$

$$\Gamma(Z(1^-) \to D_1' D^* \to D^* D^* \pi) = 46 \text{ MeV}$$

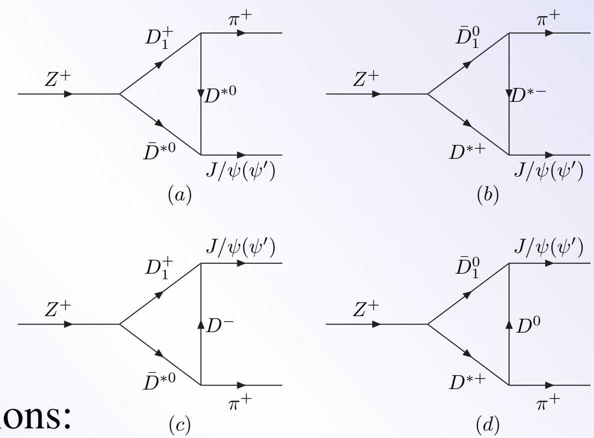
$$g^0_{ZD_1^{(\prime)}D^*} = 5 \text{ GeV}$$

$$g^1_{ZD_1^{(\prime)}D^*} = 1.5$$

$$g^0_{ZD_1^{(\prime)}D^*}/m_{D_1} \sim g^1_{ZD_1^{(\prime)}D^*} \ll g_{\psi DD} \approx 8$$

$\diamondsuit\ Z^+ \to J/\psi(\psi')\pi^+$

- Diagrams for $Z^+ \to D^{*+}\bar{D}_1^0 + \bar{D}^{*0}D_1^+ \to J/\psi(\psi')\pi^+$



- Form factor suppressions:
  - For the $DD\psi^{(\prime)}$ and $DD\pi$ vertexes:

    $\mathcal{F}(m_i, q^2) = \left(\frac{\Lambda^2 - m_i^2}{\Lambda^2 - q^2}\right)^n$

    * They favor $\psi'\pi$ over $J/\psi\pi$
    * We choose $\Lambda = 660$ MeV and $n \in (1, 2)$
  - For the ZDD vertex:

    $\mathbf{Abs}_i(s) \to \mathbf{Abs}_i(s)e^{-\beta|\vec{p}_2|^2}$

    * We choose $\beta \in (0.4, 1.0)$ GeV^{-2} for ZDD system.

◇ Numerical results:

♡ $Z^+ \to J/\psi(\psi')\pi^+$

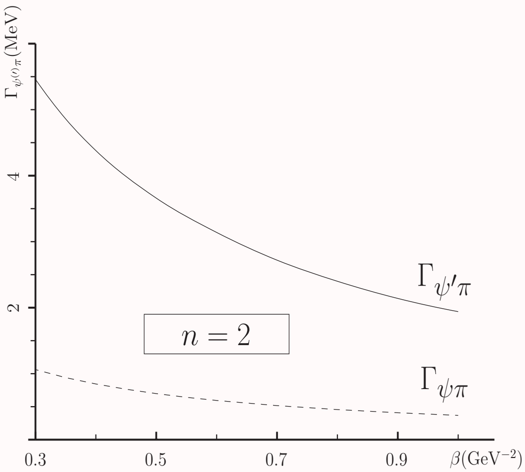
- The re-scattering effects of D^*D_1' are small due to the large width of D_1'
- The contributions arising from diagrams (a) and (b) are dominant.
- Provided $r = g_{\psi' DD}/g_{\psi DD} = 2$ ($g_{\psi(3770)DD}/g_{\psi DD} \approx 1.7$)

$(n,\ \beta \times \text{GeV}^2)$	$(1.0,\ 1.0)$		$(1.5,\ 0.6)$		$(2.0,\ 0.4)$	
$J^P(Z)$	0^-	1^-	0^-	1^-	0^-	1^-
$\Gamma_{\psi\pi^+}$(MeV)	5.3	10.6	1.5	3.2	0.42	0.84
$\Gamma_{\psi'\pi^+}$(MeV)	3.7	13.9	2.5	8.7	1.5	4.4

  – The predictions favor 1^-: $\Gamma_{\psi'\pi^+} = 4.4$ MeV, $R_{\psi'/\psi} \approx 5.2$

  – Can not rule out 0^-

- The β-dependence of $\Gamma_{\psi^{(\prime)}\pi^+}$



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3.3. Summary for $Z^+(4430)$

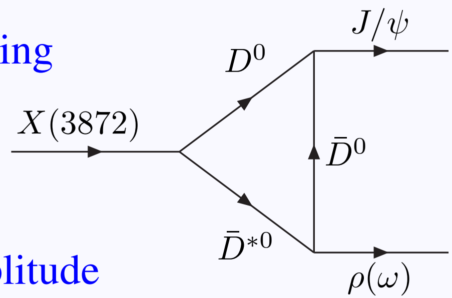
- Summary:
  - Z is most like a resonance of $D^* D_1$ in isospin-triplet.
  - The numerical results favor $J^P = 1^-$, but can not rule out 0^-.
  - $R_{\psi'/\psi} \approx 1.4r^2$ with Br$(Z^+ \to \psi'\pi^+) = 0.04r^2$.
  - Dominant decay mode should be $Z \to D^* D^* \pi$
  - Can decay to $\psi(3770)\pi$.

Thank You!

Back Up

♡ $X \to J/\psi\rho(\omega)$

- Effective Lagrangian: $\mathcal{L}_X = g_X X^\mu D_\mu^{*\dagger} D + h.c.$
- Diagram for the re-scattering



- Imaginary part of the amplitude

$$\mathbf{Abs}_n = \frac{|\vec{p}_1|}{32\pi^2 m_X} \int d\Omega \mathcal{A}(X \to D^0\bar{D}^{*0})\mathcal{A}(D^0\bar{D}^{*0} \to J/\psi\rho(\omega))$$

  - **Abs** is strongly suppressed by the tiny phase space factor.(X. Liu et al., PLB 645 185)
  - The contribution arising from real part should be dominant in this case.

- Real part of the amplitude

$$\mathbf{Dis}(m_X^2) = \frac{1}{\pi}\left(\mathcal{P}\int_{th_n^2}^{\infty} \frac{\mathbf{Abs}_n(s')}{s'-m_X^2} ds' + \mathcal{P}\int_{th_c^2}^{\infty} \frac{\mathbf{Abs}_c(s')}{s'-m_X^2} ds'\right)$$

¶ $\mathcal{P}$ denotes principal integral

¶ $th_n = m_{D^0} + m_{D^{*0}}, th_c = m_{D^+} + m_{D^{*-}}$.

  - Naive cut: $\infty \to s_{max} = 2m_{D^*}$
  - Form factor: $\mathbf{Abs} \to \mathbf{Abs} \cdot \mathrm{e}^{-\beta|\vec{p}_1|^2}$

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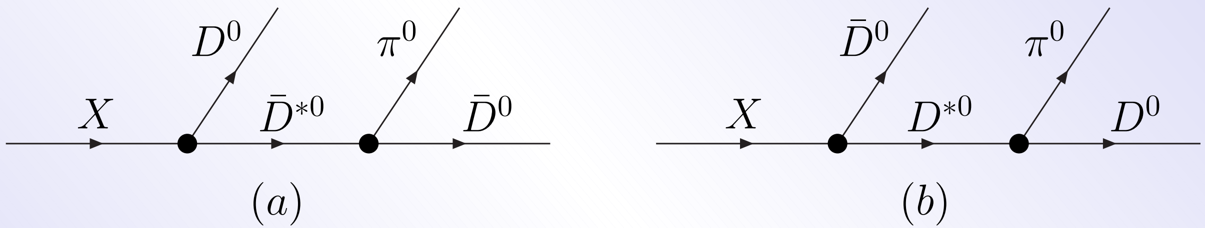
♡ $X \to D^0 \bar{D}^0 \pi^0$

- $m_X > th_n$

$$\Gamma(X \to D^0 \bar{D}^0 \pi^0) = 2\Gamma(X \to D^0 \bar{D}^{*0}) \mathrm{Br}(\bar{D}^{*0} \to \bar{D}^0 \pi^0)$$

$$\Gamma(X \to D^0 \bar{D}^{*0}) = \frac{g_X^2 |\vec{p}_1|}{24\pi m_X^2}(3 + \frac{|\vec{p}_1|^2}{m_{D^{*0}}^2}) \simeq \frac{g_X^2 |\vec{p}_1|}{8\pi m_X^2}$$

- $m_X < th_n$



$$i\mathcal{M} = i(\mathcal{M}_a + \mathcal{M}_b) = \frac{i\sqrt{2} g_X g_{D^* D\pi}}{q^2 - m_{D^{*0}}^2 + i m_{D^{*0}} \Gamma_{D^{*0}}} [\frac{(q \cdot k_\pi)(q \cdot \epsilon_X)}{m_{D^{*0}}^2} - (k_\pi \cdot \epsilon_X)]$$

- Cascade decay formula:

$$\Gamma_{X \to D^0 D^0 \pi^0} = \frac{1}{\pi} \int_{(m_{\pi^0} + m_{D^0})^2}^{(m_X - m_{D^0})^2} ds \sqrt{s} \frac{2\Gamma_{X \to D^{*0} \bar{D}^0}(s) \Gamma_{D^{*0} \to D^0 \pi^0}(s)}{(s - m_{D^{*0}}^2)^2 + (\sqrt{s}\, \Gamma_{D D^{*0}}(s))^2}$$

♠ Note: all the above formulae will be invalid when $|m_X - th_n| \sim \Gamma_{D^{*0}} \sim 70$ KeV.

Back Up

(C. Hanhart et al, PRD 76 034007)

$$F(E) = -\frac{g}{2D(E)} = \frac{1}{-\gamma+\kappa(E)},\ E = m_X - th_n \qquad (g \approx \frac{Zg_X^2}{4\pi m_X^2})$$

$$D(E) = E - E_f - \frac{g}{2}\kappa(E) - \frac{g}{2}\kappa(E-\delta) + i\frac{\Gamma(E)}{2}$$

$$\kappa(E) = \sqrt{-2\mu E - i0^+},\ \mu = \frac{m_D m_{D^*}}{m_D + m_{D^*}},\ \delta = th_c - th_n$$

- The real part of γ can be expanded around $E = 0$:

  $\mathrm{Re}\gamma = 1/a - r_s \mu E + \ldots$

  scattering length: $a = -(\sqrt{2\mu\delta} + 2E_f/g)^{-1}$, $\sqrt{2\mu\delta} \approx 125$ MeV $\sim m_\pi$

  effective range: $r_s = -(1/\sqrt{2\mu\delta} + 2/\mu g) \sim 1/m_\pi$ if $g > 0.3$

- For $a \gg 1/m_\pi$, there will be a bound state (molecule) just below the threshold with the binding energy $E_b \approx 1/(\mu a^2)$, and the line shape of $X \to J/\psi\pi\pi$ and $X \to DD\pi$ will exhibit Breit-Wigner distributions very well.

- For $a \ll -1/m_\pi$, one get a virtual state. The distribution of $X \to J/\psi\pi\pi$ will exhibit a cusp (D.V. Bugg, PLB 598 8) which is peaked exactly at $E = 0$, and the peak of the distribution of $X \to DD\pi$ will be pushed up to a few MeV above the threshold .