An "SU(8)" theory of the SM quarks and leptons

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Contents

1. Historical Reviews: *minimal* GUTs and the Flavor Puzzle
2. Flavors and Global Symmetries in the "$\mathrm{SU}(8)$" Theory
3. The SM Quark/Lepton Masses and the CKM mixing in the "$\mathrm{SU}(8)$" Theory
4. Summary

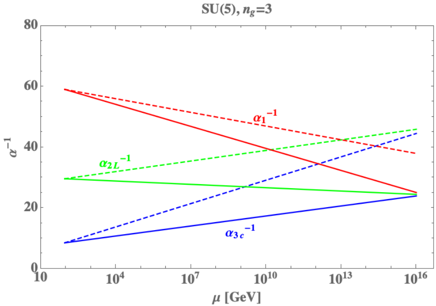
References

- “The global $B - L$ symmetry in the flavor-unified $\mathrm{SU}(N)$ theories”, JHEP in press, 2307.07921, **NC**, Ying-nan Mao, Zhaolong Teng.
- “The Standard Model quark/lepton masses and the Cabibbo-Kobayashi-Maskawa mixing in an $\mathrm{SU}(8)$ theory”, 2402.10471, **NC**, Ying-nan Mao, Zhaolong Teng.
- *in preparation*, **NC**, Ying-nan Mao, Zhaolong Teng.

Historical Reviews: *minimal* GUTs and the Flavor Puzzle

Historical reviews

- GUTs were proposed in their simplest forms of $\mathrm{SU}(5)$ with $3 \times \left[\overline{\mathbf{5_F}} \oplus \mathbf{10_F}\right]$ by [$'74$, Georgi-Glashow] (GG), and $\mathrm{SO}(10)$ with $3 \times \mathbf{16_F}$ by [$'75$, Fritzsch-Minkowski]. The main ingredients: (i) gauge symmetries $\mathcal{G}_{\rm SM} \equiv \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_W \otimes \mathrm{U}(1)_Y \subset \mathcal{G}_{\rm GUT}$, AF of the QCD, (ii) the two-generational chiral fermions (with charm quark theorized in $'70$ by Glashow-Iliopoulos-Maiani, and discovered in late $'74$).
- The supersymmetric (susy) extension to the $\mathrm{SU}(5)$ can unify three gauge couplings of the SM [$'81$, Dimopoulos-Georgi].



Historical reviews

- The chiral fermions in $\mathrm{SU}(5)$ are decomposed as
$\overline{\mathbf{5_F}} = \underbrace{(\overline{\mathbf{3}}\,,\mathbf{1}\,,+\frac{1}{3})_{\mathbf{F}}}_{{d_R}^c} \oplus \underbrace{(\mathbf{1}\,,\overline{\mathbf{2}}\,,-\frac{1}{2})_{\mathbf{F}}}_{\ell_L}$ and
$\mathbf{10_F} = \underbrace{(\mathbf{3}\,,\mathbf{2}\,,+\frac{1}{6})_{\mathbf{F}}}_{q_L} \oplus \underbrace{(\overline{\mathbf{3}}\,,\mathbf{1}\,,-\frac{2}{3})_{\mathbf{F}}}_{{u_R}^c} \oplus \underbrace{(\mathbf{1}\,,\mathbf{1}\,,+1)_{\mathbf{F}}}_{{e_R}^c}$.
- The susy GG $\mathrm{SU}(5)$ model contains Higgs fields of $\mathbf{24_H} \oplus \mathbf{5_H} \oplus \overline{\mathbf{5_H}}$, with the GUT symmetry breaking of $\mathrm{SU}(5) \xrightarrow{\langle \mathbf{24_H} \rangle} \mathcal{G}_{\mathrm{SM}}$.
- The Yukawa couplings come from the superpotential of

$$W_Y = Y_D \overline{\mathbf{5_F}} \mathbf{10_F} \overline{\mathbf{5_H}} + Y_U \mathbf{10_F} \mathbf{10_F} \mathbf{5_H}\,. \quad (1)$$

At the EW scale, the Higgs spectrum include two doublets of $\Phi_u \equiv (\mathbf{1}\,,\mathbf{2}\,,+\frac{1}{2})_{\mathbf{H}} \subset \mathbf{5_H}$ and $\Phi_d \equiv (\mathbf{1}\,,\overline{\mathbf{2}}\,,-\frac{1}{2})_{\mathbf{H}} \subset \overline{\mathbf{5_H}}$.

Historical reviews

- Besides of the well-acknowledged challenges within the *minimal* GUTs, there are two longstanding problems within the SM that have never been solved with/without the susy extension, which are: (i) the SM flavor puzzle, (ii) the PQ quality problem of the QCD axion.
- The formulation of the QM solved several fundamental puzzles in the late $19^{\rm th}$ century: (i) blackbody radiation [1900, Max Planck], (ii) photoelectric effects [1905, Albert Einstein], (iii) hydrogen spectrum [1913, Niels Bohr], by hypothesizing the quantized energies/angular momenta of particles.
- This talk: the SM flavor puzzle can be addressed by extending the *minimal* $\rm SU(5)/SO(10)$ GUTs to the "$\rm SU(8)$" GUT (minimally), with the $[N\,, k]_{\mathbf{F}}$ $(k \geq 3)$ irreps.

The flavor puzzle: origin

- What is the flavor puzzle? (i) inter-generational mass hierarchies, (ii) intra-generational mass hierarchies with non-universal splitting patterns, and (iii) mixings of flavors mediated by the EW charged currents.
- Why/how $n_g = 3$? Both the SM and the *minimal* GUTs admit the simple repetitive generational structure in terms of their irreducible anomaly-free fermion sets (IRAFFSs).

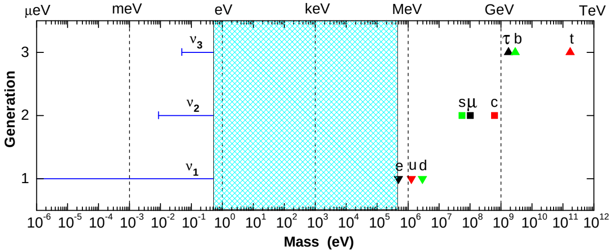


Figure: The SM fermion mass spectrum, 1909.09610, Z.Z. Xing.

The flavor puzzle: Yukawa couplings

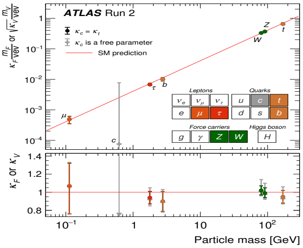


Figure: The LHC measurements of the SM Higgs boson, 2207.00092.

- The flavor puzzle: to look for the origin of the hierarchical Yukawa couplings of the *single* SM Higgs boson $y_f = \sqrt{2} m_f / v_{\rm EW}$ for all SM quarks/leptons.
- Symmetry dictates interactions [‘80, Chen-Ning Yang].

The flavor puzzle

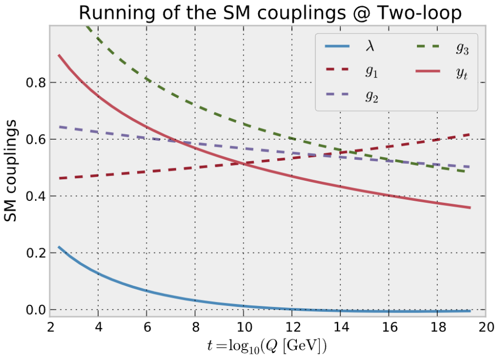


Figure: The RGEs of SM couplings, by PyR@TE.

- The RGEs cannot generate large mass hierarchies ['78, Froggatt, Nielsen].

Flavors and Global Symmetries in the “SU(8)” Theory

The origin of generations

- The main conjecture: three generations do not repeat but are non-trivially embedded in the UV theories such as the GUT ['79, Georgi, '80, Nanopolous].
- This was first considered by ['79, Georgi] based on a unified group of $\mathrm{SU}(N)$, with the anti-symmetric chiral fermions of

$$\{f_L\}_{\mathrm{SU}(N)} = \sum_k n_k \, [N\,, k]_{\mathbf{F}}\ ,\ n_k \in \mathbb{Z}\,. \quad (2)$$

  No exotic fermions in the spectrum with the $[N\,, k]_{\mathbf{F}}$.
- The anomaly-free condition can be expressed in terms of a Diophantine equation

$$\sum_k n_k \, \mathrm{Anom}([N\,, k]_{\mathbf{F}}) = 0\,, \quad (3)$$

$$\mathrm{Anom}([N\,, k]_{\mathbf{F}}) = \frac{(N-2k)\,(N-3)!}{(N-k-1)!\,(k-1)!}\,. \quad (4)$$

Georgi's counting of the SM generations

- To decompose the $\mathrm{SU}(N)$ irreps under the $\mathrm{SU}(5)$, e.g., $\mathbf{N_F} = (N-5) \times \mathbf{1_F} \oplus \mathbf{5_F}$. Decompositions of other irreps can be obtained by tensor products, ['79, Georgi].
- All fermion irreps in Eq. (2) can be decomposed into the $\mathrm{SU}(5)$ irreps of $(\mathbf{1_F}\,, \mathbf{5_F}\,, \mathbf{10_F}\,, \overline{\mathbf{10_F}}\,, \overline{\mathbf{5_F}})$, and we denote their multiplicities as $(\nu_{\mathbf{1_F}}\,, \nu_{\mathbf{5_F}}\,, \nu_{\mathbf{10_F}}\,, \nu_{\overline{\mathbf{10_F}}}\,, \nu_{\overline{\mathbf{5_F}}})$.
- Their multiplicities should satisfy $\nu_{\mathbf{5_F}} + \nu_{\mathbf{10_F}} = \nu_{\overline{\mathbf{5_F}}} + \nu_{\overline{\mathbf{10_F}}}$ from the anomaly-free condition.
- The total SM fermion generations are determined by

$$n_g = \nu_{\overline{\mathbf{5_F}}} - \nu_{\mathbf{5_F}} = \nu_{\mathbf{10_F}} - \nu_{\overline{\mathbf{10_F}}}\,. \qquad (5)$$

Georgi’s counting of the SM generations in GUTs

- The generations from a particular $\mathrm{SU}(N)$ irrep

$$\nu_{\mathbf{10_F}}[N\,,k]_{\mathbf{F}} - \nu_{\overline{\mathbf{10_F}}}[N\,,k]_{\mathbf{F}} = \frac{(N-2k)(N-5)!}{(k-2)!\,(N-k-2)!}\,. \tag{6}$$

- The usual rank-2 GG models can only give $\nu_{\mathbf{10_F}}[N\,,2]_{\mathbf{F}} - \nu_{\overline{\mathbf{10_F}}}[N\,,2]_{\mathbf{F}} = 1$. This means one can only repeat the set of anomaly-free fermion irreps to form multiple generations in rank-2 GG models.
- Alternatively, to embed multiple generations non-trivially in the GUTs, one must consider at least the rank-3 GG models. The leading candidate group must be $\mathrm{SU}(7)$, [‘79, Frampton], since the $[6\,,3]_{\mathbf{F}}$ irrep of $\mathrm{SU}(6)$ is self-conjugate.
- Note that the non-minimal GUTs are likely to loose the asymptotic freedom (AF), since $T([N\,,2]) \sim N$ and $T([N\,,3]) \sim N^2$. For the $n_g = 3$ case, we find that the GUTs with $\mathcal{G} \geq \mathrm{SU}(9)$ loose the AF.

Georgi's counting of the SM generations in GUTs

- $\mathrm{SO}(2N)$ cannot embed multiple generations non-trivially, e.g., spinor irrep of $\mathbf{64_F}$ in the $\mathrm{SO}(14)$ is decomposed into $\mathrm{SO}(10)$ as

$$\mathbf{64_F} \to \ldots \to 2 \times \left[\mathbf{16_F} \oplus \overline{\mathbf{16_F}} \right] , \tag{7}$$

  $\mathbf{16_F} = \mathbf{1_F} \oplus \overline{\mathbf{5_F}} \oplus \mathbf{10_F}$, and $\overline{\mathbf{16_F}} = \mathbf{1_F} \oplus \mathbf{5_F} \oplus \overline{\mathbf{10_F}}$. One can only obtain $n_g = 2 - 2 = 0$ at the EW scale.
- The realistic symmetry breaking patterns of the $\mathrm{SU}(N)$ usually do not follow the $\mathrm{SU}(N) \to \ldots \to \mathrm{SU}(5) \to \mathcal{G}_{\mathrm{SM}}$ one, which is dangerous in terms of the proton decay. n_g is independent of the symmetry breaking patterns.
- The zeroth stage would better be achieved by the $\mathrm{SU}(N)$ adjoint Higgs field [‘74, L.F.Li] as $\mathrm{SU}(N) \to \mathrm{SU}(k_1)_S \otimes \mathrm{SU}(k_2)_W \otimes \mathrm{U}(1)_X$, $k_1 = [\frac{N}{2}]$, since we wish to set the proton decay scale as high as possible. Currently, $\tau_p \gtrsim 10^{34}\,\mathrm{yr}$ [‘20, SuperK].

Georgi's counting of the SM generations in GUTs

- Georgi's "third law" of GUT ['79]: no repetition of a particular irrep of $\mathrm{SU}(N)$, i.e., $n_k = 0$ or $n_k = 1$ in Eq. (2). The minimal theory is

$$\{f_L\}_{\mathrm{SU}(11)} = [11\,,4]_{\mathbf{F}} \oplus [11\,,8]_{\mathbf{F}} \oplus [11\,,9]_{\mathbf{F}} \oplus [11\,,10]_{\mathbf{F}}\ . \tag{8}$$

  with $\dim_{\mathbf{F}} = 561$.
- My understanding: to prevent the simple repetitions of one generational anomaly-free fermions, such as $3 \times \left[\overline{\mathbf{5}_{\mathbf{F}}} \oplus \mathbf{10}_{\mathbf{F}}\right]$ in an $\mathrm{SU}(5)$ chiral theory. The fermions of $\left[\overline{\mathbf{5}_{\mathbf{F}}} \oplus \mathbf{10}_{\mathbf{F}}\right]$ is an irreducible anomaly-free fermion set (IRAFFS) of the $\mathrm{SU}(5)$.
- My "third law" [2307.07921]: *only distinctive IRAFFSs without simple repetitions and can lead to $n_g = 3$ at the EW scale are allowed in an $\mathrm{SU}(N)$ theory.*

The "SU(8)" theory

- The "SU(8)" theory with chiral fermions of

$$\{f_L\}^{n_g=3}_{\rm SU(8)} = \left[\overline{\mathbf{8_F}}^\omega \oplus \mathbf{28_F}\right] \bigoplus \left[\overline{\mathbf{8_F}}^{\dot\omega} \oplus \mathbf{56_F}\right] , \ \dim_{\mathbf{F}} = 156 ,$$
$$\omega = (3 , {\rm IV} , {\rm V} , {\rm VI}) , \ \dot\omega = (\dot 1 , \dot 2 , \dot{\rm VII} , \dot{\rm IIX} , \dot{\rm IX}) . \quad (9)$$

Six $(\mathbf{5_F} , \overline{\mathbf{5_F}})$ pairs, one $(\mathbf{10_F} , \overline{\mathbf{10_F}})$ pair from the $\mathbf{56_F}$, and $3 \times \left[\overline{\mathbf{5_F}} \oplus \mathbf{10_F}\right]_{\rm SM}$.

- The Higgs fields and the Yukawa couplings:

$$- \mathcal{L}_Y = Y_{\mathcal{B}} \overline{\mathbf{8_F}}^\omega \mathbf{28_F} \overline{\mathbf{8_H}}_{,\omega} + Y_{\mathcal{T}} \mathbf{28_F} \mathbf{28_F} \mathbf{70_H}$$
$$+ Y_{\mathcal{D}} \overline{\mathbf{8_F}}^{\dot\omega} \mathbf{56_F} \overline{\mathbf{28_H}}_{,\dot\omega} + \frac{c_4}{M_{\rm pl}} \mathbf{56_F} \mathbf{56_F} \overline{\mathbf{28_H}}^\dagger_{,\dot\omega} \mathbf{63_H} + H.c. . \quad (10)$$

NB: $\mathbf{56_F} \mathbf{56_F} \mathbf{28_H} = 0$ ['08, S. Barr], and one has to use $d = 5$ operator suppressed by $1/M_{\rm pl}$.

- *Gravity breaks global symmetries.*

Global symmetries in the "SU(8)" theory

- The global symmetries in the "SU(8)" theory with IRAFFSs:

$$\widetilde{\mathcal{G}}_{\rm flavor}\,[{\rm SU}(8)] = \Big[\widetilde{\rm SU}(4)_\omega \otimes \widetilde{\rm U}(1)_{T_2} \otimes \widetilde{\rm U}(1)_{{\rm PQ}_2}\Big]$$
$$\bigotimes \Big[\widetilde{\rm SU}(5)_{\dot\omega} \otimes \widetilde{\rm U}(1)_{T_3} \otimes \widetilde{\rm U}(1)_{{\rm PQ}_3}\Big]\,,$$
$$[{\rm SU}(8)]^2 \cdot \widetilde{\rm U}(1)_{T_{2\,,3}} = 0\,,\quad [{\rm SU}(8)]^2 \cdot \widetilde{\rm U}(1)_{{\rm PQ}_{2\,,3}} \neq 0\,. \qquad (11)$$

Fermions	$\overline{\mathbf{8_F}}^{\Omega=\omega\,,\dot\omega}$	$\mathbf{28_F}$	$\mathbf{56_F}$	
$\widetilde{\rm U}(1)_T$	$-3t$	$+2t$	$+t$	
$\widetilde{\rm U}(1)_{\rm PQ}$	p	q_2	q_3	
Higgs	$\overline{\mathbf{8_H}}_{\,,\omega}$	$\overline{\mathbf{28_H}}_{\,,\dot\omega}$	$\mathbf{70_H}$	$\mathbf{63_H}$
$\widetilde{\rm U}(1)_T$	$+t$	$+2t$	$-4t$	0
$\widetilde{\rm U}(1)_{\rm PQ}$	$-(p+q_2)$	$-(p+q_3)$	$-2q_2$	0

Table: The $\widetilde{\rm U}(1)_T$ and the $\widetilde{\rm U}(1)_{\rm PQ}$ charges for the "SU(8)" fermions and Higgs fields, $p:q_2 \neq -3:+2$ and $p:q_3 \neq -3:+1$.

Global symmetries in the "SU(8)" theory

- The global $\widetilde{\rm U}(1)_T$ symmetries at different stages

$$\begin{aligned} {\rm SU}(8) \to \mathcal{G}_{441_{X_0}} &: \mathcal{T}' = \mathcal{T} - 4t\mathcal{X}_0\,, \\ \mathcal{G}_{441_{X_0}} \to \mathcal{G}_{341_{X_1}} &: \mathcal{T}'' = \mathcal{T}' + 8t\mathcal{X}_1\,, \\ \mathcal{G}_{341_{X_1}} \to \mathcal{G}_{331_{X_2}} &: \mathcal{T}''' = \mathcal{T}''\,, \quad \mathcal{G}_{331_{X_2}} \to \mathcal{G}_{\rm SM} \;:\; \mathcal{B} - \mathcal{L} = \mathcal{T}'''\,. \end{aligned} \quad (12)$$

Higgs	$\mathcal{G}_{441} \to \mathcal{G}_{341}$	$\mathcal{G}_{341} \to \mathcal{G}_{331}$	$\mathcal{G}_{331} \to \mathcal{G}_{\rm SM}$	$\mathcal{G}_{\rm SM} \to {\rm SU}(3)_c \otimes {\rm U}(1)_{\rm EM}$
$\overline{\mathbf{8_H}}_{,\omega}$	✓	✓	✓	✓
$\overline{\mathbf{28_H}}_{,\dot\omega}$	✗	✓	✓	✓
$\mathbf{70_H}$	✗	✗	✗	✓

Table: The Higgs fields and their symmetry-breaking directions in the "SU(8)" GUT. The ✓and ✗represent possible and impossible symmetry breaking directions for a given Higgs field.

Global symmetries in the "SU(8)" theory

- Consistent global $\widetilde{\mathrm{U}}(1)_{B-L}$ relations of $(\mathcal{B}-\mathcal{L})(q_L)=\frac{4}{3}t$, $(\mathcal{B}-\mathcal{L})(\ell_L)=-4t$, and etc.
- The global $\widetilde{\mathrm{U}}(1)_{B-L}$

$$\mathbf{70_H} \supset \underline{(\mathbf{4}\,,\overline{\mathbf{4}}\,,+\frac{1}{2})_{\mathbf{H}}} \oplus (\overline{\mathbf{4}}\,,\mathbf{4}\,,-\frac{1}{2})_{\mathbf{H}} \supset \boxed{\underbrace{(\mathbf{1}\,,\overline{\mathbf{2}}\,,+\frac{1}{2})_{\mathbf{H}}^{\prime\prime\prime}}_{\mathcal{B}-\mathcal{L}=0}} \oplus \underbrace{(\mathbf{1}\,,\mathbf{2}\,,-\frac{1}{2})_{\mathbf{H}}^{\prime\prime\prime}}_{\mathcal{B}-\mathcal{L}=-8t}\,. \tag{13}$$

  We conjecture that $(\mathbf{1}\,,\overline{\mathbf{2}}\,,+\frac{1}{2})_{\mathbf{H}}^{\prime\prime\prime}$ is the only SM Higgs doublet.
- We also find 23 out of 27 left-handed sterile neutrinos $\check{\mathcal{N}}_L^{\Omega}$ remain massless through the 't Hooft anomaly matching of the global $\widetilde{\mathrm{U}}(1)_T$ to $\widetilde{\mathrm{U}}(1)_{B-L}$.

Fermions in the "SU(8)" theory: $\overline{\mathbf{8_F}}^{\Omega}$

"SU(8)"	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\overline{\mathbf{8_F}}^{\Omega}$	$(\overline{\mathbf{4}},\mathbf{1},+\frac{1}{4})_{\mathbf{F}}^{\Omega}$	$(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3})_{\mathbf{F}}^{\Omega}$	$(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3})_{\mathbf{F}}^{\Omega}$	$(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3})_{\mathbf{F}}^{\Omega}$: $\mathcal{D}_R^{\Omega\,c}$
		$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Omega}$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Omega}$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Omega}$: $\check{\mathcal{N}}_L^{\Omega}$
	$(\mathbf{1},\overline{\mathbf{4}},-\frac{1}{4})_{\mathbf{F}}^{\Omega}$	$(\mathbf{1},\overline{\mathbf{4}},-\frac{1}{4})_{\mathbf{F}}^{\Omega}$	$(\mathbf{1},\overline{\mathbf{3}},-\frac{1}{3})_{\mathbf{F}}^{\Omega}$	$(\mathbf{1},\overline{\mathbf{2}},-\frac{1}{2})_{\mathbf{F}}^{\Omega}$: $\mathcal{L}_L^{\Omega}=(\mathcal{E}_L^{\Omega},-\mathcal{N}_L^{\Omega})^T$
				$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Omega'}$: $\check{\mathcal{N}}_L^{\Omega'}$
			$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Omega''}$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Omega''}$: $\check{\mathcal{N}}_L^{\Omega''}$

Table: $\underline{\mathcal{D}_R^{\Omega\,c}=d_R^{\Omega\,c}}$ stand for the SM right-handed down-type quarks, and $\mathcal{D}_R^{\Omega\,c}=\mathfrak{D}_R^{\Omega\,c}$ stand for the right-handed down-type heavy partner quarks. $\underline{\mathcal{L}_L^{\Omega}=(e_L^{\Omega},-\nu_L^{\Omega})^T}$ stand for the left-handed SM lepton doublets, and $\mathcal{L}_L^{\Omega}=(\mathfrak{e}_L^{\Omega},-\mathfrak{n}_L^{\Omega})^T$ stand for the left-handed heavy lepton doublets. $\check{\mathcal{N}}_L$ are sterile neutrinos.

Fermions in the “SU(8)” theory: $\mathbf{28_F}$

“SU(8)”	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\mathbf{28_F}$	$(\mathbf{6},\mathbf{1},-\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}$ $(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}$ $(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}\ :\ \mathfrak{D}_L$ $(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}\ :\ {t_R}^c$
	$(\mathbf{1},\mathbf{6},+\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{6},+\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{3},+\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{2},+\frac{1}{2})_{\mathbf{F}}\ :\ ({\mathfrak{e}_R}^c,{\mathfrak{n}_R}^c)^T$ $(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}\ :\ \check{\mathfrak{n}}_R^c$
			$(\mathbf{1},\overline{\mathbf{3}},+\frac{2}{3})_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{2}},+\frac{1}{2})'_{\mathbf{F}}\ :\ ({\mathfrak{n}'_R}^c,-{\mathfrak{e}'_R}^c)^T$ $(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}\ :\ {\tau_R}^c$
	$(\mathbf{4},\mathbf{4},0)_{\mathbf{F}}$	$(\mathbf{3},\mathbf{4},-\frac{1}{12})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{3},0)_{\mathbf{F}}$	$(\mathbf{3},\mathbf{2},+\frac{1}{6})_{\mathbf{F}}\ :\ (t_L,b_L)^T$ $(\mathbf{3},\mathbf{1},-\frac{1}{3})'_{\mathbf{F}}\ :\ \mathfrak{D}'_L$
			$(\mathbf{3},\mathbf{1},-\frac{1}{3})''_{\mathbf{F}}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})''_{\mathbf{F}}\ :\ \mathfrak{D}''_L$
		$(\mathbf{1},\mathbf{4},+\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{3},+\frac{1}{3})''_{\mathbf{F}}$	$(\mathbf{1},\mathbf{2},+\frac{1}{2})''_{\mathbf{F}}\ :\ ({\mathfrak{e}''_R}^c,{\mathfrak{n}''_R}^c)^T$ $(\mathbf{1},\mathbf{1},0)'_{\mathbf{F}}\ :\ \check{\mathfrak{n}}'^c_R$
			$(\mathbf{1},\mathbf{1},0)''_{\mathbf{F}}$	$(\mathbf{1},\mathbf{1},0)''_{\mathbf{F}}\ :\ \check{\mathfrak{n}}''^c_R$

Table: The “SU(8)” fermion representation of $\mathbf{28_F}$.

Fermions in the “SU(8)” theory: $\mathbf{56_F}$

“SU(8)”	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\mathbf{56_F}$	$(\mathbf{1}, \overline{\mathbf{4}}, +\frac{3}{4})_{\mathbf{F}}$	$(\mathbf{1}, \overline{\mathbf{4}}, +\frac{3}{4})_{\mathbf{F}}$	$(\mathbf{1}, \overline{\mathbf{3}}, +\frac{2}{3})_{\mathbf{F}}^{\prime}$	$(\mathbf{1}, \overline{\mathbf{2}}, +\frac{1}{2})_{\mathbf{F}}^{\prime\prime\prime}$: $(\mathfrak{n}_R^{\prime\prime\prime c}, -\mathfrak{e}_R^{\prime\prime\prime c})^T$
				$(\mathbf{1}, \mathbf{1}, +1)_{\mathbf{F}}^{\prime}$: ${\mu_R}^c$
			$(\mathbf{1}, \mathbf{1}, +1)_{\mathbf{F}}^{\prime\prime}$	$(\mathbf{1}, \mathbf{1}, +1)_{\mathbf{F}}^{\prime\prime}$: ${\mathfrak{E}_R}^c$
	$(\overline{\mathbf{4}}, \mathbf{1}, -\frac{3}{4})_{\mathbf{F}}$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}}^{\prime}$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}}^{\prime}$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{\mathbf{F}}^{\prime}$: ${u_R}^c$
		$(\mathbf{1}, \mathbf{1}, -1)_{\mathbf{F}}$	$(\mathbf{1}, \mathbf{1}, -1)_{\mathbf{F}}$	$(\mathbf{1}, \mathbf{1}, -1)_{\mathbf{F}}$: $\mathfrak{E}_L$

Table: The “SU(8)” fermion representation of $\mathbf{56_F}$.

Fermions in the "SU(8)" theory: $\mathbf{56_F}$

"SU(8)"	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\mathbf{56_F}$	$(\mathbf{4}\,,\mathbf{6}\,,+\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{3}\,,\mathbf{6}\,,+\frac{1}{6})_{\mathbf{F}}$	$(\mathbf{3}\,,\mathbf{3}\,,0)_{\mathbf{F}}^{\prime}$	$(\mathbf{3}\,,\mathbf{2}\,,+\frac{1}{6})_{\mathbf{F}}^{\prime}\ :\ (c_L\,,s_L)^T$
				$(\mathbf{3}\,,\mathbf{1}\,,-\frac{1}{3})_{\mathbf{F}}^{\prime\prime\prime}\ :\ \mathfrak{D}_L^{\prime\prime\prime}$
			$(\mathbf{3}\,,\overline{\mathbf{3}}\,,+\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{3}\,,\overline{\mathbf{2}}\,,+\frac{1}{6})_{\mathbf{F}}^{\prime\prime}\ :\ (\mathfrak{d}_L\,,-\mathfrak{u}_L)^T$
				$(\mathbf{3}\,,\mathbf{1}\,,+\frac{2}{3})_{\mathbf{F}}\ :\ \mathfrak{U}_L$
		$(\mathbf{1}\,,\mathbf{6}\,,+\frac{1}{2})_{\mathbf{F}}^{\prime}$	$(\mathbf{1}\,,\mathbf{3}\,,+\frac{1}{3})_{\mathbf{F}}^{\prime}$	$(\mathbf{1}\,,\mathbf{2}\,,+\frac{1}{2})_{\mathbf{F}}^{\prime\prime\prime\prime}\ :\ (\mathfrak{e}_R^{\prime\prime\prime\prime\,c}\,,\mathfrak{n}_R^{\prime\prime\prime\prime\,c})^T$
				$(\mathbf{1}\,,\mathbf{1}\,,0)_{\mathbf{F}}^{\prime\prime\prime}\ :\ \check{\mathfrak{n}}_R^{\prime\prime\prime\,c}$
			$(\mathbf{1}\,,\overline{\mathbf{3}}\,,+\frac{2}{3})_{\mathbf{F}}^{\prime\prime}$	$(\mathbf{1}\,,\overline{\mathbf{2}}\,,+\frac{1}{2})_{\mathbf{F}}^{\prime\prime\prime\prime\prime}\ :\ (\mathfrak{n}_R^{\prime\prime\prime\prime\prime\,c}\,,-\mathfrak{e}_R^{\prime\prime\prime\prime\prime\,c})^T$
				$(\mathbf{1}\,,\mathbf{1}\,,+1)_{\mathbf{F}}^{\prime\prime\prime}\ :\ {e_R}^c$

Table: The "SU(8)" fermion representation of $\mathbf{56_F}$.

Fermions in the "SU(8)" theory: $\mathbf{56_F}$

"SU(8)"	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\mathbf{56_F}$	$(\mathbf{6}, \mathbf{4}, -\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{4}, -\frac{1}{12})'_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{3}, 0)''_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{6})'''_{\mathbf{F}}$: $(u_L, d_L)^T$
				$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})''''_{\mathbf{F}}$: $\mathfrak{D}''''_L$
			$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})'''''_{\mathbf{F}}$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})'''''_{\mathbf{F}}$: $\mathfrak{D}'''''_L$
		$(\overline{\mathbf{3}}, \mathbf{4}, -\frac{5}{12})_{\mathbf{F}}$	$(\overline{\mathbf{3}}, \mathbf{3}, -\frac{1}{3})_{\mathbf{F}}$	$(\overline{\mathbf{3}}, \mathbf{2}, -\frac{1}{6})_{\mathbf{F}}$: $(\mathfrak{d}_R{}^c, \mathfrak{u}_R{}^c)^T$
				$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})''_{\mathbf{F}}$: $\mathfrak{U}_R{}^c$
			$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})'''_{\mathbf{F}}$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})'''_{\mathbf{F}}$: $c_R{}^c$

Table: The "SU(8)" fermion representation of $\mathbf{56_F}$.

Symmetry breaking pattern in the “SU(8)” theory

- The symmetry breaking pattern [‘74, L.F.Li] of
  $\mathrm{SU}(8) \to \mathcal{G}_{441} \to \mathcal{G}_{341} \to \mathcal{G}_{331} \to \mathcal{G}_{\rm SM} \to \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{\rm EM}$.
- The intermediate symmetry breaking stages and massive vectorlike fermions:

  0 : $\mathrm{SU}(8) \xrightarrow{\mathbf{63_H}} \mathcal{G}_{441}$, all fermions remain massless.

  1 : $\mathcal{G}_{441} \xrightarrow{\overline{\mathbf{8_H}}_{,\rm IV}} \mathcal{G}_{341}$, a pair of $(\mathbf{5_F}, \overline{\mathbf{5_F}})$.

  2 : $\mathcal{G}_{341} \xrightarrow{\overline{\mathbf{8_H}}_{,\rm V}, \overline{\mathbf{28_H}}_{,\dot{1},\dot{\rm VII}}} \mathcal{G}_{331}$, two pairs of $(\mathbf{5_F}, \overline{\mathbf{5_F}})$ and a pair of $(\mathbf{10_F}, \overline{\mathbf{10_F}})$.

  3 : $\mathcal{G}_{331} \xrightarrow{\overline{\mathbf{8_H}}_{,3,\rm VI}, \overline{\mathbf{28_H}}_{,\dot{2},\dot{\rm IIX},\dot{\rm IX}}} \mathcal{G}_{\rm SM}$, three pairs of $(\mathbf{5_F}, \overline{\mathbf{5_F}})$.
- Dimensionless parameters

$$\zeta_1 \equiv \frac{W_{\overline{\mathbf{4}},\rm IV}}{M_{\rm pl}}, \quad \zeta_2 \equiv \frac{w_{\overline{\mathbf{4}},\rm V}}{M_{\rm pl}}, \quad \dot{\zeta}_2 \equiv \frac{w_{\overline{\mathbf{4}},\dot{1},\dot{\rm VII}}}{M_{\rm pl}},$$
$$\zeta_3 \equiv \frac{V_{\overline{\mathbf{3}},3,\rm VI}}{M_{\rm pl}}, \quad \dot{\zeta}_3' \equiv \frac{V'_{\overline{\mathbf{3}},\dot{2},\dot{\rm IIX}}}{M_{\rm pl}}, \quad \dot{\zeta}_3 \equiv \frac{V_{\overline{\mathbf{3}},\dot{\rm IX}}}{M_{\rm pl}},$$
$$\zeta_1 \gg \zeta_2 \sim \dot{\zeta}_2 \gg \zeta_3 \sim \dot{\zeta}_3' \sim \dot{\zeta}_3 \,. \tag{14}$$

Vectorlike fermions in the "SU(8)" theory

stages	$Q_e = -\frac{1}{3}$	$Q_e = +\frac{2}{3}$	$Q_e = -1$	$Q_e = 0$
v_{441} $\{\Omega\}$	$\mathfrak{D}$ IV	-	$(\mathfrak{e}'', \mathfrak{n}'')$ IV	$\{\check{\mathfrak{n}}', \check{\mathfrak{n}}''\}$ $\{\text{IV}', \text{IV}''\}$
v_{341} $\{\Omega\}$	$\mathfrak{d}, \{\mathfrak{D}'', \mathfrak{D}'''''\}$ $\{\text{V}, \dot{\text{V}}\text{II}\}$	$\mathfrak{u}, \mathfrak{U}$	$\mathfrak{E}, (\mathfrak{e}, \mathfrak{n}), (\mathfrak{e}'''', \mathfrak{n}'''')$ $\{\text{V}, \dot{\text{V}}\text{II}\}$	$\{\check{\mathfrak{n}}, \check{\mathfrak{n}}'''\}$ $\{\text{V}', \dot{\text{V}}\text{II}'\}$
v_{331} $\{\Omega\}$	$\{\mathfrak{D}', \mathfrak{D}''', \mathfrak{D}''''\}$ $\{\text{VI}, \text{II}\dot{\text{X}}, \text{I}\dot{\text{X}}\}$	-	$(\mathfrak{e}', \mathfrak{n}'), (\mathfrak{e}''', \mathfrak{n}'''), (\mathfrak{e}''''', \mathfrak{n}''''')$ $\{\text{VI}, \text{II}\dot{\text{X}}, \text{I}\dot{\text{X}}\}$	-

Table: The vectorlike fermions at different intermediate symmetry breaking scales in the SU(8) theory.

Symmetry breaking pattern in the “SU(8)” theory

The SM Quark/Lepton Masses and the CKM mixing in the “SU(8)” Theory

SM fermion masses in the "SU(8)" theory

- The natural top quark mass from the tree level

$$
\begin{aligned}
& Y_{\mathcal{T}} \mathbf{28_F} \mathbf{28_F} \mathbf{70_H} \supset Y_{\mathcal{T}} (\mathbf{6}\,,\mathbf{1}\,,-\frac{1}{2})_{\mathbf{F}} \otimes (\mathbf{4}\,,\mathbf{4}\,,0)_{\mathbf{F}} \otimes (\mathbf{4}\,,\overline{\mathbf{4}}\,,+\frac{1}{2})_{\mathbf{H}} \\
\supset\ & \ldots \supset Y_{\mathcal{T}} (\overline{\mathbf{3}}\,,\mathbf{1}\,,-\frac{2}{3})_{\mathbf{F}} \otimes (\mathbf{3}\,,\mathbf{2}\,,+\frac{1}{6})_{\mathbf{F}} \otimes (\mathbf{1}\,,\overline{\mathbf{2}}\,,+\frac{1}{2})_{\mathbf{H}}^{\prime\prime\prime} \\
\Rightarrow\ & \frac{1}{\sqrt{2}} Y_{\mathcal{T}} t_L {t_R}^c v_{\rm EW}\,. \qquad (15)
\end{aligned}
$$

- With the identification of $(\mathbf{3}\,,\mathbf{2}\,,+\frac{1}{6})_{\mathbf{F}} \equiv (t_L\,,b_L)^T$ and $(\overline{\mathbf{3}}\,,\mathbf{1}\,,-\frac{2}{3})_{\mathbf{F}} \equiv {t_R}^c$ within the $\mathbf{28_F}$, it is straightforward to infer that $(\mathbf{1}\,,\mathbf{1}\,,+1)_{\mathbf{F}} \equiv {\tau_R}^c$ in Tab. 4. This explains why do the third-generational SM $\mathbf{10_F}$ reside in the $\mathbf{28_F}$, while the first- and second-generational SM $\mathbf{10_F}$'s must reside in the $\mathbf{56_F}$.

SM fermion masses in the "SU(8)" theory

- To generate other lighter SM fermion masses: the gravitational effects through $d=5$ operators, which break the global symmetries in Eq. (11) explicitly.
- The direct Yukawa couplings of $\mathcal{O}_{\mathcal{F}}^{d=5}$:

$$
\begin{aligned}
&\mathcal{O}_{\mathcal{F}}^{(3\,,2)} \equiv \overline{\mathbf{8_F}}^{\dot\omega} \mathbf{56_F} \cdot \overline{\mathbf{28_H}}^{\dagger}_{\,,\dot\kappa} \cdot \mathbf{70_H}^{\dagger} \\
\Rightarrow &\left[\dot\zeta_3 (s_L \mathcal{D}_R^{\dot\omega\,c} - \mathcal{E}_L^{\dot\omega} \mu_R{}^c) + \dot\zeta_3^\prime (d_L \mathcal{D}_R^{\dot\omega\,c} - \mathcal{E}_L^{\dot\omega} e_R{}^c) \right] v_{\rm EW} \,, \\
&\mathcal{O}_{\mathcal{F}}^{(4\,,1)} \equiv \mathbf{56_F} \mathbf{56_F} \cdot \overline{\mathbf{28_H}}_{\,,\dot\omega} \cdot \mathbf{70_H} \Rightarrow \dot\zeta_2 (c_L u_R{}^c + \cancel{u_L c_R{}^c}) v_{\rm EW} \,, \\
&\mathcal{O}_{\mathcal{F}}^{(5\,,1)} \equiv \mathbf{28_F} \mathbf{56_F} \cdot \overline{\mathbf{8_H}}_{\,,\omega} \cdot \mathbf{70_H} \\
\Rightarrow &\left[\zeta_1 (u_L t_R{}^c + t_L u_R{}^c) + \zeta_2 (c_L t_R{}^c + t_L c_R{}^c) \right] v_{\rm EW} \,. \qquad (16)
\end{aligned}
$$

- All $(u\,,c\,,t)$ obtain hierarchical masses, while only the $(s\,,\mu)$ become massive.

SM fermion masses in the "SU(8)" theory

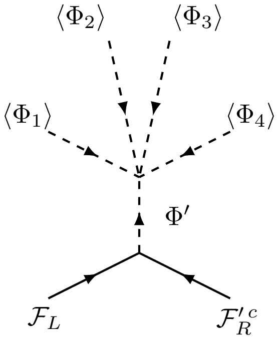


Figure: The indirect Yukawa couplings.

These are achievable through the EWSB components in renormalizable Yukawa couplings of

$$\mathcal{F}_L \mathcal{F}'^{\,c}_R \Phi' \equiv \overline{\mathbf{8_F}}^{\omega} \mathbf{28_F} \overline{\mathbf{8_H}}_{,\omega} \, , \quad \overline{\mathbf{8_F}}^{\dot{\omega}} \mathbf{56_F} \overline{\mathbf{28_H}}_{,\dot{\omega}} \, . \tag{17}$$

SM fermion masses in the "SU(8)" theory

- There are two indirect Yukawa couplings with the irreducible Higgs mixing operators of $\mathcal{O}_{\mathcal{H}}^{d=5}$:

$$\begin{aligned}
\mathcal{O}_{\mathscr{A}}^{d=5} &\equiv \epsilon_{\omega_1\omega_2\omega_3\omega_4} \overline{\mathbf{8_H}}^{\dagger}_{,\omega_1} \overline{\mathbf{8_H}}^{\dagger}_{,\omega_2} \overline{\mathbf{8_H}}^{\dagger}_{,\omega_3} \overline{\mathbf{8_H}}^{\dagger}_{,\omega_4} \mathbf{70_H}^{\dagger} \,, \\
& \mathcal{PQ} = 2(2p + 3q_2) \neq 0 \,, \qquad (18a)
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_{\mathscr{B}}^{d=5} &\supset (\overline{\mathbf{28_H}}^{\dagger}_{,\mathrm{i}} \overline{\mathbf{28_H}}_{,\dot{\mathrm{VII}}}) \cdot \overline{\mathbf{28_H}}^{\dagger}_{,\mathrm{II}\dot{\mathrm{X}}} \overline{\mathbf{28_H}}^{\dagger}_{,\mathrm{i}} \mathbf{70_H}^{\dagger} \,, \\
& (\overline{\mathbf{28_H}}^{\dagger}_{,\mathrm{i}} \overline{\mathbf{28_H}}_{,\dot{\mathrm{VII}}}) \cdot \overline{\mathbf{28_H}}^{\dagger}_{,\mathrm{I}\dot{\mathrm{X}}} \overline{\mathbf{28_H}}^{\dagger}_{,\dot{2}} \mathbf{70_H}^{\dagger} \,, \\
& \mathcal{PQ} = 2(p + q_2 + q_3) \,. \qquad (18b)
\end{aligned}$$

- Each operator of $\mathcal{O}_{\mathcal{H}}^{d=5}$
  1. *breaks the global symmetries explicitly*;
  2. *can not be further partitioned into subset of renormalizable operators, among which any of them can be allowed by both the gauge and the global symmetries.*

SM fermion masses in the “SU(8)” theory

- The $(u\,,c\,,t)$ masses

$$\mathcal{M}_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & c_5 \zeta_1 \\ c_4 \dot{\zeta}_2 & 0 & c_5 \zeta_2 \\ c_5 \zeta_1 & c_5 \zeta_2 & Y_{\mathcal{T}} \end{pmatrix} v_{\rm EW} \,. \tag{19}$$

- The $(d\,,s\,,b)$ masses

$$\mathcal{M}_d = \frac{1}{\sqrt{2}} \begin{pmatrix} c_3 \dot{\zeta}_3' & c_3 \dot{\zeta}_3' & 0 \\ c_3 \dot{\zeta}_3 & (c_3 + Y_{\mathcal{D}} d_{\mathscr{B}} \zeta_{23}^{-2}) \dot{\zeta}_3 & 0 \\ 0 & 0 & Y_{\mathcal{B}} d_{\mathscr{A}} \zeta_{23}^{-1} \zeta_1 \end{pmatrix} v_{\rm EW} \,. \tag{20}$$

The charged lepton masses are $\mathcal{M}_\ell = (\mathcal{M}_d)^T$.

SM fermion masses in the "SU(8)" theory

- The $(u\,,c\,,t)$ masses

$$m_u \approx c_4 \frac{\zeta_2^2}{\sqrt{2}\zeta_1} v_{\rm EW}\,, \quad m_c \approx c_5^2 \frac{\zeta_1^2}{\sqrt{2} Y_{\mathcal{T}}} v_{\rm EW}\,, \quad m_t \approx \frac{Y_{\mathcal{T}}}{\sqrt{2}} v_{\rm EW}\,. \tag{21}$$

- The $(d\,,s\,,b)$ and $(e\,,\mu\,,\tau)$ masses

$$m_d = m_e \approx \frac{c_3 \zeta_3 |\cos\lambda|}{\sqrt{2}} v_{\rm EW}\,, \quad m_s = m_\mu \approx Y_{\mathcal{D}} \frac{d_{\mathscr{B}} \zeta_2^2}{\sqrt{2}\zeta_3 |\sin\lambda|} v_{\rm EW}\,,$$
$$m_b = m_\tau \approx Y_{\mathcal{B}} \frac{d_{\mathscr{A}} \zeta_1 \zeta_2}{\sqrt{2}\zeta_3} v_{\rm EW}\,. \tag{22}$$

- The CKM mixing:

$$\hat{V}_{\rm CKM}\Big|_{\rm SU(8)} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & \frac{c_5}{Y_{\mathcal{T}}}\zeta_2 \\ -\lambda & 1-\lambda^2/2 & -\frac{c_5}{Y_{\mathcal{T}}}\zeta_1 \\ -\frac{c_5}{Y_{\mathcal{T}}}(\lambda\zeta_1+\zeta_2) & -\frac{c_5}{Y_{\mathcal{T}}}\zeta_1 & 1 \end{pmatrix}\,. \tag{23}$$

SM fermion masses in the "SU(8)" theory: benchmark

$\zeta_1\ (v_{441})$ 6.3×10^{-2} $(7\times 10^{17}\,\mathrm{GeV})$	$\zeta_2\ (v_{341})$ 3.8×10^{-3} $(7\times 10^{16}\,\mathrm{GeV})$	$\zeta_3\ (v_{331})$ 2.6×10^{-5} $(3\times 10^{14}\,\mathrm{GeV})$		
c_3 1	c_4 3.0×10^{-2}	c_5 1	$d_{\mathscr{A}}$ 1.4×10^{-3}	$d_{\mathscr{B}}$ 1.0×10^{-4}
m_u 1.2×10^{-3}	m_c 0.7	m_t 174.0	$m_d = m_e$ 4.4×10^{-3}	$m_s = m_\mu$ 4.4×10^{-2}
$\lvert V_{ud}\rvert$ 0.98	$\lvert V_{us}\rvert$ 0.22	$\lvert V_{ub}\rvert$ 3.8×10^{-3}		
$\lvert V_{cd}\rvert$ 0.22	$\lvert V_{cs}\rvert$ 0.98	$\lvert V_{cb}\rvert$ 6.3×10^{-2}		
$\lvert V_{td}\rvert$ 0.018	$\lvert V_{ts}\rvert$ 6.3×10^{-2}	$\lvert V_{tb}\rvert$ 1		

Table: The parameters of the $\mathrm{SU}(8)$ benchmark point and the predicted SM quark/lepton masses (in unit of GeV) as well as the CKM mixings.

Summary

Summary

- We suggest that an "SU(8)" theory has the potential to address the fundamental flavor puzzle in the SM. Different generations transform differently in the UV-complete theory and their repetitive structure only emerge in the IR, which lead to the flavor non-universality of SM quarks/leptons.
- Our construction relaxes Georgi's "third law" in 1979, and we avoid the repetitions of one IRAFFS. The global symmetries based on the IRAFFSs are vital to: (i) determine one single SM Higgs doublet through the non-anomalous $\widetilde{\mathrm{U}}(1)_{B-L}$ symmetry, (ii) count the massless left-handed sterile neutrinos precisely through the 't Hooft anomaly matching, (iii) organize the $d = 5$ operators for the SM fermion mass (mixing) terms.
- The symmetry-breaking pattern of the "SU(8)" theory is described, and all light SM fermion masses besides of the top quark are due to the inevitable gravitational effects that break the underlying global symmetries explicitly.

Summary and Outlook

- Crucial assumptions: (i) the VEV assignments in Eq. (14), (ii) the SM flavor IDs in Tabs. 3, 4, 5, 6, and 7, and (iii) the $d = 5$ operators of direct and indirect Yukawa couplings containing the SM Higgs doublet.
- The SM neutrinos $\nu_L \in \overline{\mathbf{8_F}}^{\Omega}$ are flavor-universal, to look for their masses and mixings through the $d = 5$ operators as well.
- The degenerate $m_{d^i} = m_{\ell^i}$ will be further probed based on the RGEs of $\frac{dm_f(\mu)}{d\log\mu} \equiv \gamma_{m_f} m_f(\mu)$, $\gamma_{m_f}(\alpha^{\Upsilon}) = \frac{\alpha^{\Upsilon}}{4\pi}\gamma_0(\mathcal{R}_f^{\Upsilon})$.
- The gauge coupling unification? The alternative symmetry breaking patterns? Proton lifetime?