Investigating the BNV dinucleon to dilepton decays in the EFT

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X. G.He and **XDMA**: *JHEP* 06 (2021) 047
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Outline

- **Motivation for BNV/LNV interactions**
- **EFT for the** $\Delta B = \Delta L = -2$**:** $pp \to \ell^+\ell'^+, pn \to \ell^+\bar{\nu}', nn \to \bar{\nu}\bar{\nu}'$
- **Estimation of decay rate**
- **Summary**

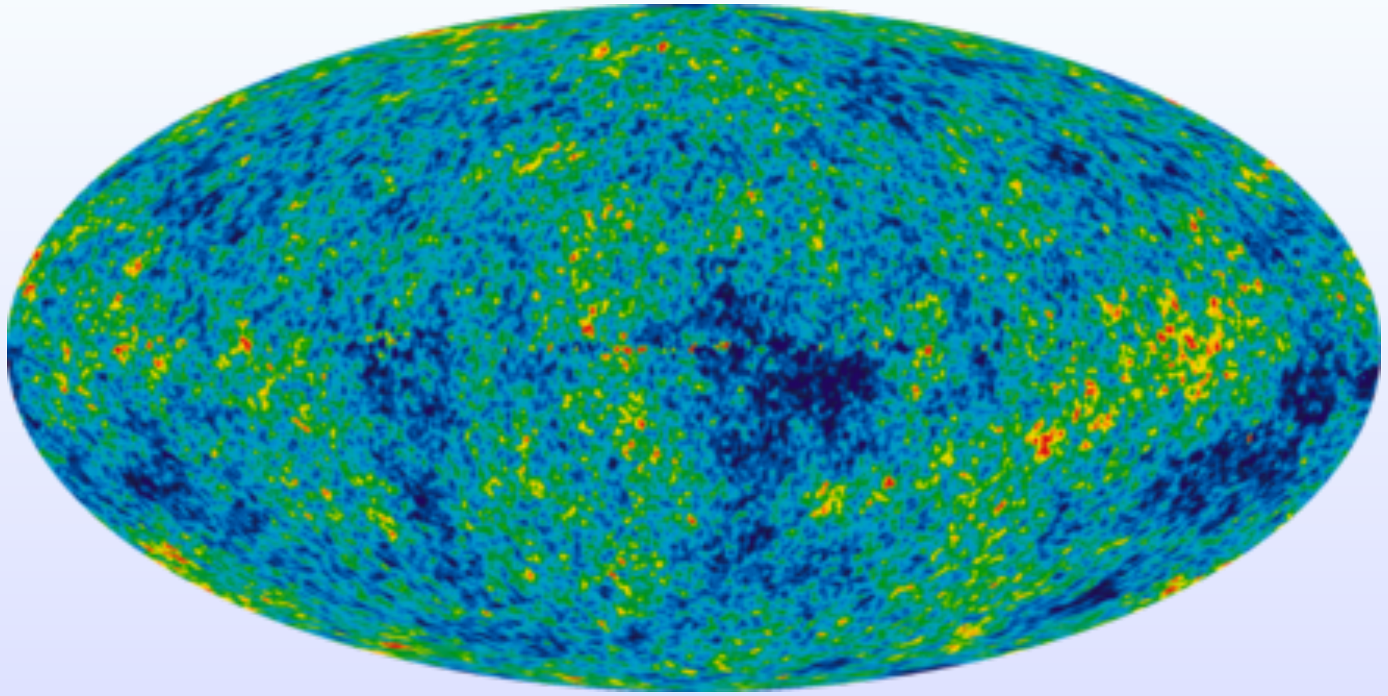
BNV is a key ingredient for the baryon asymmetry of the universe

Sakharov's conditions for baryogenesis:

1. BNV
2. C, CP violation
3. Interactions out of thermal equilibrium

Sakharov, 1967dj

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 6 \times 10^{-10}$$



https://en.wikipedia.org/wiki/Baryogenesi

LNV and the Majorana nature of neutrinos

- **SM: B/L is violated via anomaly but $B-L$ is conserved**

't Hooft, 1976

- **B & L violation is a clear signature for new physics (NP)**

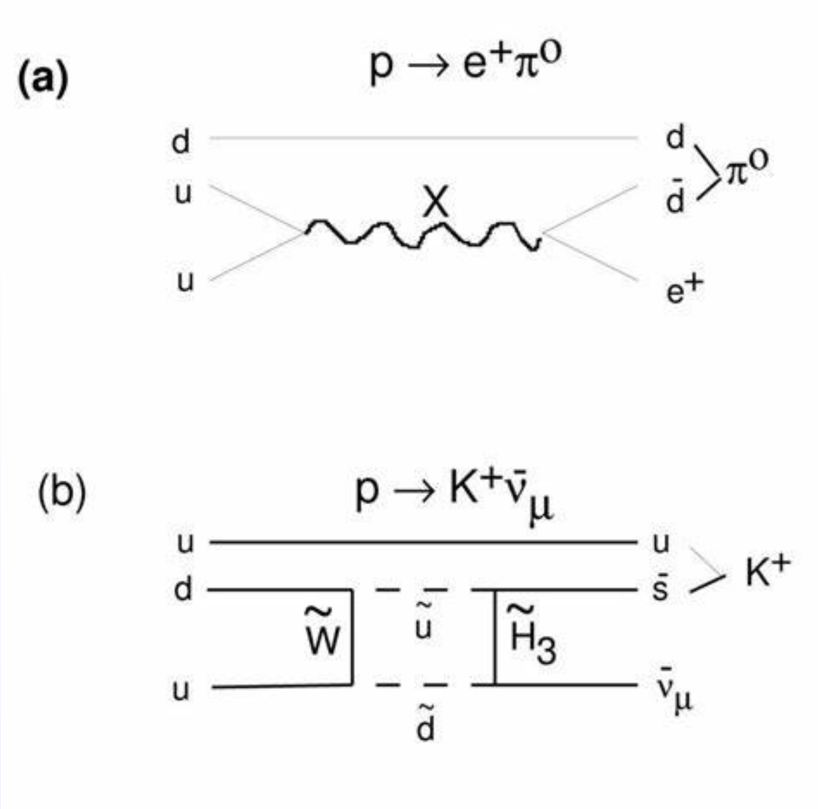
- **Theoretically: GUTs, SUSY, Extra-dim, etc $\Rightarrow$ BNV & LNV**

Low energy probes of BNV signals

$\Delta B = 1$: nucleon decay like $p \to e^+\pi^0, \pi^+\nu, \cdots$

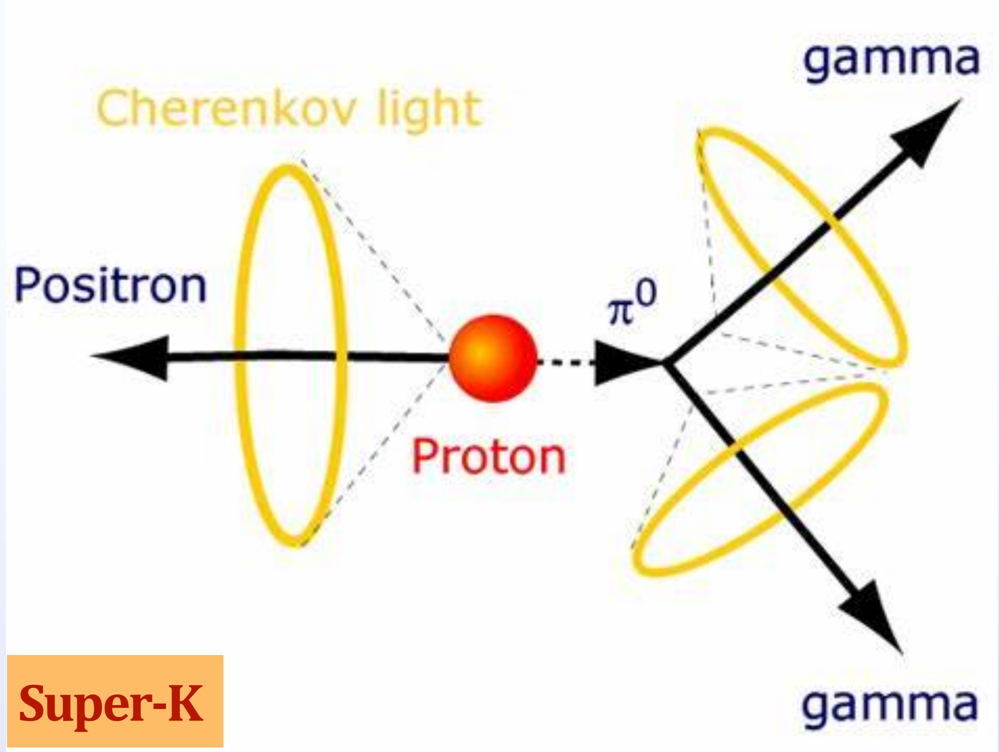
$$\mathcal{O} \sim \frac{1}{\Lambda^2}(uude)\ ,\ \frac{1}{\Lambda^2}(udd\nu), \cdots$$

Dim-6, dim-7 SMEFT



$$\Gamma \sim \frac{m_p^5}{\Lambda^4}$$

Exp: $\tau_p > 10^{34}$ yr



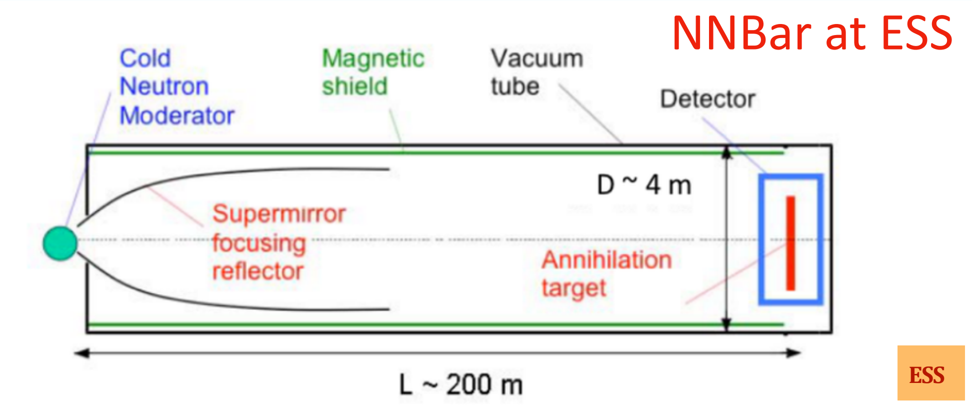
$$\Lambda \sim 10^{15}\ \text{GeV}$$

Hard to search for at colliders!

More on $\Delta B = 1$ process, see, Heeck, Takhistov, 1910.07647

Low energy probes of BNV signals

$\Delta B = 2 \,\&\, \Delta L = 0$



- $n - \bar{n}$ **oscillation**
- **dinucleon decays:** $NN' \to M_1 M_2, \ell^+\ell^-, \ell^+\nu, \bar{\nu}\nu$

$$\tau_{n-\bar{n}}^{-1} = |\delta m|, \delta m \equiv \langle \bar{n} | \mathcal{H}_{\text{eff}} | n \rangle \sim \frac{\Lambda_{\text{QCD}}^6}{\Lambda^6}$$

@ dim-9 in SMEFT

$\tau_{n-\bar{n}}^{\text{SK}} > 4.7 \times 10^8\ s$ Super-K_2012.02607

$$\Lambda_{\text{NP}} \sim \mathcal{O}(1 - 10^4)\ \text{TeV}$$

Easy to test at colliders: LHC, ...

D.G. Phillips II et al. / Physics Reports 612 (2016) 1–45

$|\Delta B| = 2$: A State of the Field, and Looking Forward, 2010.02299

$\Delta B = 2 \,\&\, \Delta L = 2$

- $H - \bar{H}$ **oscillation**

  Feinberg, Goldhaber and Steigman, 1978;

  Arnellos and W. J. Marciano , 1982; Grossman and Ng, 2018

- **Dinucleon decays:**

$$pp \to \ell_\alpha^+ \ell_\beta^+, pn \to \ell_\alpha^+ \bar{\nu}_\beta, nn \to \bar{\nu}_\alpha \bar{\nu}_\beta$$

Our goal: a systematic EFT analysis

Models suppress $\Delta B = 1$ **but not** $\Delta B = 2$

Mohapatra and Senjanovic, 1982, Perez and Wise, 2011

Arnold, Fornal, and Wise, 2012; Gardner and Yan , 2019

Helset, Murgui and Wise, 2021; Girmohanta, Shrock 2019, 2020, etc

Current experimental bounds

Decay mode	Lifetime limit
$pp \to e^+e^+$	$4.2 \times 10^{33}\mathrm{yr}$
$pp \to e^+\mu^+$	$4.4 \times 10^{33}\mathrm{yr}$
$pp \to \mu^+\mu^+$	$4.4 \times 10^{33}\mathrm{yr}$
$pp \to e^+\tau^+$	$-$

Super-Kamiokande, 2018, arXiv:1811.12430

Decay mode	Lifetime limit
$pn \to e^+\bar{\nu}'$	$2.6 \times 10^{32}\mathrm{yr}$
$pn \to \mu^+\bar{\nu}'$	$2.2 \times 10^{32}\mathrm{yr}$
$pn \to \tau^+\bar{\nu}'$	$2.9 \times 10^{31}\mathrm{yr}$

$^{16}\mathrm{O}$ *Super-Kamiokande, 2015*

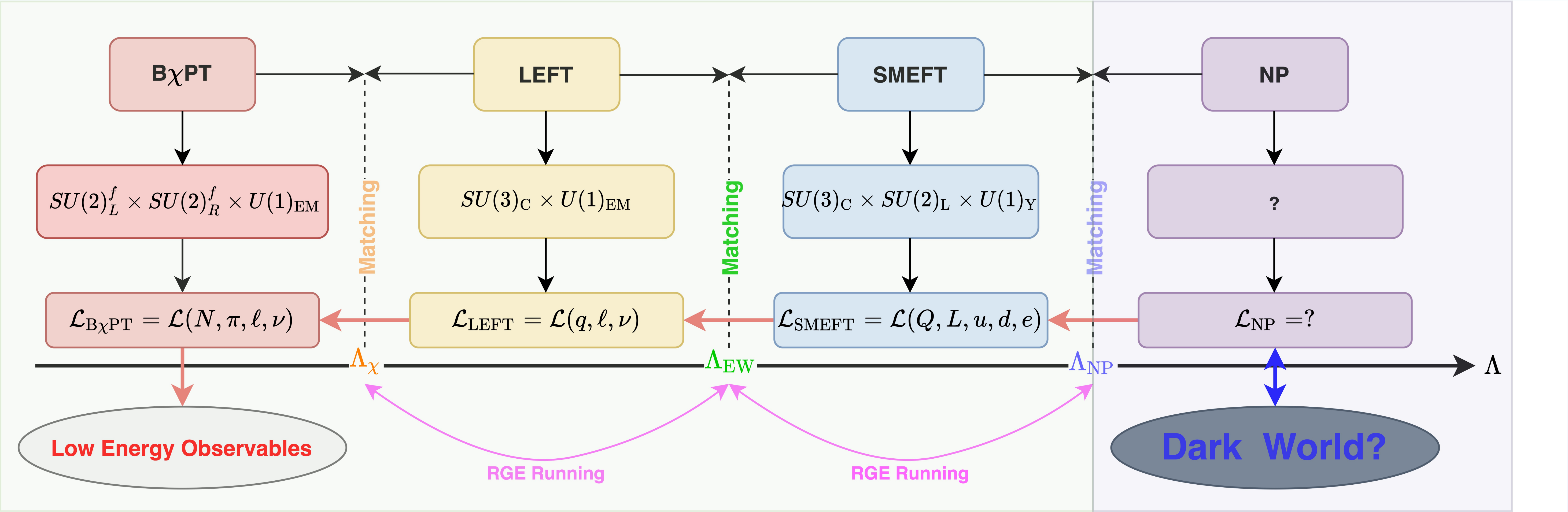
Decay mode	Lifetime limit
$nn \to \bar{\nu}\bar{\nu}'$	$1.4 \times 10^{30}\mathrm{yr}$

KamLAND, 2006 $^{12}\mathrm{C}$

* **The anti-neutrinos can be other invisible particles like neutrinos**

* **The limits on the partial lifetime are extremely large $\Rightarrow$ sensitive to NP**

EFT for $\Delta B = \Delta L = -2$ interactions



* LEFT is a more general framework since $\Lambda_{\rm NP}$ can be as low as a few GeV, but with more parameters

* SMEFT is a strong constraint for the LEFT interactions, fewer parameters, but with the assumption: $\Lambda_{\rm NP} \gg \Lambda_{\rm EW}$

* (B)χPT is a systematic way to determine the non-perturbative QCD effect

LEFT

- **Fields:** $u,\ d, s,\ c,\ b;\ \ e,\ \mu,\ \tau;\ \ \nu_e,\ \nu_\mu,\ \nu_\tau$

- **Symmetry:** $SU(3)_\mathrm{C} \times U(1)_\mathrm{EM}$

- **Power counting: canonical dimension** d

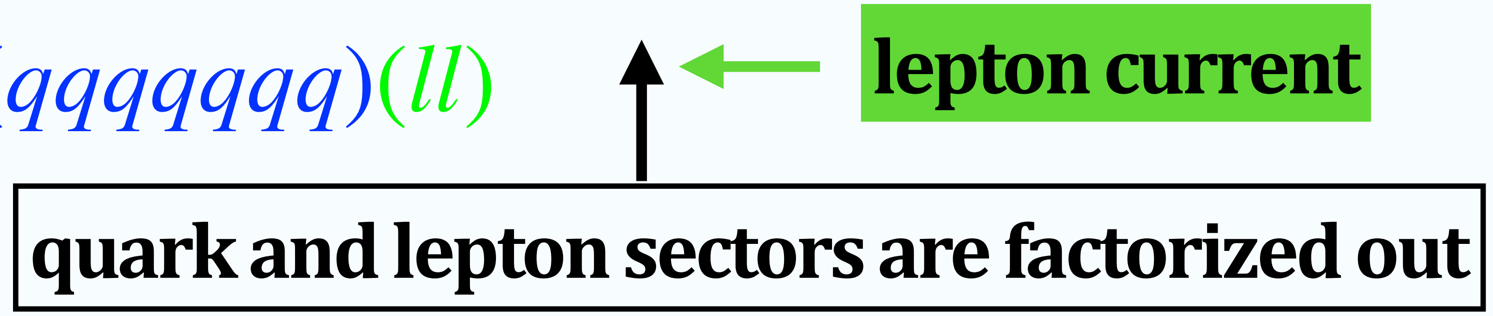
The effective operators for $\Delta B = \Delta L = -2$ interactions

6 quarks + 2 leptons

Dim-12 operators $(qqqqqqll)$ **with** $q = u, d$ & $l = \ell, \nu$

Dim-12 operators $(qqqqqqll)$ with $q = u, d$ & $l = \ell, \nu$

- **Operator structure**: $(qqqqqqll) \overset{\text{Fierz identities}}{\Longrightarrow} (qqqqqq)(ll)$



- $U(1)_{\mathrm{EM}}$: $(uuuudd)(\ell\ell'), \quad (uuuddd)(\ell\nu'), \quad (uudddd)(\nu\nu')$
- $SU(3)_C$: $\mathcal{O} \sim T_{ijklmn}(q^i q^j)(q^k q^l)(q^m q^n) j_{\mathrm{lep.}}$

color tensor

$$T^{SSS}_{\{ij\}\{kl\}\{mn\}} = \epsilon_{ikm}\epsilon_{jln} + \epsilon_{ikn}\epsilon_{jlm} + \epsilon_{ilm}\epsilon_{jkn} + \epsilon_{iln}\epsilon_{jkm}$$
$$T^{SAA}_{\{ij\}[kl][mn]} = \epsilon_{imn}\epsilon_{jkl} + \epsilon_{ikl}\epsilon_{jmn}$$
$$T^{SAA}_{\{kl\}[mn][ij]} = \epsilon_{ijk}\epsilon_{mnl} + \epsilon_{ijl}\epsilon_{mnk}$$
$$T^{SAA}_{\{mn\}[ij][kl]} = \epsilon_{ijm}\epsilon_{kln} + \epsilon_{ijn}\epsilon_{klm}$$
$$T^{AAA}_{[ij][kl][mn]} = \epsilon_{ijm}\epsilon_{kln} - \epsilon_{ijn}\epsilon_{klm}$$

Final operator's Lorentz structure:

Scalar lepton current: $(qq)(qq)(qq)(ll)$ S-S-S-S

Vector lepton current: $(qq)(qq)(q\gamma_\mu q)(l\gamma^\mu l)$ S-S-V-V

Tensor lepton current: $(qq)(qq)(q\sigma_{\mu\nu} q)(l\sigma^{\mu\nu} l)$ S-S-T-T

Total counting

- $pp \to \ell_\alpha^+ \ell_\beta^+$: **28 (S-S-S-S)+19 (S-S-V-V)+16 (S-S-T-T) = 63 operators**

$\alpha = \beta = e \Rightarrow H - \bar{H}$ **oscillation: 47 vs 60** by Caswell, Milutinovic, and Senjanovic, 1983

- $pn \to \ell_\alpha^+ \bar{\nu}_\beta$: **14 (S-S-S-S)+24 (S-S-V-V)+13 (S-S-T-T) = 51 operators**

- $nn \to \bar{\nu}_\alpha \bar{\nu}_\beta$: **14 (S-S-S-S)+8 (S-S-T-T) = 22 operators**

14 $n - \bar{n}$ oscillation operators after dropping the scalar lepton current.

A glimpse of the operators for $pp \to \ell_\alpha^+ \ell_\beta^+$

dim-12 operators with a scalar lepton current

$$\mathcal{Q}^{(pp)S,\pm}_{1LLL,a} = (u_L^{i\mathrm{T}} C u_L^j)(u_L^{k\mathrm{T}} C d_L^l)(u_L^{m\mathrm{T}} C d_L^n) j^{\ell\ell'}_{S,\pm} T^{SSS}_{\{ij\}\{kl\}\{mn\}}$$

$$\mathcal{Q}^{(pp)S,\pm}_{1LLL,b} = (u_L^{i\mathrm{T}} C u_L^j)(u_L^{k\mathrm{T}} C d_L^l)(u_L^{m\mathrm{T}} C d_L^n) j^{\ell\ell'}_{S,\pm} T^{SAA}_{\{ij\}[kl][mn]}$$

$$\mathcal{Q}^{(pp)S,\pm}_{2LLR,a} = (u_L^{i\mathrm{T}} C u_L^j)(u_L^{k\mathrm{T}} C d_L^l)(u_R^{m\mathrm{T}} C d_R^n) j^{\ell\ell'}_{S,\pm} T^{SSS}_{\{ij\}\{kl\}\{mn\}}$$

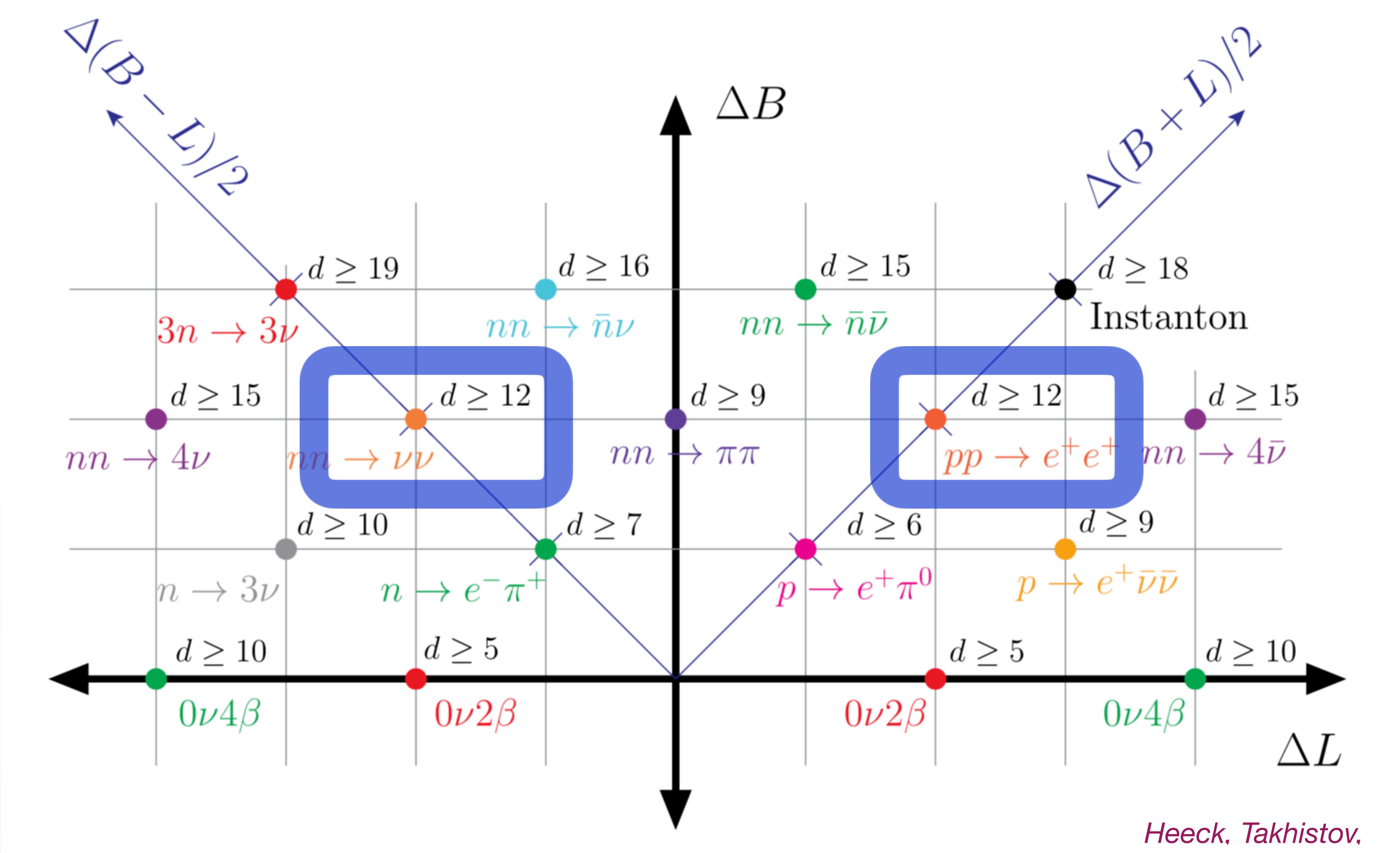
$$\mathcal{Q}^{(pp)S,\pm}_{2LLR,b} = (u_L^{i\mathrm{T}} C u_L^j)(u_L^{k\mathrm{T}} C d_L^l)(u_R^{m\mathrm{T}} C d_R^n) j^{\ell\ell'}_{S,\pm} T^{SAA}_{\{ij\}[kl][mn]}$$

$$\mathcal{Q}^{(pp)S,\pm}_{3LLR,a} = (u_L^{i\mathrm{T}} C d_L^j)(u_L^{k\mathrm{T}} C d_L^l)(u_R^{m\mathrm{T}} C u_R^n) j^{\ell\ell'}_{S,\pm} T^{SSS}_{\{ij\}\{kl\}\{mn\}}$$

$$\mathcal{Q}^{(pp)S,\pm}_{3LLR,b} = (u_L^{i\mathrm{T}} C d_L^j)(u_L^{k\mathrm{T}} C d_L^l)(u_R^{m\mathrm{T}} C u_R^n) j^{\ell\ell'}_{S,\pm} T^{SAA}_{\{mn\}[ij][kl]}$$

$$\mathcal{Q}^{(pp)S,\pm}_{4LLR} = (u_L^{i\mathrm{T}} C u_L^j)(u_L^{k\mathrm{T}} C u_L^l)(d_R^{m\mathrm{T}} C d_R^n) j^{\ell\ell'}_{S,\pm} T^{SSS}_{\{ij\}\{kl\}\{mn\}}$$

B & L quantum numbers in the SMEFT



The operator's dimension is even (odd) if its $(B\text{–}L)/2$ is even (odd) *Kobach , 2016*

The LO operators also first appear at dim 12

Focusing on the LO dim-12 operators

- $U(1)_Y$:

$u^3d^3L^2, u^2d^2Q^2L^2, udQ^4L^2, Q^6L^2$, $u^4d^2e^2, u^3dQ^2e^2, u^2Q^4e^2$, $u^3d^2QeL, u^2dQ^3eL, uQ^5eL$

- **Fierz identities** $\Rightarrow \mathcal{O}_q \times j_{\text{lep}}$: **S-S-S-S, S-S-V-V, S-S-T-T**

- $SU(2)_L$: **Levi-Civita tensor** ϵ_{ab}

- $SU(3)_C$: **color tensor** T_{ijklmn}

$$\mathcal{O} \sim T_{ijklmn}(q^i q^j)(q^k q^l)(q^m q^n) j_{\text{lep.}}$$

dim-12 operators with a scalar lepton current

$$\mathcal{O}^{S,(A)}_{u^3d^3L^21} = (u^{iT}_R C d^j_R)(u^{kT}_R C d^l_R)(u^{mT}_R C d^n_R)(L^T_a C L'_b)\epsilon_{ab} T^{SSS}_{\{ij\}\{kl\}\{mn\}} \,,$$

$$\mathcal{O}^{S,(A)}_{u^3d^3L^22} = (u^{iT}_R C d^j_R)(u^{kT}_R C d^l_R)(u^{mT}_R C d^n_R)(L^T_a C L'_b)\epsilon_{ab} T^{SAA}_{\{ij\}[kl][mn]} \,,$$

$$\mathcal{O}^{S,(S)}_{u^2d^2Q^2L^21} = (u^{iT}_R C d^j_R)(u^{kT}_R C d^l_R)(Q^{mT}_a C Q^n_b)(L^T_c C L'_d)\epsilon_{ac}\epsilon_{bd} T^{SSS}_{\{ij\}\{kl\}\{mn\}} \,,$$

$$\mathcal{O}^{S,(A)}_{u^2d^2Q^2L^22} = (u^{iT}_R C d^j_R)(u^{kT}_R C d^l_R)(Q^{mT}_a C Q^n_b)(L^T_c C L'_d)\epsilon_{ab}\epsilon_{cd} T^{SAA}_{\{ij\}[kl][mn]} \,,$$

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$$\mathcal{O}^{S,(A)}_{udQ^4L^21} = (u^{iT}_R C d^j_R)(Q^{kT}_a C Q^l_b)(Q^{mT}_c C Q^n_d)(L^T_e C L'_f)\epsilon_{ab}\epsilon_{cd}\epsilon_{ef} T^{SAA}_{\{ij\}[kl][mn]} \,,$$

$$\mathcal{O}^{S,(S)}_{udQ^4L^22} = (u^{iT}_R C d^j_R)(Q^{kT}_a C Q^l_b)(Q^{mT}_c C Q^n_d)(L^T_e C L'_f)\epsilon_{ab}\epsilon_{ce}\epsilon_{df} T^{SAA}_{\{mn\}[kl][ij]} \,,$$

$$\mathcal{O}^{S,(S)}_{Q^6L^2} = (Q^{iT}_a C Q^j_b)(Q^{kT}_c C Q^l_d)(Q^{mT}_e C Q^n_f)(L^T_g C L'_h)\epsilon_{ab}\epsilon_{cd}\epsilon_{eg}\epsilon_{fh} T^{SAA}_{\{mn\}[kl][ij]} \,,$$

$$\mathcal{O}^{S,(S)}_{u^4d^2e^21} = (u^{iT}_R C u^j_R)(u^{kT}_R C d^l_R)(u^{mT}_R C d^n_R)(e^T_R C e'_R) T^{SSS}_{\{ij\}\{kl\}\{mn\}} \,,$$

$$\mathcal{O}^{S,(S)}_{u^4d^2e^22} = (u^{iT}_R C u^j_R)(u^{kT}_R C d^l_R)(u^{mT}_R C d^n_R)(e^T_R C e'_R) T^{SAA}_{\{ij\}[kl][mn]} \,,$$

$$\mathcal{O}^{S,(S)}_{u^3dQ^2e^2} = (u^{iT}_R C u^j_R)(u^{kT}_R C d^l_R)(Q^{mT}_a C Q^n_b)(e^T_R C e'_R)\epsilon_{ab} T^{SAA}_{\{ij\}[kl][mn]} \,,$$

$$\mathcal{O}^{S,(S)}_{u^2Q^4e^2} = (u^{iT}_R C u^j_R)(Q^{kT}_a C Q^l_b)(Q^{mT}_c C Q^n_d)(e^T_R C e'_R)\epsilon_{ab}\epsilon_{cd} T^{SAA}_{\{ij\}[kl][mn]} \,,$$

12 (S-S-S-S)+7 (S-S-V-V)+10 (S-S-T-T)=29

Girmohanta and Shrock, 2020: **28**

8 redundant ones

9 missed ones

They are the starting point for the study of relevant signals at colliders:

LHC: $pp \to \ell^+\ell^+ + 4\text{jets}$

LHeC: $e^-p \to \ell^+ + 5\text{jets}$

Tree-level matching between dim-12 SMEFT and LEFT operators

SMEFT operators	$pp \to \ell\ell'$	$pn \to \ell\bar{\nu}'$	$nn \to \bar{\nu}\bar{\nu}'$
$\mathcal{O}^{S,(A)}_{u^3d^3L^21}$	-	$C^{(pn)S}_{1RRR,a} = -2C^{S,(A)}_{u^3d^3L^21}$	-
$\mathcal{O}^{S,(A)}_{u^3d^3L^22}$	-	$C^{(pn),}_{1RRR,b} = -2C^{S,(A)}_{u^3d^3L^22}$	-
$\mathcal{O}^{S,(S)}_{u^2d^2Q^2L^21}$	$C^{(pp)S,-}_{3RRL,a} = C^{S,(S)}_{u^2d^2Q^2L^21}$	$C^{(pn)S}_{3RRL,a} = -2C^{S,(S)}_{u^2d^2Q^2L^21}$	$C^{(nn)S}_{3RRL,a} = C^{S,(S)}_{u^2d^2Q^2L^21}$
$\mathcal{O}^{S,(A)}_{u^2d^2Q^2L^22}$	-	$C^{(pn)S}_{3RRL,b} = -4C^{S,(A)}_{u^2d^2Q^2L^22}$	-
$\mathcal{O}^{S,(S)}_{u^2d^2Q^2L^23}$	$C^{(pp)S,-}_{3RRL,b} = C^{S,(S)}_{u^2d^2Q^2L^23}$	$C^{(pn)S}_{3RRL,c} = -2C^{S,(S)}_{u^2d^2Q^2L^23}$	$C^{(nn)S}_{3RRL,b} = C^{S,(S)}_{u^2d^2Q^2L^23}$
$\mathcal{O}^{S,(A)}_{udQ^4L^21}$	-	$C^{(pn)S}_{3LLR,c} = -8C^{S,(A)}_{udQ^4L^21}$	-
$\mathcal{O}^{S,(S)}_{udQ^4L^22}$	$C^{(pp)S,-}_{2LLR,b} = 2C^{S,(S)}_{udQ^4L^22}$	$C^{(pn)S}_{3LLR,b} = -4C^{S,(S)}_{udQ^4L^22}$	$C^{(nn)S}_{2LLR,b} = 2C^{S,(S)}_{udQ^4L^22}$
$\mathcal{O}^{S,(S)}_{Q^6L^2}$	$C^{(pp)S,-}_{1LLL,b} = 4C^{S,(S)}_{Q^6L^2}$	$C^{(pn)S}_{1LLL,b} = -8C^{S,(S)}_{Q^6L^2}$	$C^{(nn)S}_{1LLL,b} = 4C^{S,(S)}_{Q^6L^2}$
$\mathcal{O}^{S,(S)}_{u^4d^2e^21}$	$C^{(pp)S,+}_{1RRR,a} = C^{S,(S)}_{u^4d^2e^21}$	-	-
$\mathcal{O}^{S,(S)}_{u^4d^2e^22}$	$C^{(pp)S,+}_{1RRR,b} = C^{S,(S)}_{u^4d^2e^22}$	-	-
$\mathcal{O}^{S,(S)}_{u^3dQ^2e^2}$	$C^{(pp)S,+}_{2RRL,b} = 2C^{S,(S)}_{u^3dQ^2e^2}$	-	-
$\mathcal{O}^{S,(S)}_{u^2Q^4e^2}$	$C^{(pp)S,+}_{3LLR,b} = 4C^{S,(S)}_{u^2Q^4e^2}$	-	-

SMEFT simplifies life hugely.

Unmatched LEFT operators can be generated by dim-14, dim-16 SMEFT ones.

Only a few can yield both three channels: $\mathcal{O}^{S,(S)}_{Q^6L^2}$, $\mathcal{O}^{S,(S)}_{u^2d^2Q^2L^21,3}$, $\mathcal{O}^{S,(S)}_{udQ^4L^22}$

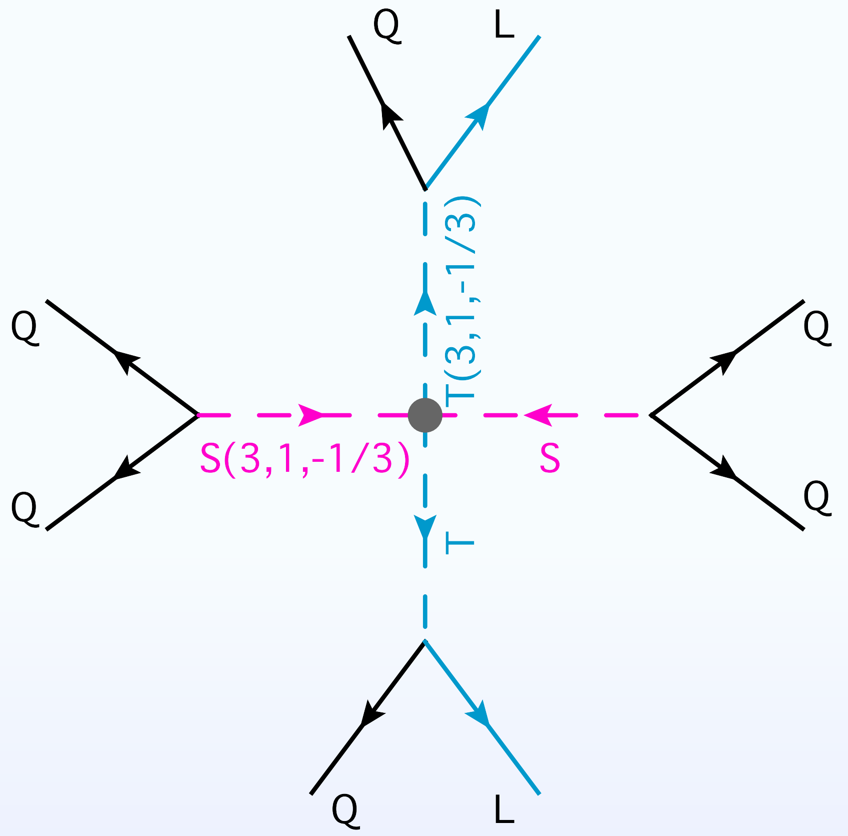
A specific model to realize one operator: $\mathcal{O}^{S,(S)}_{Q^6L^2}$

SM+ $S(3,1,-1/3)$

$+\,T(3,1,-1/3)$

$\mathbb{Z}_2 : T, L, e$ odd

$\Delta B = 1 :\ QQQL$ ✗



$\Delta B = 2$

$$\mathcal{O}^{S,(S)}_{Q^6L^2} = (Q_a^{i\mathrm{T}} C Q_b^j)(Q_c^{k\mathrm{T}} C Q_d^l)(Q_e^{m\mathrm{T}} C Q_f^n)(L_g^{\mathrm{T}} C L'_h)\epsilon_{ab}\epsilon_{cd}\epsilon_{eg}\epsilon_{fh} T^{SAA}_{\{mn\}[kl][ij]}$$ ✔

More models can be found:

Arnellos, Marciano 1982

Arnold, Fornal, and Wise, 2012;

Bramante, Kumar, Learned, 2014

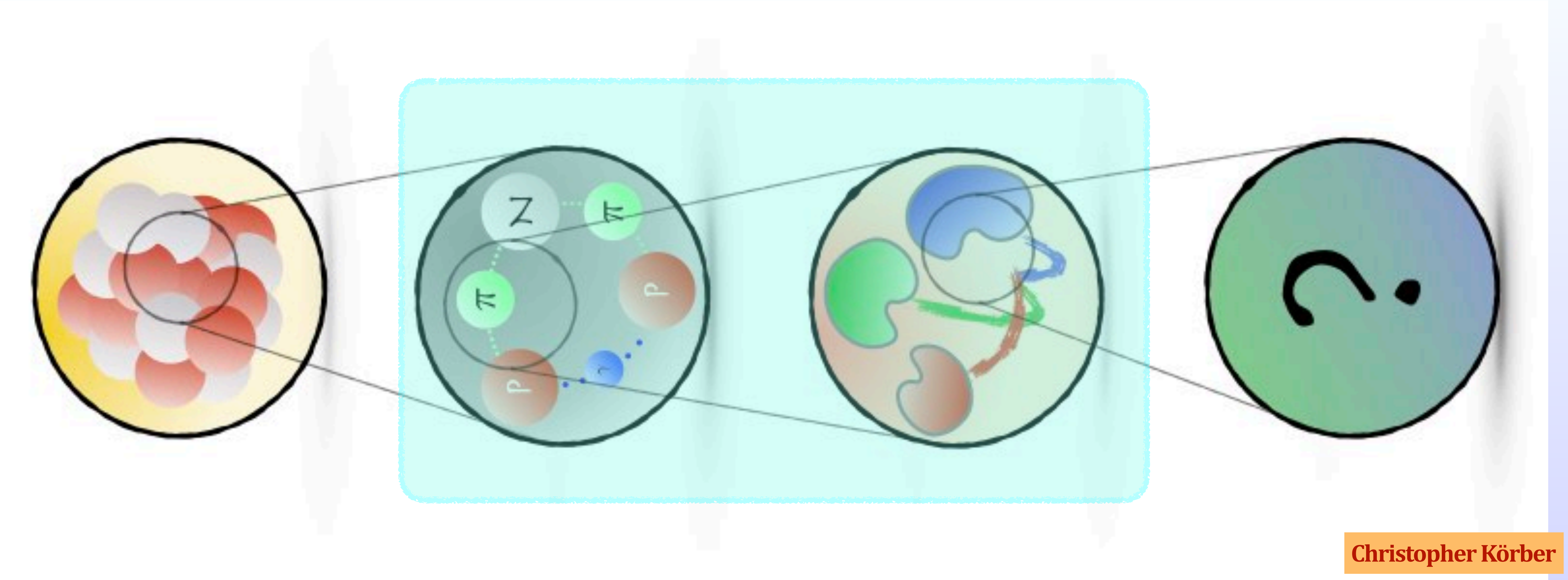
Gardner and Yan , 2019

Helset, Murgui and Wise, 2021

Girmohanta, Shrock 2019, 2020

Chiral symmetry: BChPT

Nucleon level operators ← **Quark level operators**



Christopher Körber

BχPT

- **Chiral symmetry $SU(3)_L \otimes SU(3)_R$ of three-flavor $q = (u, d, s)^{\mathrm{T}}$ QCD Lagrangian:**

$$\mathscr{L} = \mathscr{L}_{\mathrm{QCD}}^{m=0} + \overline{q_L} l_\mu \gamma^\mu q_L + \overline{q_R} r_\mu \gamma^\mu q_R - [\overline{q_R}(s - ip) q_L - \overline{q_R}\left(t_l^{\mu\nu}\sigma_{\mu\nu}\right) q_L + \mathrm{h.c.}]$$

- **Building blocks: Nucleons, pions, external sources**

$$u = \exp\left(\frac{i\Pi}{2F_0}\right), \quad \Pi = \pi^a \tau^a = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} \qquad \Psi = (p, n)^{\mathrm{T}}$$

$$u_\mu = i(u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger), \quad u_\mu^\dagger = u_\mu$$

- **Power counting: soft momentum:** $u = \mathcal{O}\left(p^0\right), \quad u_\mu = \mathcal{O}\left(p^1\right), \quad \Psi = \mathcal{O}\left(p^0\right)$

- **LO Lagrangian:** $\mathscr{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i\gamma_\mu D^\mu - m_N + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\right)\Psi$

$$D_\mu \Psi = (\partial_\mu + \Gamma_\mu)\Psi, \quad \Gamma_\mu = \frac{1}{2}(u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger)$$

BχPT realization of dim-12 LEFT operators

Chiral matching procedures

- Chiral $SU(2)_L \otimes SU(2)_R$ irrep decomposition:

$$P = \theta^{uvwxyz} \left(q^{i\,\mathrm{T}}_{\chi_1,u} C\Gamma_1 q^j_{\chi_2,v} \right) \left(q^{k\,\mathrm{T}}_{\chi_3,w} C\Gamma_2 q^l_{\chi_4,x} \right) \left(q^{m\,\mathrm{T}}_{\chi_5,y} C\Gamma_3 q^n_{\chi_6,z} \right) T^{\mathrm{color}}_{ijklmn}$$

- **Spurion fields technique**: treat θ as a field transforming under $SU(2)_L \otimes SU(2)_R$
  $\Rightarrow$ P **is chiral invariant**

- **Chiral counterparts of P**: construct chiral invariant operators out of θ, Ψ, u, ...

- **Low energy constant (LEC)**: associate an unknown LEC for each indep. operator

- **Determination of LEC**: fit to data, LQCD, chiral symmetry, NDA

Chiral basis	LEFT basis	Chiral irrep.	Chiral spurion
$P^{(pp)S,\pm}_{1,a}$	$\frac{1}{5}\left(5\mathcal{Q}^{(pp)S,\pm}_{1LLL,a} - 3\mathcal{Q}^{(pp)S,\pm}_{1LLL,b}\right)$	$(\mathbf{7}_L, \mathbf{1}_R)$	$\theta^{u_L v_L w_L x_L y_L z_L}_{(111122)}$
$P^{(pp)S,\pm}_{1,b}$	$\mathcal{Q}^{(pp)S,\pm}_{1LLL,b}$	$(\mathbf{3}_L, \mathbf{1}_R)\vert_a$	$\theta^{u_L v_L}_{(11)}$
$P^{(pp)S,\pm}_{2,a}$	$\mathcal{Q}^{(pp)S,\pm}_{2LLR,a}$	$(\mathbf{5}_L, \mathbf{3}_R)$	$\theta^{u_L v_L w_L x_L y_R z_R}_{(1112)(12)}$
$P^{(pp)S,\pm}_{2,b}$	$\mathcal{Q}^{(pp)S,\pm}_{2LLR,b}$	$(\mathbf{3}_L, \mathbf{1}_R)\vert_b$	$\theta^{u_L v_L}_{(11)}$
$P^{(pp)S,\pm}_{3,a}$	$\mathcal{Q}^{(pp)S,\pm}_{3LLR,a} - \mathcal{Q}^{(pp)S,\pm}_{3LLR,b}$	$(\mathbf{5}_L, \mathbf{3}_R)$	$\theta^{u_L v_L w_L x_L y_R z_R}_{(1122)(11)}$
$P^{(pp)S,\pm}_{3,b}$	$\mathcal{Q}^{(pp)S,\pm}_{3LLR,b}$	$(\mathbf{1}_L, \mathbf{3}_R)\vert_c$	$\theta^{u_R v_R}_{(11)}$
$P^{(pp)S,\pm}_{4}$	$\mathcal{Q}^{(pp)S,\pm}_{4LLR}$	$(\mathbf{5}_L, \mathbf{3}_R)$	$\theta^{u_L v_L w_L x_L y_R z_R}_{(1111)(22)}$
$P^{(pp)V}_{1,a}$	$\frac{1}{5}\left(5\mathcal{Q}^{(pp)V}_{1LL,a} - 6\mathcal{Q}^{(pp)V}_{1LL,b} - 3\mathcal{Q}^{(pp)V}_{1LL,c}\right)$	$(\mathbf{6}_L, \mathbf{2}_R)$	$\theta^{u_L v_L w_L x_L y_L z_R}_{(11122)1}$
$P^{(pp)V}_{1,b}$	$\frac{1}{3}\left(3\mathcal{Q}^{(pp)V}_{1LL,b} - \mathcal{Q}^{(pp)V}_{1LL,c}\right)$	$(\mathbf{4}_L, \mathbf{2}_R)\vert_a$	$\theta^{u_L v_L w_L x_R}_{(112)1}$
$P^{(pp)V}_{1,c}$	$\mathcal{Q}^{(pp)V}_{1LL,c}$	$(\mathbf{2}_L, \mathbf{2}_R)\vert_a$	$\theta^{u_L v_R}_{11}$
$P^{(pp)V}_{2,a}$	$\frac{1}{5}\left(5\mathcal{Q}^{(pp)V}_{2LL,a} - 3\mathcal{Q}^{(pp)V}_{2LL,b}\right)$	$(\mathbf{6}_L, \mathbf{2}_R)$	$\theta^{u_L v_L w_L x_L y_L z_R}_{(11112)2}$
$P^{(pp)V}_{2,b}$	$\mathcal{Q}^{(pp)V}_{2LL,b}$	$(\mathbf{4}_L, \mathbf{2}_R)\vert_a$	$\theta^{u_L v_L w_L x_R}_{(111)2}$

1. Many different chiral irreps.
2. Different irreps have different LECs
3. They do not mix under QCD renormalization.

Final matching result

Ope. type	Chi. irrep	Chi. order	Matching operator
Scalar current: $\mathcal{O}^S_{\text{quark}} \times j_S$	$(\mathbf{3}_L, \mathbf{1}_R)\|_i$	p^0	$O^S_{3\times1,i} = \theta^{u_L v_L}_{(\alpha\beta)} (u^\dagger)_{u_L a} (u^\dagger)_{v_L b} [\Psi^{\mathrm{T}}_a C (g_{3\times1,i} + \hat{g}_{3\times1,i}\gamma_5)\Psi_b]$
	$(\mathbf{5}_L, \mathbf{3}_R)$	p^0	$O^S_{5\times3} = \theta^{u_L v_L w_L x_L y_R z_R}_{(\alpha\beta\gamma\rho)(\sigma\tau)} (Ui\tau^2)_{y_R w_L} (Ui\tau^2)_{z_R x_L} (u^\dagger)_{u_L a} (u^\dagger)_{v_L b} [\Psi^{\mathrm{T}}_a C (g_{5\times3} + \hat{g}_{5\times3}\gamma_5)\Psi_b]$
	$(\mathbf{7}_L, \mathbf{1}_R)$	$p^2(\times)$	$O^S_{7\times1} = \theta^{u_L v_L w_L x_L y_L z_L}_{(\alpha\beta\gamma\rho\sigma\tau)} (u^\dagger u_\mu u i\tau^2)_{w_L x_L} (u^\dagger u^\mu u i\tau^2)_{y_L z_L} (u^\dagger)_{u_L a} (u^\dagger)_{v_L b} [\Psi^{\mathrm{T}}_a C (g_{7\times1} + \hat{g}_{7\times1}\gamma_5)\Psi_b]$
	$(\mathbf{1}_L, \mathbf{3}_R)\|_i$	p^0	$\tilde{O}^S_{1\times3,i} = \theta^{u_R v_R}_{(\alpha\beta)} u_{u_R a} u_{v_R b} [\Psi^{\mathrm{T}}_a C (g_{1\times3,i} + \hat{g}_{1\times3,i}\gamma_5)\Psi_b]$
	$(\mathbf{3}_L, \mathbf{5}_R)$	p^0	$\tilde{O}^S_{3\times5} = \theta^{u_R v_R w_R x_R y_L z_L}_{(\alpha\beta\gamma\rho)(\sigma\tau)} (Ui\tau^2)_{w_R y_L} (Ui\tau^2)_{x_R z_L} u_{u_R a} u_{v_R b} [\Psi^{\mathrm{T}}_a C (g_{3\times5} + \hat{g}_{3\times5}\gamma_5)\Psi_b]$
	$(\mathbf{1}_L, \mathbf{7}_R)$	$p^2(\times)$	$\tilde{O}^S_{1\times7} = \theta^{u_R v_R w_R x_R y_R z_R}_{(\alpha\beta\gamma\rho\sigma\tau)} (u u_\mu u^\dagger i\tau^2)_{w_R x_R} (u u^\mu u^\dagger i\tau^2)_{y_R z_R} u_{u_R a} u_{v_R b} [\Psi^{\mathrm{T}}_a C (g_{1\times7} + \hat{g}_{1\times7}\gamma_5)\Psi_b]$
Vector current: $\mathcal{O}^{V,\mu}_{\text{quark}} \times j_{V,\mu}$	$(\mathbf{2}_L, \mathbf{2}_R)\|_i$	p^0	$O^{V,\mu}_{2\times2,i} = \theta^{u_L v_R}_{\alpha\beta} (u^\dagger)_{u_L a} u_{v_R b} [\Psi^{\mathrm{T}}_a C \gamma^\mu (g_{2\times2,i} + \hat{g}_{2\times2,i}\gamma_5)\Psi_b]$
	$(\mathbf{4}_L, \mathbf{2}_R)\|_i$	p^0	$O^{V,\mu}_{4\times2,i} = g_{4\times2,i} \theta^{u_L v_L w_L x_R}_{(\alpha\beta\gamma)\rho} (Ui\tau^2)_{x_R w_L} (u^\dagger)_{u_L a} (u^\dagger)_{v_L b} [\Psi^{\mathrm{T}}_a C \gamma^\mu \gamma_5 \Psi_b]$
	$(\mathbf{4}_L, \mathbf{4}_R)$	p^0	$O^{V,\mu}_{4\times4} = \theta^{u_L v_L w_L x_R y_R z_R}_{(\alpha\beta\gamma)(\rho\sigma\tau)} (Ui\tau^2)_{y_R v_L} (Ui\tau^2)_{z_R w_L} (u^\dagger)_{u_L a} u_{x_R b} [\Psi^{\mathrm{T}}_a C \gamma^\mu (g_{4\times4} + \hat{g}_{4\times4}\gamma_5)\Psi_b]$
	$(\mathbf{6}_L, \mathbf{2}_R)$	$p^1(\times)$	$O^{V,\mu}_{6\times2} = \theta^{u_L v_L w_L x_L y_L z_R}_{(\alpha\beta\gamma\rho\sigma)\tau} (Ui\tau^2)_{z_R w_L} (u^\dagger u^\mu u i\tau^2)_{x_L y_L} (u^\dagger)_{u_L a} (u^\dagger)_{v_L b} [\Psi^{\mathrm{T}}_a C (g_{6\times2} + \hat{g}_{6\times2}\gamma_5)\Psi_b]$
	$(\mathbf{2}_L, \mathbf{4}_R)\|_i$	p^0	$\tilde{O}^{V,\mu}_{2\times4,i} = -g_{2\times4,i} \theta^{u_R v_R w_R x_L}_{(\alpha\beta\gamma)\rho} (Ui\tau^2)_{w_R x_L} u_{u_R a} u_{v_R b} (\Psi^{\mathrm{T}}_a C \gamma^\mu \gamma_5 \Psi_b)$
	$(\mathbf{2}_L, \mathbf{6}_R)$	$p^1(\times)$	$\tilde{O}^{V,\mu}_{2\times6} = -\theta^{u_R v_R w_R x_R y_R z_L}_{(\alpha\beta\gamma\rho\sigma)\tau} (Ui\tau^2)_{w_R z_L} (u u^\mu u^\dagger i\tau^2)_{x_R y_R} u_{u_R a} u_{v_R b} [\Psi^{\mathrm{T}}_a C (g_{2\times6} + \hat{g}_{2\times6}\gamma_5)\Psi_b]$
Tensor current: $\mathcal{O}^{T,\mu\nu}_{\text{quark}} \times j^{\mu\nu}_T$	$(\mathbf{1}_L, \mathbf{1}_R)\|_i$	p^0	$O^{T,\mu\nu}_{1\times1,i} = \frac{1}{2}\epsilon^{ab} [\Psi^{\mathrm{T}}_a C \sigma^{\mu\nu} (g_{1\times1,i} + \hat{g}_{1\times1,i}\gamma_5)\Psi_b]$
	$(\mathbf{3}_L, \mathbf{1}_R)$	$p^1(\times)$	$O^{T,\mu\nu}_{3\times1} = \theta^{u_L v_L}_{(\alpha\beta)} (u^\dagger u^\mu)_{u_L a} (u^\dagger)_{v_L b} [\Psi^{\mathrm{T}}_a C \gamma^\nu (g_{3\times1,T} + \hat{g}_{3\times1,T}\gamma_5)\Psi_b] - \mu \leftrightarrow \nu$
	$(\mathbf{3}_L, \mathbf{3}_R)\|_i$	p^0	$O^{T,\mu\nu}_{3\times3,i} = \theta^{u_L v_L w_R x_R}_{(\alpha\beta)(\gamma\rho)} (Ui\tau^2)_{x_R v_L} (u^\dagger)_{u_L a} u_{w_R b} [\Psi^{\mathrm{T}}_a C \sigma^{\mu\nu} (g_{3\times3,i} + \hat{g}_{3\times3,i}\gamma_5))\Psi_b]$
	$(\mathbf{5}_L, \mathbf{1}_R)\|_i$	$p^1(\times)$	$O^{T,\mu\nu}_{5\times1,i} = g_{5\times1,i} \theta^{u_L v_L w_L x_L}_{(\alpha\beta\gamma\rho)} (u^\dagger u^\mu u i\tau^2)_{w_L x_L} (u^\dagger)_{u_L a} (u^\dagger)_{v_L b} (\Psi^{\mathrm{T}}_a C \gamma^\nu \gamma_5 \Psi_b) - \mu \leftrightarrow \nu$
	$(\mathbf{1}_L, \mathbf{3}_R)$	$p^1(\times)$	$\tilde{O}^{T,\mu\nu}_{1\times3} = \theta^{u_R v_R}_{(\alpha\beta)} (u u^\mu)_{u_R a} u_{v_R b} [\Psi^{\mathrm{T}}_a C \gamma^\nu (g_{1\times3,T} + \hat{g}_{1\times3,T}\gamma_5)\Psi_b] - \mu \leftrightarrow \nu$
	$(\mathbf{1}_L, \mathbf{5}_R)\|_i$	$p^1(\times)$	$\tilde{O}^{T,\mu\nu}_{1\times5,i} = g_{1\times5,i} \theta^{u_R v_R w_R x_R}_{(\alpha\beta\gamma\rho)} (u u^\mu u^\dagger i\tau^2)_{w_R x_R} u_{u_R a} u_{v_R b} (\Psi^{\mathrm{T}}_a C \gamma^\nu \gamma_5 \Psi_b) - \mu \leftrightarrow \nu$

Expanding to the LO can lead to the nucleon -lepton interactions

$$
\begin{aligned}
pp \to \ell^+\ell'^+ :\ & \mathcal{O}^{(pp)S}_L = (p^{\mathrm{T}}Cp)(\ell^{\mathrm{T}}_L C \ell'_L)\,, && \mathcal{O}^{(pp)S}_{5L} = (p^{\mathrm{T}}C\gamma_5 p)(\ell^{\mathrm{T}}_L C \ell'_L)\,,\\
& \mathcal{O}^{(pp)S}_R = (p^{\mathrm{T}}Cp)(\ell^{\mathrm{T}}_R C \ell'_R)\,, && \mathcal{O}^{(pp)S}_{5R} = (p^{\mathrm{T}}C\gamma_5 p)(\ell^{\mathrm{T}}_R C \ell'_R)\,,\\
& \mathcal{O}^{(pp)V} = (p^{\mathrm{T}}C\gamma_\mu\gamma_5 p)(\ell^{\mathrm{T}}_R C \gamma^\mu \ell'_L)\,,\\
pn \to \ell^+\bar{\nu}' :\ & \mathcal{O}^{(pn)S}_L = (p^{\mathrm{T}}Cn)(\ell^{\mathrm{T}}_L C \nu'_L)\,, && \mathcal{O}^{(pn)S}_{5L} = (p^{\mathrm{T}}C\gamma_5 n)(\ell^{\mathrm{T}}_L C \nu'_L)\,,\\
& \mathcal{O}^{(pn)V}_L = (p^{\mathrm{T}}C\gamma_\mu n)(\ell^{\mathrm{T}}_R C \gamma^\mu \nu'_L)\,, && \mathcal{O}^{(pn)V}_{5L} = (p^{\mathrm{T}}C\gamma_\mu\gamma_5 n)(\ell^{\mathrm{T}}_R C \gamma^\mu \nu'_L)\,,\\
& \mathcal{O}^{(pn)T} = (p^{\mathrm{T}}C\sigma_{\mu\nu} n)(\ell^{\mathrm{T}}_L C \sigma_{\mu\nu} \nu'_L)\,,\\
nn \to \bar{\nu}\bar{\nu}' :\ & \mathcal{O}^{(nn)S}_L = (n^{\mathrm{T}}Cn)(\nu^{\mathrm{T}}_L C \nu'_L)\,, && \mathcal{O}^{(nn)S}_{5L} = (n^{\mathrm{T}}C\gamma_5 n)(\nu^{\mathrm{T}}_L C \nu'_L)\,,
\end{aligned}
$$

- **Function of LECs and SMEFT WCs**
- **LECs:** $g_i \sim \Lambda^6_{\text{QCD}}$
- $\hat{g}_{3\times1,a} \sim 4 \times 10^{-4}\ \text{GeV}^6$

the matrix element for $n - \bar{n}$ oscillation

Rinaldi, Syritsyn, Wagman, Buchoff, Schroeder, Wasem, 2018

Estimation of decay rate

$$\Gamma_{NN'\to l_\alpha l_\beta} = \frac{1}{(2\pi)^3\sqrt{\rho_N\rho_{N'}}}\int d^3k_1 d^3k_2\, \rho_N(k_1)\rho_{N'}(k_2) v_{\rm rel.}(1-\mathbf{v_1}\cdot\mathbf{v_2})\sigma(NN'\to l_\alpha l_\beta)$$

Goity, Sher, 1995

nucleon density distribution

$$\sigma(NN'\to l_\alpha l_\beta) = \frac{1}{S}\frac{1}{4E_1E_2v_{\rm rel.}}\int d\Pi_2 \overline{\left|\mathcal{M}_{NN'\to l_\alpha l_\beta}\right|^2}$$

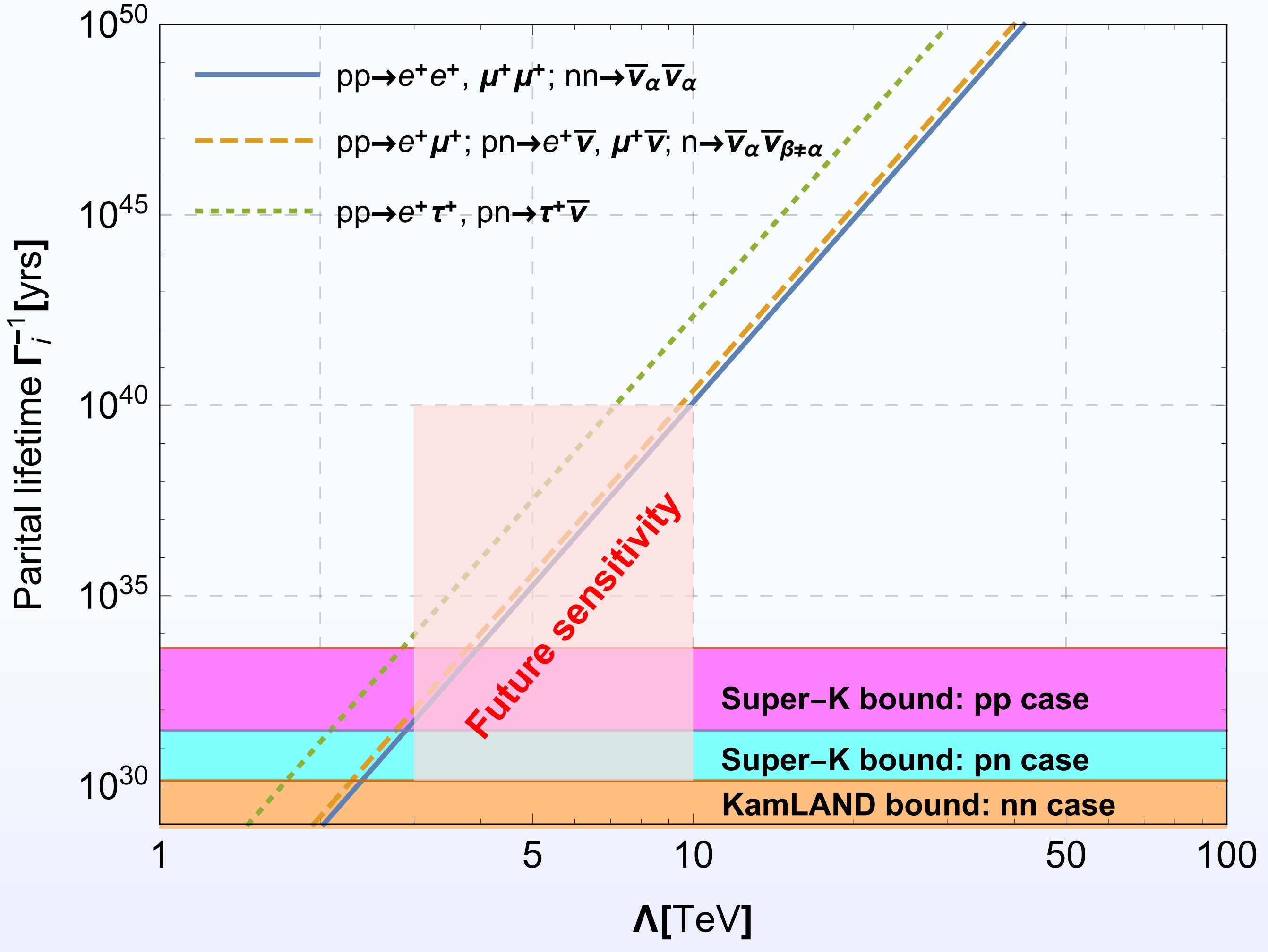
average nucleon density: $0.25\ \mathrm{fm}^{-3}$

Neglect the nucleon Fermi motion and other nuclear effects

$$\Gamma_{NN'\to l_\alpha l_\beta} = \frac{1}{S}\frac{\rho_N}{4m_N^2}\overline{\left|\mathcal{M}_{NN'\to l_\alpha l_\beta}\right|^2}\Pi_2$$

Implication for the NP scale

SMEFT WCs	$pp \to e^+e^+,\ e^+\mu^+,\ \mu^+\mu^+$ $\Lambda_{\rm NP} \equiv \|C_i\|^{-\frac{1}{8}}$ [TeV]	$pn \to e^+\nu,\ \mu^+\bar{\nu},\ \tau^+\bar{\nu}$ $\Lambda_{\rm NP} \equiv \|C_i\|^{-\frac{1}{8}}$ [TeV]	$nn \to \bar{\nu}_\alpha\bar{\nu}_\alpha,\ \bar{\nu}_\alpha\bar{\nu}_{\beta\neq\alpha}$ $\Lambda_{\rm NP} \equiv \|C_i\|^{-\frac{1}{8}}$ [TeV]
$C^{S,(A)}_{u^3d^3L^21}$	-	$2.04,\ 2.02,\ 1.34 \times \left[\frac{\hat{g}_{1\times3,a}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	-
$C^{S,(A)}_{u^3d^3L^22}$	-	$1.89,\ 1.87,\ 1.25 \times \left[\frac{\hat{g}_{1\times3,a}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	-
$C^{S,(S)}_{u^2d^2Q^2L^21,3}$	$2.35,\ 2.26,\ 2.36 \times \left[\frac{\hat{g}_{3\times1,c}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	$1.89,\ 1.87,\ 1.25 \times \left[\frac{\hat{g}_{1\times3,c}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	$1.43,\ 1.37 \times \left[\frac{\hat{g}_{3\times1,c}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$
$C^{S,(A)}_{u^2d^2Q^2L^22}$	-	$2.07,\ 2.04,\ 1.36 \times \left[\frac{\hat{g}_{1\times3,b}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	-
$C^{S,(A)}_{udQ^4L^21}$	-	$2.25,\ 2.23,\ 1.48 \times \left[\frac{\hat{g}_{1\times3,c}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	-
$C^{S,(S)}_{udQ^4L^22}$	$2.57,\ 2.46,\ 2.57 \times \left[\frac{\hat{g}_{3\times1,b}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	$2.07,\ 2.04,\ 1.36 \times \left[\frac{\hat{g}_{1\times3,b}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	$1.56,\ 1.49 \times \left[\frac{\hat{g}_{3\times1,b}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$
$C^{S,(S)}_{Q^6L^2}$	$2.80,\ 2.69,\ 2.81 \times \left[\frac{\hat{g}_{3\times1,a}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	$2.25,\ 2.23,\ 1.48 \times \left[\frac{\hat{g}_{1\times3,a}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	$1.70,\ 1.63 \times \left[\frac{\hat{g}_{3\times1,a}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$
$C^{S,(S)}_{u^4d^2e^21}$	$2.21,\ 2.12,\ 2.21 \times \left[\frac{\hat{g}_{1\times3,a}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	-	-
$C^{S,(S)}_{u^4d^2e^22}$	$2.35,\ 2.26,\ 2.36 \times \left[\frac{\hat{g}_{1\times3,a}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	-	-
$C^{S,(S)}_{u^3dQ^2e^2}$	$2.57,\ 2.46,\ 2.57 \times \left[\frac{\hat{g}_{1\times3,b}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	-	-
$C^{S,(S)}_{u^2Q^4e^2}$	$2.80,\ 2.69,\ 2.81 \times \left[\frac{\hat{g}_{1\times3,c}}{\Lambda^6_{\rm QCD}}\right]^{\frac{1}{8}}$	-	-



the matrix element for $n - \bar{n}$ oscillation

Rinaldi, Syritsyn, Wagman, Buchoff, Schroeder, Wasem, 2018

- **LECs:** $g_i \sim \Lambda^6_{\rm QCD}$
- $\hat{g}_{3\times1,a} \sim 4\times10^{-4}\ {\rm GeV}^6$

The NP scale is bound to be $\Lambda_{\rm NP} > 1 - 3\ {\rm TeV}$, which opens up the possibility to search for the signals at high energy colliders.

Summary

- The $|\Delta B = \Delta L| = 2$ dinucleon to dilepton decays have been studied in the EFT framework;
- An operator basis in the LEFT is constructed;
- An operator basis in the SMEFT is constructed.

Thanks for your attention!