



# Revisiting Metastable Cosmic String Breaking

Based on [2312.15662](#)

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Collaboration with

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# Outline

## 1. Metastable Cosmic String

Motivation.

Gravitational wave and NANOGrav result.

## 2. Breaking of Metastable String

Conventional approach via monopole pair production

## 3. Revisiting String Breaking

Explicit construction field configuration of breaking process.

Bounce solution. Numerical results.

## 4. Summary

# Topological Object in BSM

Extension of Standard Model can have larger symmetry.

$$G_{\text{BSM}} \rightarrow G_{\text{SM}} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$$

Example:

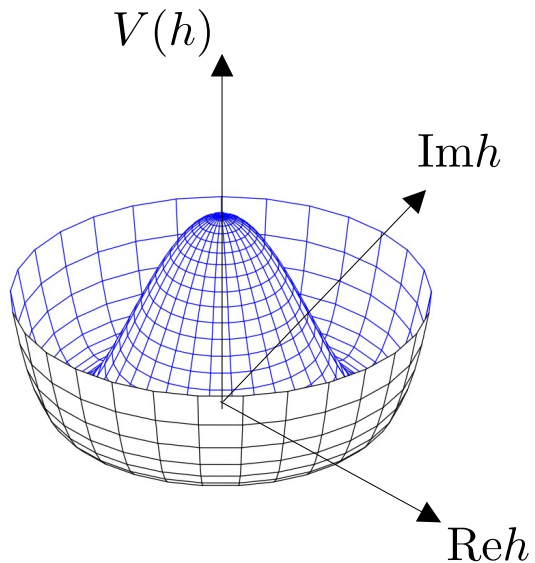
Grand Unified Theory,  $\text{SU}(5)$ ,  $\text{SO}(10)$ ....: Monopole, cosmic string...

$\text{U}(1)_{\text{PQ}}$  extension for Axion: Domain wall, cosmic string.

# Cosmic String

$U(1) \rightarrow \text{nothing}$

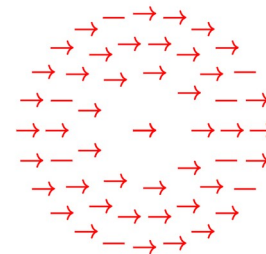
$$\langle h \rangle \neq 0$$



SSB



Phase of  $h$

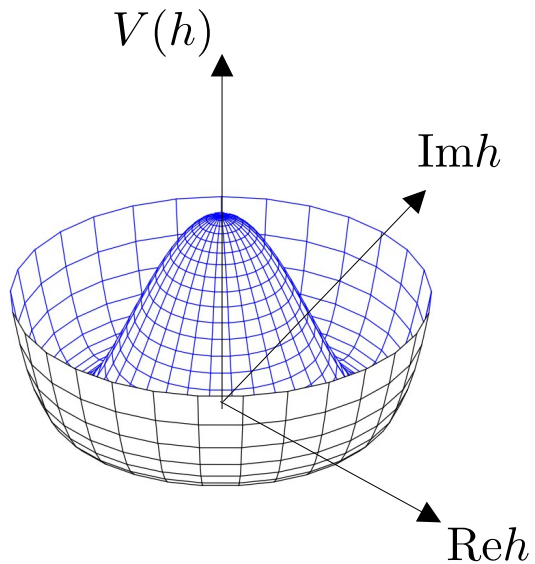


Winding number 0

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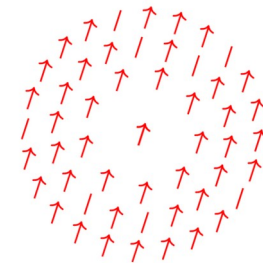
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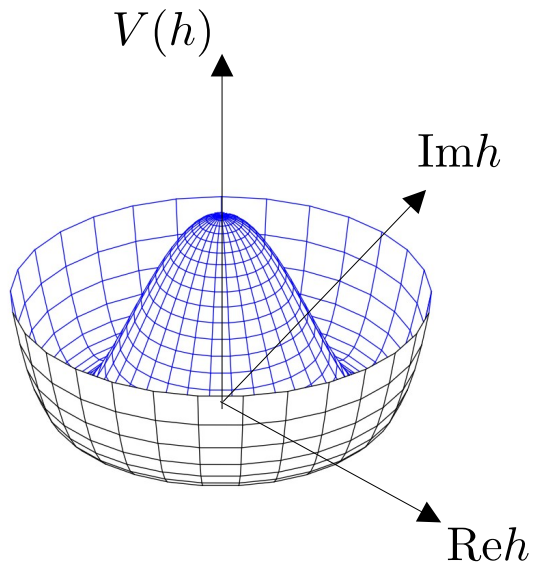


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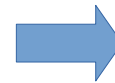
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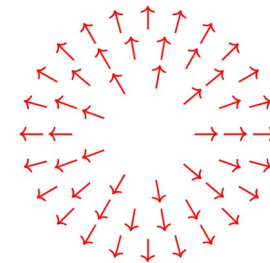
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SSB



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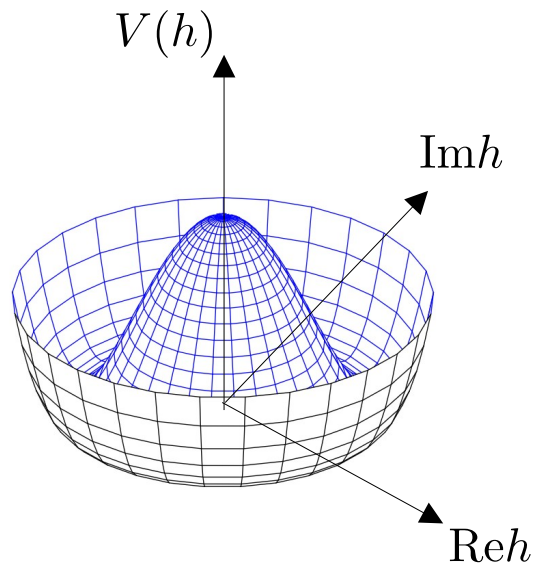


Winding number 1

# Cosmic String

$U(1) \rightarrow \text{nothing}$

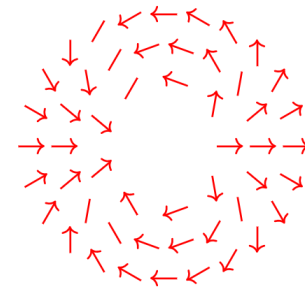
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SSB



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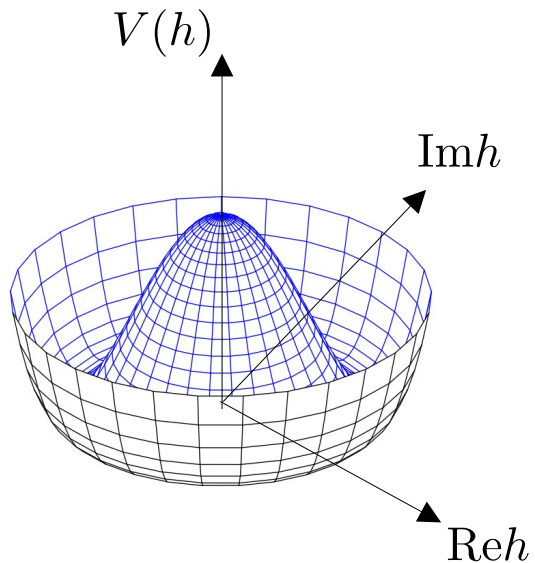


Winding number 2

# Cosmic String

$U(1) \rightarrow \text{nothing}$

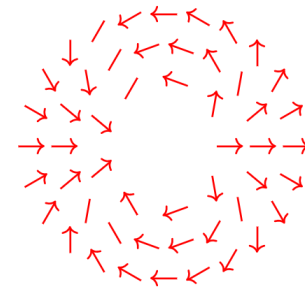
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SSB



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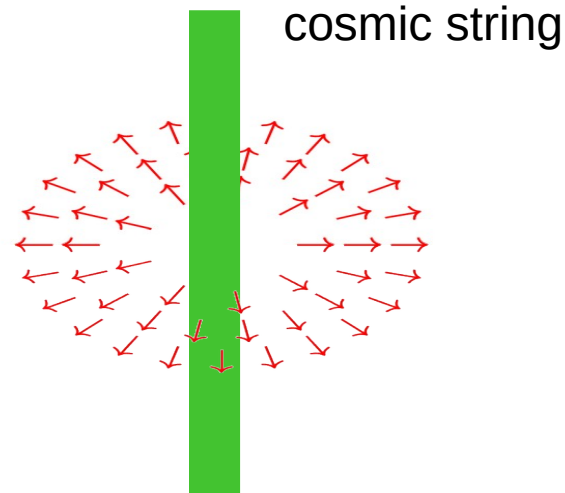
Winding number 2

$$\pi_1(U(1)) = \mathbb{Z}$$



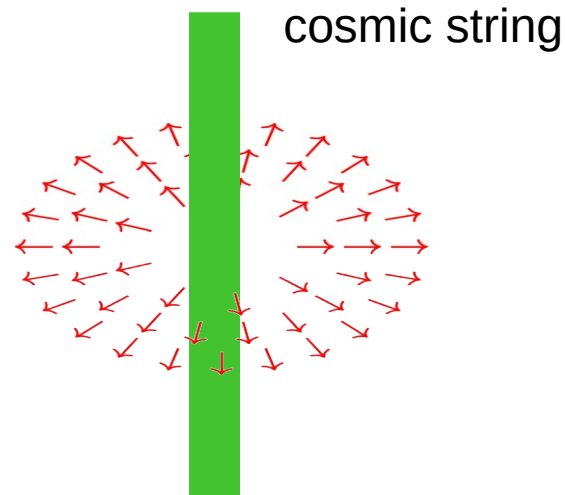
# Cosmic String

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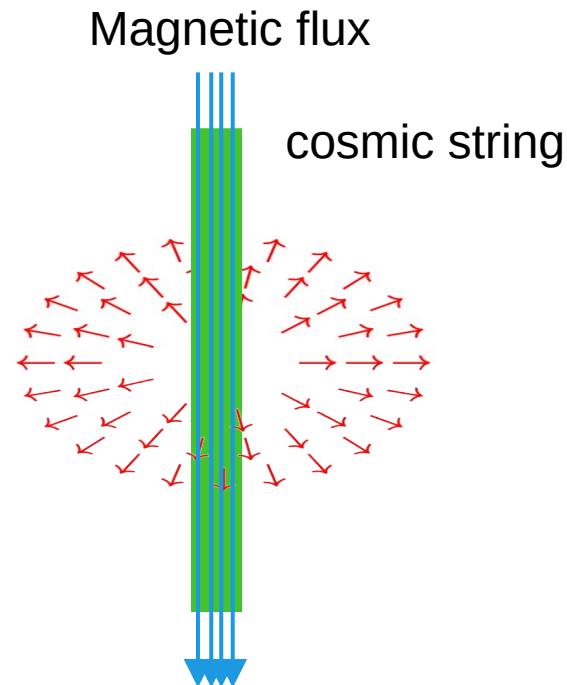
# Cosmic String

$U(1)_{\text{gauge}} \rightarrow \text{nothing}$



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$U(1)_{\text{gauge}} \rightarrow \text{nothing}$



# Multi-step SSB

$SU(2) \rightarrow \text{nothing}$

$$\pi_2(SU(2)) = \pi_1(SU(2)) = 0$$

No topological objects.

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Monopole

Cosmic string

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Monopole

  
Cosmic string



Metastable string with  $V \gg v$

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No topological objects.

$$\begin{array}{c} \text{SU}(2) \xrightarrow[V]{} \text{U}(1) \xrightarrow[v]{} \text{nothing} \\ \underbrace{\hspace{1.5cm}} \qquad \underbrace{\hspace{1.5cm}} \\ \text{Monopole} \qquad \text{Cosmic string} \end{array}$$

Example:

$$SO(10)_{\text{GUT}} \rightarrow G_{\text{SM}} \times U(1)_{B-L} \rightarrow G_{\text{SM}}$$



Metastable string with  $V \gg v$



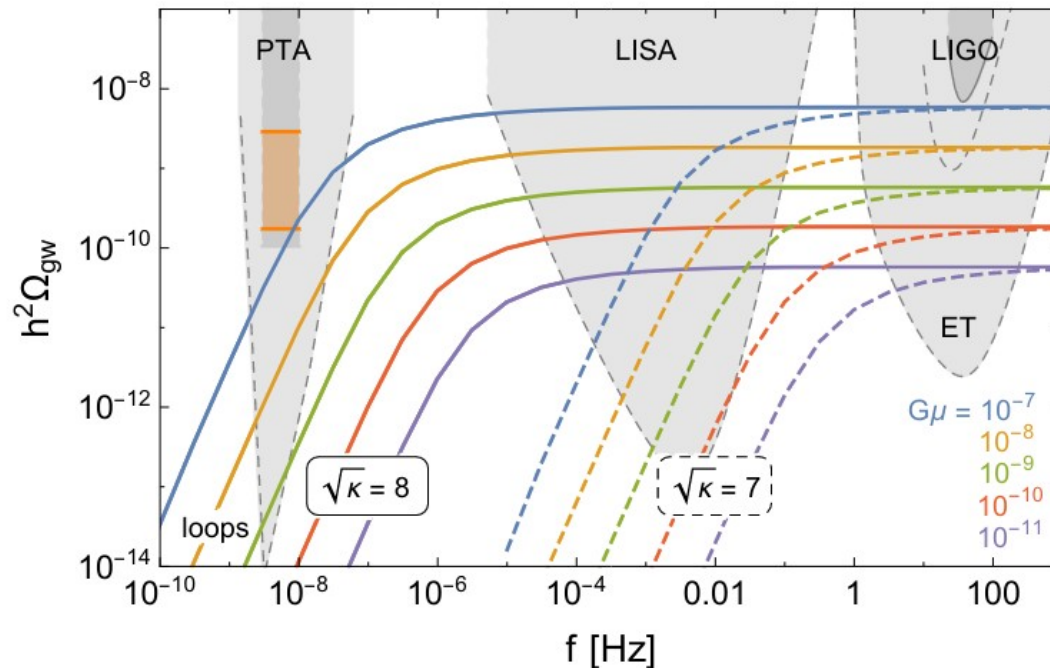
# Gravitational Wave

String breaking rate per unit length

$$\Gamma = \frac{\mu}{2\pi} e^{-\pi\kappa}$$

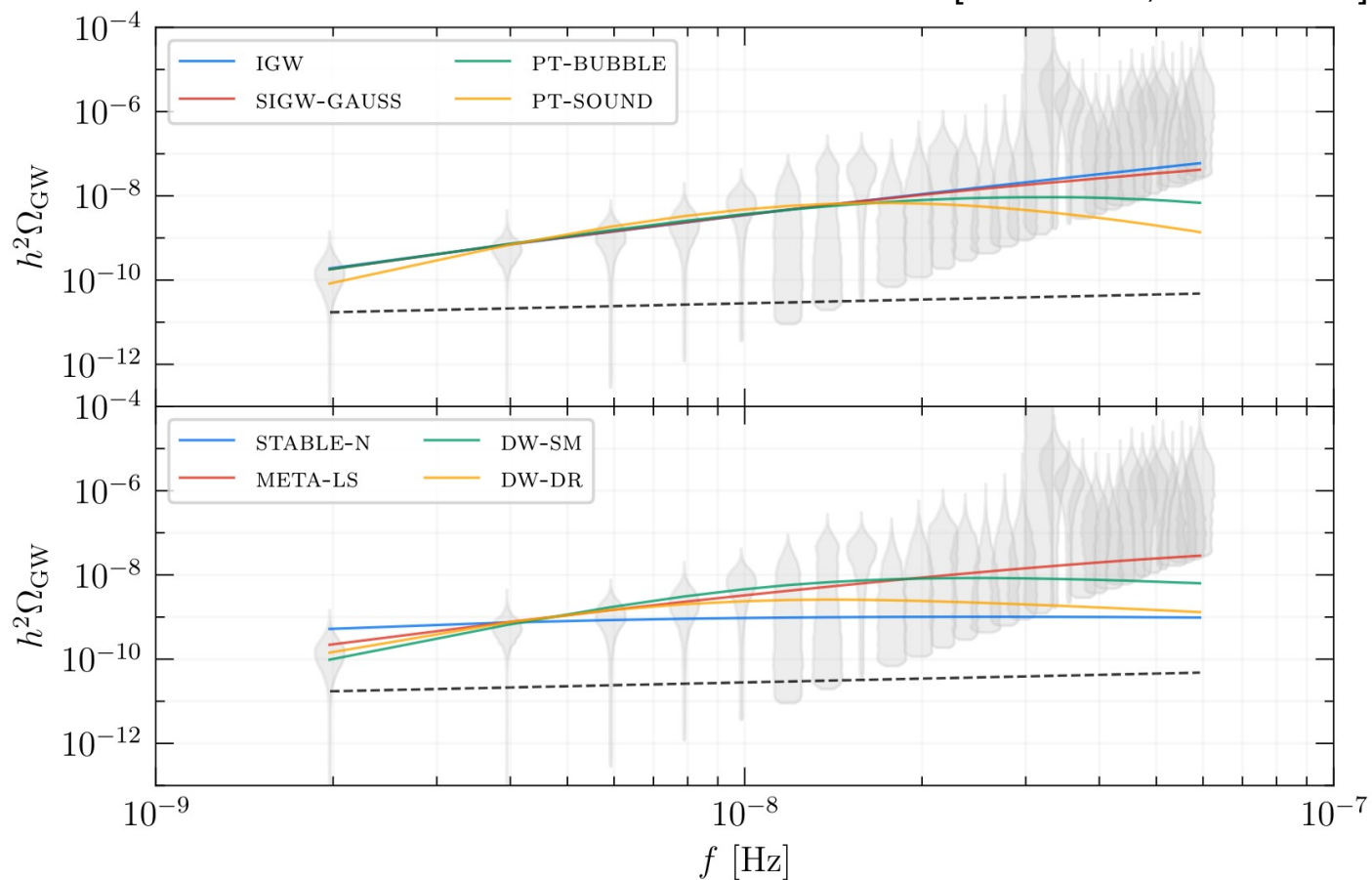
$\mu$ : String tension

[Buchmüller, Domcke, Schmitz 2107.04578]



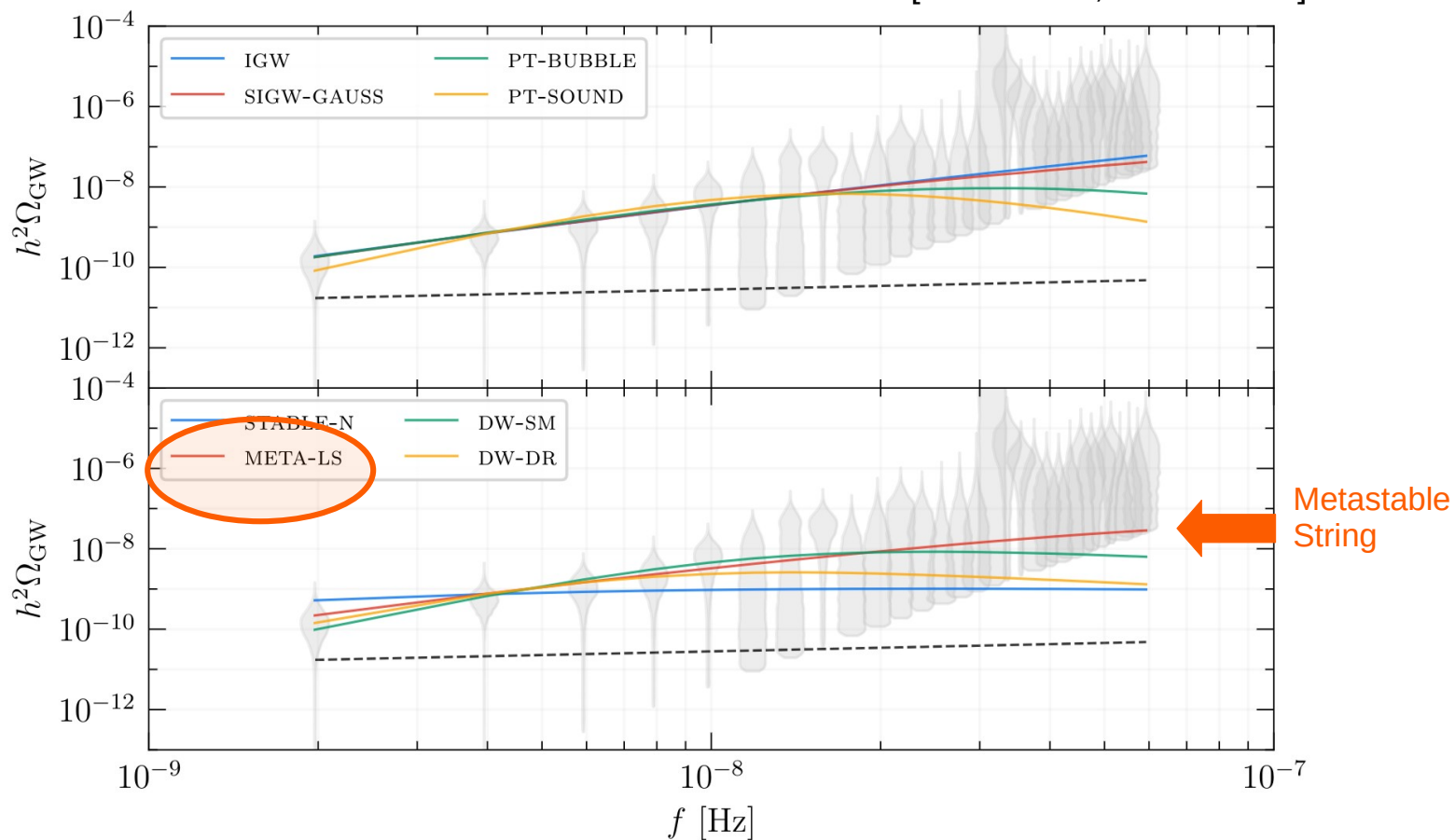
# Hint of Metastable String?

[NANOGrav, 2306.16219]



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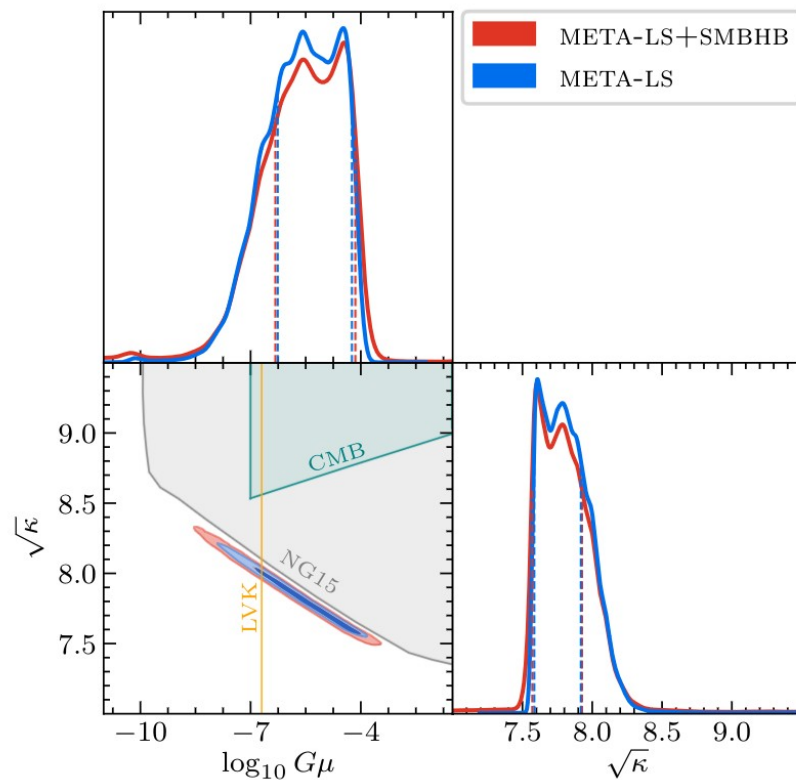
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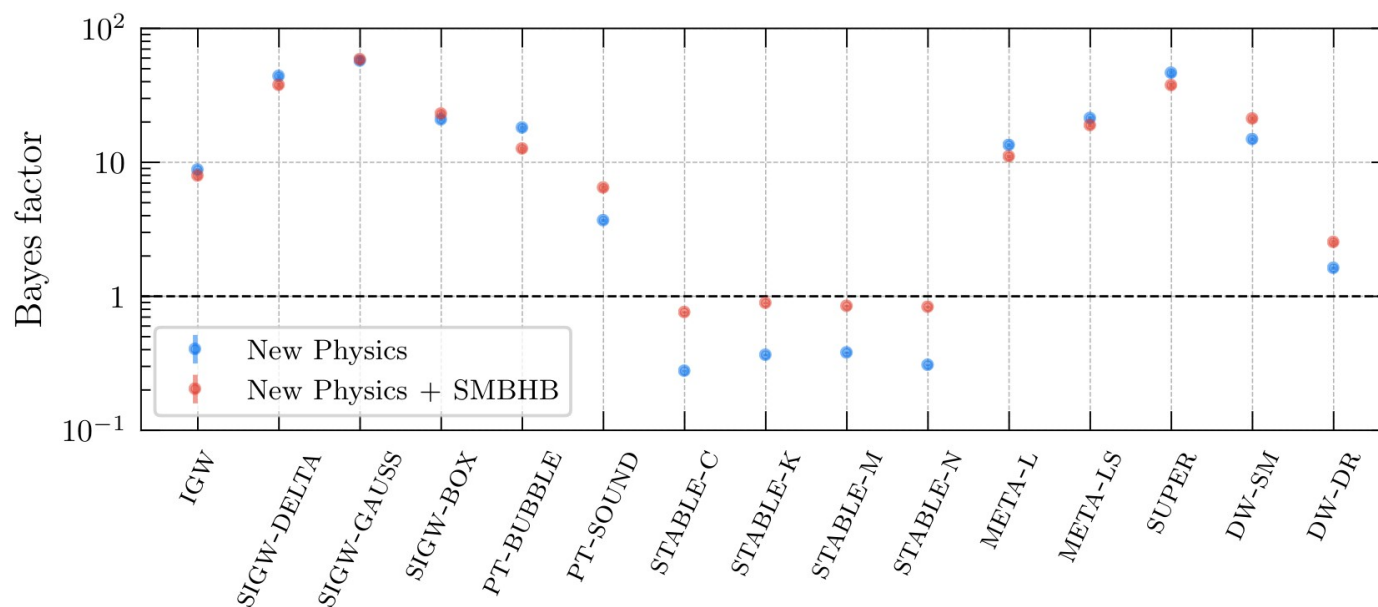
$$\Gamma = \frac{\mu}{2\pi} e^{-\pi\kappa} \quad \mu: \text{String tension}$$

[NANOGrav, 2306.16219]



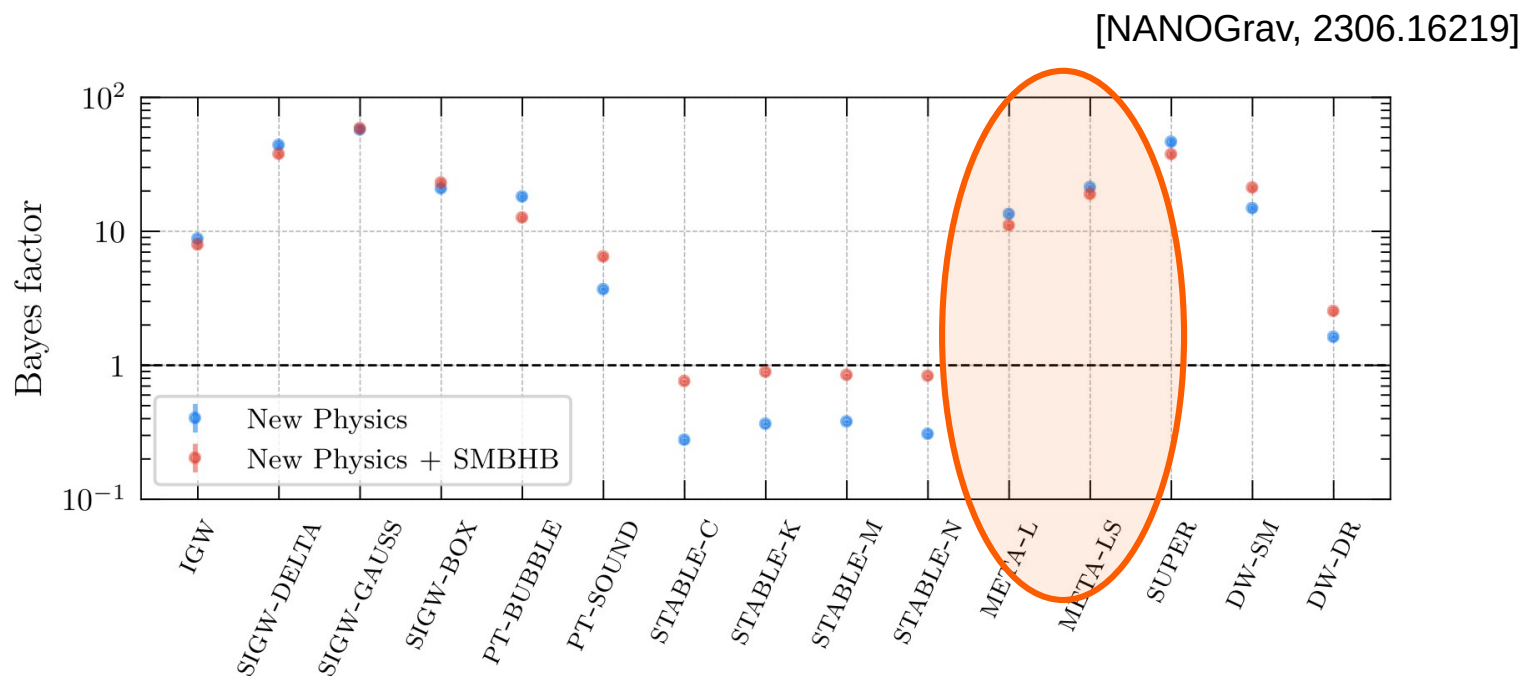
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Bayes factor  $\sim$  Fitness of model for data.

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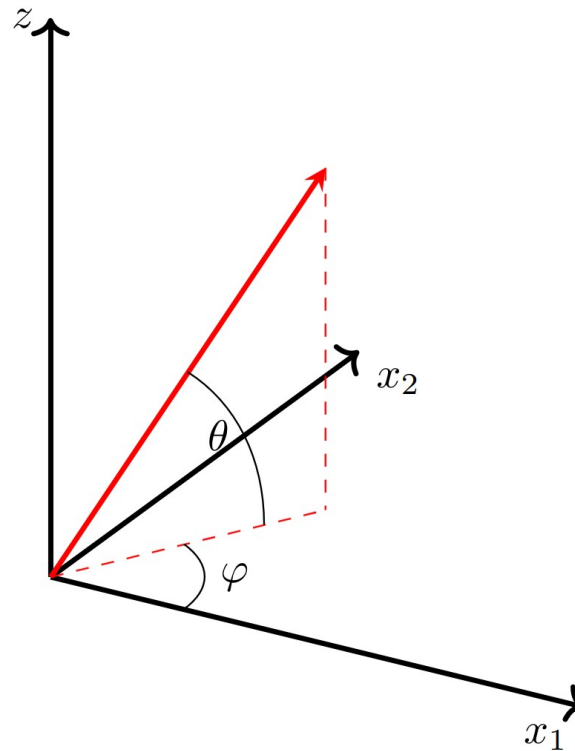
Bayes factor  $\sim$  Fitness of model for data.



# Breaking of Metastable String

# Coordinate

String direction





# Explicit Model: SU(2) Theory

Symmetry breaking pattern:

$$\text{SU}(2) \xrightarrow{V} \text{U}(1) \xrightarrow{v} \text{nothing}$$

Higgs fields:

$$\phi \quad \text{Triplet} \quad \langle \phi^a \rangle = V \delta^{a3}$$

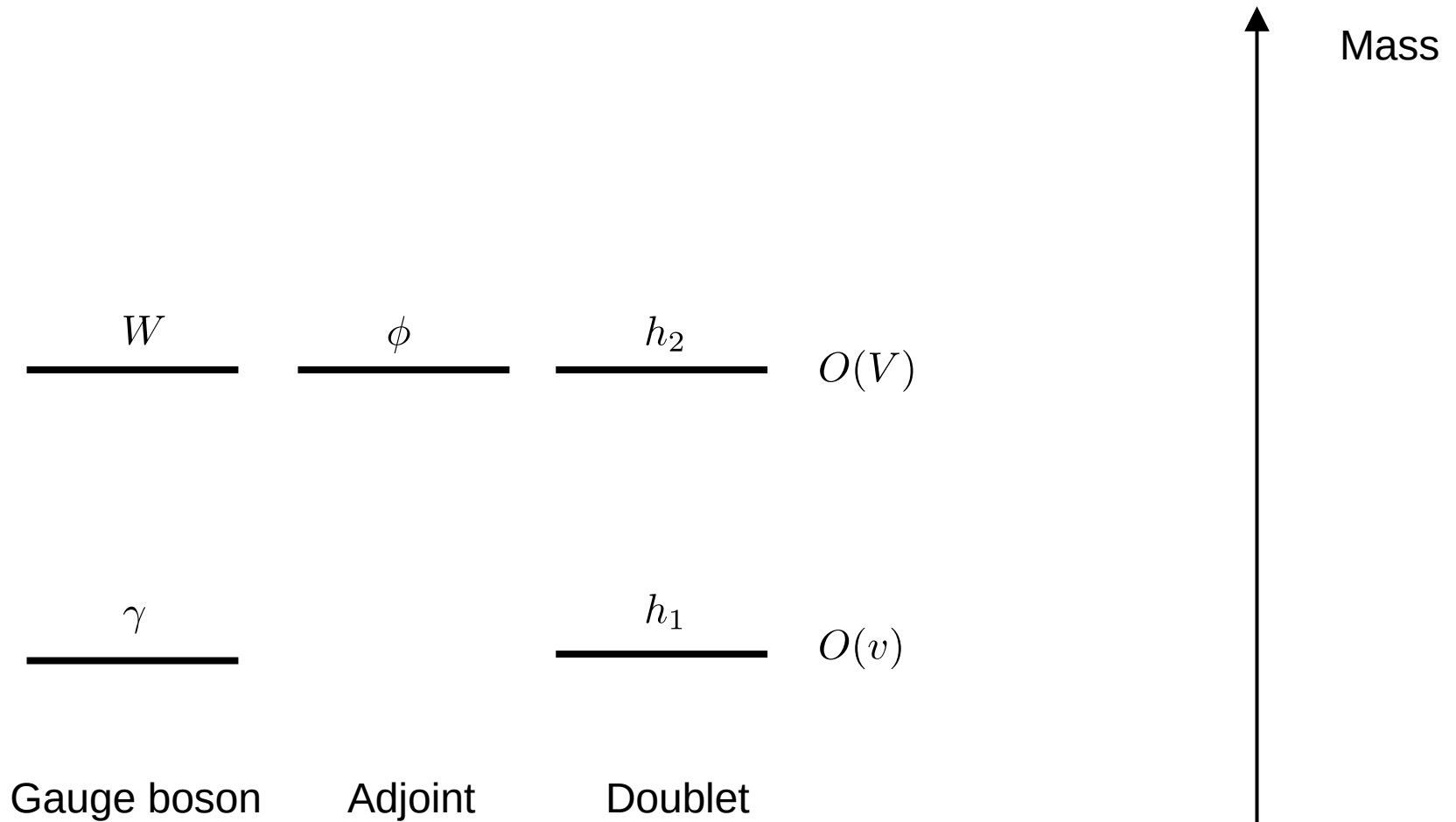
$$h \quad \text{Doublet} \quad \langle h \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

Lagrangian:

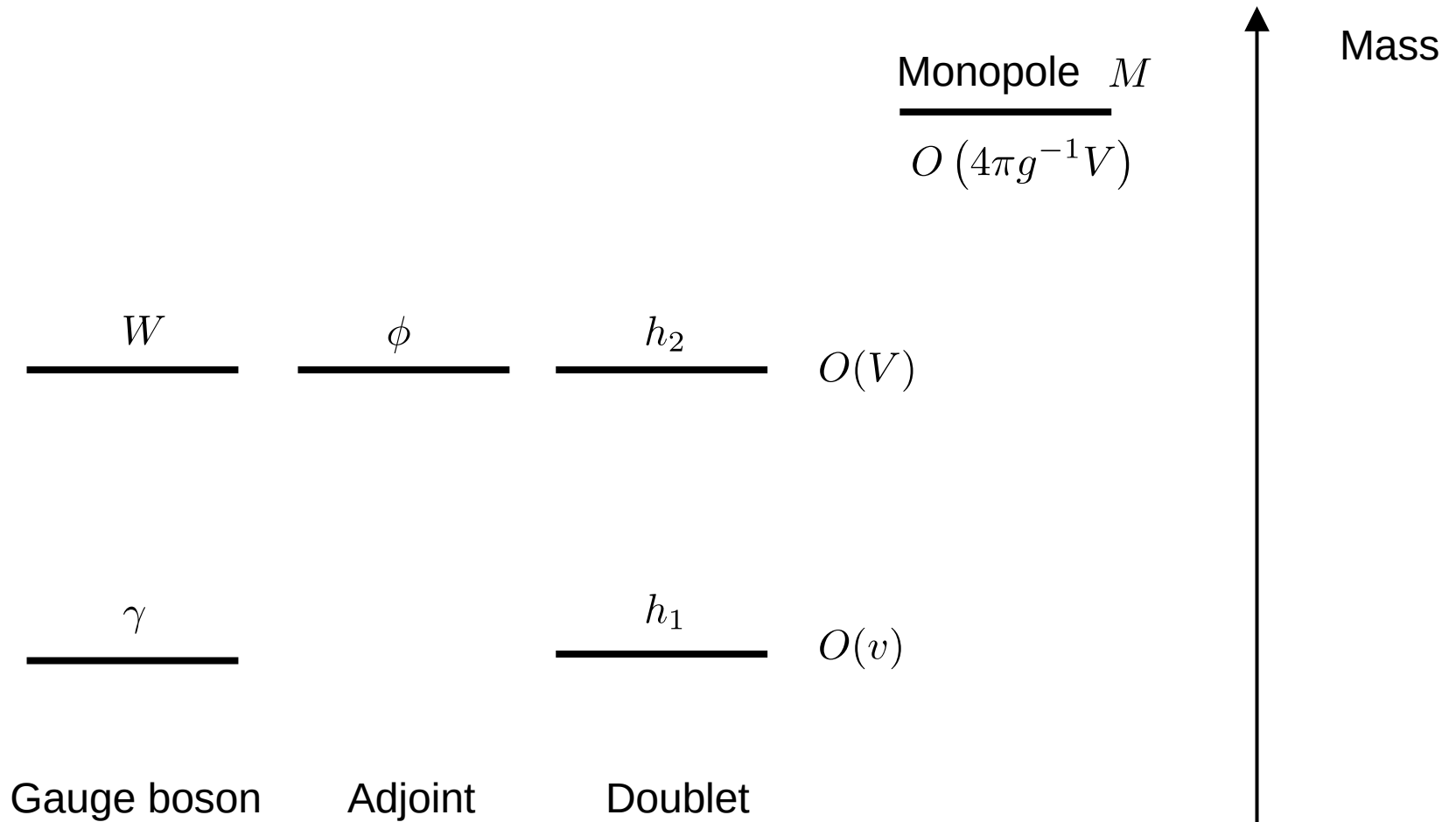
$$\mathcal{L} = -\frac{1}{2}(D_\mu \phi^a)(D^\mu \phi^a) - (D_\mu h)^\dagger (D^\mu h) - \frac{1}{4g^2} F_{\mu\nu}^a F^{a\mu\nu} - V_{\text{Higgs}}(\phi, h)$$

$$V_{\text{Higgs}}(\phi, h) := \lambda(|h|^2 - v^2)^2 + \tilde{\lambda}(\phi^a \phi^a - V^2)^2 + \gamma \left| \left( \phi - \frac{V}{2} \right) h \right|^2$$

# Mass Spectrum

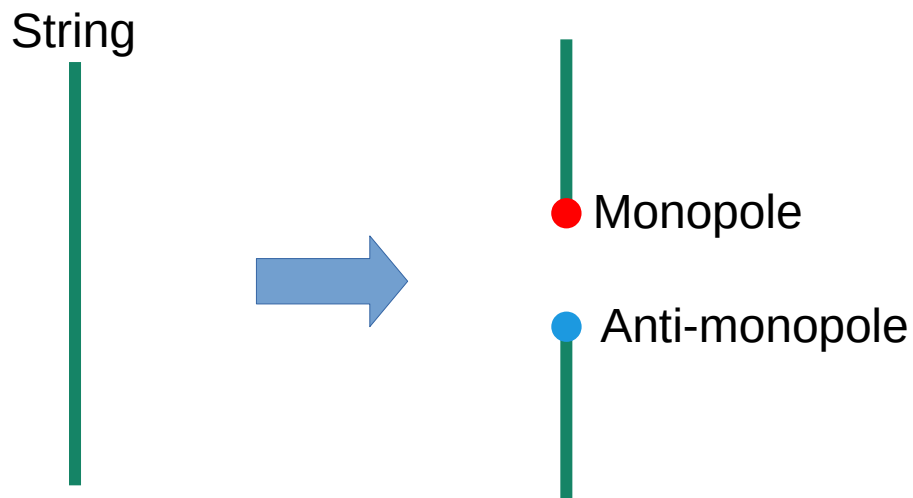


# Mass Spectrum



# String Breaking via Monopole

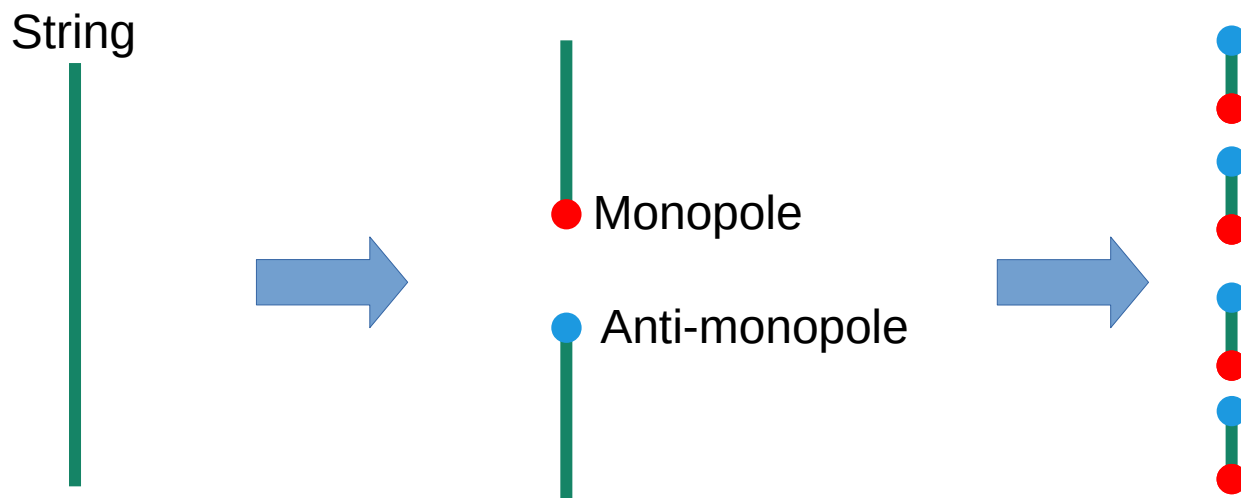
[Vilenkin, NPB 196(1982) 240]  
[Preskill & Vilenkin, hep-ph/9209210]



Monopole pair production like Schwinger effect.

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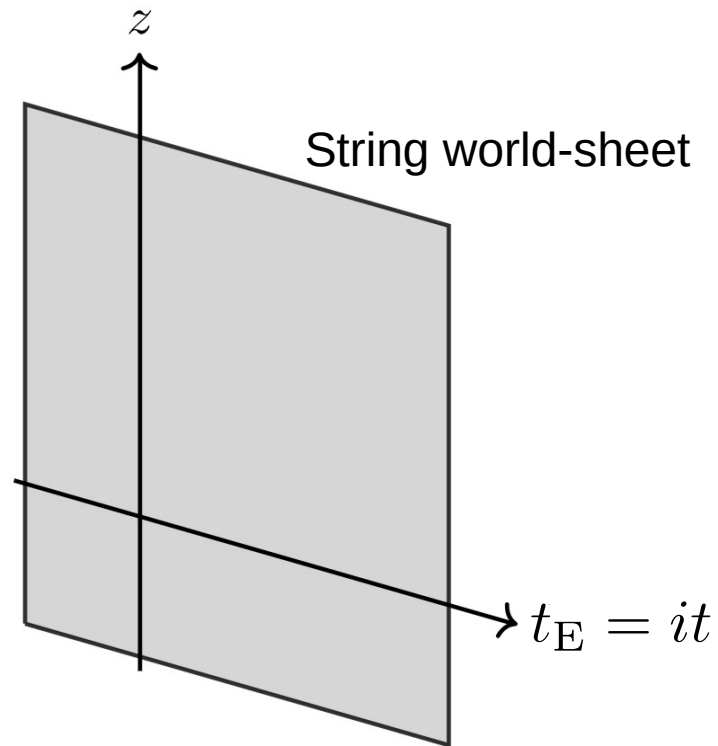
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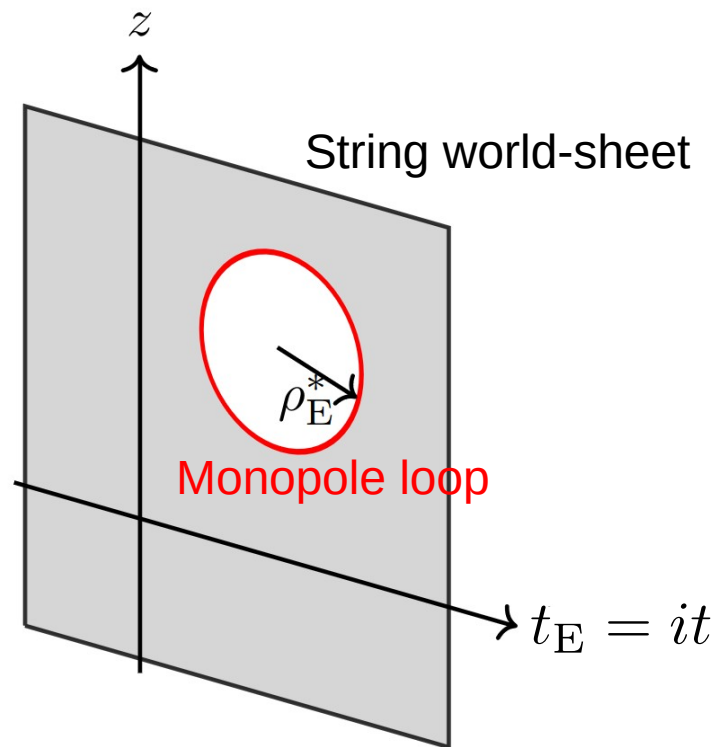
# Conventional Approach

$V \gg v$     Infinitely thin string



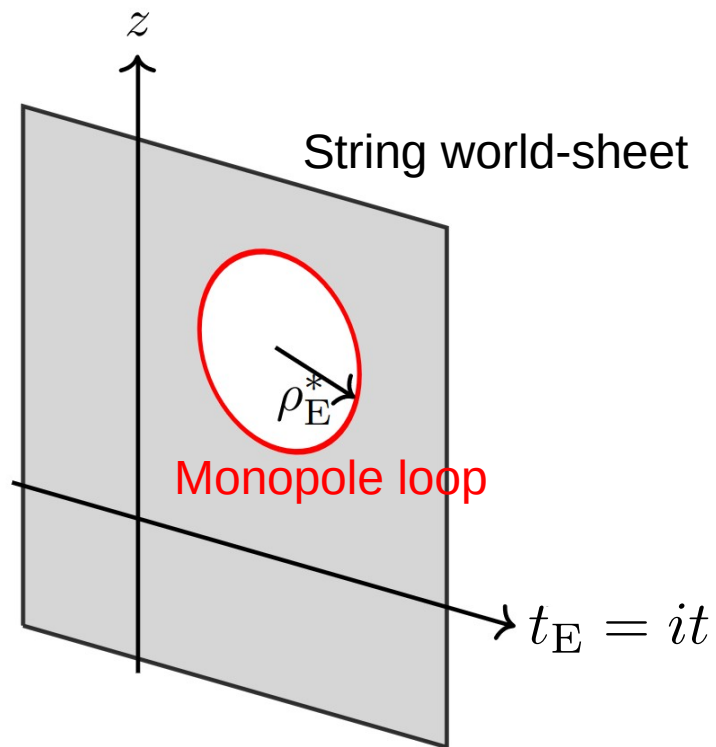
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$$S_B = m_M \int_{\text{worldline}} dx - \mu \int_{\text{hole}} d^2 S$$

$$= 2\pi \rho_E^* m_M - \pi \rho_E^{*2} \mu$$

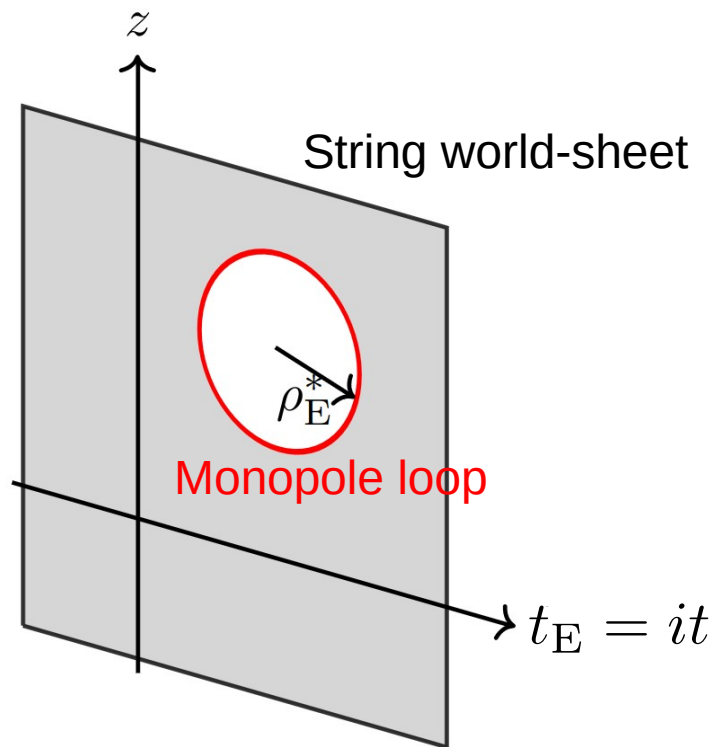
➡  $\rho_E^* = \frac{m_M}{\mu}$

maximization



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maximization

$$S_B^{(\text{thin})} = \frac{\pi m_M^2}{\mu} =: \pi \kappa$$

$$\Gamma \propto e^{-S_B^{(\text{thin})}}$$

# Nanograv Implication

$$\begin{aligned} & \text{NANOGrav} \\ & \downarrow \\ \sqrt{\kappa} = \frac{m_M}{\sqrt{\mu}} = O\left(\frac{4\pi V}{\sqrt{2\pi g v}}\right) \simeq 8 & \quad \Rightarrow \quad V/v = O(1) \\ \mu \sim 2\pi v^2 & \\ M_M \sim 4\pi g^{-1} V & \end{aligned}$$

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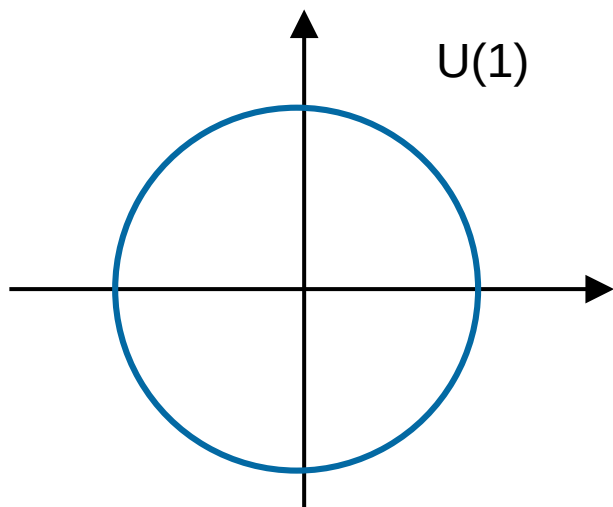
Thin-string approximation is valid?

Any other path to break the string?

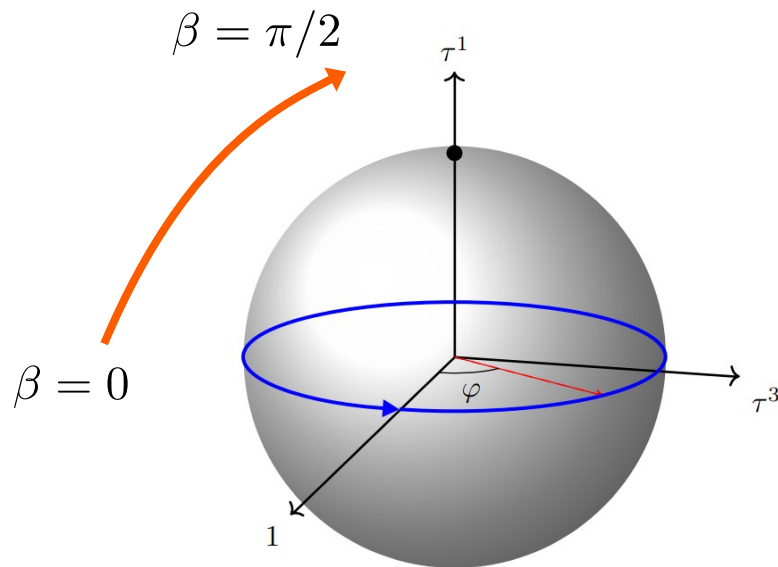


# Revisiting String Breaking

# U(1) in SU(2)



$$\pi_1(\text{U}(1)) \neq 0$$

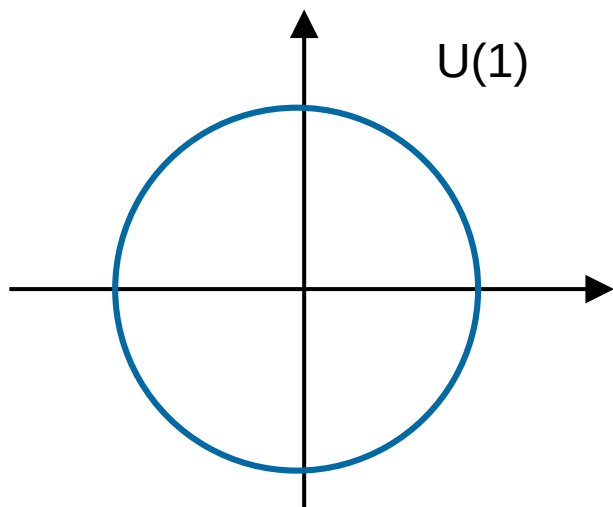


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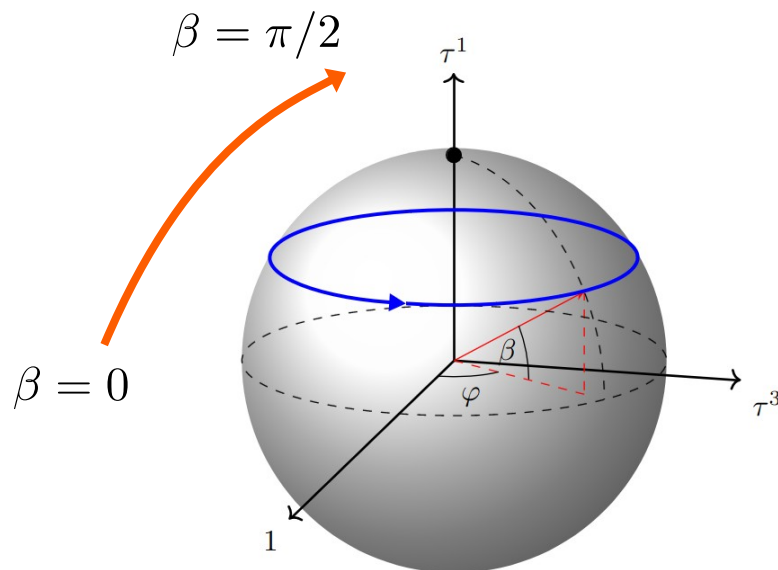
$\beta$  : Unwinding parameter.

$\beta \rightarrow \pi/2$ , no winding.

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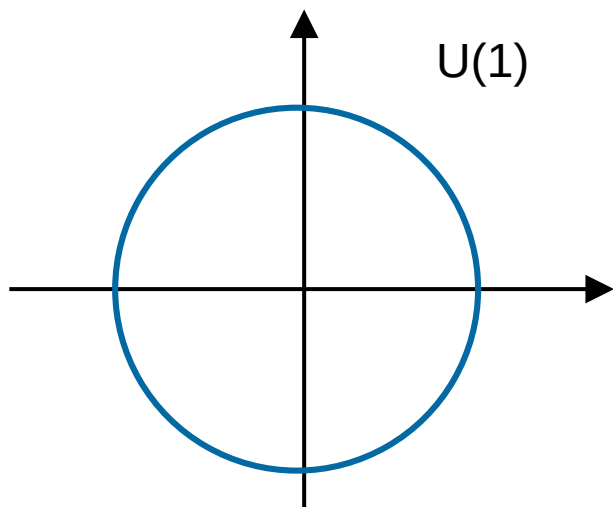
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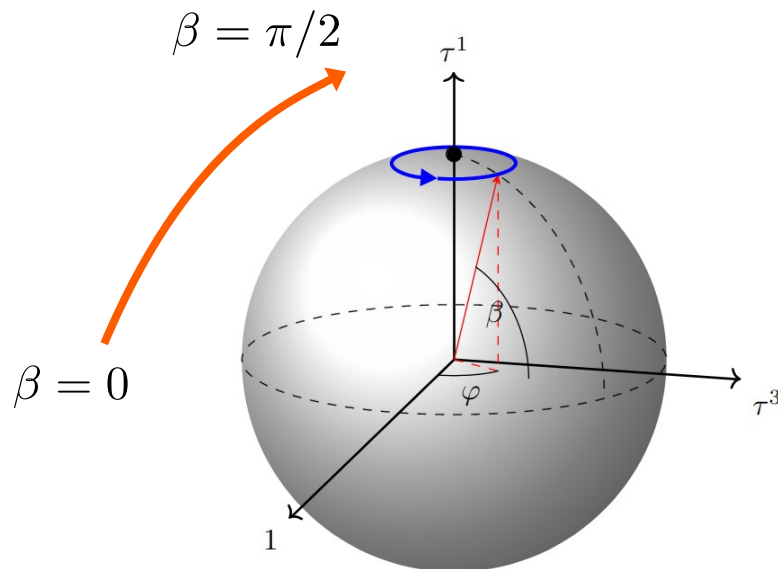
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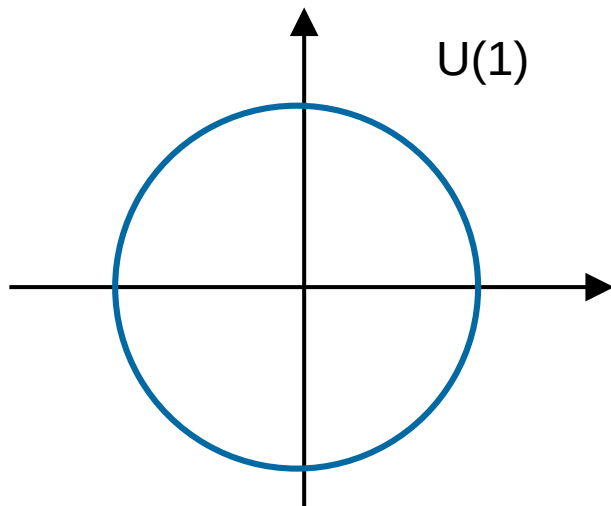


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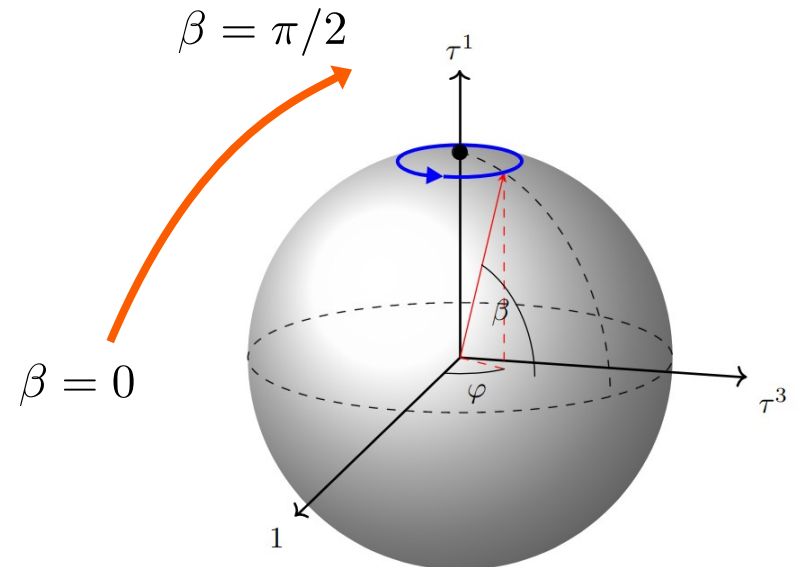
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Using  $\beta$ , explicit field configuration of string breaking is possible.

# Primitive Ansatz

[Shifman & Yung hep-th/0205025]

$\tau$  : Pauli matrices

$$\rho = \sqrt{x_1^2 + x_2^2}$$

$$U = e^{-i\varphi\tau^3} \cos \beta + i\tau^1 \sin \beta$$

$$h(x) = U \begin{pmatrix} \xi_\beta(\rho) \\ 0 \end{pmatrix}$$

$$A_\rho(x) = 0$$

$$A_\varphi(x) = iU(\partial_\varphi U^\dagger)(1 - f_\beta(\rho))$$

$$\phi(x) = VU \frac{\tau^3}{2} U^\dagger + \Delta\phi$$

$$\Delta\phi = \Phi_\beta(\rho) \left( \frac{\tau^1}{2} \sin \varphi - \frac{\tau^2}{2} \cos \varphi \right)$$

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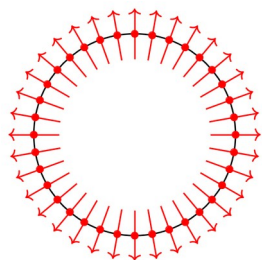
These fields yet to be determined.

$$\phi(x) = VU \frac{\tau^3}{2} U^\dagger + \Delta\phi$$

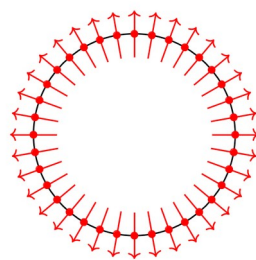
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# Field Configuration

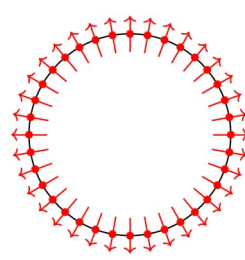
$\varphi$  dependent phase of doublet  $\mathbf{h}$



$$\beta = 0$$



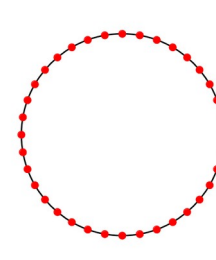
$$\beta = \pi/8$$



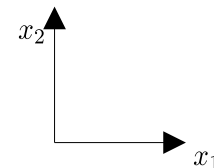
$$\beta = \pi/4$$



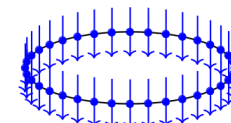
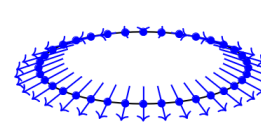
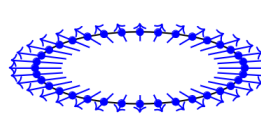
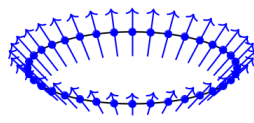
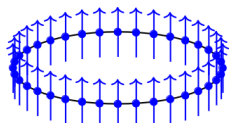
$$\beta = 3\pi/4$$



$$\beta = \pi/2$$



Direction of Adjoint field  $\Phi^a$



# Procedure

- Promote  $\beta$  as collective coordinate  $\beta(t,z)$ .

- Get effective action.

$$S_{\text{Eucl}} = 2\pi \int \rho_E d\rho_E \left( \frac{1}{2} K_{\text{eff}}(\beta) (\partial_{\rho_E} \beta(\rho_E))^2 + T(\beta) \right)$$
$$\rho_E = \sqrt{t_E^2 + z^2}$$

- Get tunneling rate with bounce solution.

# Primitive Ansatz

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$$A_{t/z}(x) = -2(\partial_{t/z}\beta(t, z)) \left( \frac{\tau^1}{2} \cos \varphi - \frac{\tau^2}{2} \sin \varphi \right) a_\beta(\rho)$$

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Substitute to original Lagrangian

(kinetic-term) +  $V_{\text{Higgs}}$

$$(\partial_{\rho_E}\beta)^2 K_{\text{eff}}[\xi_\beta, f_\beta, \Phi_\beta, a_\beta] + T[\xi_\beta, f_\beta, \Phi_\beta]$$

$$\rho_E = \sqrt{t_E^2 + z^2}$$



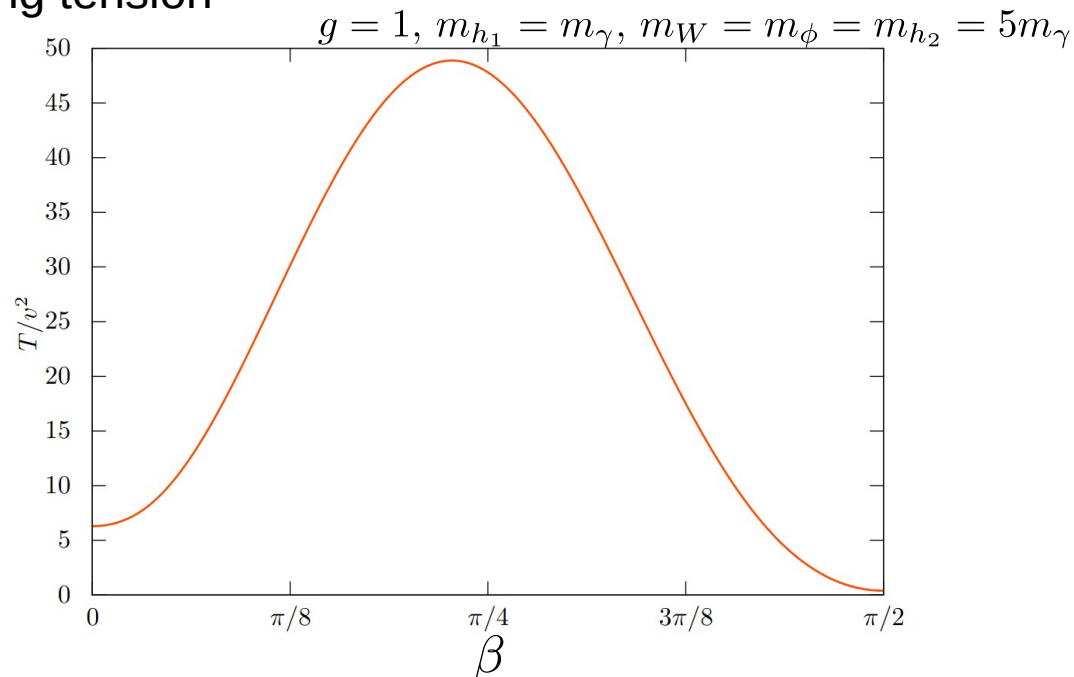
# String Tension

Determine  $f$ ,  $\Phi$ ,  $\xi$  to minimize string tension for each  $\beta$ .

$$T(\beta) = 2\pi \int_0^\infty \rho d\rho \left\{ \frac{2}{g^2} \frac{(\partial_\rho f_\beta)^2}{\rho^2} \cos^2 \beta + (\partial_\rho \xi_\beta)^2 + \frac{f_\beta^2}{\rho^2} \xi_\beta^2 \cos^2 \beta \right. \\ \left. + \frac{1}{2} (\partial_\rho \Phi_\beta)^2 + \frac{1}{2\rho^2} [\Phi_\beta (\cos 2\beta - 2f_\beta \cos^2 \beta) + V f_\beta \sin 2\beta]^2 \right. \\ \left. + \lambda (\xi_\beta^2 - v^2) + \tilde{\lambda} [\Phi_\beta^2 - 2V \Phi_\beta \sin 2\beta]^2 + \frac{\gamma}{4} \Phi_\beta^2 \xi_\beta^2 \right\}.$$

$$\rho = \sqrt{x_1^2 + x_2^2}$$

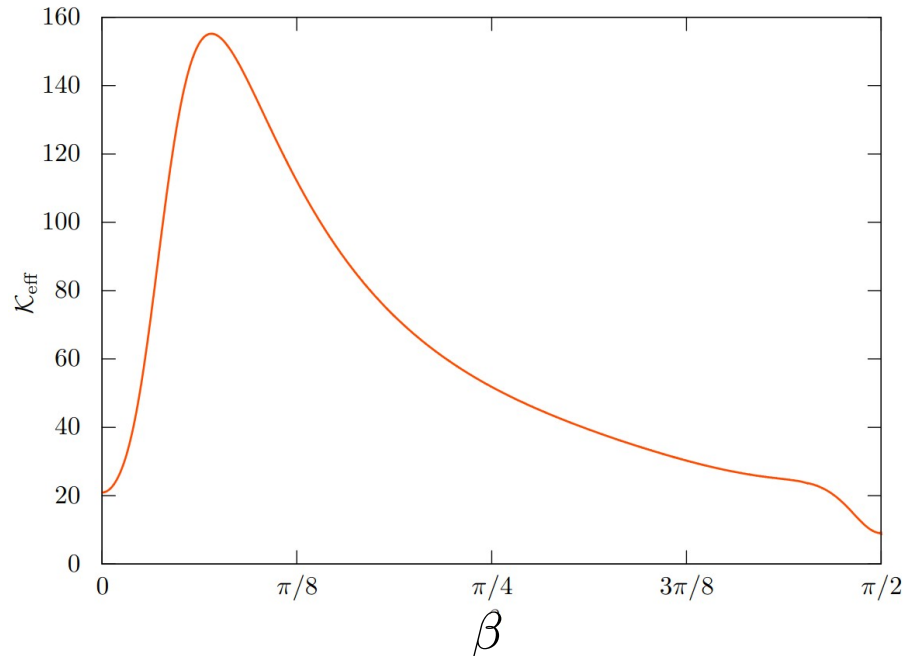
String tension



# Kinetic Term

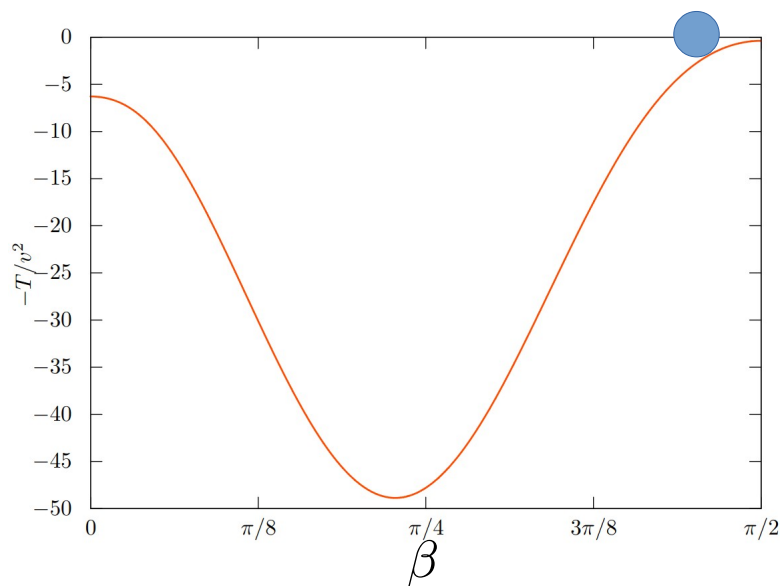
Determine  $a_\beta$  to minimize kinetic term for each  $\beta$ .

$$8\pi\rho \left[ \frac{1}{g^2} (\partial_\rho a_\beta)^2 - a_\beta \left\{ -2\Phi_\beta (V \sin 2\beta - \Phi_\beta) + \frac{1}{2} V (\partial_\beta \Phi_\beta) \cos 2\beta \right. \right. \\ \left. \left. + \frac{1}{g^2 \rho^2} \left( f_\beta^2 - \frac{1}{2} (\partial_\beta f_\beta) \sin 2\beta - f_\beta (1 - f_\beta) \cos 2\beta \right) \right\} \right. \\ \left. + a_\beta^2 \left\{ \frac{1}{g^2 \rho^2} (1 - 4f_\beta (1 - f_\beta) \cos^2 \beta) + V^2 + \frac{1}{2} \xi_\beta^2 + \Phi_\beta (-2V \sin 2\beta + \Phi_\beta) \right\} \right] \dots$$



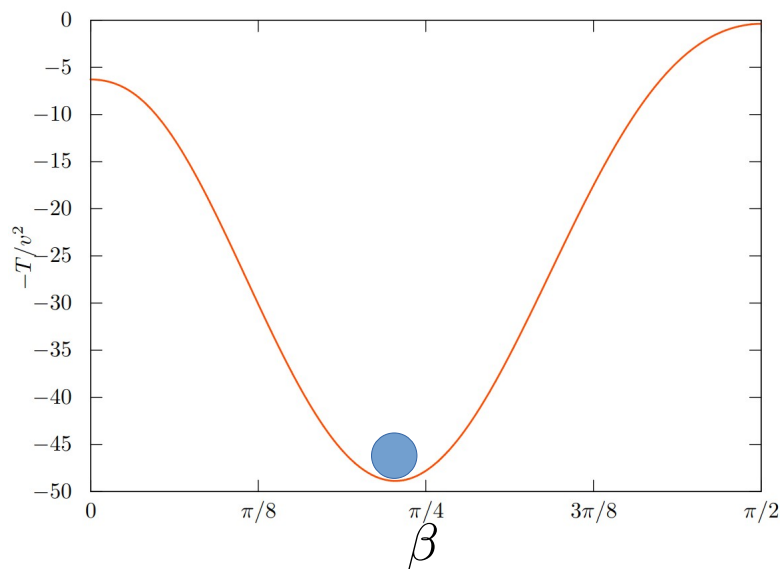
# Effective Theory of $\beta(\rho_E)$

$$S_{\text{Eucl}} = 2\pi \int \rho_E d\rho_E \left[ \frac{1}{2} \mathcal{K}_{\text{eff}}(\beta(\rho_E)) (\partial_{\rho_E} \beta(\rho_E))^2 + T(\beta(\rho_E)) \right]$$



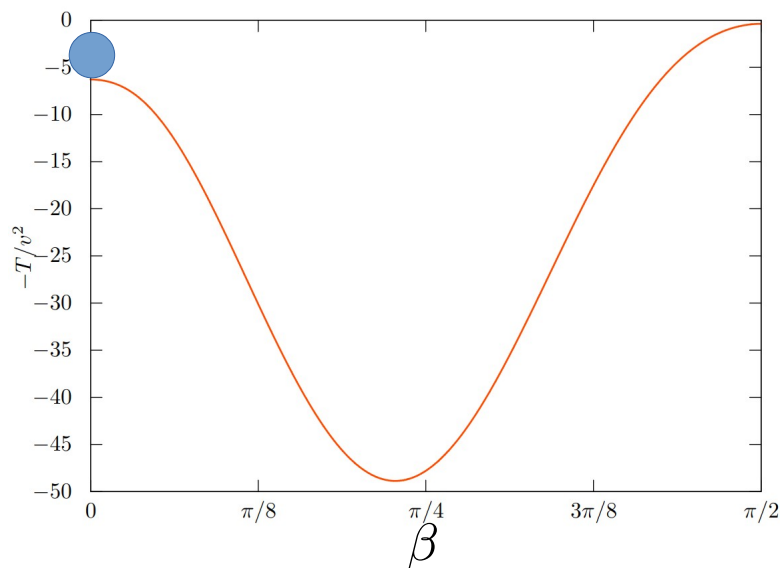
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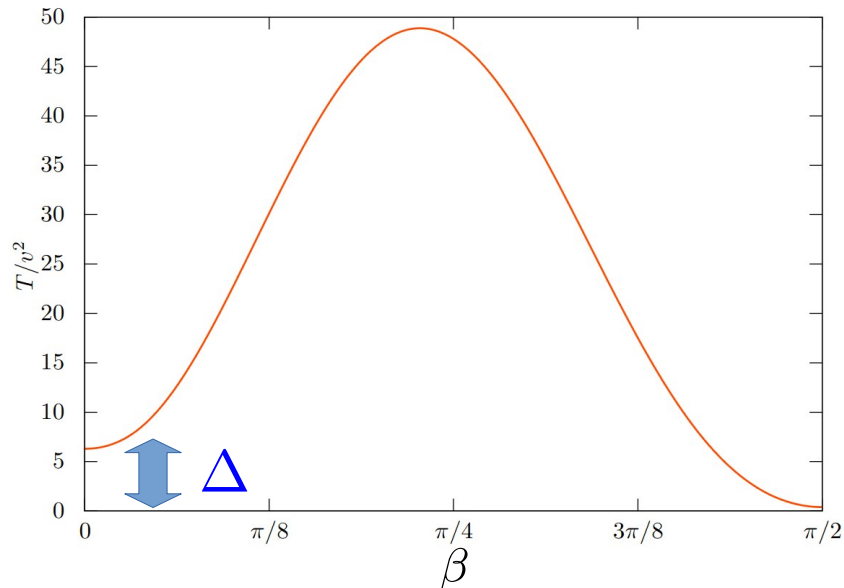
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# Thin-Wall Approximation

String tension  $T(\beta)$



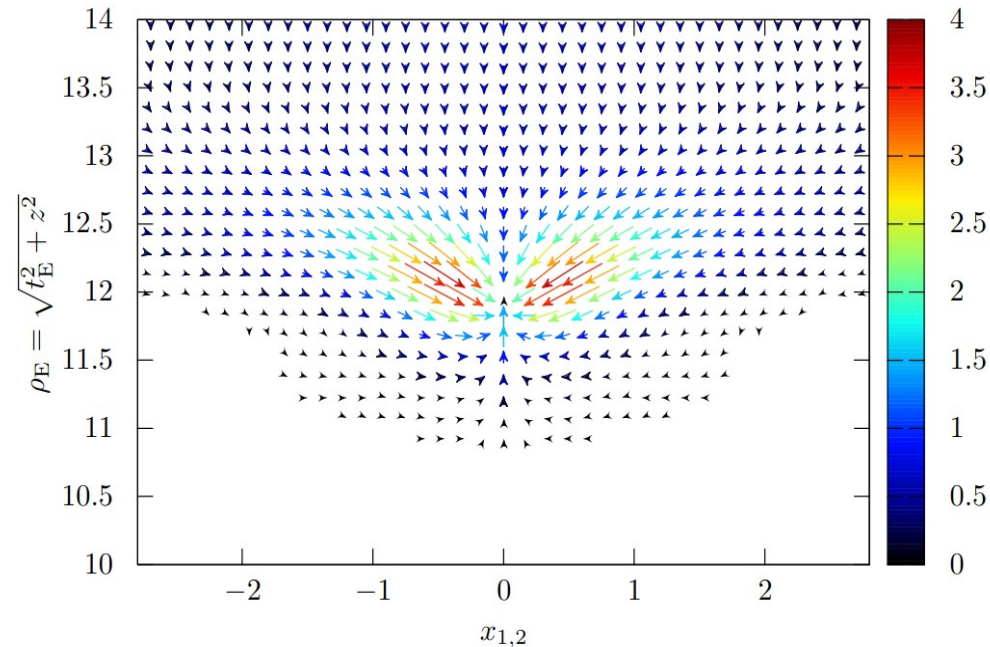
$\Delta \rightarrow 0$ , thin-wall approximation is valid.

$$S_B \sim \frac{1}{\Delta} \left[ \int_0^{2\pi} \sqrt{\mathcal{K}(\beta)(T(\beta) - \Delta)} d\beta \right]^2$$

# Configuration during Tunneling

$\beta = 0$ : String state

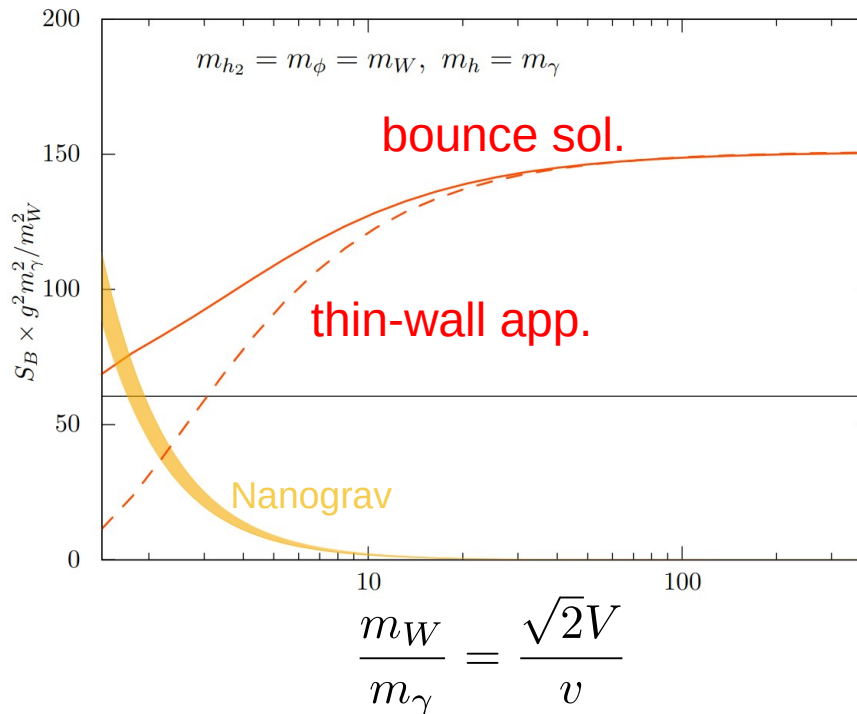
Magnetic field



$\beta = \pi/2$ : "True" vacua

# Bounce Action

Normalized  $S_B$



← Conventional estimation  
 $\frac{\pi m_M^2}{\mu}$

- The bounce solution results in larger  $S_B$ .
- Thin-wall approximation fails when  $V/v \lesssim 10$ .



# Improved Ansatz

[Shifman & Yung hep-th/0205025]

Primitive Ansatz

$$h(x) = U \begin{pmatrix} \xi_\beta(\rho) \\ 0 \end{pmatrix}$$

$$A_\rho(x) = 0$$

$$A_\varphi(x) = iU(\partial_\varphi U^\dagger)(1 - f_\beta(\rho))$$

$$\phi(x) = VU \frac{\tau^3}{2} U^\dagger + \Delta\phi$$

# Improved Ansatz

[Shifman & Yung hep-th/0205025]

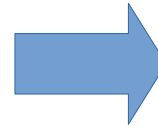
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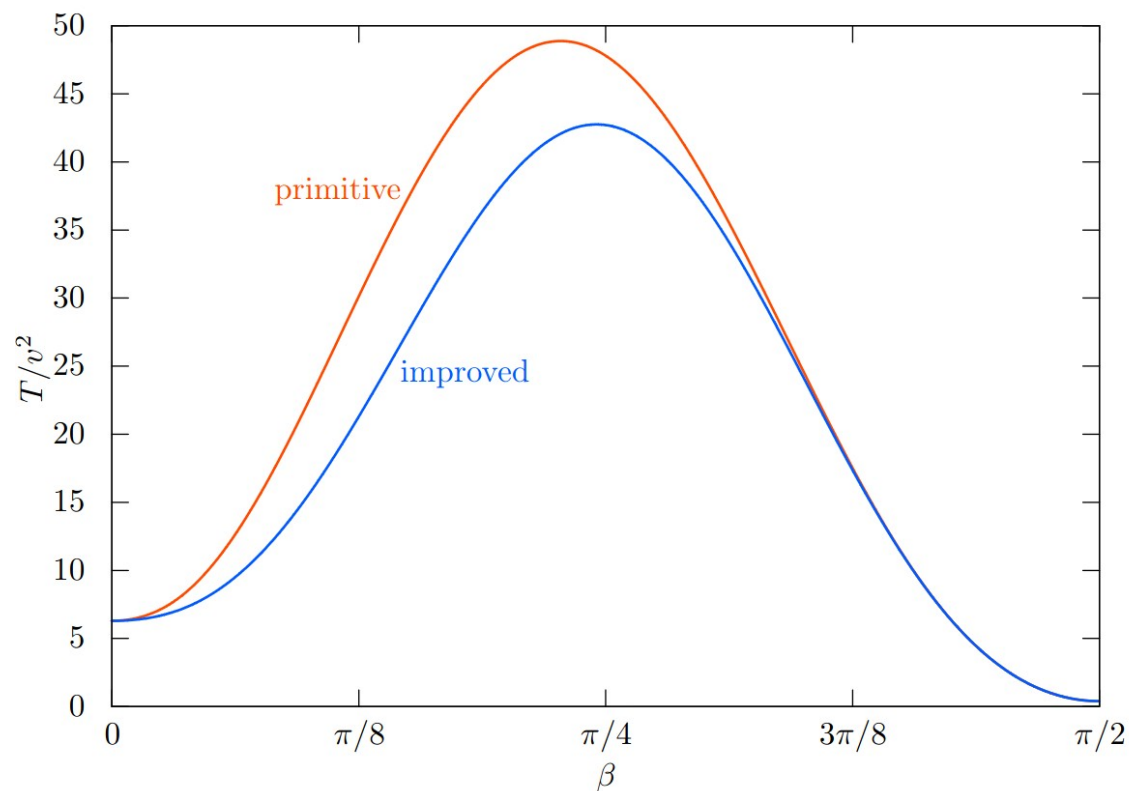
$$\phi(x) = VU \frac{\tau^3}{2} U^\dagger + \Delta\phi$$



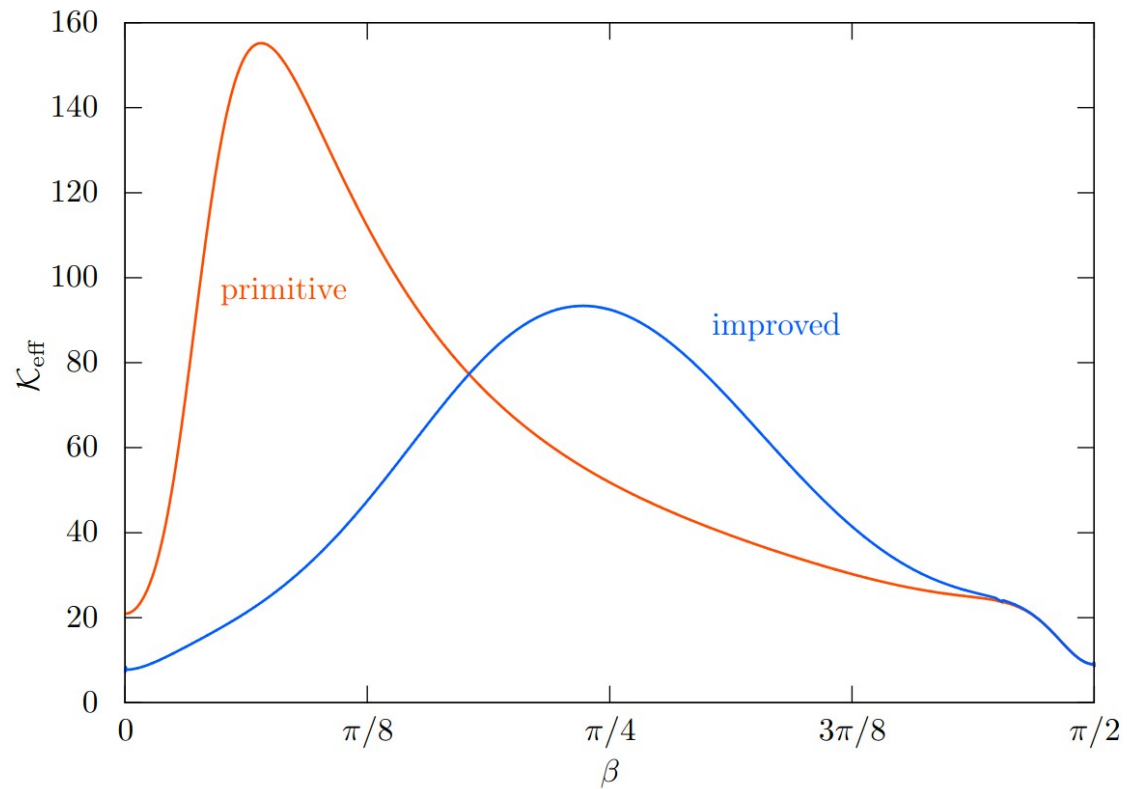
$$A \rightarrow A_W + A_\gamma$$

Separation of  $\gamma$  and  $W$

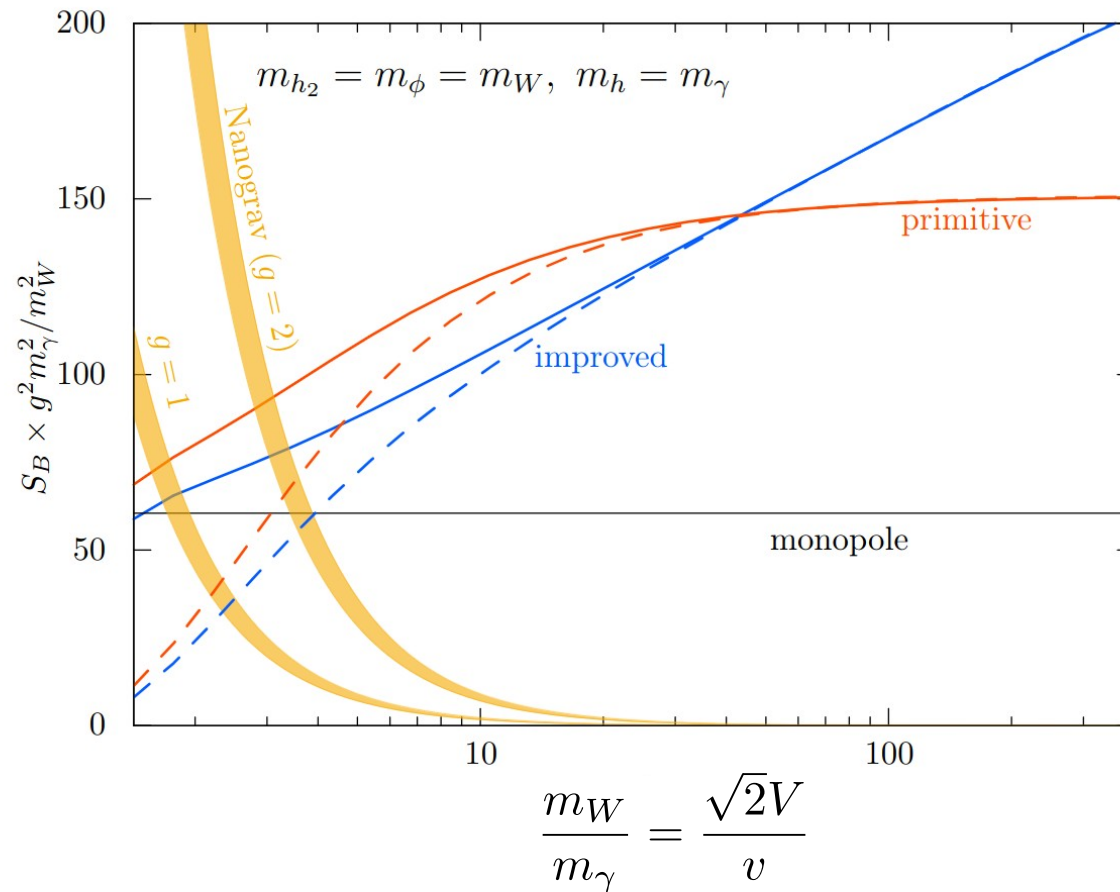
# String Tension



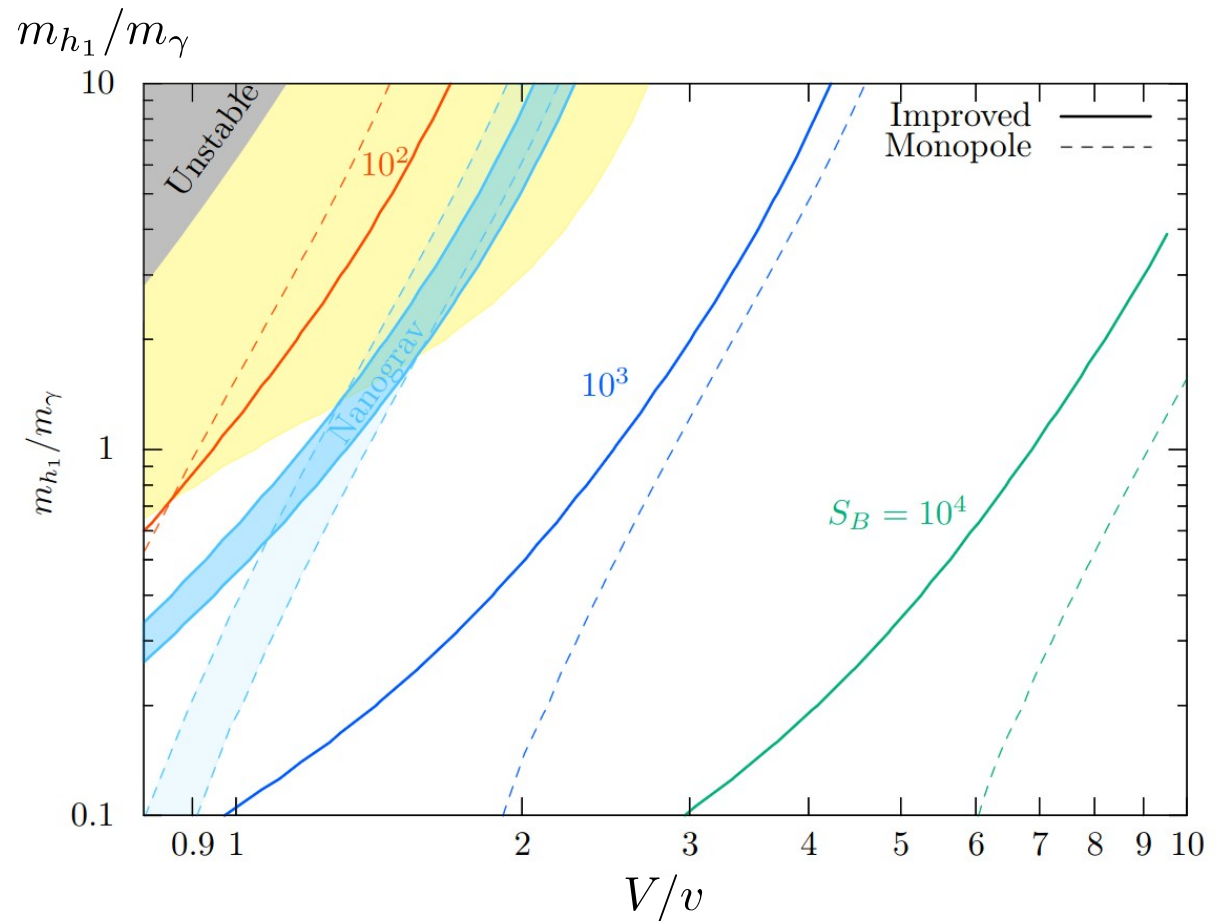
# Kinetic Term



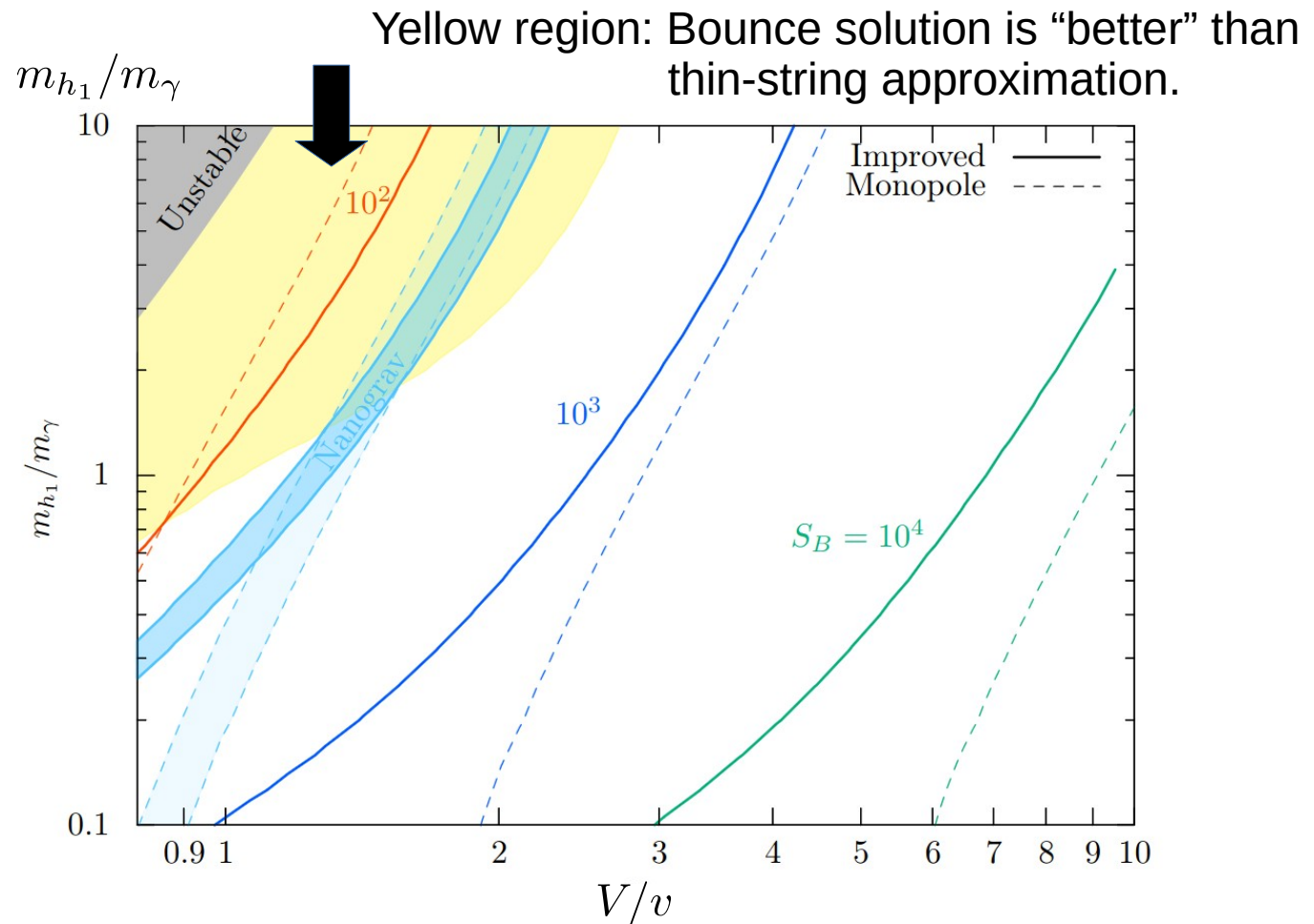
# Result



# Higgs Mass Effect



# Higgs Mass Effect



# Summary and Prospect

- Metastable string can appear from hierarchical symmetry breaking.
  - GW from metastable string gives good fit to NANOGrav data.
- Breaking rate of metastable string is estimated based on two Ansätze.
  - Give robust lower limit of breaking rate even with  $V/v = O(1)$ .
  - Validity of conventional estimation?
- The present procedure is not best way to fastest path to string breaking.
  - Complete analysis without effective  $\beta$  description.
- Application to other metastable strings.