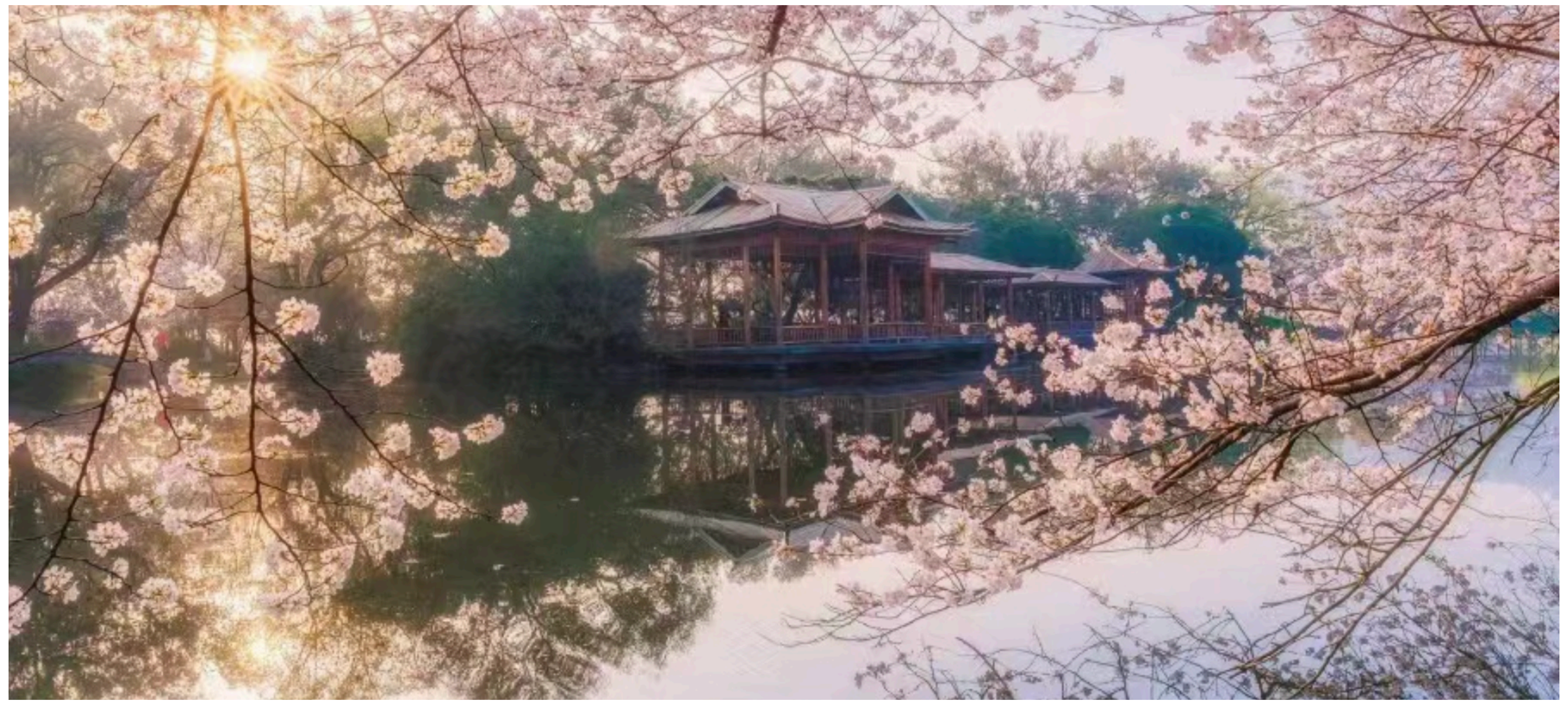
Yukawa coupling unification in SO(10) GUTs and the origin of Yukawa hierarchy of third-generation fermions

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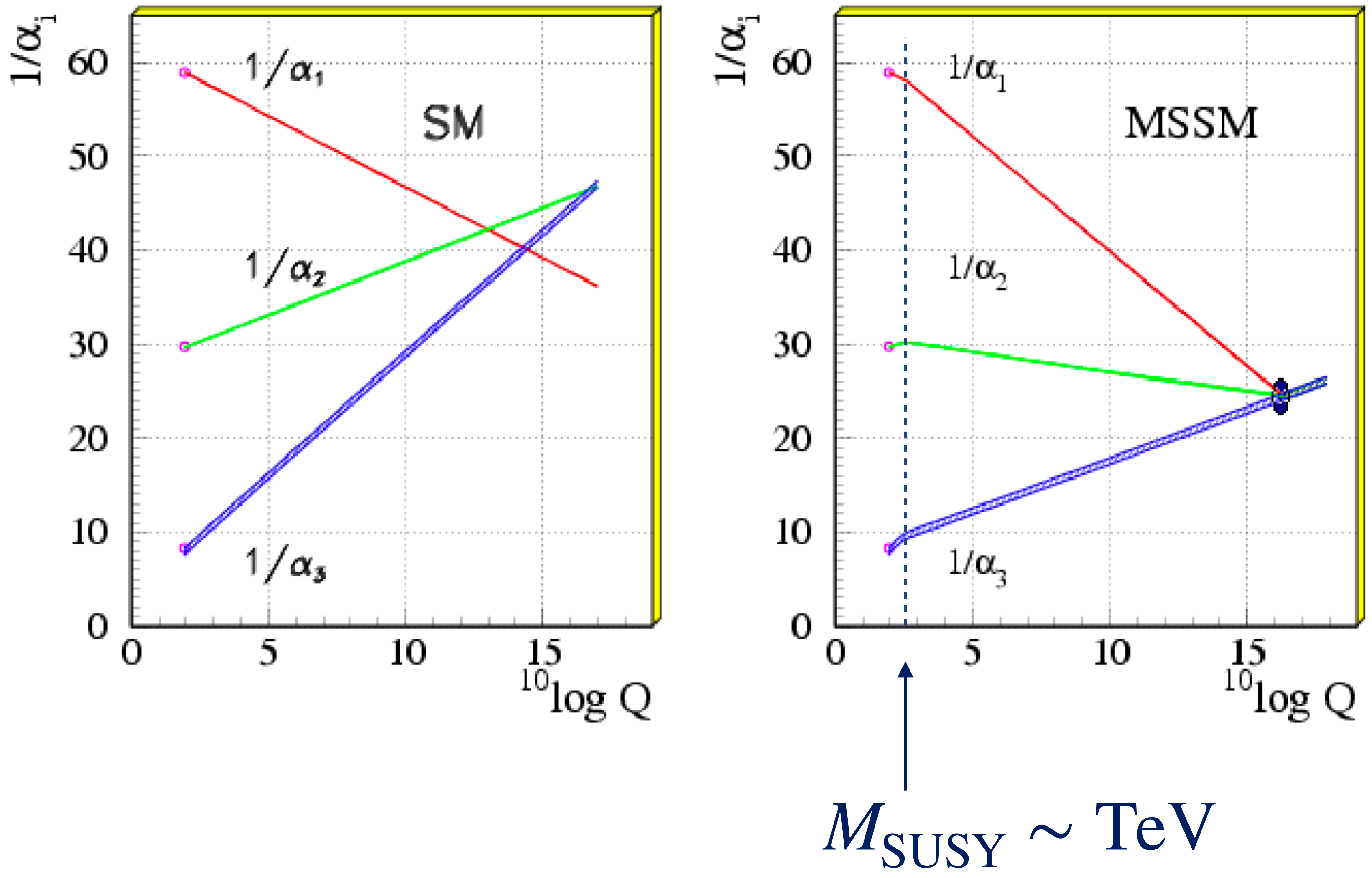
Outline

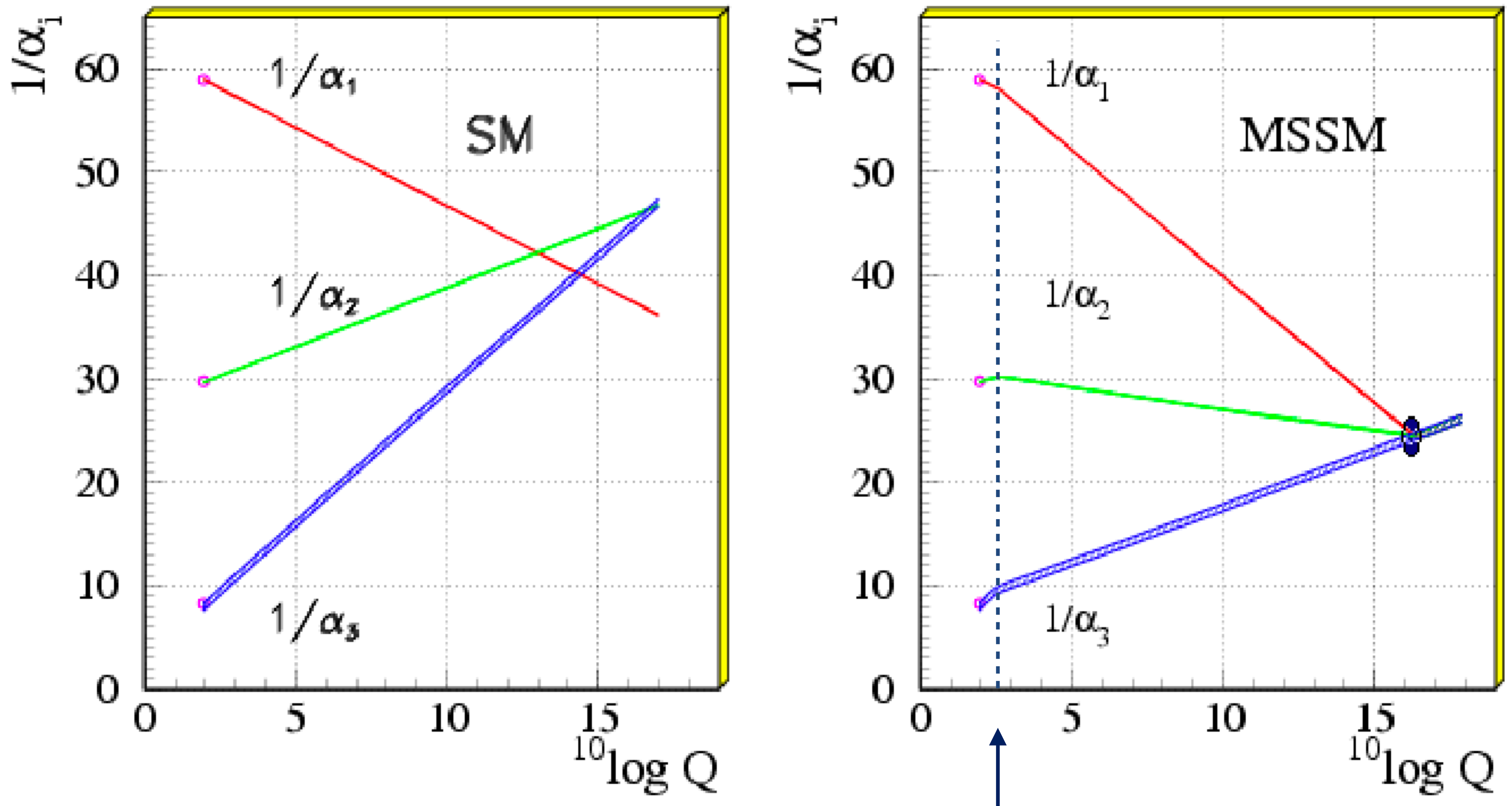
1. Introduction & motivation
2. Yukawa couplings in SO(10)
3. Unification of Yukawa couplings in SO(10)
4. Conclusions

1. Introduction & motivation

Unification of fundamental couplings



Unification of fundamental couplings

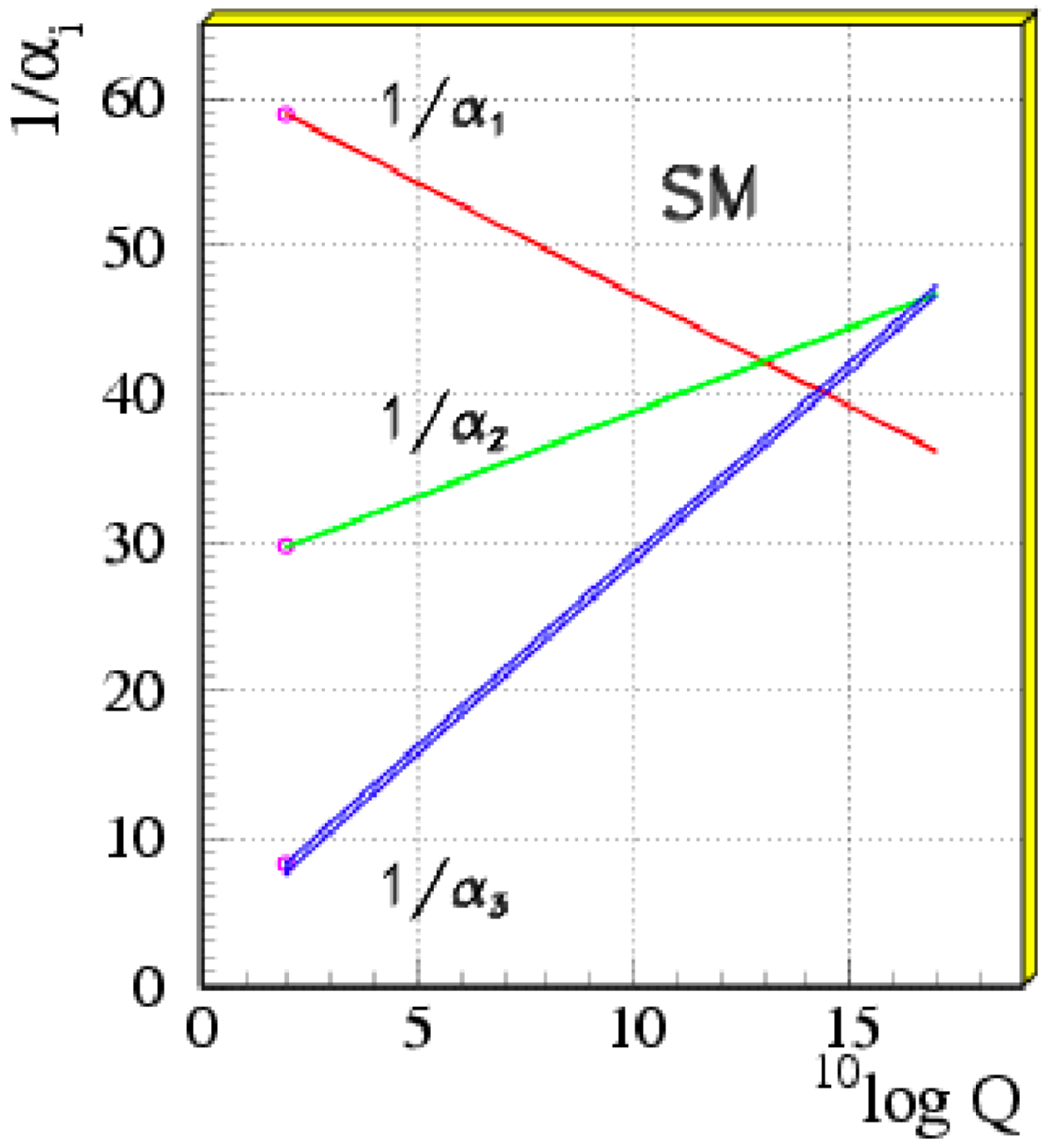


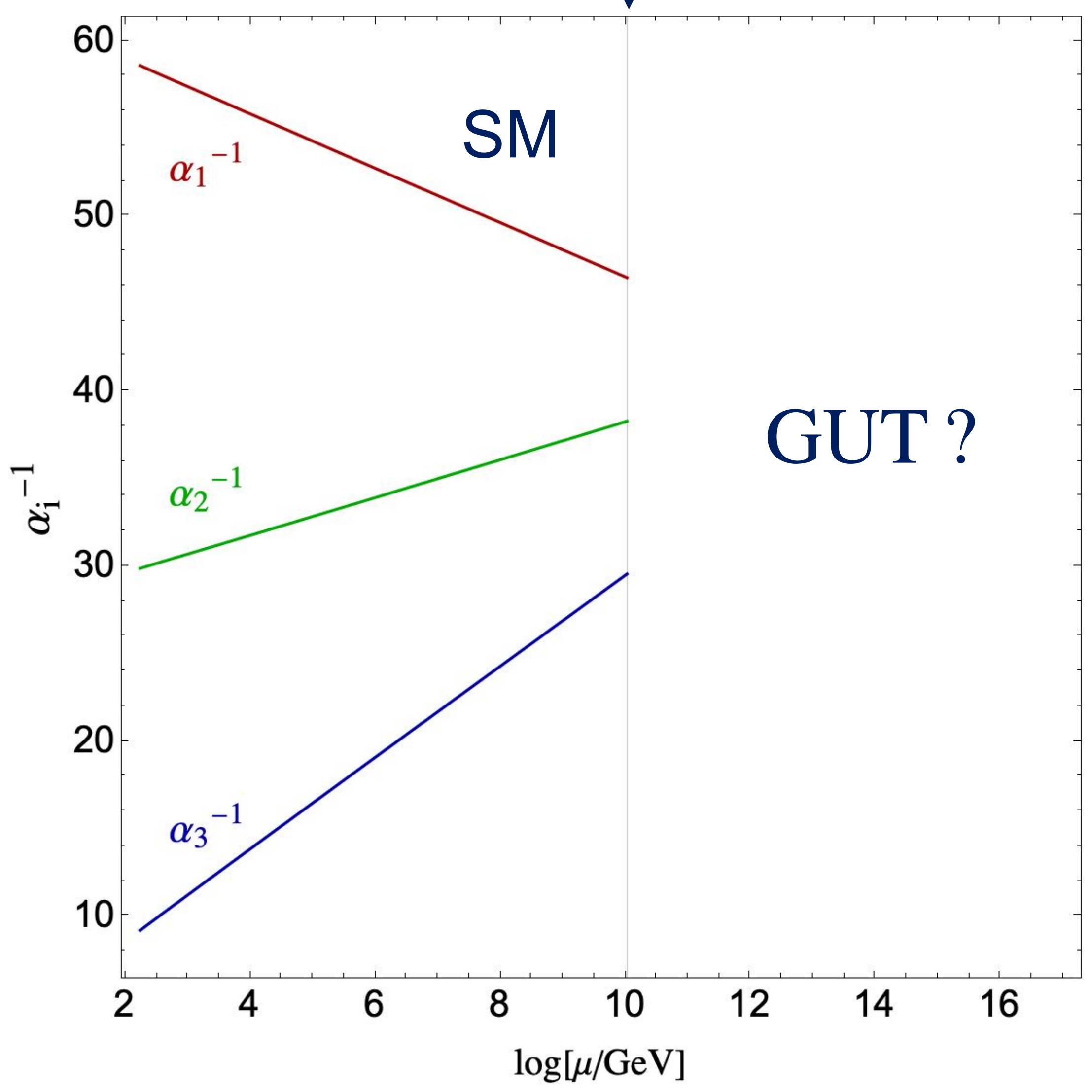
~~$M_{\mathrm{SUSY}} \sim \mathrm{TeV}$~~ →

1. No weak scale SUSY
2. Simple SU(5) unification doesn't work

Unification of fundamental couplings

$M_{\text{intermediate}} = ?$





~~$M_{\text{SUSY}} \sim \text{TeV}$~~ →

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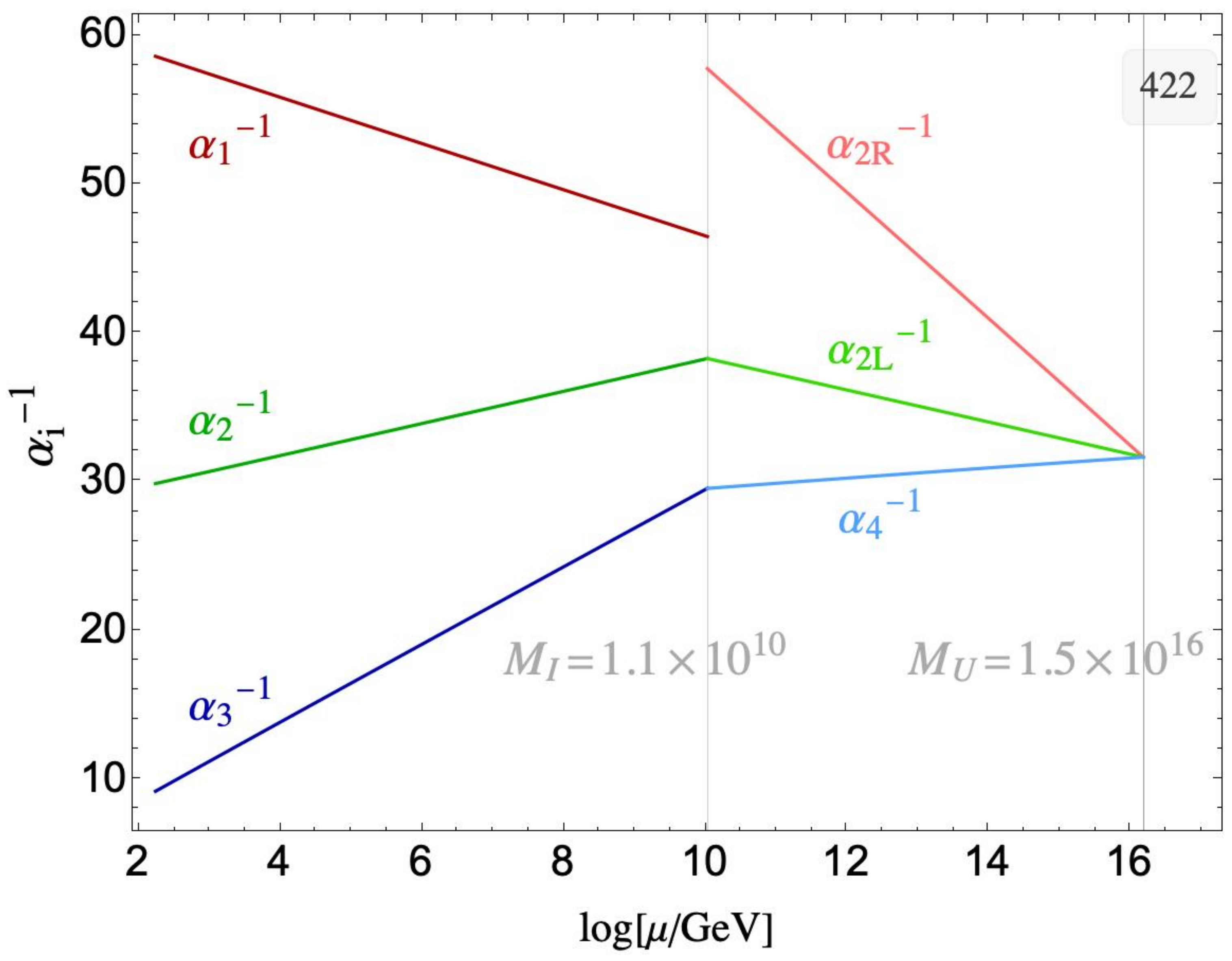
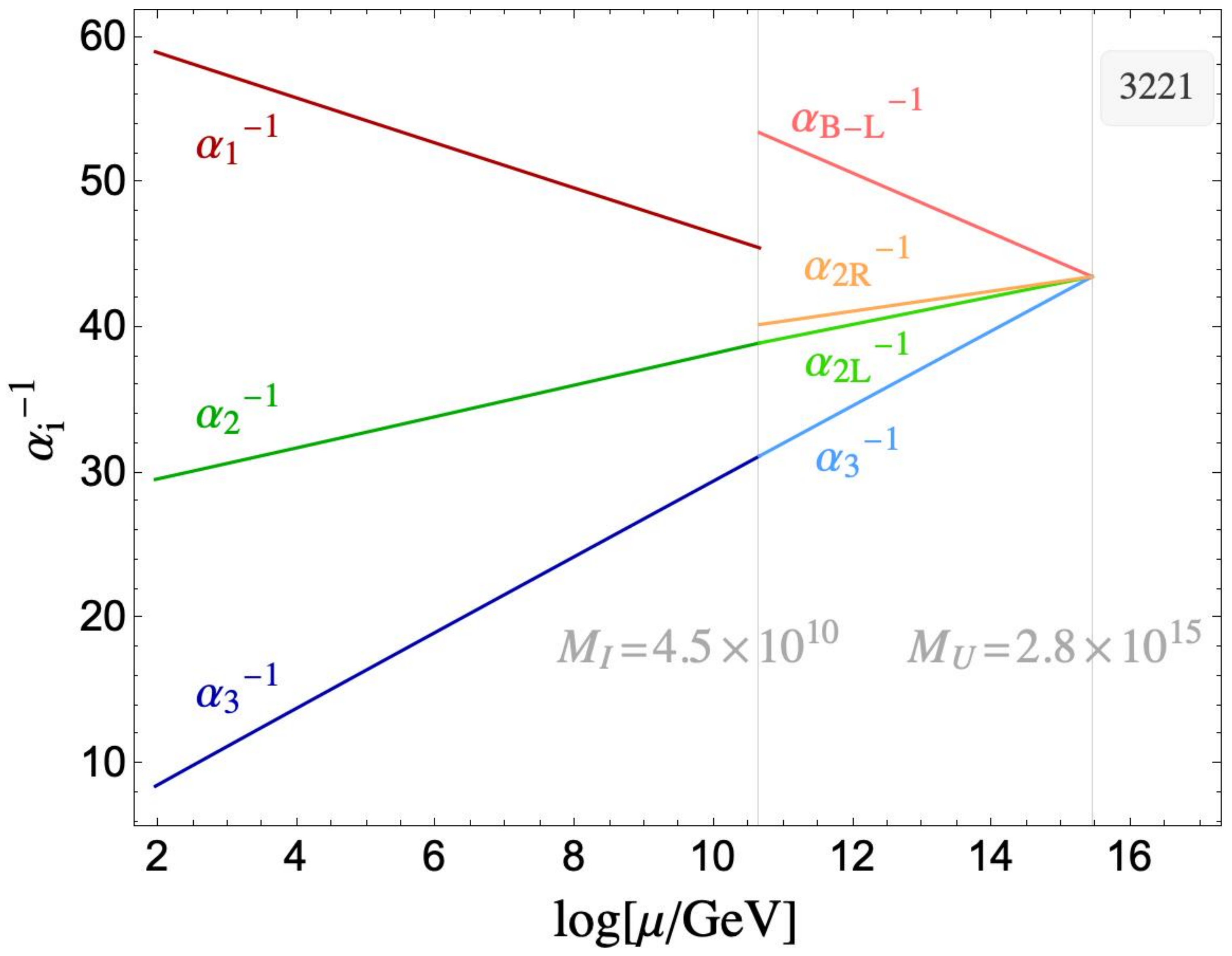
Unification of gauge couplings in non-SUSY SO(10)



Pati-Salam (422)

$$\mathscr{G}_{422} = \mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$$



Left-Right Symmetry (3221)

$$\mathscr{G}_{3221} = \mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$$

[Djouadi, Fonseca, RO, Raidal, '22]

The unification scheme depends on matter representations!

The theme of grand unification

- Unification of Gauge coupling
- Unification of matter representation: $\bar{\mathbf{5}} + \mathbf{10}, \mathbf{16}, \mathbf{27}, \ldots$

Fermion representations of SO(10)

- Counting SM chiral fermions of a single generation:

  8 Left-handed fermions: $u_L^{c_1}, d_L^{c_1}, u_L^{c_2}, d_L^{c_2}, u_L^{c_3}, d_L^{c_3}, \ell_L, \nu_L^{\ell}$

  7 Right-handed fermions: $u_R^{c_1}, d_R^{c_1}, u_R^{c_2}, d_R^{c_2}, u_R^{c_3}, d_R^{c_3}, \ell_R$

- All these fermion can be embedded into a single 16-dimensional spinor representation of SO(10) group: $\mathbf{16_F}$, with an additional right-handed fields identified as the right-handed neutrino: ν_R^{ℓ}

$$\mathbf{16_F} \supset \left(u_L^{c_1}, d_L^{c_1}, u_R^{c_1}, d_R^{c_1}, u_L^{c_2}, d_L^{c_2}, u_R^{c_2}, d_R^{c_2}, u_L^{c_3}, d_L^{c_3}, u_R^{c_3}, d_R^{c_3}, \ell_L, \nu_L^{\ell}, \ell_R, \nu_R^{\ell} \right)$$

Motivations

- Essentially, different representations leads to different unifications.
- The representation problem is a fundamental problem existing in any 4D gauge models:

- e.g. In SM, there are three generations of five chiral fermion representations and one scalar representation:

$$(\mathbf{3},\mathbf{2})_{1/6}+(\bar{\mathbf{3}},\mathbf{1})_{-2/3}+(\bar{\mathbf{3}},\mathbf{1})_{1/3}+(\mathbf{1},\mathbf{2})_{-1/2}+(\mathbf{1},\mathbf{1})_{1} \quad \text{and} \quad (\mathbf{1},\mathbf{2})_{1/2}\,.$$

- e.g. In SO(10), fermions of a single generation can be embedded into one $\mathbf{16}_F$.

- The question is: Is it possible to use only one representation in the UV to obtain the SM spectrum for each generation?
- This question could be answered by means of Yukawa coupling unification.

The theme of grand unification

- Unification of Gauge coupling

- Unification of matter representation: $\bar{\mathbf{5}} + \mathbf{10}, \mathbf{16}, \mathbf{27}, \ldots$

$$\downarrow$$

- Unification of Yukawa couplings?

Unification of Yukawa couplings

- In SUSY SO(10), there is a long history of discussing the possibility of unifying the Yukawa couplings, for example:
- The Yukawa flows to different values in IR because of RGEs.
- Yukawa unification are regarded as boundary conditions of RGEs

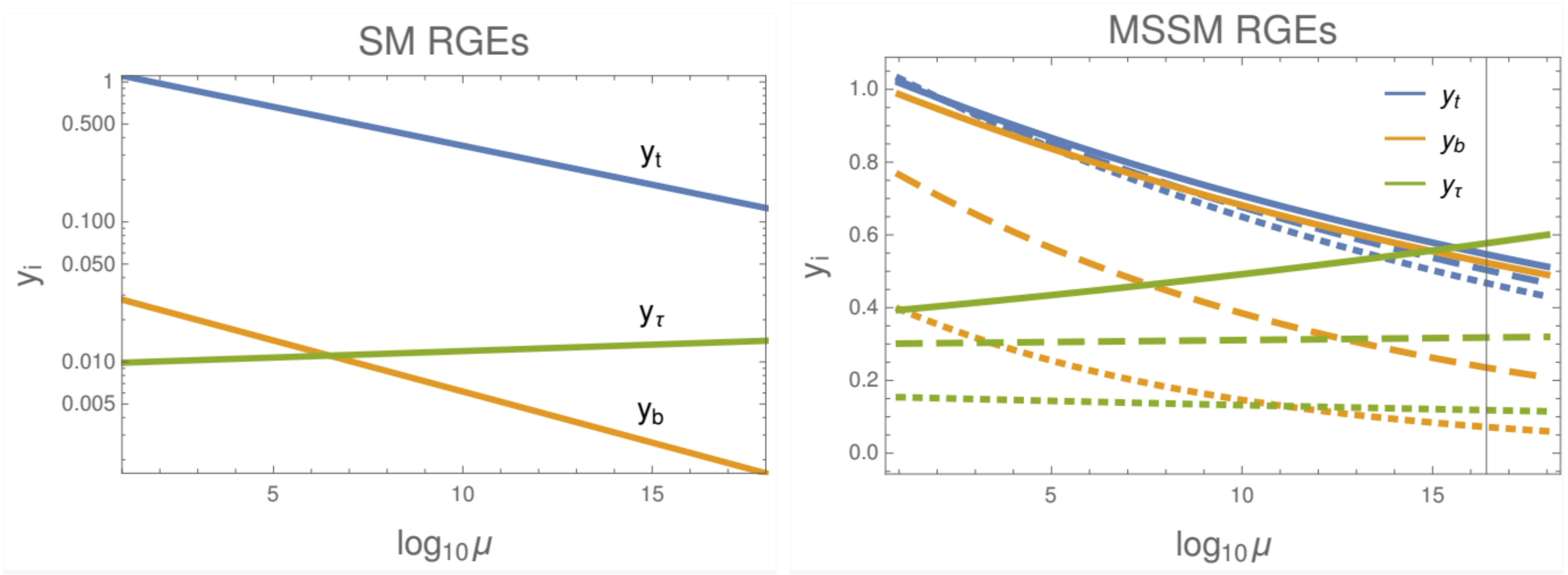


Figure 5: *One loop renormalisation group flow of the SM (left) and MSSM (right) Yukawa couplings, with* $m_0 = 2$ *TeV*, $m_{1/2} = 3$ *TeV*, $A_0 = 0$ *and* $\tan\beta = 40$ *(solid)*, $\tan\beta = 30$ *(dashed) and* $\tan\beta = 15$ *(dotted).*

[Croon, Gonzalo, Graf, Košnik, White '19]

For SUSY GUTs, see e.g. [PDG '22];
[Ananthanarayan, Lazarides, Shafi '91];
[Kelley, Lopez, Nanopoulos, '92];
[Rattazzi, Sarid, Hall '94];
[Baer, Ferrandis '01];
[Blazek, Dermisek, Raby] '02;
[Bajc, Senjanovic, Vissani] '03;
[Hebbar, Leontaris, Shafi] '16;
...

2. Yukawa couplings in SO(10)

Constructing Yukawa sector in SO(10)

- Consider a single fermion representation $\mathbf{16}_F$ in a SO(10) GUT, the mass is computed from the Yukawa couplings between a pair of spinor product (like $\bar{\psi}\psi$) and a scalar field Φ:

$$\mathscr{L}_Y \sim y(\mathbf{16}_F \mathbf{16}_F)\Phi$$

- To ensure SO(10) invariance, Φ must be a scalar representation from the Clebsch–Gordan decomposition on tensor product of spinor representation:

$$\mathbf{16}_F \times \mathbf{16}_F = \mathbf{10}_H + \overline{\mathbf{126}}_H + \mathbf{120}_H$$

- Note that in principle the scalar representation $\mathbf{10}_H$ must be real while $\overline{\mathbf{126}}_H$ must be complex.

Flowing to low-energy

- The SO(10) breaking follows the patterns:

$$\mathrm{SO}(10)(M_U) \longrightarrow \mathrm{EFT}(M_I) \longrightarrow \mathrm{SM}$$

- To obtain the masses/break EW symmetry, this scalar field Φ must include a Higgs field which acquires a vev at EW scale. Thus we must also decompose the scalar representation under the SM group.
- All 3 scalar representations can contain SM Higgs, for example, if the $\mathrm{EFT}(M_I)$ is chosen to be $\mathscr{G}_{422} = \mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$:

$$\mathbf{10}_H \supset (\mathbf{1}, \mathbf{2}, \mathbf{2}) \oplus \ldots \supset \boxed{(\mathbf{1}, \mathbf{2})_{1/2}} \oplus \mathbf{1}, \mathbf{2}_{-1/2} \oplus \ldots$$

$$\overline{\mathbf{126}}_H \supset (\mathbf{15}, \mathbf{2}, \mathbf{2}) \oplus (\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3}) \oplus \ldots \supset \boxed{(\mathbf{1}, \mathbf{2})_{1/2}} \oplus \mathbf{1}, \mathbf{2}_{-1/2} \oplus \ldots$$

$$\mathbf{120}_H \supset (\mathbf{1}, \mathbf{2}, \mathbf{2}) \oplus (\mathbf{15}, \mathbf{2}, \mathbf{2}) \oplus (\mathbf{6}, \mathbf{1}, \mathbf{3}) \oplus \ldots \supset \boxed{(\mathbf{1}, \mathbf{2})_{1/2}} \oplus \mathbf{1}, \mathbf{2}_{-1/2} \oplus \boxed{(\mathbf{1}, \mathbf{2})_{1/2}} \oplus \mathbf{1}, \mathbf{2}_{-1/2} \oplus \ldots$$

Flowing to low-energy

- If only one scalar among $\mathbf{10}_H$, $\overline{\mathbf{126}}_H$ and $\mathbf{120}_H$ is present, it is inevitable that some fermion masses are related at the GUT scale, where the difference comes from different vevs and CG-coefficients, such as:

$\mathbf{10}_H$ case: $m_d = m_e, m_u = m_\nu$

$\overline{\mathbf{126}}_H$ case: $m_e = 3m_d$

(Early prototype of "Yukawa unification")

- The above toy model clearly contradicts to the experiment because of neutrinos.
- A more acceptable model is to use a combination of $\mathbf{10}_H$ and $\overline{\mathbf{126}}_H$ to implement the seesaw mechanism by assigning a vev for $\overline{\mathbf{126}}_H$ at the intermediate scale to break the right-handed symmetry.
- However, a real $\mathbf{10}_H$ and a complex $\overline{\mathbf{126}}_H$ leads to an unrealistic spectrum if there are more than one generation [Bajc, Melfo, Senjanovic, Vissani 07'].

Fermion masses in minimal SO(10)

- The "minimal SO(10) model" have the following Yukawa couplings:

$$-\mathscr{L}_{\text{Yukawa}} = \mathbf{16}_F(Y_{10}\mathbf{10} + Y_{10*}\mathbf{10}^* + Y_{126}\overline{\mathbf{126}}_H)\mathbf{16}_F \qquad \text{(without U(1) PQ)}$$

- The real field $\mathbf{10}$ and $\mathbf{10}^*$ can be combined into a single complex field $\mathbf{10}_H$ by introducing an additional U(1) PQ symmetry, reducing the above Yukawa to:

$$-\mathscr{L}_{\text{Yukawa}} = \mathbf{16}_F(Y_{10}\mathbf{10}_H + Y_{126}\overline{\mathbf{126}}_H)\mathbf{16}_F$$

- Extensive numerical fits to fermion masses and mixings are carried out for the above model (Joshipura *et al.* '11, Dueck *et al.* '13, Altarelli *et al.* '13, Meloni *et al.* '14)
- The mass formulas for quarks and leptons (of the third-generation) are:

$$m_t = v_{10}^u Y_{10} + v_{126}^u Y_{126}, \qquad m_b = v_{10}^d Y_{10} + v_{126}^d Y_{126},$$

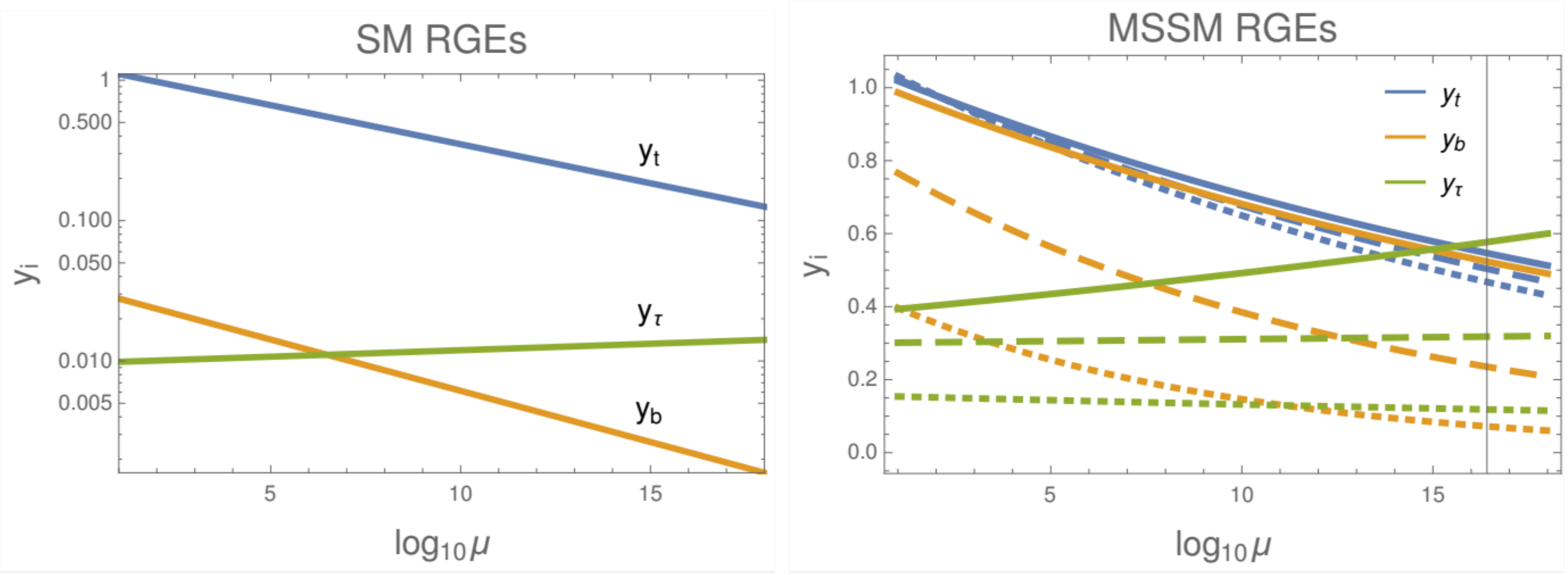
$$m_{\nu_D} = v_{10}^u Y_{10} - 3v_{120}^u Y_{126}, \qquad m_\tau = v_{10}^d Y_{10} - 3v_{126}^d Y_{126}.$$

If v_{126}^d is small enough, we can have $b-\tau$ unification.

3. Unification of Yukawa couplings in SO(10)

Unification of Yukawa couplings

- Yukawa unification are regarded as boundary conditions for the RGEs.
- Yukawa couplings flows to different values in IR because of RGEs.



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[Croon, Gonzalo, Graf, Košnik, White '19]

Can we extend the idea of Yukawa unification to non-supersymmetric case?

[Djouadi, RO, Raidal, '21]

Unification of Yukawa couplings

- Yukawa unification are regarded as boundary conditions for the RGEs.
- Yukawa couplings flows to different values in IR because of RGEs.

What is the implication of Yukawa unification? ← How to motivate the Yukawa unification? ← Can we extend the idea of Yukawa unification to non-supersymmetric case?

[Djouadi, RO, Raidal, ’21]

What is the implication of Yukawa unification?
↓
There is a common origin for Yukawa hierarchy for a single generation.

How to motivate the Yukawa unification?
↓
The original motivation of GUT:

Unification of matter representation:

Fermions: $\mathbf{16} \longrightarrow \mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1}\ (E_6)$

Scalars: $\mathbf{10} + \overline{\mathbf{126}} \longrightarrow ?\ (E_6)$

[Djouadi, Fonseca, RO, Raidal, ’22]

Common origin of Yukawas in minimal SO(10)

- In E_6, we calculate the CG decomposition of spinor product $\mathbf{27}\times\mathbf{27}$ and found:

$$\mathbf{351'} \supset \mathbf{10} + \overline{\mathbf{126}} + \cdots$$

$$Y \times \mathbf{27}_F \cdot \mathbf{27}_F \cdot \mathbf{351'}_H \supset c_{10} Y \times \mathbf{16}_F \cdot \mathbf{16}_F \cdot \mathbf{10}_H + c_{126} Y \times \mathbf{16}_F \cdot \mathbf{16}_F \cdot \overline{\mathbf{126}}_H + \cdots$$

- As $\mathbf{351'}$ is a complex representation, $\mathbf{10}_H$ must be associated to a complex field.
- An E_6-symmetric Yukawa section does not involve the coupling $\mathbf{16_F 16_F 10^*}$, hence, there is no such an interaction at leading order. Its absence can be understood by the fact that E_6 contains an extra U(1) subgroup which commutes with SO(10).
- After CG decomposition, the SO(10) Yukawa couplings are unified by:

$$\frac{Y_{10}}{Y_{126}} = \frac{c_{10} Y}{c_{126} Y} = \frac{c_{10}}{c_{126}} = \sqrt{\frac{3}{5}}$$

[Fonseca, '21]
[Babu, Bajc, Susič, '15]

Flowing to low-energy

- The SO(10) breaking follows the patterns:

$$\mathrm{SO}(10)(M_U) \longrightarrow \mathrm{EFT}(M_I) \longrightarrow \mathrm{SM}$$

- For convenience, the $\mathrm{EFT}(M_I)$ is chosen to be $\mathscr{G}_{422} = \mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$.
- It is important to note that it is natural to have two Higgses at the EW scale:

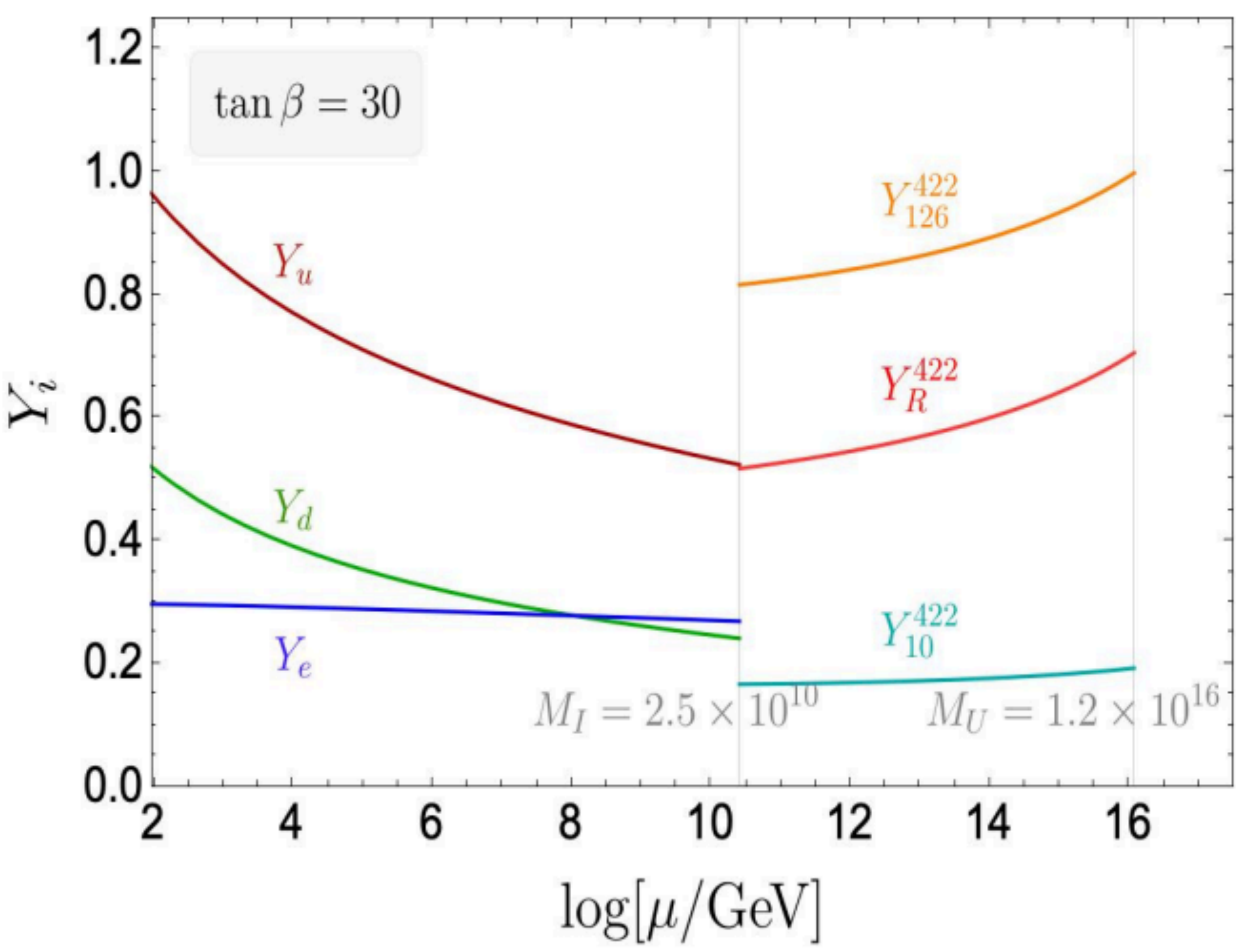
$$\mathbf{10}_H \supset (\mathbf{1}, \mathbf{2}, \mathbf{2}) \oplus \ldots \supset \boxed{(\mathbf{1}, \mathbf{2})_{1/2} \oplus \mathbf{1}, \mathbf{2}_{-1/2}} \oplus \ldots$$

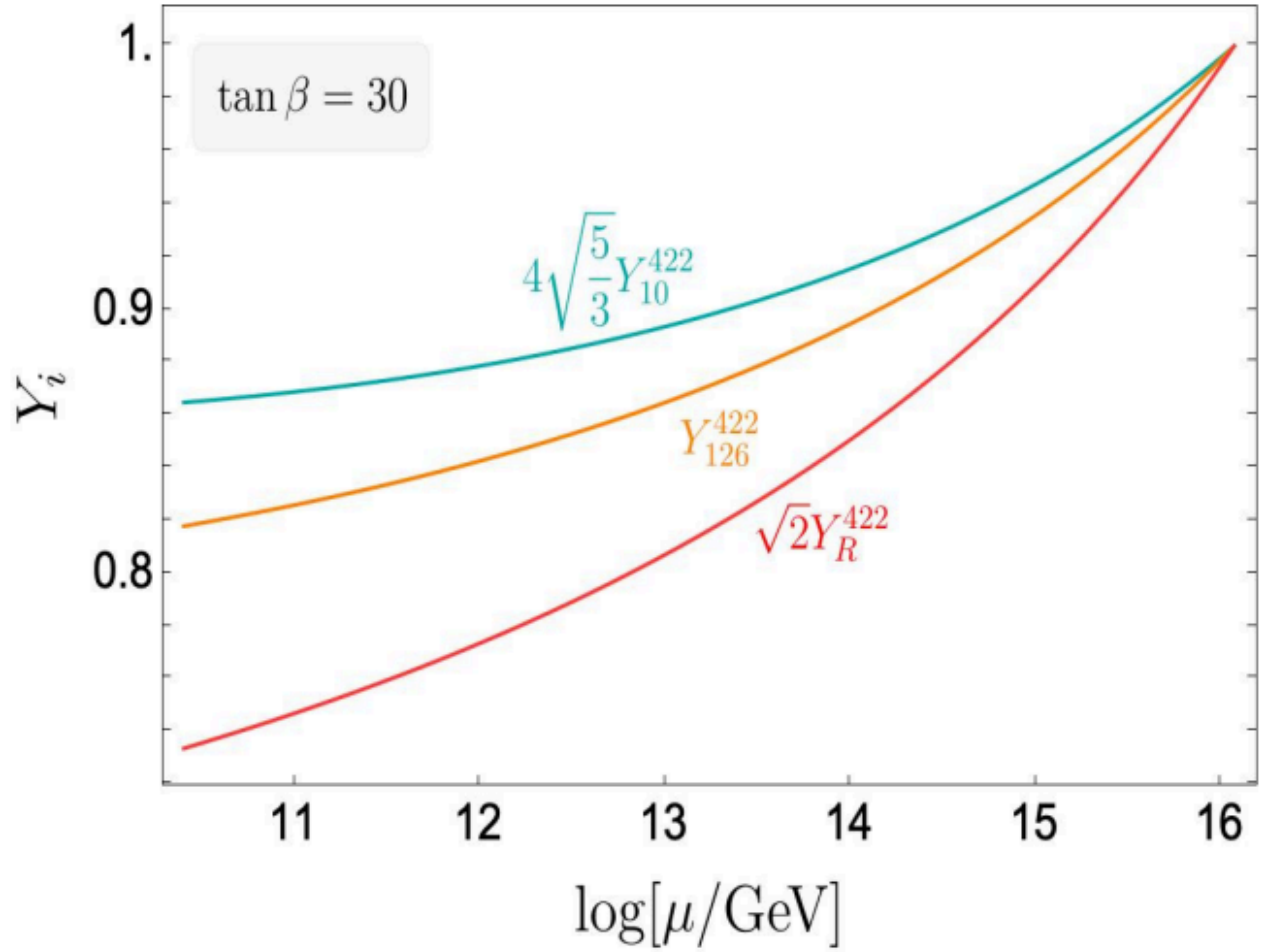
$$\overline{\mathbf{126}}_H \supset (\mathbf{15}, \mathbf{2}, \mathbf{2}) \oplus (\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3}) \oplus \ldots \supset \boxed{(\mathbf{1}, \mathbf{2})_{1/2} \oplus \mathbf{1}, \mathbf{2}_{-1/2}} \oplus \ldots$$

- Therefore, the low-energy model is a two-Higgs doublet model (2HDM)

Unification of fundamental couplings

- In non-SUSY SO(10) case, after solving the RGEs of gauge and Yukawa couplings and implementing the desired boundary conditions for the RGEs, the models are very constrained with the only two free parameter identified with the $\tan\beta$ of low energy 2HDM and GUT-scale unified Yukawa coupling Y_U





What happens at the intermediate scale?

- The mass should be continuous at the intermediate scale M_I. Therefore some matching conditions can be deduced for Yukawa couplings in both EFTs above or below M_I.

- From 422 model: $m_t = \frac{v_{10}^u}{\sqrt{2}} \boxed{Y_{10}^{422}} + \frac{v_{126}^u}{4\sqrt{2}} \boxed{Y_{126}^{422}}, \; m_b = \frac{v_{10}^d}{\sqrt{2}} \boxed{Y_{10}^{422}} + \frac{v_{126}^d}{4\sqrt{2}} \boxed{Y_{126}^{422}}, \; m_\tau = \frac{v_{10}^d}{\sqrt{2}} \boxed{Y_{10}^{422}} - \frac{3v_{126}^d}{4\sqrt{2}} \boxed{Y_{126}^{422}}.$

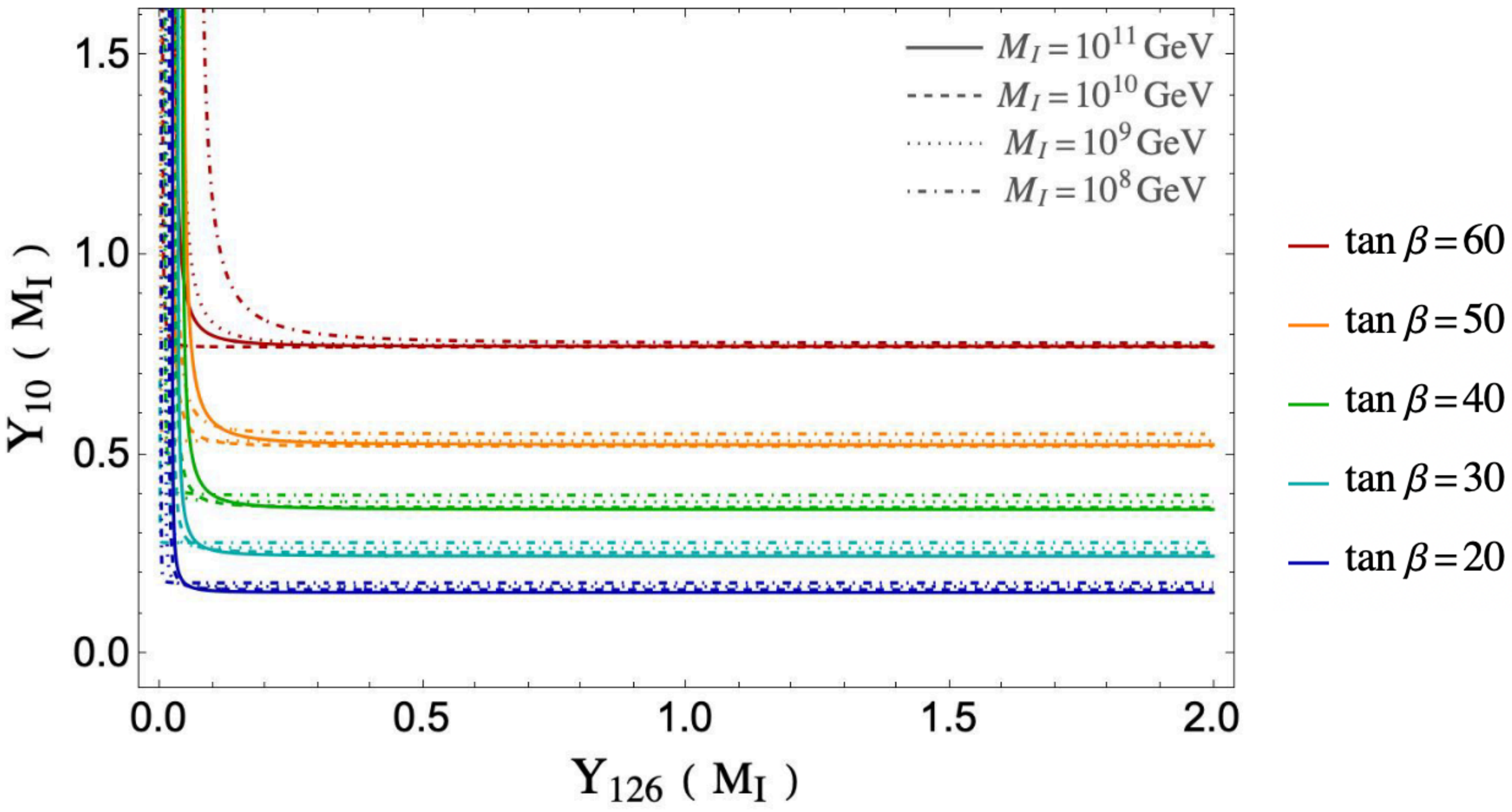
- From 2HDM: $m_t = \frac{1}{\sqrt{2}} \boxed{Y_t} v_u \,, \quad m_b = \frac{1}{\sqrt{2}} \boxed{Y_b} v_d \,, \quad m_\tau = \frac{1}{\sqrt{2}} \boxed{Y_\tau} v_d \,.$

What happens at the intermediate scale?

- These relations can be simplified to be (assuming no tree-level FCNCs):

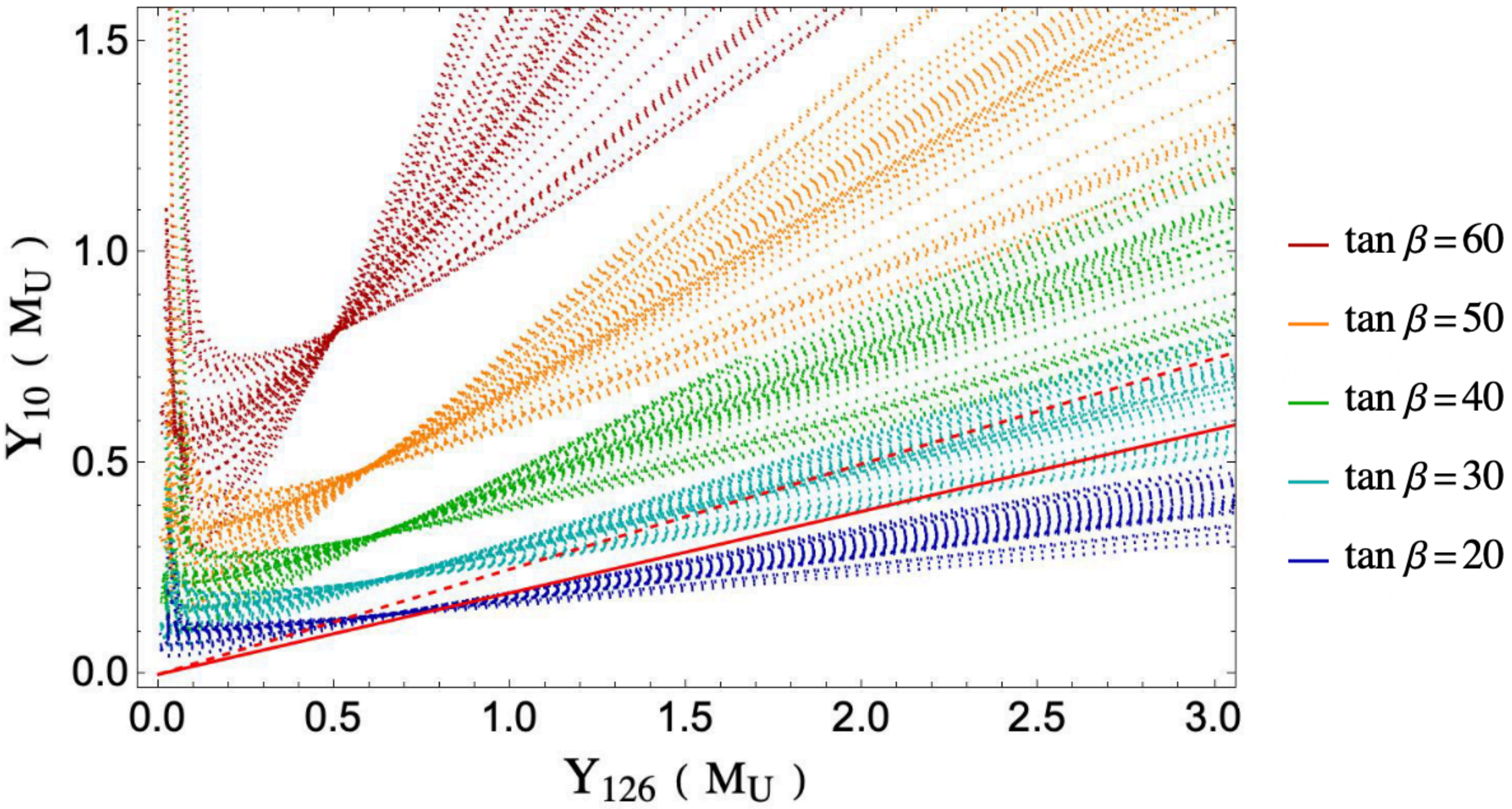
$$\left(Y_{10}^{422}(M_I)\right)^2 = \frac{\left(Y_{126}^{422}(M_I)\right)^2 \left(3Y_b(M_I) + Y_\tau(M_I)\right)^2}{16\left[\left(Y_{126}^{422}(M_I)\right)^2 - \left(Y_b(M_I) - Y_\tau(M_I)\right)^2\right]}$$

Constraints from Yukawa unification



Visualizing the matching conditions

Constraints from Yukawa unification



(Numerical) Solutions of RGEs + matching conditions

Implications of Yukawa unification

- The constraint from unification of Yukawa couplings imposes non-trivial relations on the parameters of the scalar sector, which is described by the (numerical) solution of RGEs of Yukawa couplings with particular boundary conditions and matching conditions.
- The original dimesionless parameters (Yukawa couplings) will be related to the ratio of vevs ($\tan\beta$). The unification of Yukawa couplings in our model implies that $\tan\beta \lesssim 30$, which can be tested in future collider experiment. [e.g. PDG '23]
- Yukawa unification implies that the Yukawa hierarchy of a single generation can be explained dynamically by higher rank symmetry and RGEs.

Conclusions

Conclusions

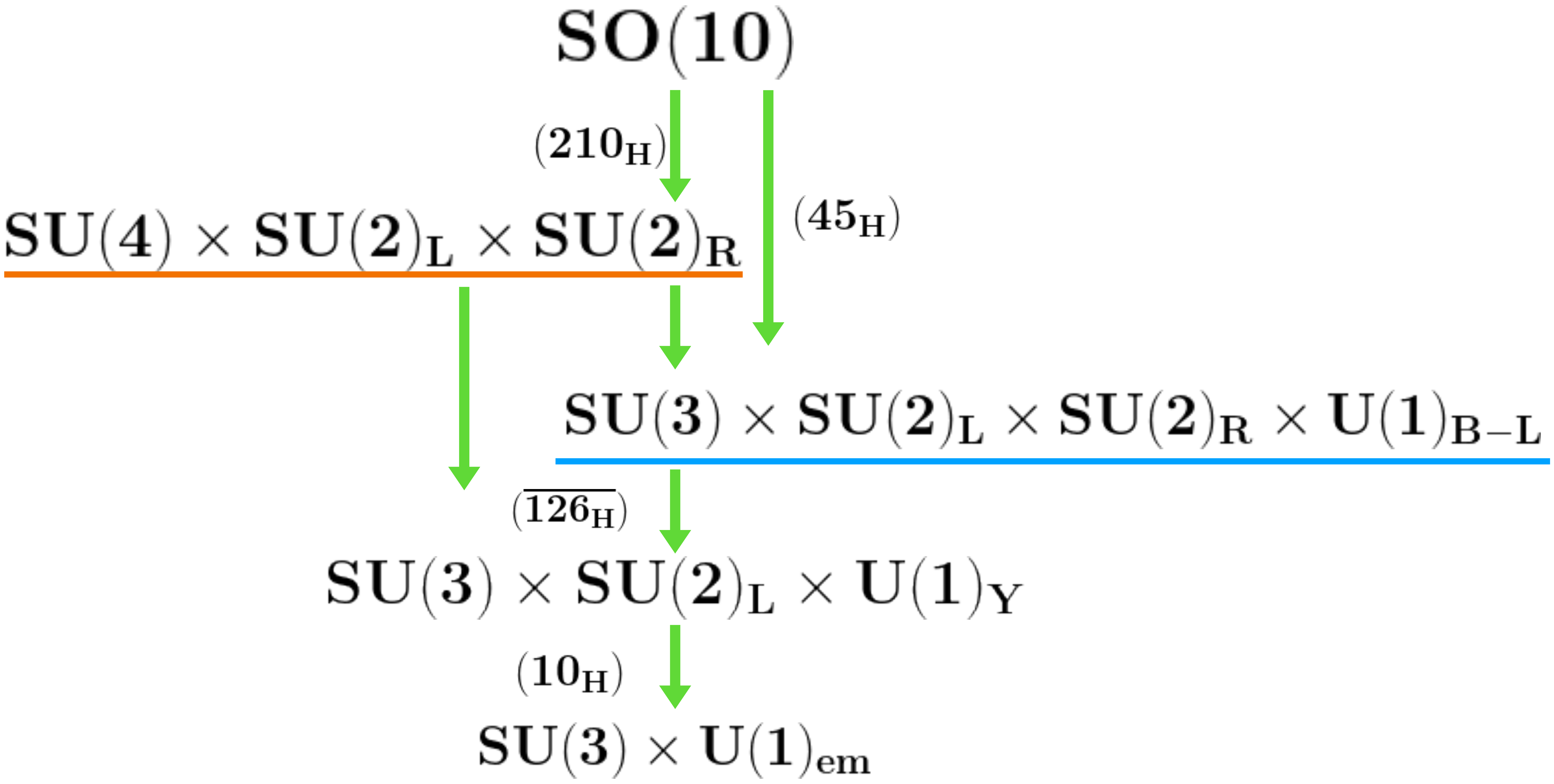
- We motivate and realize the unification of gauge and Yukawa couplings in non-supersymmetric SO(10) models.
- We introduce the Yukawa section in minimal SO(10) models in details.
- The discuss the implications from unification of Yukawa couplings.
- Finally, we are still lack of understanding of the analytical structures of RGEs in these cases, especially for the Yukawa couplings for non-trivial BSM models.

Thank you very much for your attention!

Outline

1. Introduction & Motivation
2. Non-SUSY SO(10) grand unified theory
3. Constraints from unification of fundamental couplings
4. Conclusions

Unification of fundamental couplings



$$\mathrm{PS}: \quad \mathrm{SO}(10)|_{M_U} \xrightarrow{\langle \mathbf{210_H} \rangle} \mathcal{G}_{422}|_{M_I} \xrightarrow{\langle \overline{\mathbf{126}}_\mathbf{H} \rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle \mathbf{10_H} \rangle} \mathcal{G}_{31} \quad \mathrm{LR}: \quad \mathrm{SO}(10)|_{M_U} \xrightarrow{\langle \mathbf{45_H} \rangle} \mathcal{G}_{3221}|_{M_I} \xrightarrow{\langle \overline{\mathbf{126}}_\mathbf{H} \rangle} \mathcal{G}_{321}|_{M_Z} \xrightarrow{\langle \mathbf{10_H} \rangle} \mathcal{G}_{31}$$

Principles for EFT model building

Agmon, Bedroya, Kang, Vafa '22

- Symmetry principle: all terms allowed by symmetries are allowed. Renormalizability is certainly not required. The symmetry $\mathscr{G}_{\text{Lorentz}} \times \mathscr{G}_{\text{Gauge}}$ is a free parameter.

- UV/IR decoupling principle: low-energy physics can be effectively described independently of high-energy physics within the EFT framework. (Wilson's Renormalization group)

- Naturalness principle: coupling constants in a theory are of order one in the appropriate mass scale. Therefore, if any parameter is unusually small or large, a good explanation, such as an underlying symmetry, is require.

The survival hypothesis

- The survival hypothesis: scalars should have masses of order 1 at the symmetry breaking scale (the GUT scale), unless there are symmetries to protect their masses. (Again motivated by Naturalness)

- Only certain scalar components from $\mathbf{10_H}$ and $\overline{\mathbf{126}}_\mathbf{H}$ representations can acquire small vevs, so they can stay light below the GUT scale;

The EFT at intermediate scale

- The EFT at the intermediate scale should be left-right symmetric in the discussed breaking chains: it is a left-right model where the left-handed and right-handed fermions are coupled via a bi-doublet scalar field as

$$\bar{F}_L(Y_{10}\Phi_{10} + Y_{126}\Sigma_{126})F_R + Y_R F_R^T C\overline{\Delta_R} F_R + \mathrm{h.c.}$$

- The $\mathrm{SU}(2)_R$ right-handed symmetry will be broken by the right-handed triplet field Δ_R, which acquires an intermediate scale masses.

- Below the intermediate scale, we can integrate out the heavy gauge bosons and decouple most scalars except for the (two) Higgs doublet fields. So we should end up with a two Higgs doublet model (2HDM) at lower energy.

SO(10) as BSM model

- SO(10) models generalize the gauge group of SM to a larger gauge symmetry. The vacuum structure is much more complicated with many different phases. We can have different intermediate breaking patterns.

- The fermion within one generation plus a right-handed neutrino can all be embedded into a single representation $\mathbf{16_F}$ of SO(10).

- The SM Higgs field, with hypercharge +1/2, come from a decomposition of the SO(10) scalar field (can be a mixing of Φ_{10} and Σ_{126}).

- At the intermediate scale, we will have a left-right model, which is broken by the vev of Δ_R. The right-handed neutrinos can thus get Majorana masses at the scale Δ_R, and triggers the seesaw mechanism in this scenario.

What happens at the intermediate scale?

- We assume that the mass should be continuous at the intermediate scale. We will then have a matching conditions coming from the mass relations from low-energy EFT and intermediate scale models:

  In 422 intermediate scale model:

$$m_t = \frac{v_{10}^u}{\sqrt{2}} Y_{10}^{422} + \frac{v_{126}^u}{4\sqrt{2}} Y_{126}^{422}, \; m_b = \frac{v_{10}^d}{\sqrt{2}} Y_{10}^{422} + \frac{v_{126}^d}{4\sqrt{2}} Y_{126}^{422}, \; m_\tau = \frac{v_{10}^d}{\sqrt{2}} Y_{10}^{422} - \frac{3v_{126}^d}{4\sqrt{2}} Y_{126}^{422},$$

  In 2HDM:

$$m_t = \frac{1}{\sqrt{2}} Y_t v_u \,, \quad m_b = \frac{1}{\sqrt{2}} Y_b v_d \,, \quad m_\tau = \frac{1}{\sqrt{2}} Y_\tau v_d \,.$$

Proton decay

- The proton decay is a function of unification scale as well as the unified coupling, for example:

$$\tau(p \to e^+\pi^0) \simeq (7.47 \times 10^{35}\text{yr})\left(\frac{M_U}{10^{16}\ \text{GeV}}\right)^4 \left(\frac{0.03}{\alpha_U}\right)^2$$

Meloni-Ohlsson-Pernow ’20

Proton decay

- Numerical result: proton decay only preferred the Pati-Salam (422) and Minimal Left-Right (3221) breaking chains of SO(10).

Breaking chain	$\log\left(\frac{M_{Ic}}{\mathrm{GeV}}\right)^{2-\mathrm{loop}}$	$\log\left(\frac{M_{Uc}}{\mathrm{GeV}}\right)^{2-\mathrm{loop}}$	$\alpha_U^{2-\mathrm{loop}}$	$\tau(p \to e^+\pi^0)/\mathrm{yr}$
422	10.03	16.19	0.032	3.82×10^{36}
3221	10.66	15.45	0.023	7.84×10^{33}
422D	13.65	14.66	0.026	4.22×10^{30}
3221D	11.82	14.63	0.024	3.89×10^{30}

Table 3: A summary table of the numerical results of the intermediate scale, the unification scale, and the universal gauge coupling at the two-loop level, neglecting all the threshold corrections as well as the estimated proton lifetimes obtained for each considered breaking chain with two Higgs doublets at the electroweak scale. The ratio of vevs is fixed to $\tan\beta = 65$ as the results do not change significantly for lower values of $\tan\beta$.

Scalar multiplets in different breaking chains

Intermediate symmetry	Scalar Multiplets
422	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_R \oplus \Delta_{45R}$
422D	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_L \oplus \Delta_R \oplus \Delta_{45L} \oplus \Delta_{45R}$
3221	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_R$
3221D	$\Phi_{10} \oplus \Sigma_{126} \oplus \Delta_L \oplus \Delta_R \oplus \Delta_{45L} \oplus \Delta_{45R}$

Table 1: List of scalar multiplets containing light fields, for each intermediate symmetry. They are the only ones which are not integrated out below the SO(10) symmetry breaking scale mass M_U.