

Cosmic superstrings vs gauge
strings: gravitational wave signals

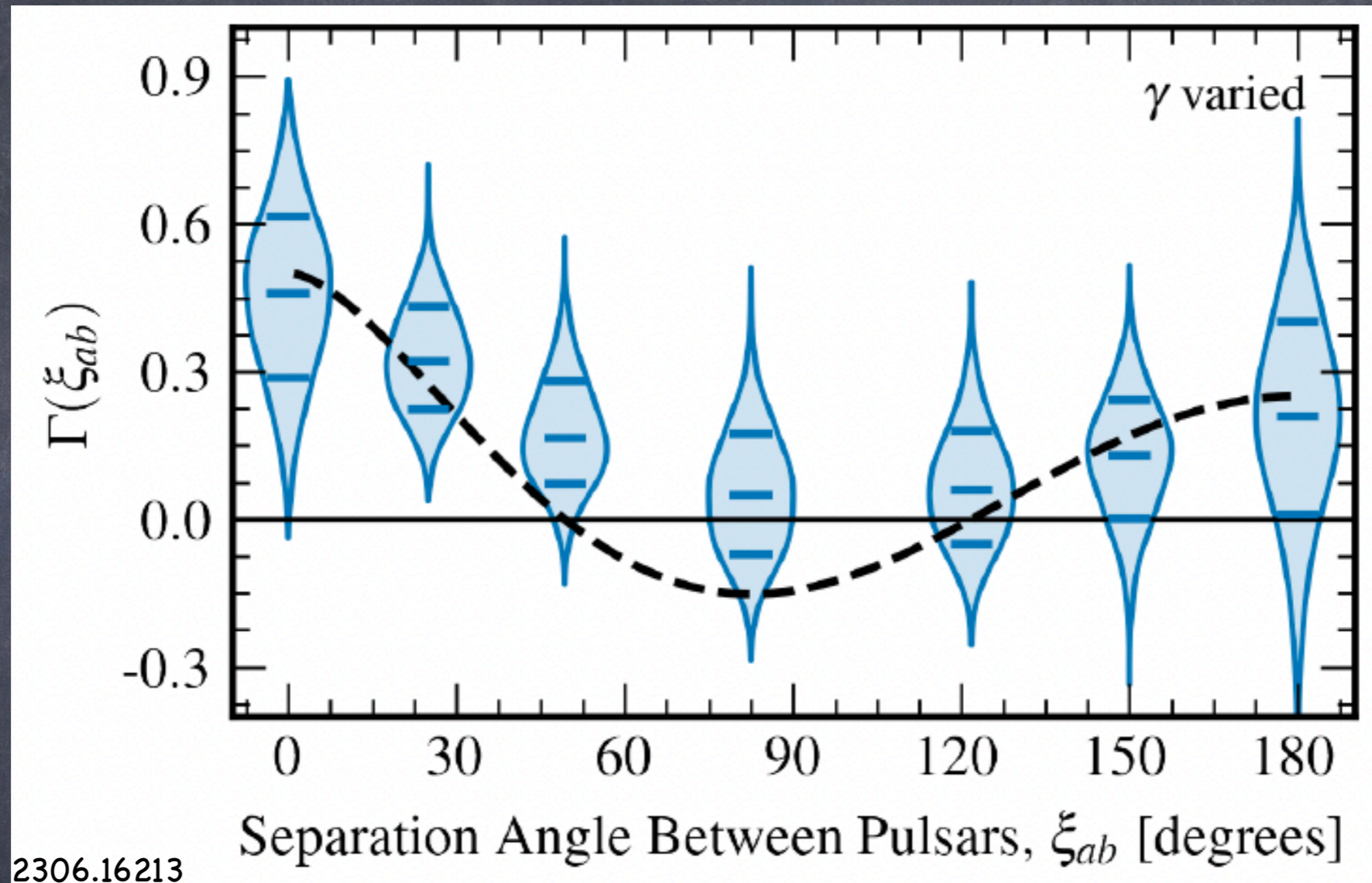
Pulsar Timing Arrays



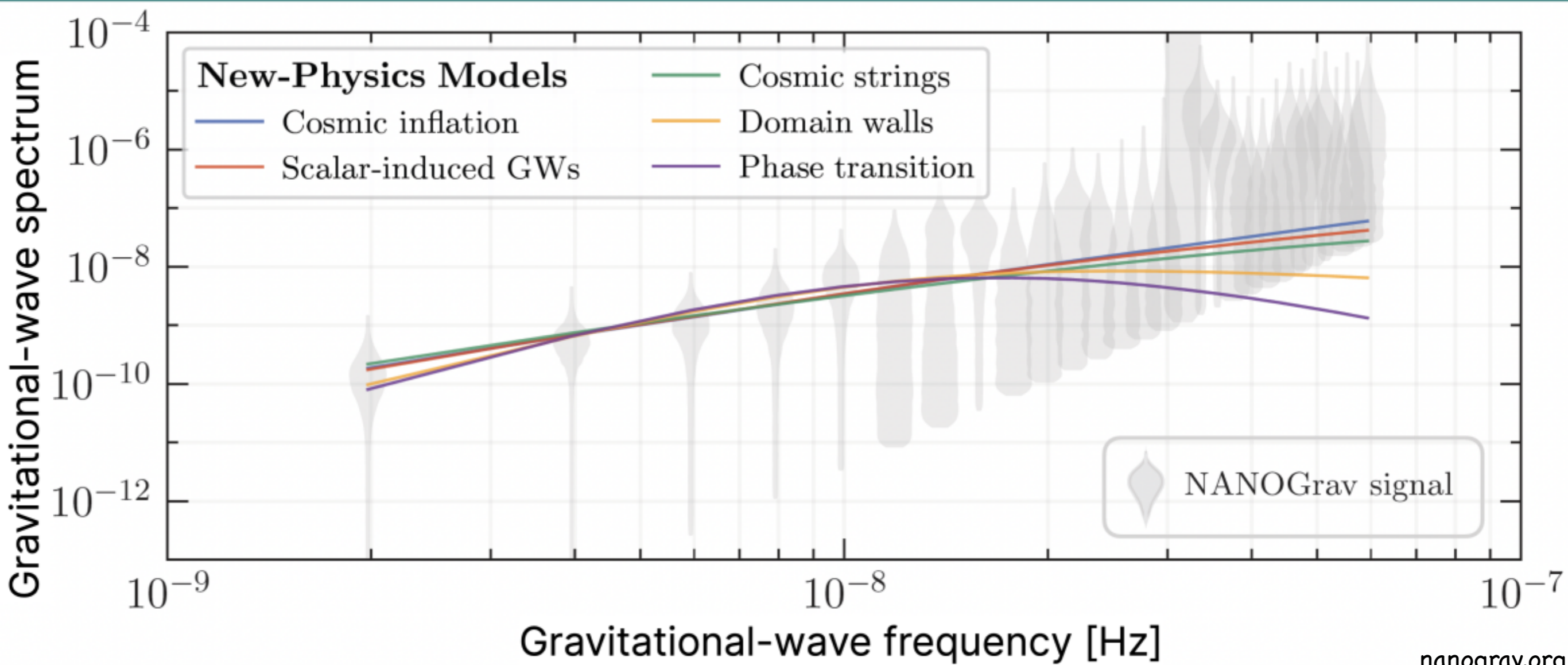
- Galactic size GW detector made of millisecond pulsar array
- Precise rotation periods and radio pulses from poles of pulsars make them ultra-precise clocks

- GW perturbs spacetime between the pulsar and Earth and changes the time of arrival (TOA) of pulses
- Measure residuals in TOA: $R^a = TOA_{measured}^a - TOA_{model}^a$
- Cross-correlate timing residuals of pairs of pulsars separated by angle ξ_{ab}
- Sensitive to frequencies between $1/(\text{total observation time})$ and $1/\text{cadence}$ i.e., [1/years, 1/weeks]

Hellings-Downs curve

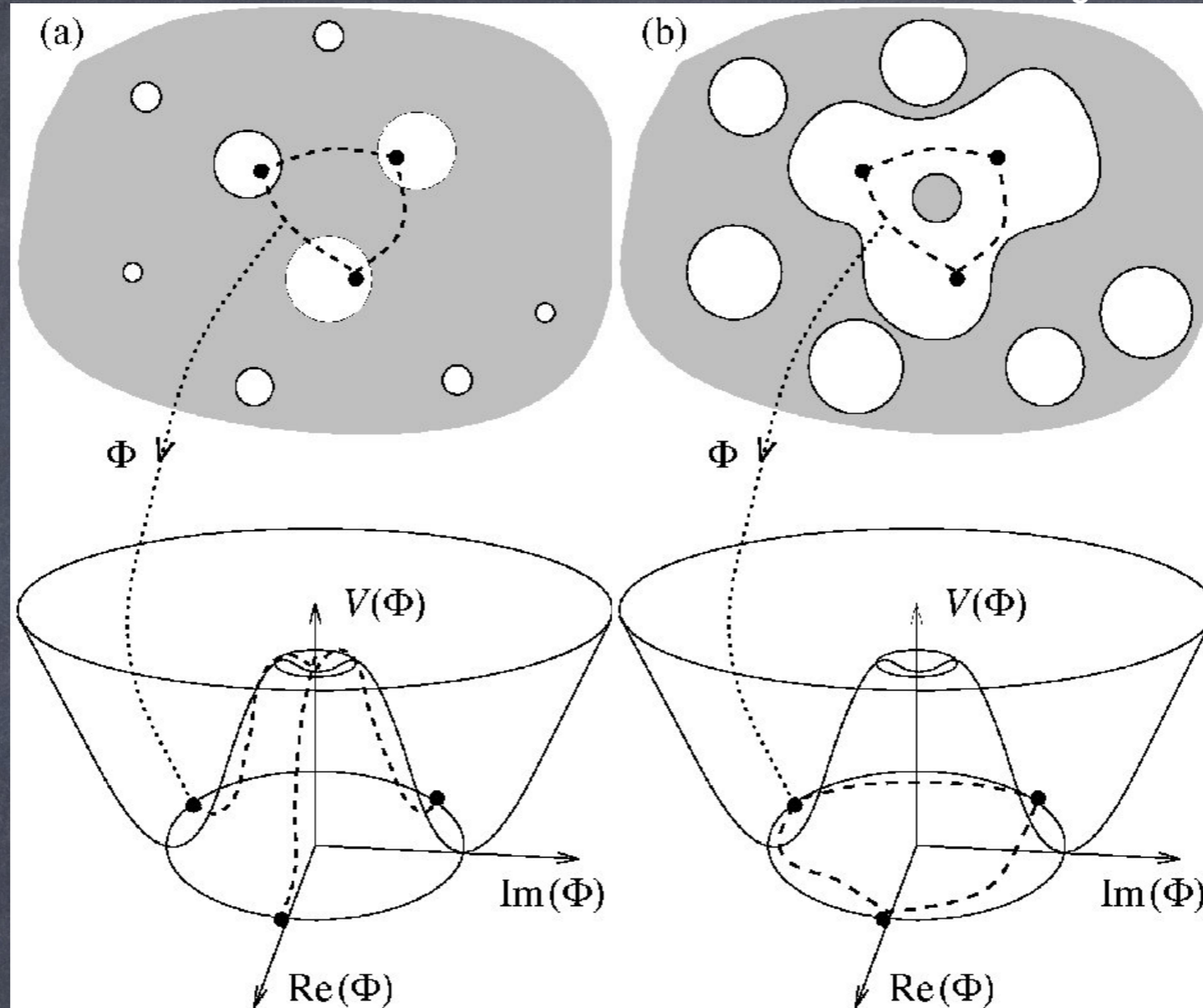


- PTAs see $\sim 3\sigma$ quadrupole correlation of timing residuals
- Smoking gun signal of stochastic GW background



Cosmic strings

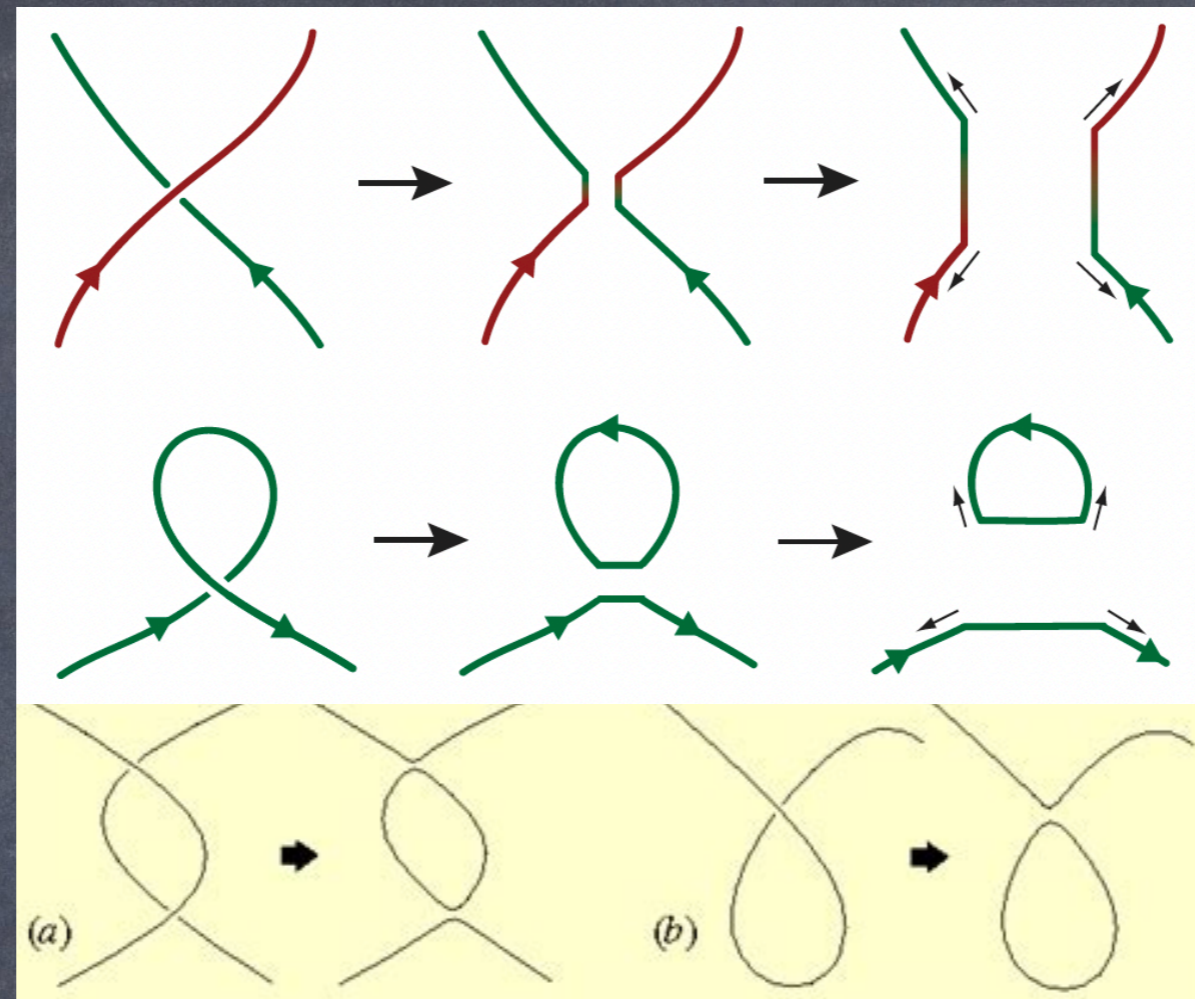
Jeong, Smoot



- Can have field theoretic origin, e.g., from spontaneous $U(1)$ symmetry breaking
- Can be fundamental strings of superstring theory stretched to cosmic scales

String interactions

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- Strings collide and reconnect (with segments exchanged) with intercommutation probability P
- Produce loops by self-interactions or pair-wise interactions
- Loops can have kinks or cusps

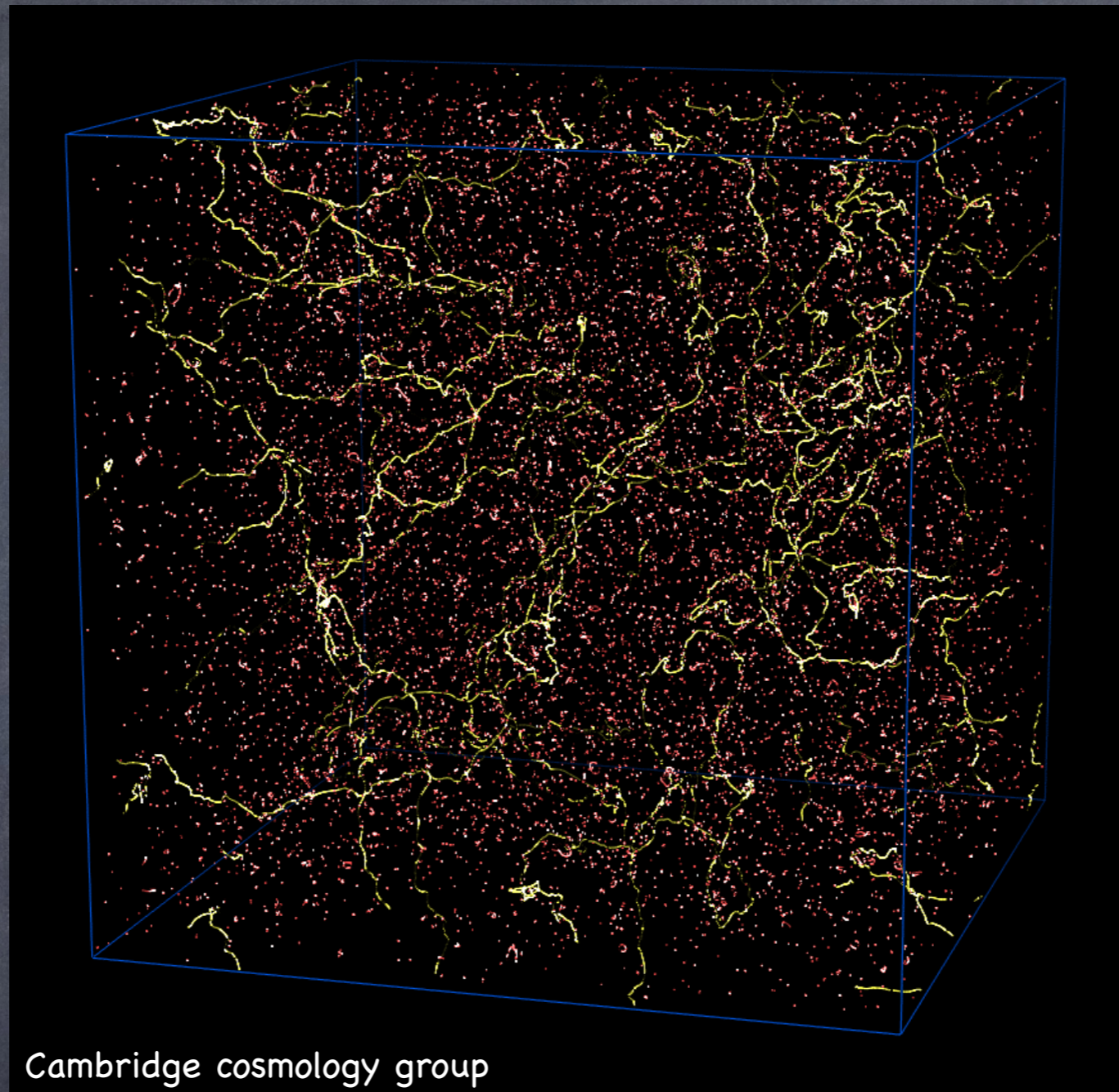
Velocity-dependent one-scale model

- Infinitely thin strings with no couplings to matter and energy/length (tension) μ
- Network evolution described by characteristic length scale L and rms velocity v of string segments
- In scaling regime (self-similar evolution),
$$\rho = \mu/L^2 \sim \mu/t^2 \sim \mu H^2 \implies \Omega = \text{constant}$$
- So cosmic strings don't overclose the Universe
- Scaling solution is an attractor solution

$$\dot{\rho} = - \left[2H(1 + v^2) + \frac{\tilde{c}v}{L} \right] \rho \implies t \dot{\xi} = \beta(1 + v^2)\xi - \xi + \frac{1}{2}\tilde{c}v$$

$$\dot{v} = (1 - v^2) \left[\frac{k(v)}{L} - 2Hv \right] \implies t \dot{v} = (1 - v^2) \left[\frac{k(v)}{\xi} - 2\beta v \right]$$

- $\xi = L/t$ is constant in the scaling regime
- $\beta = Ht$ is $\sim 1/2$ in the radiation era and $2/3$ in the matter era
- $\tilde{c} \simeq 0.23$ is the efficiency of chopping loops from the network
- $k(v)$ accounts for acceleration due to curvature of strings



CMB bound $G\mu \lesssim 10^{-7}$

GWs from oscillating loops

- Energy lost by network ends up in loops
- Oscillating loops lose energy mainly by GW emission
- According to Einstein quadrupole formula, GW power emitted by string loop is $\dot{E} = \frac{G}{5c^5} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle$
- Loop of length ℓ , mass $M = \mu\ell$, quadrupole moment $Q \sim M\ell^2$ oscillates with frequency $\omega \sim 1/\ell$ and emits GWs with power $\dot{E} \sim GQ^2\omega^6 \sim GM^2\ell^4\ell^{-6} \sim G\mu^2$

Loop number density distribution

• All initial loop lengths are a fraction $\alpha_L = 0.37$ of L

• New loops produced from the network with rate

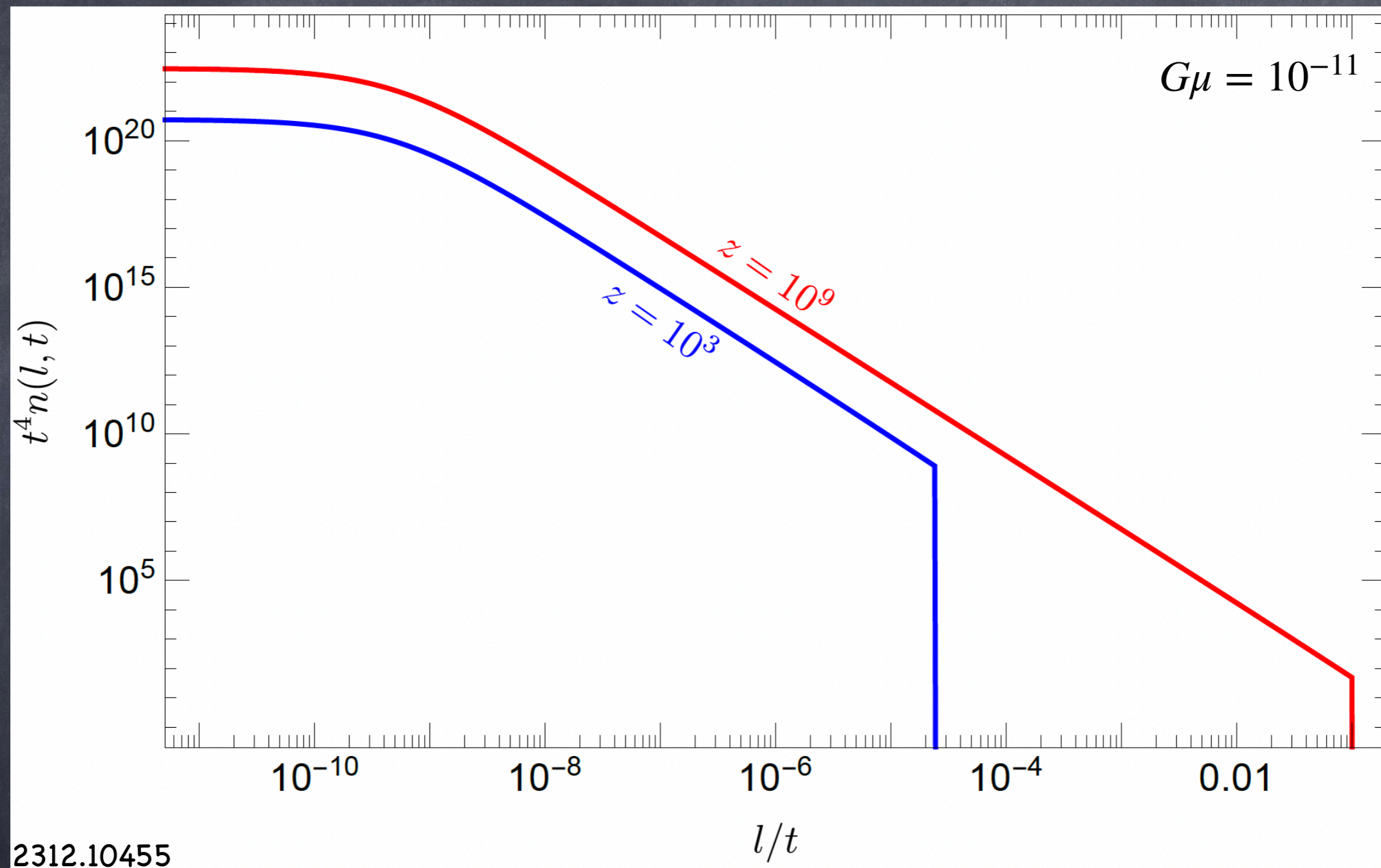
$$\dot{\rho}_0 \propto \frac{\tilde{c}v}{L}\rho$$

• Loops shrink at a constant rate $\dot{\ell} = -\Gamma G\mu$ (where $\Gamma = \dot{E}/(G\mu^2)$ is peaked at ≈ 50)

• Number density scales as $1/a^3$

$$t^4 n(\ell, t) = \mathcal{F} \frac{\tilde{c}v_\star}{\gamma_{v\star} \alpha_L \xi_\star^4} \frac{1}{\alpha_L \xi_\star + \alpha_L \dot{\xi}_\star t_\star + \Gamma G\mu} \left[\frac{a(t_\star)}{a(t)} \right]^3 \left[\frac{t}{t_\star} \right]^4$$

★ \equiv values at production



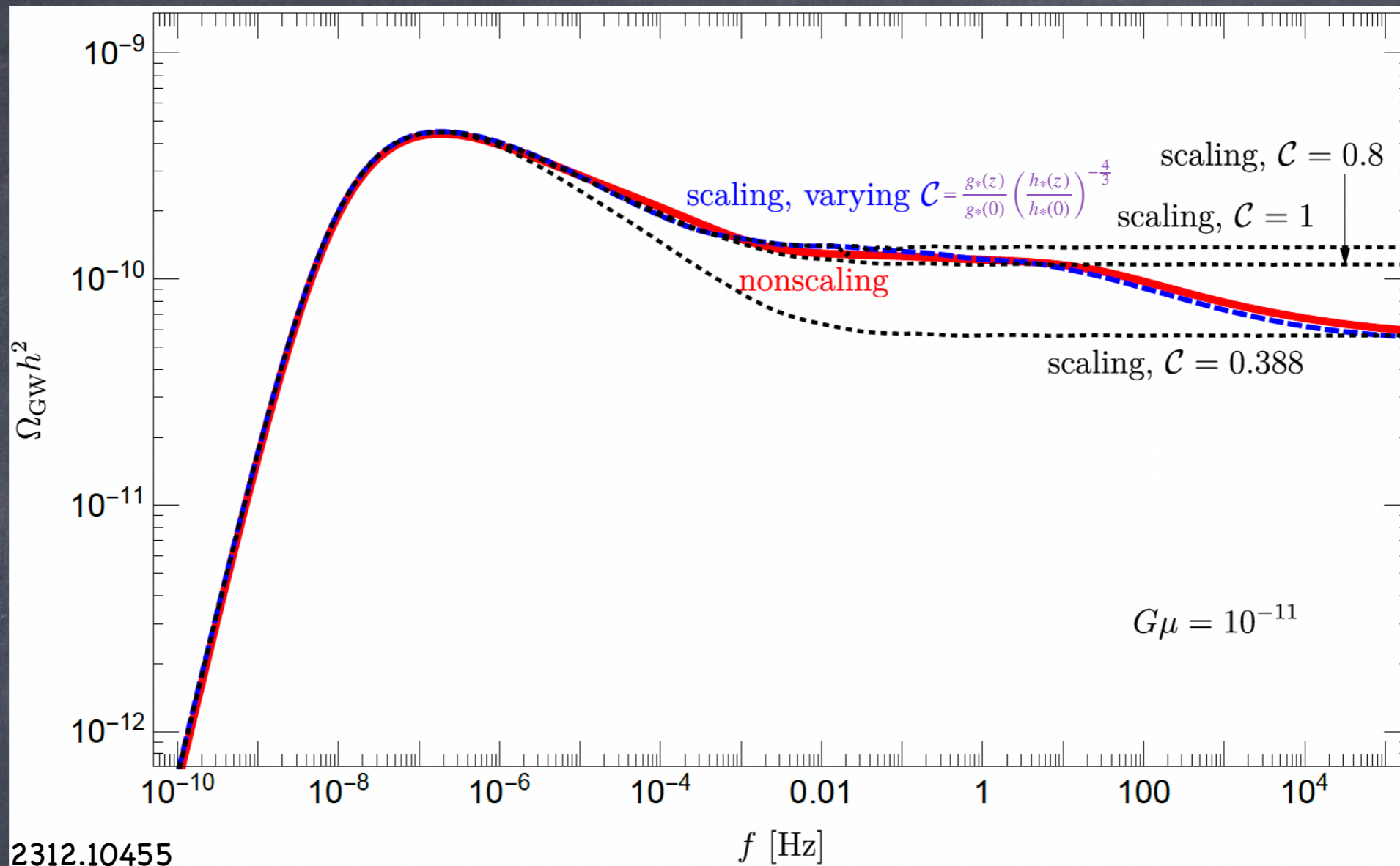
GW spectrum

• GW signal at frequency f today $\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}(t_0, f)}{df}$

• Signal at frequency f arises from superposition of GW emission from all harmonic modes k of loops of length ℓ at redshift z such that the redshifted frequency is f today: $\ell = \frac{2k}{(1+z)f}$

• $\frac{d\rho_{\text{GW}}(t_0, f)}{df} = G\mu^2 \sum_k C_k(f) P_k$ where $P_k \simeq \Gamma k^{-q} / \zeta(q)$. $G\mu^2 P_k$

is the GW power emitted by a loop in mode k , and $C_k(f)$ is the time-integrated weight function of loops in mode k that emit GWs detected with frequency f



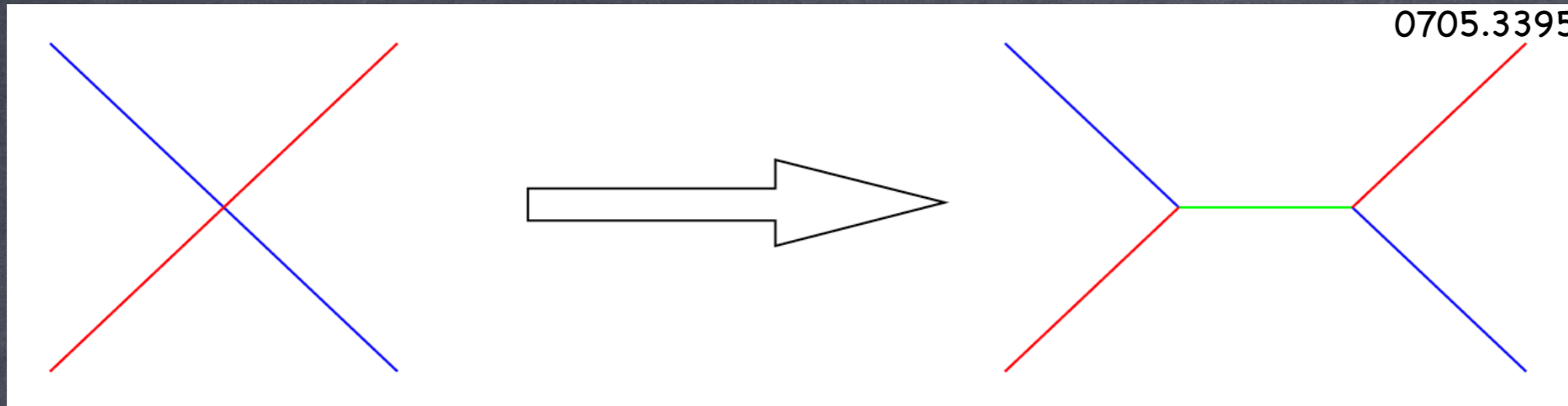
- Power-law dependence in PTA band (1–100 nHz) with $\Omega_{\text{GW}}(f)h^2 \propto f^{3/2}(G\mu)^2$
- Flat spectrum in interferometer band ($10^{-3} - 10^4$ Hz) with $\Omega_{\text{GW}}(f)h^2 \propto \sqrt{G\mu}$

Cosmic superstrings

- Network comprised more than one type of string

$$\begin{aligned} \text{string 1 (F-string)} &: \mu_1 = \mu_F, \\ \text{string 2 (D-string)} &: \mu_2 = \mu_F/g_s, \\ \text{string 3 (FD-string)} &: \mu_3 = \mu_F\sqrt{1 + 1/g_s^2}. \end{aligned}$$

- Since strings evolve in higher-dimensional space and reconnection is a quantum process that depends on g_s , intercommutation probability $P_{ij} < 1$
- For F-F string interactions, $P_{11} \sim g_s^2 \in (10^{-3}, 1)$
- For D-D and FD-FD interactions, $P_{22}, P_{33} \in (0.1, 1)$ obtained non-perturbatively
- For F-D and F-FD interactions, $P_{12}, P_{13} \sim g_s \in (10^{-2}, 1)$



- Network evolution is more complex because different string types can zip together to form a segment (zipper) of another string type

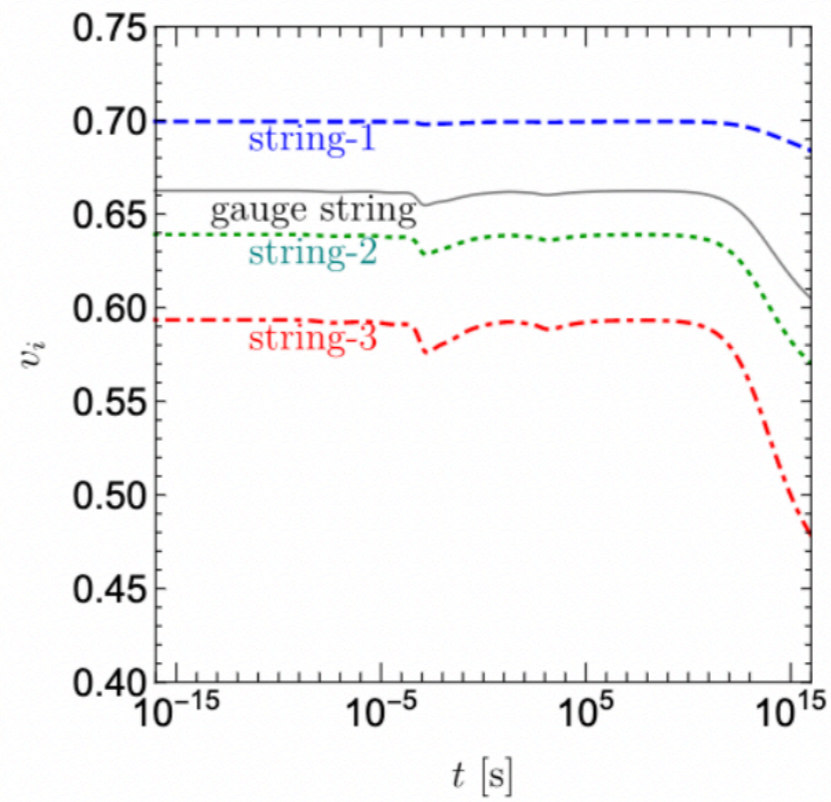
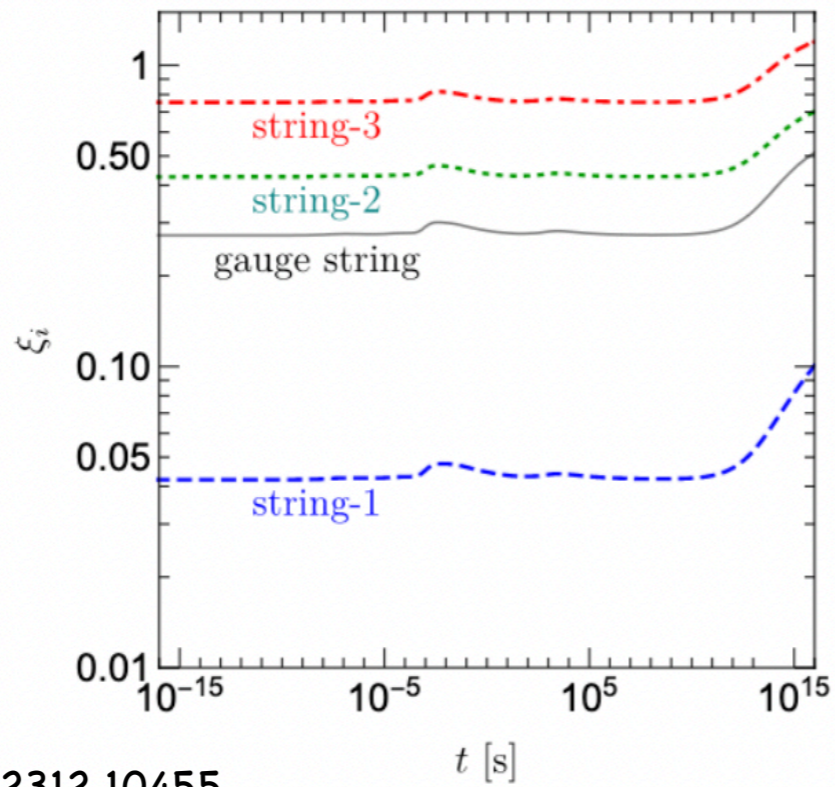
$$\dot{\rho}_i = - \left[2H(1 + v_i^2) + \frac{\tilde{c}_i v_i}{L} \right] \rho_i + \sum_{j,k} \dot{\rho}_{j,k \rightarrow i} - \sum_{j,k} \dot{\rho}_{i,j \rightarrow k}$$

- ... parameterized by chopping efficiency $\tilde{c}_i = \tilde{c} P_{ii}^{1/3}$ and **cross-interaction efficiency** $\tilde{d}_{jk}^i \propto P_{jk}^{1/3}$ to produce a type- i zipper from a collision of a type- j and type- k string

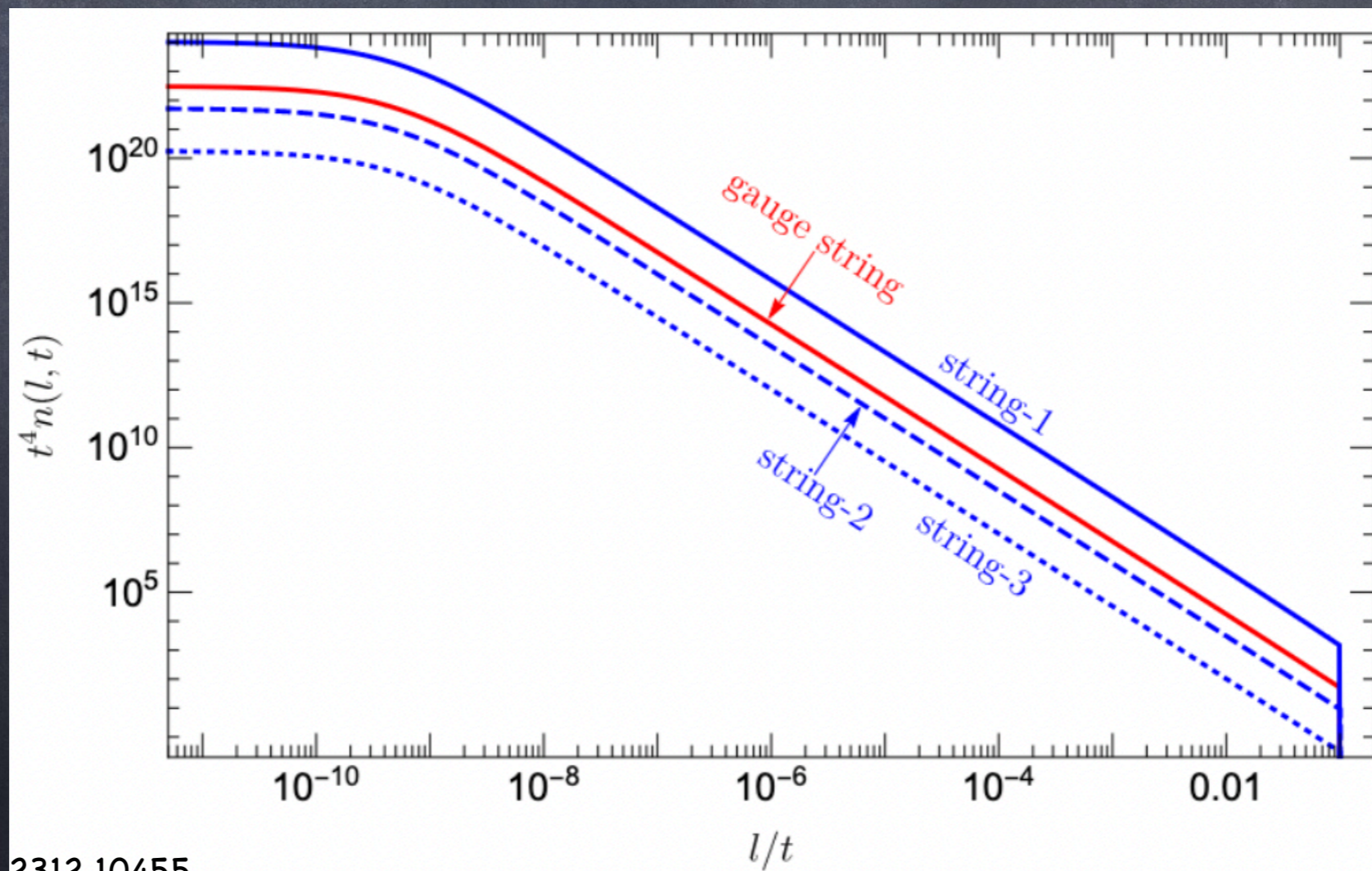
- These parameters depend on the string coupling constant g_s and a **volume suppression factor** w
- Energy loss is less efficient than for gauge strings so network is denser
- Loop number density of type- i string is enhanced/suppressed compared to gauge strings by a factor

$$N_i \simeq 0.04 P_{ii}^{1/3} \frac{V_i}{\gamma_{v_i} \xi_i^3}$$

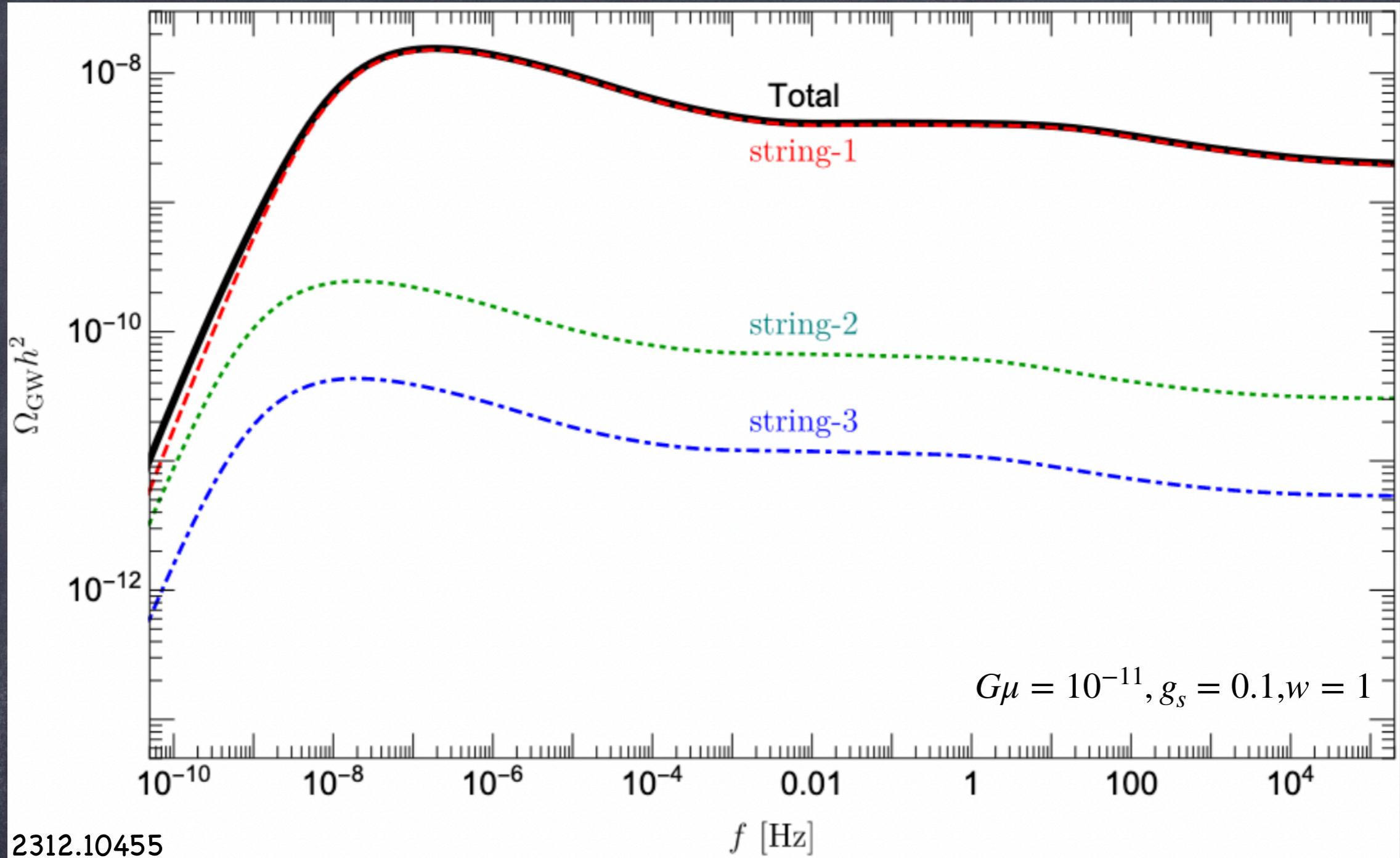
g_s	$w = 1$			$w = 0.1$		
	N_1	N_2	N_3	N_1	N_2	N_3
0.04	71.3	0.116	0.0543	238	0.101	0.0602
0.1	33.8	0.180	0.0336	73.6	0.156	0.0365
0.2	12.9	0.332	0.0220	75.4	0.259	0.0264
0.3	6.81	0.412	0.0188	35.0	0.422	0.0184
0.5	3.62	0.703	0.00975	13.5	0.789	0.00899
0.7	2.58	0.752	0.00814	9.29	1.38	0.0142
0.9	1.74	0.825	0.00669	6.69	2.17	0.0169



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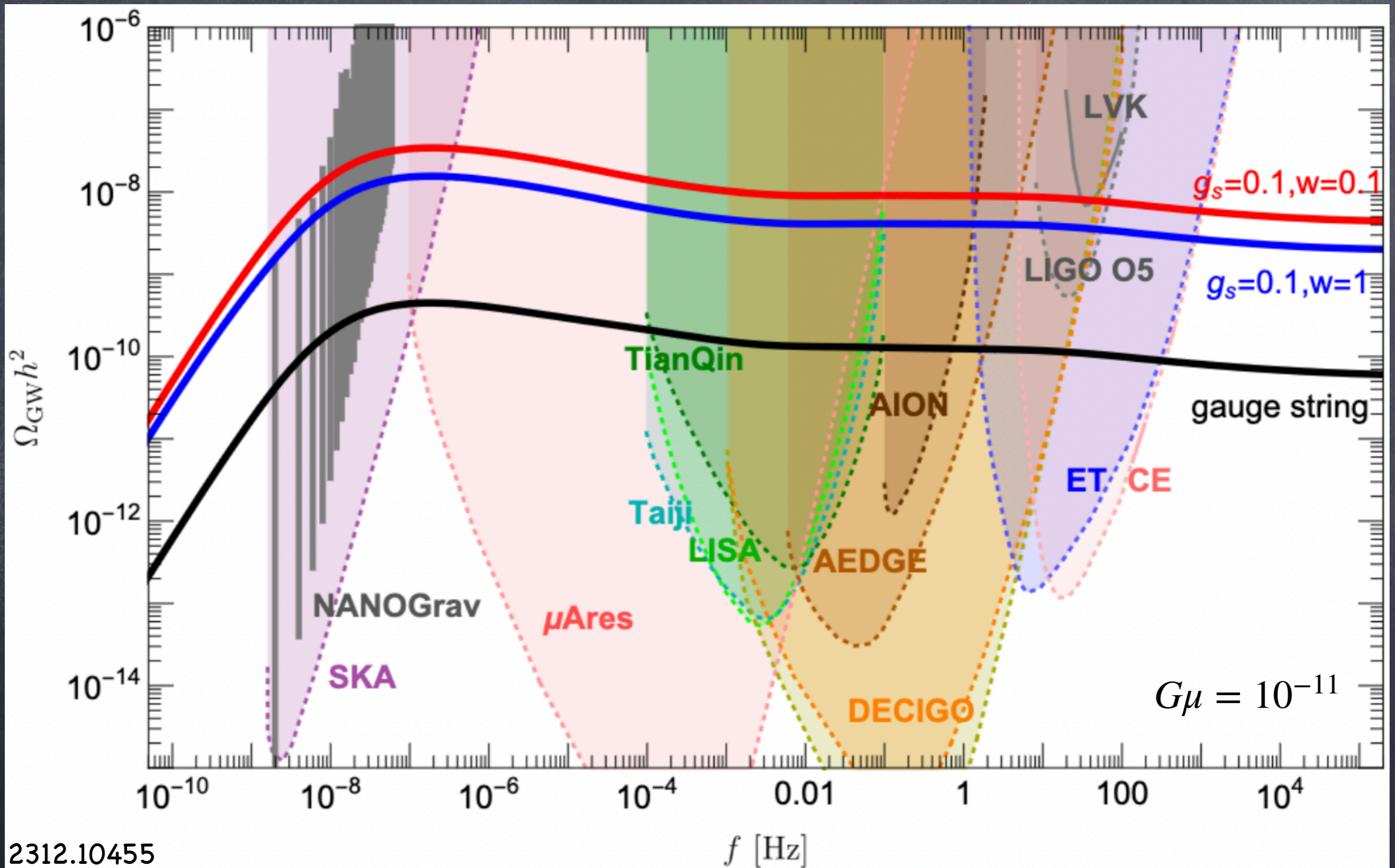


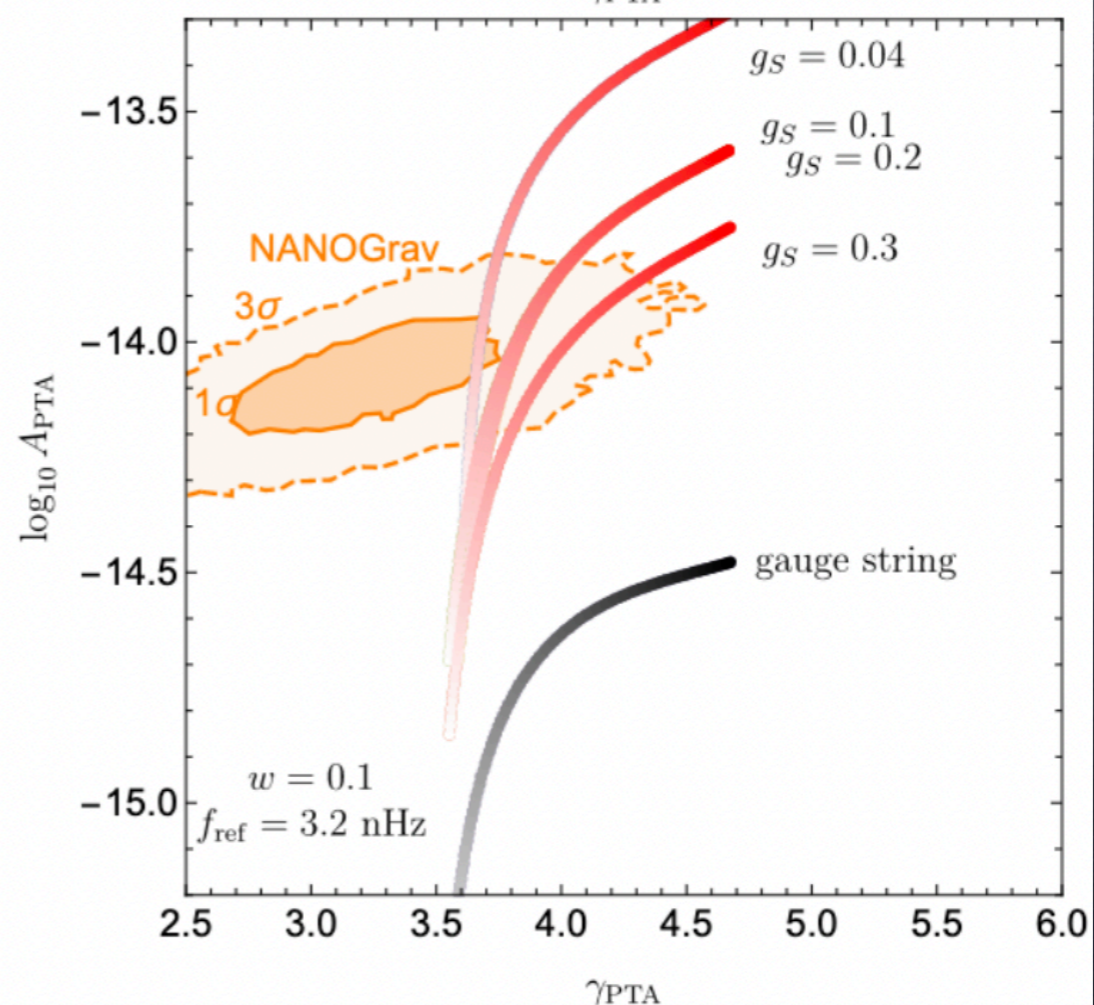
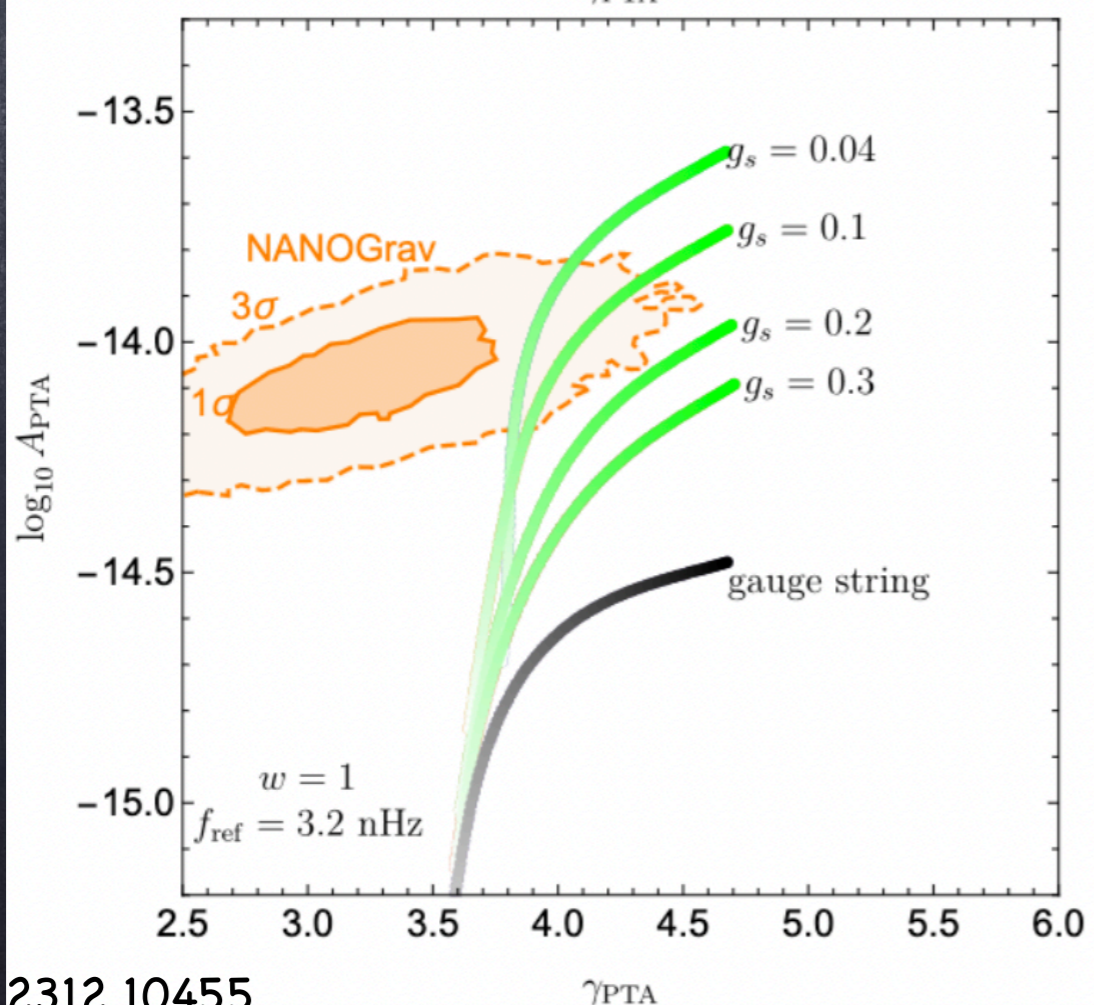
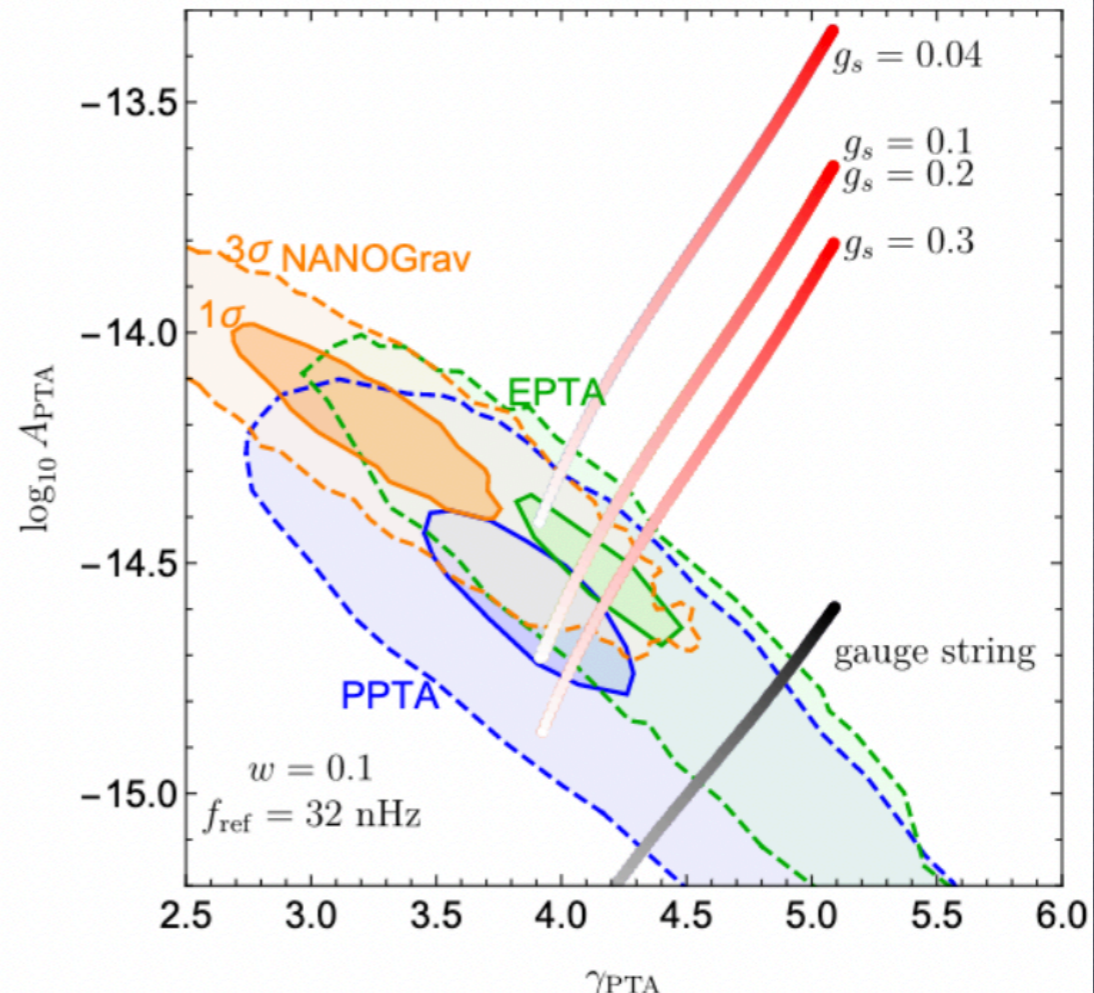
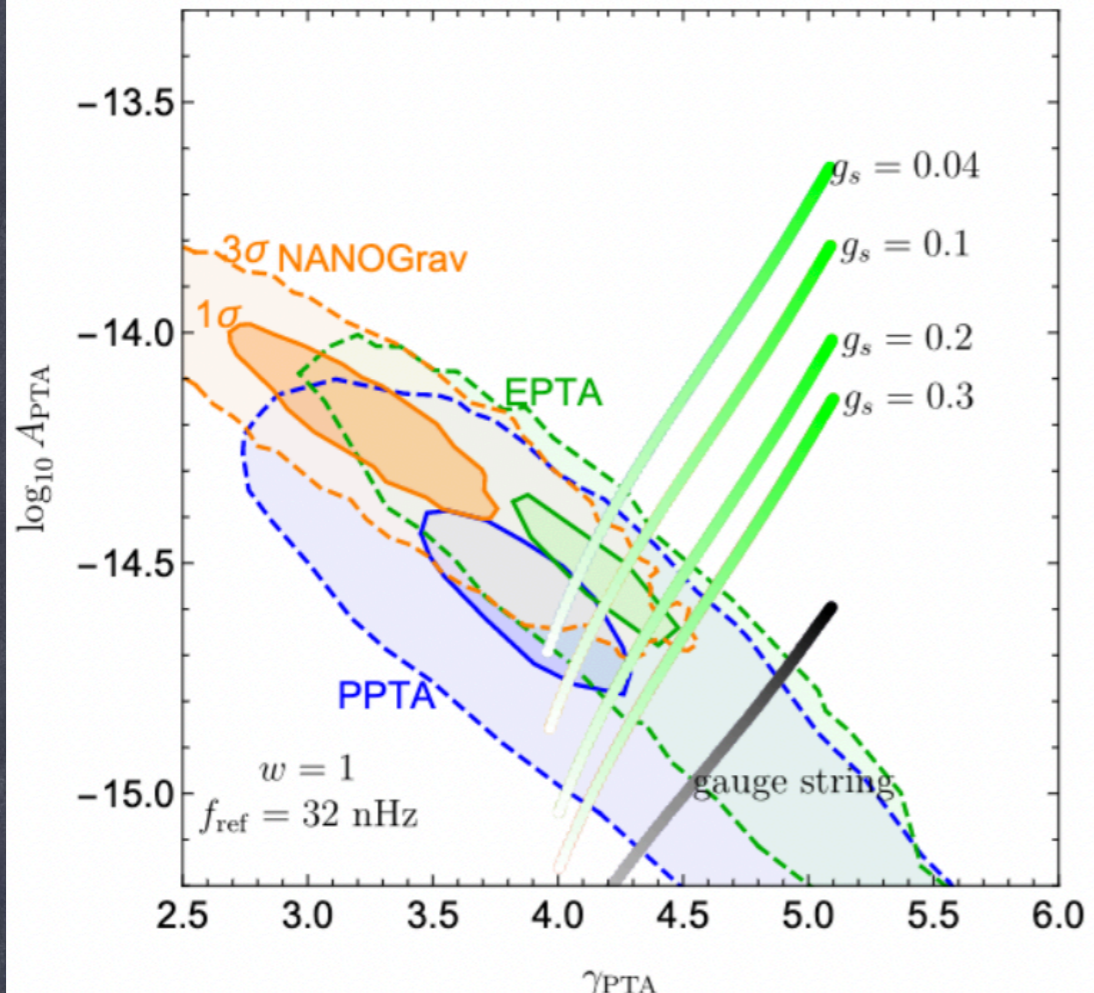
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- $\xi_i \propto \tilde{c}_i \propto P_{ii}^{1/3}$ so smaller loops are produced which emit GWs with higher frequency $\propto P_{ii}^{-1/3}$. Shape of spectrum in PTA band affected by multiple string types

- $\Omega_{\text{GW},i}(f) \propto n_i(\ell, t) \propto N_i$

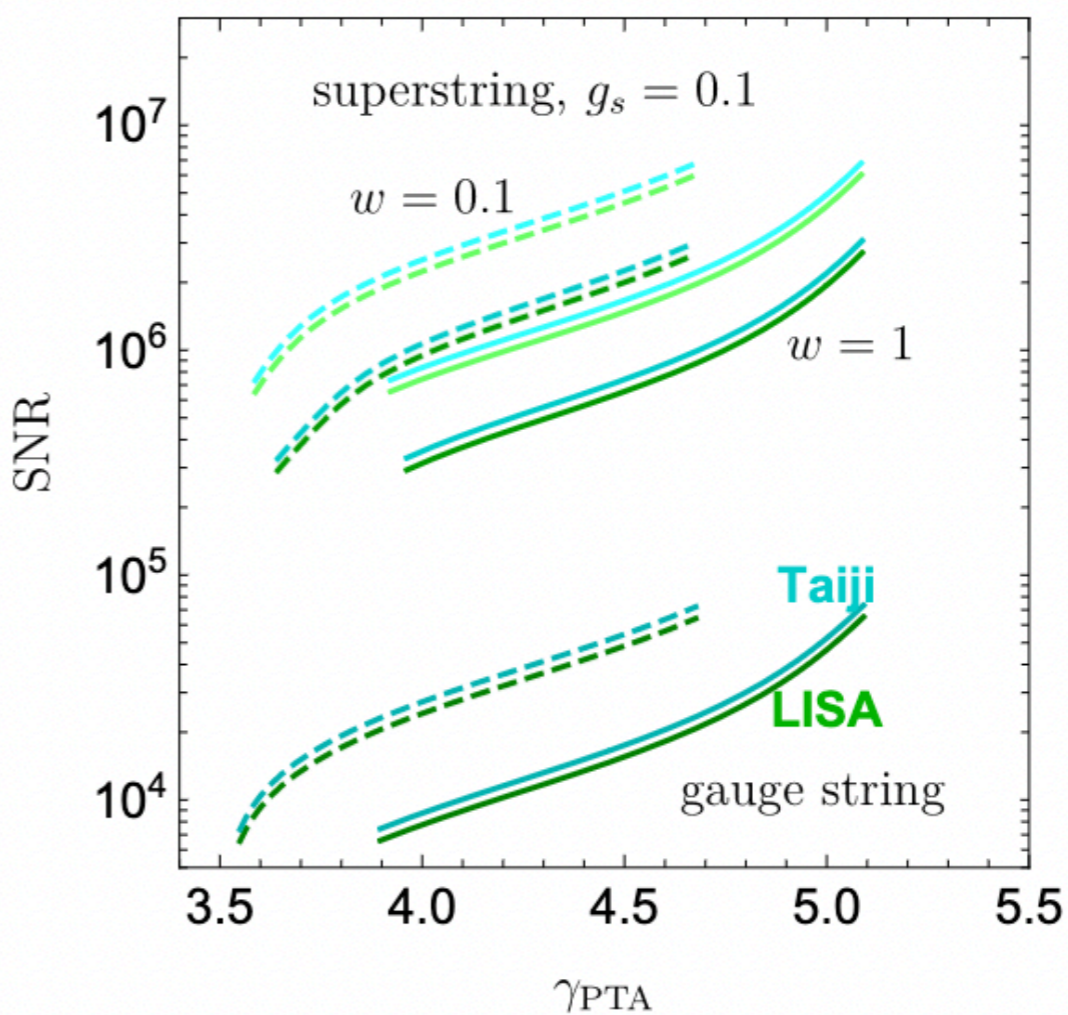
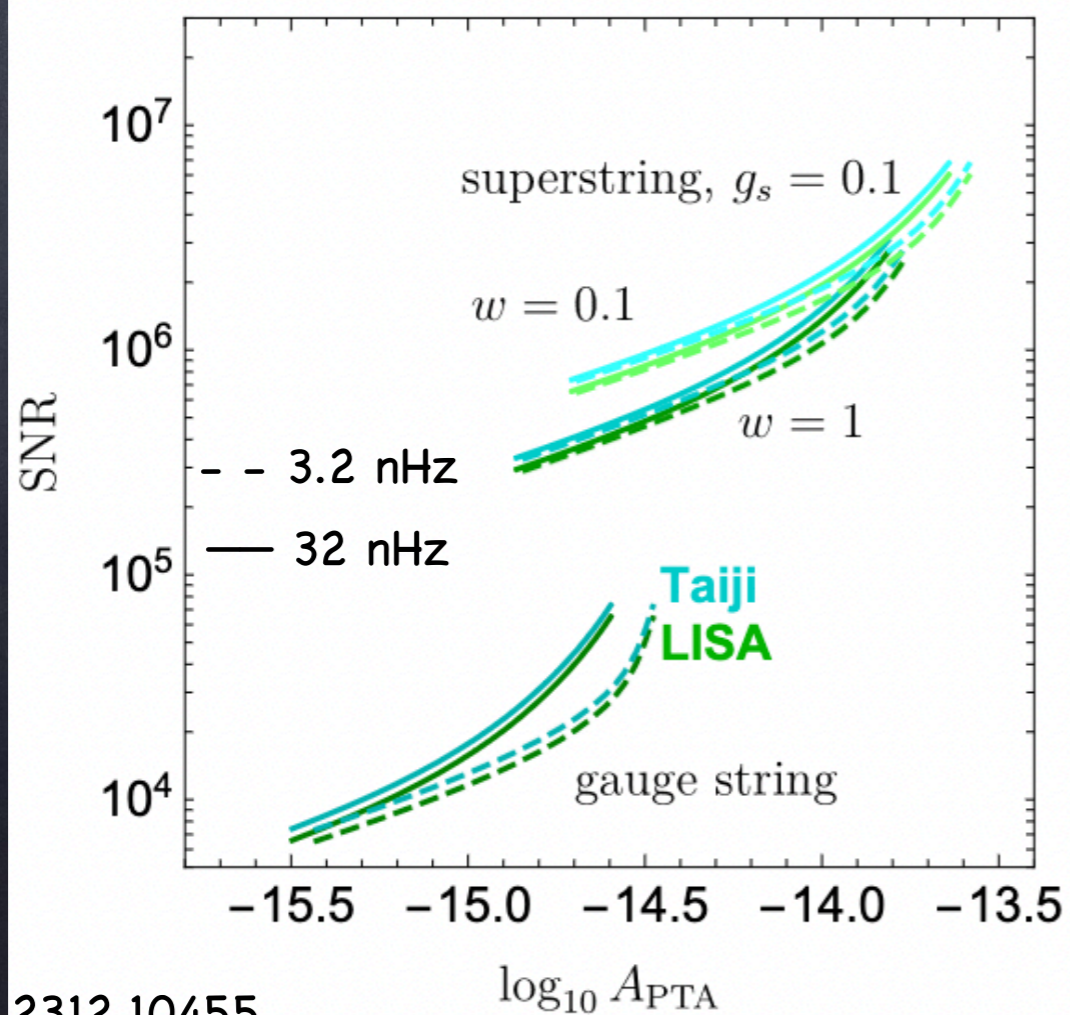
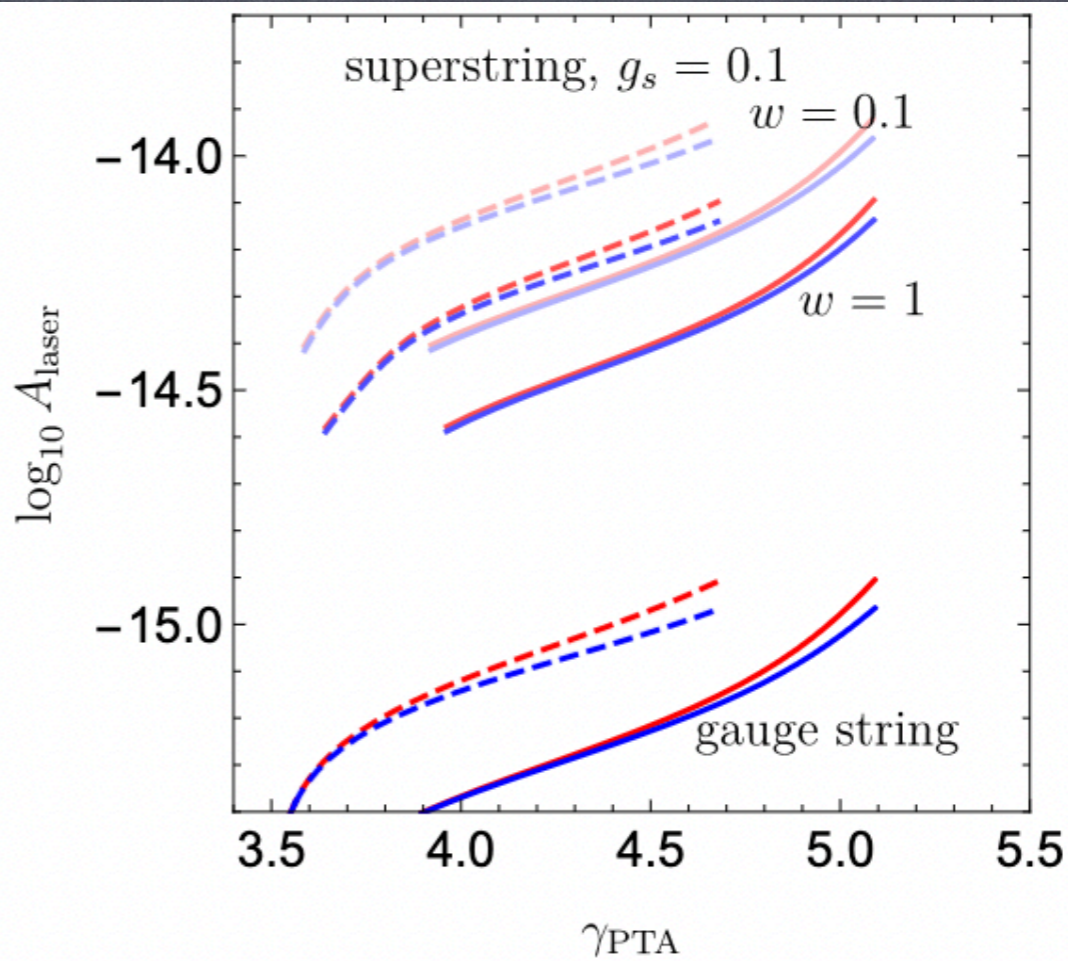
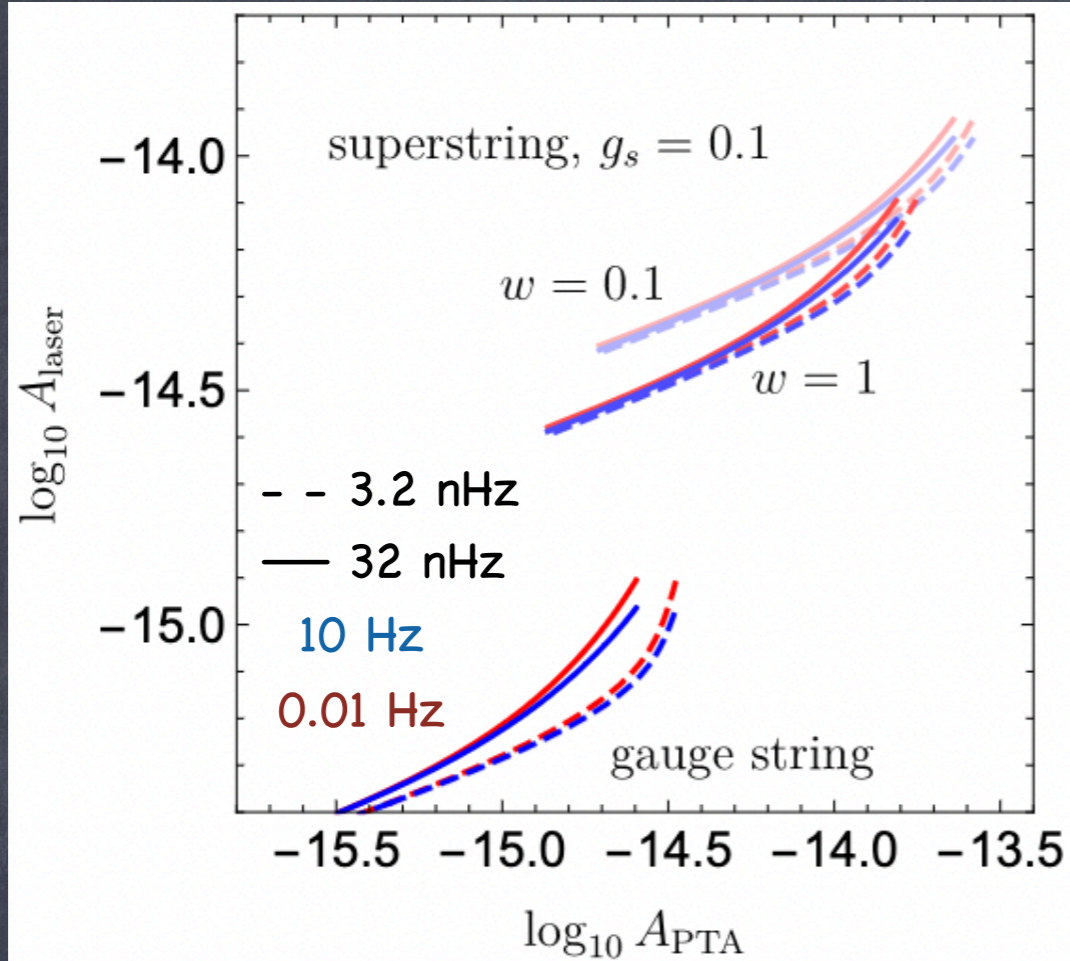
Comparison with data





10^{-10}
 \uparrow
 $G\mu_1$
 \uparrow
 10^{-12}

- LIGO–Virgo–Kagra (LVK) data at 25 Hz places a 2σ bound $\Omega_{\text{GW}}(f)h^2 \lesssim 7.8 \times 10^{-9}$
- Gauge string networks are excluded by NANOGrav at 3σ
- ... but compatible with EPTA and PPTA data
- Superstring networks are consistent with 32 nHz data from NANOGrav
- ... but excluded by 3.2 nHz data at 3σ unless $g_s < 0.2$ or strings evolve in about 10% of the space



- Strong correlations between PTA and interferometer signals for gauge string networks
- More parameters for superstring networks weakens correlations
- Clear separation between dashed and solid curves in right panels indicates that power-law approx is not good in the PTA band
- Signal amplitude is a clear discriminator of superstring and gauge string networks

Main messages

- Gauge string networks not yet excluded by PTA data
- GW spectrum for superstring networks shifted to higher frequency and enhanced compared to gauge string networks
- Spectrum in PTA band affected by multiple string types
- Not surprising that superstring networks are favored given their extra freedom