Theory and phenomenology of Flavoured Trinification

Roman Pasechnik
Lund U.

The SM is a tremendously successful theory that explains "boringly" well all its predictions!

However, it fails to...

- Explain neutrino masses
- Explain dark matter
- Explain CP violation and matter/anti-matter assymetry
- Explain the observed flavour structure

What Grand Unification can teach us about these problems?

Example: SM fermion sector features

Large mass and mixing hierarchies found in Nature

- $m_t\,,\ m_h\,, m_z\,, m_w \sim 10^2$ GeV (EW scale)
- $m_t \sim 10^2 m_c \sim 10^{4.6} m_u$
- $m_t \sim 10^{1.6} m_b \sim 10^{3.2} m_s \sim 10^{4.8} m_d$
- $m_t \gtrsim 10^{11} m_\nu$
- $m_e \gtrsim 10^5 m_\nu$
- $m_\tau \sim 10^{1.2} m_\mu \sim 10^{3.5} m_e$

$$V_{\rm CKM} \sim \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} + \text{Perturbations}\ .$$

Top-down approach: the story of Trinification

- The trinification gauge group (Glashow, '84)

$$[\mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}} \times \mathrm{SU}(3)_{\mathrm{C}}] \rtimes \mathbb{Z}_3^{(\mathrm{LRC})}$$
$$\downarrow$$
$$\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{L+R}}$$
$$\downarrow$$
$$\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}$$

- Subgroup of $\mathrm{E}_6 \supset [\mathrm{SU}(3)]^3$

- SM fields sit in chiral superfields that are bi-fundamental representatic of the gauge group: $\mathbf{L} \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})$, $\mathbf{Q}_{\mathrm{L}} \sim (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3})$, and $\mathbf{Q}_{\mathrm{R}} \sim (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$:

$$(\mathbf{L}^i)^l{}_r = \begin{pmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} & \nu_{\mathrm{L}} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \mathbf{e}_{\mathrm{L}} \\ \nu^c_{\mathrm{R}} & \mathbf{e}^c_{\mathrm{R}} & \phi \end{pmatrix}^i , \quad \begin{matrix} (\mathbf{Q}^i_{\mathrm{L}})^x{}_l = \begin{pmatrix} \mathbf{u}^x_{\mathrm{L}} & \mathbf{d}^x_{\mathrm{L}} & \mathbf{D}^x_{\mathrm{L}} \end{pmatrix}^i , \\ (\mathbf{Q}^i_{\mathrm{R}})^r{}_x = \begin{pmatrix} \mathbf{u}^c_{\mathrm{R}x} & \mathbf{d}^c_{\mathrm{R}x} & \mathbf{D}^c_{\mathrm{R}x} \end{pmatrix}^{\top\ i} , \end{matrix}$$

- Each family can be arranged into an E_6 **27**-plet:

$$\mathbf{27}^i = (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})^i \otimes (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})^i \otimes (\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3})^i$$

Why Trinification

Positives:

- The model accommodates any quark and lepton masses and mixings (Sayre et al 2016)
- Naturally light neutrinos via radiative see-saw with split-SUSY (Cauet et al 2011)
- Gauge symmetry preserve baryon number, i.e. no gauge-mediated proton decay (Achiman&Stech'78; Glashow&Kang'84)
- Motivated as a low-energy version of E8xE8 heterotic string theory (Gross et al 1985)

- GUT scale fermion masses through $L \cdot L' \cdot L''$ type operators
  - Higher dimensional operators needed **(Cauet et al. 2011)**

Negatives:

- Considerable amount of particles and many couplings involved
  - Realistic calculations cumbersome
- Unmotivated Hierarchy between the trinification and the EW breaking scales (common to most GUTs)

Trinification-based models were left as the least developed GUT scenarios

“Flavoured” T-GUT approach

Build a SUSY GUT-scale framework in the top-down approach that:

> Features all **the basic advantages of the trinification GUTs** and resolves their major issues;

> Addresses **the μ-problem** of conventional MSSM-based approaches;

> Generates **larger masses** and **Cabibbo mixing** at **tree-level**;

> **Full CKM and light fermion masses** to be **radiatively generated**;

> **Adopts a seesaw mechanism** for **light active neutrinos, with no strong PMNS hierarchies**;

> **Unifies gauge interactions** and reduces parametric freedom in the Yukawa sector **(Yukawa unification)**.

References:

2004.114550,

2001.06383, 2001.04804,

1711.05199, 1610.03642,

1606.03492

“Flavoured” T-GUT with gauged family symmetry

Consider embedding Trinification into E6:

$$\mathcal{G} \xrightarrow{M_{\rm GUT}} \mathrm{E}_6 \times \mathrm{SU}(2)_{\rm F} \times \mathrm{U}(1)_{\rm F} \xrightarrow{M_6} [\mathrm{SU}(3)]^3 \times \mathrm{SU}(2)_{\rm F} \times \mathrm{U}(1)_{\rm F} \xrightarrow{M_3} \mathrm{SU}(3)_{\rm C} \times [\mathrm{SU}(2) \times \mathrm{U}(1)]^2 \times \mathrm{SU}(2)_{\rm F} \times \mathrm{U}(1)_{\rm F} \xrightarrow{M_{\rm S}} \ldots .$$

Scale hierarchy:

$$M_{\rm GUT} \gtrsim M_6 \gtrsim M_3 \qquad M_{\rm S} \ll M_3$$

SU(2)F Anomaly-free content:

$\mathbb{Z}_2$-even	$\mathbb{Z}_2$-odd
$\psi^{\mu\, i} = (\mathbf{27},\mathbf{2})_{(1)}$, $\psi^{\mu\, 3} = (\mathbf{27},\mathbf{1})_{(-2)}$	
$\mathcal{H}_{\mathcal{U}} = (\mathbf{1},\mathbf{2})_{(-1)}$, $\mathcal{H}_{\mathcal{D}} = (\mathbf{1},\mathbf{2})_{(+1)}$	$\mathcal{L}_k = (\mathbf{1},\mathbf{2})_{(-1)}$
$\mathcal{A} = (\mathbf{78},\mathbf{1})_{(0)}$	$\mathcal{E}_k = (\mathbf{1},\mathbf{1})_{(+2)}$
$\Sigma,\Sigma' = (\mathbf{650},\mathbf{1})_{(0)}$	$\mathcal{N}_k = (\mathbf{1},\mathbf{1})_{(0)}$
$\Psi = (\mathbf{2430},\mathbf{1})_{(0)}$	

possible source for Dark Matter

Z2-odd sector is massive:

$$W_{\mathcal{H}\mathcal{N}} = \mu_{\mathcal{H}} \mathcal{H}_{\mathcal{U}} \mathcal{H}_{\mathcal{D}} + y_{\mathcal{L}} \mathcal{H}_{\mathcal{U}} \mathcal{L}\mathcal{E} + y_{\mathcal{N}} \mathcal{H}_{\mathcal{D}} \mathcal{L}\mathcal{N} + \mu_{\mathcal{N}} \mathcal{N}\mathcal{N}$$

Massless sector dim-3 superpotential with universal Yukawa coupling:

$$W_{27} = \frac{1}{2}\lambda_{27} d_{\mu\nu\lambda}\varepsilon_{ij}\psi^{\mu\, i}\psi^{\nu\, j}\psi^{\lambda\, 3} = 0$$

$d_{\mu\nu\lambda}$ — completely symmetric $\qquad$ ε_{ij} — totally anti-symmetric

$$(\mathbf{27},\mathbf{2})_{(1)} \equiv \psi^{\mu\, i}, \qquad (\mathbf{27},\mathbf{1})_{(-2)} \equiv \psi^{\mu\, 3} \qquad \mu = 1, \ldots, 27 \qquad i = 1,2$$

Effects from higher dimensional operators become dominant!

The role of E6 dim-5 operators in the gauge sector

Consider the two breaking steps:

$$\begin{aligned} E_8 &\xrightarrow{M_8} E_6 \times SU(2)_F \times U(1)_F \\ &\xrightarrow{M_6} [SU(3)]^3 \times SU(2)_F \times U(1)_F . \end{aligned}$$

If M_8 and M_6 are close dim-5 corrections to gauge-kinetic terms may become relevant

$$\mathcal{L}_{5D} = -\frac{\xi}{M_8}\left[\frac{1}{4C} Tr\left(\boldsymbol{F}_{\mu\nu} \cdot \tilde{\Phi}_{E_6} \cdot \boldsymbol{F}^{\mu\nu}\right)\right]$$

$$\tilde{\Phi}_{E_6} \sim (\mathbf{78} \otimes \mathbf{78})_{\text{sym}} = \mathbf{1} \oplus \mathbf{650} \oplus \mathbf{2430}$$

Since $\mathbf{650}$ contains two orthogonal trinification singlets we consider corrections from

$$\Sigma \equiv \mathbf{650}\,, \qquad \Sigma' \equiv \mathbf{650'}\,, \qquad \Psi \equiv \mathbf{2430}$$

E6 and Trinification breaking

$$\mathcal{L}_{5D} = -\frac{\xi}{M_{\rm GUT}}\left[\frac{1}{4C}\,{\rm Tr}\left(\boldsymbol{F}_{\mu\nu}\cdot\tilde{\Phi}_{\rm E_6}\cdot\boldsymbol{F}^{\mu\nu}\right)\right] \qquad \tilde{\Phi}_{\rm E_6} \in (\mathbf{78}\otimes\mathbf{78})_{\rm sym} = \mathbf{1}\oplus\mathbf{650}\oplus\mathbf{2430}$$

$\boldsymbol{\Sigma}$, $\boldsymbol{\Sigma}'$ and Ψ allow quadratic and cubic superpotential interactions

$$W_{\rm E_6} \supset M_\Sigma {\rm Tr}\Sigma^2 + M_{\Sigma'}{\rm Tr}\Sigma'^2 + M_\Psi {\rm Tr}\Psi^2 + \lambda_\Sigma {\rm Tr}\Sigma^3 + \lambda_{\Sigma'}{\rm Tr}\Sigma'^3 + \lambda_\Psi {\rm Tr}\Psi^3$$
$$+ \text{crossed terms}$$

and can develop VEVs obeying the relation

$$v^2_{\rm E_6} = v^2_\Sigma + v^2_{\Sigma'} + v^2_\Psi \equiv \left(k^2_\Sigma + k^2_{\Sigma'} + k^2_\Psi\right) v^2_{\rm E_6}\,, \qquad k^2_\Sigma + k^2_{\Sigma'} + k^2_\Psi = 1$$

$$k_\Psi \propto \frac{\langle\mathbf{2430}\rangle}{M_6}\,, \qquad k_\Sigma \propto \frac{\langle\mathbf{650}\rangle}{M_6}\,, \qquad k_{\Sigma'} \propto \frac{\langle\mathbf{650'}\rangle}{M_6}$$

$$\alpha^{-1}_{3\rm C}(1+\zeta\delta_{\rm C})^{-1} = \alpha^{-1}_{3\rm L}(1+\zeta\delta_{\rm L})^{-1} = \alpha^{-1}_{3\rm R}(1+\zeta\delta_{\rm R})^{-1}\,, \qquad \zeta\sim 1$$
$$\alpha^{-1}_{3A} = \frac{4\pi}{g^2_A}\,, \qquad \delta_{\rm C} = -\frac{1}{\sqrt{2}}k_\Sigma - \frac{1}{\sqrt{26}}k_\Psi\,, \qquad \delta_{\rm L,R} = \frac{1}{2\sqrt{2}}k_\Sigma \pm \frac{3}{2\sqrt{2}}k_{\Sigma'} - \frac{1}{\sqrt{26}}k_\Psi$$

Chakrabortty, Raychaudhuri Phys.Lett. B673 (2009) 57-62

Below E6 breaking scale:

	$\rm SU(3)_L$	$\rm SU(3)_R$	$\rm SU(3)_C$	$\rm SU(2)_F$	$\rm U(1)_F$
$\boldsymbol{\Delta}_{\rm L}$	$\mathbf{8}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0
$\boldsymbol{\Delta}_{\rm R}$	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{1}$	$\mathbf{1}$	0
$\boldsymbol{\Delta}_{\rm C}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{1}$	0
$\boldsymbol{\Xi}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$	0
$\boldsymbol{\Xi}'$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	0

$$W_{\mathbf{78}} = \sum_{A=\rm L,R,C}\left[\frac{1}{2}\mu_{78}{\rm Tr}\boldsymbol{\Delta}^2_A + \frac{1}{3!}\mathcal{Y}_{78}{\rm Tr}\boldsymbol{\Delta}^3_A\right] + \mu_{78}{\rm Tr}(\boldsymbol{\Xi}\boldsymbol{\Xi}') + \sum_{A=\rm L,R,C}\mathcal{Y}_{78}{\rm Tr}(\boldsymbol{\Xi}\boldsymbol{\Xi}'\boldsymbol{\Delta}_A)$$

$$\rm SU(3)_L\times SU(3)_R \overset{v_{L,R}}{\to} SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R \qquad v_{\rm L} = v_{\rm R} \equiv M_3$$

Trinification EFT: Yukawa sector

$$W_{4D} = \frac{1}{2}\frac{1}{M_8}\varepsilon_{ij}\boldsymbol{\psi}^{\mu\,i}\boldsymbol{\psi}^{\nu\,j}\boldsymbol{\psi}^{\lambda\,3}\Big[\tilde{\lambda}_1\boldsymbol{\Sigma}^{\alpha}_{\mu}d_{\alpha\nu\lambda} + \tilde{\lambda}_2\boldsymbol{\Sigma}^{\alpha}_{\nu}d_{\alpha\mu\lambda} + \tilde{\lambda}_3\boldsymbol{\Sigma}^{\alpha}_{\lambda}d_{\alpha\mu\nu} + \tilde{\lambda}_4\boldsymbol{\Sigma}'^{\alpha}_{\mu}d_{\alpha\nu\lambda} + \tilde{\lambda}_5\boldsymbol{\Sigma}'^{\alpha}_{\nu}d_{\alpha\mu\lambda} + \tilde{\lambda}_6\boldsymbol{\Sigma}'^{\alpha}_{\lambda}d_{\alpha\mu\nu}\Big]$$

E_6 **27**-plet contains three trinification $SU(3)_L \times SU(3)_R \times SU(3)_C$ bi-triplets:

$$\mathbf{27} \supset (\mathbf{3},\bar{\mathbf{3}},\mathbf{1}) \oplus (\bar{\mathbf{3}},\mathbf{1},\mathbf{3}) \oplus (\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) \equiv \boldsymbol{L} \oplus \boldsymbol{Q}_L \oplus \boldsymbol{Q}_R$$

Accidental symmetries

	$U(1)_W$	$U(1)_B$
$\boldsymbol{L}$	$+1$	0
$\boldsymbol{Q}_L$	$-1/2$	$+1/3$
$\boldsymbol{Q}_R$	$-1/2$	$-1/3$

After $\langle\boldsymbol{\Sigma}\rangle$ and $\langle\boldsymbol{\Sigma}'\rangle$ VEVs the massless superpotential reduces to

$$W_{\rm eff} = \varepsilon_{ij}\left(\mathcal{Y}_1\boldsymbol{L}^i\cdot\boldsymbol{Q}_L^3\cdot\boldsymbol{Q}_R^j - \mathcal{Y}_2\boldsymbol{L}^i\cdot\boldsymbol{Q}_L^j\cdot\boldsymbol{Q}_R^3 + \mathcal{Y}_2\boldsymbol{L}^3\cdot\boldsymbol{Q}_L^i\cdot\boldsymbol{Q}_R^j\right)$$

$$\mathcal{Y}_1 = \zeta\frac{k_{\Sigma'}}{\sqrt{6}}\tilde{\lambda}_{45}, \quad \mathcal{Y}_2 = \zeta\frac{k_{\Sigma}}{2\sqrt{2}}(\tilde{\lambda}_{21} - \tilde{\lambda}_{45})$$

$$\tilde{\lambda}_{ij} \equiv \tilde{\lambda}_i - \tilde{\lambda}_j \qquad \zeta \simeq M_6/M_{3F}$$

$$\zeta \sim 1 \qquad k_{\Sigma} \simeq -k_{\Sigma'} \qquad \tilde{\lambda}_{21} \simeq \tilde{\lambda}_{45}$$

Compressed hierarchy + steep E6 RG evolution suggest:

$$\mathcal{Y}_2 \ll \mathcal{Y}_1 \sim 1$$

tree-level quark hierarchies are secured!

$$\frac{\mathcal{Y}_1}{\mathcal{Y}_2} = \frac{m_t}{m_c} \approx \frac{m_b}{m_s} \approx \frac{m_B}{m_{D,S}} \sim \mathcal{O}(100)$$

- SUSY unifies Higgs and Leptons in $\boldsymbol{L}$
- Only two universal Yukawa couplings at trinification scale
- Only two quark generations acquire tree-level masses

Trinification EFT: SSB

Particle content and charges

	$\mathrm{SU(3)_L}$	$\mathrm{SU(3)_R}$	$\mathrm{SU(3)_C}$	$\mathrm{SU(2)_F}$	$\mathrm{U(1)_F}$	$\mathrm{U(1)_W}$	$\mathrm{U(1)_B}$
$\boldsymbol{L}^i$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{2}$	1	1	0
$\boldsymbol{L}^3$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	-2	1	0
$\boldsymbol{Q}_\mathrm{L}^i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{2}$	1	$-1/2$	$1/3$
$\boldsymbol{Q}_\mathrm{L}^3$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	-2	$-1/2$	$1/3$
$\boldsymbol{Q}_\mathrm{R}^i$	$\mathbf{1}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{2}$	1	$-1/2$	$-1/3$
$\boldsymbol{Q}_\mathrm{R}^3$	$\mathbf{1}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	-2	$-1/2$	$-1/3$
$\boldsymbol{\Delta}_\mathrm{L}$	$\mathbf{8}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	0	0
$\boldsymbol{\Delta}_\mathrm{R}$	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{1}$	$\mathbf{1}$	0	0	0
$\boldsymbol{\Delta}_\mathrm{C}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{1}$	0	0	0
$\boldsymbol{\Xi}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$	0	0	0
$\boldsymbol{\Xi}'$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	0	0	0

Massive super fields sector:

$\mu_{78} \simeq M_\mathrm{GUT}$

$$W_{\mathbf{78}} = \sum_{A=\mathrm{L,R,C}} \left[\frac{1}{2}\mu_{78}\mathrm{Tr}\,\boldsymbol{\Delta}_A^2 + \frac{1}{3!}\mathcal{Y}_{78}\mathrm{Tr}\,\boldsymbol{\Delta}_A^3\right] + \mu_{78}\mathrm{Tr}(\boldsymbol{\Xi}\boldsymbol{\Xi}') + \sum_{A=\mathrm{L,R,C}} \mathcal{Y}_{78}\mathrm{Tr}(\boldsymbol{\Xi}\boldsymbol{\Xi}'\boldsymbol{\Delta}_A)$$

The breaking of the trinification symmetry takes place once the heavy adjoint scalar octets $\tilde{\Delta}_\mathrm{L}$ and $\tilde{\Delta}_\mathrm{R}$ acquire VEVs of the order $v_\mathrm{L,R} \sim M_3$ (Morais, Pasechnik et al. Phys.Rev. D99 (2019) no.3, 035041)

$$\mathrm{SU(3)_L} \times \mathrm{SU(3)_R} \xrightarrow{v_\mathrm{L,R}} \mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \times \mathrm{U(1)_L} \times \mathrm{U(1)_R}$$

Generate SUSY Left-Right model (tree-level matched)

$$\begin{aligned} W_\mathrm{LR} = & \lambda_1 \varepsilon_{ij}\left[\chi^i \cdot \boldsymbol{q}_\mathrm{L}^3 \cdot \boldsymbol{q}_\mathrm{R}^j + \boldsymbol{\ell}_\mathrm{R}^i \cdot \boldsymbol{D}_\mathrm{L}^3 \cdot \boldsymbol{q}_\mathrm{R}^j + \boldsymbol{\ell}_\mathrm{L}^i \cdot \boldsymbol{q}_\mathrm{L}^3 \cdot \boldsymbol{D}_\mathrm{R}^j + \phi^i \cdot \boldsymbol{D}_\mathrm{L}^3 \cdot \boldsymbol{D}_\mathrm{R}^j\right] \\ & -\lambda_2 \varepsilon_{ij}\left[\chi^i \cdot \boldsymbol{q}_\mathrm{L}^j \cdot \boldsymbol{q}_\mathrm{R}^3 + \boldsymbol{\ell}_\mathrm{R}^i \cdot \boldsymbol{D}_\mathrm{L}^j \cdot \boldsymbol{q}_\mathrm{R}^3 + \boldsymbol{\ell}_\mathrm{L}^i \cdot \boldsymbol{q}_\mathrm{L}^j \cdot \boldsymbol{D}_\mathrm{R}^3 + \phi^i \cdot \boldsymbol{D}_\mathrm{L}^j \cdot \boldsymbol{D}_\mathrm{R}^3\right] \\ & +\lambda_2 \varepsilon_{ij}\left[\chi^3 \cdot \boldsymbol{q}_\mathrm{L}^i \cdot \boldsymbol{q}_\mathrm{R}^j + \boldsymbol{\ell}_\mathrm{R}^3 \cdot \boldsymbol{D}_\mathrm{L}^i \cdot \boldsymbol{q}_\mathrm{R}^j + \boldsymbol{\ell}_\mathrm{L}^3 \cdot \boldsymbol{q}_\mathrm{L}^i \cdot \boldsymbol{D}_\mathrm{R}^j + \phi^3 \cdot \boldsymbol{D}_\mathrm{L}^i \cdot \boldsymbol{D}_\mathrm{R}^j\right]. \end{aligned}$$

Quark spectrum: more details

The most generic VeV setting:

$$\left\langle \tilde{L}^1 \right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u_1 & 0 & 0 \\ 0 & d_1 & e_1 \\ 0 & \omega & s_1 \end{pmatrix}, \quad \left\langle \tilde{L}^2 \right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u_2 & 0 & 0 \\ 0 & d_2 & e_2 \\ 0 & s_2 & f \end{pmatrix}, \quad \left\langle \tilde{L}^3 \right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u_3 & 0 & 0 \\ 0 & d_3 & e_3 \\ 0 & s_3 & p \end{pmatrix}$$

Up-quark sector:

light up-type quarks!

$$M_{\rm u} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & u_3\mathcal{Y}_2 & u_2\mathcal{Y}_2 \\ -u_3\mathcal{Y}_2 & 0 & -u_1\mathcal{Y}_2 \\ -u_2\mathcal{Y}_1 & u_1\mathcal{Y}_1 & 0 \end{pmatrix} \qquad m_{\rm u} = 0 \qquad m_{\rm c}^2 = \tfrac{1}{2}\mathcal{Y}_2^2\left(u_1^2 + u_2^2 + u_3^2\right) \qquad m_{\rm t}^2 = \tfrac{1}{2}\left[\mathcal{Y}_1^2\left(u_1^2+u_2^2\right) + \mathcal{Y}_2^2 u_3^2\right]$$

vector-like quarks!

Down-quark sector (before EWSB):

$$\begin{pmatrix} d_{\rm L}^i & D_{\rm L}^i \end{pmatrix}^\top M_d \begin{pmatrix} d_{\rm R}^i & D_{\rm R}^i \end{pmatrix}$$

$$M_{\rm d}^{6\times6} \simeq \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p\mathcal{Y}_2 & f\mathcal{Y}_2 \\ 0 & 0 & -\omega\mathcal{Y}_2 & -p\mathcal{Y}_2 & 0 & 0 \\ 0 & w\mathcal{Y}_1 & 0 & -f\mathcal{Y}_1 & 0 & 0 \end{pmatrix}$$

$$m_{\rm D/S}^2 \simeq \frac{1}{2}(f^2+p^2)\mathcal{Y}_2^2\,, \quad m_{\rm S/D}^2 \simeq \frac{\omega^2(f^2+p^2+\omega^2)}{2(f^2+\omega^2)}\mathcal{Y}_2^2\,,$$

$$m_{\rm B}^2 \simeq \frac{1}{2}(f^2+\omega^2)\mathcal{Y}_1^2 + \frac{f^2p^2}{2(f^2+\omega^2)}\mathcal{Y}_2^2\,.$$

Scenarios	ω [TeV]	f [TeV]	p [TeV]	$m_{\rm D}$ [TeV]	$m_{\rm S}$ [TeV]	$m_{\rm B}$ [TeV]
$\omega \sim f \sim p$	$100-1000$	$100-1000$	$100-1000$	$1-10$	$1-10$	$100-1000$
$\omega \sim f \ll p$	$10-100$	$10-100$	$100-1000$	$1-10$	$1-10$	$10-100$
$\omega \ll f \sim p$	100	1000	1000	1	10	1000

Down-quark sector (after EWSB):

$$M_{\rm d}^{6\times6} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & d_3\mathcal{Y}_2 & d_2\mathcal{Y}_2 & 0 & 0 & 0 \\ -d_3\mathcal{Y}_2 & 0 & -d_1\mathcal{Y}_2 & 0 & 0 & 0 \\ -d_2\mathcal{Y}_1 & d_1\mathcal{Y}_2 & 0 & 0 & 0 & 0 \\ 0 & s_3\mathcal{Y}_2 & s_2\mathcal{Y}_2 & 0 & p\mathcal{Y}_2 & f\mathcal{Y}_2 \\ -s_3\mathcal{Y}_2 & 0 & -\omega\mathcal{Y}_2 & -p\mathcal{Y}_2 & 0 & -s_1\mathcal{Y}_2 \\ -s_2\mathcal{Y}_1 & w\mathcal{Y}_1 & 0 & -f\mathcal{Y}_1 & s_1\mathcal{Y}_1 & 0 \end{pmatrix}$$

$$M_{\rm d} \approx \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & \mathcal{Y}_2\frac{d_3f-d_2p}{\sqrt{f^2+p^2+\omega^2}} \\ -d_3\mathcal{Y}_2 & 0 & d_1\mathcal{Y}_2\frac{p}{\sqrt{f^2+p^2+\omega^2}} \\ -d_2\mathcal{Y}_1 & 0 & d_1\mathcal{Y}_1\frac{f}{\sqrt{f^2+p^2+\omega^2}} \end{pmatrix}$$

light down-type quarks!

$$m_{\rm d} = 0\,, \qquad m_{\rm s}^2 = \frac{(d_3f-d_2p)^2}{2(f^2+p^2+\omega^2)}\mathcal{Y}_2^2\,, \qquad m_{\rm b}^2 = \frac{1}{2}(d_2^2\mathcal{Y}_1^2 + d_3^2\mathcal{Y}_2^2)$$

Quark mixing

$d_1 = 0$

CKM mixing:

$$V_{\rm CKM} \equiv L_{\rm u} L_{\rm d}^\dagger = \begin{pmatrix} \frac{d_2 u_2 \mathcal{Y}_1^2 + d_3 u_3 \mathcal{Y}_2^2}{\sqrt{\mathcal{AB}}} & -\frac{u_1 \mathcal{Y}_1}{\sqrt{\mathcal{A}}} & \frac{(d_2 u_3 - d_3 u_2)\mathcal{Y}_1 \mathcal{Y}_2}{\sqrt{\mathcal{AB}}} \\ -\frac{d_2 u_1 \mathcal{Y}_1}{\sqrt{\mathcal{BC}}} & -\frac{u_2}{\sqrt{\mathcal{C}}} & \frac{d_3 u_1 \mathcal{Y}_2}{\sqrt{\mathcal{BC}}} \\ \frac{(\mathcal{C} d_3 - d_2 u_2 u_3)\mathcal{Y}_1 \mathcal{Y}_2}{\sqrt{\mathcal{ABC}}} & \frac{u_1 u_3 \mathcal{Y}_2}{\sqrt{\mathcal{AC}}} & \frac{\mathcal{C} d_2 \mathcal{Y}_1^2 + d_3 u_2 u_3 \mathcal{Y}_2^2}{\sqrt{\mathcal{ABC}}} \end{pmatrix}$$

$$\mathcal{A} = \mathcal{C}\mathcal{Y}_1^2 + u_3^2 \mathcal{Y}_2^2 \,, \qquad \mathcal{B} = d_2^2 \mathcal{Y}_1^2 + d_3^2 \mathcal{Y}_2^2 \,, \qquad \mathcal{C} = u_1^2 + u_2^2 \,.$$

For consistency with the up-quark spectrum, we require $\mathcal{Y}_2 \ll \mathcal{Y}_1$

$$V_{tb} \simeq 1 - \left(\frac{\mathcal{Y}_2}{\mathcal{Y}_1}\right)^2 \frac{d_3^2 \mathcal{C} + d_2 u_3 (d_2 u_3 - 2 d_3 u_2)}{2 d_2^2 \mathcal{C}}$$

Minimal 3HDM limit:

$u_3 \to 0 \qquad d_3 \to 0$

$$|V_{\rm CKM}| = \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ \sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \theta_C = \arctan\left(\frac{u_1}{u_2}\right)$$

Fully compressed $\omega \sim f \sim p$ scenario

$$p = 220 \text{ TeV}\,, \quad f = 210 \text{ TeV}\,, \quad \omega = 200 \text{ TeV}$$

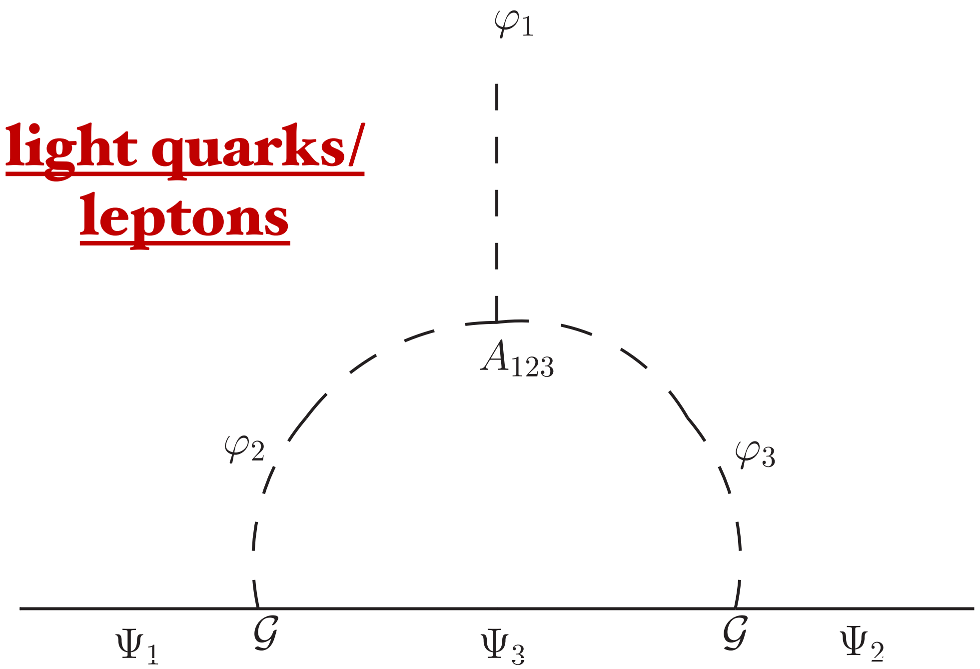
$$m_{\rm s} = 0.017 \text{ GeV}\,, \quad m_{\rm b} = 4.15 \text{ GeV}\,, \quad m_{\rm D} = 1.3 \text{ TeV}\,, \quad m_{\rm S} = 1.5 \text{ TeV}\,, \quad m_{\rm B} = 211.0 \text{ TeV}$$

$$|V_{\rm CKM}| \simeq \left(\begin{array}{ccc|ccc} 0.97 & 0.24 & 2.31\times 10^{-5} & 4.36\times 10^{-6} & 7.29\times 10^{-7} & \sim 0 \\ 0.24 & 0.97 & 9.23\times 10^{-5} & 1.74\times 10^{-5} & 2.92\times 10^{-6} & \sim 0 \\ 0 & 9.51\times 10^{-5} & 1 & 5.55\times 10^{-5} & 1.15\times 10^{-5} & 6.47\times 10^{-7} \end{array}\right)$$

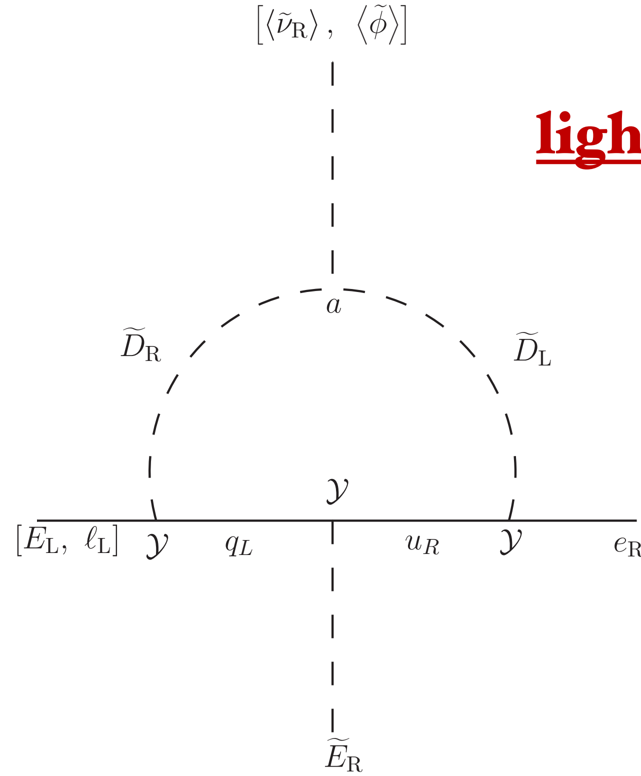
Good opportunity to probe the model at the LHC or future colliders

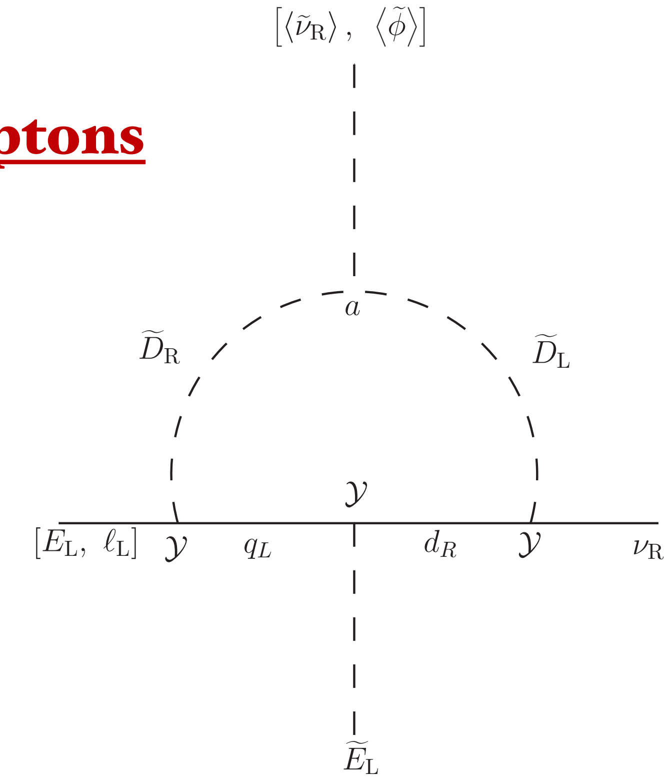
Radiative generation of charge lepton spectra

light quarks/ leptons



light leptons

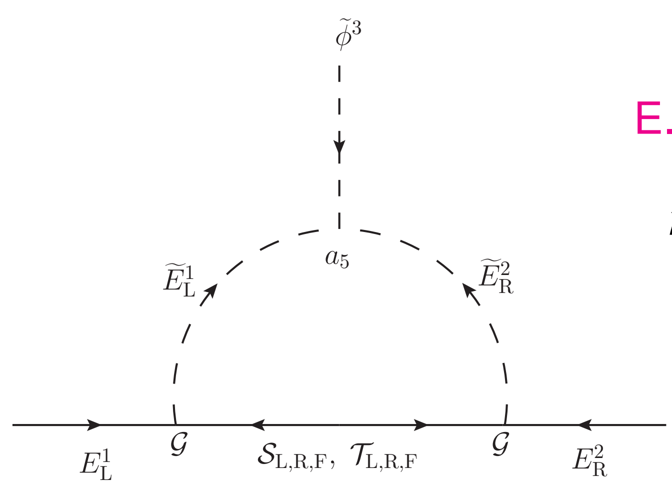


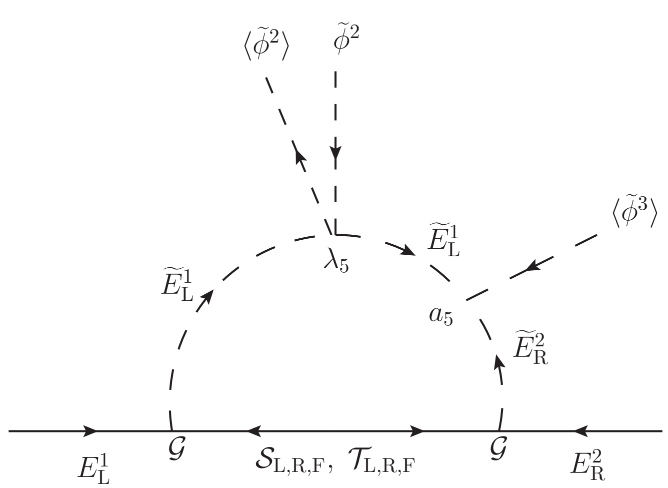


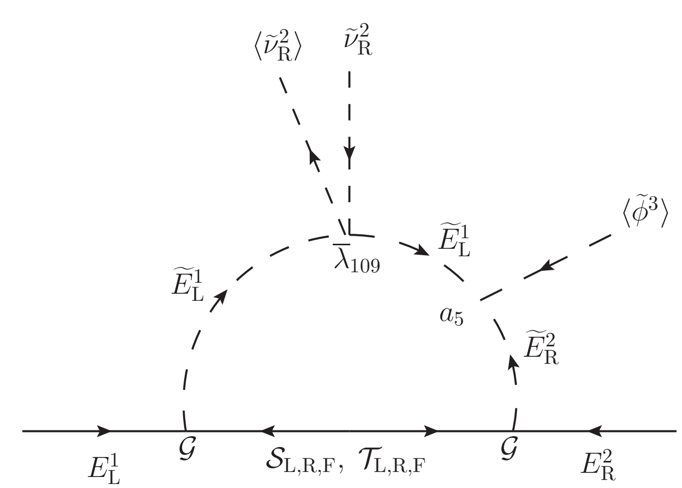
E.g.: $p \sim 10^3$ TeV, $\omega \sim 100$ TeV and $\kappa_i \sim 10^{-3}$ to 10^{-2}:

$m_{T,M} \sim 1$ to 10 TeV $\quad m_E \sim 0.1$ to 1 TeV

heavy vector-like leptons (VLLs)







Neutrino sector

$$\Psi_N = \left(\nu_L^1\ \nu_L^2\ \nu_L^3\ \mathcal{N}_L^1\ \mathcal{N}_L^2\ \mathcal{N}_L^3\ \mathcal{N}_R^1\ \mathcal{N}_R^2\ \mathcal{N}_R^3\ \phi^1\ \phi^2\ \phi^3\ \nu_R^1\ \nu_R^2\ \nu_R^3\right)$$

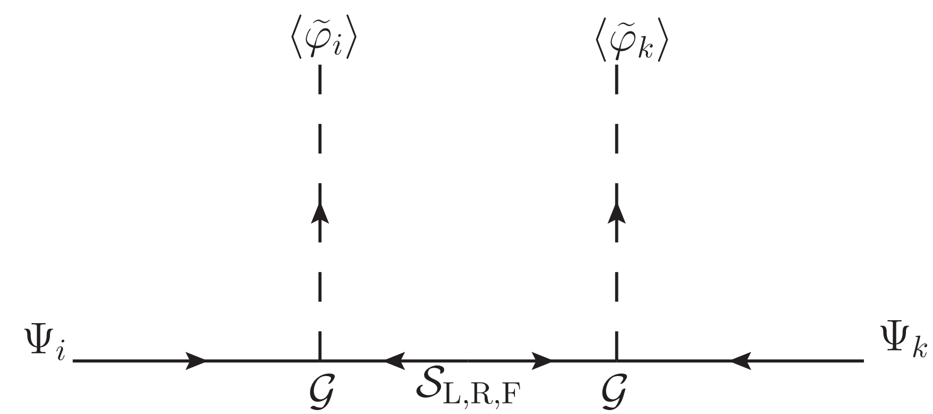
SU(2)L-doublet neutrinos

$$\mathcal{M}_N = \begin{pmatrix} \overline{\boldsymbol{M}}_{9\times 9} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_{6\times 6} \end{pmatrix}$$

SU(2)L-singlet neutrinos

$$M_{ik} = \frac{1}{2}\mathcal{G}^2 \frac{\langle\widetilde{\varphi}_i\rangle\,\langle\widetilde{\varphi}_k\rangle}{M_{\mathcal{S}}}$$

$$\langle\widetilde{\varphi}_i\rangle = s_{1,2,3},\ \omega,\ f,\ p$$



Loop-generated mass-form (before EWSB):

$$\overline{M} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_7\omega & \kappa_5\omega \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_6\omega & \kappa_8\omega \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa_1 p & \kappa_3 f \\
0 & 0 & 0 & 0 & 0 & 0 & \kappa_2 p & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \kappa_4 f & 0 & 0 \\
0 & 0 & 0 & 0 & \kappa_2 p & \kappa_4 f & 0 & 0 & 0 \\
0 & \kappa_7\omega & \kappa_6\omega & \kappa_1 p & 0 & 0 & 0 & 0 & 0 \\
0 & \kappa_5\omega & \kappa_8\omega & \kappa_3 f & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$\overline{m}^2_{N_{1,2,3}} = 0 \qquad \overline{m}^2_{N_{4,5}} = m_E^2$$

$$\overline{m}^2_{N_{6,7}} = m_M^2 \qquad \overline{m}^2_{N_{8,9}} = m_T^2$$

12 massive states!

Loop-generated mass-form (after EWSB):

$$m_\nu = \begin{pmatrix} \boldsymbol{0}_{3\times 3} & \frac{v_{\rm EW}}{\sqrt{2}}\left(\boldsymbol{y}_\nu\right)_{12\times 3} \\ \frac{v_{\rm EW}}{\sqrt{2}}\left(\boldsymbol{y}_\nu^\top\right)_{3\times 12} & \left(\boldsymbol{\mu}_N\right)_{12\times 12} \end{pmatrix}$$

Type-I seesaw structure!

Grand Unification

SU(2)L-singlet neutrinos

$$\alpha_6^{-1}(M_6) = \alpha_8^{-1} + \frac{b_6}{2\pi}\log\left(\frac{M_6}{M_8}\right)$$

$$b_6 = -1095$$

$$p = f = \omega = s_{1,2,3} \equiv M_{\rm S}$$

$$M_{Z'} = g_{\rm T} M_{\rm S}/2$$

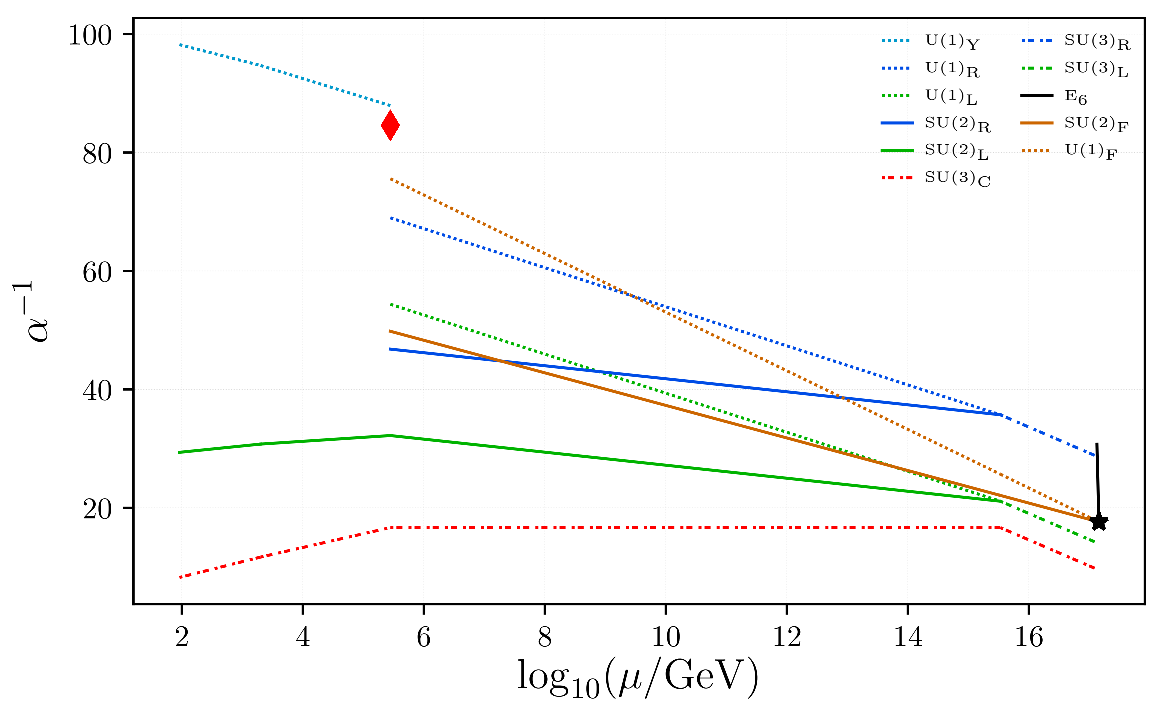
$$\zeta = M_6/M_8$$

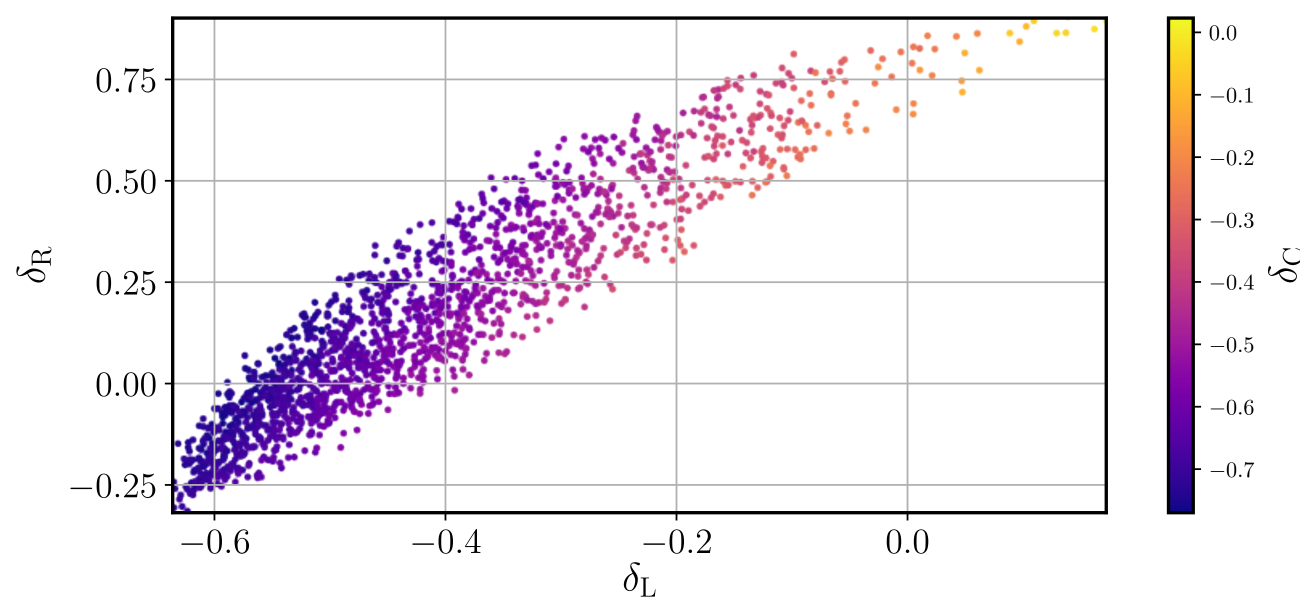
$$\alpha_{\rm C,L}^{-1}(M_Z) = \alpha_6^{-1}(M_6)\left(1+\zeta\delta_{\rm C,L}\right) + \frac{b_3}{2\pi}\log\left(\frac{M_3}{M_6}\right) + \frac{b_{\rm C,L}^{(3)}}{2\pi}\log\left(\frac{M_{\rm S}}{M_3}\right) + \frac{b_{\rm C,L}^{(4)}}{2\pi}\log\left(\frac{M_{\rm VLF}}{M_{\rm S}}\right) + \frac{b_{\rm C,L}^{(5)}}{2\pi}\log\left(\frac{M_Z}{M_{\rm VLF}}\right),$$

$$\alpha_{\rm Y}^{-1}(M_Z) = \frac{1}{3}\alpha_6^{-1}(M_6)\left(5+\zeta\delta_{\rm L}+4\zeta\delta_{\rm R}\right) + \frac{5b_3}{6\pi}\log\left(\frac{M_3}{M_6}\right) + \left(\frac{b_{\rm R}^{\prime(3)}+b_{\rm L}^{\prime(3)}}{6\pi} + \frac{b_{\rm R}^{(3)}}{2\pi}\right)\log\left(\frac{M_{\rm S}}{M_3}\right) + \frac{b_{\rm Y}^{(4)}}{2\pi}\log\left(\frac{M_{\rm VLF}}{M_{\rm S}}\right) + \frac{b_{\rm Y}^{(5)}}{2\pi}\log\left(\frac{M_Z}{M_{\rm VLF}}\right),$$

$$\alpha_{\rm T}^{-1}(M_{\rm S}) = \frac{1}{27}\left[16\alpha_8^{-1} + \alpha_6^{-1}(M_6)\left(29+\zeta\delta_{\rm L}+28\zeta\delta_{\rm R}\right) + \frac{29b_3}{2\pi}\log\left(\frac{M_3}{M_6}\right) + \frac{2b_{\rm F}^{\prime(1)}+6b_{\rm F}^{(1)}}{\pi}\log\left(\frac{M_3}{M_8}\right)\right] + \left(\frac{2b_{\rm F}^{\prime(2)}}{27\pi} + \frac{b_{\rm L}^{\prime(3)}+b_{\rm R}^{\prime(3)}}{57\pi} + \frac{2b_{\rm F}^{(2)}}{9\pi} + \frac{b_{\rm R}^{(3)}}{2\pi}\right)\log\left(\frac{M_{\rm S}}{M_3}\right),$$

$$\alpha_{\rm C}^{-1}(M_Z) = 8.4\,,\quad \alpha_{\rm L}^{-1}(M_Z) = 29.6\,,\quad \alpha_{\rm Y}^{-1}(M_Z) = 98.5$$





$M_{\rm S}$ [GeV]	M_3 [GeV]	$\alpha_{\rm T}^{-1}(M_{Z'})$	$\alpha_8^{-1}(M_8)$
$10^4 - 10^6$	$10^4 - 10^{18}$	$10 - 200$	$5 - 200$

t_8	t_3	$t_{\rm S}$	ζ	$\alpha_8^{-1}(M_8)$	$\alpha_{\rm T}^{-1}(M_{\rm S})$	$\delta_{\rm L}$	$\delta_{\rm R}$	$\delta_{\rm C}$	k_Ψ	k_Σ	k'_Σ	k_σ
17.16	15.53	5.446	0.928	17.63	84.58	−0.583	−0.0714	−0.740	0.384	−0.418	0.790	0.224

Emergence of SM-like EFT

$E_6 \times SU(2)_F \times U(1)_F$

↓ $\langle \Phi_{E_6} \rangle \equiv M_6$

$SU(3)_L \times SU(3)_R \times SU(3)_C \times SU(2)_F \times U(1)_F \times U(1)_W \times U(1)_B$

↓ $\left\langle \widetilde{\Delta}^8_{L,R} \right\rangle \equiv M_3$

$SU(3)_C \times SU(2)_L \times SU(2)_R \times SU(2)_F \times U(1)_L \times U(1)_R \times U(1)_F \times U(1)_W \times U(1)_B$

↓ $\left\langle \widetilde{\phi}^3 \right\rangle \equiv p \sim M_{Soft}$

$SU(3)_C \times SU(2)_L \times SU(2)_R \times SU(2)_F \times U(1)_{L+R} \times U(1)_S$

↓ $\left\langle \widetilde{\phi}^2 \right\rangle \equiv f$

$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{L+R} \times U(1)_V$

↓ $\langle \widetilde{\nu}^1_R \rangle \equiv \omega$

$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_T$

↓ $\langle \widetilde{\nu}^{2,3}_R \rangle , \left\langle \widetilde{\phi}^1 \right\rangle \equiv s_{1,2,3}$

$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$

No proton decay below E6 scale!

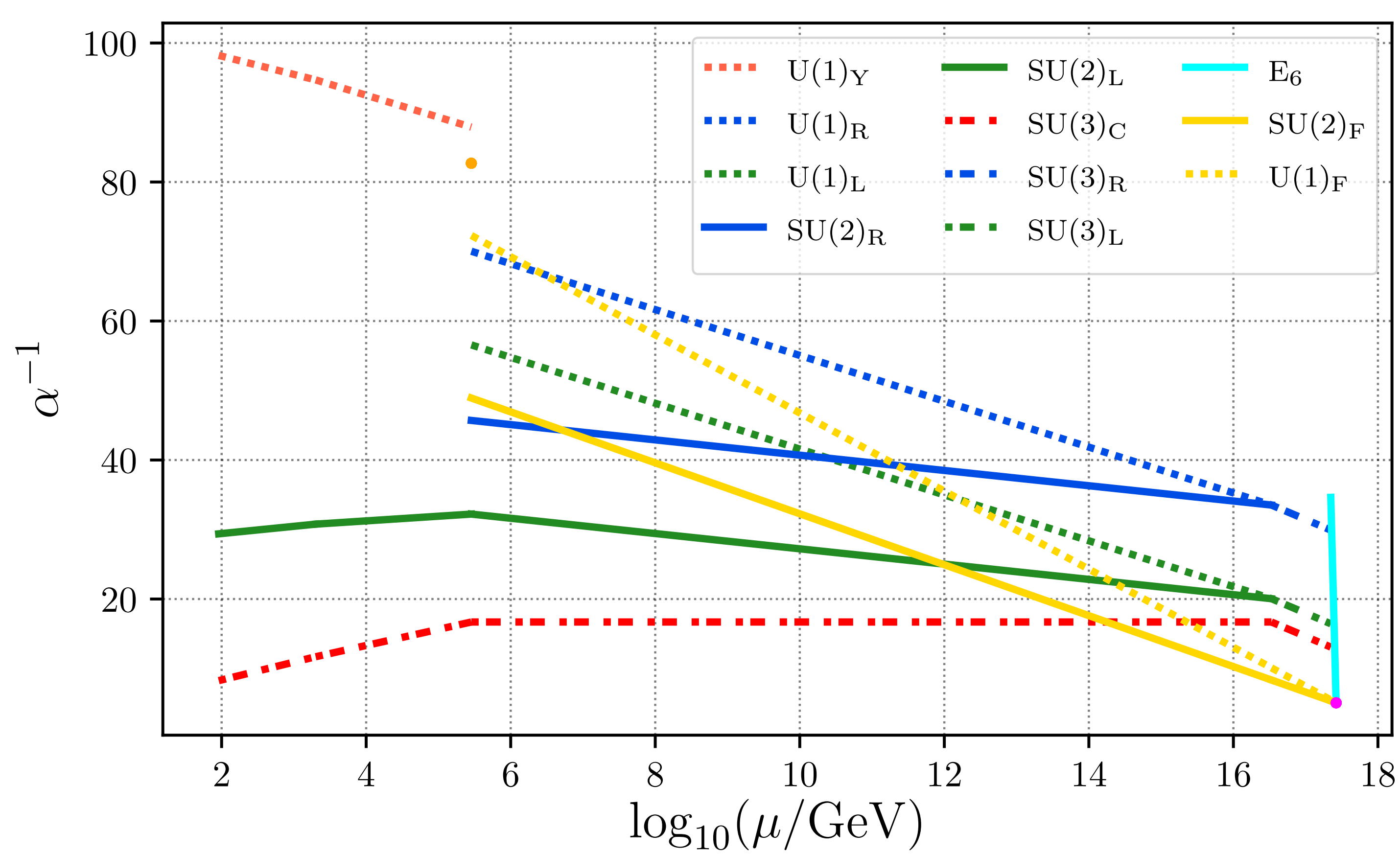
$\mathbb{P}_B$-parity $\quad \mathbb{P}_B = (-1)^{2W+2S} = (-1)^{3B+2S}$

- SUSY theory, broken SUSY, broken flavour
- Between M_6 and M_3: **Steep running** due to $\mathbf{\Psi}$, $\mathbf{\Sigma}$, $\mathbf{\Sigma'}$
- Between M_3 and M_{Soft}: Trinification running including $\boldsymbol{L}$, $\boldsymbol{Q}_L$, $\boldsymbol{Q}_R$, $\boldsymbol{\Delta}_{L,R,C}$
- **Soft scales compressed** $s = \omega = f = p$

Low scale G_{SM} theory

1. Three Higgs doublets - 3HDM
2. Two generations of VLQ below the soft scale
3. Three generations of VLL below the soft scale

$M_S \lesssim 10^3$ TeV, $\quad M_{GUT} = 10^{16} - 10^{18}$ GeV $\quad M_{EW} \ll M_S$



Concluding remarks

- We developed a novel Flavoured Trinification GUT framework giving rise to a SM-like EFT, with a realistic flavour structure in charged fermion and neutrino sectors

- The framework offers interesting implications for flavour and collider physics, primarily through vector-like fermions and scalar leptoquarks (LQs)