Proton Decay and Gravitational Waves As Complementary Tests of Grand Unification

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Workshop on Grand Unified Theories: Phenomenology & Cosmology
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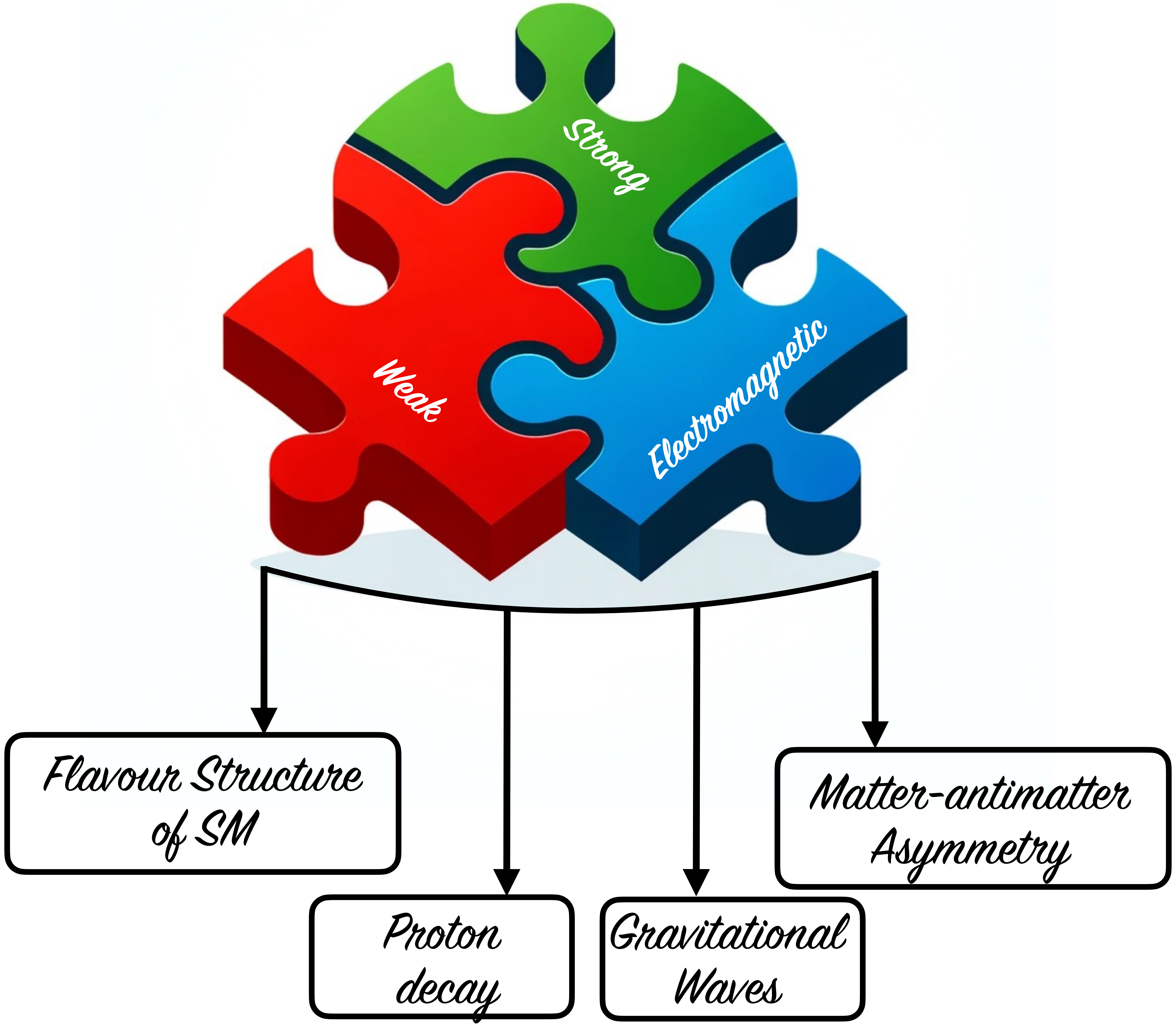
Durham University

2005.13549, 2106.15634, 2209.00021

IP3

GUTs: motivation



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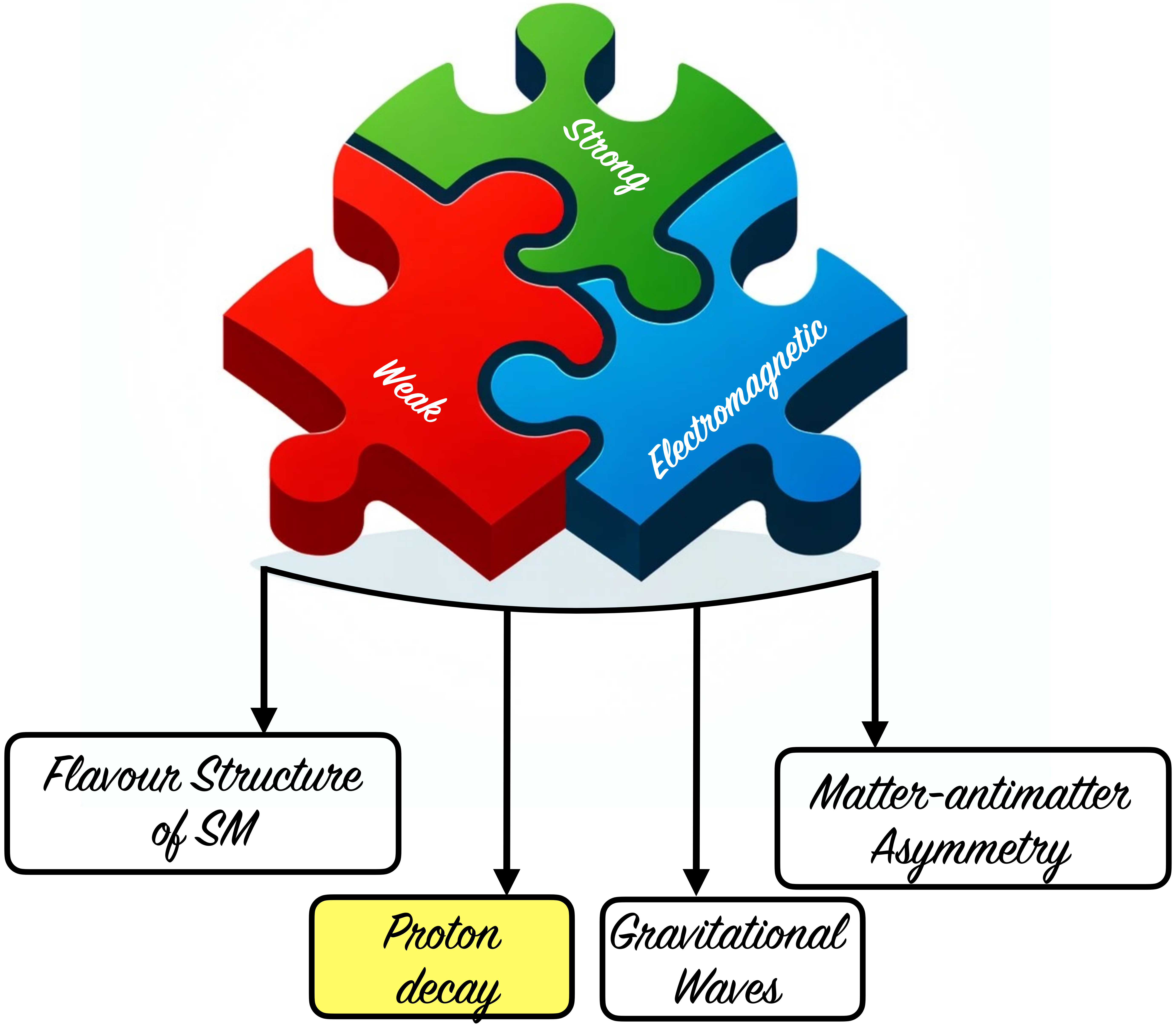
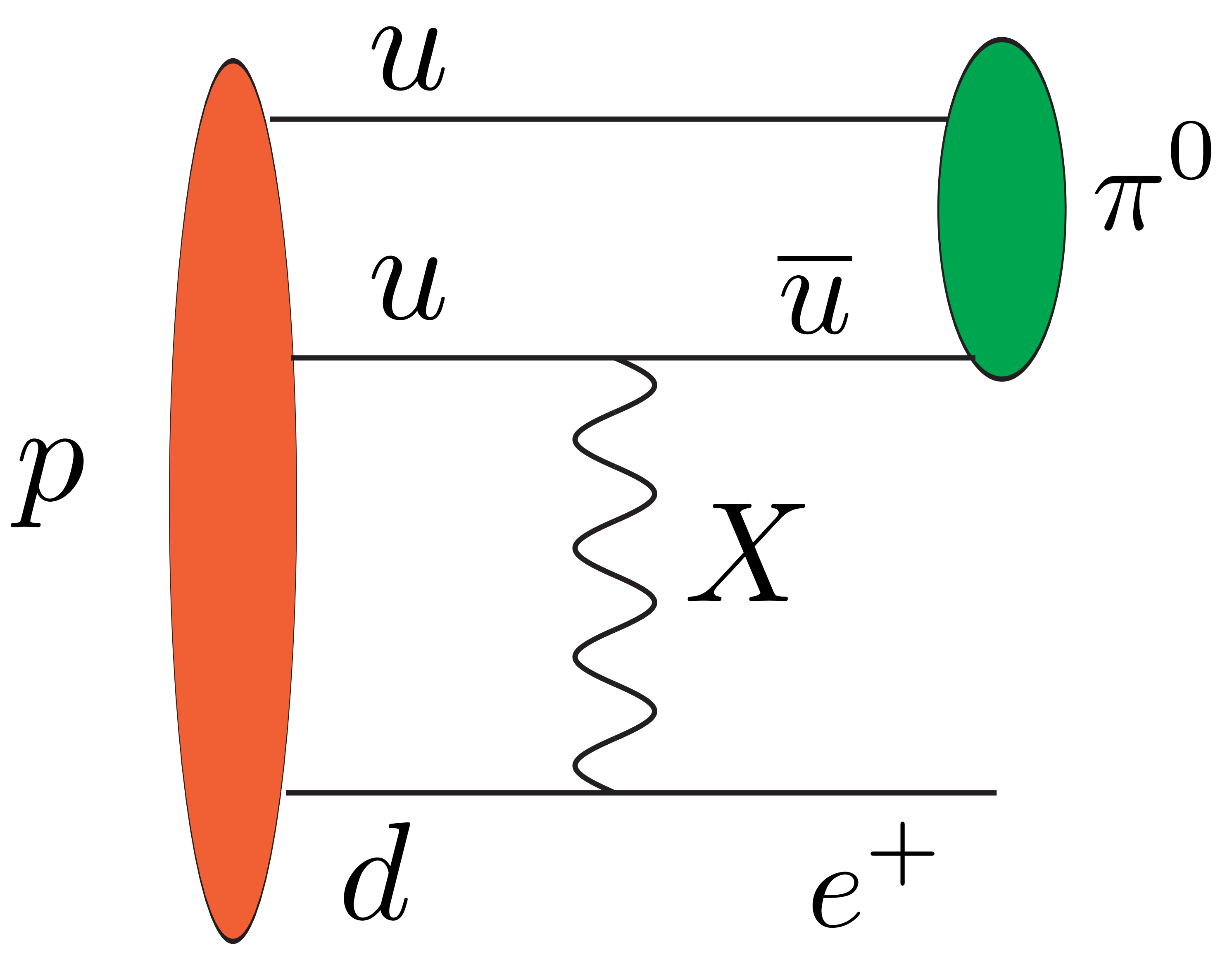


Proton Decay from GUTs

- GUTs unify leptons and quarks into common multiplets $\Longrightarrow$ B & L not conserved

non-SUSY GUT

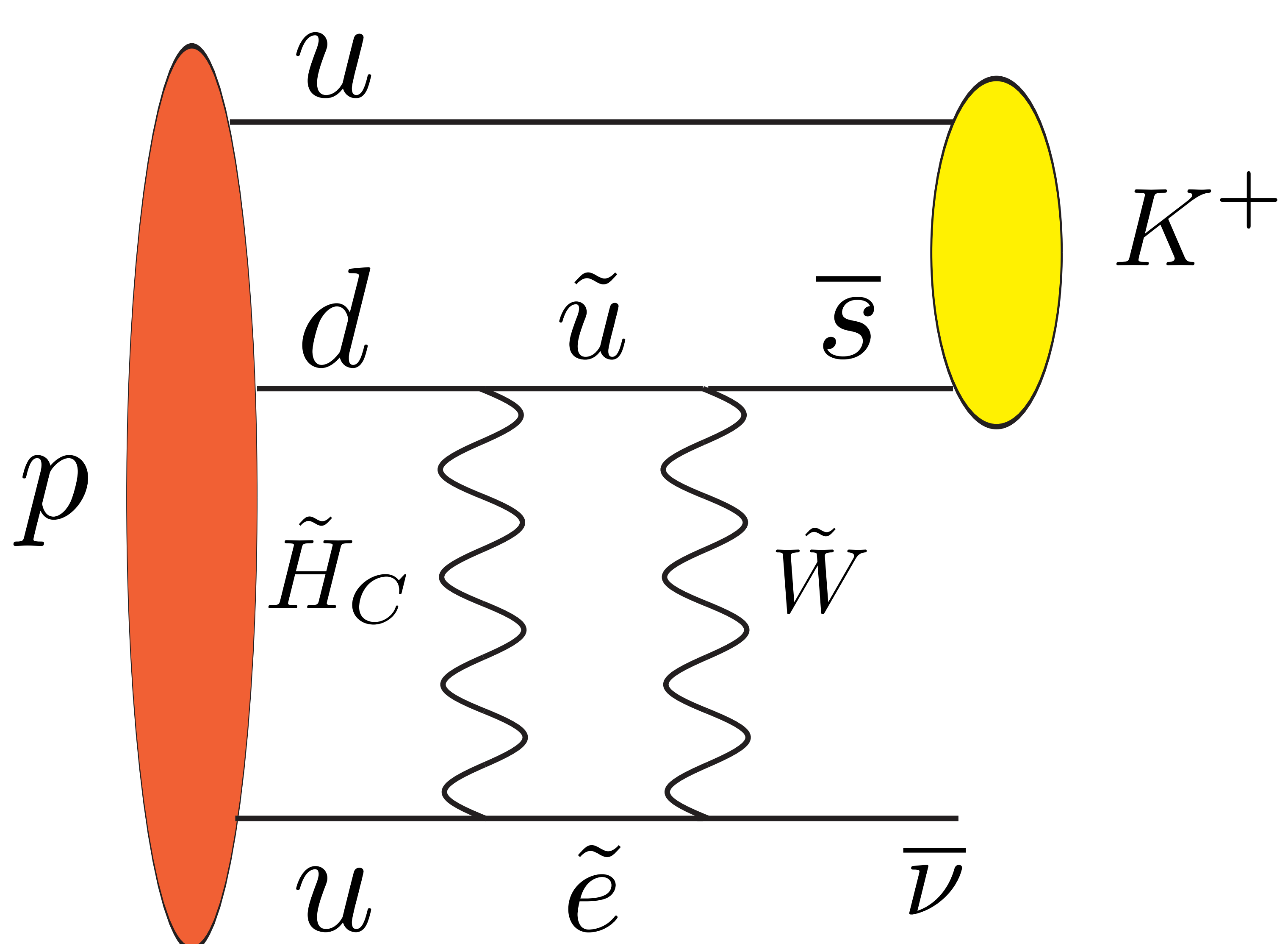
$$p \to \pi^0 \, e^+$$



$$\tau \propto \frac{1}{\alpha_X^2} \left(\frac{M_X}{10^{16} \mathrm{GeV}} \right)^4$$

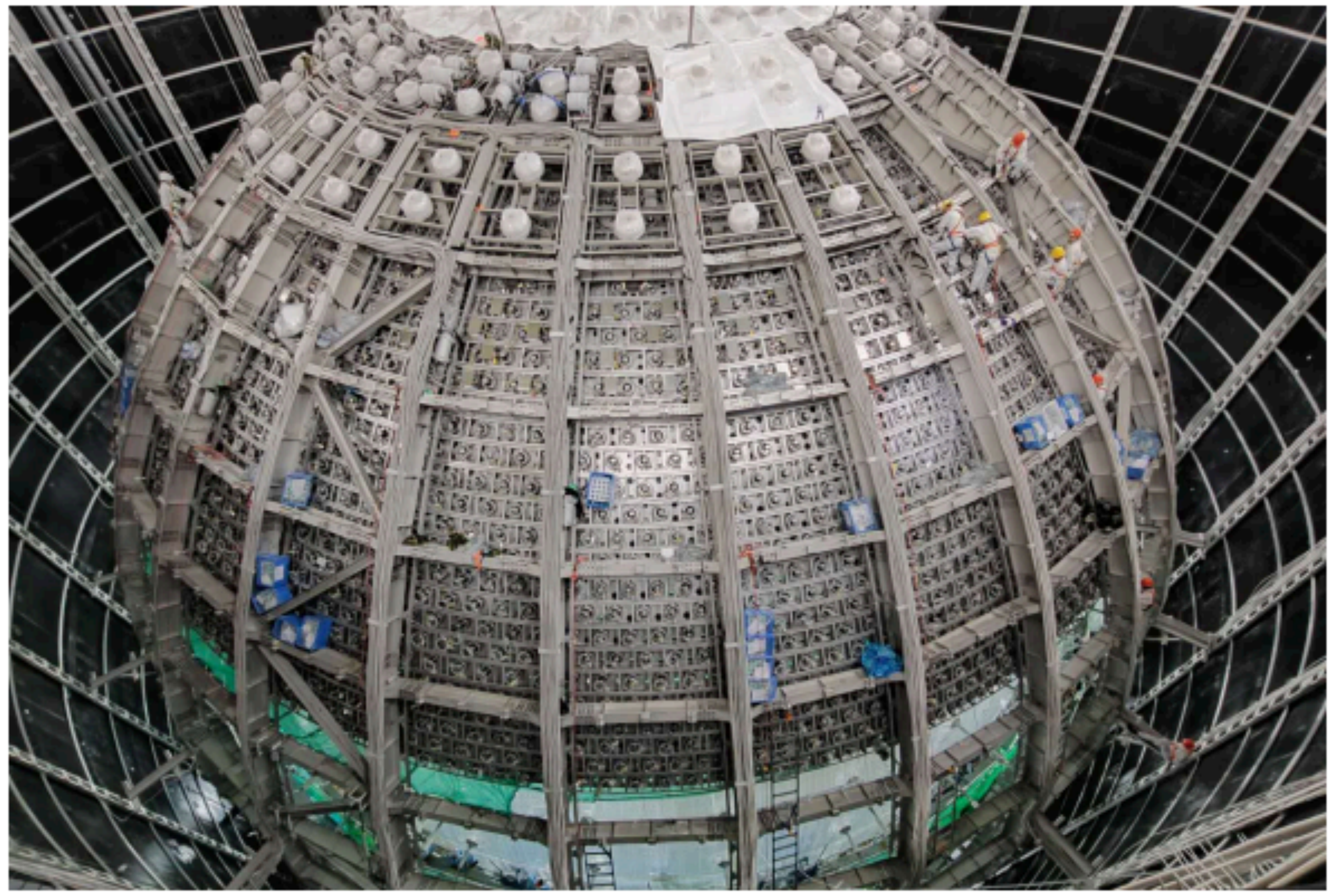
SUSY GUT

$$p \to K^+ \, \overline{\nu}$$

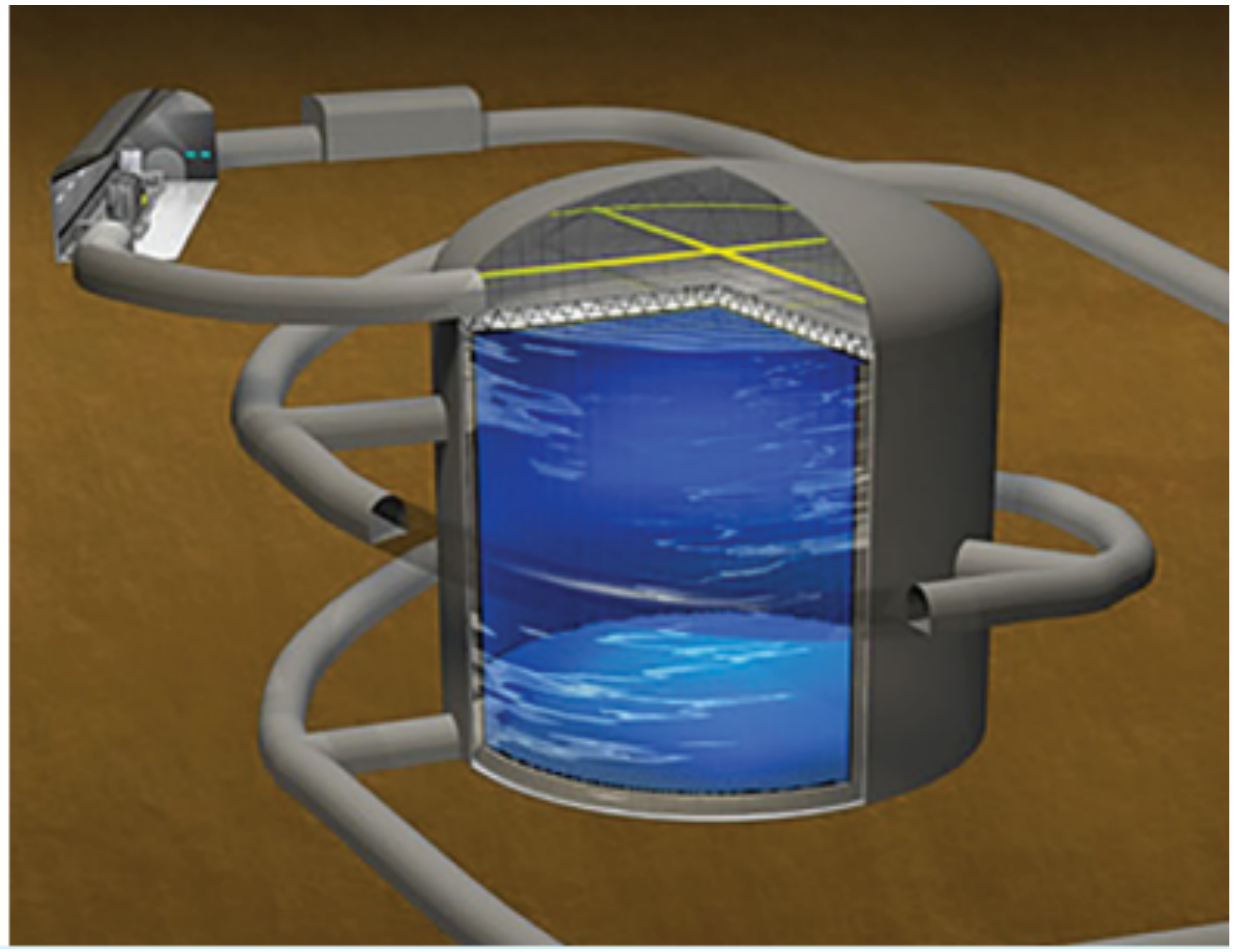


$$\tau \propto \frac{1}{\tilde{\alpha}^2} \left(\frac{M_{\tilde{W}} M_{\tilde{H}_C}}{10^{32} \, \mathrm{GeV}^2} \right)^2$$

Reason 1: Proton Decay from GUTs



See talk by Jason Evans & Akira Takenaka



Hyper-Kamiokande
2027 expected data taking

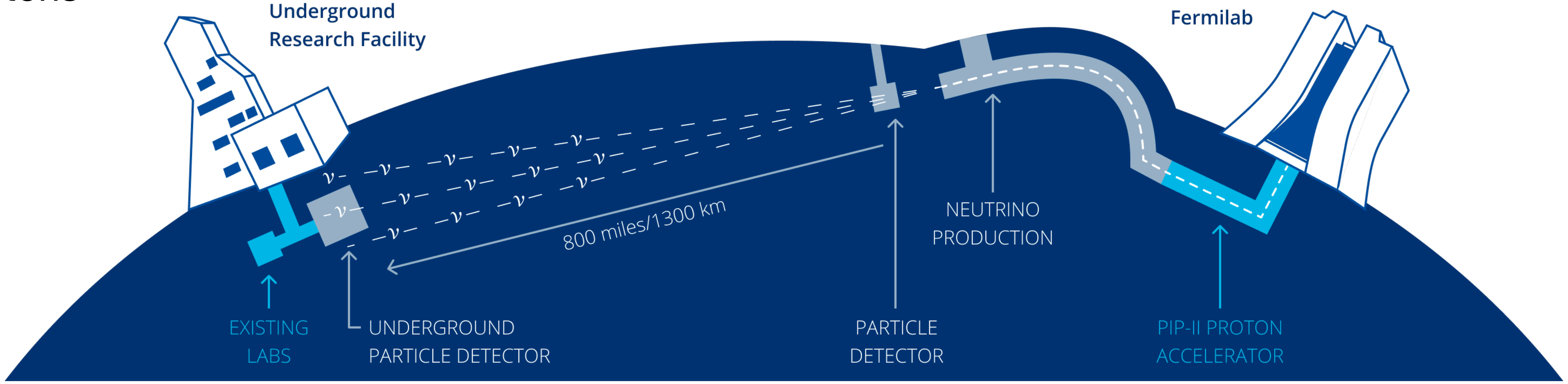
188 kiloton $\sim 7 \times 10^{34}$ protons

JUNO, data taking end this year

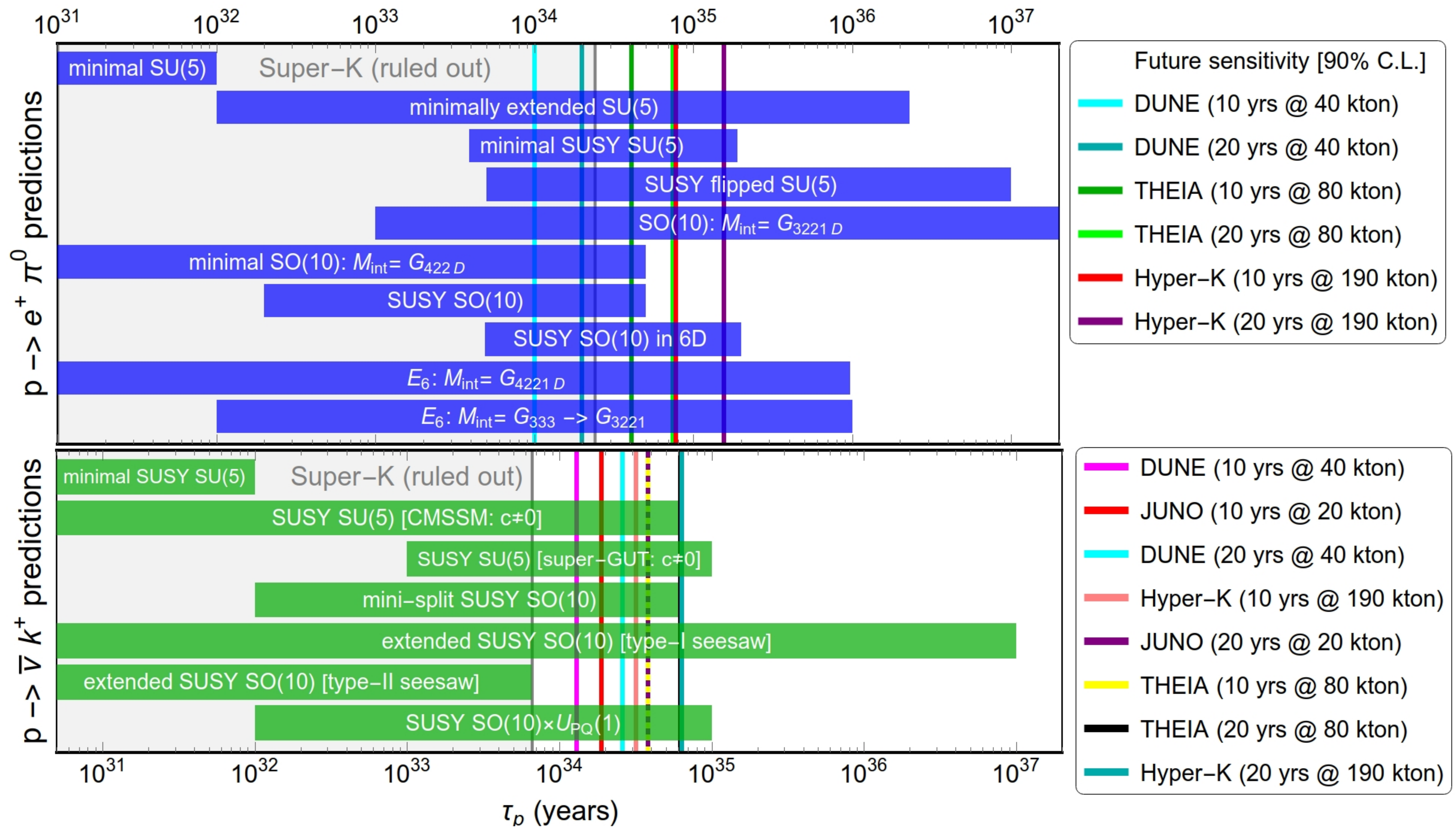
20 kiloton $\sim 7 \times 10^{33}$ protons

DUNE
2030(ish) expected data taking

40 kiloton $\sim 10^{34}$ protons

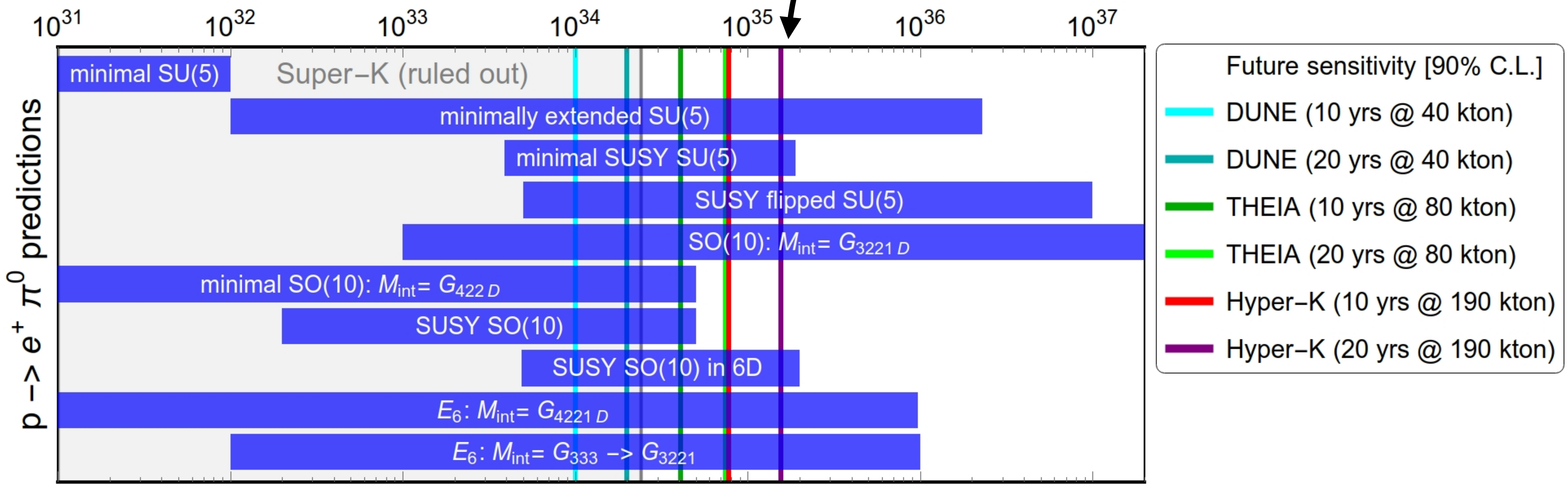


Searches for Baryon Number Violation in Neutrino Experiments: A White Paper

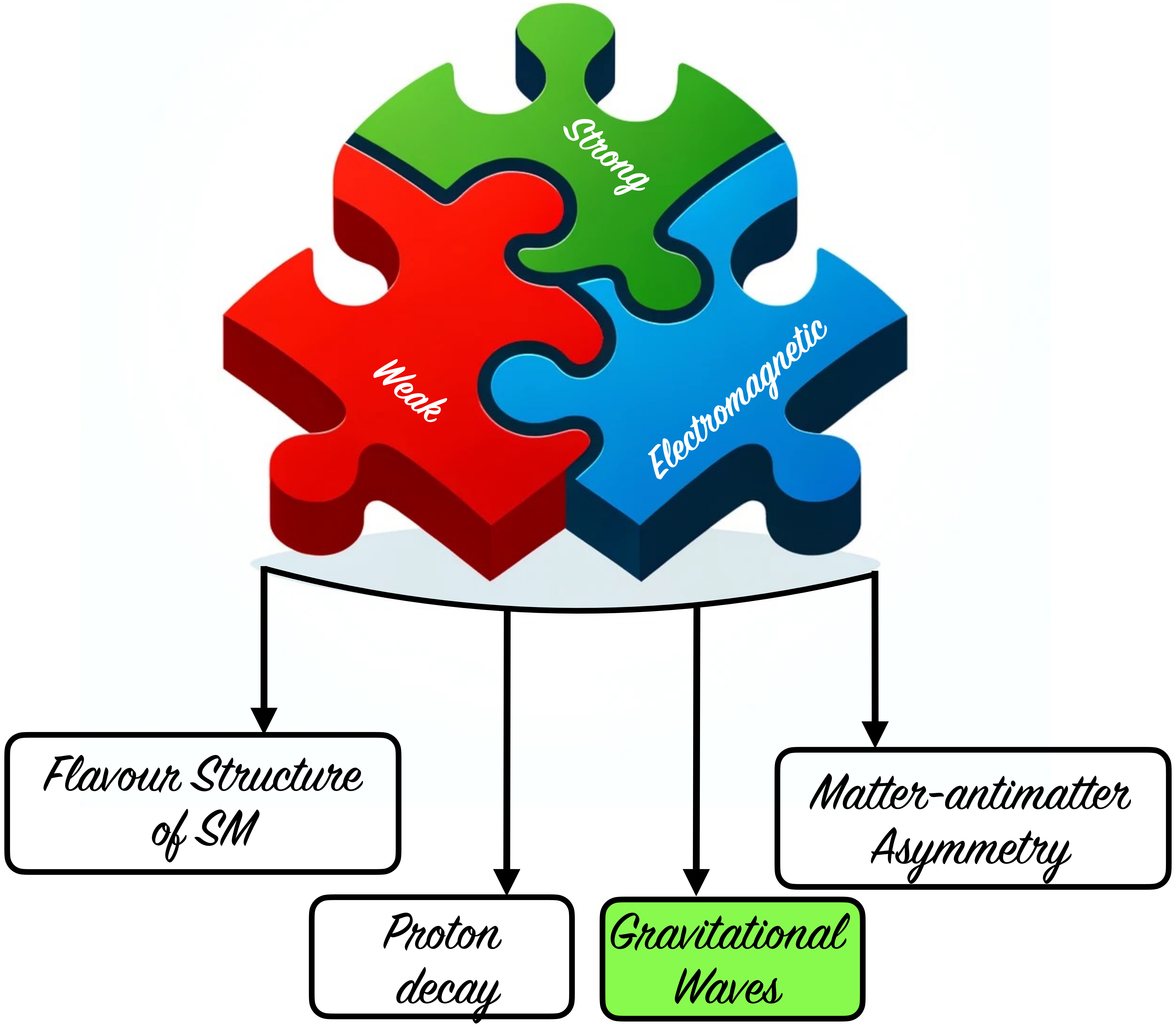


Searches for Baryon Number Violation in Neutrino Experiments: A White Paper

Hyper-Kamiokande
Will be most sensitive to
$p \rightarrow e^+\pi^0$

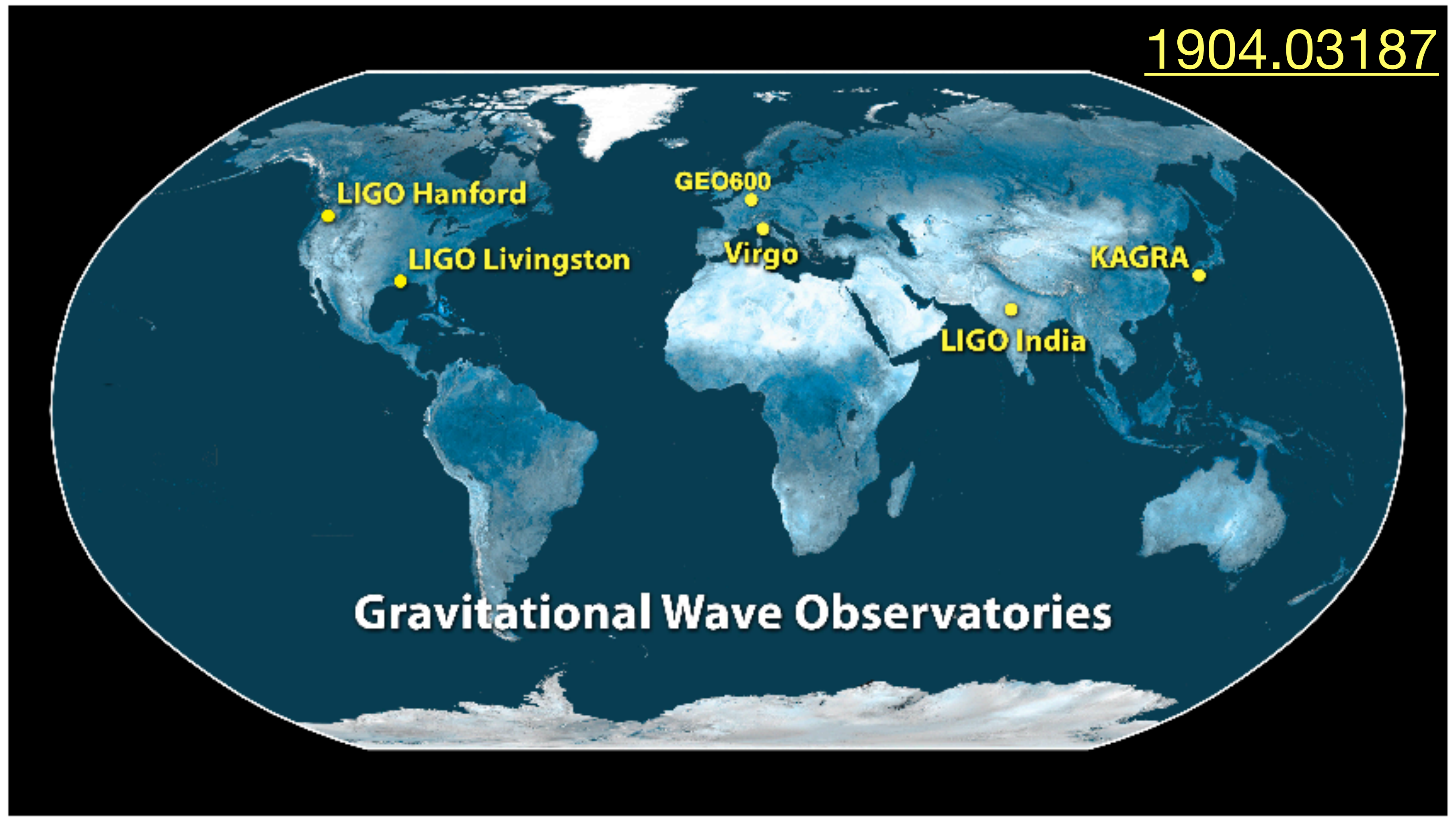


GUTs: motivation



Gravitational waves from GUTs

Ground Based Interferometers



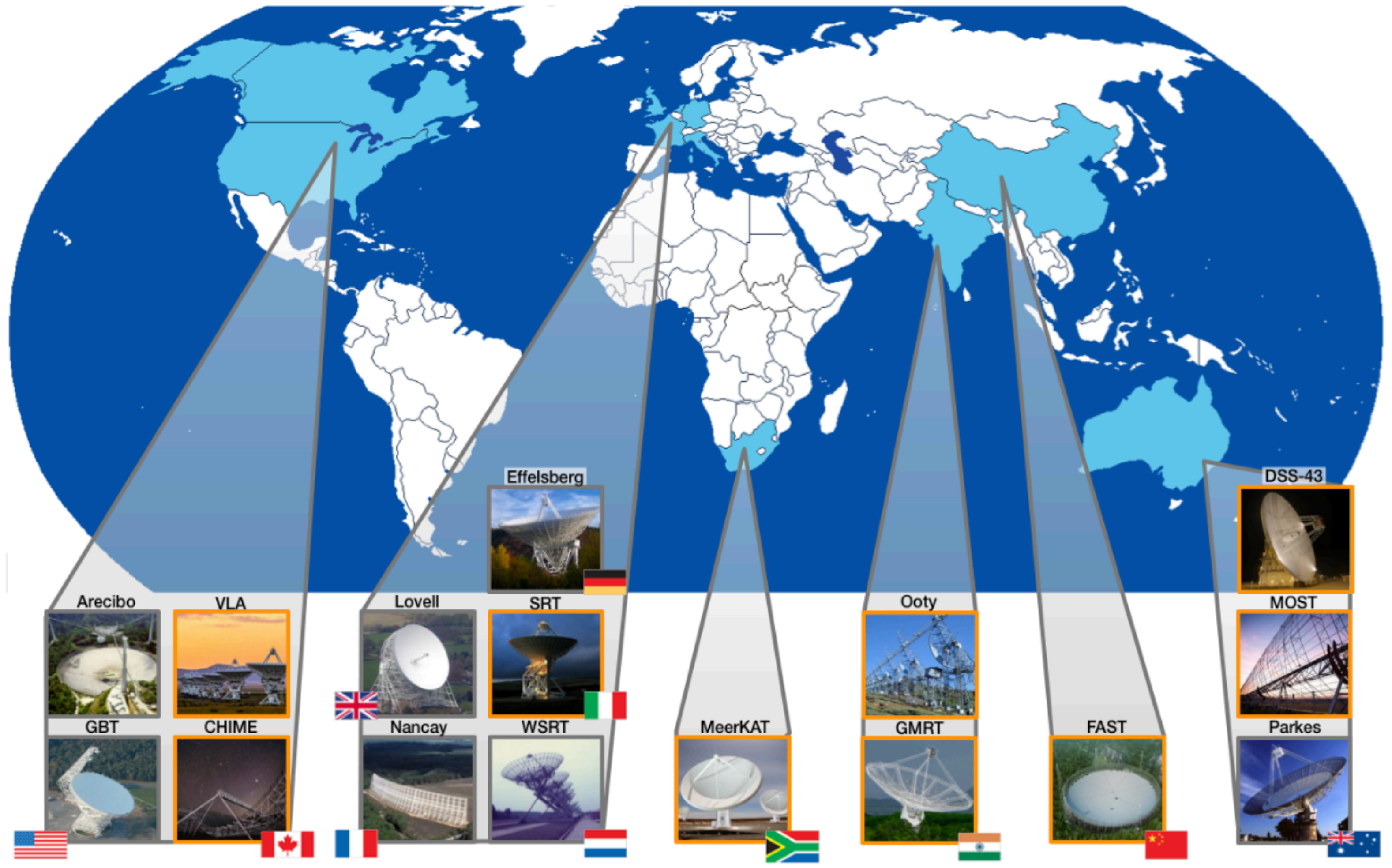
$\mathcal{O}(1-10^3)$ Hz

June 2023

NANOGrav 15 year dataset, European Pulsar Timing Array
Parkes Pulsar Timing Array, Chinese Pulsar Timing Array
has evidence of gravitational wave in nanoHertz regime

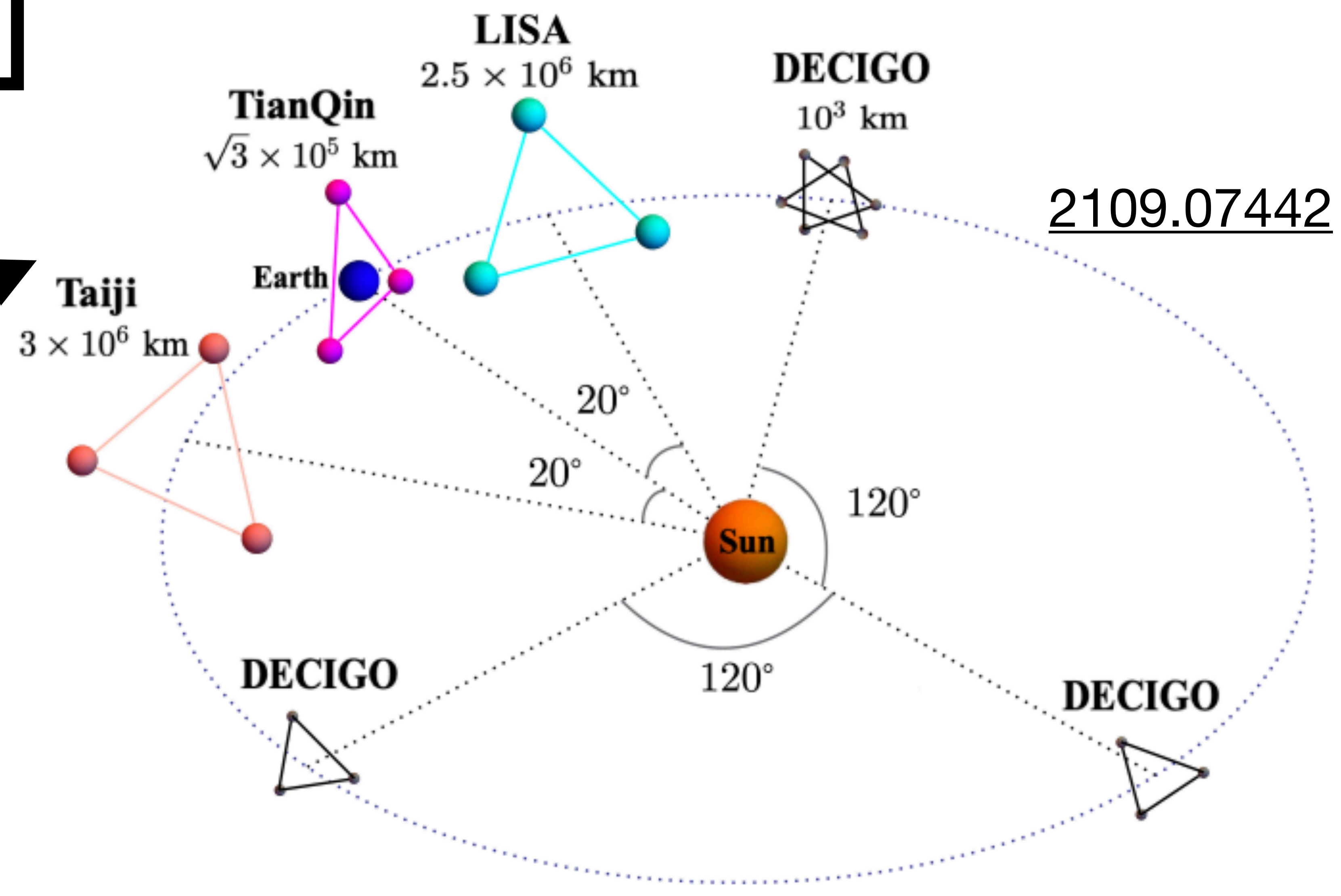
Some experimental developments based here!

Pulsar Timing Arrays



$\mathcal{O}(10^{-9}-10^{-6})$ Hz

Space Based Interferometers



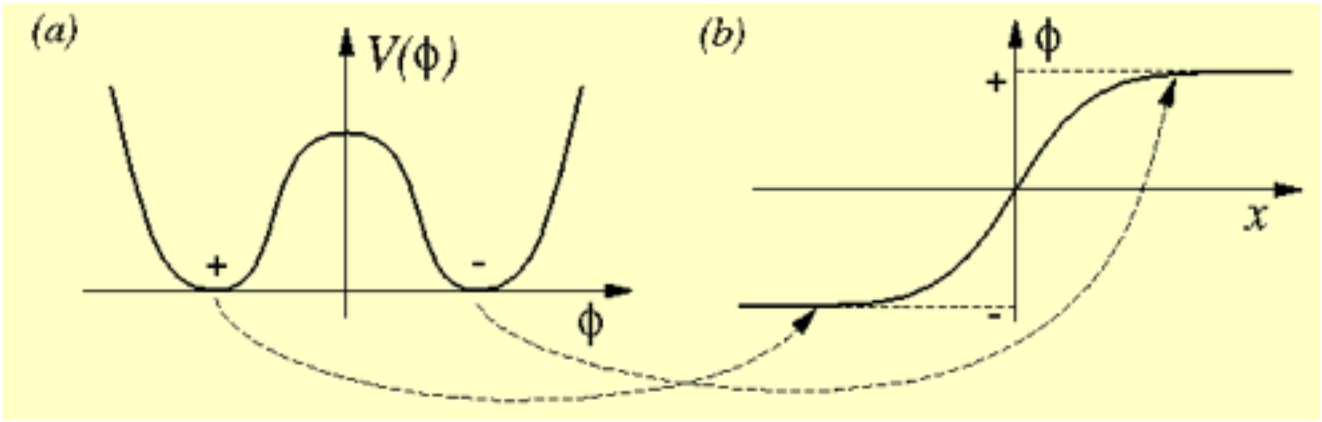
2109.07442

$\mathcal{O}(10^{-4}-1)$ Hz

GUTs prediction: topological defects

During SSB from $G_{GUT} \rightarrow \cdots \rightarrow G_{SM}$ topological defects may form.

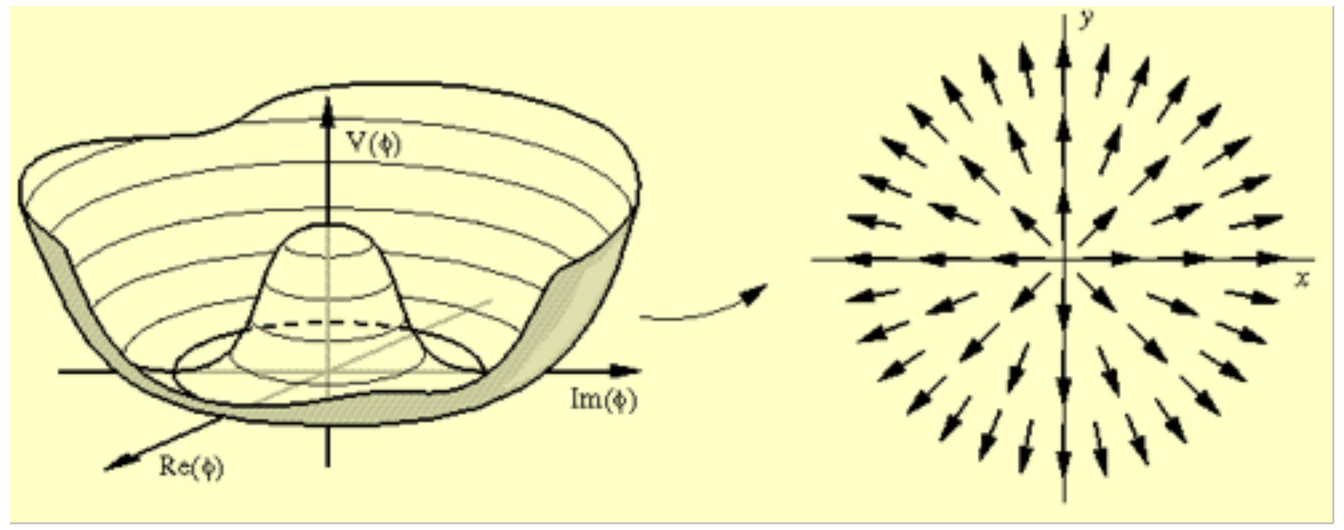
Domain wall



$\pi_0(\mathrm{G/H}) \neq 0$

Disconnected

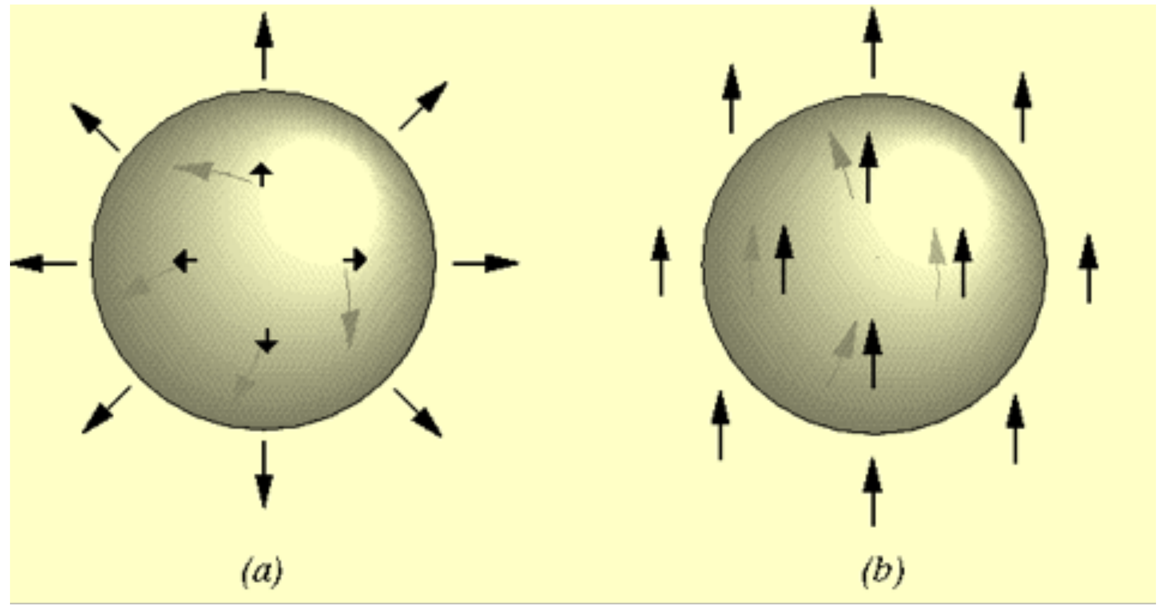
Cosmic strings



$\pi_1(\mathrm{G/H}) \neq 0$

Non-contractible loop

Monopoles



$\pi_2(\mathrm{G/H}) \neq 0$

cambridge cosmic structures

Two-sphere

GUTs prediction: topological defects

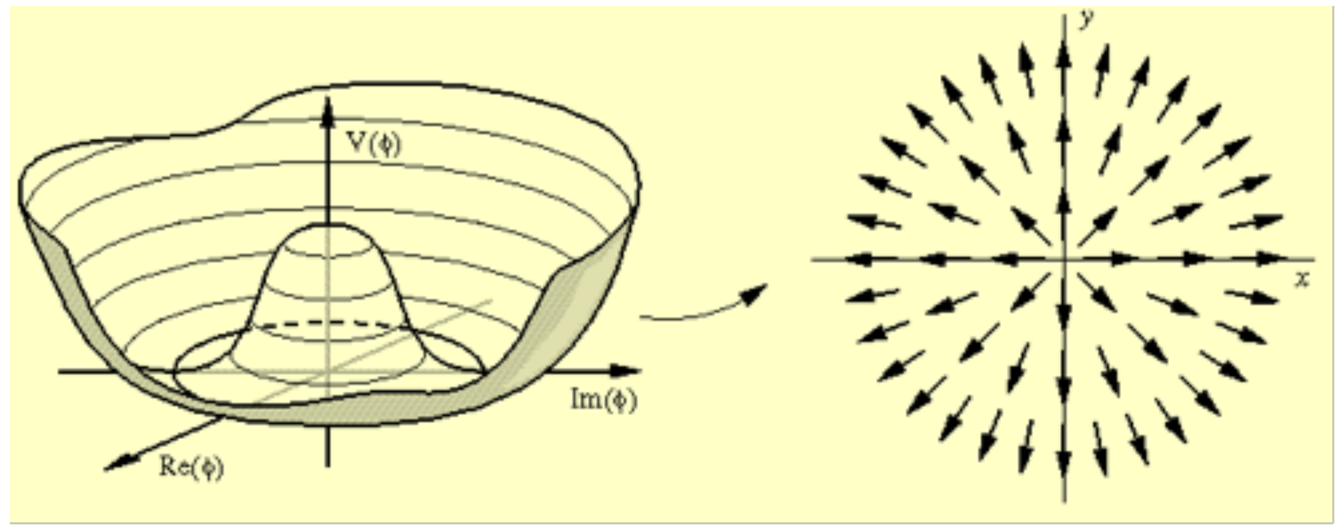
During SSB from $G_{GUT} \rightarrow \cdots \rightarrow G_{SM}$ topological defects may form.

Domain wa

Inflation to dilute

$\pi_0(\mathrm{G/H}) \neq 0$

Cosmic strings



$\pi_1(\mathrm{G/H}) \neq 0$

onopoles

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cambridge cosmic structures

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Domain w

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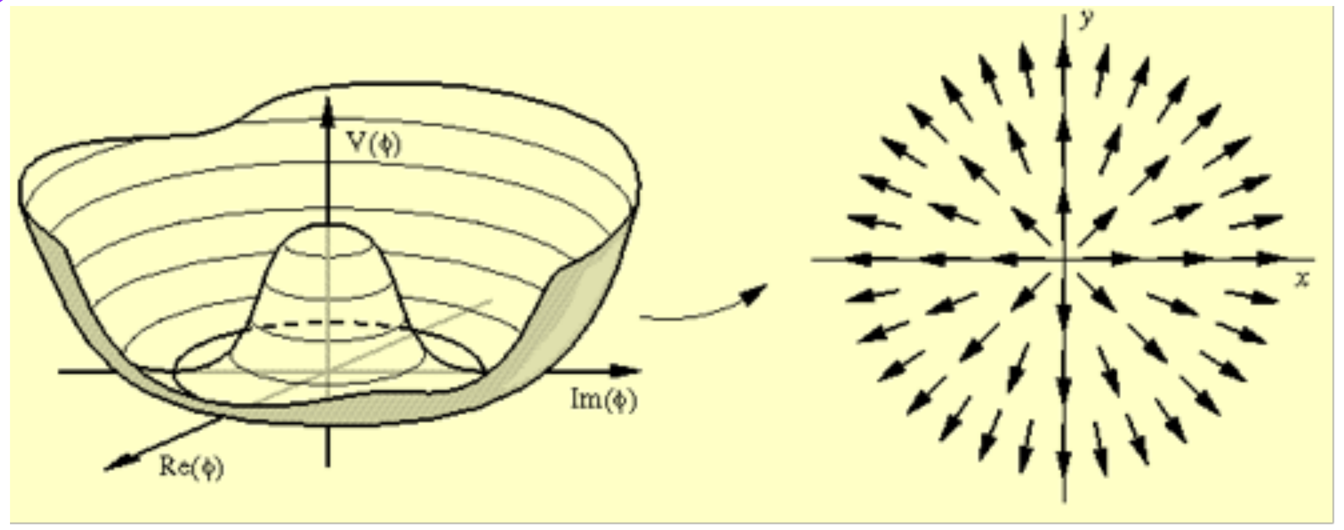
Cosmic strings



$\pi_1(\mathrm{G/H}) \neq 0$

onopoles

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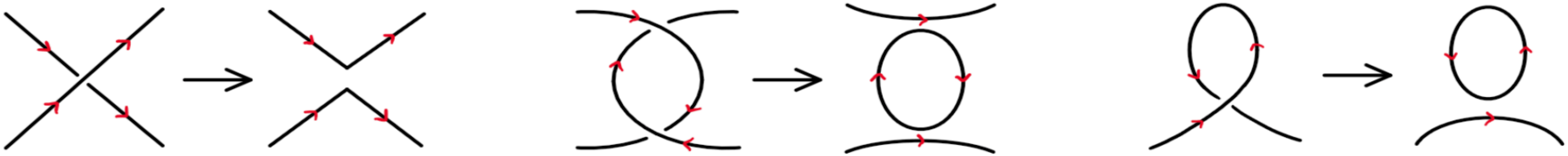
cambridge cosmic structures

Cosmic strings induced via U(1) breaking are ubiquitously as GUT breaks to SM

GUTs prediction: topological defects

Vachaspati & Vilenkin

- Assume Nambu-Goto strings, only coupling to massless mode is gravity
- String properties controlled by symmetry breaking scale, η
- $\eta = 10^{16}$ GeV $\implies$ $\delta = 10^{-30}$ cm and $\mu = 10^{22}$ gm/cm (string parameter)
- String intercommute, can swap partners and create loops



- Gravitational effect of GUT scale string $G\mu = \left(\frac{\eta}{M_{Pl}}\right)^2 \sim 10^{-6}$
- Emission of gravitational radiation by loops: $P_k = \frac{dE_{(k)}}{dt} = -\Gamma_{(k)} G\mu^2$

- Inflation occurs **before** string formation → string network gives “scaling” solution
- Inflation occurs **after** string formation → string network diluted and **no GW signal**
- Inflation occurs **during** string formation → partly diluted string network → **GW spectrum broken power law behaviour** (Cui, Lewicki, Morrissey) *1912.08832*

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$$\Omega_{\mathrm{GW}}(f) = \frac{G\mu^2}{\rho_{\mathrm{crit}}} \sum_{k=1}^{\infty} C_k(f) P_k$$

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- GW power emitted by single loop oscillating at $f = 2k/l$

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- GW signal is superposition of GW emission from all oscillation modes, need to sum All modes for reliable result

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- GW power emitted by single loop oscillating at $f = 2k/l$
- Number of loops that emit GW at frequency f today
- GW signal is superposition of GW emission from all oscillation modes, need to sum All modes for reliable result
- All non-trivial physics contained in loop density function

Cui, Lewicki, Morrissey, Wells

- Inflation occurs **before** string formation → string network gives “scaling” solution

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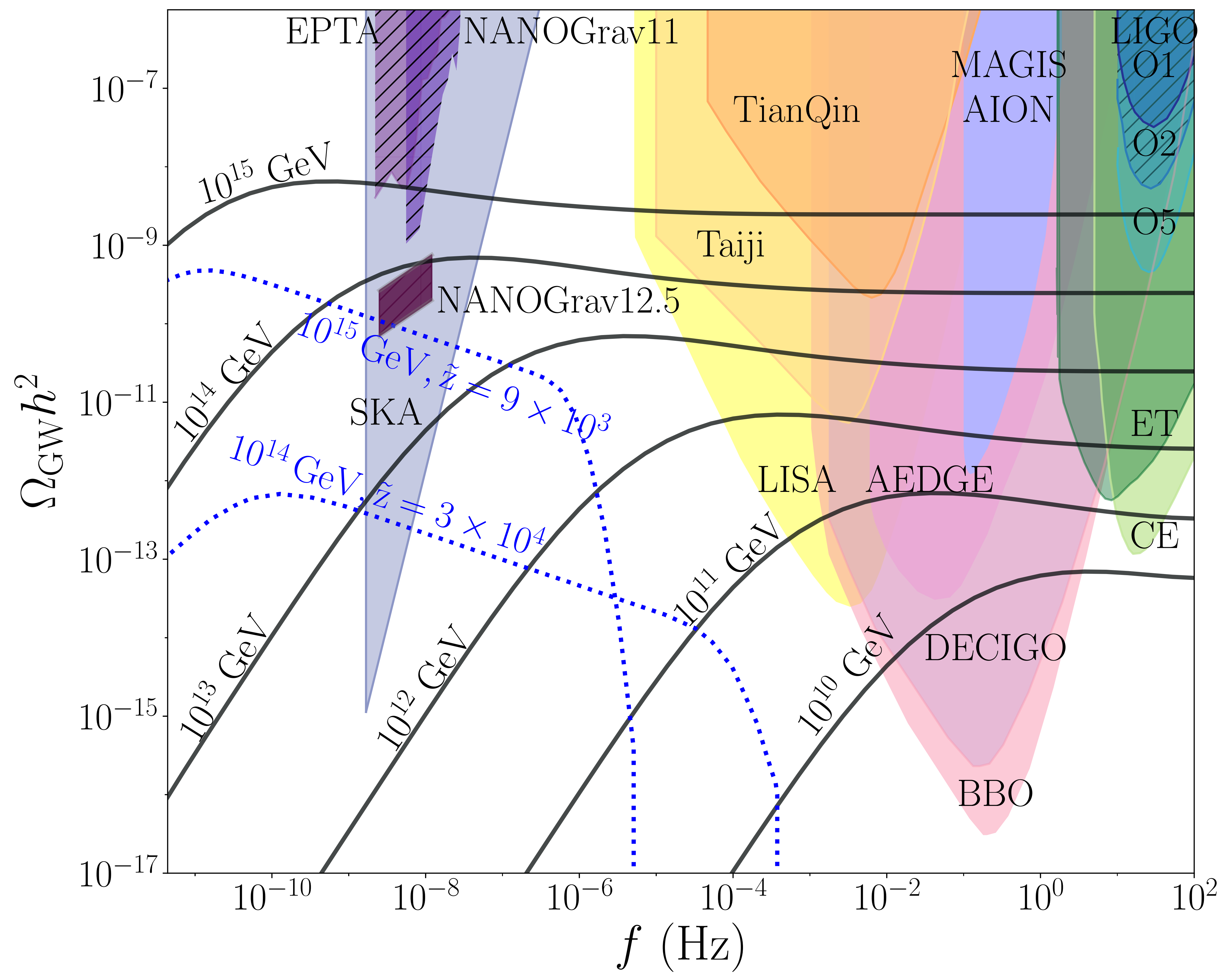


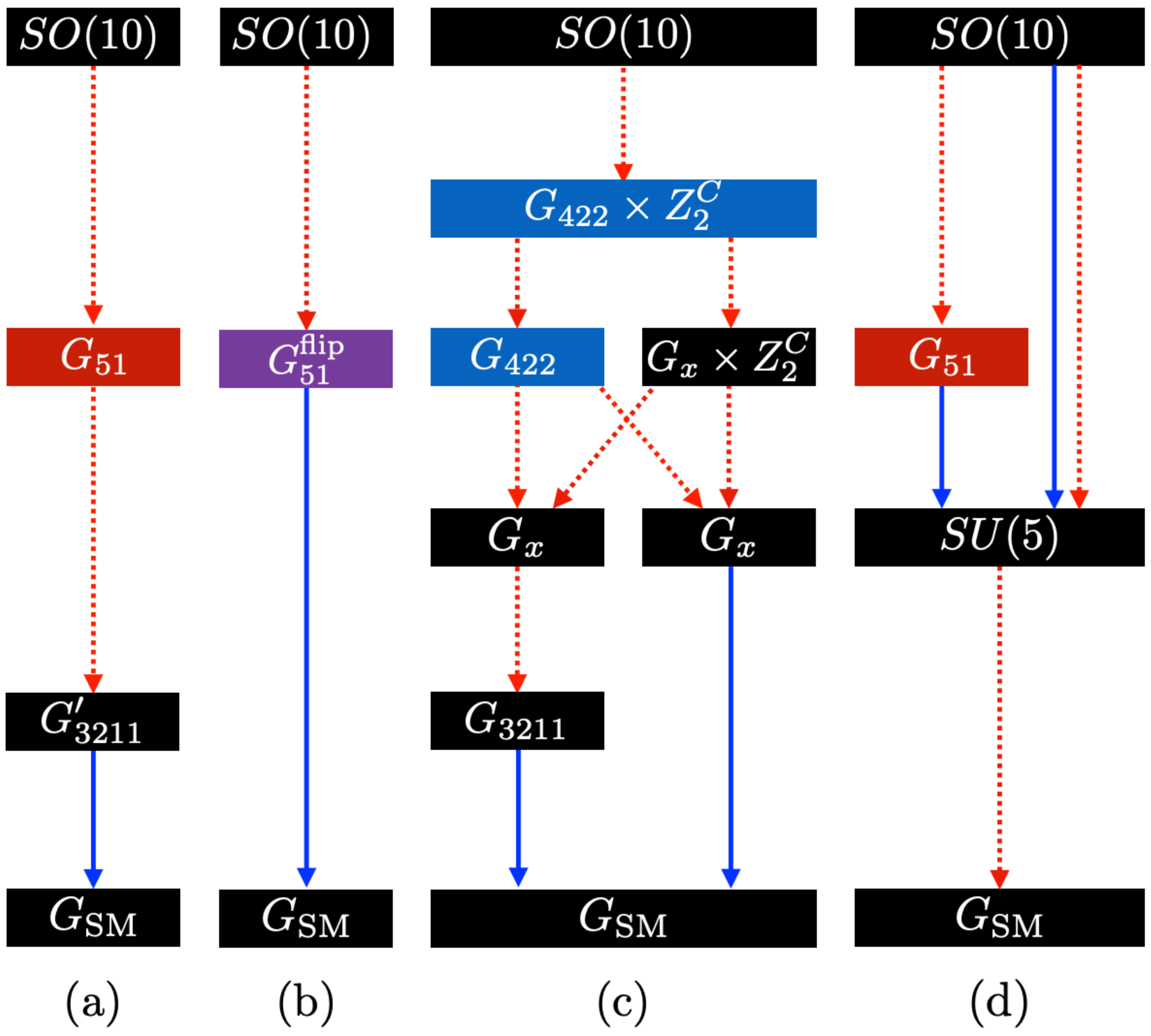
SO(10) phenomenological predictions

- $SO(10)$ provides unification without SUSY and can explain neutrino masses
- 51 "breaking chains" of $SO(10)$ to SM! *Jeannerot, 0308134*

$$\underbrace{SO(10)}_{G_X} \xrightarrow{M_X} \underbrace{G_{422}}_{G_2} \xrightarrow{M_2} \underbrace{G_{3221}}_{G_1} \xrightarrow{M_1} G_{SM}$$

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R$$
$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

SO(10) phenomenological predictions



$$X = \sqrt{\frac{3}{4}}B - L$$

$$G_{51} = SU(5) \times U(1)_X$$

$$G_{51}^{\rm flip} = SU(5)_{\rm flip} \times U(1)_{\rm flip}$$

$$G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$$

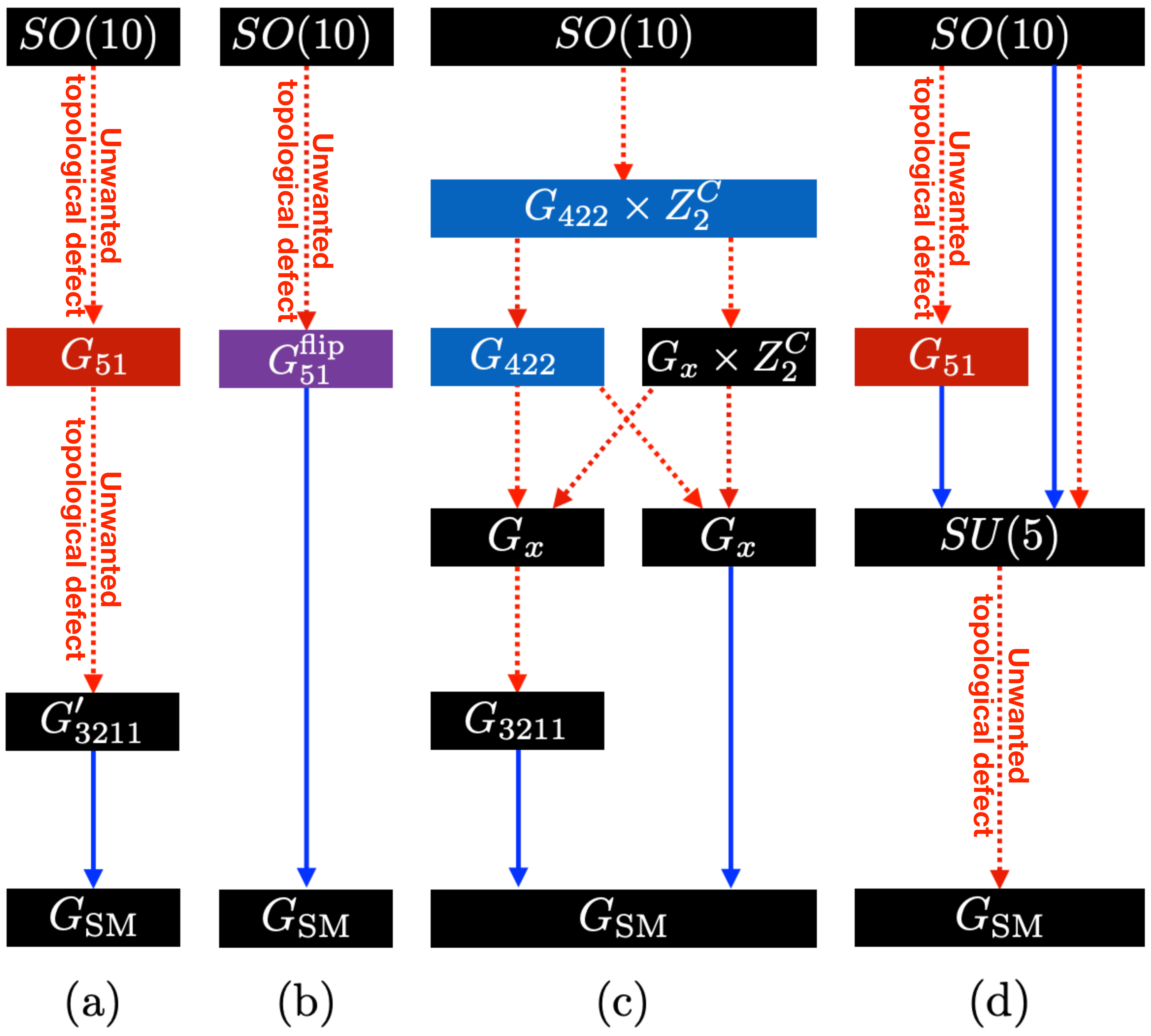
$$G'_{3211} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$$

$$G_{421} = SU(4)_C \times SU(2)_L \times U(1)_Y$$

$$G_{422} = SU(4)_C \times SU(2)_L \times SU(2)_R\,.$$

2005.13549 King, Pascoli, JT, Zhou

SO(10) phenomenological predictions



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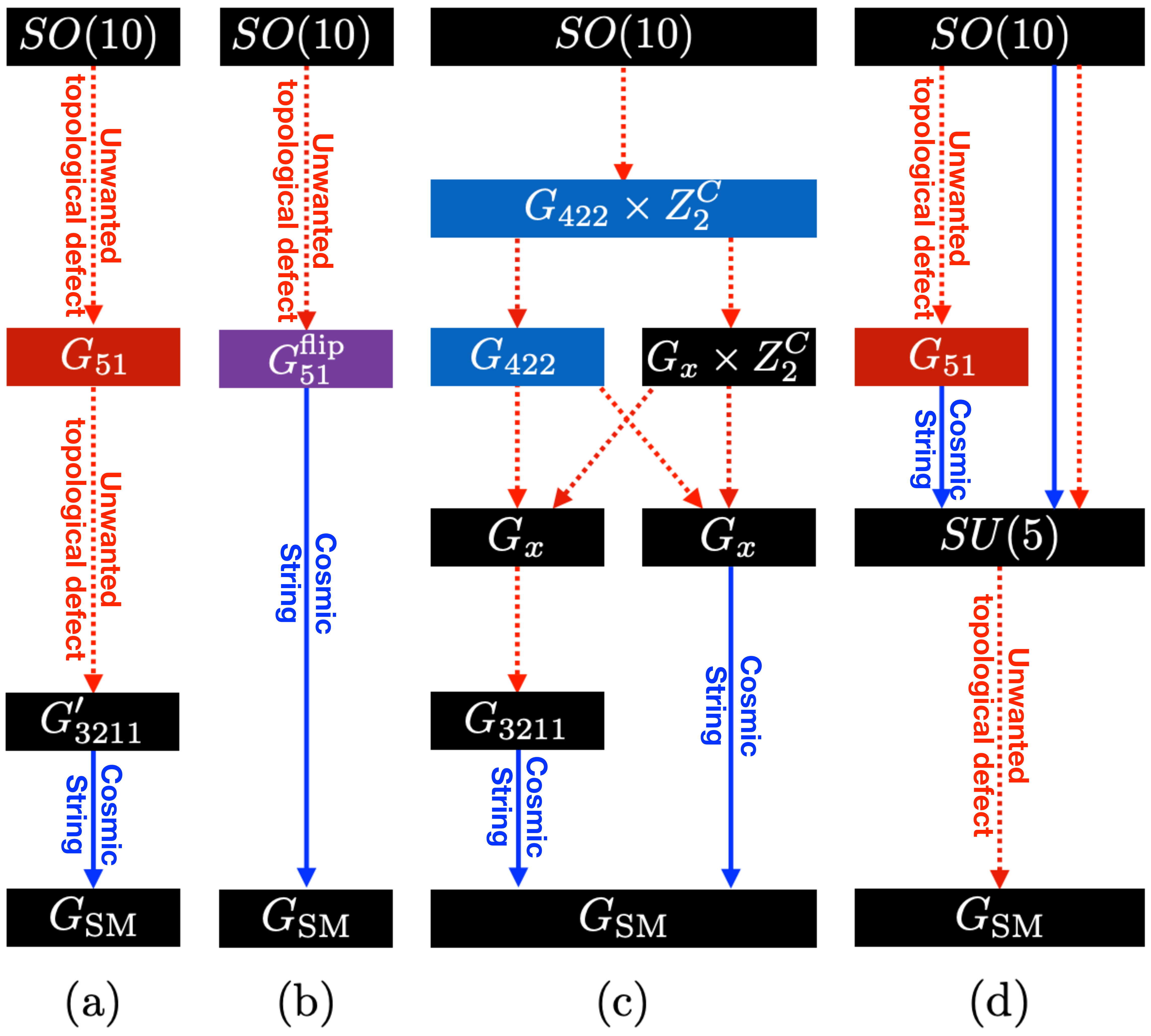
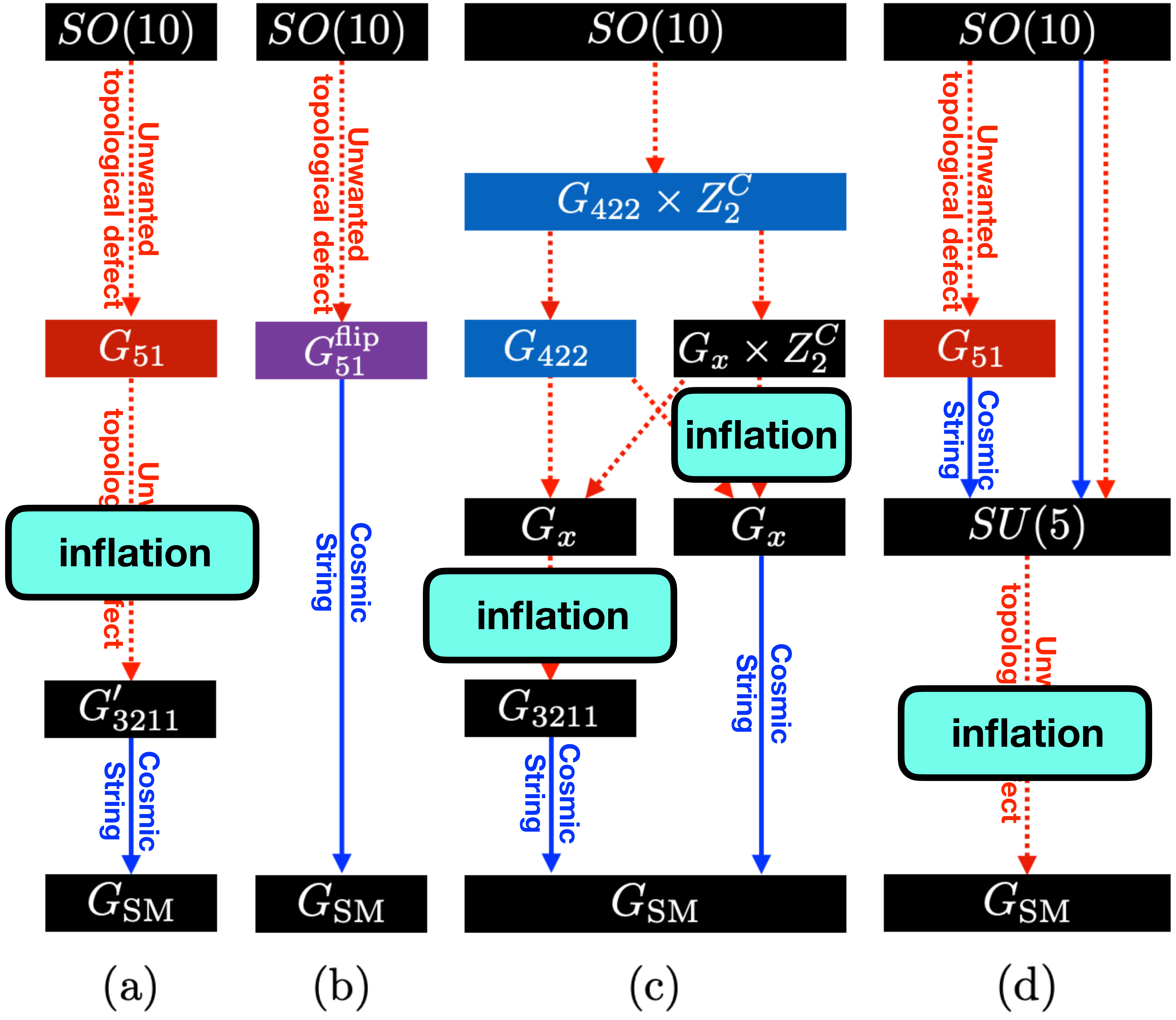
SO(10) phenomenological predictions



2005.13549 King, Pascoli, JT, Zhou

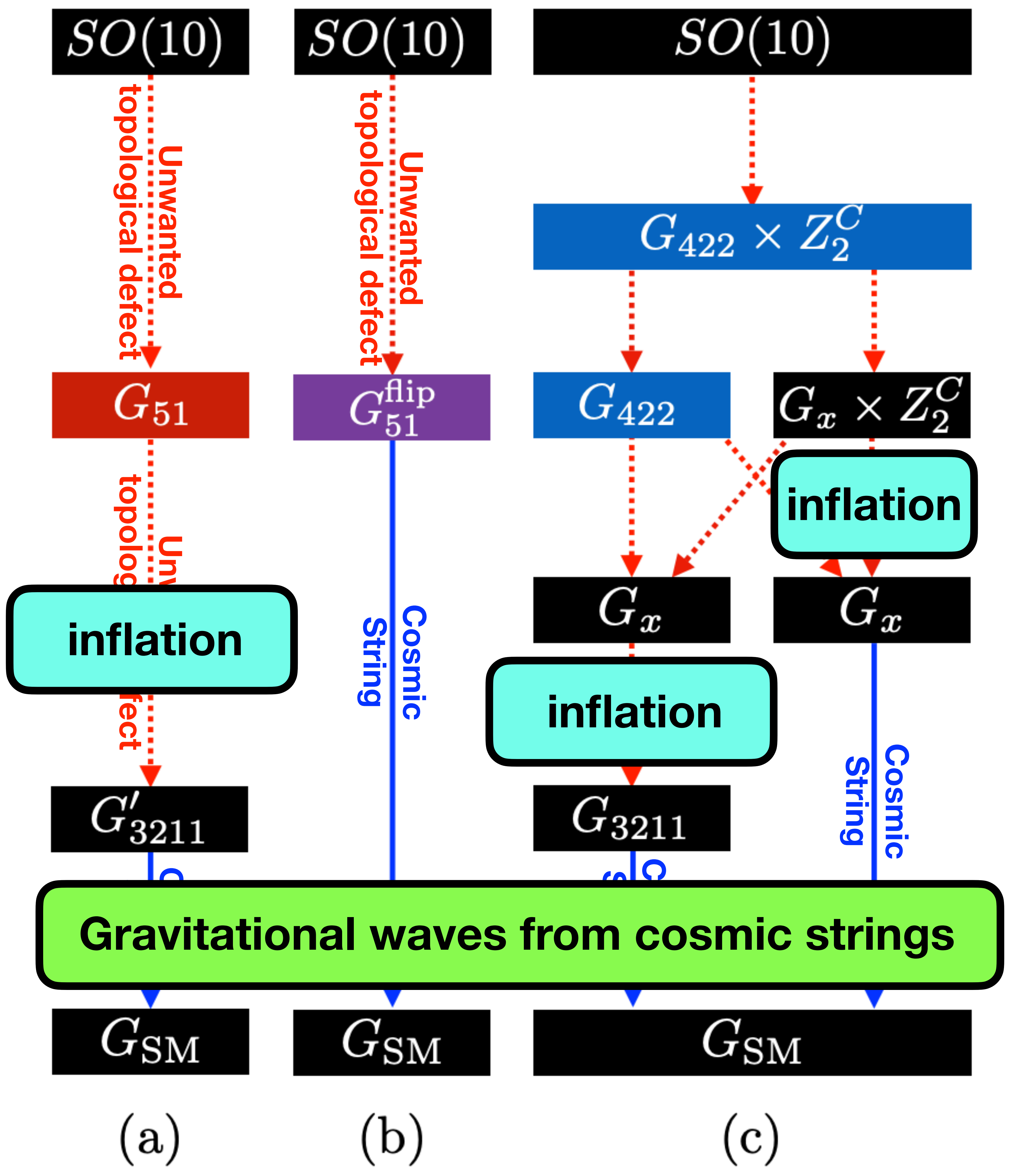
SO(10) phenomenological predictions



CMB data: $\Lambda_{\text{inf}} \lesssim 10^{16}\,\text{GeV}$

2005.13549 King, Pascoli, JT, Zhou

SO(10) phenomenological predictions



CMB data: $\Lambda_{\text{inf}} \lesssim 10^{16}\,\text{GeV}$

Non-observation GW: $\Lambda_{\text{cs}} \lesssim 5 \times 10^{14}\,\text{GeV}$

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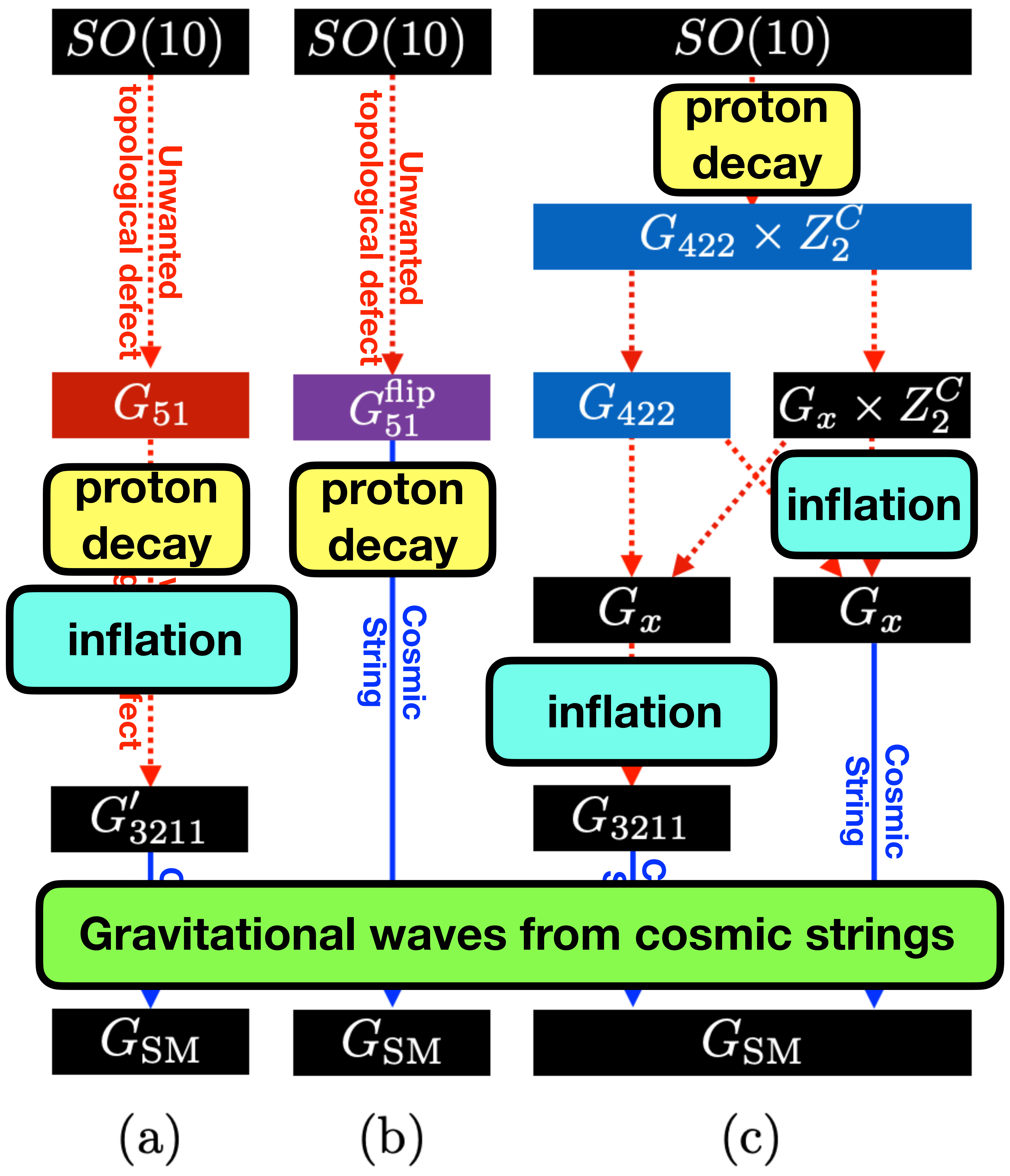

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Non-observation proton decay: $\Lambda_{\rm pd} \gtrsim 10^{15}\,{\rm GeV}$

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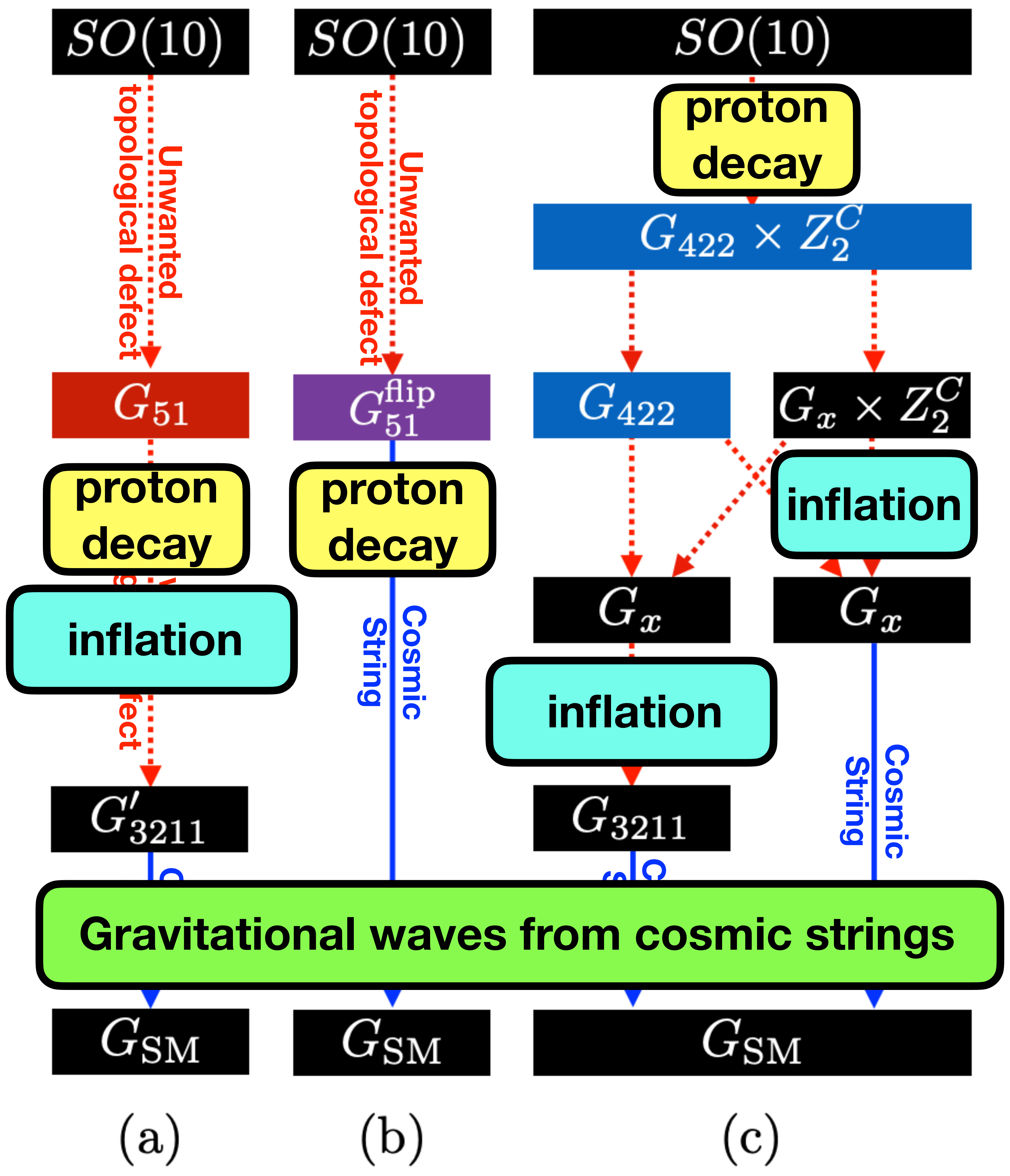

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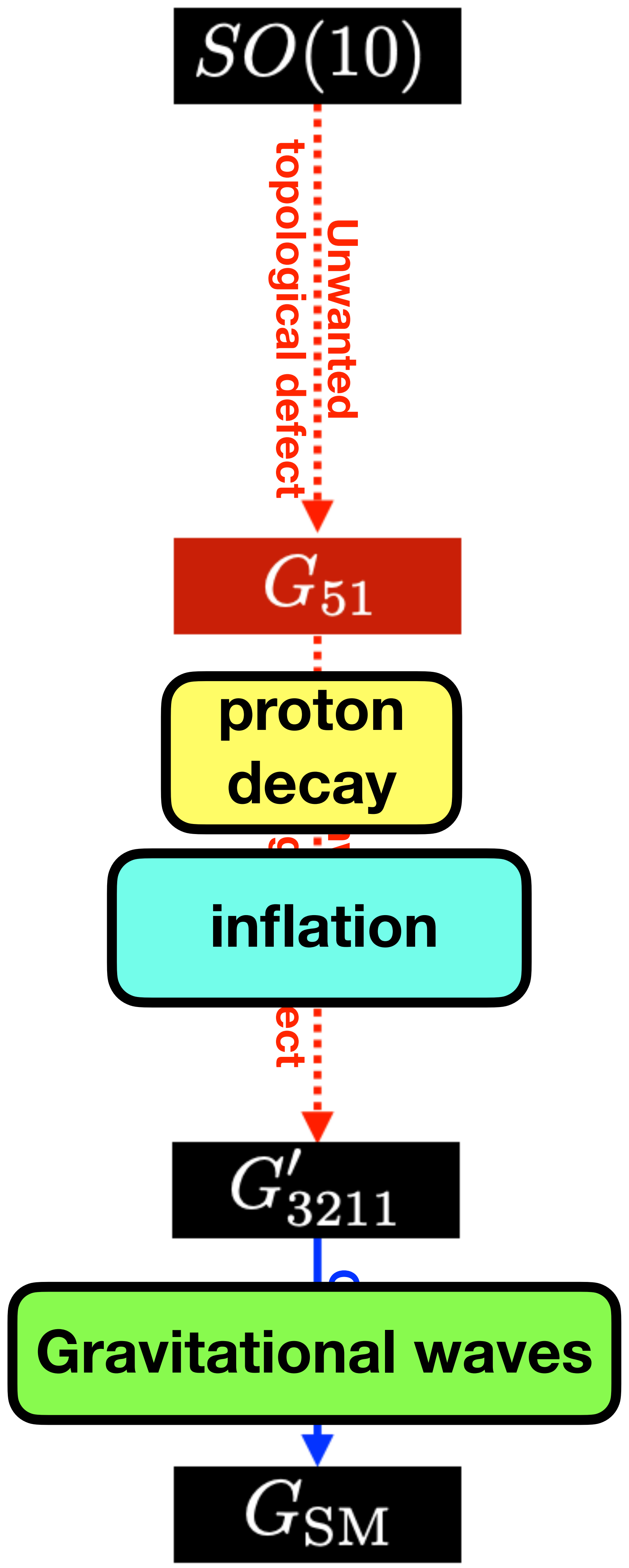
Non-observation proton decay: $\Lambda_{\rm pd} \gtrsim 10^{15}\,{\rm GeV}$

Certain scale ordering excluded e.g

$$\Lambda_{\rm inf} \gg \Lambda_{\rm cs} \gg \Lambda_{\rm pd}$$

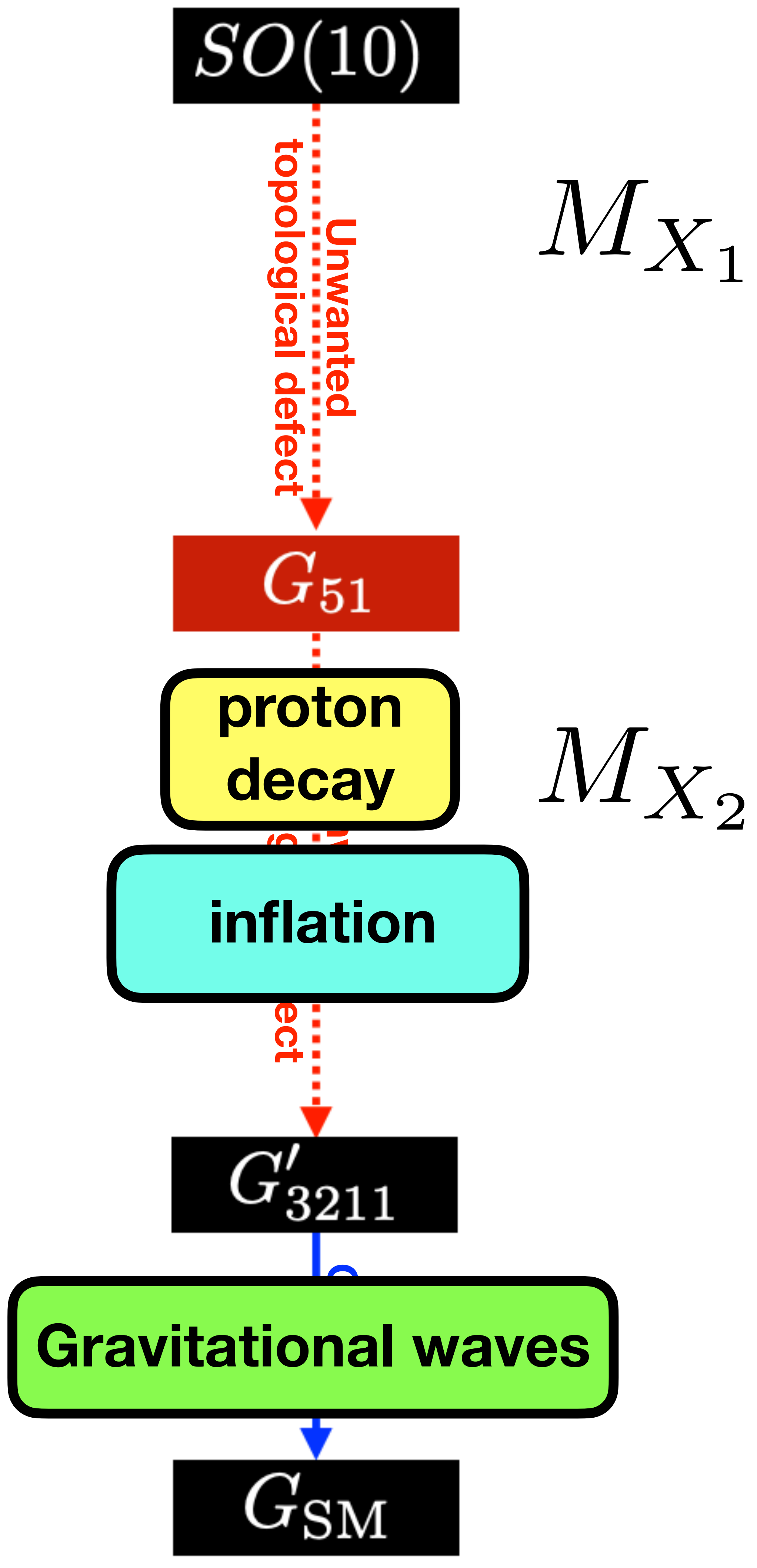
2005.13549 King, Pascoli, JT, Zhou

SO(10) phenomenological predictions



(a)

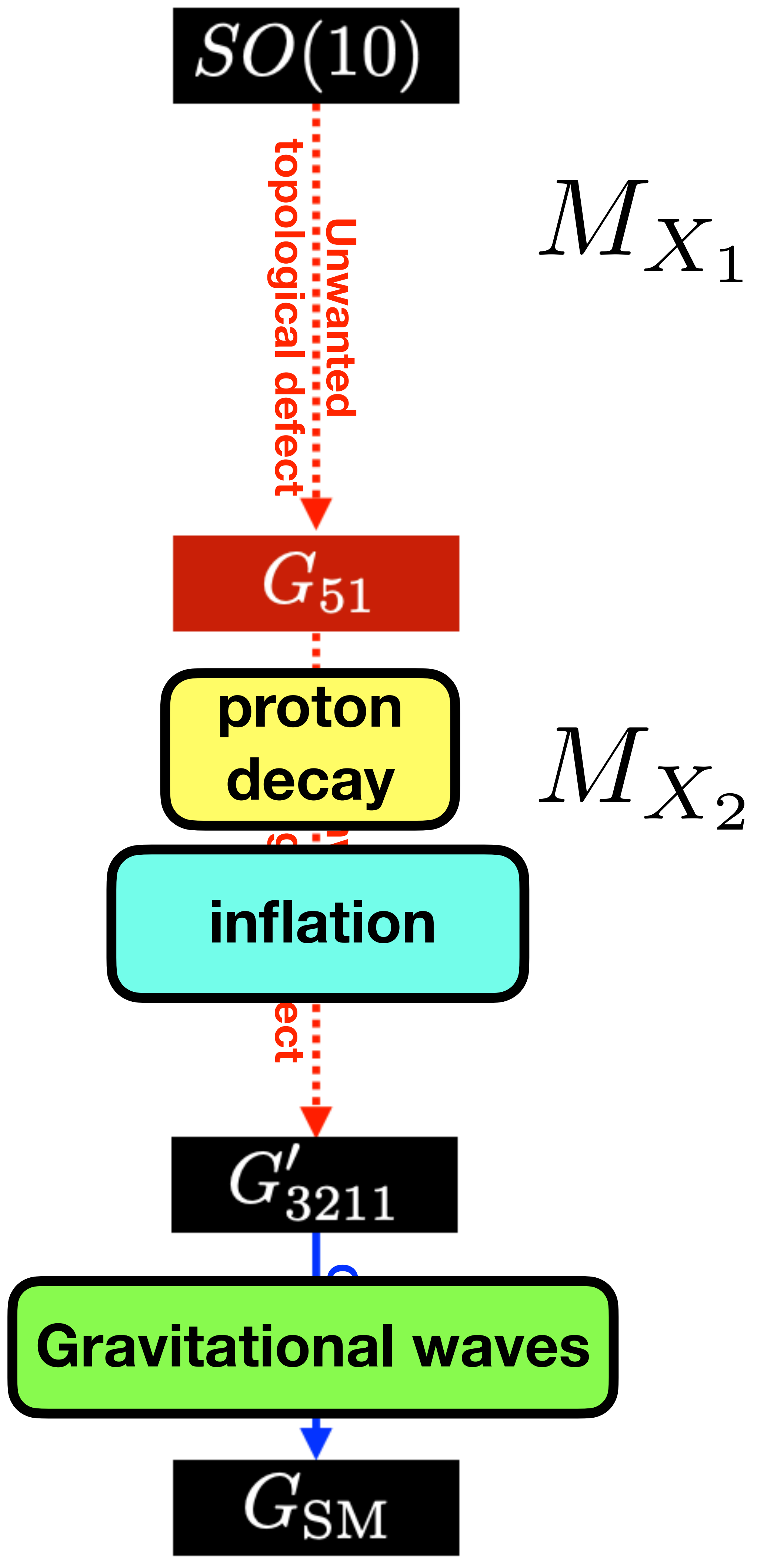
SO(10) phenomenological predictions



(a)

- Proton decay operators induced at M_{X_1} and M_{X_2}

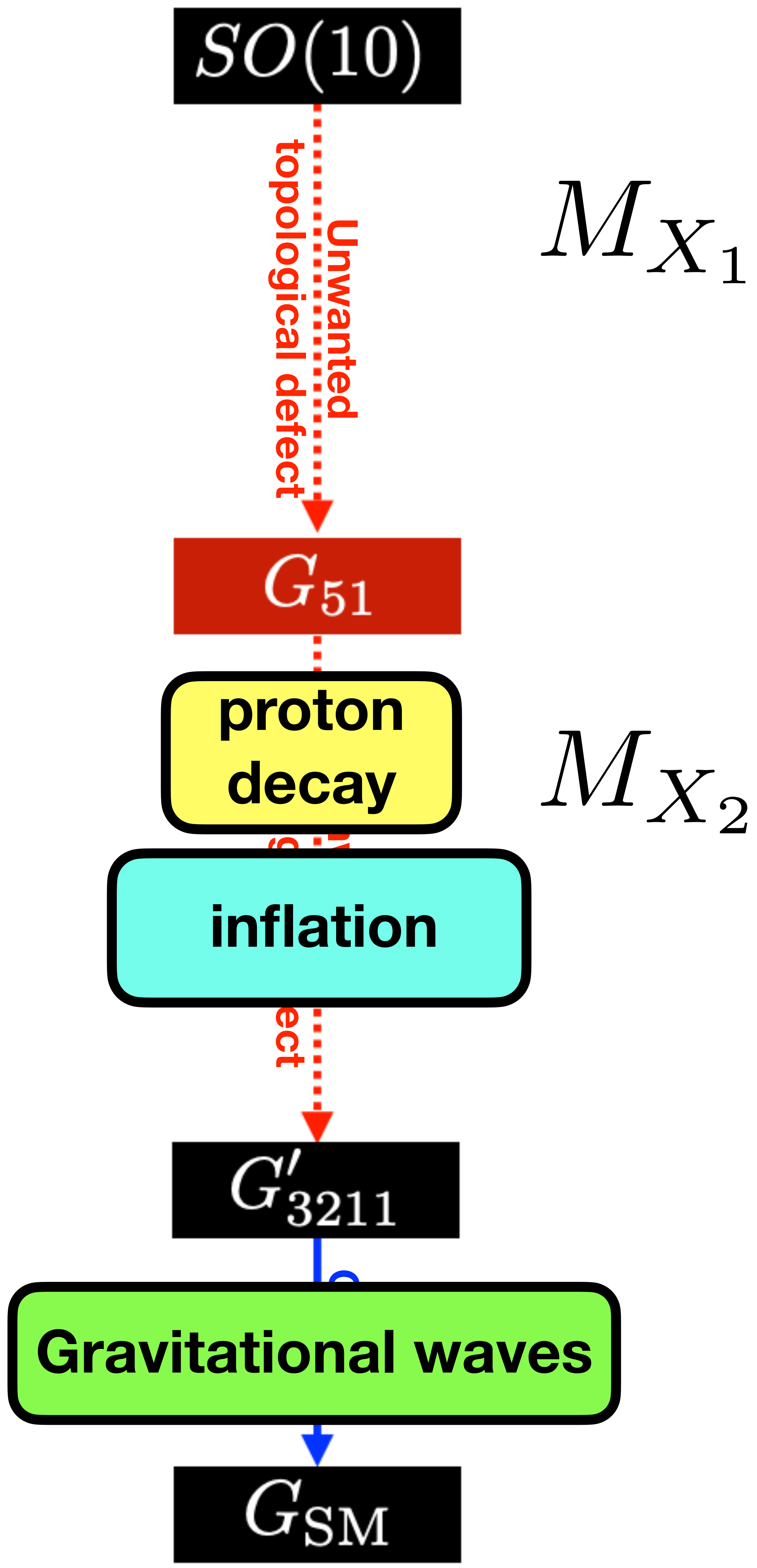
SO(10) phenomenological predictions



(a)

- Proton decay operators induced at M_{X_1} and M_{X_2}
- $M_{X_2} < M_{X_1} \implies$ main proton decay channel: $p \to e^+\pi^0$ at scale $\Lambda_{\rm pd} = M_{X_2}$

SO(10) phenomenological predictions



(a)

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- $M_{X_2} < M_{X_1} \implies$ main proton decay channel: $p \to e^+\pi^0$ at scale $\Lambda_{pd} = M_{X_2}$

1. $\Lambda_{pd} > \Lambda_{inf} > \Lambda_{cs}$: PD + undiluted GW observed (ideal case)
2. $\Lambda_{pd} > \Lambda_{inf} \sim \Lambda_{cs}$: PD + diluted GW observed
3. $\Lambda_{pd} > \Lambda_{cs} > \Lambda_{inf}$: PD + no associated GW

Proton decay and GWs as complementary windows

- Type (a): $\Lambda_{\rm pd} > \Lambda_{\rm cs}$
- Type (b): $\Lambda_{\rm pd} \sim \Lambda_{\rm cs}$
- Type (c): $\Lambda_{\rm pd} > \Lambda_{\rm cs}$
- Type (d): no GWs

Observables		**Proton decays**
		$p \to \pi^0 e^+$ observed $\Rightarrow$ non-SUSY contribution indicated
GWs	Observed	• types (a) and (c) favoured • types (b) and (d) excluded
	Marginal	• types (a) and (c) favoured • type (d) excluded • type (b) allowed if $p \to K^+\bar{\nu}$ not observed and $\Lambda_{\rm pd} \sim \Lambda_{\rm cs}$

Proton decay and GWs as complementary windows

- Type (a): $\Lambda_{\rm pd} > \Lambda_{\rm cs}$
- Type (b): $\Lambda_{\rm pd} \sim \Lambda_{\rm cs}$
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<table>
<tr><td colspan="2" rowspan="2">Observables</td><td>Proton decays</td></tr>
<tr><td>$p \to \pi^0 e^+$ observed $\Rightarrow$ non-SUSY contribution indicated</td></tr>
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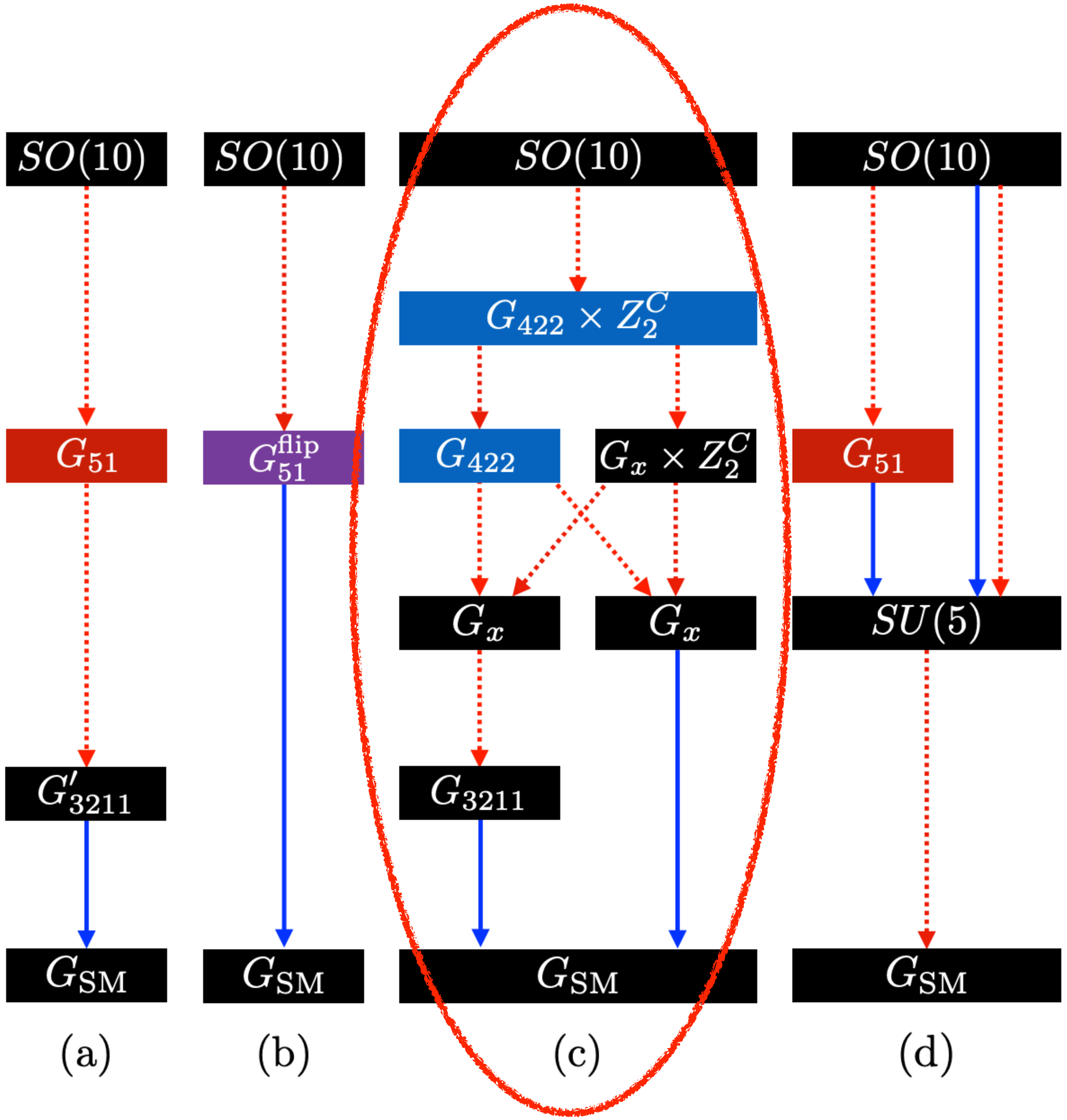
- Distinguish (a) & (c) requires model dependent study



- (d) cannot be tested with GWs since unwanted defects formed in last SSB step
- Gauge unification not possible in (a) & (b) without SUSY
- Study (c) in more detail in **2106.15634** **King, Pascoli, JT, Zhou**

Proton decay and GWs as complementary windows

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
I1:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
I2:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G_{3221}^{C}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s,w}}$		✗
I3:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
I4:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\overline{\mathbf{126}},\mathbf{45}]{\text{m}}$		✗
I5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\overline{\mathbf{126}},\mathbf{45}]{\text{m,w}}$		✗
I6:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
II1:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3221}^{C}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s,w}}$		✗
II3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II4:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G_{3221}^{C}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II5:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II7:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\overline{\mathbf{126}},\mathbf{45}]{\text{m}}$		✗
II8:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II9:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G_{3221}^{C}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II11:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
II12:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
III1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III4:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3221}^{C}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3221}^{C}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III7:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G_{3221}^{C}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III8:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III9:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
III10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_4	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
IV1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3221}^{C}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
IV2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓
IV3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G_{422}^{C}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\overline{\mathbf{126}}]{\text{s}}$		✓

Proton decay and GWs as complementary windows

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
I1:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I2:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
I3:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
I4:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m}}$		✗
I5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m,w}}$		✗
I6:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
II1:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{126}]{\text{s,w}}$		✗
II3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II4:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II5:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II7:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{126},\mathbf{45}]{\text{m}}$		✗
II8:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II9:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II11:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
II12:	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
III1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III4:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III5:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III6:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m,w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III7:	$\xrightarrow[\mathbf{210}]{\text{m,s}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III8:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III9:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
III10:	$\xrightarrow[\mathbf{210}]{\text{m}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

$SO(10)$	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_4	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_3	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_2	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_1	$\xrightarrow[\text{Higgs}]{\text{defect}}$	G_{SM}	Observable strings?
IV1:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{m}}$	G^C_{3221}	$\xrightarrow[\mathbf{45}]{\text{w}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV2:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3221}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓
IV3:	$\xrightarrow[\mathbf{54}]{\text{m,s}}$	G^C_{422}	$\xrightarrow[\mathbf{210}]{\text{w}}$	G_{422}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{421}	$\xrightarrow[\mathbf{45}]{\text{m}}$	G_{3211}	$\xrightarrow[\mathbf{126}]{\text{s}}$		✓

Proton decay and GWs as complementary windows

$$\mathrm{SO}(10) \xrightarrow{M_X} \mathrm{SU}(4)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$$

Monopole formation
Proton decay operators induced

Proton decay and GWs as complementary windows

$$\mathrm{SO}(10)\xrightarrow{M_X}\mathrm{SU}(4)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}\xrightarrow{M_2}\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SU}(2)_\mathrm{R}\times\mathrm{U}(1)_{\mathrm{B-L}}$$

Monopole formation
Proton decay operators induced

Monopole formation

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

Monopole formation

String formation

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

Monopole formation

String formation

Assumptions

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

Monopole formation

String formation

Assumptions

- Inflation after monopole formation & before cosmic string formation $\Longrightarrow$ observable GW

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

Monopole formation

String formation

Assumptions

- Inflation after monopole formation & before cosmic string formation $\Longrightarrow$ observable GW
- Minimal particle content: SM, RH neutrinos and Higgs multiplet required for SSB

Proton decay and GWs as complementary windows

$$\mathrm{SO(10)} \xrightarrow{M_X} \mathrm{SU(4)_C \times SU(2)_L \times SU(2)_R} \xrightarrow{M_2} \mathrm{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \xrightarrow{M_1} \mathrm{G_{SM}}$$

Monopole formation
Proton decay operators induced

Monopole formation

String formation

Assumptions

- Inflation after monopole formation & before cosmic string formation $\Longrightarrow$ observable GW
- Minimal particle content: SM, RH neutrinos and Higgs multiplet required for SSB
- For each, chain perform 2-loop RGE analysis to determine couplings & $M_X\,, M_2\,, M_1$

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma\left(p\to\pi^0e^+\right)=\frac{m_p}{32\pi}\left(1-\frac{m_{\pi^0}^2}{m_p^2}\right)A_L^2\times\Big[A_{SL}\Lambda_1^{-2}\left(1+\left|V_{ud}\right|^2\right)\left|\left\langle\pi^0\left|(ud)_Ru_L\right|p\right\rangle\right|^2$$
$$+A_{SR}\left(\Lambda_1^{-2}+\left|V_{ud}\right|^2\Lambda_2^{-2}\right)\left|\left\langle\pi^0\left|(ud)_Lu_L\right|p\right\rangle\right|^2\Big]$$

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma\left(p \to \pi^0 e^+\right) = \frac{m_p}{32\pi}\left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times \left[A_{SL}\Lambda_1^{-2}\left(1 + |V_{ud}|^2\right)\left|\langle \pi^0 \left|(ud)_R u_L\right| p\rangle\right|^2 \right.$$
$$\left. + A_{SR}\left(\Lambda_1^{-2} + |V_{ud}|^2 \Lambda_2^{-2}\right)\left|\langle \pi^0 \left|(ud)_L u_L\right| p\rangle\right|^2\right]$$

Hadronic matrix element from lattice

1705.01338

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma\left(p\to\pi^0e^+\right)=\frac{m_p}{32\pi}\left(1-\frac{m_{\pi^0}^2}{m_p^2}\right)A_L^2\times\left[A_{SL}\Lambda_1^{-2}\left(1+|V_{ud}|^2\right)\left|\langle\pi^0\,|(ud)_Ru_L|\,p\rangle\right|^2\right.$$
$$\left.+A_{SR}\left(\Lambda_1^{-2}+|V_{ud}|^2\,\Lambda_2^{-2}\right)\left|\langle\pi^0\,|(ud)_Lu_L|\,p\rangle\right|^2\right]$$

$$\Lambda_1\simeq\Lambda_2=\frac{g_XM_X}{2}$$

Hadronic matrix element from lattice

1705.01338

Proton Decay

- From RGE, α_X and M_X determined

$$\Gamma\left(p \to \pi^0 e^+\right) = \frac{m_p}{32\pi}\left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times \left[A_{SL}\Lambda_1^{-2}\left(1 + |V_{ud}|^2\right)\left|\left\langle \pi^0 \left| (ud)_R u_L \right| p\right\rangle\right|^2 + A_{SR}\left(\Lambda_1^{-2} + |V_{ud}|^2 \Lambda_2^{-2}\right)\left|\left\langle \pi^0 \left| (ud)_L u_L \right| p\right\rangle\right|^2\right]$$

Long & short range effects from renormalisation

$$\Lambda_1 \simeq \Lambda_2 = \frac{g_X M_X}{2}$$

Hadronic matrix element from lattice

1705.01338

Gravitational Waves

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
- Correlate vev of Higgs breaking $U(1)$ with string tension, μ
- Assume ideal Nambu-Goto string $\Longrightarrow$ gravitational radiation primary emission

$$\mu \approx 2\pi v^2$$

Vilenkin & Shellard

Gravitational Waves

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1
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Vilenkin & Shellard

$$M_1^2 \sim g_1^2 v^2 \implies G\mu = \frac{1}{M_{\rm Pl}^2}\frac{2\pi M_1^2}{g_1^2} = \frac{M_1^2}{2\alpha_1 M_{\rm Pl}^2}$$

Gravitational Waves

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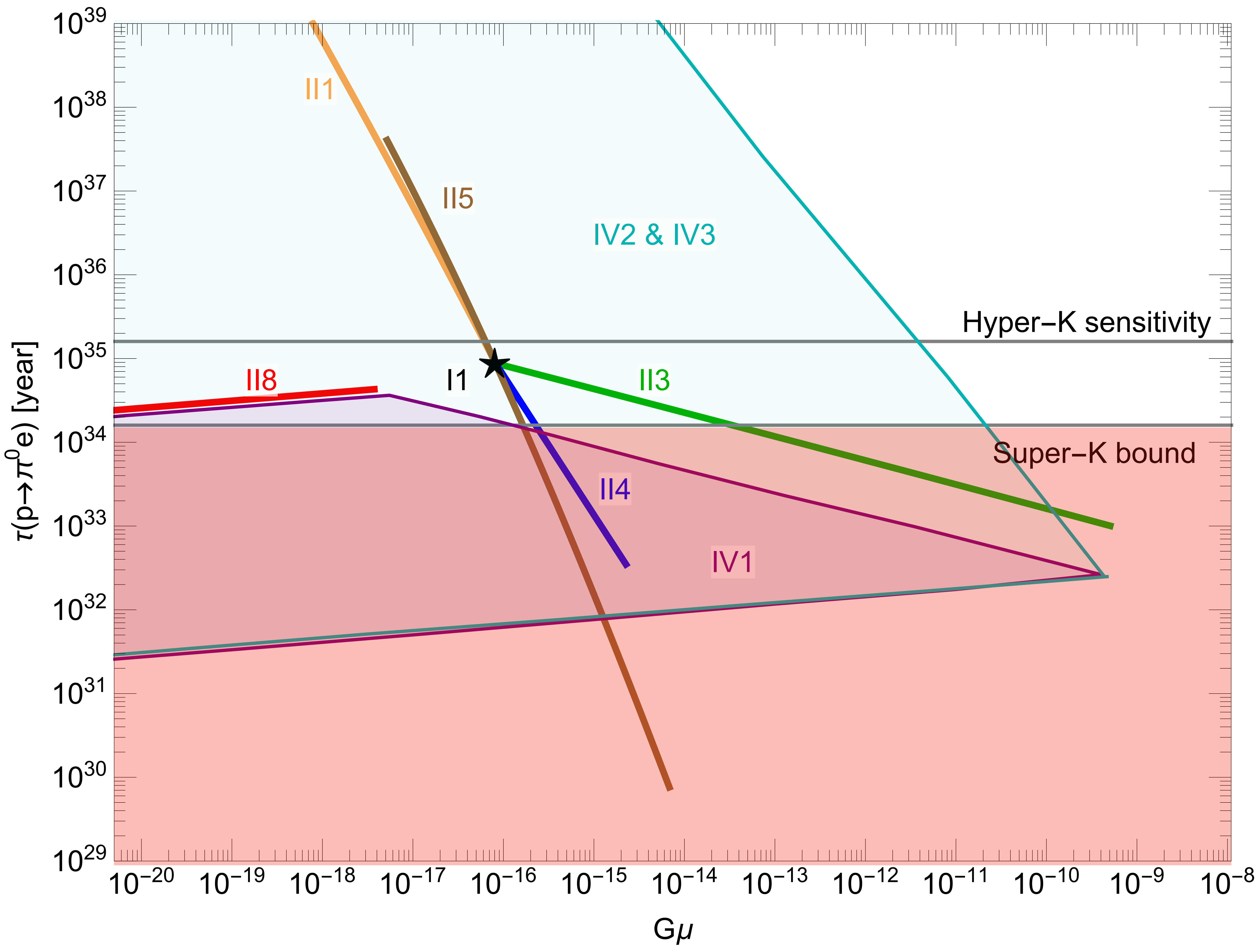
Vilenkin & Shellard

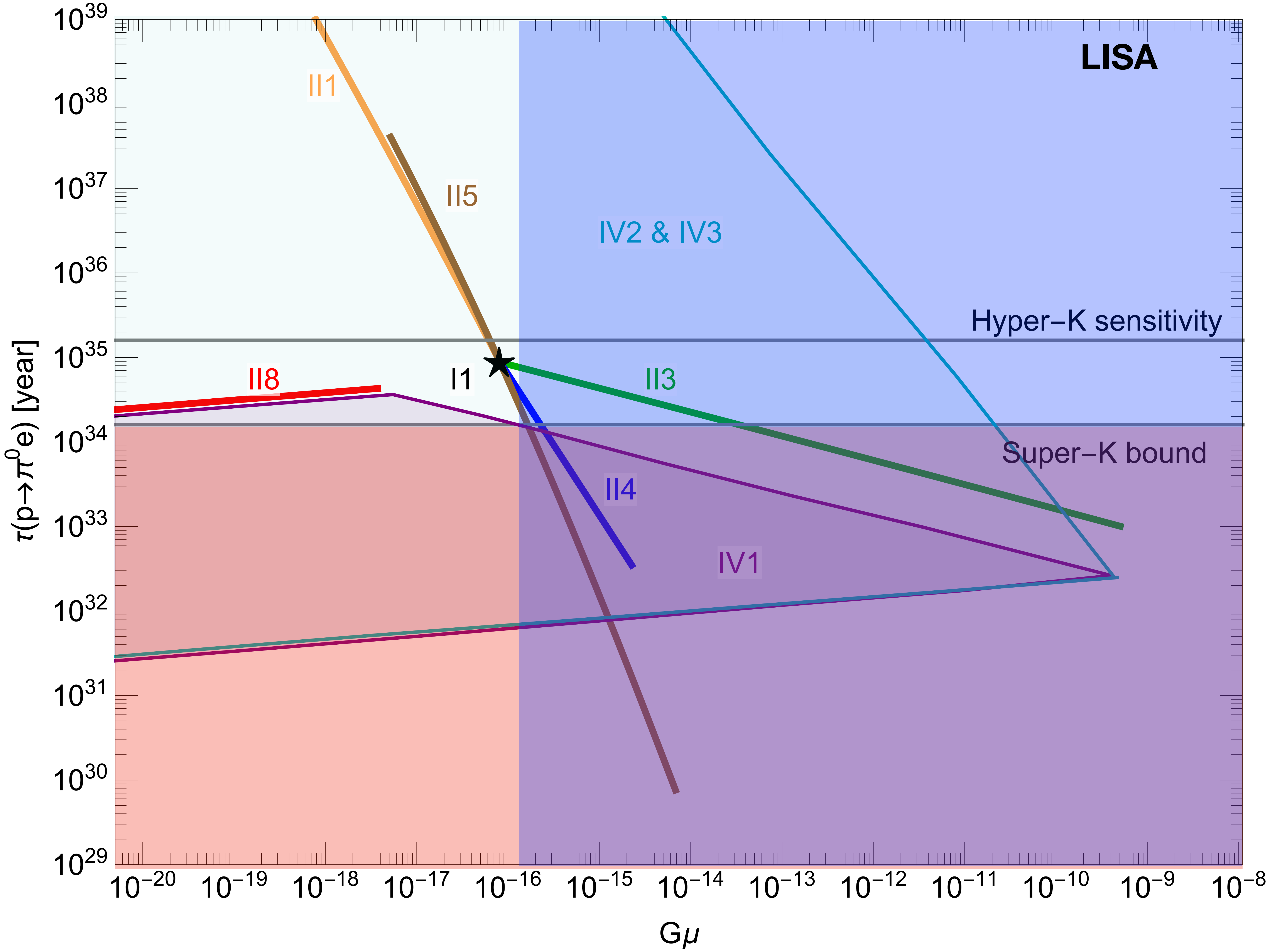
$$M_1^2 \sim g_1^2 v^2 \quad \Longrightarrow \quad G\mu = \frac{1}{M_{\rm Pl}^2}\frac{2\pi M_1^2}{g_1^2} = \frac{M_1^2}{2\alpha_1 M_{\rm Pl}^2}$$

Determined from RGE

Correlation of GW and PD signals



Correlation of GW and PD signals



Correlation of GW and PD signals

- non-SUSY $\mathrm{SO}(10)$ Pati Salam type provide unification: **31 breaking chains**
- Two-loop RGE, **17 not excluded** by Super-K bound PD.

Chain	$G\mu$ after Hyper-K (no proton decay)
I1	excluded
II1:	$G\mu \lesssim 1.5 \times 10^{-17}$
II3:	excluded
II4:	excluded
II5:	$G\mu \simeq 5.1 \times 10^{-18}$–$6.3 \times 10^{-17}$
II8:	excluded
III1:	$G\mu \simeq 1.3 \times 10^{-18}$–$1.6 \times 10^{-15}$
III2:	$G\mu \lesssim 5.0 \times 10^{-12}$
III3:	$G\mu \lesssim 6.2 \times 10^{-14}$
III4:	excluded
III6:	excluded
III7:	excluded
III8:	excluded
III10:	$G\mu \lesssim 1.1 \times 10^{-21}$
IV1:	excluded
IV2:	$G\mu \lesssim 9.4 \times 10^{-13}$
IV3:	$G\mu \lesssim 9.4 \times 10^{-13}$

Testable by LIGO, DECIGO, AEDGE, C, ET, MAGIS..

- If HyperK **does not** observe PD $\Longrightarrow$ 9 chains excluded
- **8 survivors!** If we observe GW signal **larger than upper bounds** $\Longrightarrow$ exclude those breaking chains
- If we observe PD $\Longrightarrow$ M_1 determined so is GW signal. Correlations between observables matter and need to be compared on case by case basis.

Correlation of GW and PD signals

- non-SUSY $SO(10)$ Pati Salam type provide unification: **31 breaking chains**
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III3:	$G\mu \lesssim 6.2 \times 10^{-14}$
III4:	excluded
III6:	excluded
III7:	excluded
III8:	excluded
III10:	$G\mu \lesssim 1.1 \times 10^{-21}$
IV1:	excluded
IV2:	$G\mu \lesssim 9.4 \times 10^{-13}$
IV3:	$G\mu \lesssim 9.4 \times 10^{-13}$

Study specific breaking chain **2209.00021** with **Fu, King, Marsili, Pascoli, JT & Zhou**

Why? **Can be tested by Hyper-K** & has an associated GW signal

SO(10) Model confronting data

- Treatment has been **model-independent**

SO(10) Model confronting data

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- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

SO(10) Model confronting data

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$$SO(10)$$
$$\downarrow$$
$$SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$\downarrow$$
$$SU(3)_c \times SU(2)_L \times U(1)_Y \, .$$

SO(10) Model confronting data

- Treatment has been **model-independent**

- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

$$SO(10)$$

$$\mathbf{54} \downarrow \quad \longleftarrow \text{Proton Decay}$$

$$SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C$$

$$\mathbf{210} \downarrow$$

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C$$

$$\mathbf{45} \downarrow$$

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\overline{\mathbf{126}} \downarrow$$

$$SU(3)_c \times SU(2)_L \times U(1)_Y \, .$$

SO(10) Model confronting data

- Treatment has been **model-independent**
- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

$$SO(10)$$

$$\mathbf{54}\ \downarrow$$

Proton Decay

$$SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C$$

$$\mathbf{210}\ \downarrow$$

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C$$

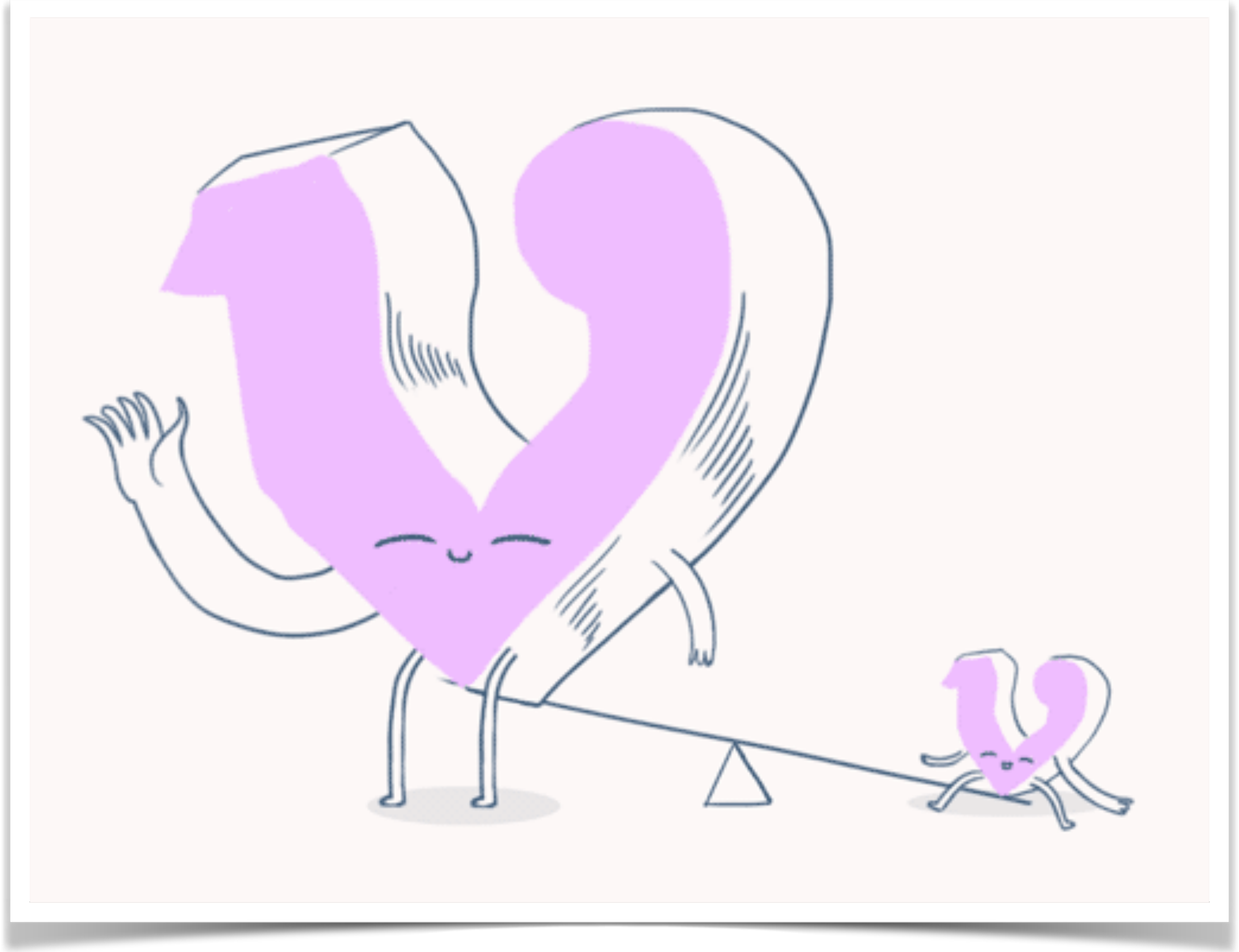
$$\mathbf{45}\ \downarrow$$

$$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\overline{\mathbf{126}}\ \downarrow$$

$U(1)_{B-L}$ Breaking
cosmic string & seesaw

$$SU(3)_c \times SU(2)_L \times U(1)_Y\ .$$



SO(10) Model confronting data

- Model of Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y^*_{\overline{\mathbf{126}}}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y^*_{\mathbf{120}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\mathrm{h.c.}\,,$$

SO(10) Model confronting data

- Model of Altarelli & Blankenburg

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$$Y_{\mathbf{10}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y_{\overline{\mathbf{126}}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y_{\mathbf{120}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\mathrm{h.c.}\,,$$

- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\rm SM}u_R+Y_d\bar{Q}h_{\rm SM}d_R+Y_\nu\bar{L}\tilde{h}_{\rm SM}\nu_R+Y_e\bar{L}h_{\rm SM}e_R+\ \mathrm{h.c.}$$

SO(10) Model confronting data

- Model of Altarelli & Blankenburg

- Above GUT scale, Yukawa sector

$$Y_{10}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y_{\overline{\mathbf{126}}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y_{120}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\mathrm{h.c.}\,,$$

Generates RHN mass

- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R+Y_d\bar{Q}h_{\mathrm{SM}}d_R+Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R+Y_e\bar{L}h_{\mathrm{SM}}e_R+\ \mathrm{h.c.}$$

- Majorana mass term for right-handed neutrino:

$$M_{\nu_R}=Y_{\overline{\mathbf{126}}}v_S\xrightarrow{\text{Seesaw Mechanism}}M_\nu=\frac{Y_\nu Y_\nu^T v_{\mathrm{SM}}^2}{M_{\nu_R}}$$

- Coupling to leptonic and Higgs doublet Y_ν predicted $\Longrightarrow$ leptogenesis prediction

SO(10) Model confronting data

- Model of Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y_{\mathbf{10}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y_{\overline{\mathbf{126}}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y_{\mathbf{120}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\text{h.c.}\,,$$

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$$Y_u=Y_{\mathbf{10}}V_{11}^{*}+\frac{1}{\sqrt{3}}Y_{\overline{\mathbf{126}}}V_{12}^{*}+Y_{\mathbf{120}}\left(V_{13}^{*}+\frac{1}{\sqrt{3}}V_{14}^{*}\right)$$

SM up Yukawa

SO(10) Model confronting data

- Model of Altarelli & Blankenburg

- Above GUT scale, Yukawa sector

$$Y_{\mathbf{10}}^{*}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10} + Y_{\overline{\mathbf{126}}}^{*}\, \mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}} + Y_{\mathbf{120}}^{*}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120} + \mathrm{h.c.}\,,$$

- After breaking to SM:

$$Y_u \bar{Q}\tilde{h}_{\mathrm{SM}} u_R + Y_d \bar{Q} h_{\mathrm{SM}} d_R + Y_\nu \bar{L}\tilde{h}_{\mathrm{SM}} \nu_R + Y_e \bar{L} h_{\mathrm{SM}} e_R + \ \mathrm{h.c.}$$

GUT Yukawa Parameter

$$Y_u = Y_{\mathbf{10}} V_{11}^{*} + \frac{1}{\sqrt{3}} Y_{\overline{\mathbf{126}}} V_{12}^{*} + Y_{\mathbf{120}} \left(V_{13}^{*} + \frac{1}{\sqrt{3}} V_{14}^{*} \right)$$

SM up Yukawa

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$$Y_{\mathbf{10}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y_{\overline{\mathbf{126}}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y_{\mathbf{120}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\mathrm{h.c.}\,,$$

- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R+Y_d\bar{Q}h_{\mathrm{SM}}d_R+Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R+Y_e\bar{L}h_{\mathrm{SM}}e_R+\ \mathrm{h.c.}$$

GUT Yukawa Parameter

$$Y_u=Y_{\mathbf{10}}V_{11}^{*}+\frac{1}{\sqrt{3}}Y_{\overline{\mathbf{126}}}V_{12}^{*}+Y_{\mathbf{120}}\left(V_{13}^{*}+\frac{1}{\sqrt{3}}V_{14}^{*}\right)$$

SM up Yukawa

Mixing between Higgs doublets

SO(10) Model confronting data

- Model of Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10} + Y^*_{\overline{\mathbf{126}}}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}} + Y^*_{\mathbf{120}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120} + \mathrm{h.c.}\,,$$

- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R + Y_d\bar{Q}h_{\mathrm{SM}}d_R + Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R + Y_e\bar{L}h_{\mathrm{SM}}e_R + \ \mathrm{h.c.}$$

GUT Yukawa Parameter

$$Y_u = Y_{\mathbf{10}}V^*_{11} + \frac{1}{\sqrt{3}}Y_{\overline{\mathbf{126}}}V^*_{12} + Y_{\mathbf{120}}\left(V^*_{13} + \frac{1}{\sqrt{3}}V^*_{14}\right)$$

SM up Yukawa

Mixing between Higgs doublets

$$\hat{h}_i = \sum_j V_{ij}h_j \qquad h_{\mathrm{SM}} \equiv \hat{h}_1$$

SO(10) Model confronting data

$$Y_u = Y_{10} V_{11}^* + \frac{1}{\sqrt{3}} Y_{\overline{126}} V_{12}^* + Y_{120} \left(V_{13}^* + \frac{1}{\sqrt{3}} V_{14}^* \right)$$

$$Y_d = Y_{10} V_{15} + \frac{1}{\sqrt{3}} Y_{\overline{126}} V_{16} + Y_{120} \left(V_{17} + \frac{1}{\sqrt{3}} V_{18} \right)$$

$$Y_\nu = Y_{10} V_{11}^* - \sqrt{3} Y_{\overline{126}} V_{12}^* + Y_{120} \left(V_{13}^* - \sqrt{3} V_{14}^* \right)$$

$$Y_e = Y_{10} V_{15} - \sqrt{3} Y_{\overline{126}} V_{16} + Y_{120} \left(V_{17} - \sqrt{3} V_{18} \right) .$$

- Reduce free parameters by considering hermitian Yukawa matrices
- Y_e and Y_ν can be expressed as functions of Y_u and Y_d

SO(10) Model confronting data

- **Input:** quark masses & mixing, charged lepton Yukawa matrix
- **Theory Model parameters:** $\mathscr{P}_m \in \left\{a_1, a_2, c_\nu, m_0, \eta_{q_{u,c,t,d,s,b}}\right\}$
- **Output:** predictions for $\mathscr{O}_m \in \left\{\theta_{12}, \theta_{23}, \theta_{12}, \delta, \alpha_{21}, \alpha_{31}, \Delta m^2_{21}, \Delta m^2_{31}\right\}$
- **BP1:** consistent with all flavour data, $\eta_B \sim \eta_{B_{cmb}}$ ($\chi^2 < 10$)
- **BP1:** $M_X = 5.6 \times 10^{15}\,\mathrm{GeV} \implies \tau(\mathrm{p} \to \pi^0 \mathrm{e}^+)$ testable by HK.
- **BP1:** $M_1 = 2 \times 10^{13}\,\mathrm{GeV}$

SO(10) Model confronting data

- **Input:** quark masses & mixing, charged lepton Yukawa matrix
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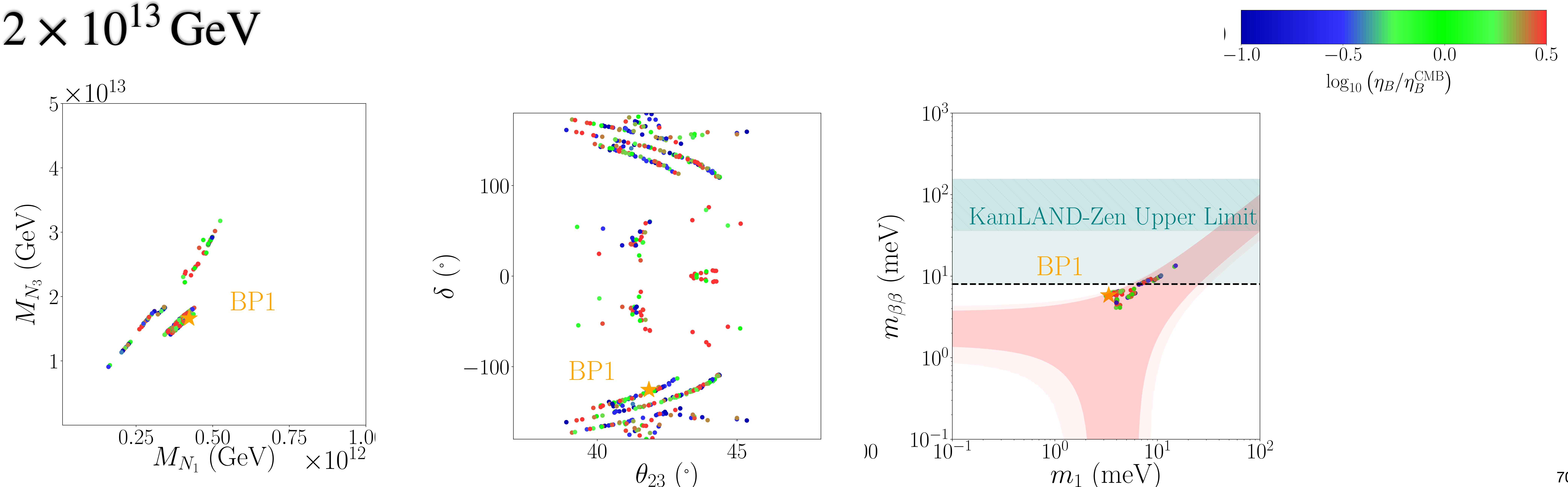
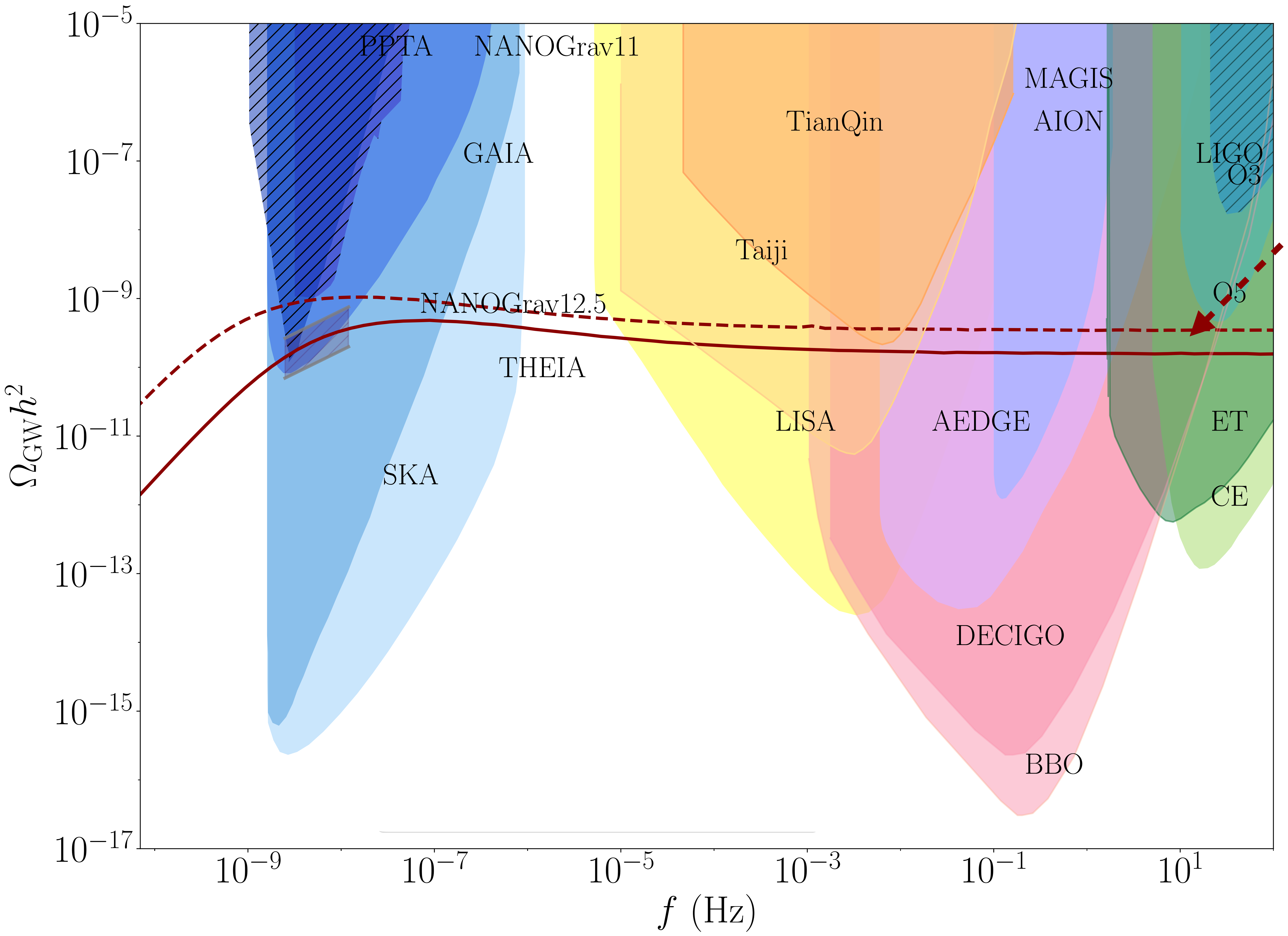
- **BP1:** $M_1 = 2 \times 10^{13}$ GeV

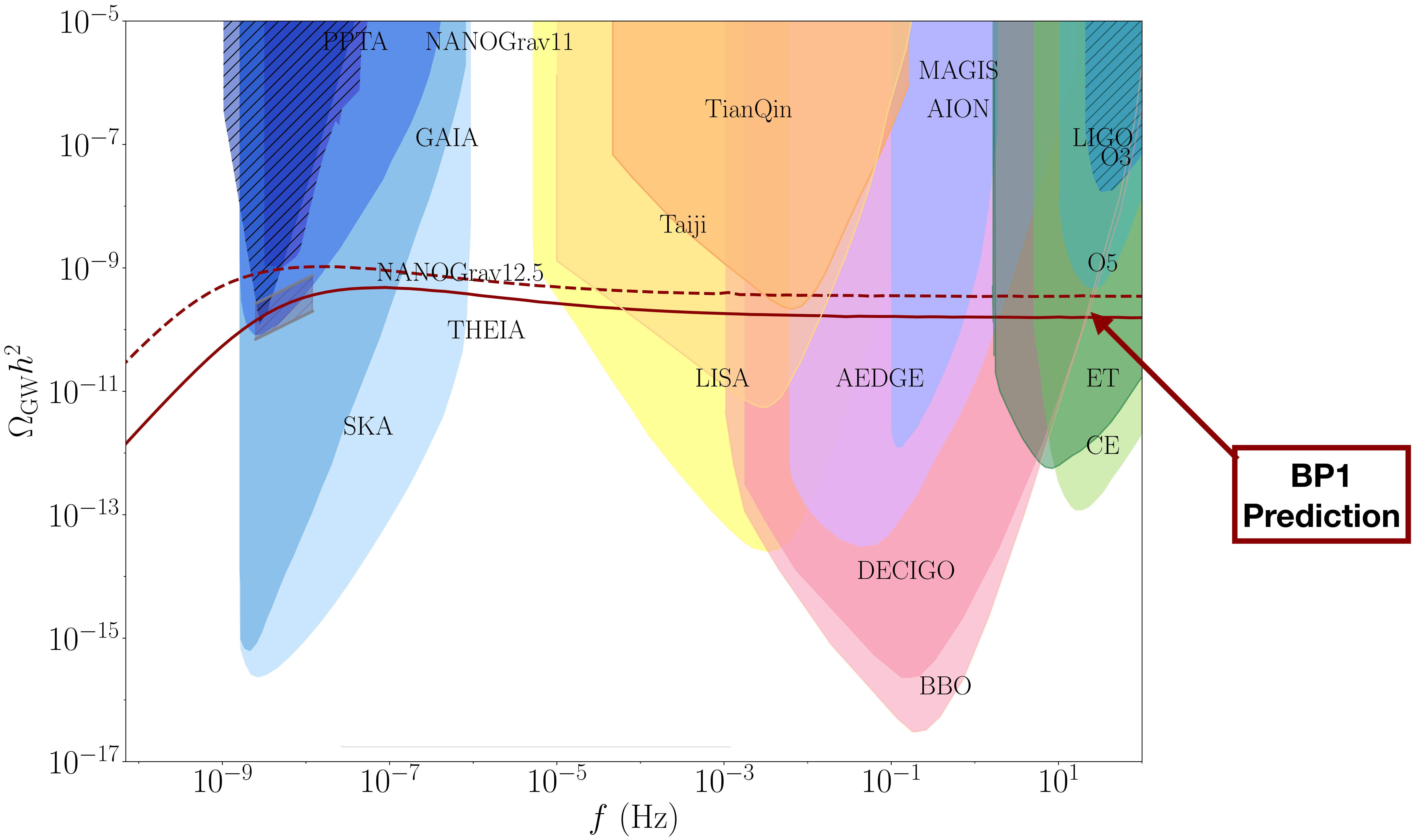


SO(10) Model confronting data

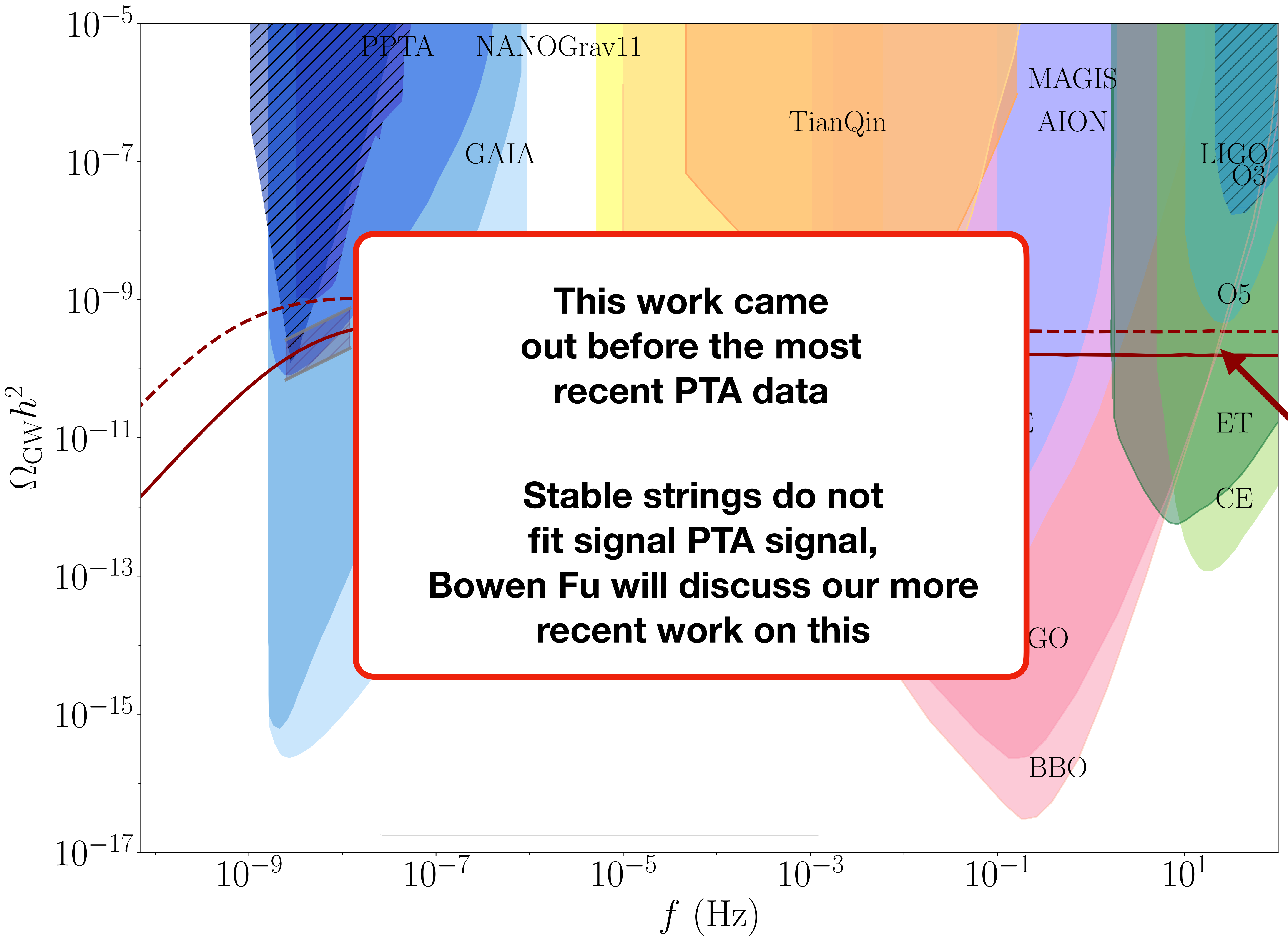


Upper bound
On $G\mu$ allowed
by proton decay limits

SO(10) Model confronting data



SO(10) Model confronting data



This work came out before the most recent PTA data

Stable strings do not fit signal PTA signal, Bowen Fu will discuss our more recent work on this

BP1 Prediction

Summary

- GUTs generically predict nucleon decay and the formation of topological defects. Interplay of these observables is a powerful way of constraining GUTs.
- Coming decade is an exciting time for GUTs as neutrino and GW experiments will constrain nucleon decay, the presence of GWs and neutrinoless double beta decay ($0\nu\beta\beta$).
- Studied non-SUSY & SUSY SO(10) breaking chains which can be tested by Hyper-K, GW detectors and $0\nu\beta\beta$.
- Parameter space consistent with fermionic masses and mixing & successful leptogenesis.

"we have entered an exciting era where new observations of GWs from the heavens and proton decay experiments from under the Earth can provide complementary windows to reveal the details of the unification of matter and forces at the highest energies."

Thank you for listening

Renormalisation Group Equations

Beta function coefficients 1 and 2-loop respectively

$$b_i = -\frac{11}{3} C_2(H_i) + \frac{2}{3} \sum_F T(F_i) + \frac{1}{3} \sum_S T(S_i) \,,$$

$$b_{ij} = -\frac{34}{3} [C_2(H_i)]^2 \delta_{ij} + \sum_F T(F_i)[2C_2(F_j) + \frac{10}{3} C_2(H_i)\delta_{ij}] + \sum_S T(S_i)[4C_2(S_j) + \frac{2}{3} C_2(H_i)\delta_{ij}] \,,$$

Two-loop RGE equation

Bertolini, di Luzio, Malinsky

$$\alpha_i(\mu)^{-1} = \alpha_i(\mu_0)^{-1} - \frac{b_i}{2\pi} \log \frac{\mu}{\mu_0} + \sum_j \frac{b_{ij}}{4\pi b_i} \log \left(1 - b_j \alpha_j(\mu_0) \log \frac{\mu}{\mu_0}\right) \,,$$

Matching condition

$$H_i \to H_j \,, \quad \frac{1}{\alpha_{H_i}(M_I)} - \frac{C_2(H_i)}{12\pi} = \frac{1}{\alpha_{H_j}(M_I)} - \frac{C_2(H_j)}{12\pi} \,.$$

- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales $\Longrightarrow$ PD rate and GW signal

Breaking chains with **one intermediate scale** has fixed prediction from unification

I1: $SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$

Mass of GB from SSB
$\Lambda_{\rm pd}$

- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales $\Longrightarrow$ PD rate and GW signal

Breaking chains with **one intermediate scale** has fixed prediction from unification

I1: $SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$

↑

Mass of GB from $U(1)$ SSB parametrises Λ_{cs}

- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales $\Longrightarrow$ PD rate and GW signal

Breaking chains with **one intermediate scale** has fixed prediction from unification

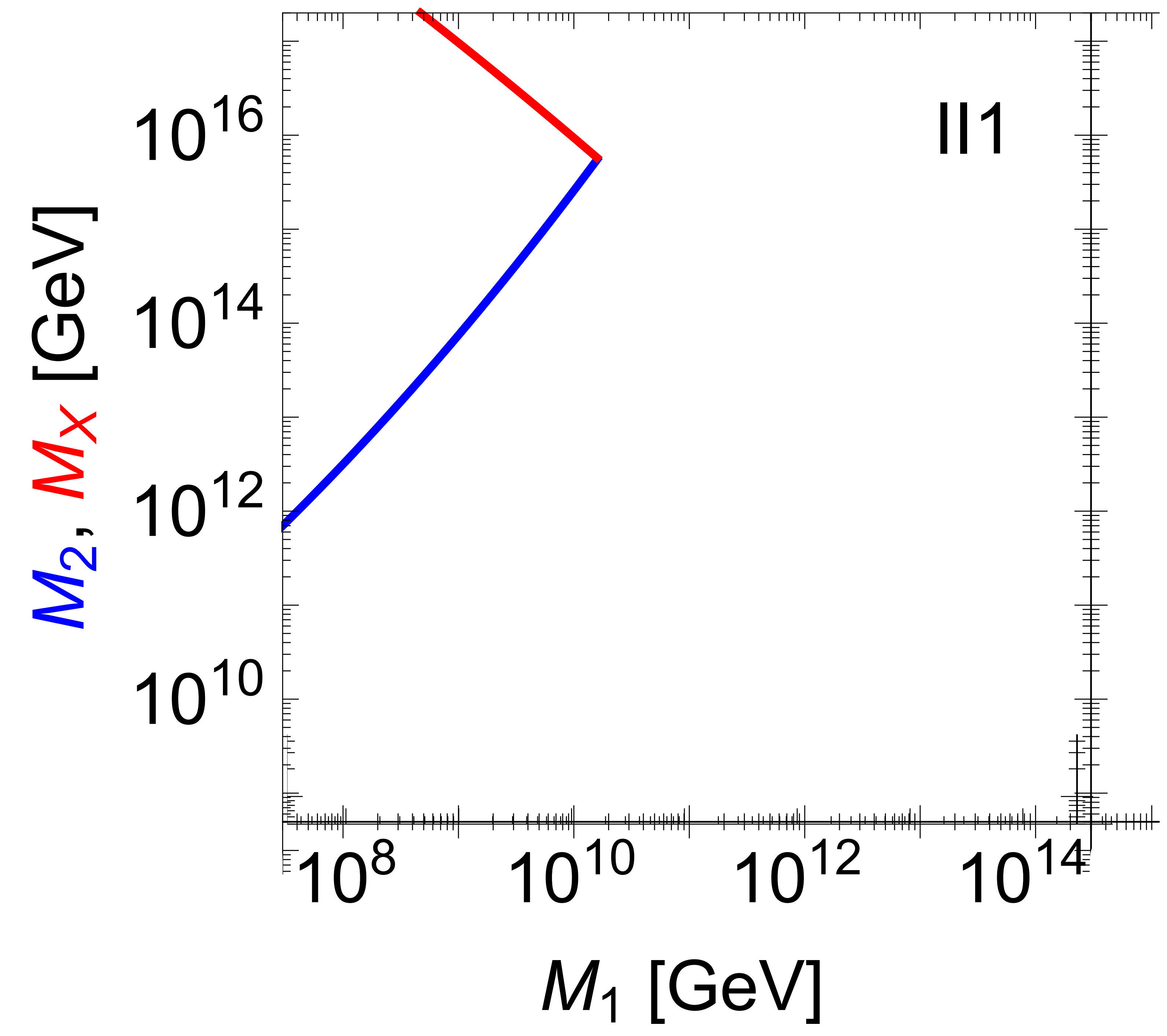
I1: $SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$

Chains	M_X[GeV]	M_1[GeV]
I1	5.660×10^{15}	1.617×10^{10}
I2	1.410×10^{15}	8.630×10^{10}
I3	2.902×10^{14}	1.634×10^{11}
I4	3.500×10^{16}	4.368×10^{9}
I5	2.722×10^{14}	1.143×10^{13}
I6	excluded	

- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales $\Longrightarrow$ PD rate and GW signal

Breaking chains with **two intermediate scales** can have a range of scales

II1 : $SO(10) \xrightarrow[M_X]{} G_{422} \xrightarrow[M_2]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$



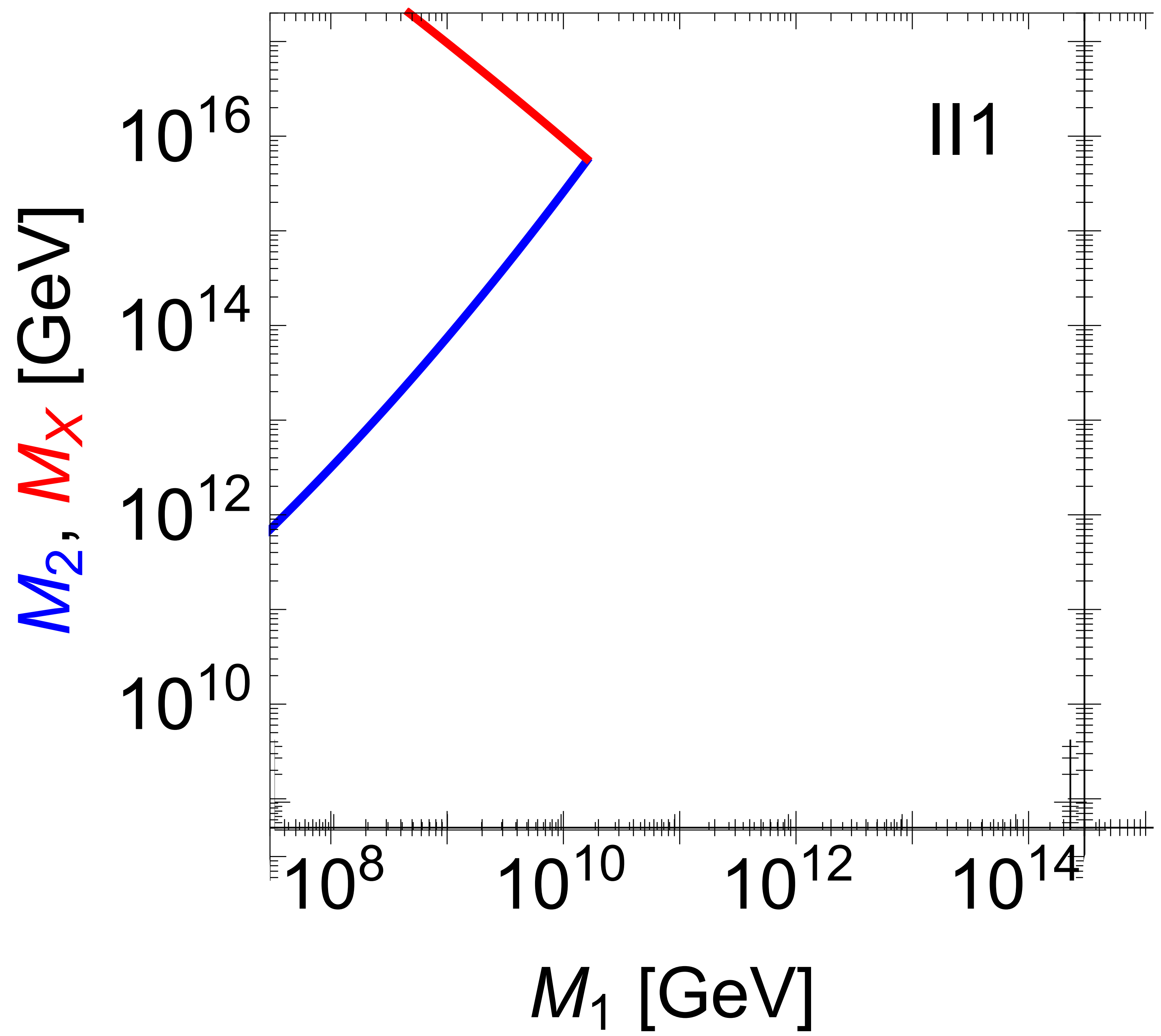
- For each chain perform two-loop RGE analysis to determine GUT scale, M_X and intermediate scales $\Longrightarrow$ PD rate and GW signal

Breaking chains with **two intermediate scales** can have a range of scales

II1 : $SO(10) \xrightarrow[M_X]{} G_{422} \xrightarrow[M_2]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$

$M_2 = M_X$ recover I1

$$SO(10) \xrightarrow[M_X]{} G_{3221} \xrightarrow[M_1]{} G_{SM}$$



- From M_X (GB mass associated to GUT SSB) we can determine proton decay rate

$$\Gamma\left(p \to \pi^0 e^+\right) = \frac{m_p}{32\pi}\left(1 - \frac{m_{\pi^0}^2}{m_p^2}\right) A_L^2 \times \Big[A_{SL}\Lambda_1^{-2}\left(1 + |V_{ud}|^2\right)\left|\langle\pi^0\,|(ud)_R u_L|\,p\rangle\right|^2$$
$$+A_{SR}\left(\Lambda_1^{-2} + |V_{ud}|^2\,\Lambda_2^{-2}\right)\left|\langle\pi^0\,|(ud)_L u_L|\,p\rangle\right|^2\Big]$$

- Cosmic string generated in final $U(1)$ symmetry breaking step at scale M_1

- Correlate vev of Higgs breaking $U(1)$ with string tension, μ

- Assume ideal Nambu-Goto string $\implies$ gravitational radiation primary emission

Vilenkin & Shellard

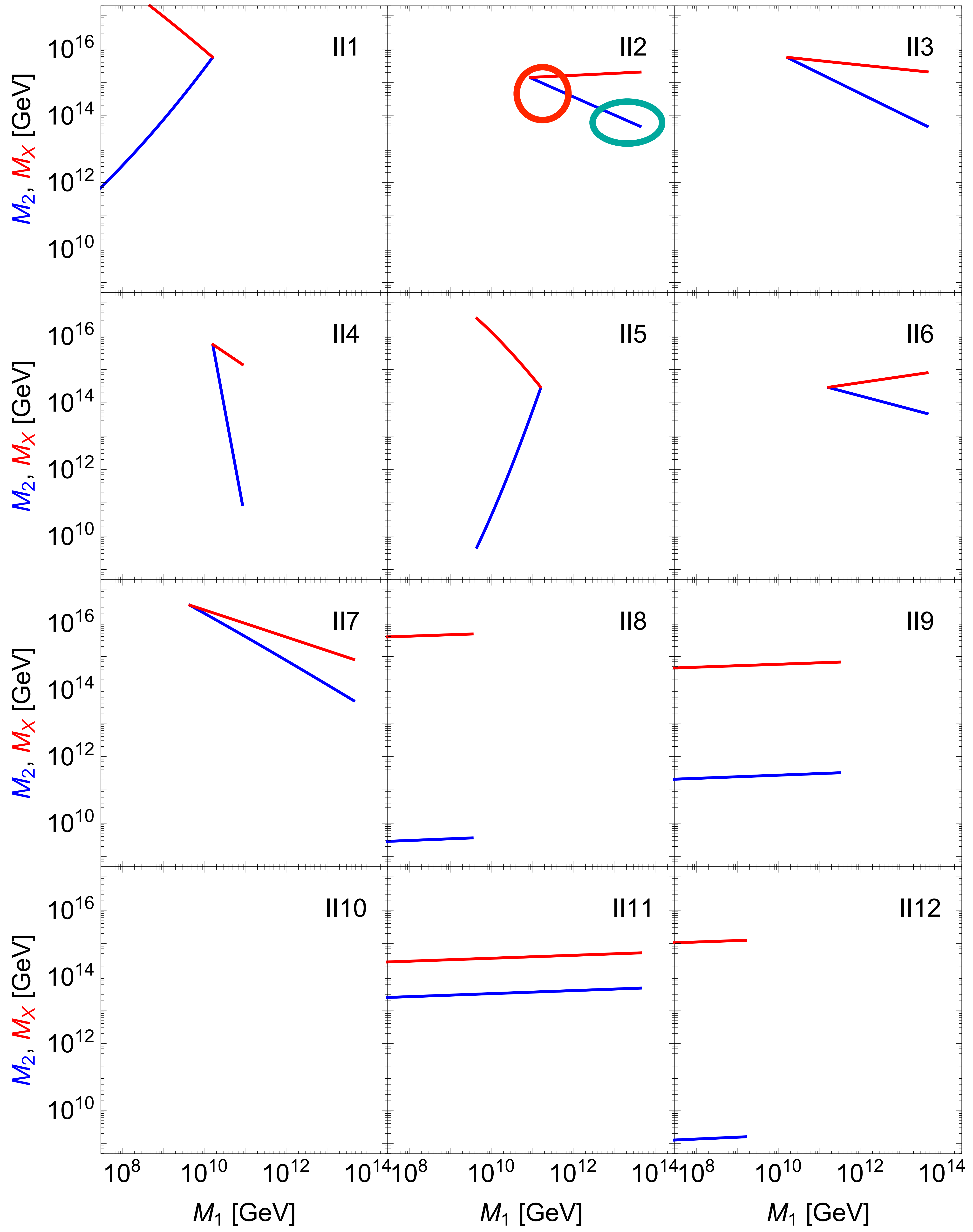
$$\mu \approx 2\pi v^2$$

$$M_1^2 = M_{Z'}^2 \sim g^2 v^2 \implies G\mu \approx \frac{1}{M_{\rm PL}^2} \cdot \frac{2\pi M_1^2}{g^2} = \frac{M_1^2}{2\alpha M_{\rm pl}^2}$$

Example

$$G_{3211} \longrightarrow G_{SM} \quad U(1)_R \times U(1)_X \to U(1)_Y$$

$$G\mu \simeq \frac{1}{2\left(\alpha_{1R}\left(M_1\right) + \alpha_{1X}\left(M_1\right)\right)} \frac{M_1^2}{M_{\rm pl}^2}$$



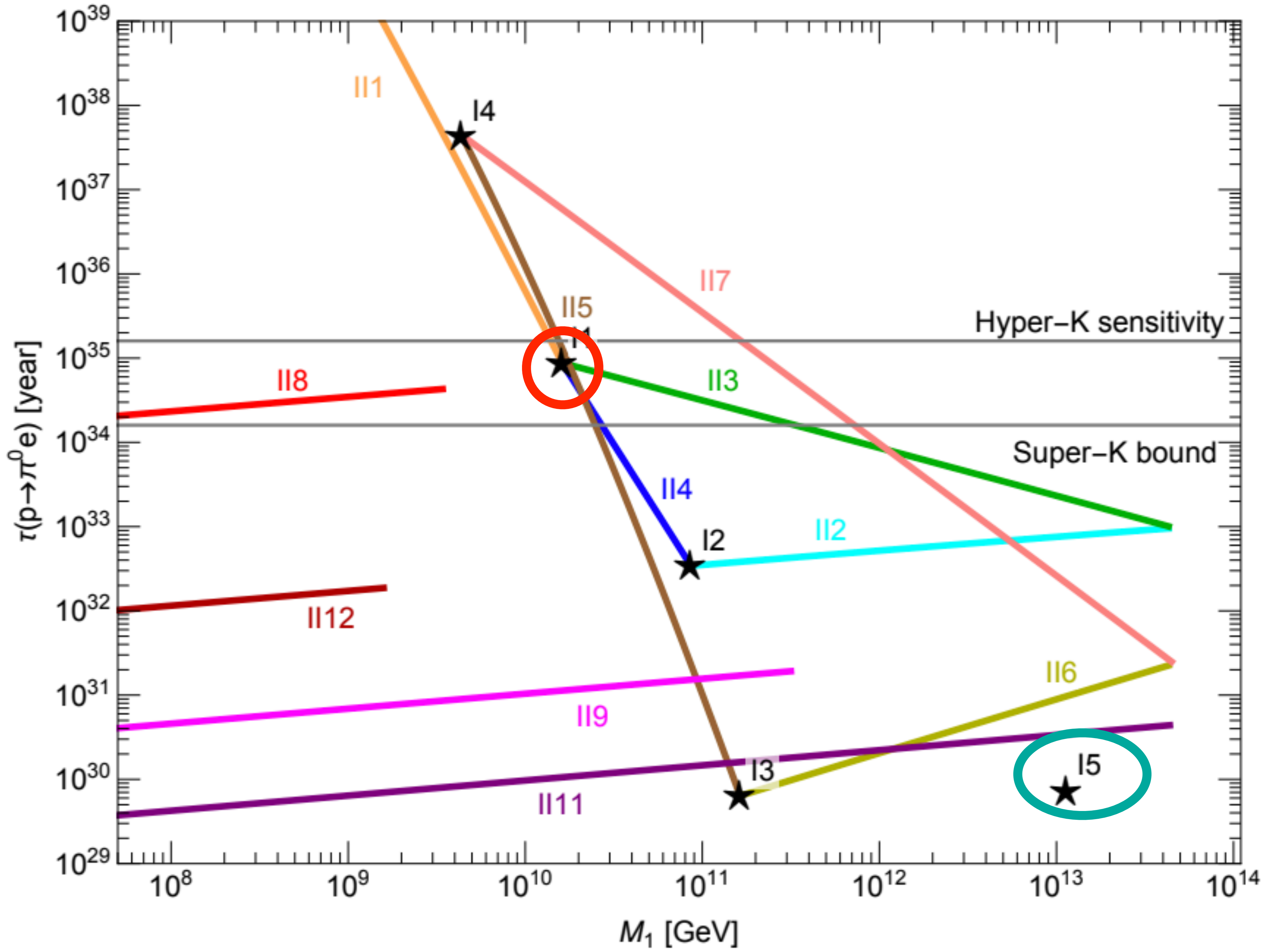
$$\mathrm{II2}: SO(10) \xrightarrow{M_X} G^C_{422} \xrightarrow{M_2} G^C_{3221} \xrightarrow{M_1} G_{\mathrm{SM}}$$

Intersection of M_2 and M_X reduces II2 to I2

$$\mathrm{I2}: SO(10) \xrightarrow{M_X} G^C_{3221} \xrightarrow{M_1} G_{\mathrm{SM}} \qquad M_X \equiv M_2$$

At right side blue curve II2 becomes I5

$$\mathrm{I5}: SO(10) \rightarrow G^C_{422} \rightarrow G_{\mathrm{SM}} \qquad M_2 \equiv M_1$$



Proton Lifetime

$$\epsilon^{ijk}\epsilon_{\alpha\beta}\Big(\frac{1}{\Lambda_1^2}(\overline{u_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{d_R^{ic}}\gamma_\mu L_\beta)+\frac{1}{\Lambda_1^2}(\overline{u_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{e_R^{c}}\gamma_\mu Q_\beta^i)$$
$$+\frac{1}{\Lambda_2^2}(\overline{d_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{u_R^{ic}}\gamma_\mu L_\beta)+\frac{1}{\Lambda_2^2}(\overline{d_R^{jc}}\gamma^\mu Q_\alpha^k)(\overline{\nu_R^{c}}\gamma_\mu Q_\beta^i)+\text{h.c.}\Big)$$

$$\Lambda_1=\Lambda_2\simeq\left(g_X M_X\right)/2$$

$$\Gamma(p\to\pi^0+e^+)=\frac{m_p}{32\pi}\left(1-\frac{m_{\pi^0}^2}{m_p^2}\right)^2 A_L^2\times\Big[A_{SL}\Lambda_1^{-2}(1+|V_{ud}|^2)\,|\langle\pi^0\,|(ud)_R u_L|p\rangle|^2$$
$$+A_{SR}(\Lambda_1^{-2}+|V_{ud}|^2\Lambda_2^{-2})\,\left|\langle\pi^0|(ud)_L u_L|p\rangle\right|^2\Big]$$

$$A_{SL(R)}=\prod_{A}^{M_Z\leqslant M_A\leqslant M_X}\prod_i\left[\frac{\alpha_i(M_{A+1})}{\alpha_i(M_A)}\right]^{\frac{\gamma_{iL(R)}}{b_i}}$$

Anomalous dimension

One-loop Beta coefficient

Gravitational Wave Calculation

$$l(t) = l_i - \Gamma G\mu\,(t - t_i) \qquad\qquad l_i = \alpha t_i \text{ with } \alpha \simeq 0.1$$

Frequencies of GW released from the loops are given by $2k/l_i$ where $k = 1,2,\cdots$

Loops are found to emit energy in the form of gravitational radiation at a constant rate

$$\frac{dE}{dt} = -\Gamma G\mu^2 \qquad\qquad \Gamma \sim 50$$

Assuming the fraction of the energy transfer in the form of large loops is $F_\alpha \sim 0.1$

$$\Omega_{\rm GW}(f) = \sum_k \Omega_{\rm GW}^{(k)}(f) = \frac{1}{\rho_c}\frac{2k}{f}\frac{\mathcal{F}_\alpha \Gamma^{(k)} G\mu^2}{\alpha(\alpha+\Gamma G\mu)}\int_{t_F}^{t_0} dt \frac{C_{\rm eff}\left(t_i^{(k)}\right)}{t_i^{(k)4}}\frac{a^2(t)a^3\left(t_i^{(k)}\right)}{a^5(t_0)}\theta\left(t_i^{(k)} - t_F\right)$$

$$C_{\rm eff} = 5.7, 0.5$$

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GUT Model

In the Yukawa sector, couplings above the GUT scale are given by

$$Y^*_{\mathbf{10}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y^*_{\overline{\mathbf{126}}}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y^*_{\mathbf{120}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\text{h.c.}\,,$$

After breaking to G_{SM}

$$Y_{\mathbf{10}}\Big[(\overline{Q}u_R+\overline{L}\nu_R)h^u_{\mathbf{10}}+(\overline{Q}d_R+\overline{L}e_R)h^d_{\mathbf{10}}\Big]+\frac{1}{\sqrt{3}}Y_{\overline{\mathbf{126}}}\left[(\overline{Q}u_R-3\overline{L}\nu_R)h^u_{\overline{\mathbf{126}}}+(\overline{Q}d_R-3\overline{L}e_R)h^d_{\overline{\mathbf{126}}}\right]$$

$$+Y_{\mathbf{120}}\Big[(\overline{Q}u_R+\overline{L}\nu_R)h^u_{\mathbf{120}}+(\overline{Q}d_R+\overline{L}e_R)h^d_{\mathbf{120}}+\frac{1}{\sqrt{3}}(\overline{Q}u_R-3\overline{L}\nu_R)h^{u'}_{\mathbf{120}}+(\overline{Q}d_R-3\overline{L}e_R)h^{d'}_{\mathbf{120}}\Big]+\text{h.c.}$$

Rotating the Higgs fields to their mass basis, we derive Yukawa couplings to the SM Higgs

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R+Y_d\bar{Q}h_{\mathrm{SM}}d_R+Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R+Y_e\bar{L}h_{\mathrm{SM}}e_R+\text{ h.c.}$$

$$Y_u=Y_{10}V^*_{11}+\frac{1}{\sqrt{3}}Y_{\overline{126}}V^*_{12}+Y_{120}\left(V^*_{13}+\frac{1}{\sqrt{3}}V^*_{14}\right)$$

$$Y_d=Y_{10}V_{15}+\frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16}+Y_{120}\left(V_{17}+\frac{1}{\sqrt{3}}V_{18}\right)$$

$$Y_\nu=Y_{10}V^*_{11}-\sqrt{3}Y_{\overline{126}}V^*_{12}+Y_{120}\left(V^*_{13}-\sqrt{3}V^*_{14}\right)$$

$$Y_e=Y_{10}V_{15}-\sqrt{3}Y_{\overline{126}}V_{16}+Y_{120}\left(V_{17}-\sqrt{3}V_{18}\right).$$

GUT Model

$$Y_u = Y_{10}V_{11}^* + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{12}^* + Y_{120}\left(V_{13}^* + \frac{1}{\sqrt{3}}V_{14}^*\right)$$
$$Y_d = Y_{10}V_{15} + \frac{1}{\sqrt{3}}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} + \frac{1}{\sqrt{3}}V_{18}\right)$$
$$Y_\nu = Y_{10}V_{11}^* - \sqrt{3}Y_{\overline{126}}V_{12}^* + Y_{120}\left(V_{13}^* - \sqrt{3}V_{14}^*\right)$$
$$Y_e = Y_{10}V_{15} - \sqrt{3}Y_{\overline{126}}V_{16} + Y_{120}\left(V_{17} - \sqrt{3}V_{18}\right).$$

$$Y_u = h + r_2 f + i r_3 h', \quad Y_d = r_1\left(h + f + ih'\right), \quad Y_\nu = h - 3r_2 f + i c_\nu h'$$
$$Y_e = r_1\left(h - 3f + i c_e h'\right), \quad M_{\nu_R} = f\,\frac{\sqrt{3}r_1}{V_{16}}v_S$$

$$h = Y_{\mathbf{10}}V_{11}\,,\ f = Y_{\overline{\mathbf{126}}}\frac{V_{16}}{\sqrt{3}}\frac{V_{11}^*}{V_{15}}\,,\ c_e = \frac{V_{17} - \sqrt{3}V_{18}}{V_{17} + V_{18}/\sqrt{3}}\,, \qquad c_\nu = \frac{V_{13}^* - \sqrt{3}V_{14}^*}{V_{17} + V_{18}/\sqrt{3}}\frac{V_{15}}{V_{11}^*}\,,$$
$$r_1 = \frac{V_{15}}{V_{11}^*}\,, \quad r_2 = \frac{V_{12}^*}{V_{16}}\frac{V_{15}}{V_{11}^*}\,, \qquad r_3 = \frac{V_{13}^* + V_{14}^*/\sqrt{3}}{V_{17} + V_{18}/\sqrt{3}}\frac{V_{15}}{V_{11}^*}\,,\ h' = -i\,Y_{\mathbf{120}}\left(V_{17} + V_{18}/\sqrt{3}\right)\frac{V_{11}^*}{V_{15}}\,,$$

$$Y_u = h + r_2 f = \mathrm{diag}\{\eta_u y_u, \eta_c y_c, \eta_t y_t\}$$

$$Y_d = P_a V_{\mathrm{CKM}} \,\mathrm{diag}\{\eta_d y_d, \eta_s y_s, \eta_b y_b\}\, V_{\mathrm{CKM}}^\dagger P_a^* \qquad P_a = \mathrm{diag}\{e^{ia_1}, e^{ia_2}, 1\}$$

$$Y_\nu = -\frac{3r_2+1}{r_2-1} Y_u + \frac{4r_2}{r_1\,(r_2-1)} \,\mathrm{Re}\, Y_d + i\frac{c_\nu}{r_1}\,\mathrm{Im}\, Y_d$$

$$Y_e = -\frac{4r_1}{r_2-1} Y_u + \frac{r_2+3}{r_2-1}\,\mathrm{Re}\, Y_d + i c_e \,\mathrm{Im}\, Y_d$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_q} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_q} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_q} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_q} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_q} & c_{13}c_{23} \end{pmatrix},$$

$$M_\nu = m_0 \Bigg(\frac{8r_2\,(r_2+1)}{r_2-1} Y_u - \frac{16 r_2^2}{r_1\,(r_2-1)}\,\mathrm{Re}\, Y_d + \frac{r_2-1}{r_1}\,(r_1 Y_u + i c_\nu\,\mathrm{Im}\, Y_d)\,(r_1 Y_u - \mathrm{Re}\, Y_d)^{-1}\,(r_1 Y_u - i c_\nu \,\mathrm{Im}\, Y_d)\Bigg)$$

RHN mass matrix obtained from inverting Seesaw Fomula: i.e. we have light neutrino Yukawa, light neutrino masses

GUT Model Particle Content

	Multiplet	Role in the model
Fermions	$\mathbf{16}$	Contains all SM fermions and RH neutrinos
Higgses	$\mathbf{10}$	Generates fermion masses
	$\mathbf{45}$	Triggers intermediate symmetry breaking
	$\mathbf{54}$	Triggers GUT symmetry breaking
	$\mathbf{120}$	Generates fermion masses
	$\overline{\mathbf{126}}$	Generates fermion masses & intermediate symmetry breaking
	$\mathbf{210}$	Triggers intermediate symmetry breaking

$SO(10)$	$\mathbf{54}$	$\mathbf{210}$	$\mathbf{45}$	$\overline{\mathbf{126}}$
G_3	$(\mathbf{1},\mathbf{1},\mathbf{1})$	$(\mathbf{15},\mathbf{1},\mathbf{1})_1$	$(\mathbf{15},\mathbf{1},\mathbf{1})_2$	$(\mathbf{10},\mathbf{1},\mathbf{3})+(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})$
G_2	–	$(\mathbf{1},\mathbf{1},\mathbf{1},0)_1$	$(\mathbf{1},\mathbf{1},\mathbf{1},0)_2$	$(\mathbf{1},\mathbf{1},\mathbf{3},-1)+(\mathbf{1},\mathbf{3},\mathbf{1},1)$
G_1	–	–	$(\mathbf{1},\mathbf{1},\mathbf{1},0)_2$	$(\mathbf{1},\mathbf{1},\mathbf{3},-1)$
$G_{\rm SM}$	–	–	–	$(\mathbf{1},\mathbf{1},0)_S$

SO(10) Higgs reps for SSB

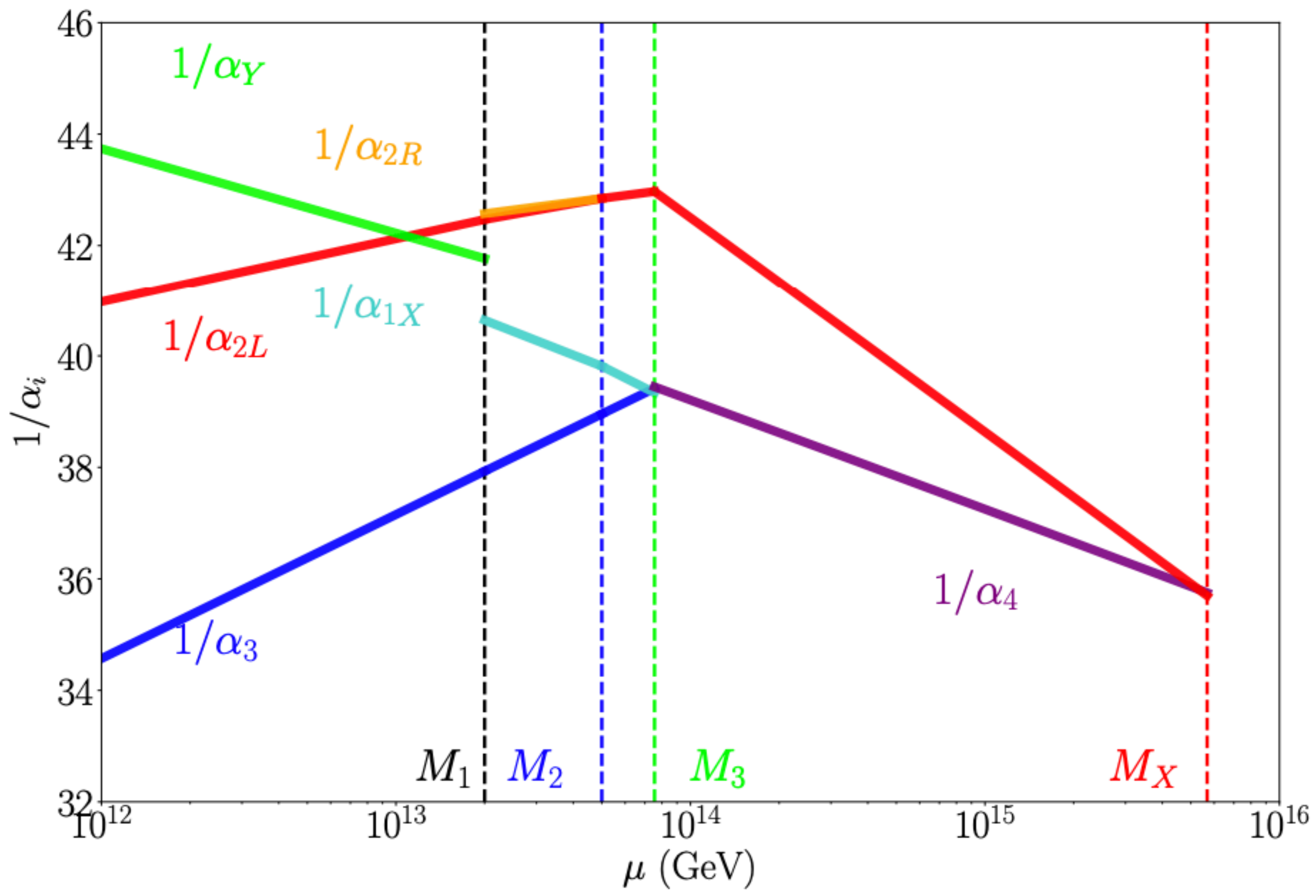
$SO(10)$	$\mathbf{16}$
G_3	$(\mathbf{4},\mathbf{2},\mathbf{1})_L+(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})_{R^c}$
G_2	$(\mathbf{3},\mathbf{2},\mathbf{1},1/6)_{Q_L}+(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},-1/6)_{Q_R^c}$ $+(\mathbf{1},\mathbf{2},\mathbf{1},-1/2)_{l_L}+(\mathbf{1},\mathbf{1},\mathbf{2},1/2)_{l_R^c}$
G_1	$(\mathbf{3},\mathbf{2},\mathbf{1},1/6)_{Q_L}+(\overline{\mathbf{3}},\mathbf{1},\mathbf{2},-1/6)_{Q_R^c}$ $+(\mathbf{1},\mathbf{2},\mathbf{1},-1/2)_{l_L}+(\mathbf{1},\mathbf{1},\mathbf{2},1/2)_{l_R^c}$
$G_{\rm SM}$	$(\mathbf{3},\mathbf{2},1/6)_{Q_L}+(\overline{\mathbf{3}},\mathbf{1},-2/3)_{u_R^c}+(\overline{\mathbf{3}},\mathbf{1},1/3)_{d_R^c}$ $+(\mathbf{1},\mathbf{2},-1/2)_{l_L}+(\mathbf{1},\mathbf{1},0)_{\nu_R^c}+(\mathbf{1},\mathbf{1},1)_{e_R^c}$

Matter field decomposition

$SO(10)$	$\mathbf{10}$	$\overline{\mathbf{126}}$	$\mathbf{120}$
G_3	$(\mathbf{1},\mathbf{2},\mathbf{2})_1$	$(\mathbf{15},\mathbf{2},\mathbf{2})_1$ $+(\mathbf{10},\mathbf{1},\mathbf{3})+(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2},\mathbf{2})_2+(\mathbf{15},\mathbf{2},\mathbf{2})_2$
G_2	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_1$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_2$ $+(\mathbf{1},\mathbf{1},\mathbf{3},-1)+(\mathbf{1},\mathbf{3},\mathbf{1},1)$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_{3,4}$
G_1	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_1$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_2$ $+(\mathbf{1},\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\mathbf{2},\mathbf{2},0)_{3,4}$
$G_{\rm SM}$	$(\mathbf{1},\mathbf{2},-1/2)_{h^u_{\mathbf{10}}}$ $+(\mathbf{1},\mathbf{2},+1/2)_{h^d_{\mathbf{10}}}$	$(\mathbf{1},\mathbf{2},-1/2)_{h^u_{\overline{\mathbf{126}}}}$ $+(\mathbf{1},\mathbf{2},+1/2)_{h^d_{\overline{\mathbf{126}}}}$ $+(\mathbf{1},\mathbf{1},0)_S$	$(\mathbf{1},\mathbf{2},-1/2)_{h^u_{\mathbf{120}},h^{u\prime}_{\mathbf{120}}}$ $+(\mathbf{1},\mathbf{2},+1/2)_{h^d_{\mathbf{120}},h^{d\prime}_{\mathbf{120}}}$

SO(10) Higgs reps for fermion mass generation

BP1



$$M_1 = 2 \times 10^{13}\ \text{GeV}\,, \quad M_2 = 5 \times 10^{13}\ \text{GeV}\,, \tag{1}$$

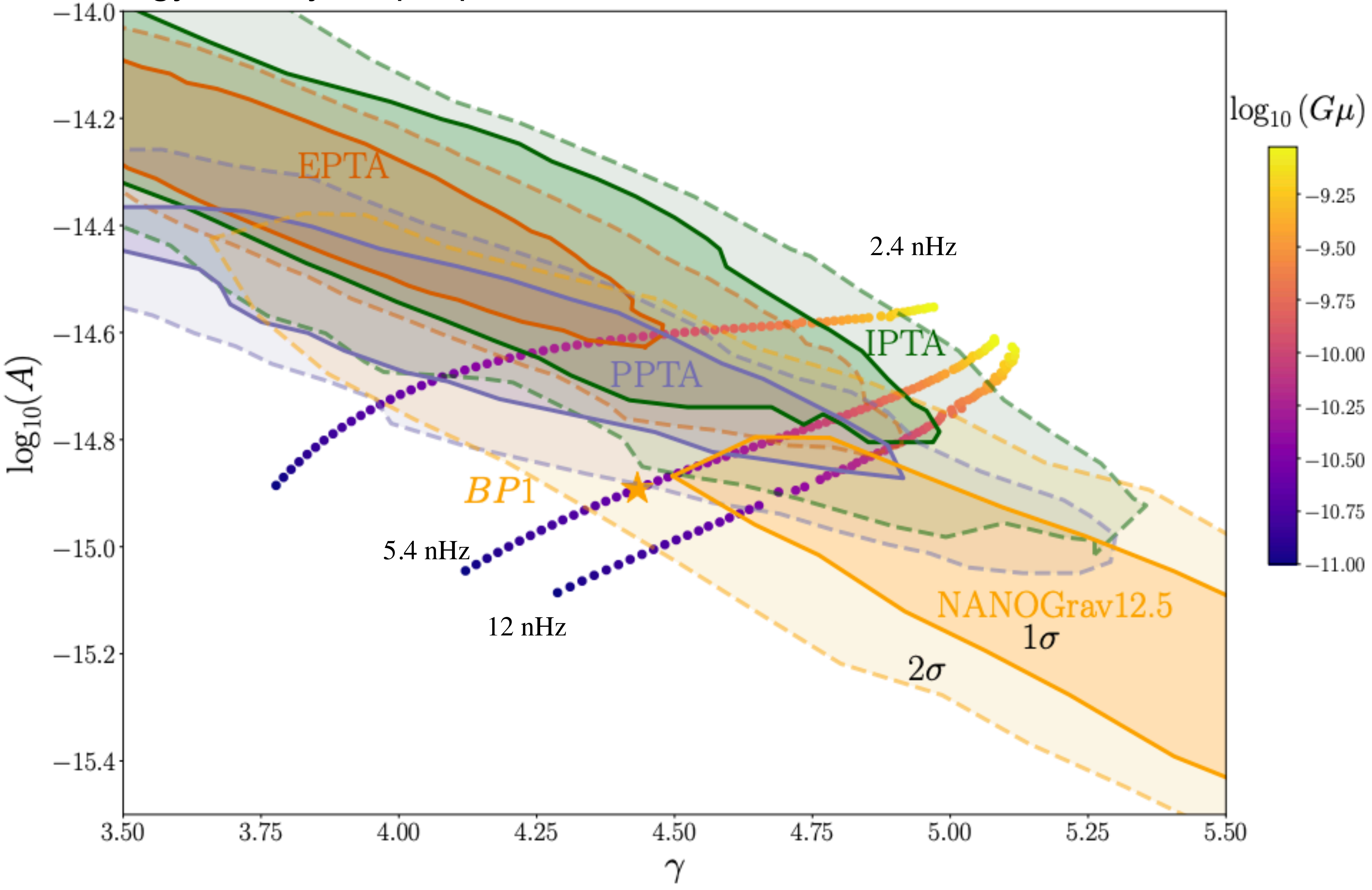
where the remaining scales and gauge coupling α_X, are then determined via the gauge unification,

$$M_3 = 7.55 \times 10^{13}\ \text{GeV}\,, \quad M_X = 5.68 \times 10^{15}\ \text{GeV}\,, \quad \alpha_X = 0.0279\,. \tag{2}$$

Overlap with PTA experiments

A ≡ amplitude parameter of correlation between pulsars.

γ ≡ related to GW energy density freq dependence

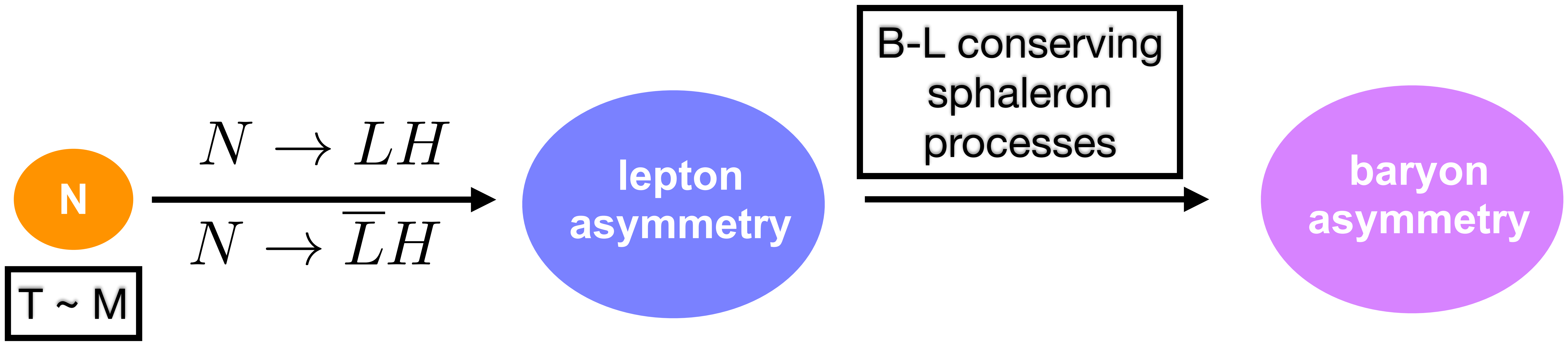


Leptogenesis Equations

$$\frac{dN^{B-L}_{\alpha\beta}}{dz} = \sum_{i=1}^{3} \varepsilon^{(i)}_{\alpha\beta} D_i \left(N_{N_i} - N^{\rm eq}_{N_i}\right) - \frac{1}{2} W_i \left\{\mathcal{P}^{(i)0}, N^{B-L}\right\}_{\alpha\beta}$$
$$- \frac{\mathrm{Im}\left(\Lambda_\tau\right)}{Hz} \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L}\right]\right]_{\alpha\beta}$$
$$- \frac{\mathrm{Im}\left(\Lambda_\mu\right)}{Hz} \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, N^{B-L}\right]\right]_{\alpha\beta} ,$$

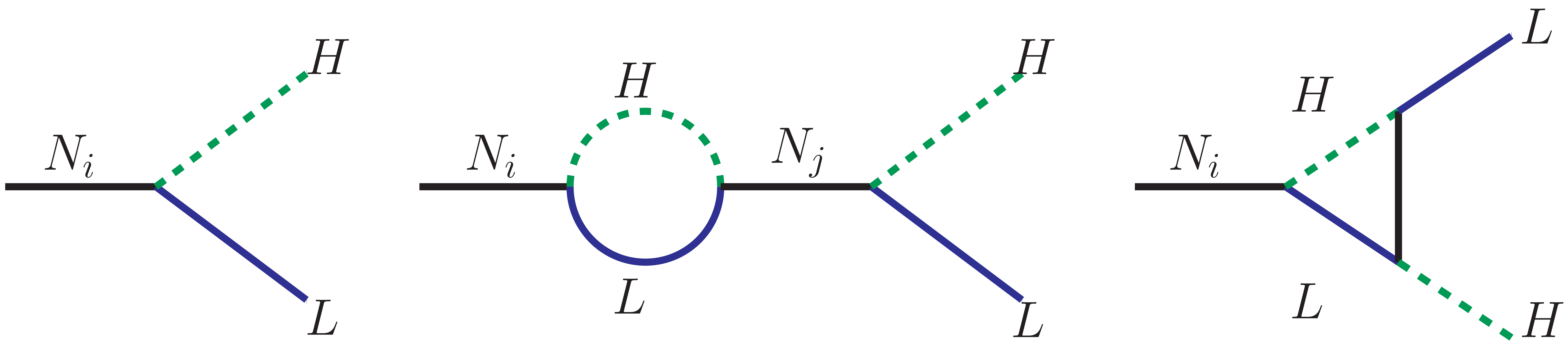
$$N^{B-L} = \begin{pmatrix} N_{\tau\tau} & N_{\tau\mu} & N_{\tau e} \\ N_{\mu\tau} & N_{\mu\mu} & N_{\mu e} \\ N_{e\tau} & N_{e\mu} & N_{ee} \end{pmatrix} , \quad \mathcal{P}^{(i)0} = \frac{1}{\left(\tilde{Y_\nu}^\dagger \tilde{Y_\nu}\right)_{ii}} \begin{pmatrix} \left|\tilde{Y}_{\nu\tau i}\right|^2 & \tilde{Y}_{\nu\tau i}\tilde{Y}^\star_{\nu\mu i} & \tilde{Y}_{\nu\tau i}\tilde{Y}^\star_{\nu e i} \\ \tilde{Y}^\star_{\nu\tau i}\tilde{Y}_{\nu\mu i} & \left|\tilde{Y}_{\nu\mu i}\right|^2 & \tilde{Y}^\star_{\nu\tau i}\tilde{Y}_{\nu e i} \\ \tilde{Y}_{\nu e i}\tilde{Y}^\star_{\nu\tau i} & \tilde{Y}_{\nu\mu i}\tilde{Y}^\star_{\nu\tau i} & \left|\tilde{Y}_{\nu e i}\right|^2 \end{pmatrix} .$$

Thermal leptogenesis



Decay asymmetry from interference between tree and loop level diagrams

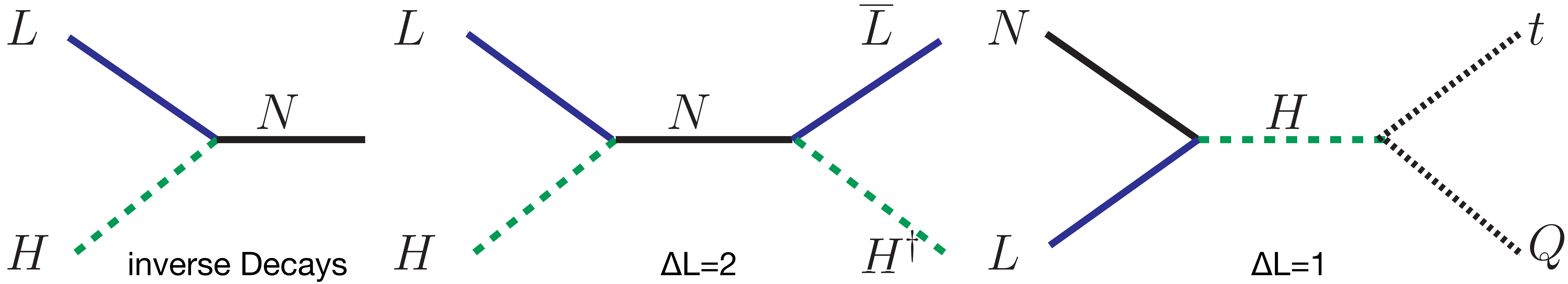
Covi, Roulet, Vissani

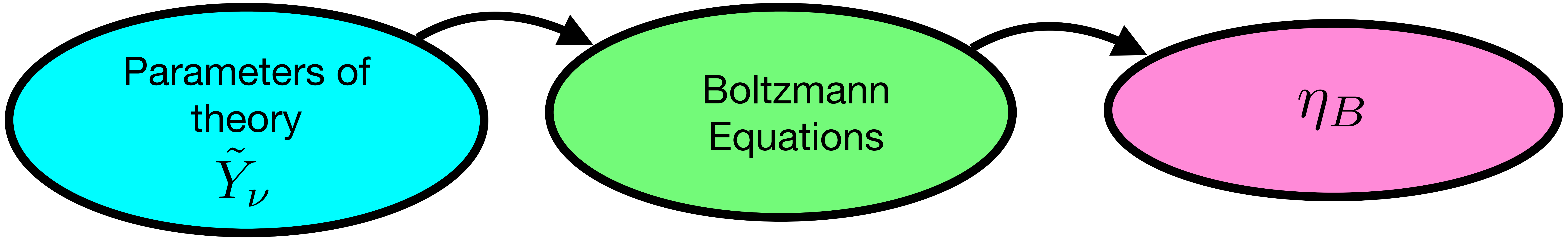


$$\epsilon_i = \frac{\Gamma_i - \overline{\Gamma_i}}{\Gamma_i + \overline{\Gamma_i}}$$

Thermal leptogenesis

Washout and scattering processes





$$\frac{dn_{N_i}}{dz} = -\, D_i(n_{N_i} - n_{N_i}^{\mathrm{eq}}),$$

source **sink**

$$\frac{dn_{B-L}}{dz} = \sum_{i=1}^{3} \left(\epsilon^{(i)} D_i (n_{N_i} - n_{N_i}^{\mathrm{eq}}) - W_i n_{B-L} \right).$$

SO(10) with leptogenesis

- Comprehensive study of chain III4: **Fu, King, Marsili, Pascoli, JT, Zhou** 2209.00021

$$
\begin{array}{c}
SO(10) \\
\mathbf{54} \downarrow \\
SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C \\
\mathbf{210} \downarrow \\
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times Z_2^C \\
\mathbf{45} \downarrow \\
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \\
\overline{\mathbf{126}} \downarrow \\
SU(3)_c \times SU(2)_L \times U(1)_Y \,.
\end{array}
$$

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$$

Proton Decay

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$$\overline{\mathbf{126}} \downarrow$$

$$SU(3)_c \times SU(2)_L \times U(1)_Y \, .$$

Proton Decay

$U(1)_{B-L}$ Breaking
cosmic string & seesaw

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$$SO(10)$$

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Proton Decay

$$SU(4) \times SU(2)_L \times SU(2)_R \times Z_2^C$$

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$$\downarrow \overline{\mathbf{126}}$$

$U(1)_{B-L}$ Breaking
cosmic string & seesaw

$$SU(3)_c \times SU(2)_L \times U(1)_Y \,.$$

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Fermions	$\mathbf{16}$	Contains SM fermions and RH neutrinos
Higgses	$\mathbf{45}$	symmetry breaking
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	$\mathbf{120}$	fermion masses
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$U(1)_{B-L}$ Breaking
cosmic string & seesaw

$$SU(3)_c \times SU(2)_L \times U(1)_Y \,.$$

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Fermions	**16**	Contains SM fermions and RH neutrinos
Higgses	**45**	symmetry breaking
	210	symmetry breaking
	54	symmetry breaking
	$\overline{\mathbf{126}}$	fermion masses & symmetry breaking
	120	fermion masses
	10	fermion masses

Added to generate SM fermion masses and mixing

SO(10) with leptogenesis

- Model based on work on work by **Altarelli & Blankenburg**
- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y^*_{\overline{\mathbf{126}}}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y^*_{\mathbf{120}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\mathrm{h.c.}\,,$$

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- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R+Y_d\bar{Q}h_{\mathrm{SM}}d_R+Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R+Y_e\bar{L}h_{\mathrm{SM}}e_R+\ \mathrm{h.c.}$$

SO(10) with leptogenesis

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- Above GUT scale, Yukawa sector

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- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R+Y_d\bar{Q}h_{\mathrm{SM}}d_R+Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R+Y_e\bar{L}h_{\mathrm{SM}}e_R+\ \mathrm{h.c.}$$

$$Y_u=Y_{\mathbf{10}}V_{11}^{*}+\frac{1}{\sqrt{3}}Y_{\overline{\mathbf{126}}}V_{12}^{*}+Y_{\mathbf{120}}\left(V_{13}^{*}+\frac{1}{\sqrt{3}}V_{14}^{*}\right)$$

SM up Yukawa

SO(10) with leptogenesis

- Model based on work on work by Altarelli & Blankenburg
- Above GUT scale, Yukawa sector

$$Y_{\mathbf{10}}^* \, \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + Y_{\overline{\mathbf{126}}}^* \, \mathbf{16} \cdot \mathbf{16} \cdot \overline{\mathbf{126}} + Y_{\mathbf{120}}^* \, \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{120} + \text{h.c.} \, ,$$

- After breaking to SM:

$$Y_u \bar{Q} \tilde{h}_{\text{SM}} u_R + Y_d \bar{Q} h_{\text{SM}} d_R + Y_\nu \bar{L} \tilde{h}_{\text{SM}} \nu_R + Y_e \bar{L} h_{\text{SM}} e_R + \text{ h.c.}$$

GUT Yukawa Parameter

$$Y_u = Y_{\mathbf{10}} V_{11}^* + \frac{1}{\sqrt{3}} Y_{\overline{\mathbf{126}}} V_{12}^* + Y_{\mathbf{120}} \left(V_{13}^* + \frac{1}{\sqrt{3}} V_{14}^* \right)$$

SM up Yukawa

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$$Y_u = Y_{\mathbf{10}} V_{11}^{*} + \frac{1}{\sqrt{3}} Y_{\overline{\mathbf{126}}} V_{12}^{*} + Y_{\mathbf{120}} \left(V_{13}^{*} + \frac{1}{\sqrt{3}} V_{14}^{*} \right)$$

SM up Yukawa

Mixing between Higgs doublets

$$\hat{h}_i = \sum_j V_{ij} h_j \qquad h_{\text{SM}} \equiv \hat{h}_1$$

SO(10) with leptogenesis

- Model based on work on work by [Altarelli & Blankenburg]
- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10} + Y^*_{\overline{\mathbf{126}}}\, \mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}} + Y^*_{\mathbf{120}}\, \mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120} + \text{h.c.}\,,$$

Generates RHN mass

- After breaking to SM:

$$Y_u \bar{Q}\tilde{h}_{\text{SM}} u_R + Y_d \bar{Q} h_{\text{SM}} d_R + Y_\nu \bar{L}\tilde{h}_{\text{SM}} \nu_R + Y_e \bar{L} h_{\text{SM}} e_R + \text{ h.c.}$$

- Majorana mass term for right-handed neutrino:

$$M_{\nu_R} = Y_{\overline{\mathbf{126}}}\, v_S \xrightarrow{\text{Type-I seesaw}} M_\nu = \frac{Y_\nu Y_\nu^T v_{\text{SM}}^2}{M_{\nu_R}}$$

SO(10) with leptogenesis

- Model based on work on work by Altarelli & Blankenburg

- Above GUT scale, Yukawa sector

$$Y_{10}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10}+Y_{\overline{126}}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}}+Y_{120}^{*}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120}+\text{h.c.}\,,$$

Generates RHN mass

- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\rm SM}u_R+Y_d\bar{Q}h_{\rm SM}d_R+Y_\nu\bar{L}\tilde{h}_{\rm SM}\nu_R+Y_e\bar{L}h_{\rm SM}e_R+\ \text{h.c.}$$

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$$M_{\nu_R}=Y_{\overline{\mathbf{126}}}v_S \xrightarrow{\text{Type-I seesaw}} M_\nu=\frac{Y_\nu Y_\nu^T v_{\rm SM}^2}{M_{\nu_R}}$$

$$Y_{\overline{126}}\lesssim\mathcal{O}(1)$$

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- Above GUT scale, Yukawa sector

$$Y^*_{\mathbf{10}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{10} + Y^*_{\overline{\mathbf{126}}}\,\mathbf{16}\cdot\mathbf{16}\cdot\overline{\mathbf{126}} + Y^*_{\mathbf{120}}\,\mathbf{16}\cdot\mathbf{16}\cdot\mathbf{120} + \mathrm{h.c.}\,,$$

- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R + Y_d\bar{Q}h_{\mathrm{SM}}d_R + Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R + Y_e\bar{L}h_{\mathrm{SM}}e_R + \ \mathrm{h.c.}$$

- Simplify GUT parameters which are not all independent of each other:

$$Y_u = h + r_2 f + i r_3 h', \quad Y_d = r_1\left(h + f + ih'\right), \quad Y_\nu = h - 3r_2 f + i c_\nu h'$$
$$Y_e = r_1\left(h - 3f + i c_e h'\right), \quad M_{\nu_R} = f\,\frac{\sqrt{3}r_1}{V_{16}}v_S$$

SO(10) with leptogenesis

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- After breaking to SM:

$$Y_u\bar{Q}\tilde{h}_{\mathrm{SM}}u_R+Y_d\bar{Q}h_{\mathrm{SM}}d_R+Y_\nu\bar{L}\tilde{h}_{\mathrm{SM}}\nu_R+Y_e\bar{L}h_{\mathrm{SM}}e_R+\ \mathrm{h.c.}$$

- Simplify GUT parameters which are not all independent of each other:

$$Y_u=h+r_2f+ir_3h',\quad Y_d=r_1\left(h+f+ih'\right),\quad Y_\nu=h-3r_2f+ic_\nu h'$$
$$Y_e=r_1\left(h-3f+ic_eh'\right),\quad M_{\nu_R}=f\frac{\sqrt{3}r_1}{V_{16}}v_S$$

Data

SO(10) with leptogenesis

- Model based on work on work by **Altarelli & Blankenburg**
- Above GUT scale, Yukawa sector

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- After breaking to SM:

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$$Y_u=h+r_2f+ir_3h',\quad Y_d=r_1\,(h+f+ih')\,,\quad Y_\nu=h-3r_2f+ic_\nu h'$$
$$Y_e=r_1\,(h-3f+ic_eh')\,,\quad M_{\nu_R}=f\,\frac{\sqrt{3}r_1}{V_{16}}v_S$$

Data

Model

SO(10) with leptogenesis

- Assume Y_u , Y_d , Y_e, Y_ν **hermitian** reduces model parameters **Altarelli et al 1012.2697**
- Assume Y_u purely real

$$\mathcal{P}_m \in \{a_1, a_2, c_e, m_1, \eta\}$$

- Quark mass, CKM parameter, charged lepton masses treated as input
- Neutrino sector is predicted (hence also RHN spectrum)

$$\mathcal{O}_n \in \{\theta_{12}, \theta_{13}, \theta_{23}, \delta, \alpha_{21}, \alpha_{31}, \Delta m^2_{21}, \Delta m^2_{31}\}$$

- Perform χ^2 analysis, current grid based analysis

- For each point in scan we have RHN mass scale, Yukawa coupling of RHN to leptonic and Higgs doublet $\Longrightarrow$ thermal leptogenesis prediction (see backup for details)

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- **BP1** consistent with measured fermionic data, $\eta_B \sim \eta_{B_{cmb}}$ & testable by HyperK

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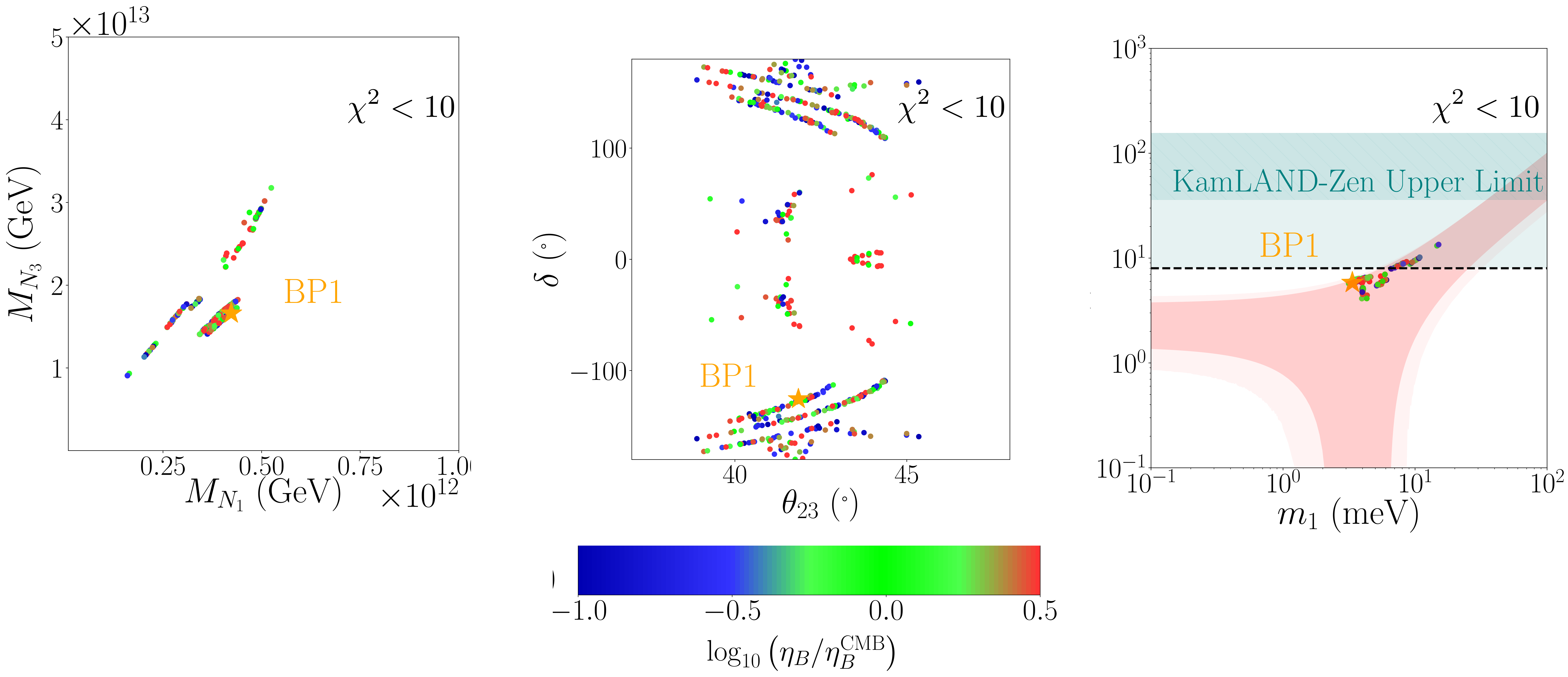
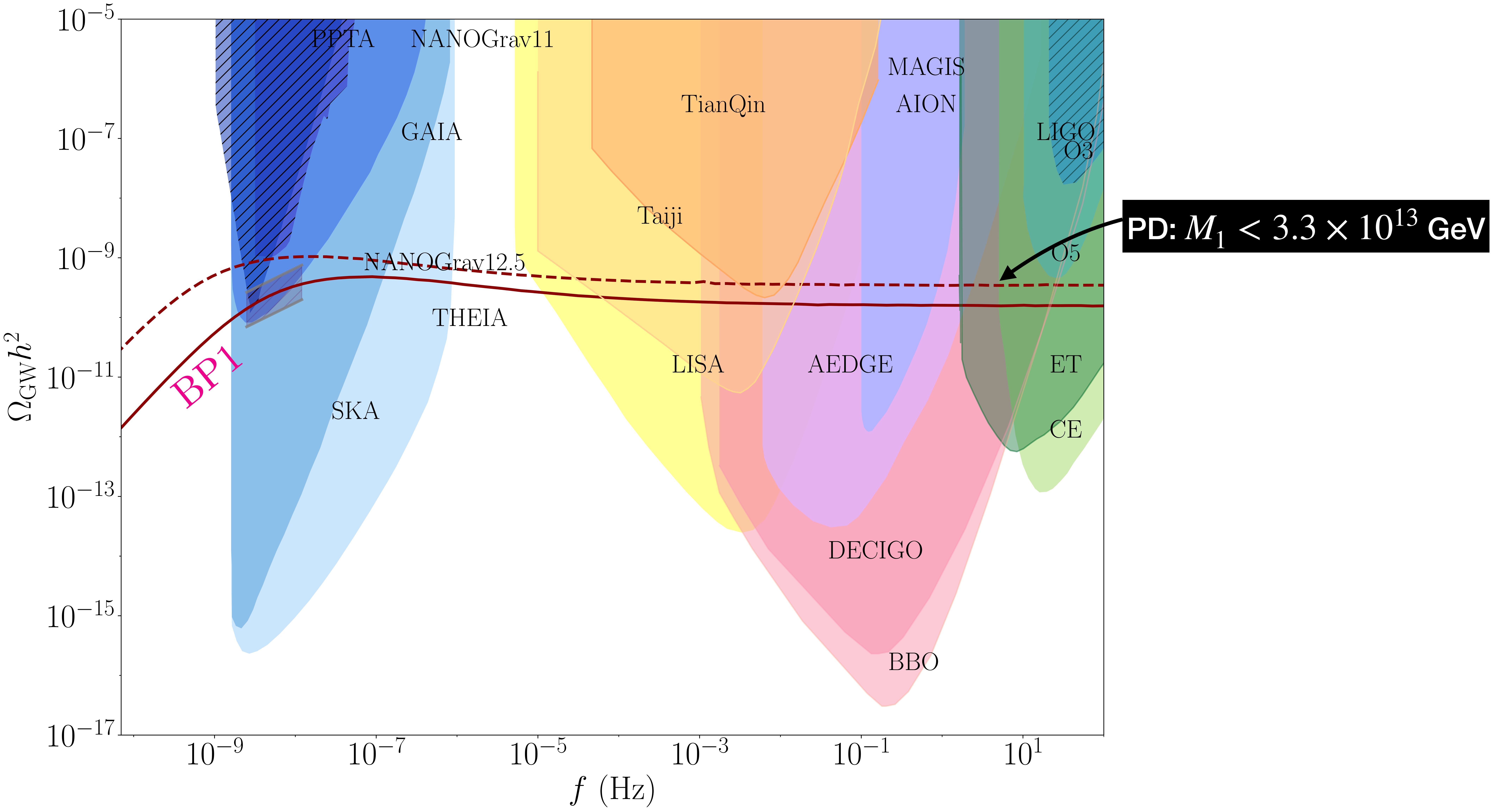
- **BP1** consistent with measured fermionic data, $\eta_B \sim \eta_{B_{cmb}}$ & testable by HyperK




PPTA
NANOGrav11
GAIA
NANOGrav12.5
THEIA
SKA
BP1
TianQin
Taiji
LISA
MAGIS
AION
AEDGE
DECIGO
BBO
LIGO
O3
O5
ET
CE
PD: $M_1 < 3.3 \times 10^{13}$ GeV
$\Omega_{\rm GW} h^2$
f (Hz)
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-9}
10^{-7}
10^{-5}
10^{-3}
10^{-1}
10^{1}