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国际理论物理中心-亚太地区

Gravitational Waves from Early Universe Symmetry Breakings

Huaike Guo

April 10, 2024

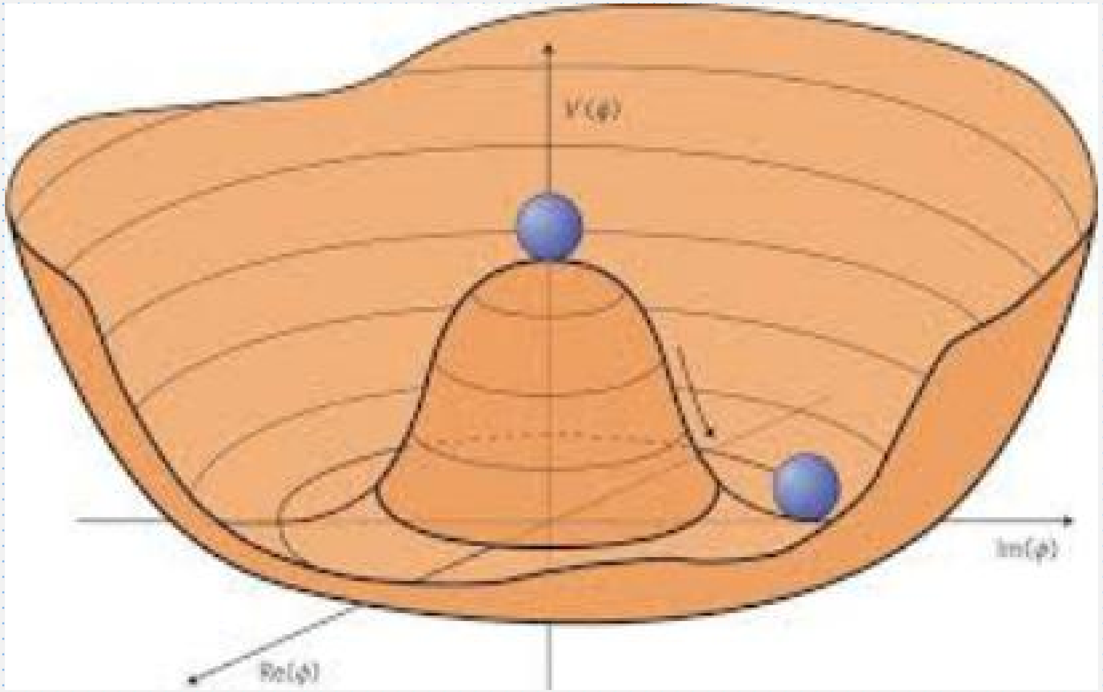
大统一理论的唯象学和宇宙学研讨会
Workshop on Grand Unified Theories:
Phenomenology and Cosmology (GUTPC)

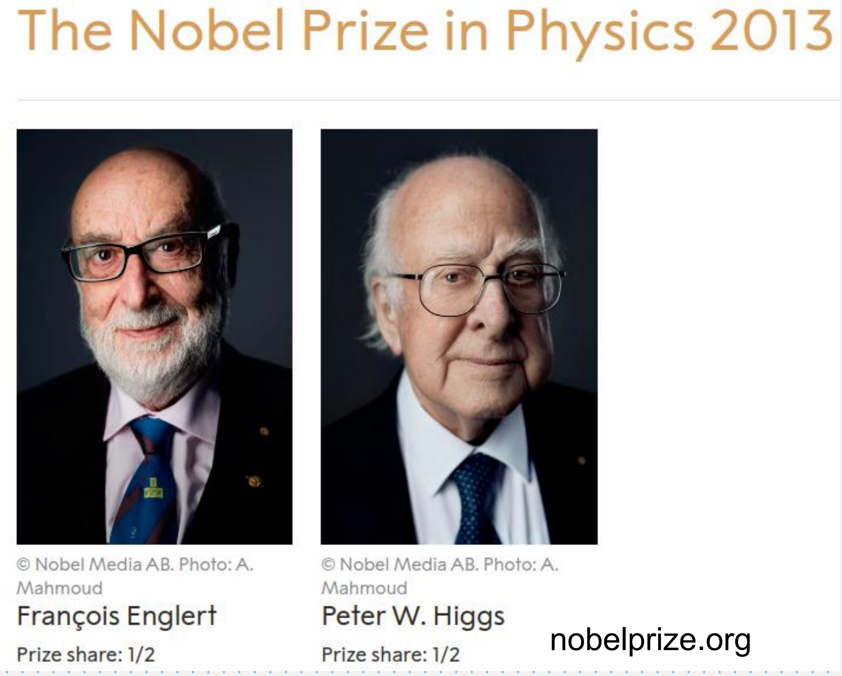
杭州 · Hangzhou, April 8–12, 2024

Electroweak Symmetry Breaking

Spontaneous symmetry breaking

$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{\mathrm{EM}}$$





The Nobel Prize in Physics 2013

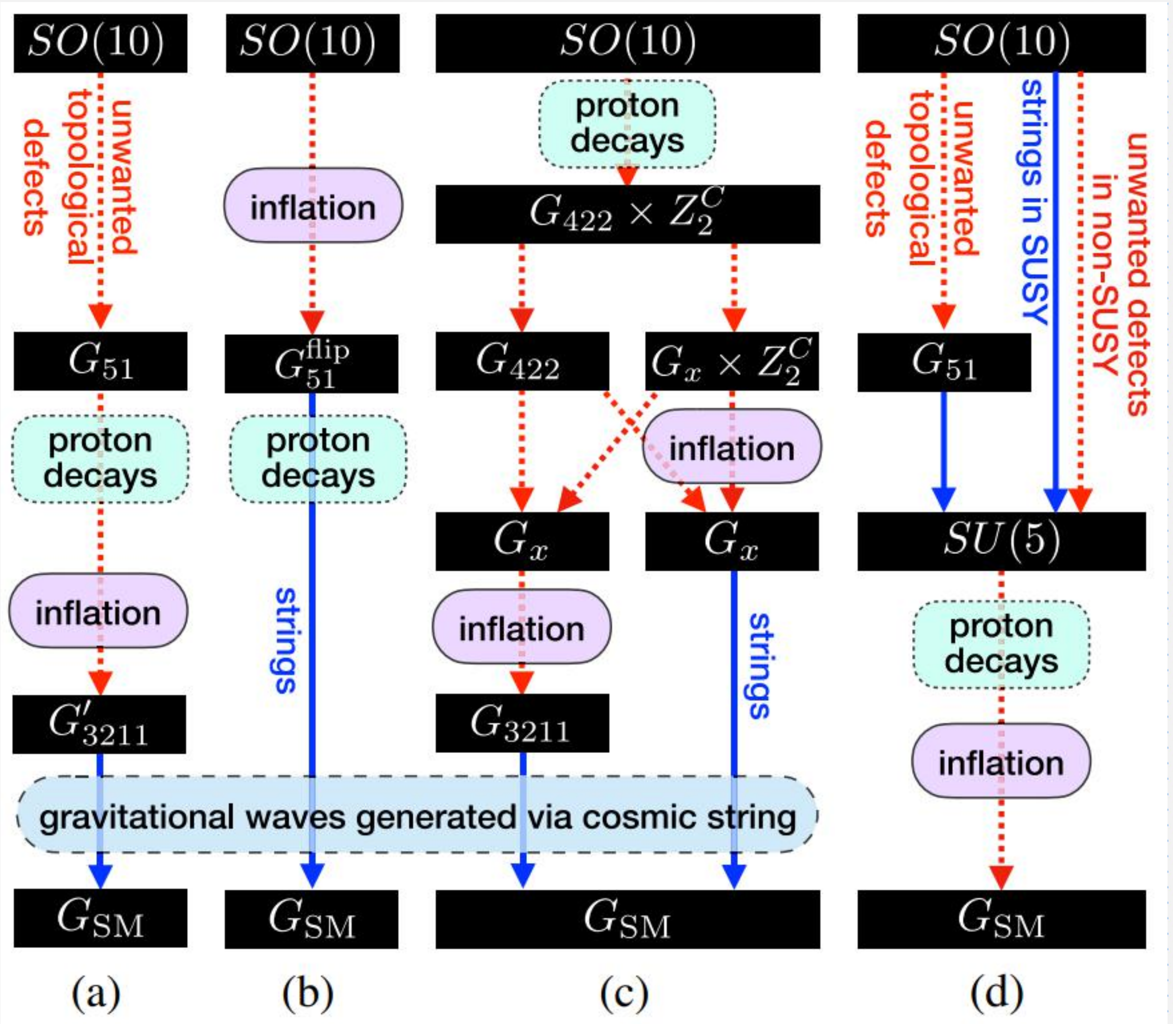
© Nobel Media AB. Photo: A. Mahmoud
François Englert
Prize share: 1/2

© Nobel Media AB. Photo: A. Mahmoud
Peter W. Higgs
Prize share: 1/2

nobelprize.org

See also Danny, Jessica, Peter, Zhenmin's talks

Symmetry breakings are inherent in GUT:

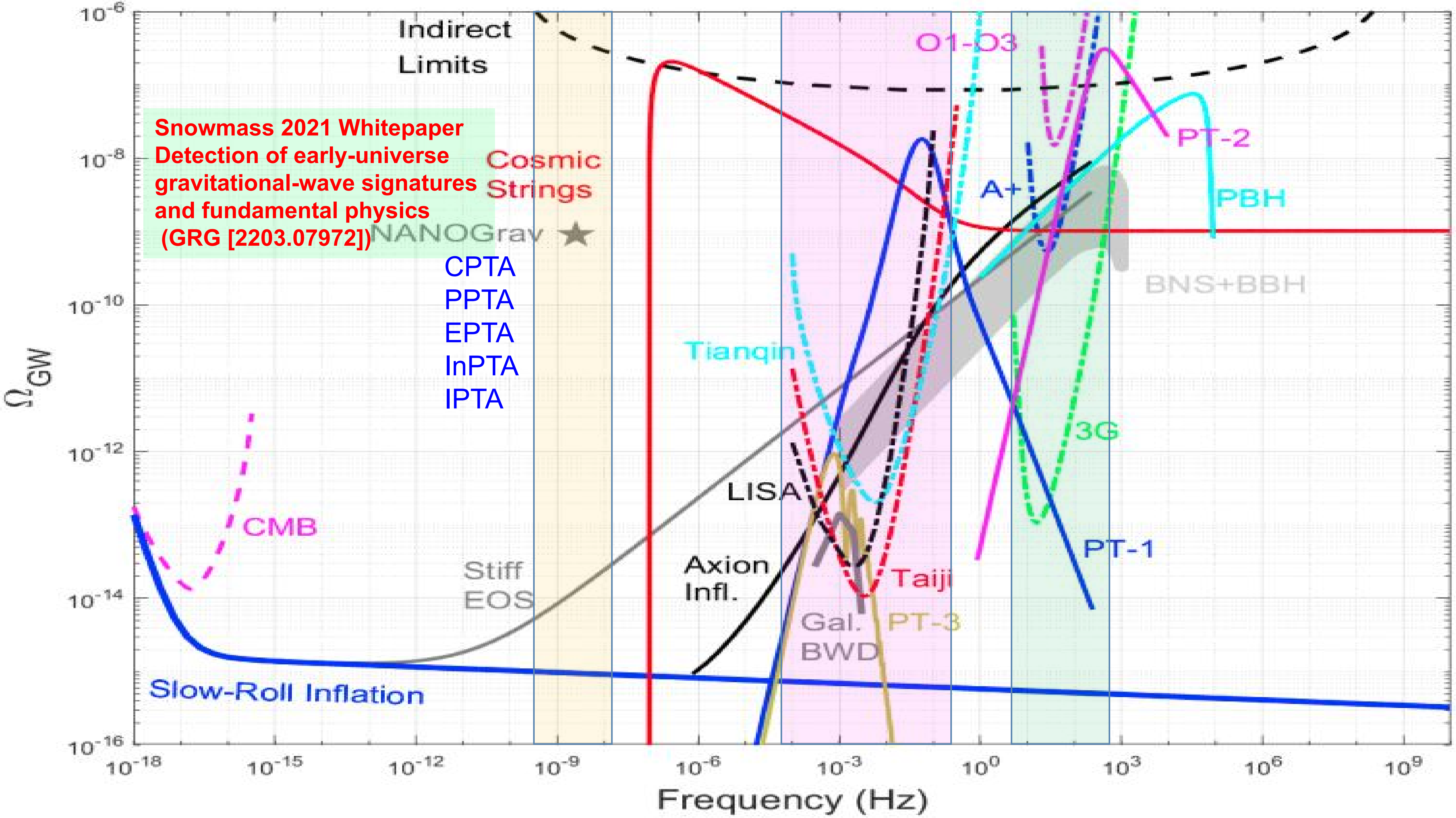


King, Pascoli, Turner, Zhou, PRL [2005.13549], JHEP [2106.15634]

Consequences of symmetry breakings:

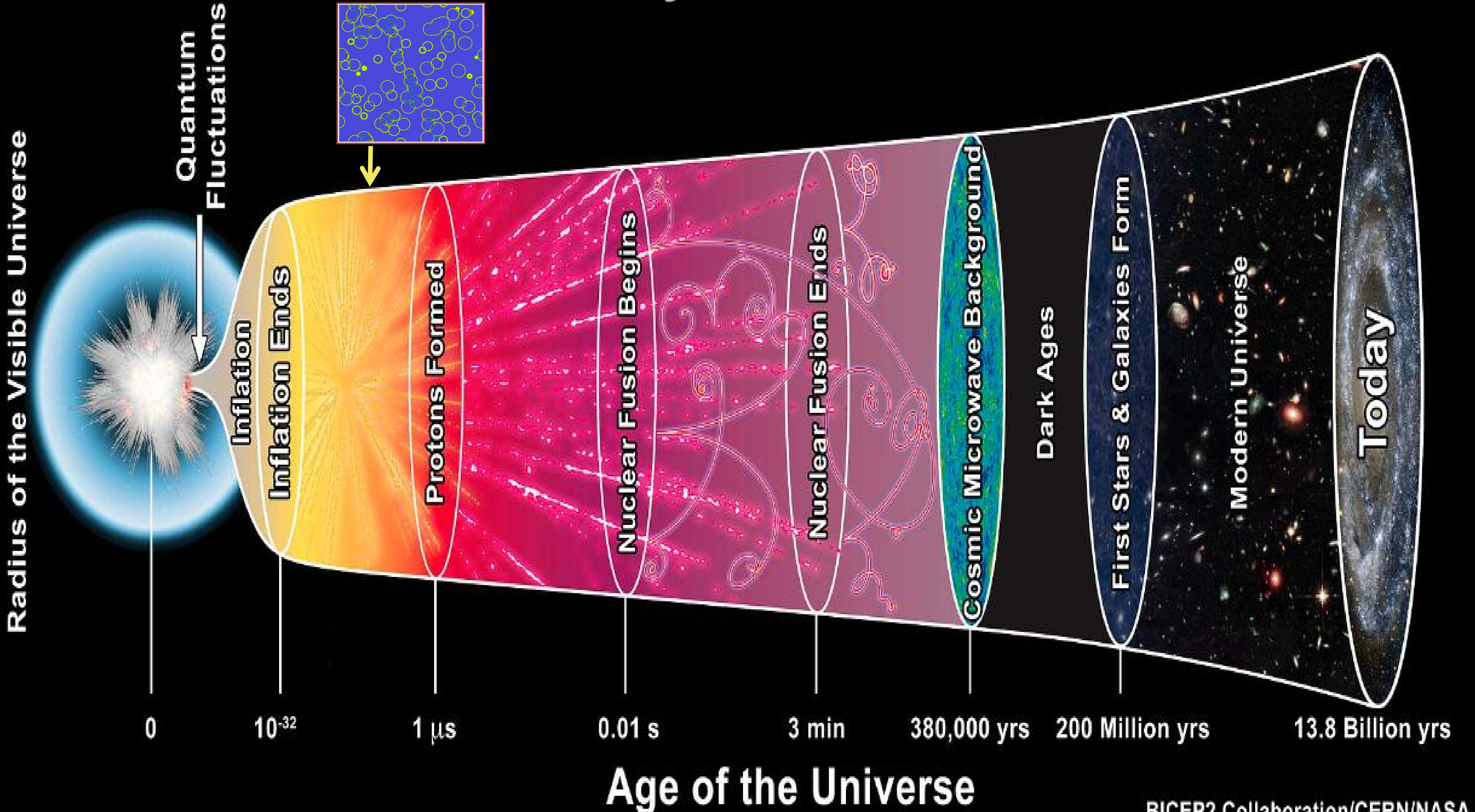
- Phase transitions
- Topological solitons (topological defects)
  monopoles, cosmic strings, domain walls, ...
- Non-Topological solitons
  Fermiballs (or PBH), ...

Multiple Sources for gravitational waves!



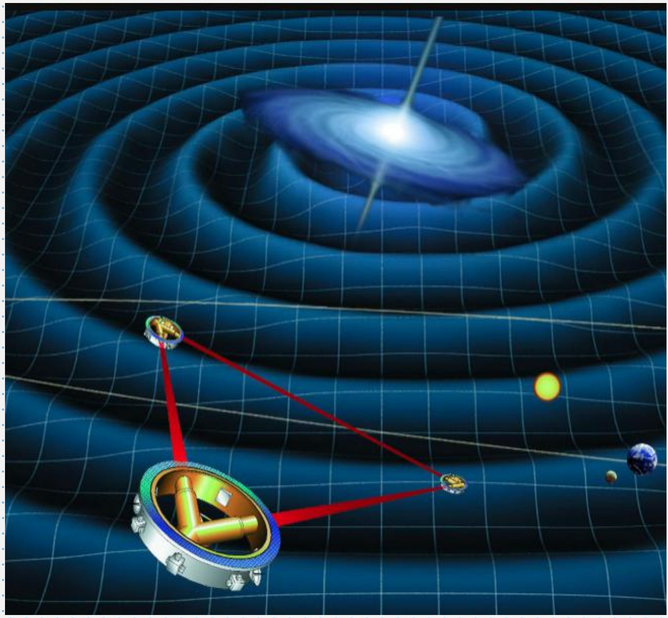
Snowmass 2021 Whitepaper
Detection of early-universe gravitational-wave signatures and fundamental physics
(GRG [2203.07972])

CPTA
PPTA
EPTA
InPTA
IPTA

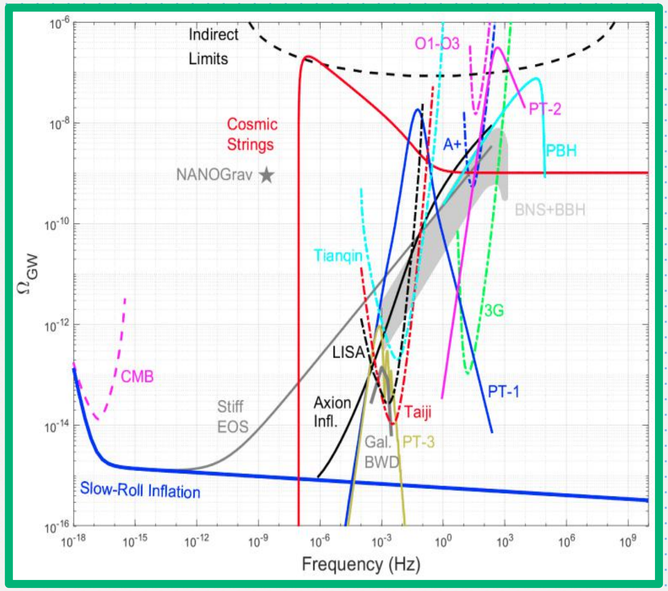


BICEP2 Collaboration/CERN/NASA

From Theory to Experiment



LIGO, LISA/Taiji/Tianqin, PTA, ...



Gravitational Wave Spectrum

α

β

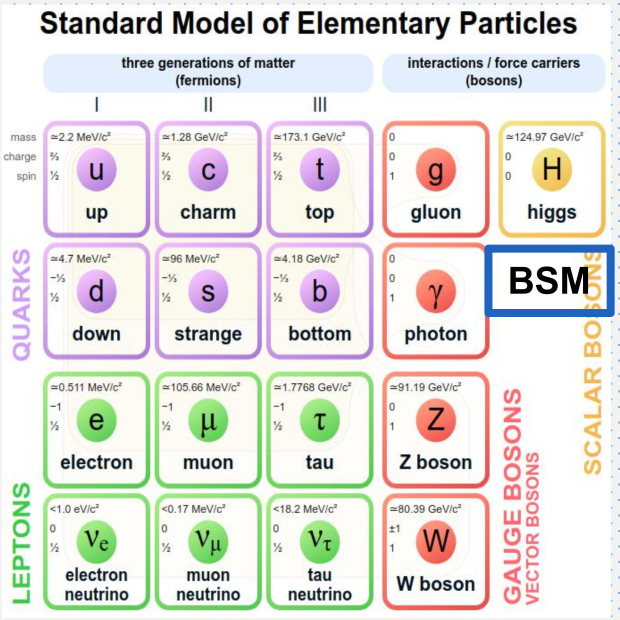
v_w

T_*

g_s

$\dots$

Phase Transition Parameters



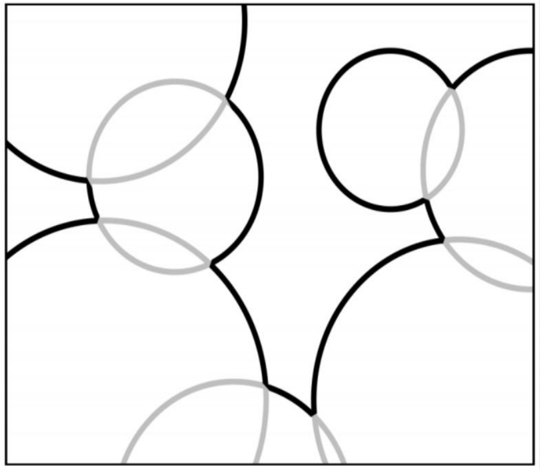
Particle Physics Model

Gravitational Wave Sources

The current understanding:

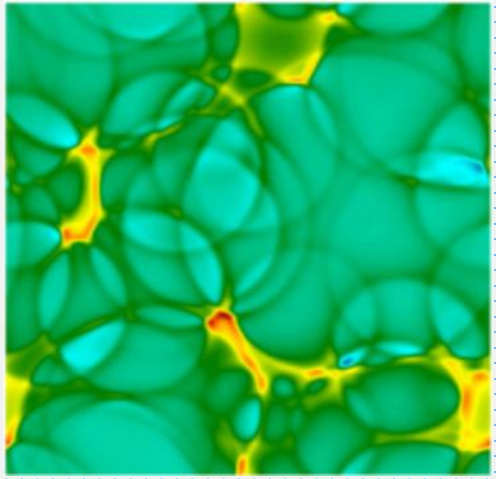
$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} \boxed{T_{\mu\nu}}$$

energy near the wall



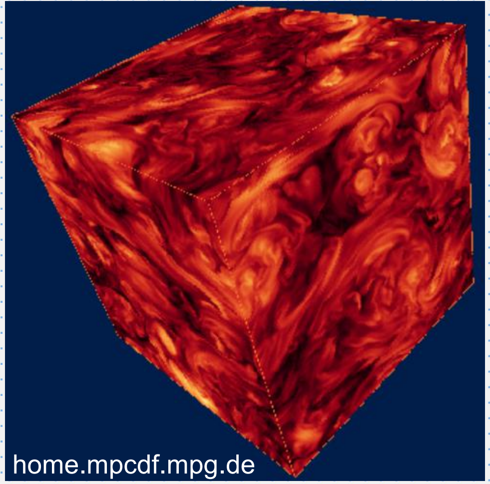
Bubble Collisions

fluid kinetic energy



Sound Waves

turbulent fluid + magnetic field



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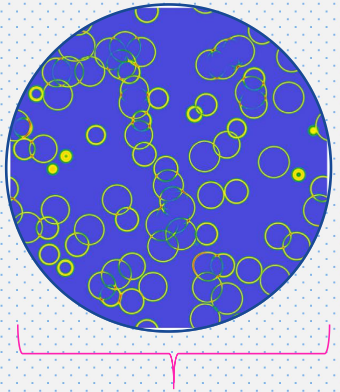
Magnetohydrodynamic Turbulence

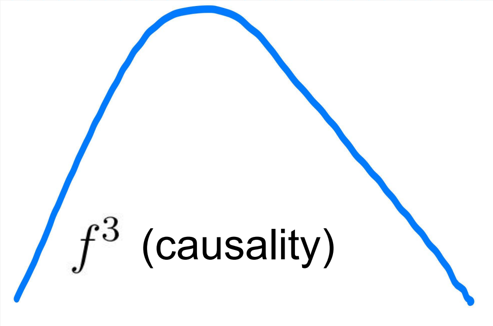
Basic Properties



Hubble size: 1/H*

$$f_{\rm now} = 1.65 \times 10^{-5} \boxed{\left(\frac{f_{\rm PT}}{\beta}\right)\left(\frac{\beta}{H_*}\right)} \left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} {\rm Hz}$$

~100-1000



Cai, Pi, Sasak, PRD [1909.13728]

nHz (~100MeV) QCD scale

~mHz : (~100GeV) weak scale

~100Hz (~PeV - EeV) high scale

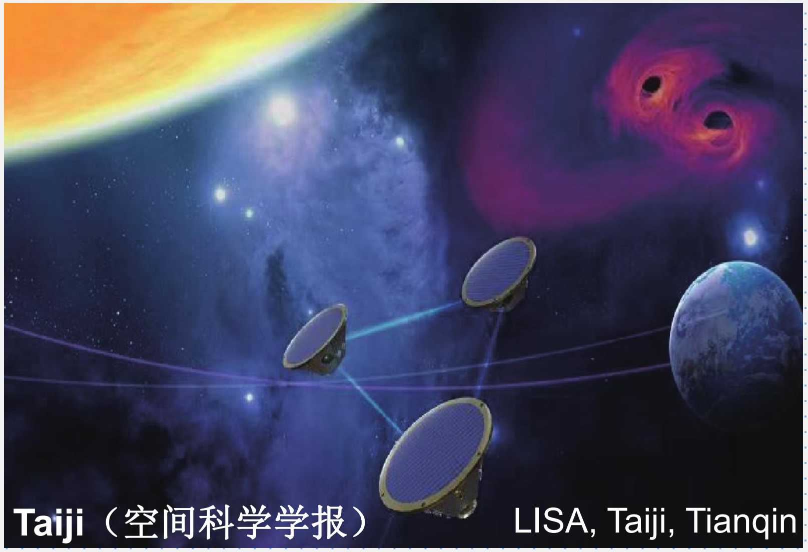
EPTA

NANOGrav Physics Frontiers Center

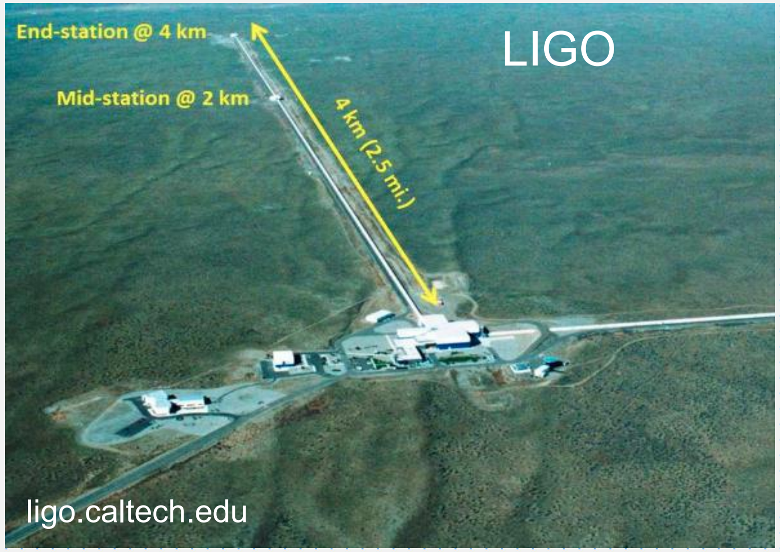
PPTA

InPTA Indian Pulsar Timing Array

中国脉冲星测时阵列（CPTA）



Taiji（空间科学学报） LISA, Taiji, Tianqin



ligo.caltech.edu

Bubble Collisions

Envelope approximation:
Kosowsky, Turner, Watkins, Kamionkowski,
PRL69,2026(1992), PRD45,4514(1992), PRD47,4372(1993), PRD [9310044]

$$h^2\Omega_{\rm BC}(f) = 1.67\times 10^{-5}\left(\frac{100}{g_*}\right)^{1/3}\Delta(v_w)\left(\frac{H_n}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2 S_{\rm env}(f)$$

simulation

$$\Delta = \frac{0.11 v_w^3}{0.42+v_w^2},$$

$$\frac{f_*}{\beta} = \frac{0.62}{1.8-0.1v_w+v_w^2},$$

$$S_{\rm env} = \left[\frac{3.8(f/f_{\rm env})^{2.8}}{1+2.8(f/f_{\rm env})^{3.8}}\right]$$

Huber, Konstandin, JCAP [0806.1828]

Chiara Caprini et al JCAP [1512.06239]

analytical

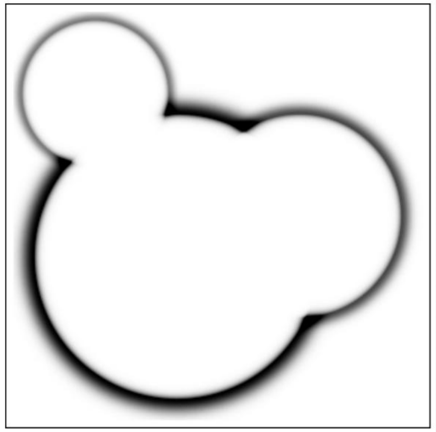
$$\Delta = \frac{0.48 v_w^3}{1+5.3v_w^2+5v_w^4},$$

$$\frac{f_*}{\beta} = \frac{0.35}{1+0.069v_w+0.69v_w^4},$$

$$S_{\rm env} = \left[c_l\left(\frac{f}{f_{\rm env}}\right)^{-3}+(1-c_l-c_h)\left(\frac{f}{f_{\rm env}}\right)^{-1}+c_h\left(\frac{f}{f_{\rm env}}\right)\right]^{-1}$$

$$(c_l = 0.064, \quad c_h = 0.48)$$

Jinno, Takimoto, PRD [1605.01403]



thin shell of uncollided walls

$$\Omega_{\rm BC}(f \gtrsim f_{\rm peak}) \propto f^{-1}$$

$$\Omega_{\rm BC}(f \lesssim f_{\rm peak}) \propto f^3$$

Bubble Collisions: Recent Development

- Wall thickness (probe effective potential)

  Cutting et al, PRD [2005.13537], Gould et al, PRD [2107.05657], Mégevand,Membiela, JCAP [2302.13349]

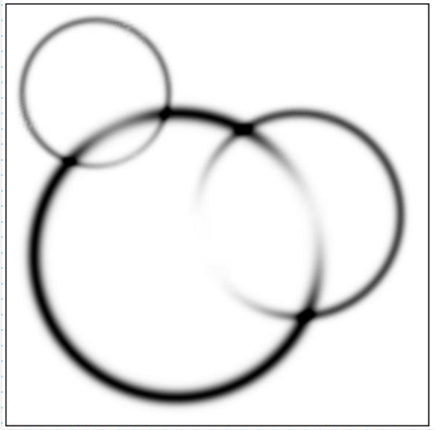
- Duration and Expanding Universe

  Zhong, Gong, Qiu, JHEP [2107.01845]

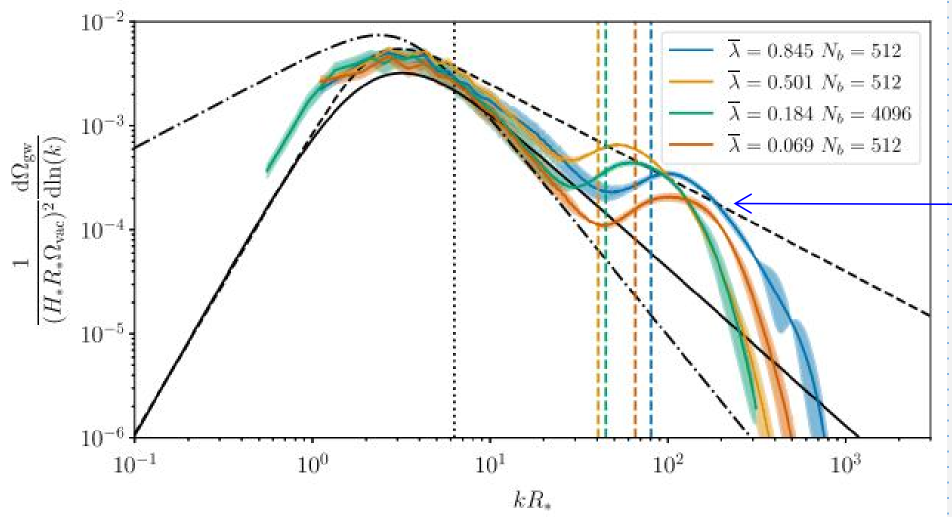
- Scalar + Gauge

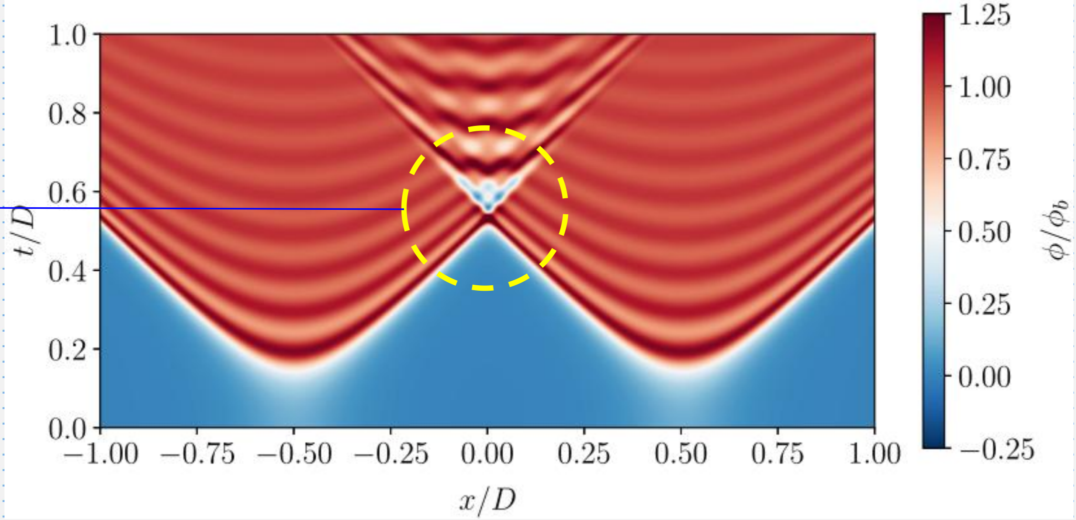
  Di, Wang, Zhou, Bian, Cai, Liu, PRL [2012.15625], Yang, Bian, PRD [2102.01398],
  Lewicki, Vaskonen, EPJC [2007.04967]

Bulk flow model



Jinno,Takimoto, JCAP [1707.03111],
Konstandin, JCAP [1712.06869]



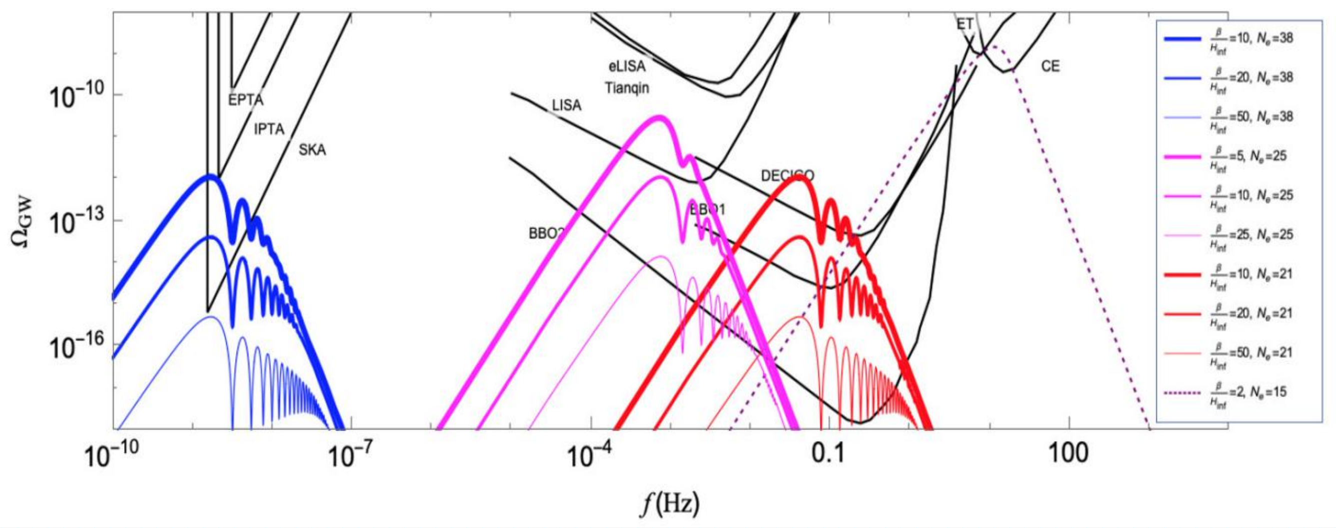


Bubble Collisions during Inflation

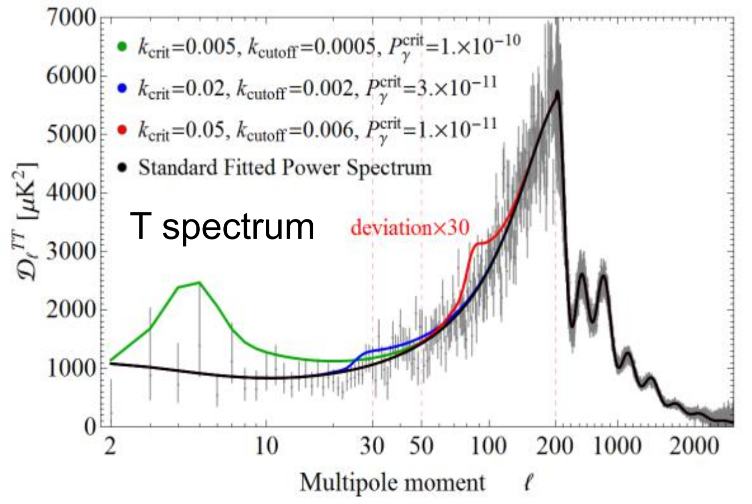
PT in the spectator field (sigma), with negligible energy density

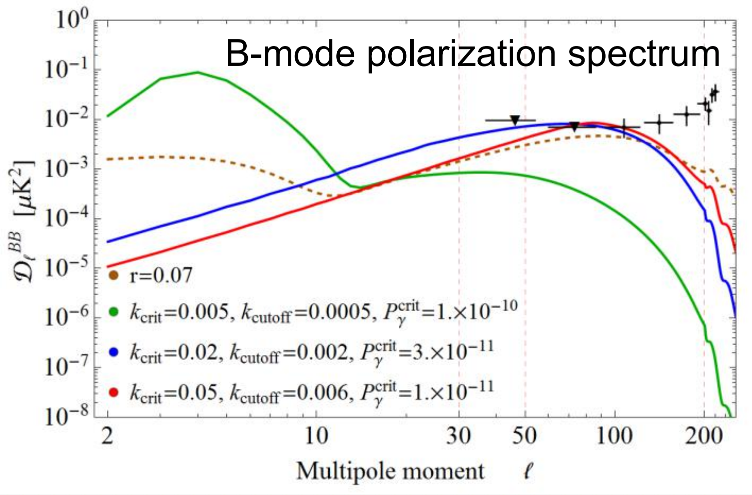
$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi,\sigma)$$

An, Lyu, Wang, Zhou, CPC [2009.12381], JHEP [2201.05171]
An, Tong, Zhou, [2208.14857]
An, Yang, [2304.02361]

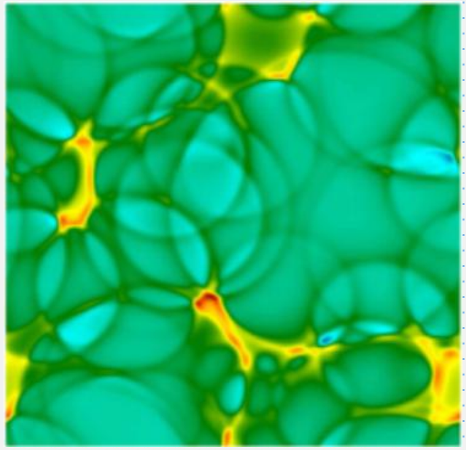


Jiang,Liu,Sun,Wang, PLB [1512.07538]


T spectrum


B-mode polarization spectrum

Sound Waves



Hindmarsh,Huber,Rummukainen,Weir, PRL [1304.2433]

$$T^{ij} \propto (p+e)v^i v^j$$

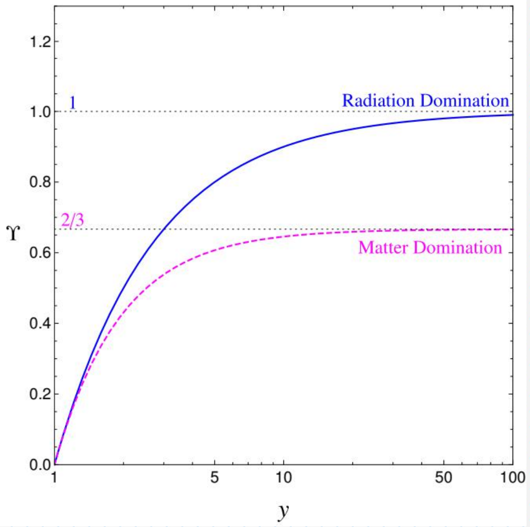
$$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w S_{\rm sw}(f)\Upsilon(\tau_{\rm sw})$$

$$S_{\rm sw}(f) = \left(\frac{f}{f_{\rm sw}}\right)^3\left[\frac{7}{4+3(f/f_{\rm sw})^2}\right]^{7/2} \qquad f_* = \frac{2\beta}{\sqrt{3}v_w} \approx \frac{3.4}{R_*}$$

Hindmarsh,Huber,Rummukainen,Weir, PRD [1504.03291]

$$\Omega_{\rm SW}(f \gtrsim f_{\rm peak}) \propto f^{-4}$$

$$\Omega_{\rm SW}(f \lesssim f_{\rm peak}) \propto f^{3}$$



Slight different fit obtained by the same group, PRD [1704.05871]

$$\Upsilon = 1-(1+2\tau_{\rm sw}H_{\rm pt})^{-1/2}$$ (radiation domination)

HG, Sinha, Vagie, White, JCAP [2007.08537]

Sound Waves: Recent Development

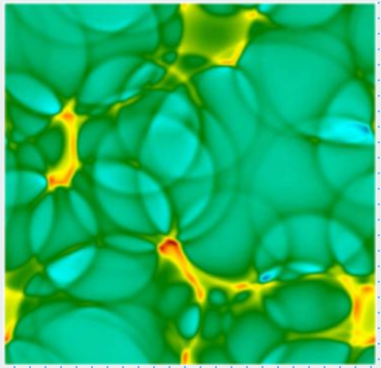
Analytical Modelling

- Refine the sound shell model
- Synergy with simulations

Sound Shell Model

Hindmarsh, PRL [1608.04735]
Hindmarsh, Hijazi, JCAP [1909.10040]
HG, Sinha, Vagie, White, JCAP [2007.08537]
Cai, Wang, Yuwen, PRD Letter [2305.00074]
Pol, Procacci, Caprini [2308.12943]

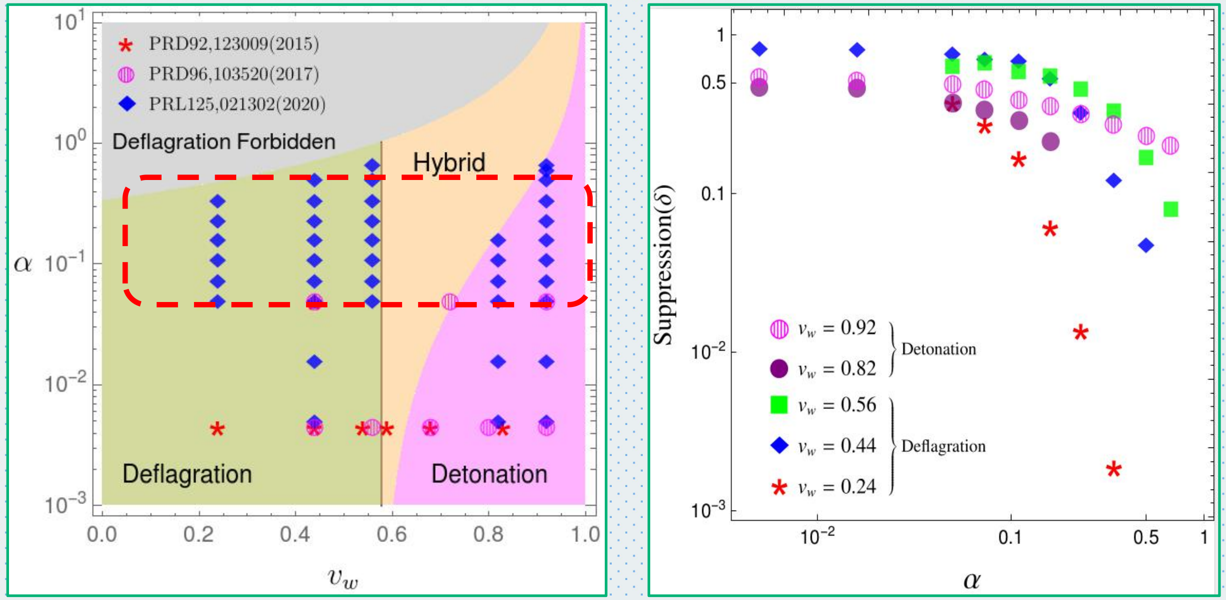
$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$



Numerical Simulation

- Suppression found for strong transitions with small vw
- Need to cover more parameter space (very strong PT)

$$h^2\Omega_{\rm sw}(f) = 2.65\times10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w S_{\rm sw}(f)\Upsilon(\tau_{\rm sw})$$



Cutting, Hindmarsh, Weir, PRL [1906.00480]

Magnetohydrodynamic Turbulence

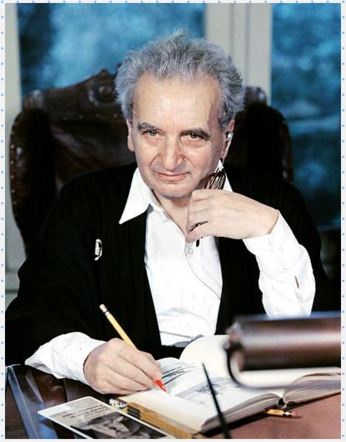
Earlier studies based on Kolmogorov spectrum:
Kamionkowski,Kosowsky,Turner, PRD [9310044]
Kosowsky,Mack,Kahniashvili, PRD [0111483]
Gogoberidze,Kahniashvili,Kosowsky, PRD [0705.1733]

$$T^{ij} \sim (p+e)v^i v^j - B_i B_j$$

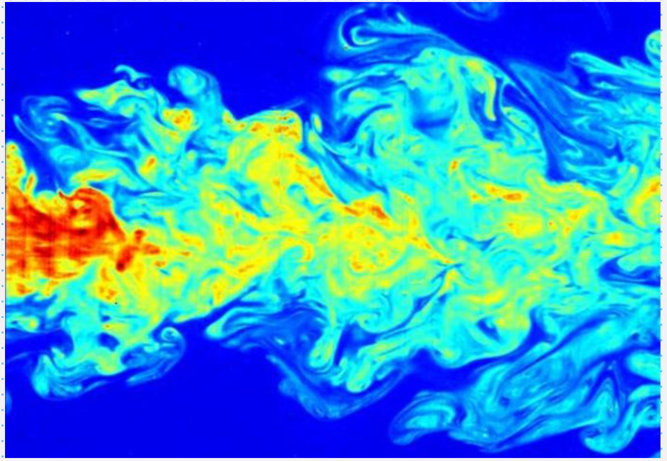
$$\boxed{h^2\Omega_{\rm turb}(f) = 3.35 \times 10^{-4} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}} \left(\frac{100}{g_*}\right)^{1/3} v_w\, S_{\rm turb}(f)}$$

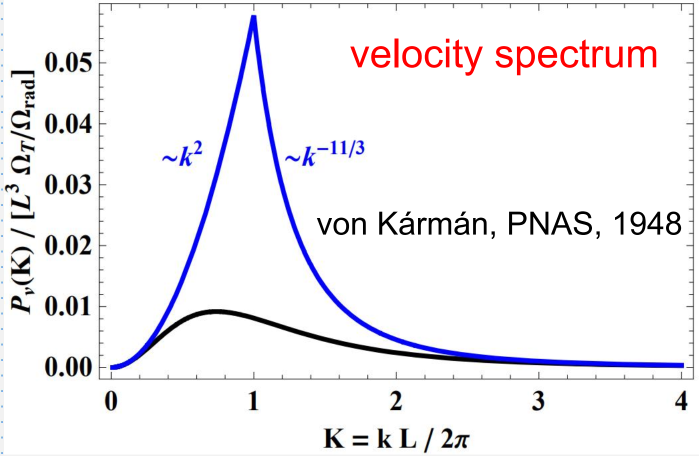
Caprini,Durrer,Servant, JCAP [0909.0622] (used von Kármán's spectrum)

Andrey Nikolaevich Kolmogorov



Theodore von Kármán

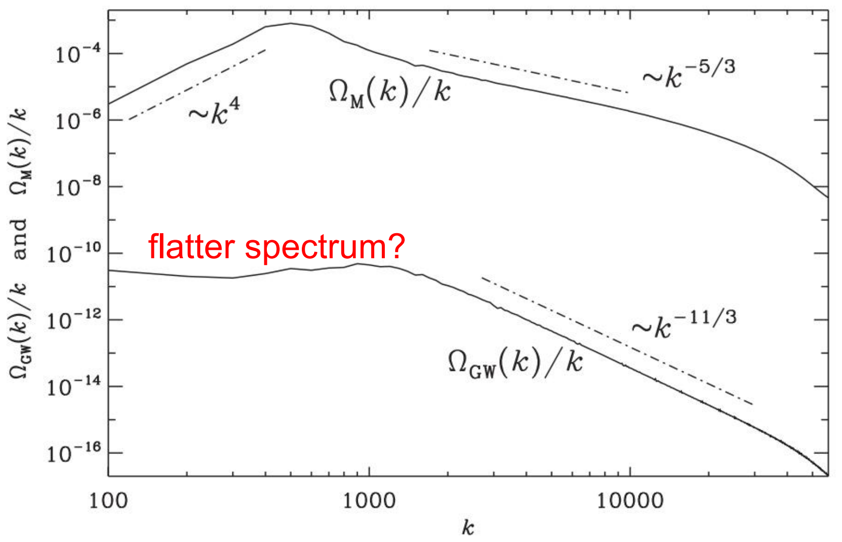




Magnetohydrodynamic Turbulence: Recent Development

Progress on numerical simulations, and analytical modellings

- Strong dependence on initial conditions
- Flatter spectrum at low frequency (violate causality?)

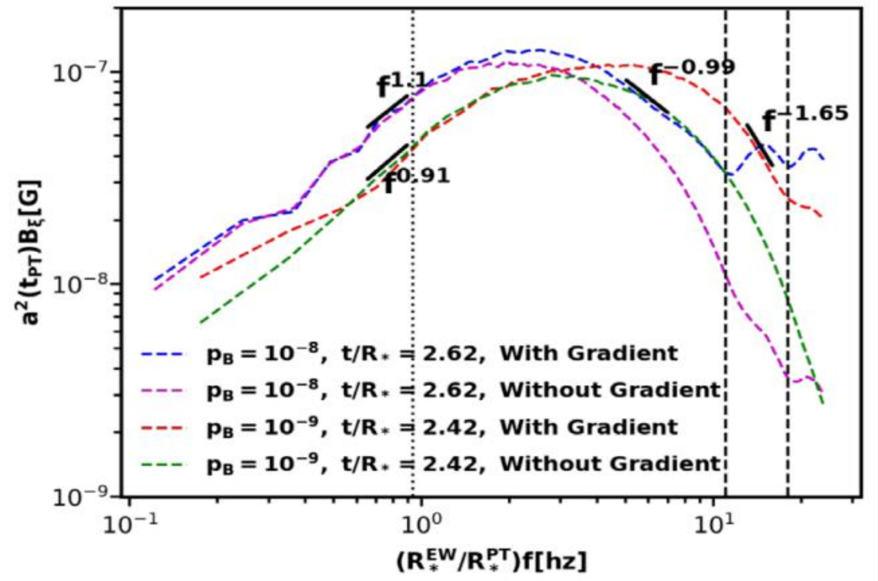


flatter spectrum?

Pol et al, PRD [1903.08585]

as initial conditions? ←

Magnetic Field Generation (simulation)



Di,Wang,Zhou,Bian,Cai, PRL [2012.15625]
Yang,Bian,PRD [2102.01398]

Modelling: Sharma, Brandenburg, PRD [2206.00055]
Time decorrelation: Auclair et al, JCAP [2205.02588]
Decay, viscosity: Dahl et al, PRD [2112.12013]
Polarization: Pol et al, JCAP [2107.05356]

high-scale PT

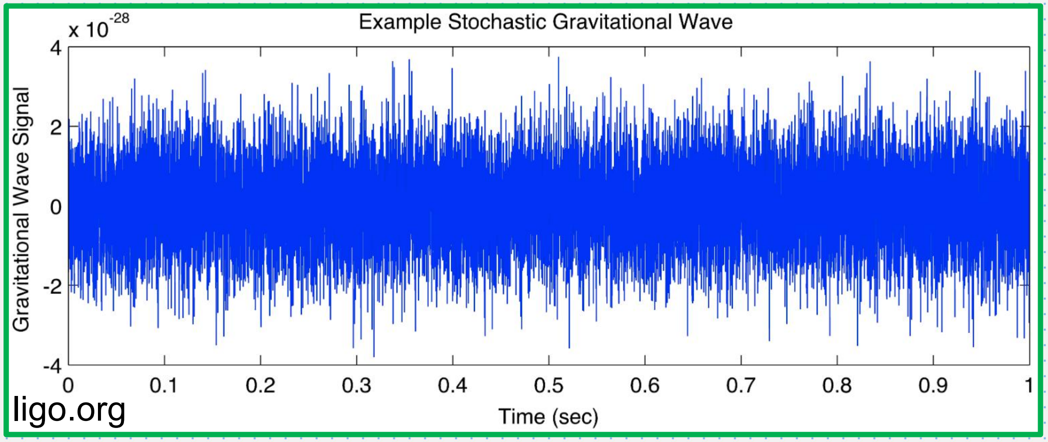
Detection at LIGO

Romero,Martinovic,Callister,HG,Martínez,Sakellariadou,
Yang,Zhao, PRL [2102.01714]

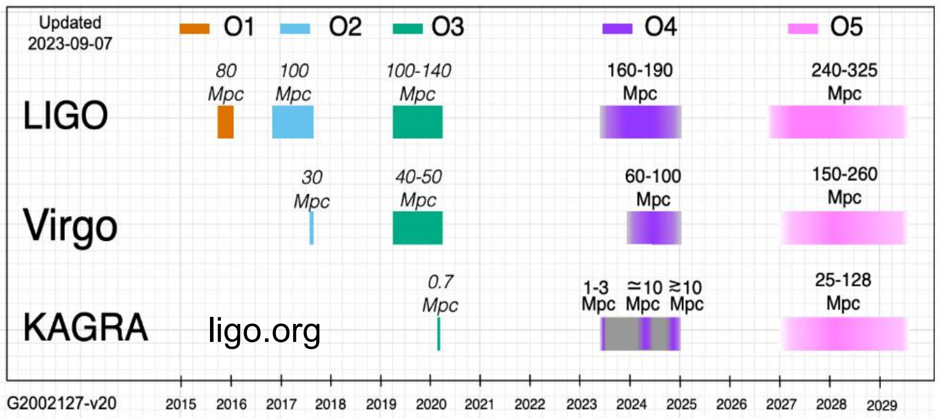
- No Evidence for Broken Power Law Signal
- No Evidence for Bubble Collision Domination Signal
- No Evidence for Sound Waves Domination Signal

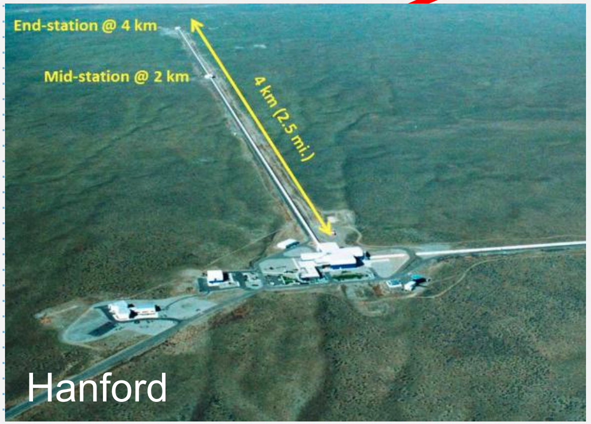
See also: Jiang, Huang, JCAP [2203.11781], Yu, Wang, PRD [2211.13111]

stochastic GWs: noise-like



O1+O2+O3@LIGO (H1, L1), Virgo



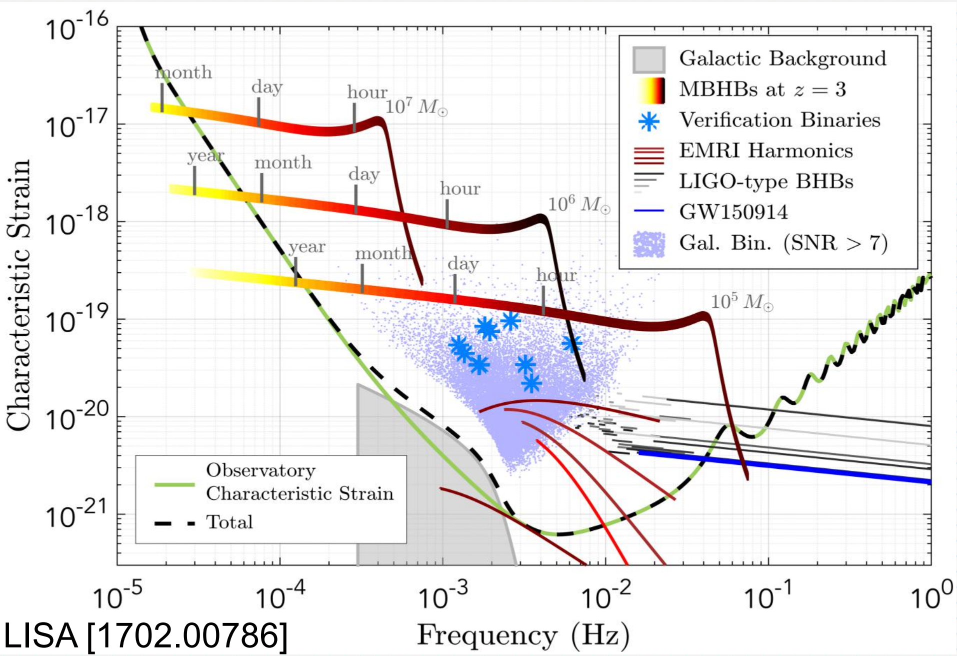


solution:
cross-correlation

Detection at LISA/Taiji/Tianqin

Detection with a single detector

- Complicated, and correlated noise
- Complications from time-delay interferometry
- Solution: null channel method, or with a network



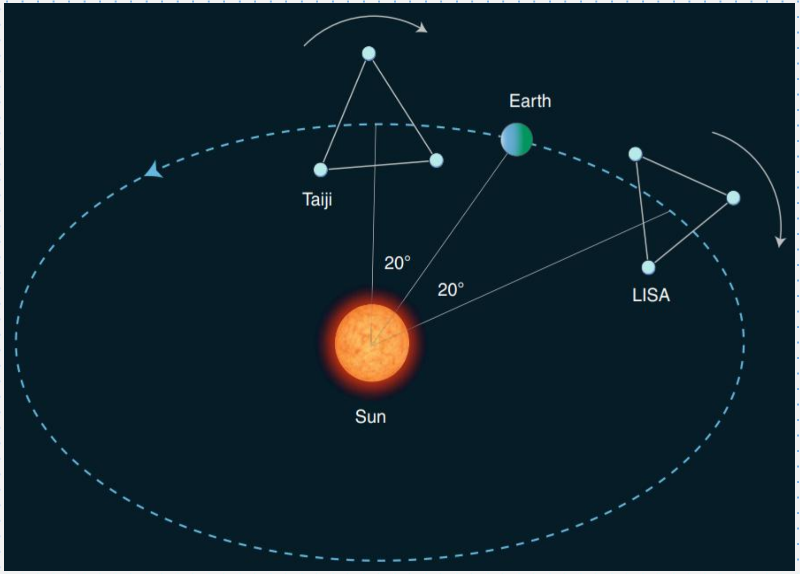
LISA [1702.00786]

galactic foreground + astro background + cosmic background

SGWB detectable down to $\Omega_{GW} \sim O(10^{-13})$

Boileau et al, MNRAS [2105.04283]

The LISA–Taiji network



Ruan, Liu, Guo, Wu, Cai, Nature Astron [2002.03603]
Cai et al [2305.04551]

Gowling et al, JCAP [2209.13551, 2106.05984]
LISA: Caprini et al [2403.03723]
TDI optimization: Wang, Li, Xu, Fan, PRD [2201.10902]

QCD-scale PT

PTA

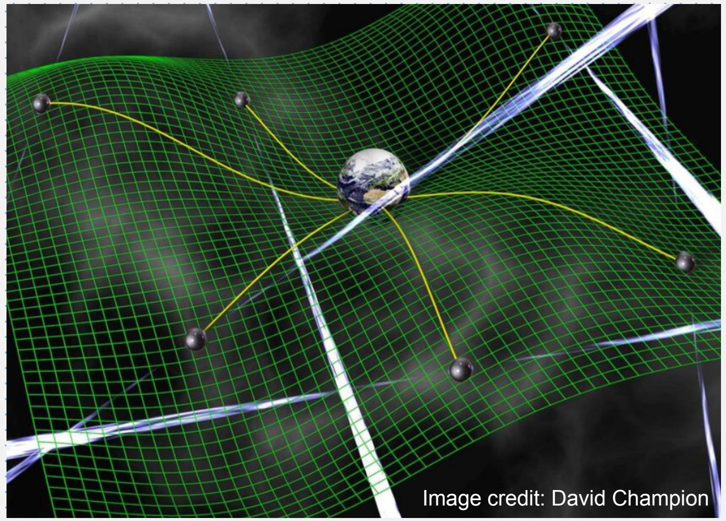
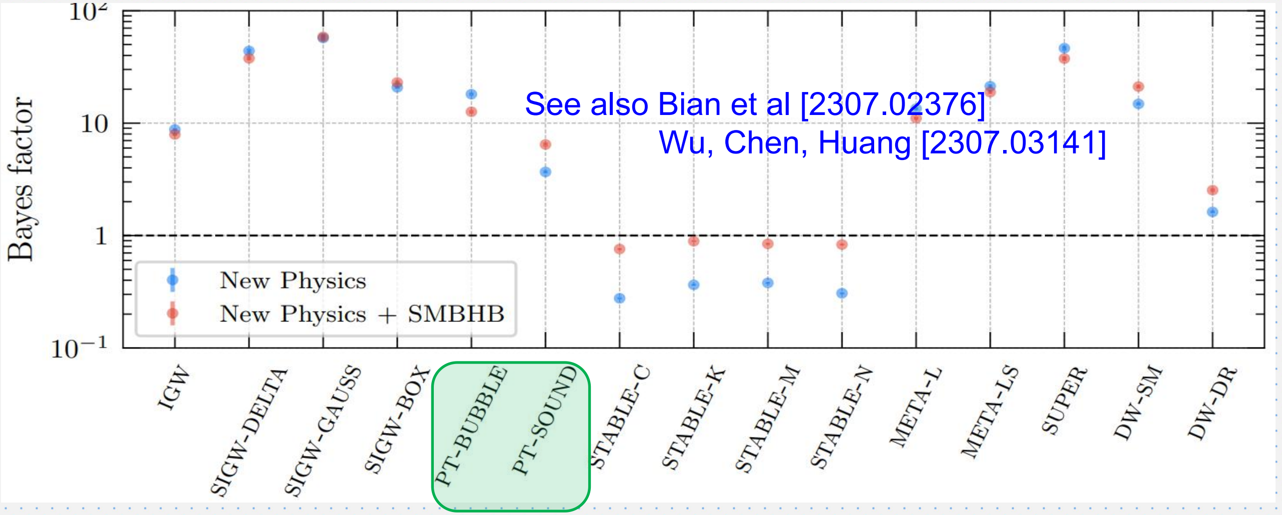


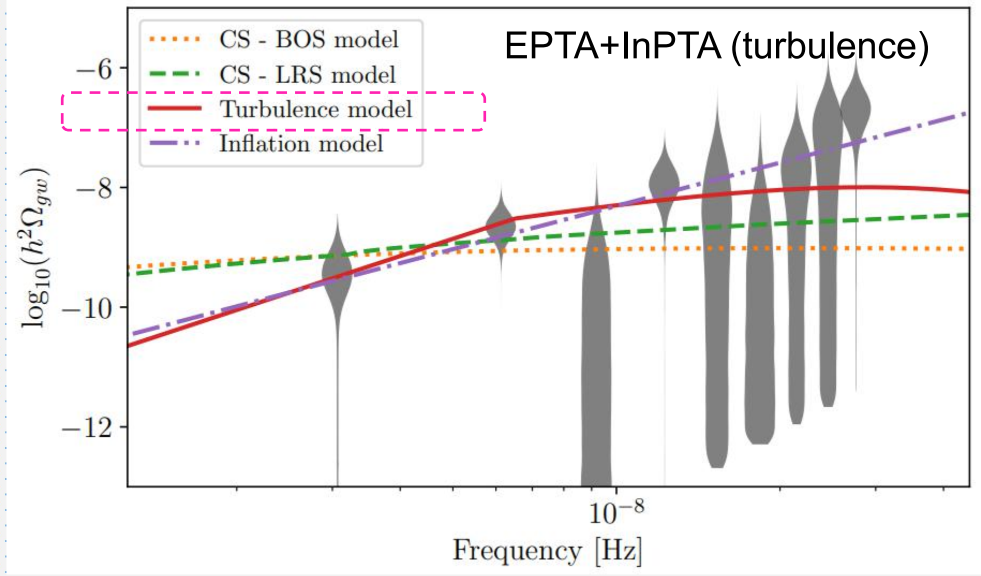
Image credit: David Champion

NANOGrav, ApjL [2306.16219]



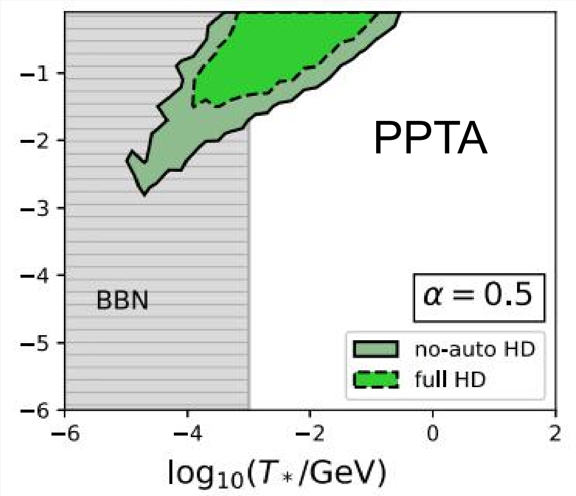
See also Bian et al [2307.02376]

Wu, Chen, Huang [2307.03141]



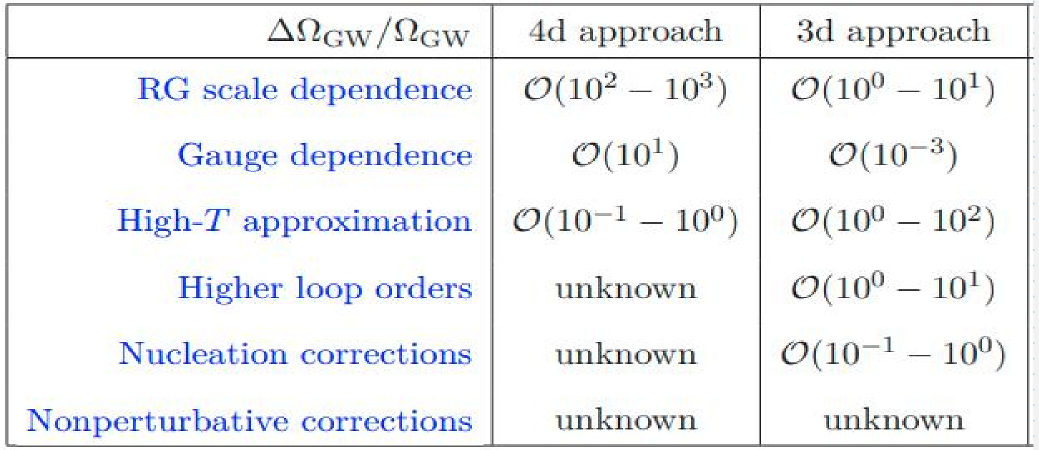
EPTA+InPTA (turbulence)

Xue,Bian,Shu,Yuan,Zhu, et al,
PRL [2110.03096]



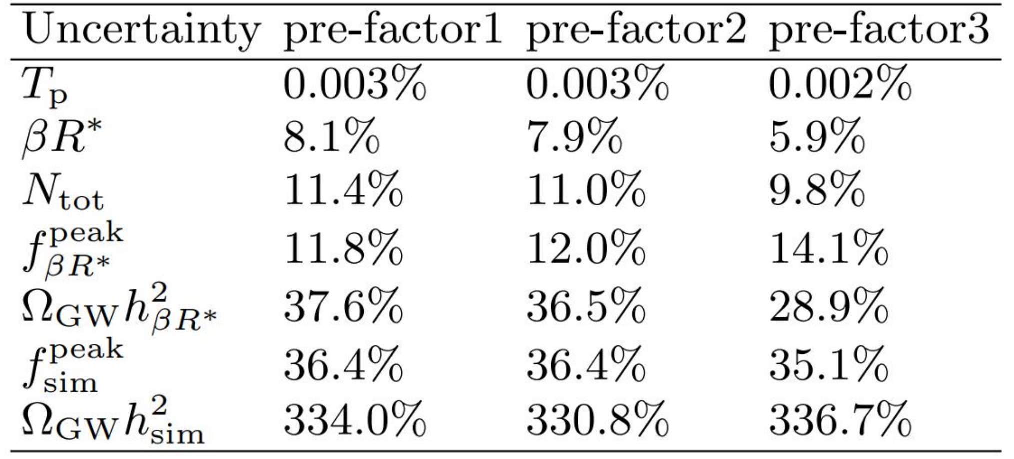
Uncertainties

- Finite T effective potential calculations
- Phase transition parameter calculations (vw)
- GW spectra calculations (simulations, modellings)



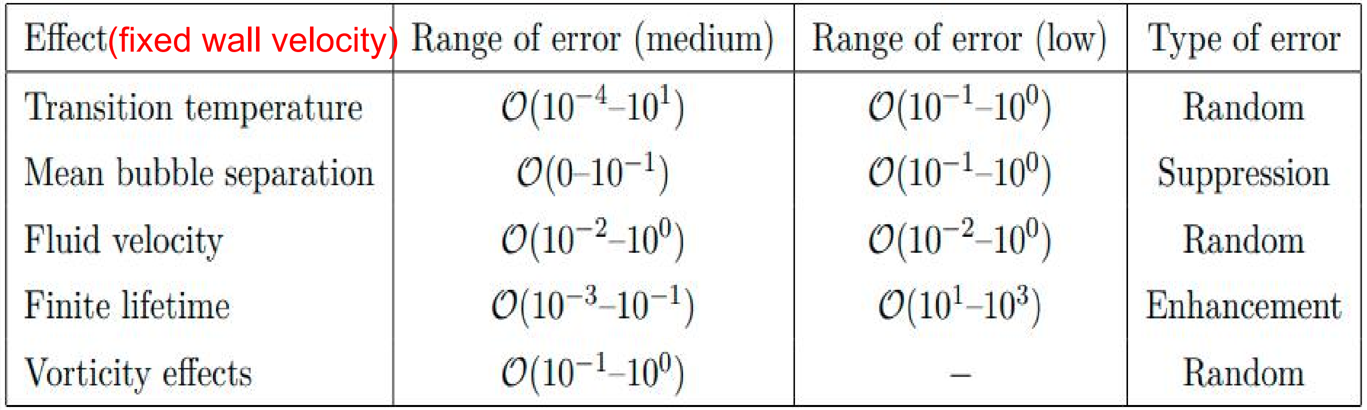
$\Delta\Omega_{\rm GW}/\Omega_{\rm GW}$	4d approach	3d approach
RG scale dependence	$\mathcal{O}(10^2 - 10^3)$	$\mathcal{O}(10^0 - 10^1)$
Gauge dependence	$\mathcal{O}(10^1)$	$\mathcal{O}(10^{-3})$
High-T approximation	$\mathcal{O}(10^{-1} - 10^0)$	$\mathcal{O}(10^0 - 10^2)$
Higher loop orders	unknown	$\mathcal{O}(10^0 - 10^1)$
Nucleation corrections	unknown	$\mathcal{O}(10^{-1} - 10^0)$
Nonperturbative corrections	unknown	unknown

Croon, Gould, Schicho,Tenkanen, White, JHEP [2009.10080]



Uncertainty	pre-factor1	pre-factor2	pre-factor3
$T_{\rm p}$	0.003%	0.003%	0.002%
βR^*	8.1%	7.9%	5.9%
$N_{\rm tot}$	11.4%	11.0%	9.8%
$f^{\rm peak}_{\beta R^*}$	11.8%	12.0%	14.1%
$\Omega_{\rm GW}h^2_{\beta R^*}$	37.6%	36.5%	28.9%
$f^{\rm peak}_{\rm sim}$	36.4%	36.4%	35.1%
$\Omega_{\rm GW}h^2_{\rm sim}$	334.0%	330.8%	336.7%

HG, Xiao, Yang, Zhang [2310.04654]



Effect(fixed wall velocity)	Range of error (medium)	Range of error (low)	Type of error
Transition temperature	$\mathcal{O}(10^{-4}\text{–}10^1)$	$\mathcal{O}(10^{-1}\text{–}10^0)$	Random
Mean bubble separation	$\mathcal{O}(0\text{–}10^{-1})$	$\mathcal{O}(10^{-1}\text{–}10^0)$	Suppression
Fluid velocity	$\mathcal{O}(10^{-2}\text{–}10^0)$	$\mathcal{O}(10^{-2}\text{–}10^0)$	Random
Finite lifetime	$\mathcal{O}(10^{-3}\text{–}10^{-1})$	$\mathcal{O}(10^1\text{–}10^3)$	Enhancement
Vorticity effects	$\mathcal{O}(10^{-1}\text{–}10^0)$	–	Random

HG, Sinha, Vagie, White, JHEP [2103.06933]

Sound speed: Wang,Huang,Li, PRD [2112.14650], etc
See also: Athron, Balazs, Fowlie, Morris, White, Yang Zhang, JHEP [2208.01319]

The Problem of parameter degeneracy

Models	Strong 1^{st} order phase transition	GW signal	Cold DM	Dark Radiation and small scale structure
SM charged				
Triplet [20–22]	✓	✓	✓	✗
complex and real Triplet [23] (Georgi-Machacek model)	✓	✓	✓	✗
Multiplet [24]	✓	✓	✓	
2HDM [25–30]	✓	✓		✗
MLRSM [31]	✓	✓	✗	✗
NMSSM [32–36]	✓	✓	✓	✗
SM uncharged				
S_r (xSM) [37–49]	✓	✓	✗	✗
2 S_r's [50]	✓	✓	✓	✗
S_c (cxSM) [49, 51–54]	✓	✓	✓	✗
$\mathrm{U(1)_D}$ (no interaction with SM) [55]	✓	✓	✓	✗
$\mathrm{U(1)_D}$ (Higgs Portal) [56]	✓	✓	✓	
$\mathrm{U(1)_D}$ (Kinetic Mixing) [57]	✓	✓	✓	
Composite SU(7)/SU(6) [58]	✓	✓	✓	
$\mathrm{U(1)_L}$ [59]	✓	✓	✓	✗
$\mathrm{SU(2)_D} \to$ global $SO(3)$ by a doublet [60–62]			✓	✗
$\mathrm{SU(2)_D} \to \mathrm{U(1)_D}$ by a triplet [63–65]			✓	✓
$\mathrm{SU(2)_D} \to Z_2$ by two triplets [66]			✓	✗
$\mathrm{SU(2)_D} \to Z_3$ by a quadruplet [67, 68]			✓	✗
$\mathrm{SU(2)_D} \times \mathrm{U(1)_{B\text{-}L}} \to Z_2 \times Z_2$ by a quintuplet and a S_c [69]			✓	✗
$\mathrm{SU(2)_D}$ with two dark Higgs doublets [70]	✓	✓	✗	✗
$\mathrm{SU(3)_D} \to Z_2 \times Z_2$ by two triplets [62, 71]			✓	✗
$\mathrm{SU(3)_D}$ (dark QCD) (Higgs Portal) [72, 73]	✓	✓	✓	
$G_{\mathrm{SM}} \times G_{\mathrm{D,SM}} \times Z_2$ [74]	✓	✓	✓	
$G_{\mathrm{SM}} \times G_{\mathrm{D,SM}} \times G_{\mathrm{D,SM}} \cdots$ [75]	✓	✓	✓	
Current work				
$\mathrm{SU(2)_D} \to \mathrm{U(1)_D}$ (see the text)	✓	✓	✓	✓

Ghosh,HG,Han,Liu, JHEP [2012.09758]

Solutions: New Observables

- Anisotropy

Geller, Hook, Sundrum, Yuhsin Tsai, PRL [1803.10780]
Li, Huang, Wang, Zhang, PRD [2112.01409]
Li, Yan, Huang, PRD [2211.03368]

- Primordial magnetic field

Di,Wang,Zhou,Bian,Cai, PRL [2012.15625]
Yang,Bian,PRD [2102.01398], ...

- Primordial black holes and solitons

Hong, Jung, Xie, PRD [2008.04430]
Kawana,Xie,PLB [2106.00111]
Liu,Bian,Cai,Guo,Wang, PRD [2106.05637]
Lu,Kawana,Xie, PRD [2202.03439]

- Curvature perturbations

Liu,Bian,Cai,Guo,Wang,PRL[2208.14086]
Jiang,Liu,Sun,Wang, PLB [1512.07538]

Anything directly readable from the isotropic GW spectrum?

Dissipative Effects as New Observables

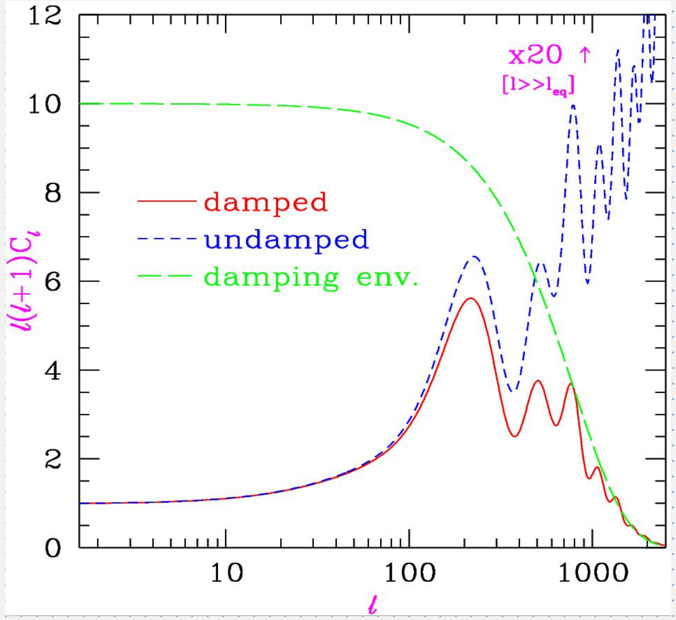
- Going beyond the perfect fluid approximation (viscosity, heat conduction)
- Particle physics origin of dissipations (very weak interactions)
- Can be searched for at LIGO, PTA, LISA/Taiji/Tianqin ...

Weinberg, ApJ, 1971

$$\Delta T^{ij} = -\eta\left(\frac{\partial U_i}{\partial x^j} + \frac{\partial U_j}{\partial x^i} - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{U}\right) - \zeta\;\delta_{ij}\nabla\cdot\mathbf{U},$$

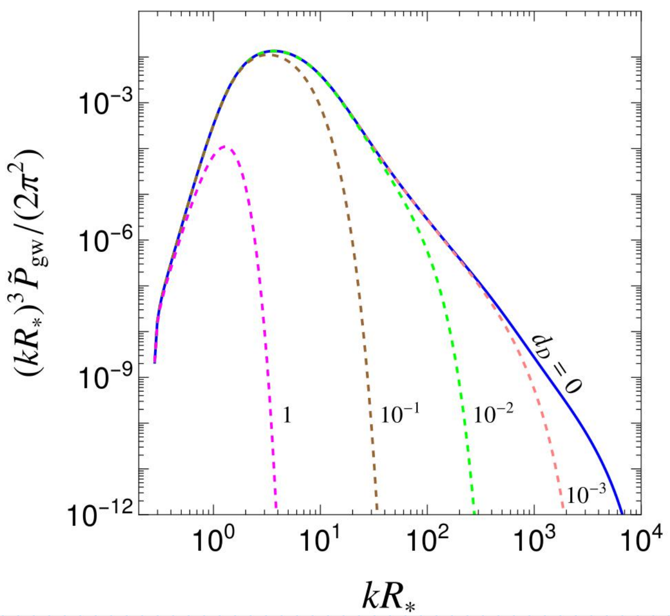
$$\Delta T^{i0} = -\chi\left(\frac{\partial T}{\partial x^i} + T\dot{U}_i\right). \qquad (1)$$

Silk damping of CMB Anisotropy



Hu,White, ApJ [9609079]

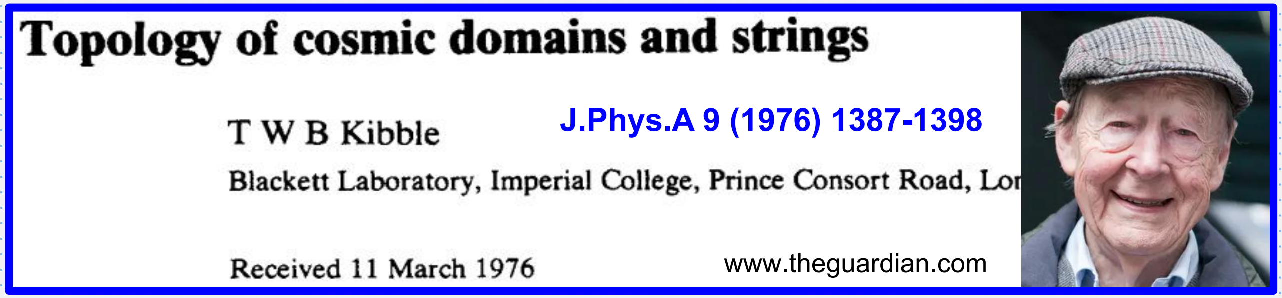
damping of GW



HG [2310.10927]

Phase Transitions: Topological Defects

- Produced during phase transitions in the early universe (not necessarily first order)
- Analogues found in condensed matter systems (Kibble-Zurek mechanism)



Topology of cosmic domains and strings

T W B Kibble

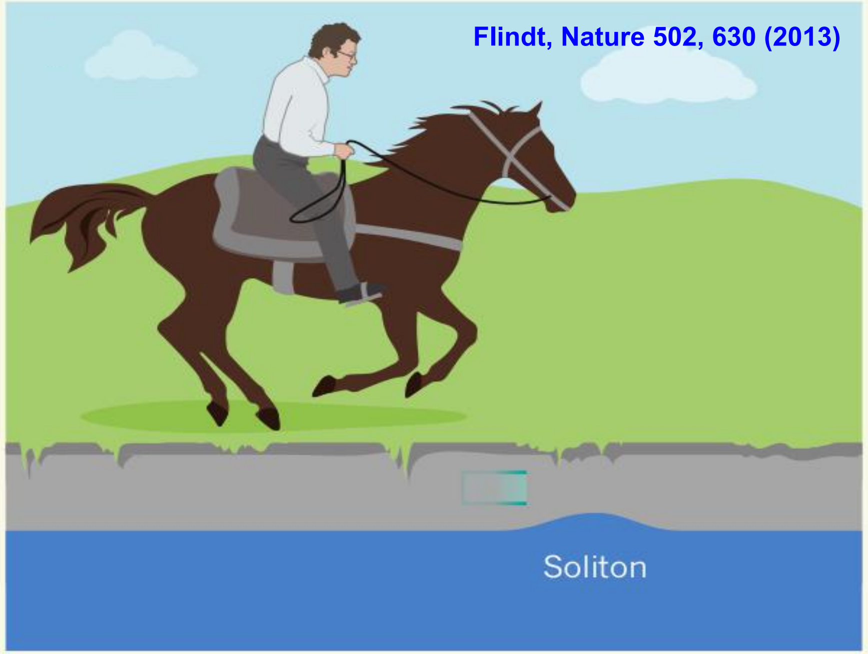
Blackett Laboratory, Imperial College, Prince Consort Road, Lo

Received 11 March 1976

J.Phys.A 9 (1976) 1387-1398

www.theguardian.com

topological defects(solitons)



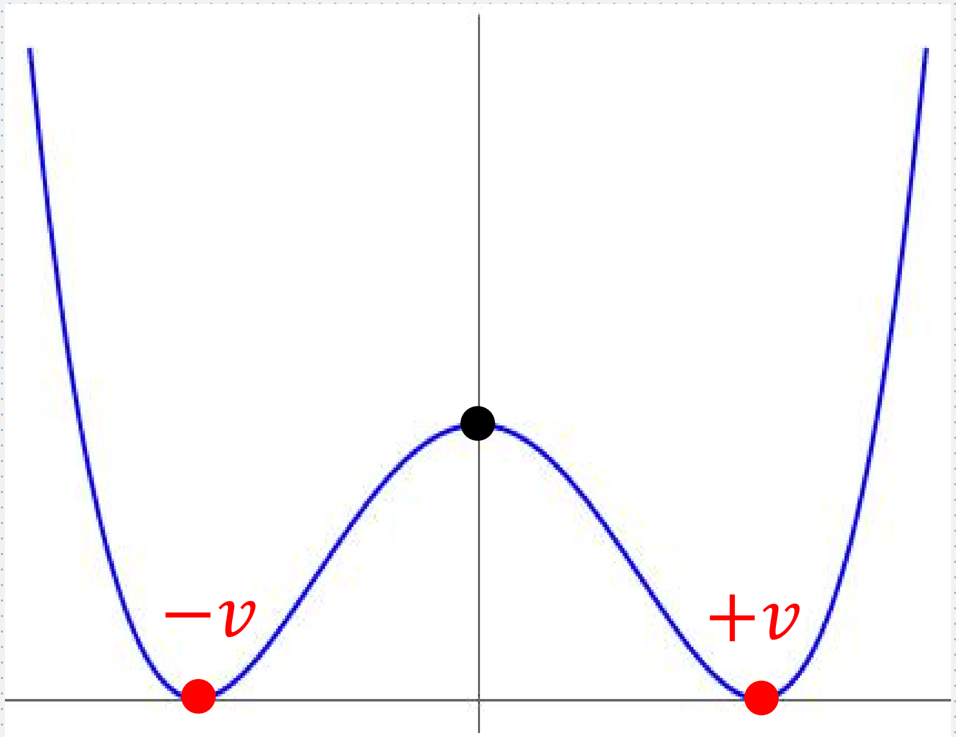
Flindt, Nature 502, 630 (2013)

monopoles, cosmic strings, domain walls, etc

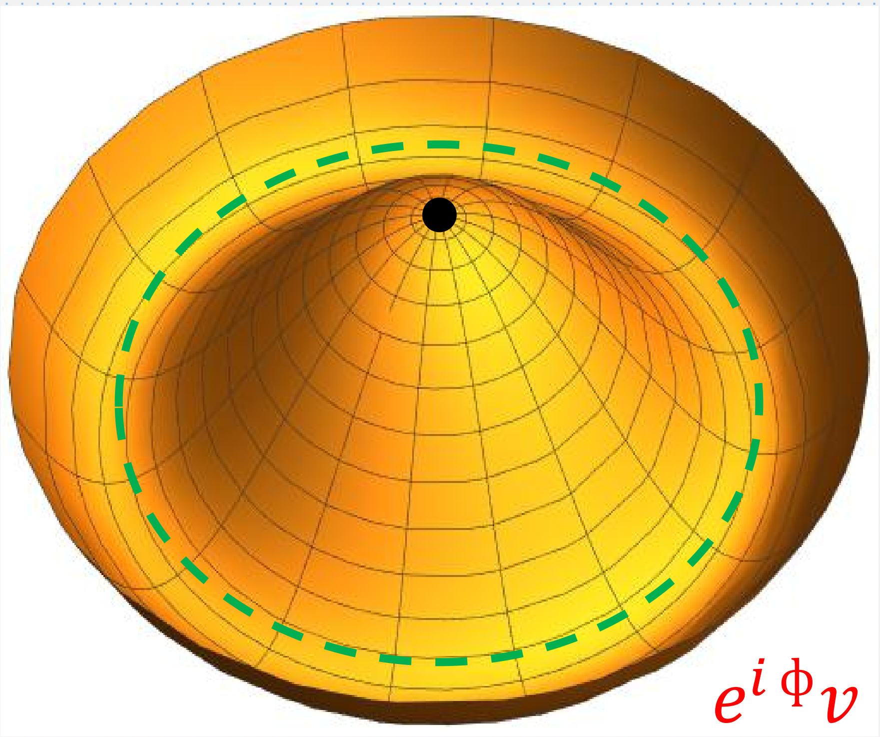
Degenerate Vacuum States

$$V(\phi) = \frac{1}{4}(\phi^2 - v^2)^2$$

$$V(\Phi) = \frac{1}{4}(|\Phi|^2 - \eta^2)^2$$



Domain wall

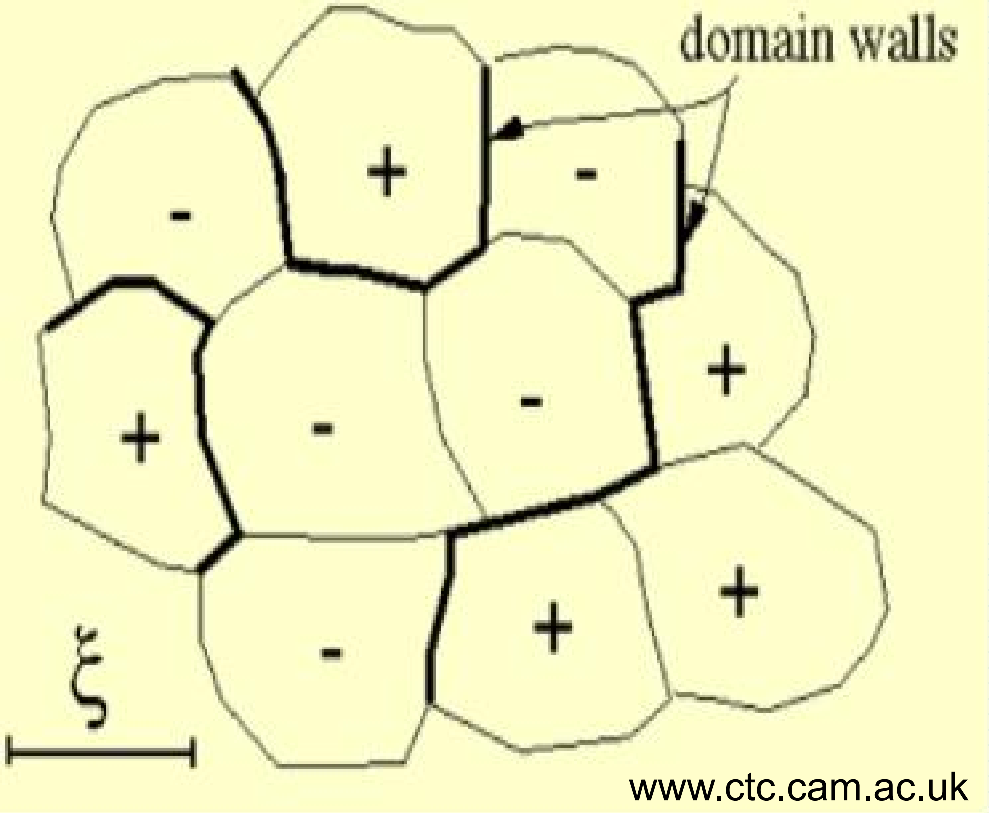


Cosmic String

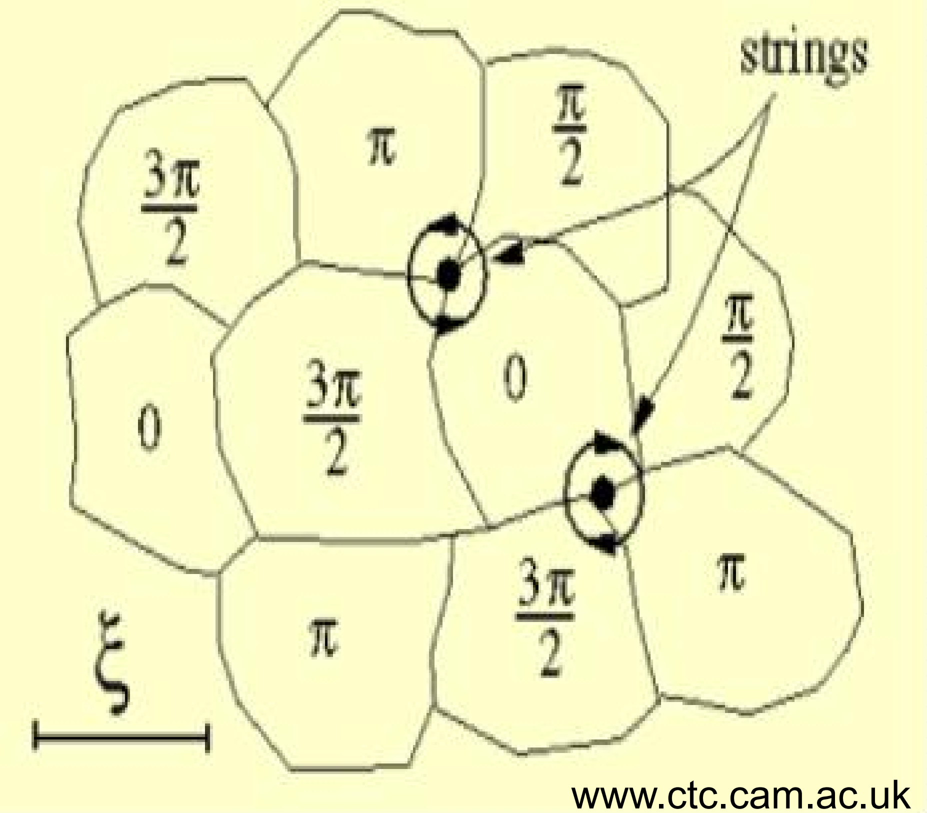
Degenerate Vacuum States

$$V(\phi) = \frac{1}{4}(\phi^2 - v^2)^2$$

$$V(\Phi) = \frac{1}{4}(|\Phi|^2 - \eta^2)^2$$



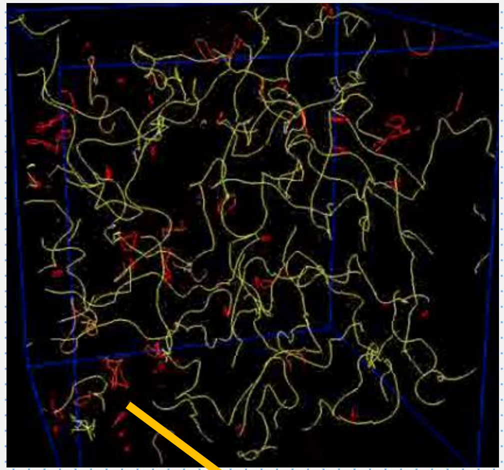
www.ctc.cam.ac.uk



www.ctc.cam.ac.uk

Will focus on cosmic strings.

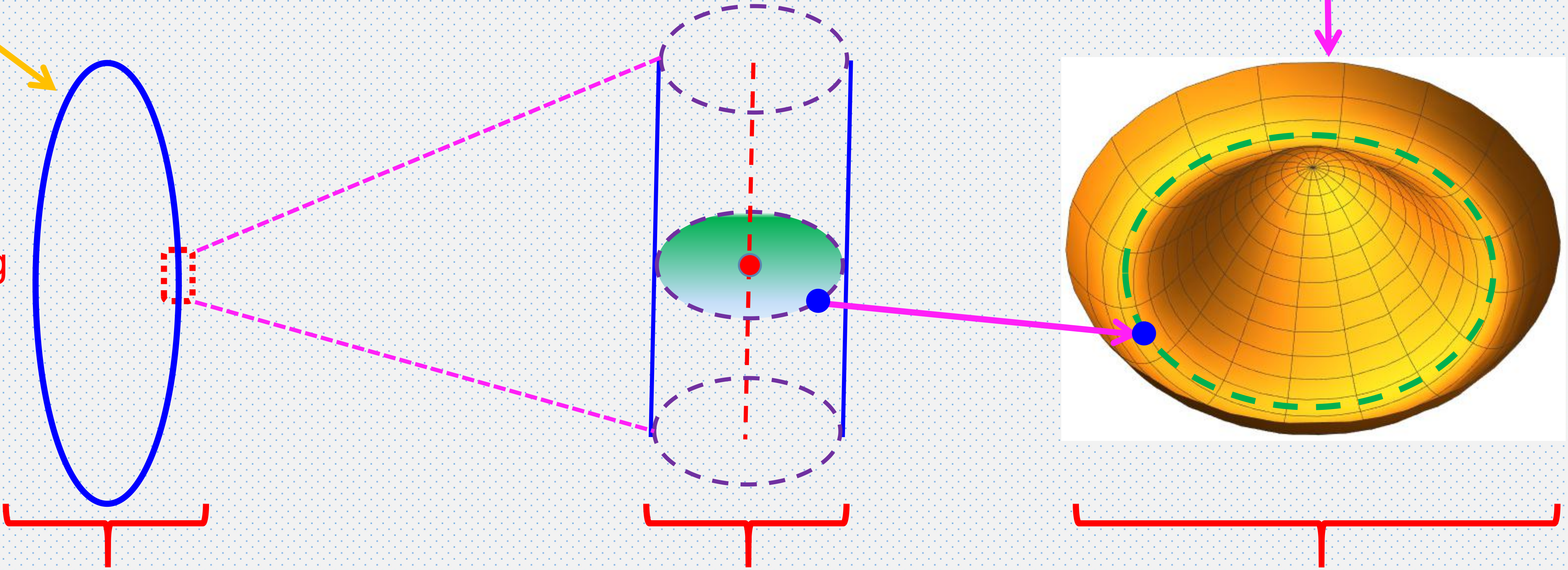
Cosmic String



$$G\mu \sim (\eta/M_{\rm Pl})^2$$

Example: the Abelian Higgs Model

$$\mathcal{L} = |(\partial_\mu - igA_\mu)\Phi|^2 - \frac{1}{4}\lambda(|\Phi|^2 - \eta^2)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



closed string (loop)

cosmological scale $\gg$ $O(1/\eta)$

degenerate vacua with nontrivial topology

Gravitational Wave Production

Burst types: cusps, kinks and kink-kink collisions (Damour, Vilenkin, PRL 85,3671, PRD 64, 064008).

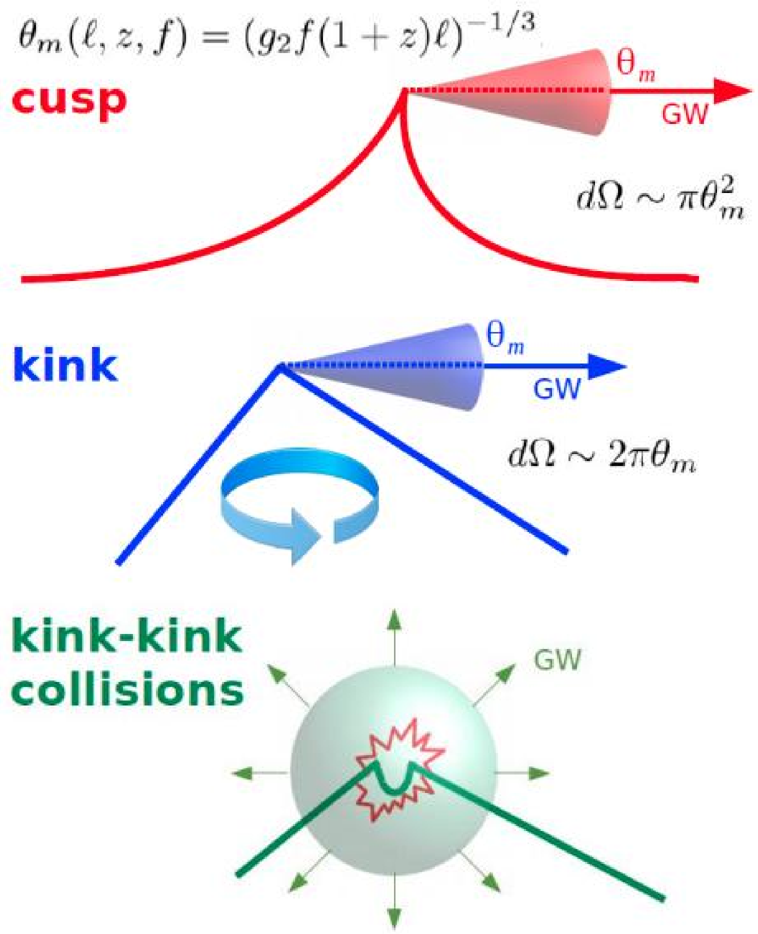


Image credit: Florent Robinet

simple waveforms in frequency domain

$$h_i(\ell, z, f) = A_i(\ell, z) f^{-q_i}$$

$$A_i(\ell, z) = g_{1,i} \frac{G\mu\, \ell^{2-q_i}}{(1+z)^{q_i-1} r(z)}$$

$q = 4/3, 5/3, 2$ for cusp, kink, and kk

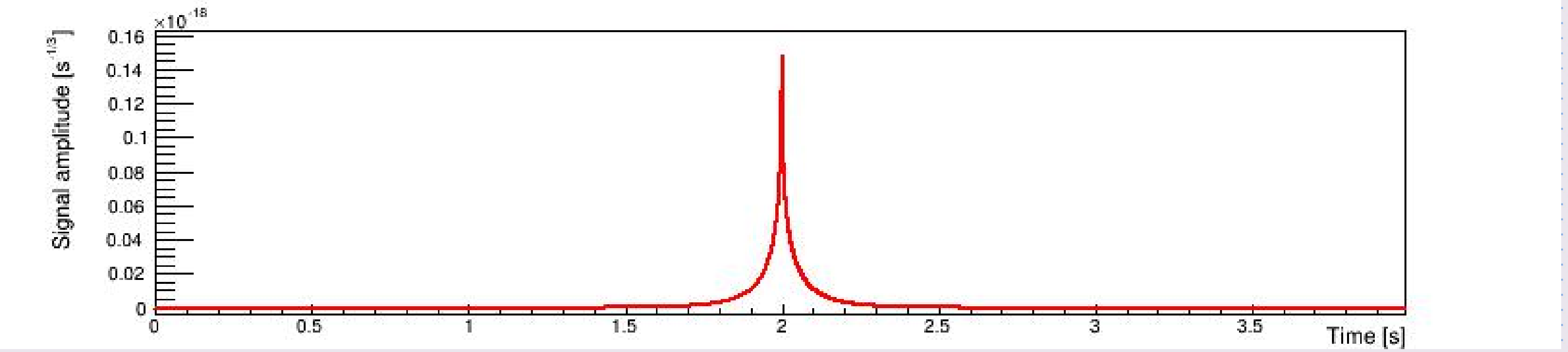
scale

↓

$$G\mu \sim (\eta/M_{\rm Pl})^2$$

particle physics model dependence

example waveform in time domain (cusp)



https://www.ligo.org/science/Publication-S5S6CosmicStrings

Loop Distribution Function

3 models considered

- Model A: (Blanco-Pillado et al., PRD 89,023512) (simulations)
- Model B: Lorentz et al., JCAP 10 (2010) 003 (analytical modelling, matched onto simulation result)
- Model C: Auclair et al., JCAP 06 (2019) 015 (interpolation between above 2)

C-1 (C-2) reproduces LDF of Model A (B) in the radiation era
and LDF of Model B (A) in the matter era

modelling loop distribution (F, dimensionless)

$$\frac{\partial}{\partial t}\left[a^3 F(l,t)\right] = a^3\mathcal{P}(l,t) + a^3\gamma_d \frac{\partial}{\partial t}F(l,t)$$

production from long strings

decay due to GW radiation

Large Nc or Nk does not necessarily lead
to large signal (loops decays faster)

$$\gamma_d = \Gamma_d G\mu$$

$$\Gamma_{\rm d} \equiv \frac{P_{\rm gw}}{G\mu^2} = \sum_i \frac{P_{{\rm gw},i}}{G\mu^2}$$

dimensionless decay constant

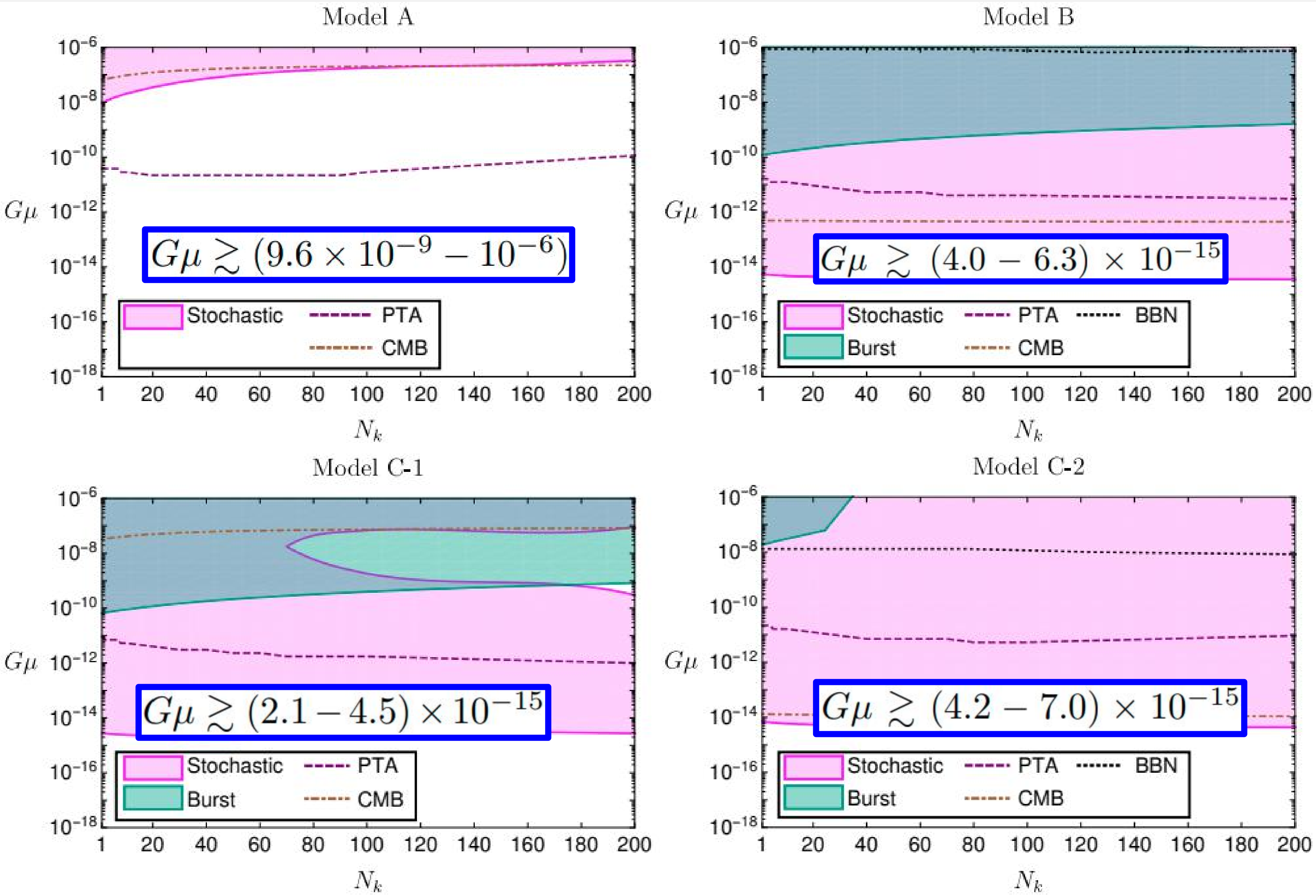
$$= N_{\rm c}\frac{3\pi^2 g_{1,{\rm c}}^2}{(2\delta)^{1/3} g_2^{2/3}} + N_{\rm k}\frac{3\pi^2 g_{1,{\rm k}}^2}{(2\delta)^{2/3} g_2^{1/3}} + \boxed{N_{\rm kk} 2\pi^2 g_{1,{\rm kk}}^2}$$

cusp

kink

kink-kink collision

LIGO Constraints



$$G\mu \gtrsim (9.6 \times 10^{-9} - 10^{-6})$$

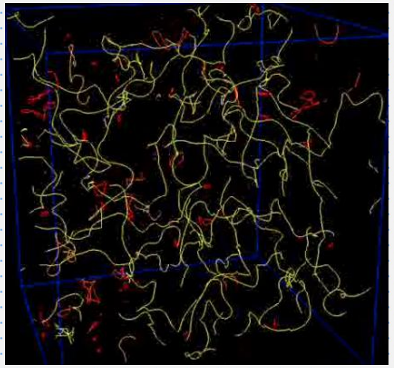
$$G\mu \gtrsim (4.0 - 6.3) \times 10^{-15}$$

$$G\mu \gtrsim (2.1 - 4.5) \times 10^{-15}$$

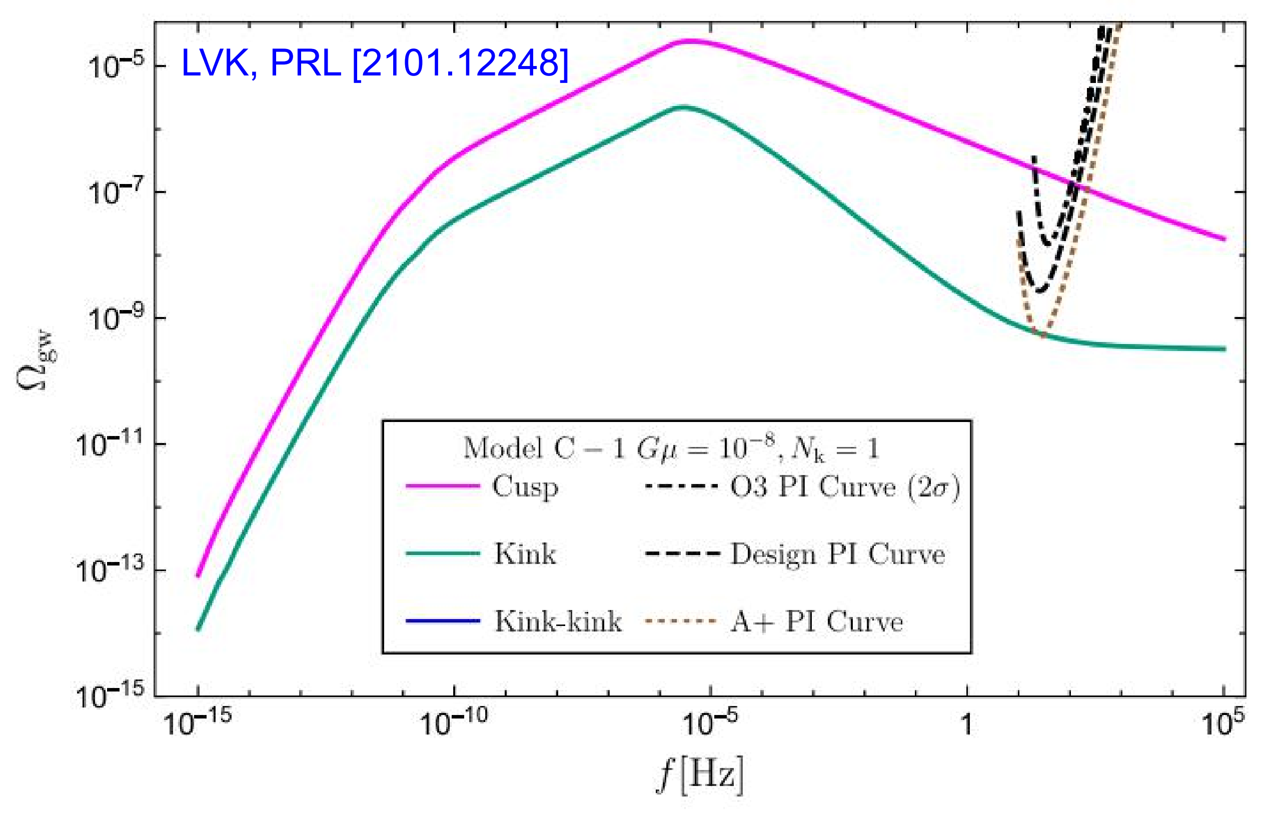
$$G\mu \gtrsim (4.2 - 7.0) \times 10^{-15}$$

LVK, PRL [2101.12248]

Multiband Probes



detectable at multiple frequency bands


LVK, PRL [2101.12248]

LIGO

O1: LIGO-Virgo, PRD [1712.01168]
O2: LIGO-Virgo, PRD [1903.02886]
O3: LVK, PRL [2101.12248]

LISA/Taiji/Tianqin

Auclair et al, JCAP [1909.00819]
Chen,Huang,Liu, et al JCAP [2310.00411]
Wang,Li, PRD [2311.07116]
...

PTA

Zhu, et al (PPTA) MNRAS [2011.13490]
Blasi, Brdar, Schmitz, PRL [2009.06607]
Bian,Shu,Wang,Yuan,Zong (PPTA) PRD [2205.07293]
Chen,Huang (PPTA) ApJ [2205.07194]
NANOGrav, ApjL [2306.16219]
EPTA [2306.16227]
...

Summary

GW is an effective new tool in probing GUT

- Cosmological first order phase transitions
- Topological defects (cosmic strings, etc)
- Multiband probes (LVK, LISA/Taiji/Tianqin, PTA)

Acknowledgement: This material is based upon work supported by NSF's LIGO Laboratory which is a major facility fully funded by the National Science Foundation.

Thanks!