基础物理与数学科学学院
School of Fundamental Physics and Mathematical Sciences

國科大杭州高等研究院
Hangzhou Institute for Advanced Study,UCAS

A phenomenological study of SU(5) with Type-I+III seesaw

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2024-04-10

Based on GXF, Y.L. Zhou, arXiv:2404.xxxxx

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GUT theory

SM: $G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$, neutrinos are massless

↓ Unify three fundamental forces into a single force

GUTs: $SU(5)$ GUTs, $SO(10)$ GUTs

$$G_{\rm GUT} \supset G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

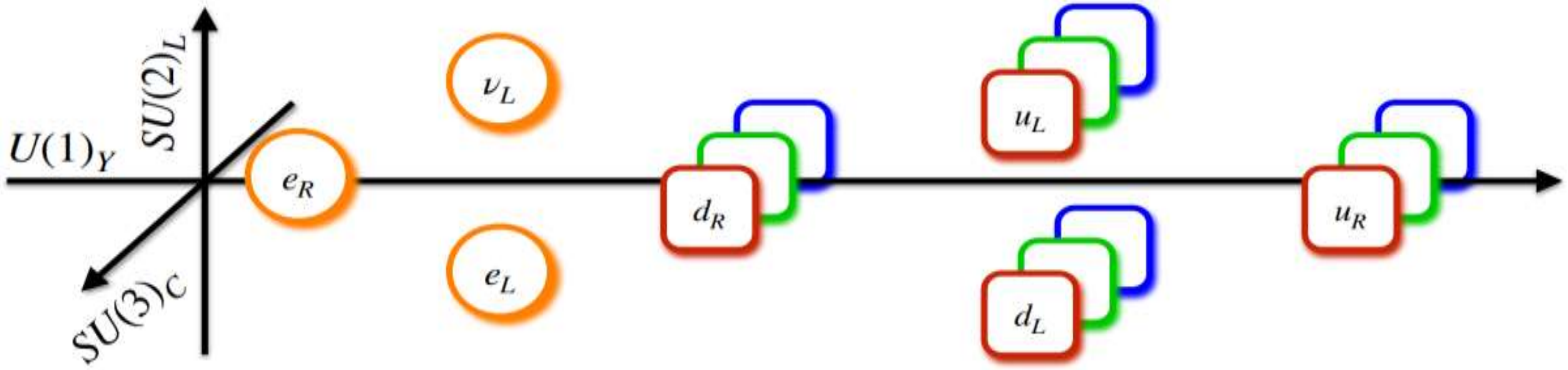
$$g_3 = g_2 = g_1$$

- Unification of symmetries
- Unification of gauge couplings

  Three gauge couplings are unified at scale called $M_{\rm GUT}$

- Unification of matters

Zhou' SUSY2023



Weak hypercharge: Y=1 Y= -$\frac{1}{2}$ Y= $\frac{1}{3}$ Y= $\frac{1}{6}$ Y=-$\frac{2}{3}$

$SU(5)$ GUTs

Bajc, Nemevsek, Senjanovic, 0703080
Perez, Gross, Murgui, 1804.07831
Calibbi, Gao, 2206.10682
Senjanović, Zantedeschi, 2402.19224

Georgi-Glashow model(1974) $SU(5)$

- Gauge couplings do not unify
- Neutrinos are massless and predicts wrong mass relations $m_d = m_e, m_s = m_\mu, m_t = m_\tau$

$$\overline{\mathbf{5}}_F = \begin{pmatrix} d^c_{rR} \\ d^c_{gR} \\ d^c_{bR} \\ e_L \\ -\nu_L \end{pmatrix}, \quad \mathbf{10}_F = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^c_{bR} & -u^c_{gR} & -u^r_L & -d^r_L \\ -u^c_{bR} & 0 & u^c_{rR} & -u^g_L & -d^g_L \\ u^c_{gR} & -u^c_{rR} & 0 & -u^b_L & -d^b_L \\ u^r_L & u^g_L & u^b_L & 0 & -e^c_R \\ d^r_L & d^g_L & d^b_L & e^c_R & 0 \end{pmatrix}$$

Minimally extended $SU(5)$

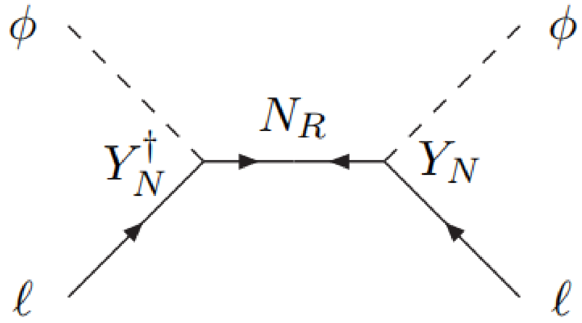
- Generating neutrino mass： type-I+III seesaw 24_F; type-II seesaw15_H
- Achieving gauge unification and predicting correct mass of quarks and leptons：

dimension=5 operators; 45_H; vector-like fermions($5_F + \bar{5}_F, 10_F + \overline{10}_F, 15_F + \overline{15}_F$)

Seesaw mechanism

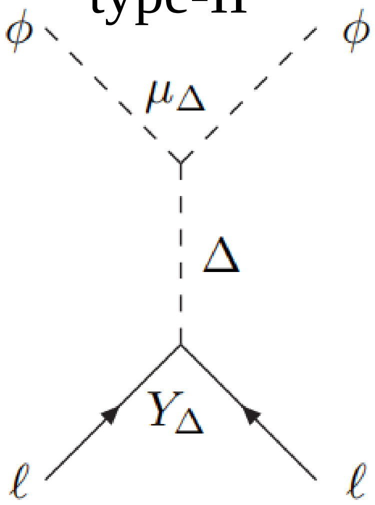
Weinberg operator: $\frac{\mathcal{L}_{d=5}}{\Lambda} = c_5 \overline{l_L} \tilde{\phi} \tilde{\phi}^T (l_L)^c + h.c.$

type-I



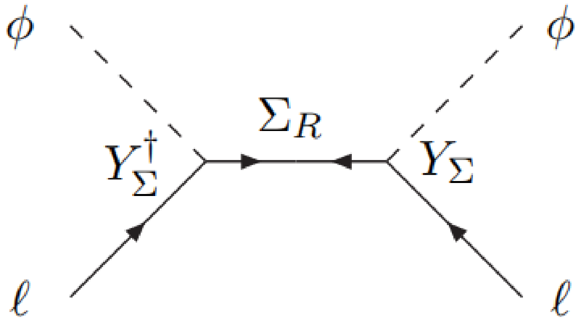
SM singlet fermions

type-II



SM triplet scalars

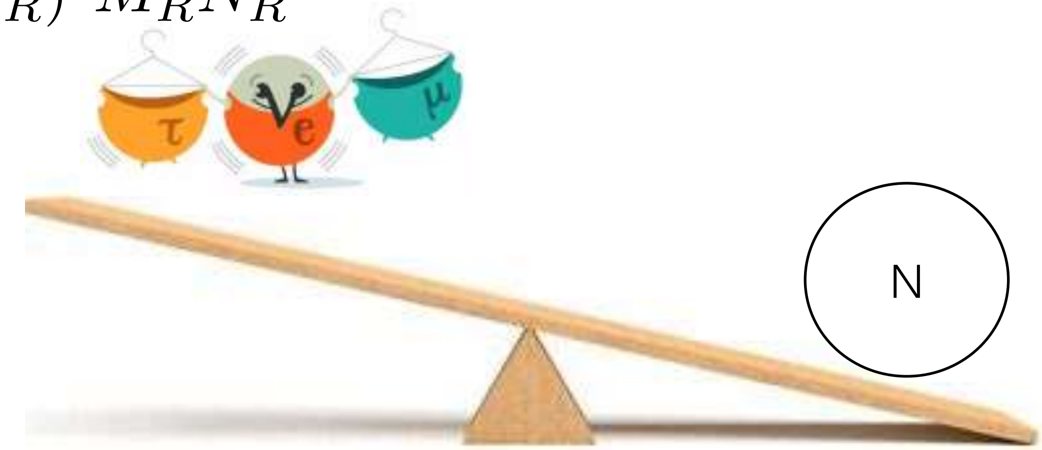
type-III



SM triplet fermions

Majorana mass term: $-\mathcal{L} = \overline{\nu_L} M_D N_R + \frac{1}{2} \overline{(N_R)^c} M_R N_R$

Majorana neutrino: $M_\nu = -\frac{M_D M_D^T}{M_R}$



Proton decay

Theory:

Non-SUSY dimension=6 operators:

$$\frac{1}{\Lambda_1^2}[\underbrace{(\overline{u_R^c}\gamma^\mu Q)(\overline{d_R^c}\gamma_\mu L)+(\overline{u_R^c}\gamma^\mu Q)(\overline{e_R^c}\gamma_\mu Q)}_{\text{non-SUSY SU(5)}}]+\frac{1}{\Lambda_2^2}[(\overline{d_R^c}\gamma^\mu Q)(\overline{u_R^c}\gamma_\mu L)+(\overline{d_R^c}\gamma^\mu Q)(\overline{\nu_R^c}\gamma_\mu Q)]$$

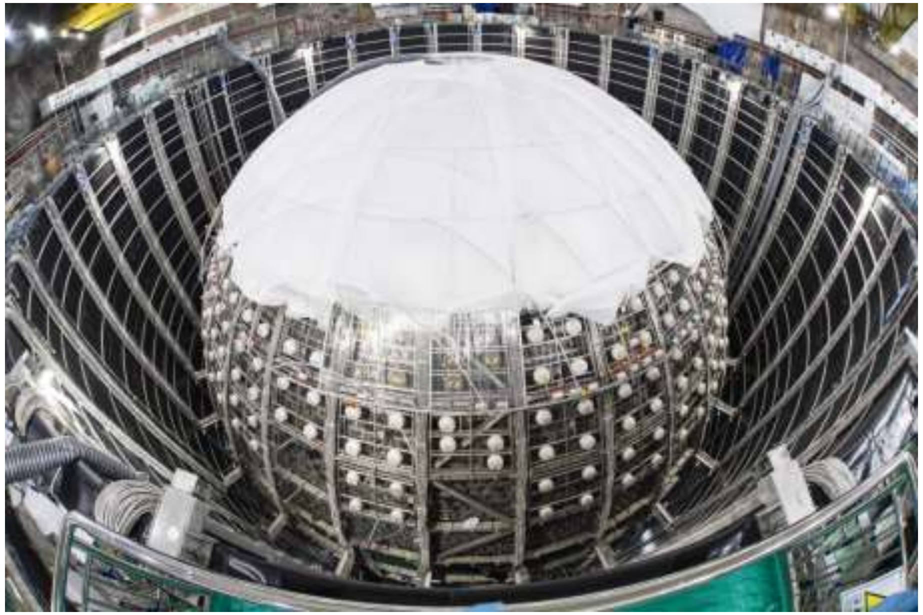
Experiments:

Current Super-K:

$\tau(p\to\pi^0e^+)>2.4\times10^{34}\text{yr}$,

$\tau(p\to K^+\bar\nu)>6.6\times10^{33}\,\text{yr}$

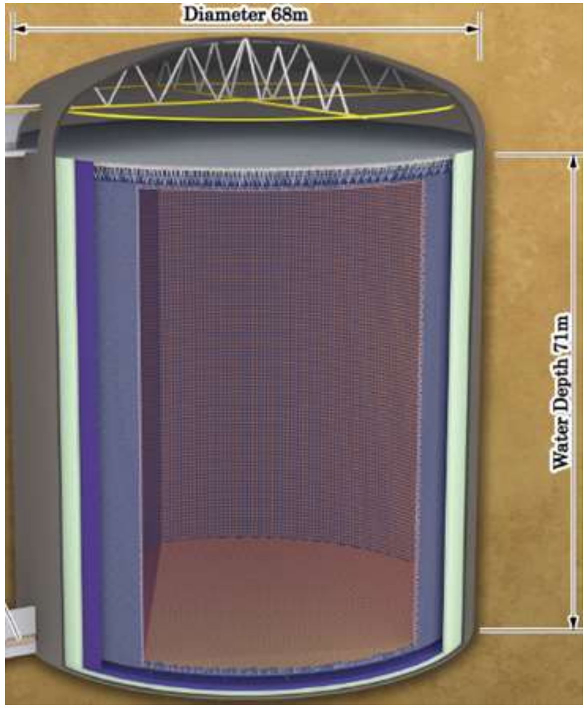
SK, 2010.16098



Future JUNO:

$\tau(p\to K^+\bar\nu)>9.6\times10^{33}\,\text{yr}$

JUNO, 2212.08502



Future HK:

$\tau(p\to K^+\bar\nu)>3.2\times10^{34}\,\text{yr}$

HK, 1805.04163

Field contents

An economical model:

Fields	$SU(5)$	$\supset SU(3)_c \times SU(2)_L \times U(1)_Y$
Fermion	$\mathbf{10}_F$	$\to (\mathbf{3},\mathbf{2},+\frac{1}{6})_{q_L} + (\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{u_R^c} + (\mathbf{1},\mathbf{1},+1)_{e_R^c}$
	$\bar{\mathbf{5}}_F$	$\to (\bar{\mathbf{3}},\mathbf{1},+\frac{1}{3})_{d_R^c} + (\mathbf{1},\mathbf{2},-\frac{1}{2})_{\ell_L}$
	$\mathbf{24}_F$	$\to (\mathbf{1},\mathbf{1},0)_N + (\mathbf{1},\mathbf{3},0)_\Sigma + (\mathbf{3},\mathbf{2},-\frac{5}{6})_{Q_L} + (\bar{\mathbf{3}},\mathbf{2},\frac{5}{6})_{Q_R^c} + (\mathbf{8},\mathbf{1},0)_{Q_8}$
Higgs	$\mathbf{5}_H$	$\to (\mathbf{1},\mathbf{2},\frac{1}{2})_{h_1}$
	$\mathbf{45}_H$	$\to (\mathbf{1},\mathbf{2},\frac{1}{2})_{h_2}$
	$\mathbf{24}_\Phi$	$\to (\mathbf{1},\mathbf{1},0)_{\phi_1}$
	$\mathbf{75}_\Phi$	$\to (\mathbf{1},\mathbf{1},0)_{\phi_2}$

$$-\mathcal{L} \supset \bar{\mathbf{5}}_F(Y_1\mathbf{5}_H^\dagger + Y_2\mathbf{45}_H^\dagger)\mathbf{10}_F + \mathbf{10}_F(Y_3\mathbf{5}_H + Y_4\mathbf{45}_H)\mathbf{10}_F$$
$$+\bar{\mathbf{5}}_F(Y_5\mathbf{5}_H + Y_6\mathbf{45}_H)\mathbf{24}_F + \mathbf{24}_F(M_1 + \kappa_1\mathbf{24}_\Phi + \kappa_2\mathbf{75}_\Phi)\mathbf{24}_F + \text{h.c.}$$

$$M_N = M_1 - M_{24} + 5M_{75}\,,$$
$$M_\Sigma = M_1 - 3M_{24} - 3M_{75}\,,$$
$$M_Q = M_1 - \frac{1}{2}M_{24} + M_{75}\,,$$
$$M_{Q_8} = M_1 + 2M_{24} - M_{75}\,.$$

New particles

$$M_e = aY_1^* - 3bY_2^*\,,$$
$$M_d = aY_1^\dagger + bY_2^\dagger\,,$$
$$M_u = cY_3^* + dY_4^*\,,$$
$$M_\mathrm{I} = \sqrt{3}fY_5^* + \sqrt{5}gY_6^*\,,$$
$$M_\mathrm{III} = \sqrt{5}fY_5^* - \sqrt{3}gY_6^*\,,$$

SM particles

type I+III seesaw

$$M_\nu = M_\mathrm{I} M_N^{-1} M_\mathrm{I}^T + M_\mathrm{III} M_\Sigma^{-1} M_\mathrm{III}^T$$

Renormalization group equation (RGE)

RGE at one-loop level: $\frac{d\alpha_i(t)}{dt} = \frac{a_i}{2\pi}\alpha_i^2$

↓ analytical solution

$$\alpha_i^{-1}(t_{\mathrm{GUT}}) = \alpha_i^{-1}(0) - \frac{a_i^{\mathrm{SM}}}{2\pi}t_{\mathrm{GUT}} - \sum_I \frac{a_i^I}{2\pi}(t_{\mathrm{GUT}} - t_I)$$

Fields	$SU(5)$	$\supset SU(3)_c \times SU(2)_L \times U(1)_Y$
Fermion	$\mathbf{10}_F$	$= (\mathbf{3}, \mathbf{2}, +\frac{1}{6})_{q_L} + (\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_{u_R^c} + (\mathbf{1}, \mathbf{1}, +1)_{e_R^c}$
	$\overline{\mathbf{5}}_F$	$= (\overline{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_{d_R^c} + (\mathbf{1}, \mathbf{2}, -\frac{1}{2})_{\ell_L}$
	$\mathbf{24}_F$	$= (\mathbf{1}, \mathbf{1}, 0)_N + (\mathbf{1}, \mathbf{3}, 0)_\Sigma + (\mathbf{3}, \mathbf{2}, -\frac{5}{6})_{Q_L} + (\overline{\mathbf{3}}, \mathbf{2}, \frac{5}{6})_{Q_R^c} + (\mathbf{8}, \mathbf{1}, 0)_{Q_8}$

Only considering fermion's contribution in RGE

Beta cofficients at one-loop level:

$$\{a_i^{\mathrm{SM}}\} = \begin{pmatrix} -7 \\ -\frac{19}{6} \\ \frac{41}{10} \end{pmatrix}$$ SM contribution

$$\{a_i^{\Sigma}\} = \begin{pmatrix} 0 \\ \frac{4}{3} \\ 0 \end{pmatrix} \quad \{a_i^{Q_8}\} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$$\{a_i^{Q}\} = \begin{pmatrix} \frac{4}{3} \\ 2 \\ \frac{10}{3} \end{pmatrix}$$

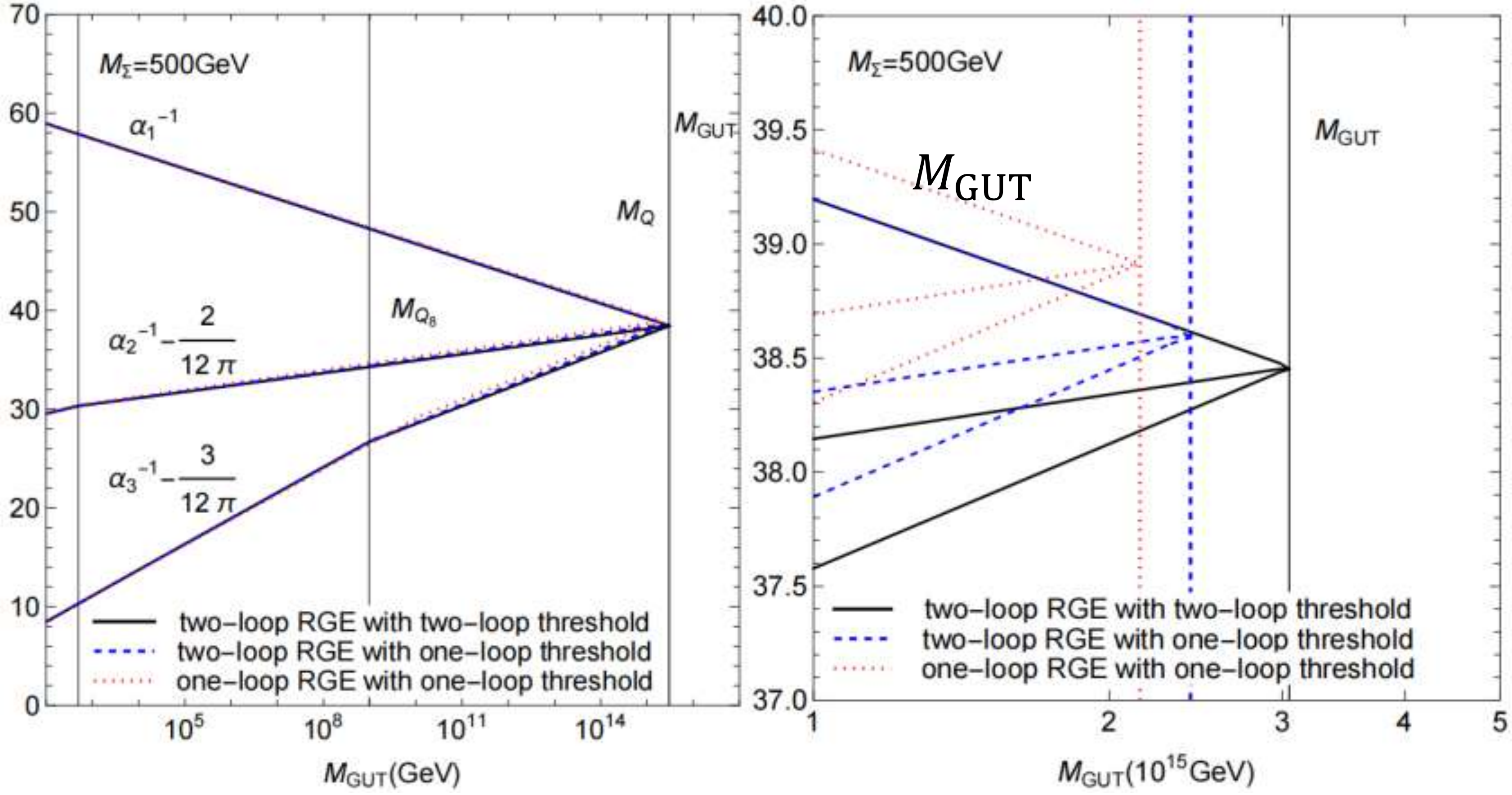
New particles contribution

Gauge coupling unification

We only focus on the situation where $\mathbf{24}_F$ has only one copy

The precise measurement of experiments requires us to calculate RGE at two-loop level

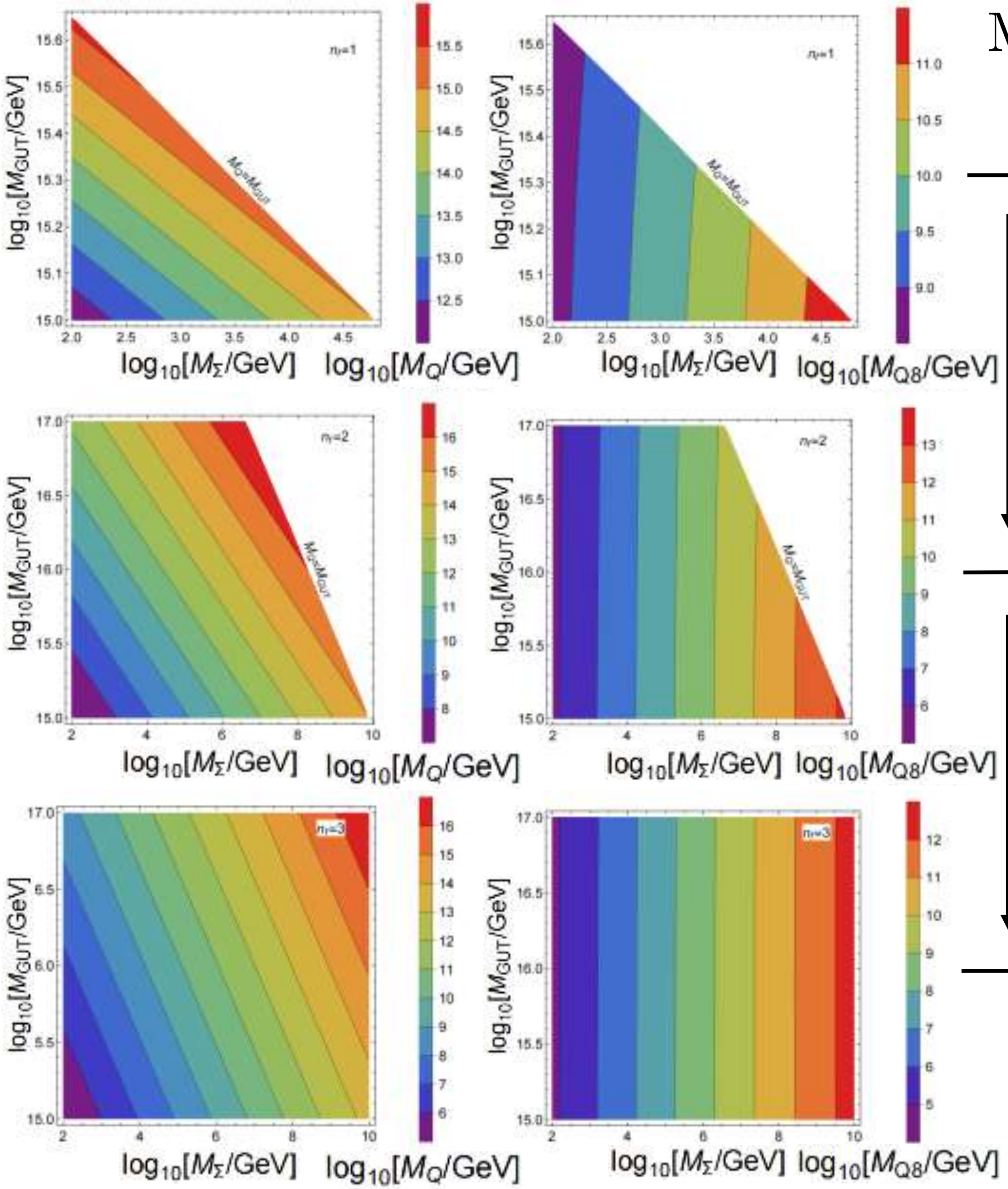
Two-loop threshold effect increase $M_{\rm GUT}$ $\quad M_{\rm GUT} = 3.05 \times 10^{15}$ GeV



one-loop matching condition:

$$\alpha_3^{-1}(t_{\rm GUT}) - \frac{3}{12\pi} = \alpha_2^{-1}(t_{\rm GUT}) - \frac{2}{12\pi} = \alpha_1^{-1}(t_{\rm GUT})$$

Scan Parameter Space



Mass hierarchy: $M_\Sigma < M_{Q_8} < M_Q \leq M_{\mathrm{GUT}}$

→ $\mathbf{24}_F$ has one copy

↓ Parameter space increases

→ $\mathbf{24}_F$ has two copy

↓ Parameter space increases

→ $\mathbf{24}_F$ has three copy

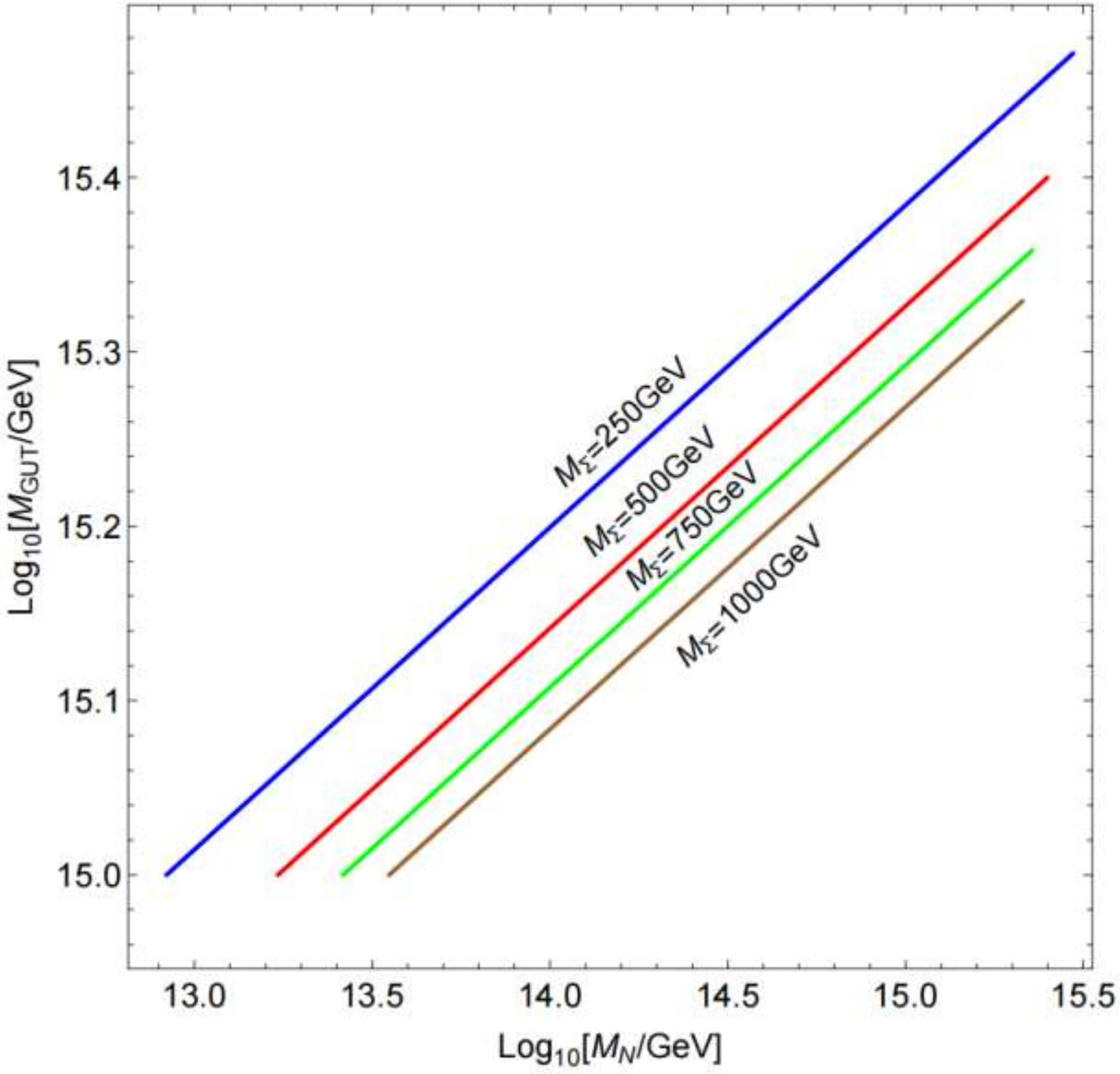
$\mathbf{24}_F$ has one copy,
if $M_{\mathrm{GUT}} \geq 10^{15}$ GeV,
$M_\Sigma \in (100\mathrm{GeV}, 63\mathrm{TeV})$

Beta cofficients at two-loop level:

$$\{b_{ij}^{\mathrm{SM}}\} = \begin{pmatrix} -26 & \frac{9}{2} & \frac{11}{10} \\ 12 & \frac{35}{6} & \frac{9}{10} \\ \frac{44}{5} & \frac{27}{10} & \frac{199}{50} \end{pmatrix} \{b_{ij}^{Q_8}\} = \begin{pmatrix} 48 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\{b_{ij}^{Q}\} = \begin{pmatrix} \frac{76}{3} & 3 & \frac{5}{3} \\ 8 & \frac{49}{2} & \frac{5}{2} \\ \frac{40}{3} & \frac{15}{2} & \frac{25}{6} \end{pmatrix} \{b_{ij}^{\Sigma}\} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{64}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Correlation between M_N and $M_{\rm GUT}$



$$M_N = M_1 - M_{24} + 5M_{75}\,,$$
$$M_\Sigma = M_1 - 3M_{24} - 3M_{75}\,,$$
$$M_Q = M_1 - \frac{1}{2}M_{24} + M_{75}\,,$$
$$M_{Q_8} = M_1 + 2M_{24} - M_{75}\,.$$

↓

$$M_N = \frac{12}{5}M_Q - \frac{4}{5}M_{Q_8} - \frac{3}{5}M_\Sigma$$

↓

M_Σ ↑ M_N ↓ $M_{\rm GUT}$ ↓

When $M_\Sigma = 500$ GeV, the maximal value of $M_{\rm GUT}$ is 2.51×10^{15} GeV, which is allowed by gauge unification

Flavor mixing

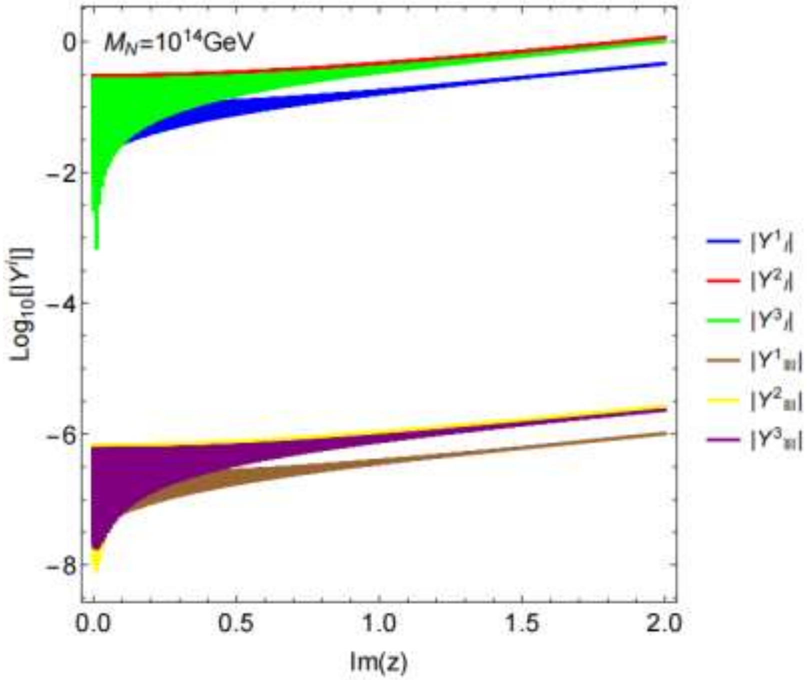
Casas-Ibarra parametrization: Casas, Ibarra, 0103065

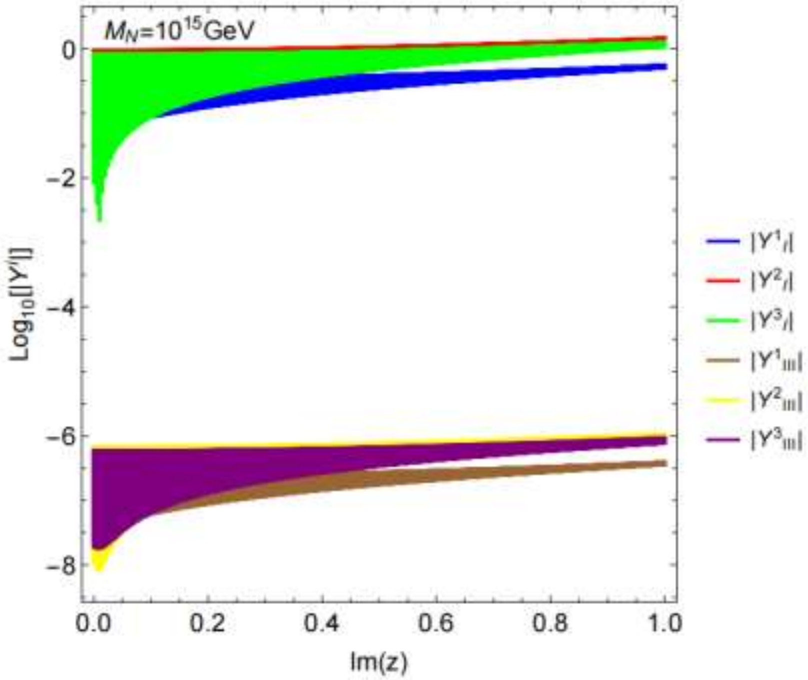
$$Y_{\rm I}^{\alpha} = \frac{\sqrt{2M_N}}{v}(\sqrt{m_2}\,\cos z\, U_{\alpha 2}^* + \sqrt{m_3}\,\sin z\, U_{\alpha 3}^*)$$

$$Y_{\rm III}^{\alpha} = \frac{\sqrt{2M_\Sigma}}{v}(-\sqrt{m_2}\,\sin z\, U_{\alpha 2}^* + \sqrt{m_3}\,\cos z\, U_{\alpha 3}^*)$$

The smallness of three left-handed neutrinos is attributed to the smallness of $|Y_{\rm III}|$ and the largeness of M_N

$$M_\nu = \frac{v^2}{2M_N}Y_{\rm I}Y_{\rm I}^T + \frac{v^2}{2M_\Sigma}Y_{\rm III}Y_{\rm III}^T$$





$$M_N \gg M_\Sigma$$

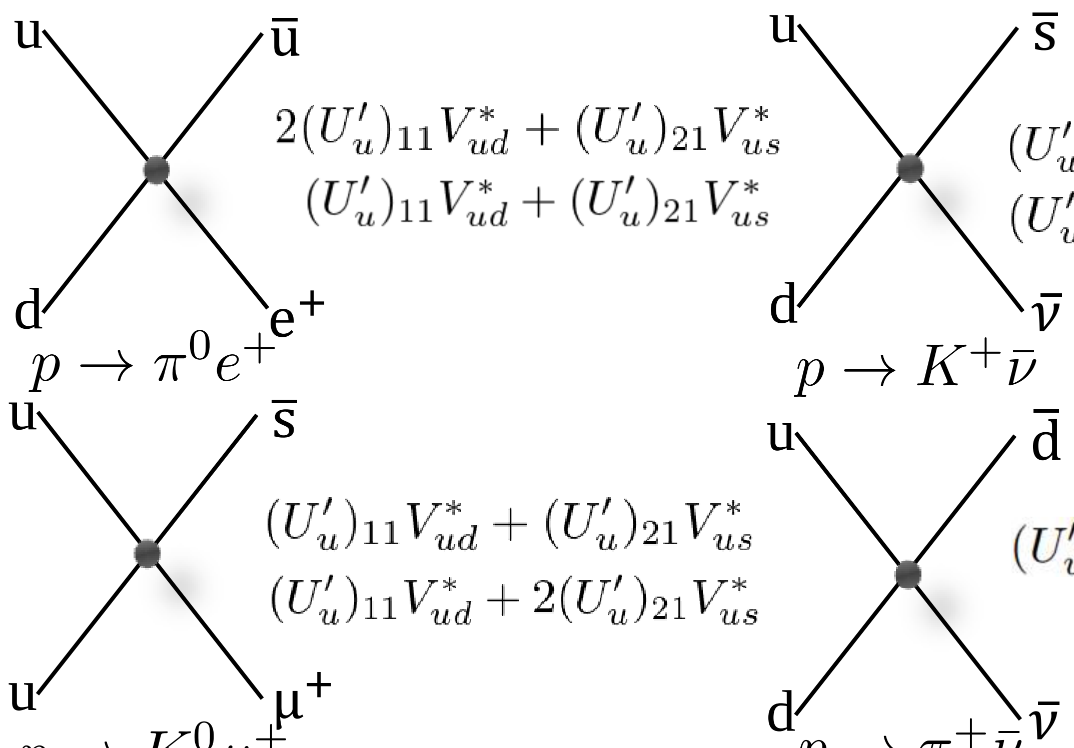
$$|Y_{\rm I}| \gg |Y_{\rm III}|$$

$\mathrm{Im}(z)$ actually affects $|Y|$

Proton decay

$$\underbrace{|V_{ub}| < |V_{td}|}_{\lambda^3} \ll \underbrace{|V_{ts}| < |V_{cb}|}_{\lambda^2} \ll \underbrace{|V_{cd}| < |V_{us}|}_{\lambda} \ll \underbrace{|V_{cs}| < |V_{ud}| < |V_{tb}|}_{\mathcal{O}(1)}$$

Scenario 1: $Y_d = \hat{Y}_d$



$p \to \pi^0 e^+$: $2(U'_u)_{11}V^*_{ud} + (U'_u)_{21}V^*_{us}$, $(U'_u)_{11}V^*_{ud} + (U'_u)_{21}V^*_{us}$

$p \to K^+ \bar{\nu}$: $(U'_u)_{11}$, $(U'_u)_{21}$

$p \to K^0 \mu^+$: $(U'_u)_{11}V^*_{ud} + (U'_u)_{21}V^*_{us}$, $(U'_u)_{11}V^*_{ud} + 2(U'_u)_{21}V^*_{us}$

$p \to \pi^+ \bar{\nu}$: $(U'_u)_{11}$

Unitarity: $|(U'_u)_{11}|^2 + |(U'_u)_{21}|^2 \leq 1$

numerical solution (V_{ud}, V_{us} are taken from PDG)

$$\tau(p \to \pi^0 e^+) = \log_{10}\left(\frac{6.88 \times 10^{32}}{(U'_u)^2_{11} + 0.27(U'_u)_{11}(U'_u)_{21} + 0.02(U'_u)^2_{21}}\right)$$

$$\tau(p \to K^+ \bar{\nu}) = \log_{10}\left(\frac{4.62 \times 10^{34}}{(U'_u)^2_{11} + 7.46(U'_u)^2_{21}}\right)$$

$$\tau(p \to K^0 \mu^+) = \log_{10}\left(\frac{5.59 \times 10^{33}}{(U'_u)^2_{11} + 0.69(U'_u)_{11}(U'_u)_{21} + 0.13(U'_u)^2_{21}}\right)$$

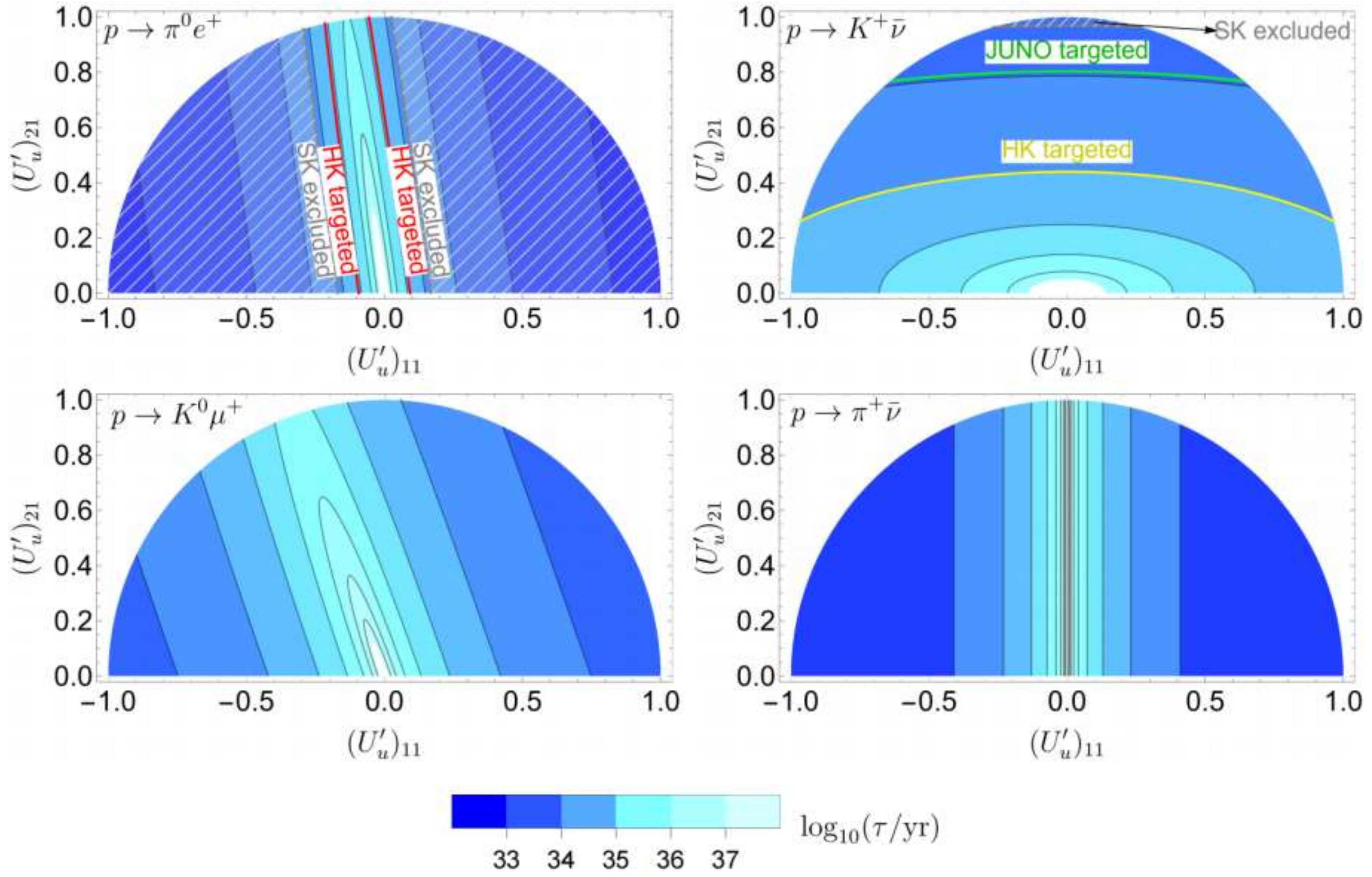
$$\tau(p \to \pi^+ \bar{\nu}) = \log_{10}\left(\frac{1.68 \times 10^{33}}{(U'_u)^2_{11}}\right)$$

Only two free parameters $(U'_u)_{11}, (U'_u)_{21}$

$(U'_u)_{11} \to 0, (U'_u)_{21} \to 0, \tau \to +\infty$

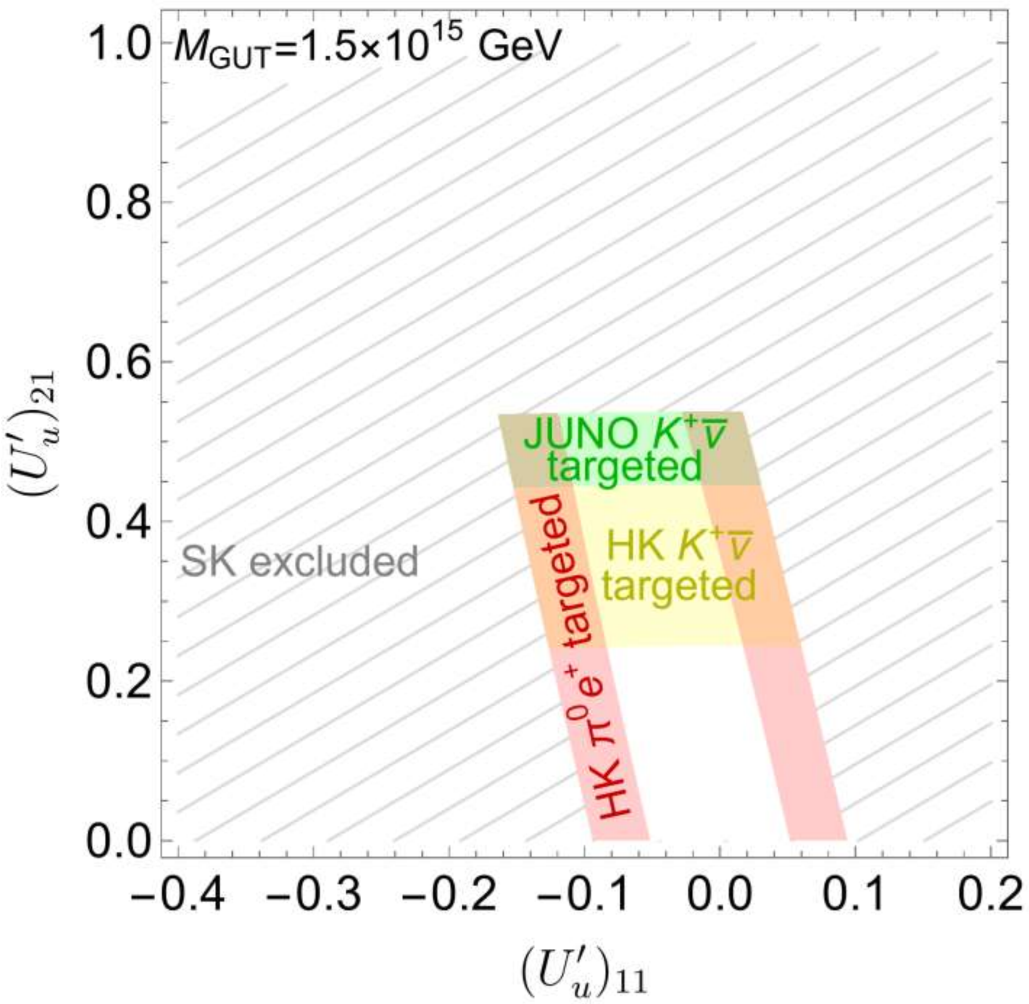
Proton decay

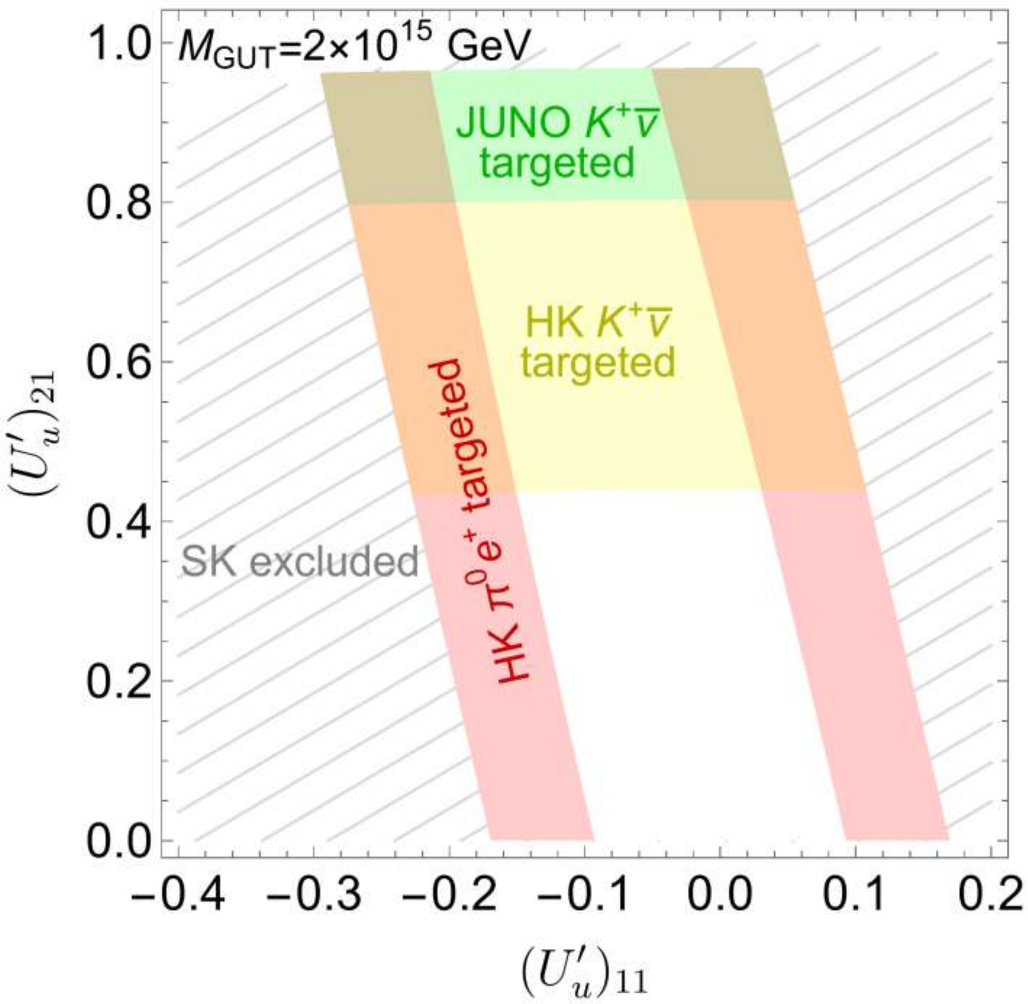
$M_\Sigma = 500$ GeV, $M_{\mathrm{GUT}} = 2\times 10^{15}$ GeV are determined



Proton decay

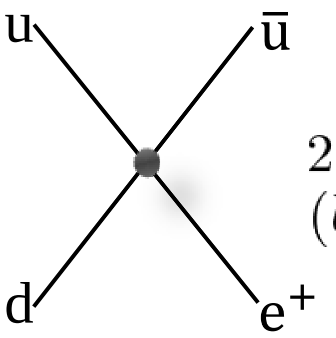
M_{GUT} ⬆ Proton lifetime ⬆ Parameter Space ⬆





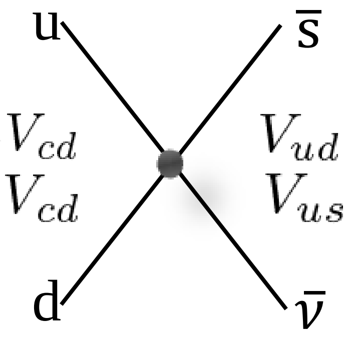
Proton decay

Scenario 2: $Y_d^\dagger = Y_d$ and $Y_u^\dagger = Y_u$



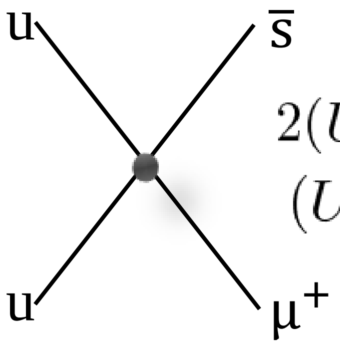
$2(U_u)_{11}V_{ud} + (U_u)_{12}V_{cd}$
$(U_u)_{11}V_{ud} + (U_u)_{12}V_{cd}$

$p \to \pi^0 e^+$



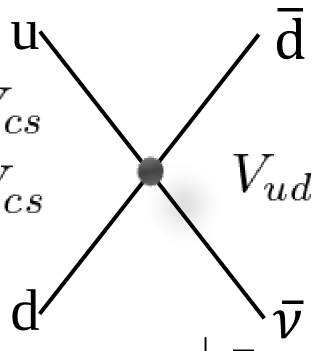
V_{ud}
V_{us}

$p \to K^+ \bar{\nu}$



$2(U_u)_{11}V_{us} + (U_u)_{12}V_{cs}$
$(U_u)_{11}V_{us} + (U_u)_{12}V_{cs}$

$p \to K^0 e^+$



V_{ud}

$p \to \pi^+ \bar{\nu}$

numerical solution:

$$\tau(p \to \pi^0 e^+) = \log_{10}\left(\frac{6.88 \times 10^{32}}{(U_u)_{11}^2 + 0.27(U_u)_{11}(U_u)_{12} + 0.02(U_u)_{12}^2}\right)$$

$$\tau(p \to K^0 e^+) = \log_{10}\left(\frac{4.22 \times 10^{34}}{(U_u)_{11}^2 + 5.22(U_u)_{11}(U_u)_{12}| + 7.56(U_u)_{12}^2}\right)$$

$\tau(p \to K^+ \bar{\nu}) = 3.5 \times 10^{34}$ years > HK bound

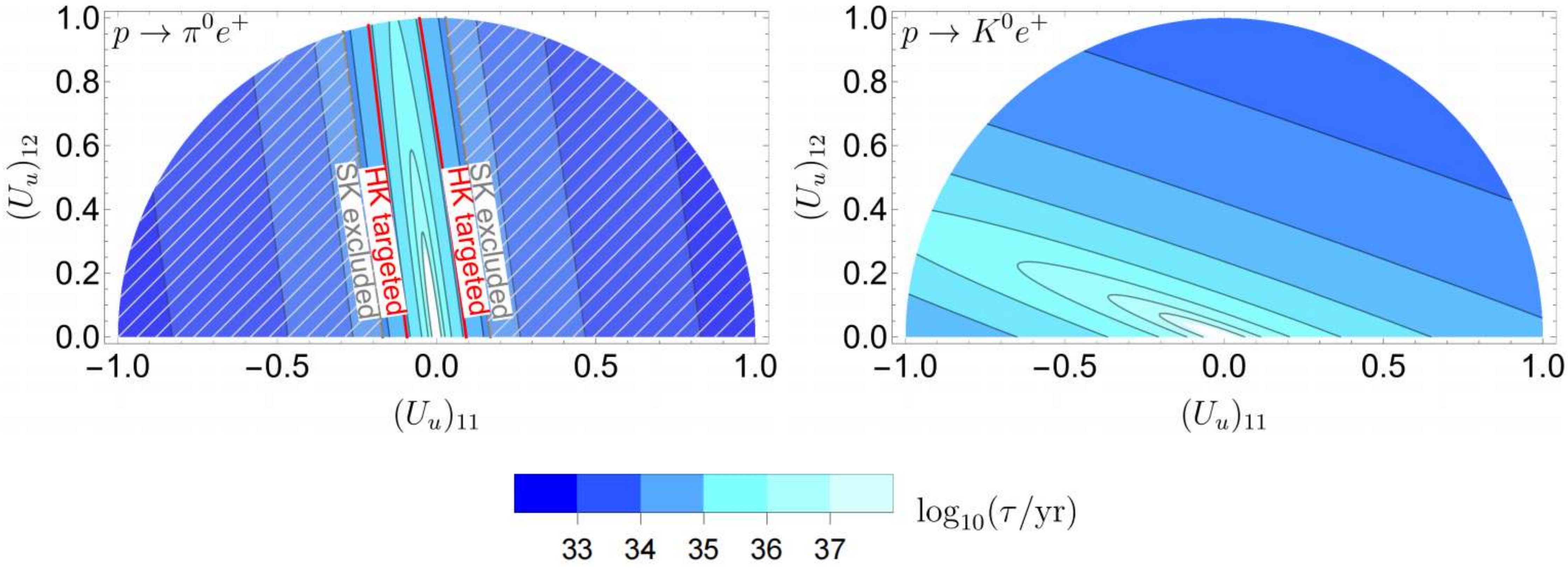
$\tau(p \to \pi^+ \bar{\nu}) = 1.8 \times 10^{33}$ years > SK bound

Only two free parameters: $(U_u)_{11}, (U_u)_{12}$

$(U_u)_{11} \to 0, (U_u)_{12} \to 0, \tau \to +\infty$

Proton decay

Two free parameters: $(U_u)_{11}, (U_u)_{12}$



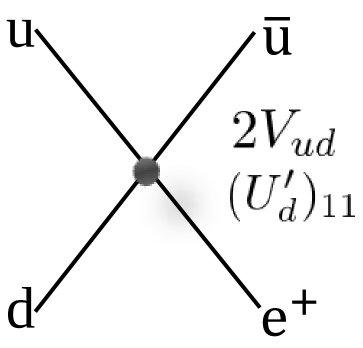
Similar to S1, there is hope to test this assumption in future proton decay experiments

Proton decay

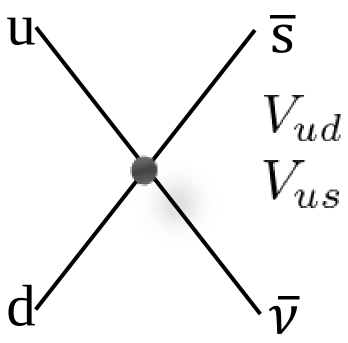
Scenario 3: $Y_u = \hat{Y}_u$

$\tau(p \to \pi^0 e^+)_{\max} = 8.5 \times 10^{32}\,\text{years} < 2.4 \times 10^{34}\,\text{years(SK bound)}$

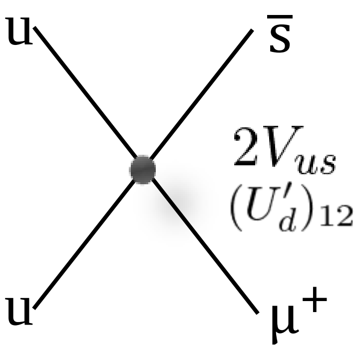
Scenario 3 is excluded



$p \to \pi^0 e^+$



$p \to K^+ \bar{\nu}$



$p \to K^0 e^+$

numerical solution:

$$\tau(p \to \pi^0 e^+) = \log_{10}\left(\frac{3.3865 \times 10^{33}}{(U'_d)^2_{11} + 3.9737}\right),$$

$$\tau(p \to K^0 e^+) = \log_{10}\left(\frac{1.0607 \times 10^{34}}{(U'_d)^2_{12} + 0.2012}\right),$$

$$\tau(p \to K^+ \bar{\nu}) = 3.49 \times 10^{34}\,\text{years} > 3.2 \times 10^{34}\,\text{years(HK targeted)}$$

Conclusion

1. New particles in $\mathbf{24}_F$ satisfy the following mass hierarchy $M_\Sigma < M_{Q_8} < M_Q$.
2. M_Σ should be lighter than $10^{4.8}$GeV in ordeer to make $M_{\rm GUT} > 10^{15}$GeV when $\mathbf{24}_F$ has only one copy.
3. There is hope to test this economical model in future proton decay experiments.
4. Future neutrino experiments can provide multiple tests of GUTs via multiple proton decay channels.

Thanks！