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BESIII上粲介子纯轻衰变研究

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未来展望	{	<ul style="list-style-type: none">• 总结与展望

物理动机

- 在SM中，CKM矩阵元描述W玻色子发生弱相互作用的耦合强度，是待定的参数，只能由实验来测量

粒子物理标准模型

三代物质粒子 (费米子)

	I	II	III		
质量	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 125.09 \text{ GeV}/c^2$
电荷	2/3	2/3	2/3	0	0
自旋	1/2	1/2	1/2	1	0
	u 上	c 粲	t 顶	g 胶子	H 希格斯玻色子
	$\approx 4.7 \text{ MeV}/c^2$	$\approx 96 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	d 下	s 奇	b 底	γ 光子	
	$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 91.19 \text{ GeV}/c^2$	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e 电子	μ μ子	τ τ子	Z Z玻色子	
	$< 2.2 \text{ eV}/c^2$	$< 1.7 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$	
	0	0	0	±1	
	1/2	1/2	1/2	1	
	ν_e 电中微子	ν_μ μ中微子	ν_τ τ中微子	W W玻色子	

夸克 (Quarks)

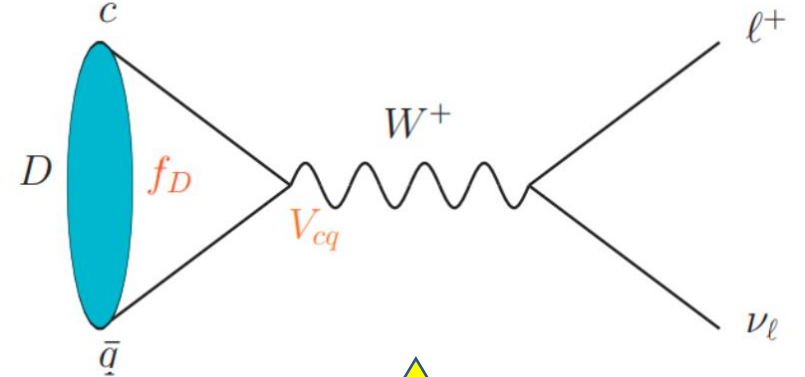
轻子 (Leptons)

标量玻色子 (Scalar Bosons)

规范玻色子 (Gauge Bosons)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

么正性: $V^\dagger V = 1$



- 研究夸克强相互作用力和弱相互作用力的理想衰变道

$$\Gamma(D_{(s)}^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2}{8\pi} |V_{cd(s)}|^2 f_{D_{(s)}^+}^2 m_\ell^2 m_{D_{(s)}^+}^2 \left(1 - \frac{m_\ell^2}{m_{D_{(s)}^+}^2}\right)^2$$

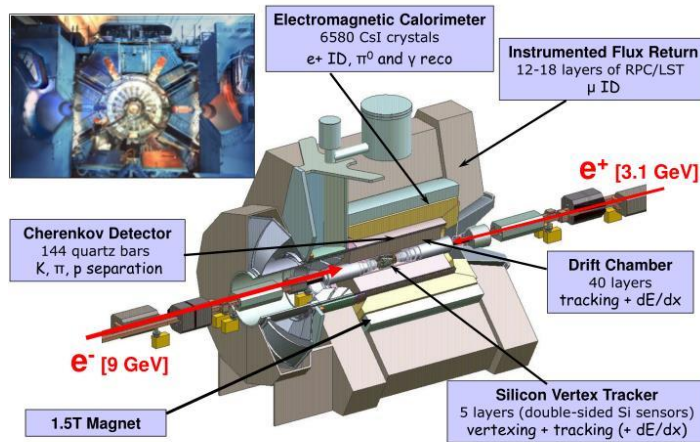
- 测量 f_D 检验LQCD; 精确测量 $|V_{cs}|$ 和 $|V_{cd}|$; 还可检验轻子普适性, 寻找新物理迹象

物理动机

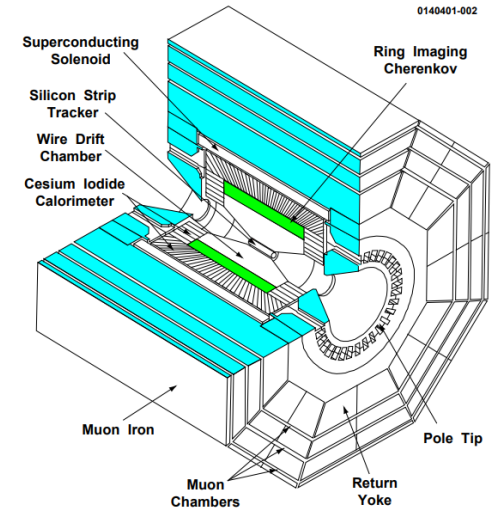
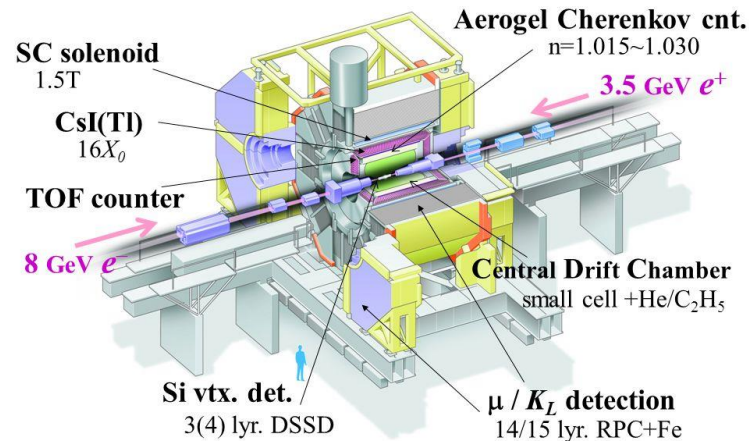
- 在BESIII之前, CLEO-c, BaBar以及Belle, 测量了 $|V_{cs}|$ 和 $|V_{cd}|$ 。

Collaboration	V_{cs}	V_{cd}
BaBar	4.3%	-
Belle	3.3%	-
CLEO-c	4.3%	4.4%

The BaBar Detector



The Belle detector



Data samples:

- $\triangleright 0.52\text{ ab}^{-1}@10.6\text{ GeV } (\gamma (4S))$
- $\triangleright \sigma(e^+e^- \rightarrow cc) = 1.3\text{ nb}$
- $\triangleright L_{\text{peak}} = 1 \times 34\text{ cm}^{-2}\text{s}^{-1}$

Data samples:

- $\triangleright 0.98\text{ ab}^{-1}@10.6\text{ GeV } (\gamma (4S))$
- $\triangleright \sigma(e^+e^- \rightarrow cc) = 1.3\text{ nb}$
- $\triangleright L_{\text{peak}} = 2 \times 34\text{ cm}^{-2}\text{s}^{-1}$

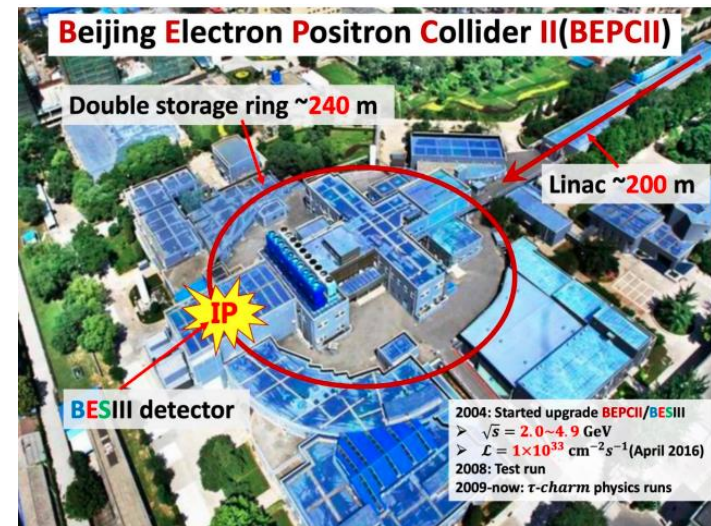
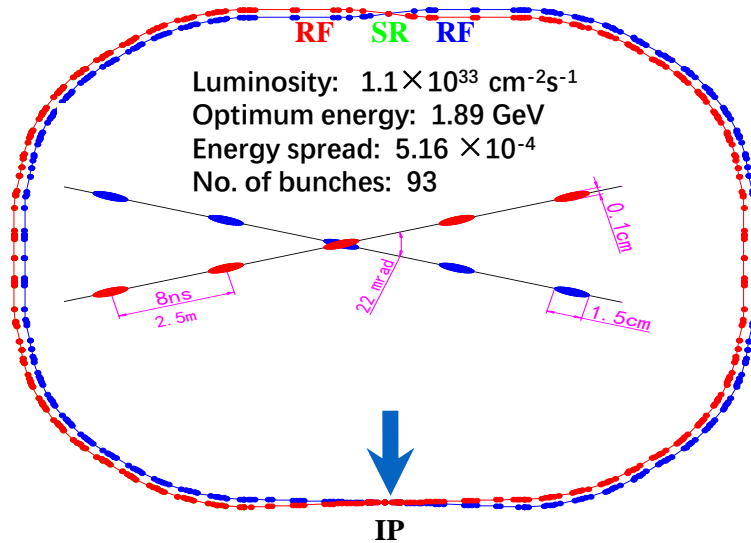
Data samples:

- $\triangleright 0.8\text{ fb}^{-1}@3.774\text{ GeV}$
- $\triangleright 0.6\text{ fb}^{-1}@4.170\text{ GeV}$

BEPC-II 和 BES-III

北京正负电子对撞机-II:

- 高亮度双储存环对撞机
- 质心系能量:
- 1.8 – 4.95 GeV



北京谱仪-III:

- BES-III是大型通用探测器，其原理主要采用现代粒子探测技术。
- 用于分析和记录质心能量对撞产生的末态粒子的信息。

MUC

超导磁铁(1 Tesla)
 电磁量能器(EMC):

$$\frac{\sigma_E}{E} = 2.5\% \text{ @ } 1 \text{ GeV (桶部)}$$

$$\frac{\sigma_E}{E} = 5\% \text{ @ } 1 \text{ GeV (端盖)}$$

飞行时间计数器(TOF):

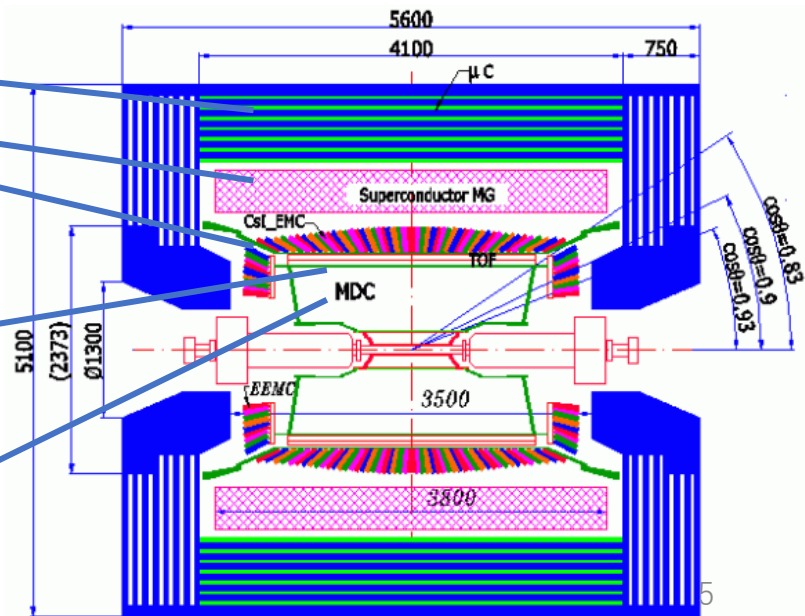
$$\sigma_t = 68 \text{ ps (桶部)}$$

$$\sigma_t = 60 \text{ ps (端盖2015)}$$

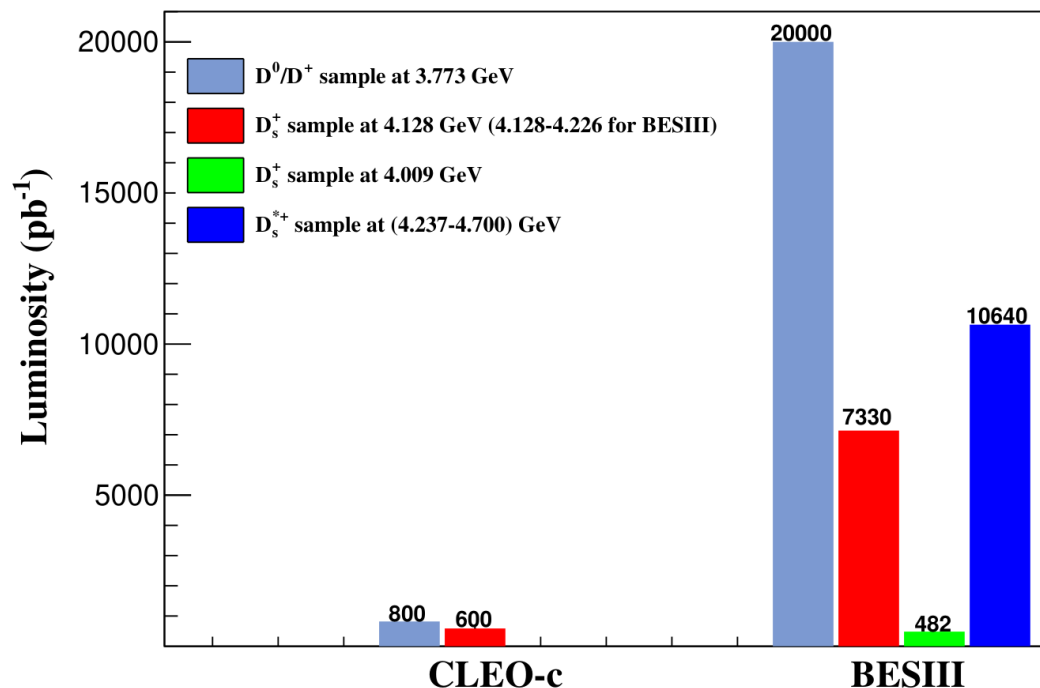
主漂移室(MDC):

$$\sigma_{\gamma\phi} = 130 \mu\text{m (single wire)}$$

$$\sigma_{p_t}/p_t = 0.5\% \text{ @ } 1 \text{ GeV}$$



数据样本及双标记方法



双标记方法

- 粲介子 D^+D^- , $D_s^+D_s^{*-}$ 成对产生, 可以选择双标记方法;
- 得到较为干净的背景, 减小系统误差;
- 分支比:

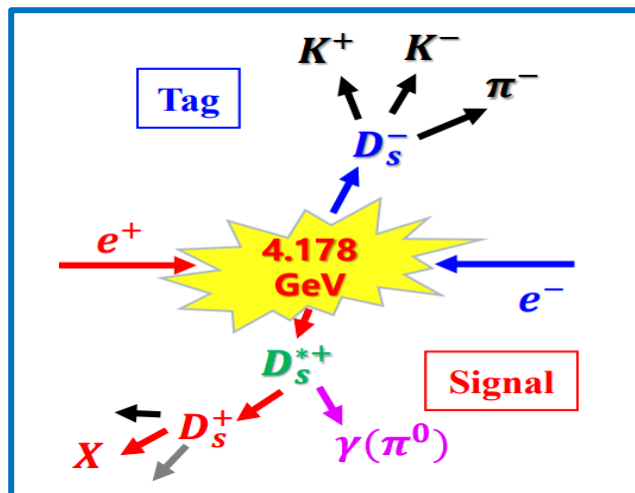
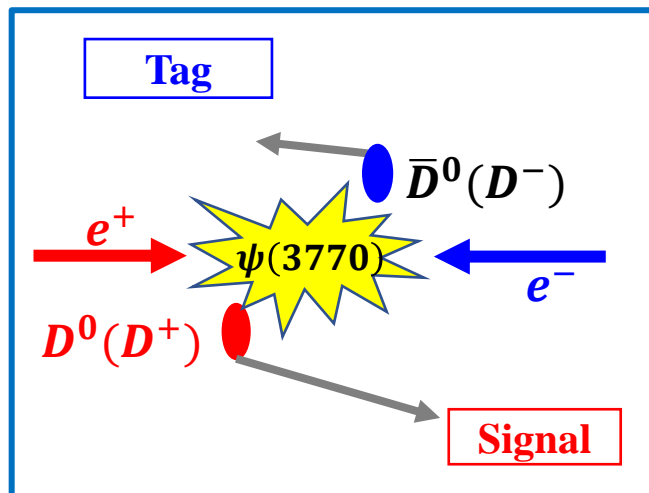
$$B_{\text{sig}} = \frac{N_{\text{sig}}}{N_{\text{ST}} \times \epsilon_{\text{sig}}}$$

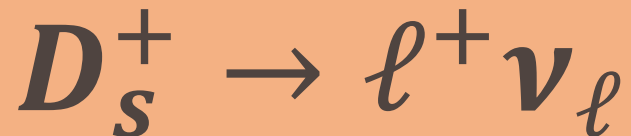
- 产额, 拟合中微子运动学变量:

$$M_{\text{miss}}^2 = E_{\text{miss}}^2 - |\vec{p}_{\text{miss}}|^2$$

$$E_{\text{miss}}^2 = E_{\text{cm}} - E_{\text{tag}} - E_X$$

$$\vec{p}_{\text{miss}} = -\vec{p}_{\text{tag}} - \vec{p}_X$$

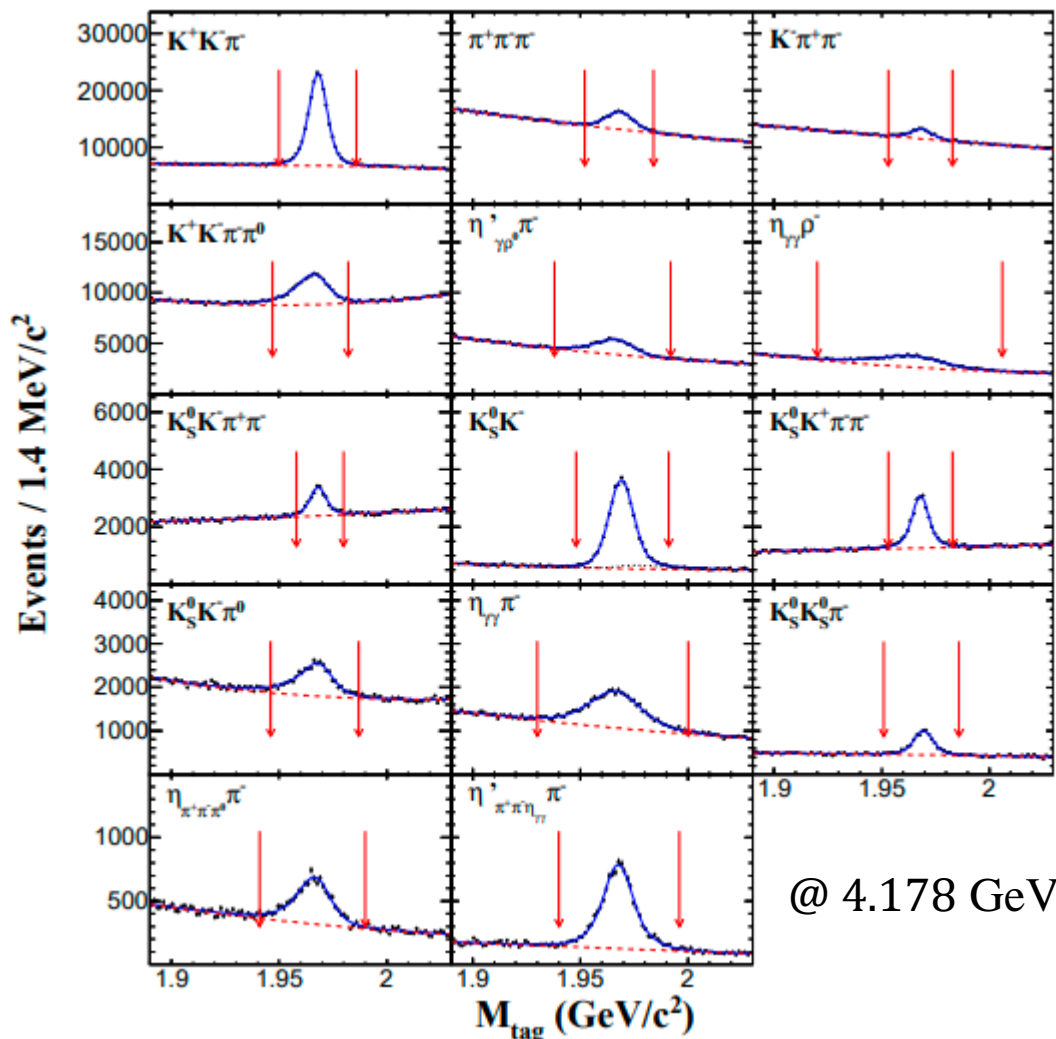




数据1: 7.33 fb⁻¹ @ 4.128-4.226 GeV

数据2: 482 pb⁻¹ @ 4.009 GeV

14个单标记道 @ 4.128-4.226 GeV: $N_{ST} \sim 0.89 M$



信号侧重建 $X(X = e^+, \pi^+, \pi^+\pi^0, \mu^+)$,
丢失掉的中微子的运动学信息:

$$M_{\text{miss}}^2 = E_{\text{miss}}^2 - |\vec{p}_{\text{miss}}|^2$$

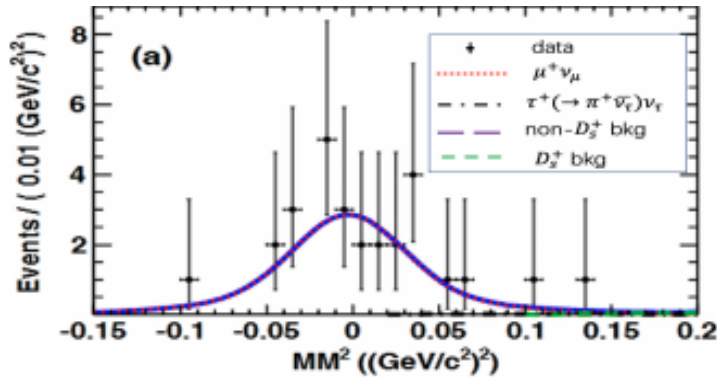
$$E_{\text{miss}}^2 = E_{\text{cm}} - \sqrt{|\vec{p}_{\text{tag}}|^2 + m_{D_s}^2} - E_{\gamma(\pi^0)} - E_X$$

$$\vec{p}_{\text{miss}} = -\vec{p}_{\text{tag}} - \vec{p}_{\gamma(\pi^0)} - \vec{p}_X$$

$$D_S^+ \rightarrow \ell^+ \nu_\ell : \ell = \mu^+, \tau^+$$

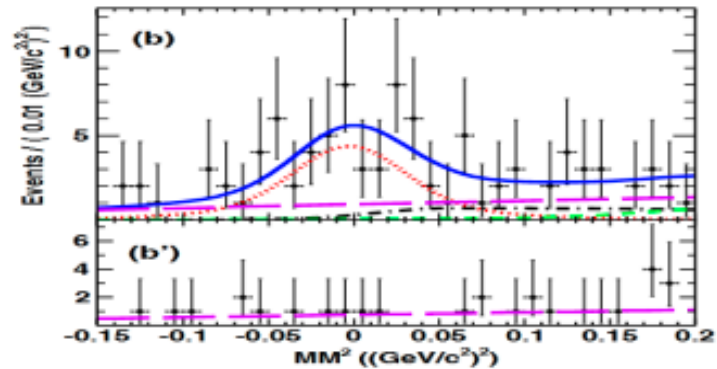
数据: 482 pb⁻¹ @ 4.009 GeV

PRD 94, 072004 (2016)



μ -like

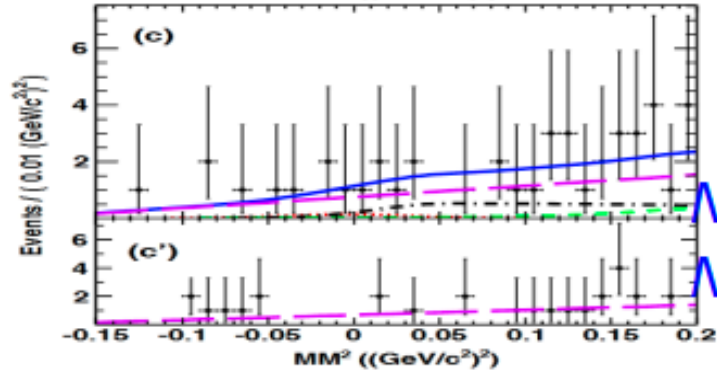
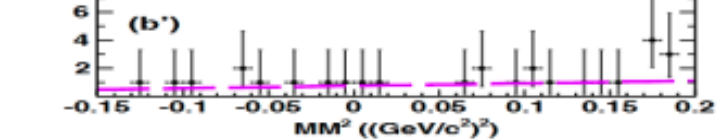
衰变链: $e^+e^- \rightarrow D_S^+ D_S^-$



non- μ , π -like

$$\mathcal{B}(D_S^+ \rightarrow \mu^+ \nu_\mu) = (0.517 \pm 0.075_{\text{stat}} \pm 0.021_{\text{syst}})\%$$

$$\mathcal{B}(D_S^+ \rightarrow \tau^+ \nu_\tau) = (3.28 \pm 1.83_{\text{stat}} \pm 0.37_{\text{syst}})\%$$



π -like

- BF ($\mu^+ \nu_\mu$) 精度: $\sim 15.1\%$
- BF ($\tau^+ \nu_\tau$) 精度: $\sim 56.9\%$

$$N(\mu^+ \nu_\mu) = 72.4 \pm 10.4$$

$$N(\tau^+ \nu_\tau) = 22.1$$

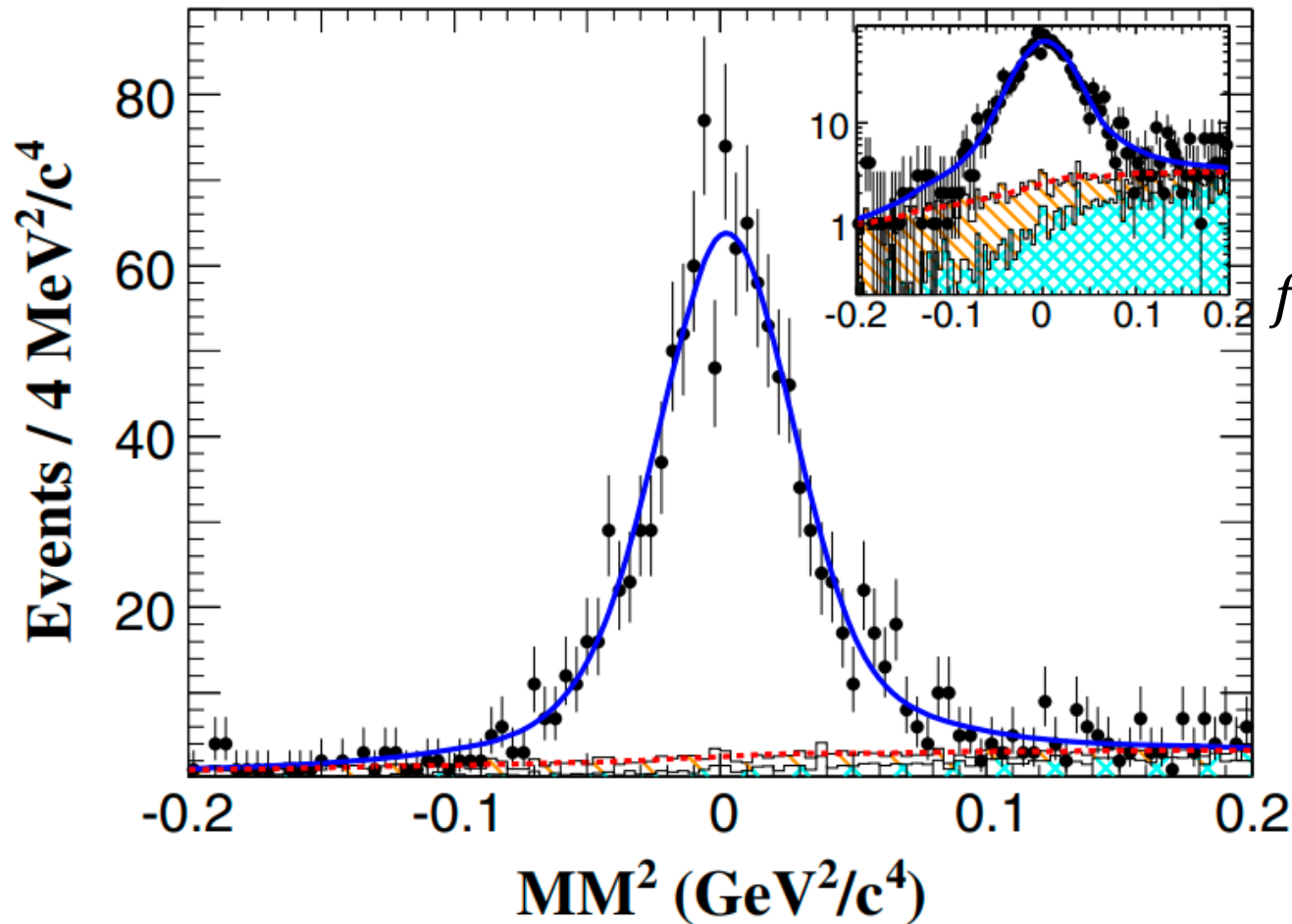
$$D_s^+ \rightarrow \ell^+ \nu_\ell : \ell = \mu^+$$

数据: 3.19 fb⁻¹ @ 4.178 GeV

$e^+e^- \rightarrow D_s^+ D_s^{*-} \rightarrow \gamma(\pi^0) D_s^+ D_s^-$

PRL 122, 071802 (2019)

双标拟合: 误差棒为数据, 直方图为蒙卡模拟的背景



$$N_{D_s^+ \rightarrow \mu^+ \nu} = 1135.9 \pm 33.1$$

$$\mathcal{B} = (0.549 \pm 0.016_{\text{stat}} \pm 0.015_{\text{syst}})\%$$

$$f_{D_s^+} |V_{cs}| = (246.2 \pm 3.6_{\text{stat}} \pm 3.5_{\text{syst}}) \text{ MeV}$$

$$f_{D_s^+} = (252.9 \pm 3.7_{\text{stat}} \pm 3.6_{\text{syst}}) \text{ MeV}$$

$$|V_{cs}| = 0.985 \pm 0.014_{\text{stat}} \pm 0.014_{\text{syst}}$$

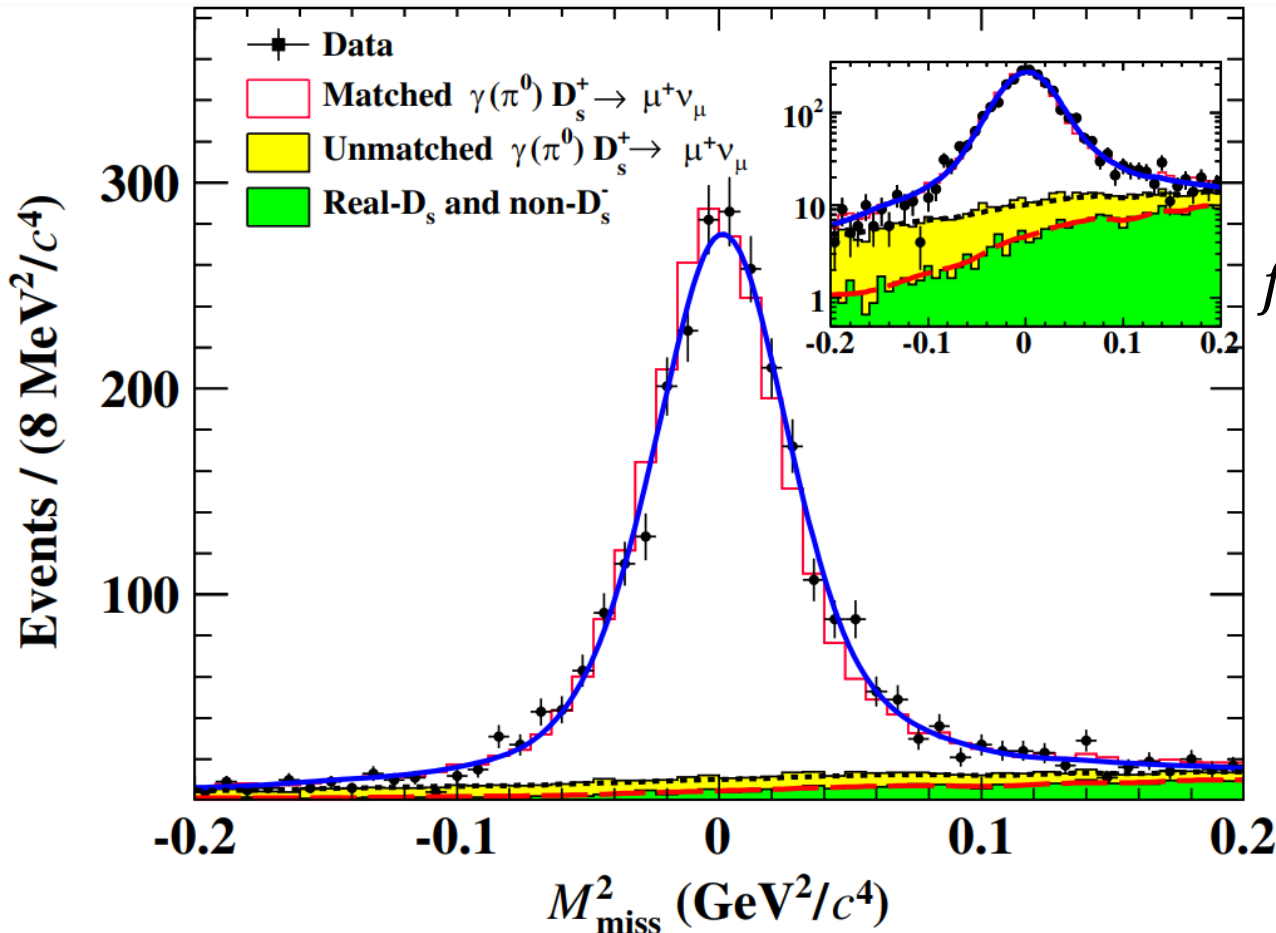
- 实验上极大改善了BF精度: 3.99 %
- $|V_{cs}|$ 精度: ~2 %

$$D_s^+ \rightarrow \ell^+ \nu_\ell : \ell = \mu^+$$

数据: 7.33 fb^{-1} @ $4.128\text{--}4.226 \text{ GeV}$ $e^+e^- \rightarrow D_s^+ D_s^{*-} \rightarrow \gamma(\pi^0) D_s^+ D_s^-$

PRD 108, 112001(2023)

双标拟合: 误差棒为数据, 直方图为蒙卡模拟的背景



$$N_{D_s^+ \rightarrow \mu^+ \nu} = 2514.5 \pm 51.6$$

$$\mathcal{B} = (0.5294 \pm 0.0108_{\text{stat}} \pm 0.0085_{\text{syst}})\%$$

$$f_{D_s^+} |V_{cs}| = (241.8 \pm 2.5_{\text{stat}} \pm 2.2_{\text{syst}}) \text{ MeV}$$

$$f_{D_s^+} = (248.4 \pm 2.5_{\text{stat}} \pm 2.2_{\text{syst}}) \text{ MeV}$$

$$|V_{cs}| = 0.968 \pm 0.010_{\text{stat}} \pm 0.009_{\text{syst}}$$

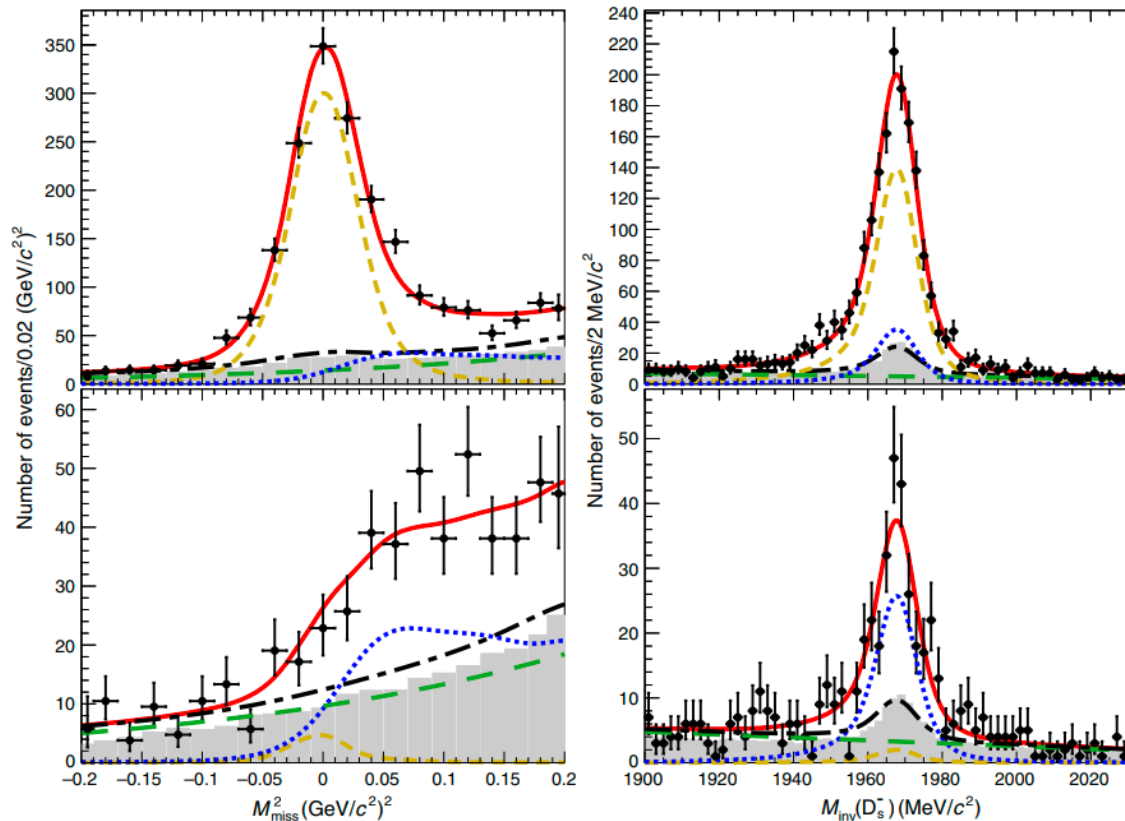
- BF的精度: 2.6 %
- 迄今为止最高的 $|V_{cs}|$ 精度: $\sim 1.4\%$

$$D_s^+ \rightarrow \ell^+ \nu_\ell: \ell = \mu^+, \tau^+ \rightarrow \pi^+ \bar{\nu}$$

数据: 6.32 fb⁻¹ @4.178–4.226 GeV

PRD 104, 052009(2021)

拟合(@4.178 GeV):



- μ -like (upper):
 $E_{\text{EMC}} \leq 300$ MeV, mixture
of $D_s^+ \rightarrow \tau^+ (\rightarrow \pi^+ \bar{\nu}_\tau) \nu_\tau$
and $D_s^+ \rightarrow \mu^+ \nu_\mu$

• BF精度: ~5.8%

- π -like (lower):
 $E_{\text{EMC}} > 300$ MeV,
dominated of
 $D_s^+ \rightarrow \tau^+ (\rightarrow \pi^+ \bar{\nu}_\tau) \nu_\tau$

$$N_{D_s^+ \rightarrow \mu^+ \nu} = 2198 \pm 55, \quad N_{D_s^+ \rightarrow \tau^+ \nu} = 946^{+46}_{-45}$$

$$\mathcal{B}(D_s^+ \rightarrow \mu^+ \nu_\mu) = (0.535 \pm 0.013 \pm 0.016)\%$$

$$\mathcal{B}(D_s^+ \rightarrow \tau^+ \nu_\tau) = (5.21 \pm 0.25 \pm 0.17)\%$$

$|V_{cs}|$ 精度

$$f_{D_s^+} = 249.8 \pm 3.0 \pm 3.7 \pm 1.0 \text{ MeV},$$

$$249.7 \pm 6.0 \pm 4.1 \pm 1.0 \text{ MeV},$$

$$249.9 \pm 2.4 \pm 3.4 \pm 1.0 \text{ MeV},$$

$$|V_{cs}| = 0.973 \pm 0.012 \pm 0.015 \pm 0.004,$$

$$0.972 \pm 0.023 \pm 0.016 \pm 0.004,$$

$$0.973 \pm 0.009 \pm 0.013 \pm 0.004,$$

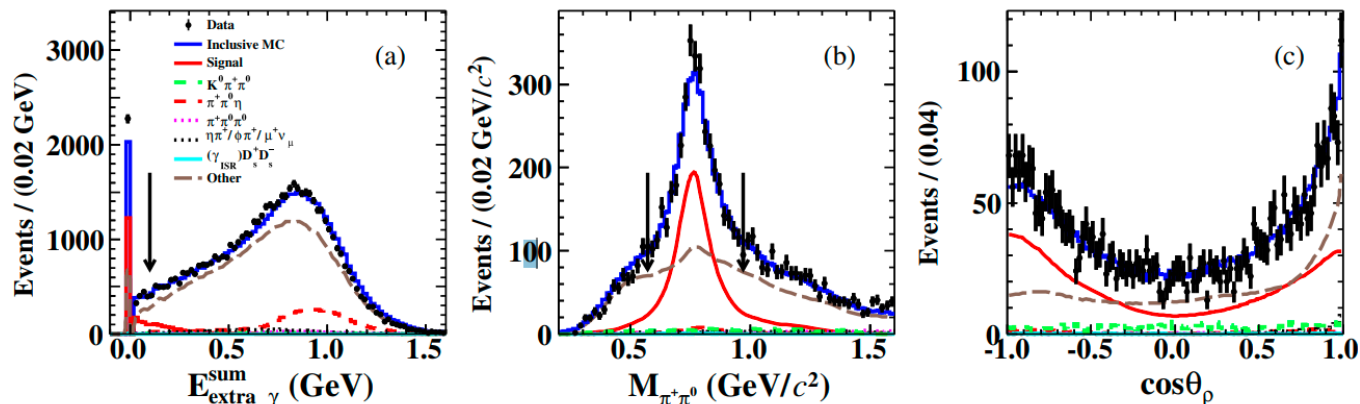
→	$\mu^+ \nu_\mu$	→	~2.0 %
→	$\tau^+ \nu_\tau$	→	~2.9 %
→	SM约束	→	~1.7 %

$$D_s^+ \rightarrow \ell^+ \nu_\ell : \ell = \tau^+, \tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}$$

数据: 6.32 fb^{-1} @ 4.178 GeV

数据蒙卡比较图

PRD 104, 032001(2021)



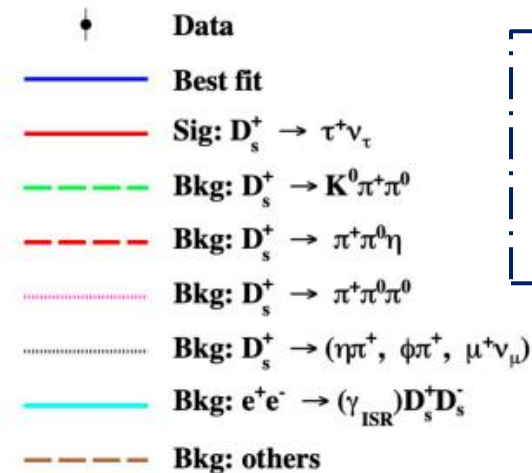
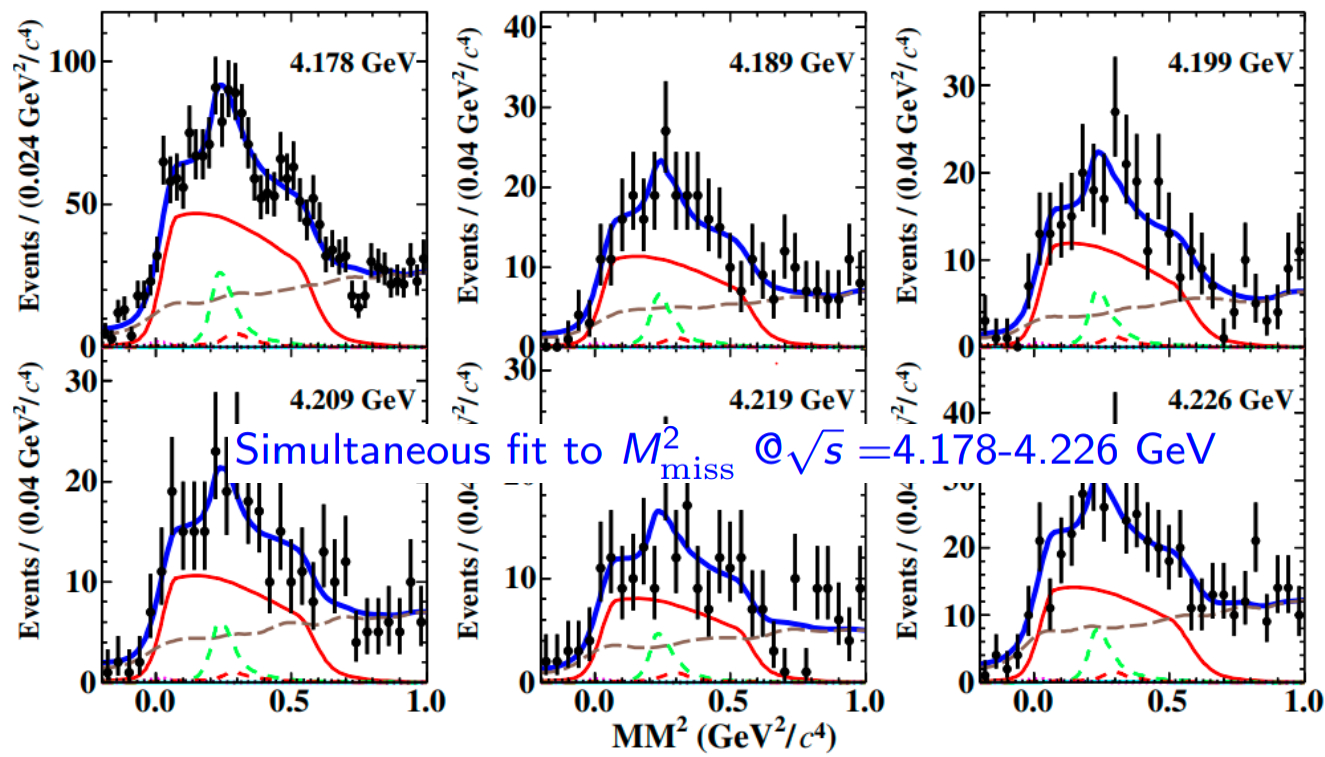
$$N_{D_s^+ \rightarrow \tau^+ \nu} = 1745 \pm 84$$

$$B = (5.29 \pm 0.25_{\text{stat}} \pm 0.20_{\text{syst}})\%$$

$$f_{D_s^+} |V_{CS}| = (244.8 \pm 5.8_{\text{stat}} \pm 4.8_{\text{syst}}) \text{ MeV}$$

$$f_{D_s^+} = (251.6 \pm 5.9_{\text{stat}} \pm 4.9_{\text{syst}}) \text{ MeV}$$

$$|V_{CS}| = 0.980 \pm 0.023_{\text{stat}} \pm 0.019_{\text{syst}}$$

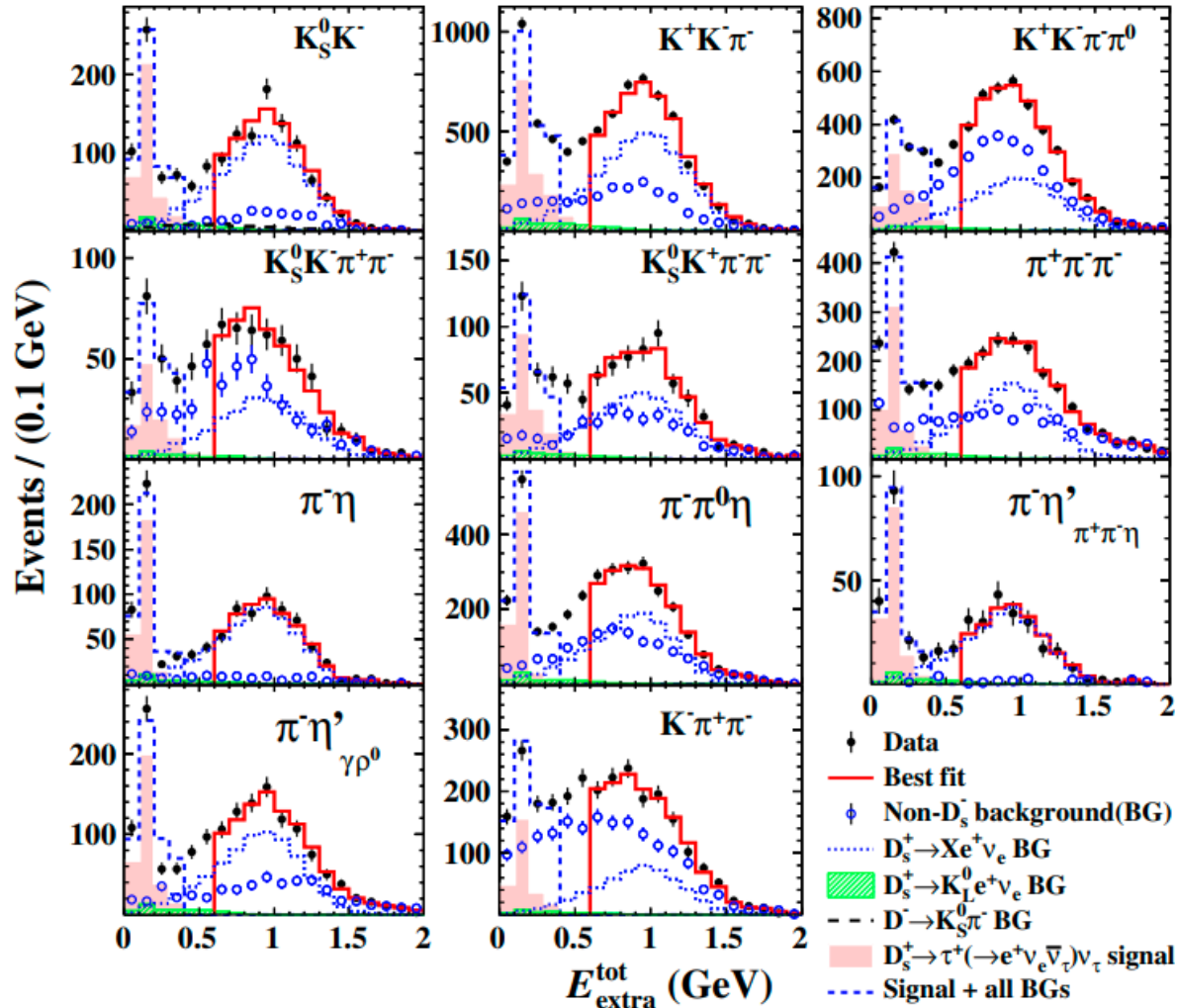


- 分支比和之前结果相符;
- $|V_{CS}|$ 精度: $\sim 3.1\%$

$$D_s^+ \rightarrow \ell^+ \nu_\ell : \ell = \tau^+, \tau^+ \rightarrow e^+ \nu_e \bar{\nu}_\tau$$

数据: 6.32 fb⁻¹ @ 4.178-4.226 GeV

PRL 127, 171801(2021)



外推拟合

$E_{\text{extra}}^{\text{tot}}$: 未使用的径迹 信号区 $\in (0, 0.4)$ GeV
在EMC中沉积的总能量 背景区 $\in (0.6, 2.0)$ GeV

$$B = (5.27 \pm 0.10_{\text{stat}} \pm 0.12_{\text{syst}})\%$$

$$f_{D_s^+} |V_{cs}| = (244.4 \pm 2.3_{\text{stat}} \pm 2.9_{\text{syst}}) \text{ MeV}$$

$$f_{D_s^+} = (251.1 \pm 2.4_{\text{stat}} \pm 3.0_{\text{syst}}) \text{ MeV}$$

$$|V_{cs}| = 0.978 \pm 0.009_{\text{stat}} \pm 0.012_{\text{syst}}$$

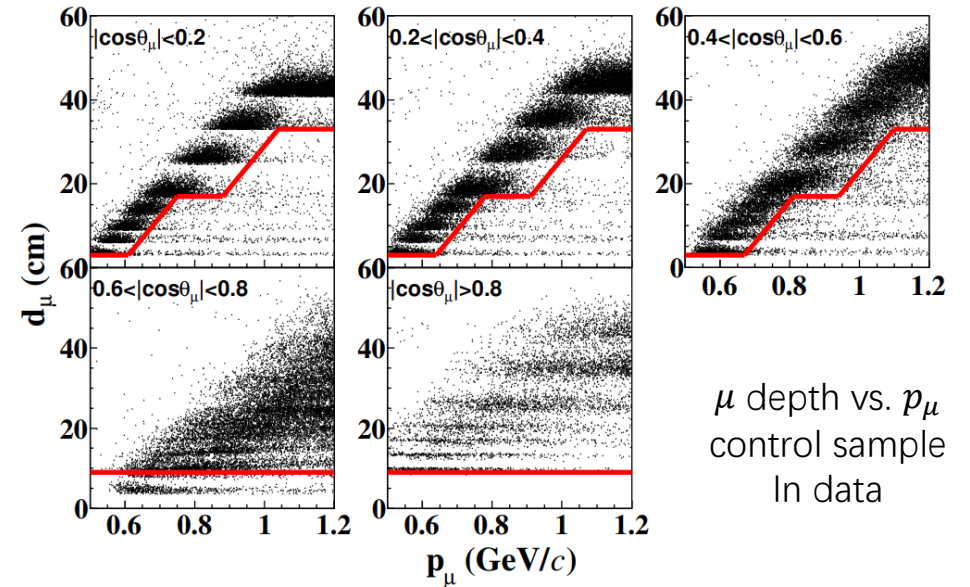
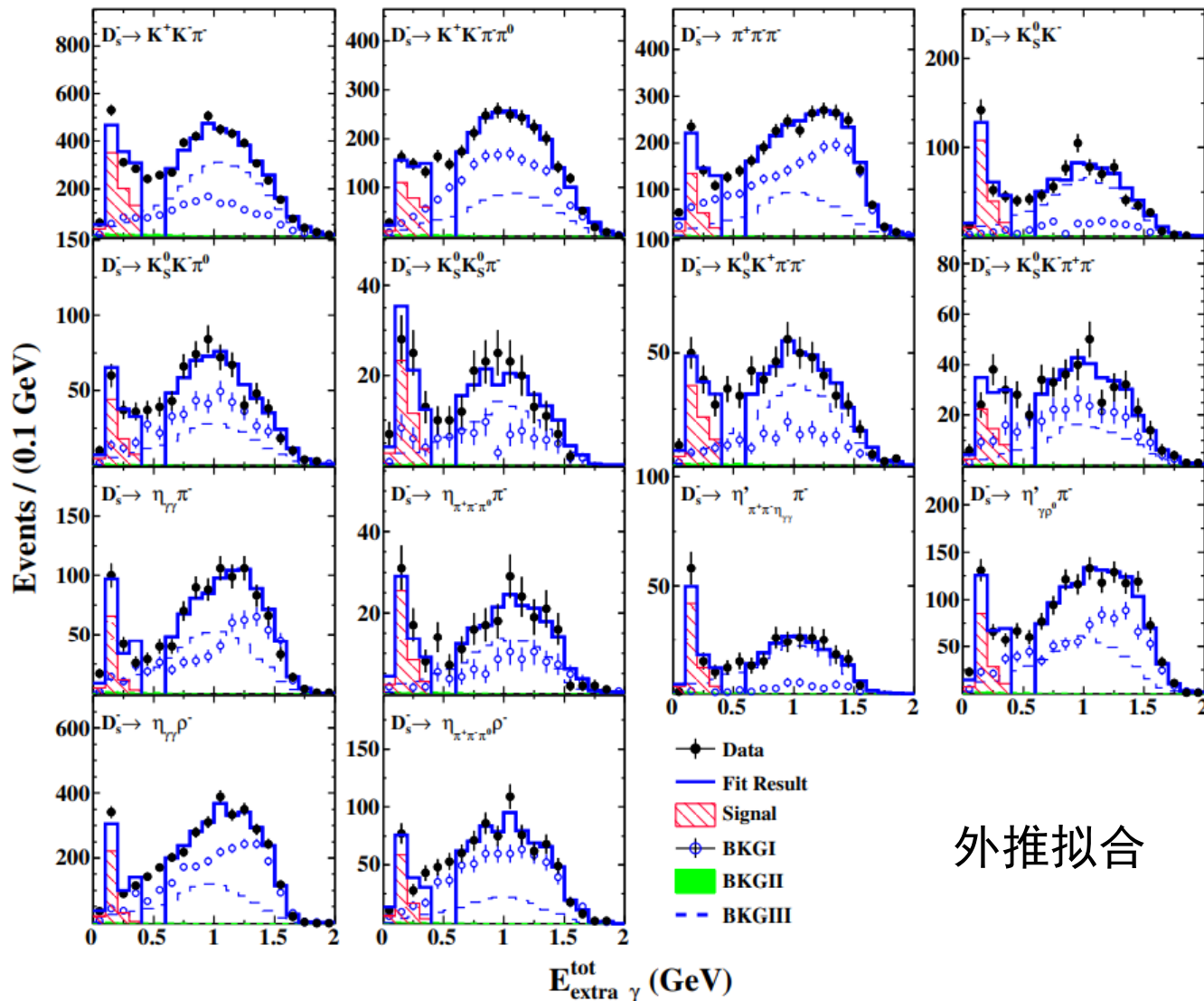
• $|V_{cs}|$ 精度: ~1.5%

$E_{\text{extra}}^{\text{tot}}$: the total energy of the good EMC showers, excluding FSR and those associated in ST

$$D_s^+ \rightarrow \ell^+ \nu_\ell: \ell = \tau^+, \tau^+ \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau$$

数据: $7.33 \text{ fb}^{-1} @ 4.128\text{-}4.226 \text{ GeV}$

JHEP 09 (2023) 124



$$B = (5.37 \pm 0.17_{\text{stat}} \pm 0.15_{\text{syst}})\%$$

$$f_{D_s^+} |V_{CS}| = (246.7 \pm 3.9_{\text{stat}} \pm 3.6_{\text{syst}}) \text{ MeV}$$

$$f_{D_s^+} = (253.4 \pm 4.0_{\text{stat}} \pm 3.7_{\text{syst}}) \text{ MeV}$$

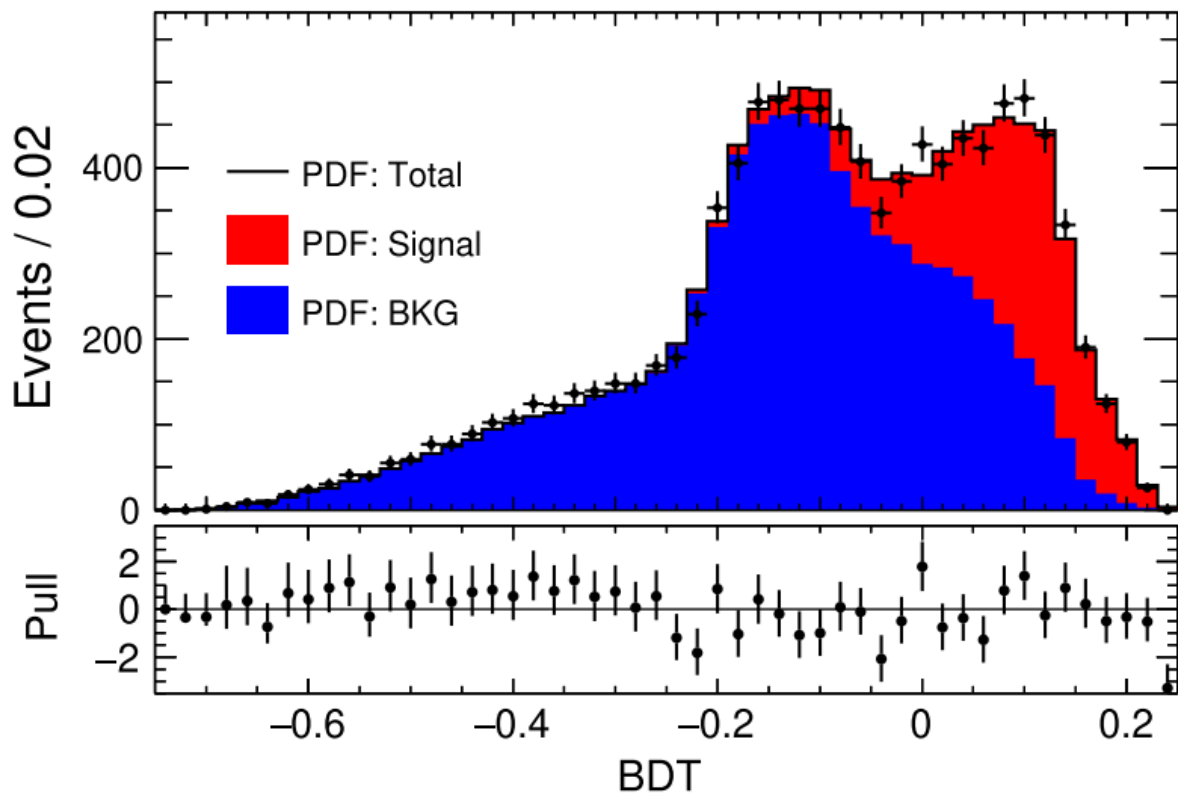
$$|V_{CS}| = 0.987 \pm 0.016_{\text{stat}} \pm 0.014_{\text{syst}}$$

• $|V_{CS}|$ 精度: $\sim 1.9\%$

$$D_s^+ \rightarrow \ell^+ \nu_\ell: \ell = \tau^+, \tau^+ \rightarrow \pi^+ \bar{\nu}$$

数据: 7.33fb^{-1} @ 4.128-4.226 GeV

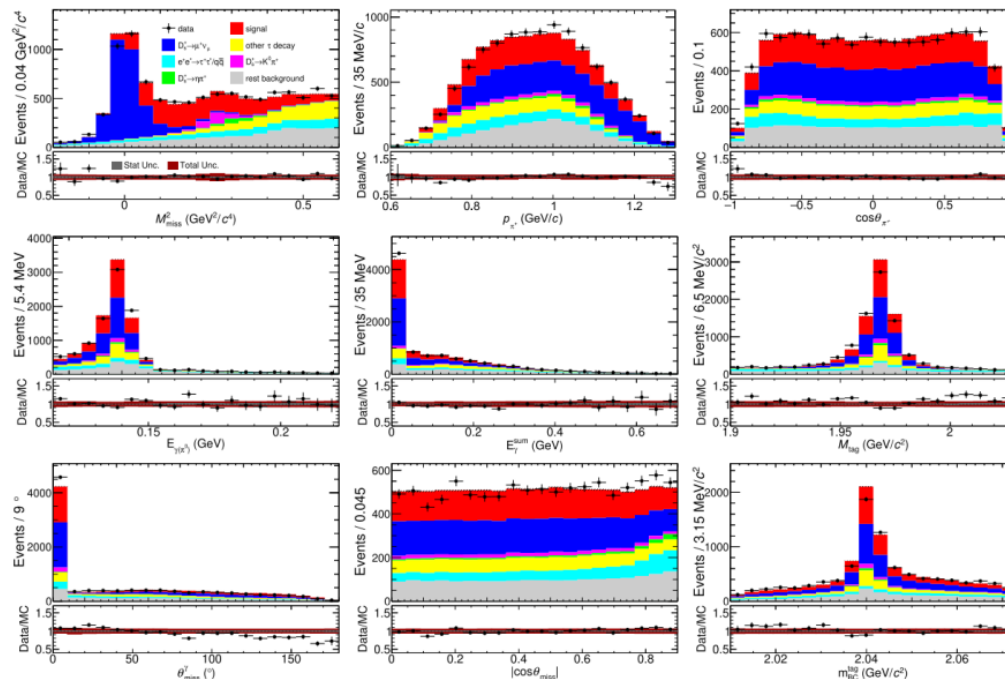
采用增强法分类的决策树集 (BDT)



$$N_{D_s^+ \rightarrow \tau^+ \nu} = 2411 \pm 75$$

BDT输入

PRD 108, 092014(2023)



$$\mathcal{B} = (5.44 \pm 0.17_{\text{stat}} \pm 0.13_{\text{syst}})\%$$

$$f_{D_s^+} |V_{CS}| = (248.3 \pm 3.9_{\text{stat}} \pm 3.1_{\text{syst}} \pm 1.0_{\text{input}})\text{MeV}$$

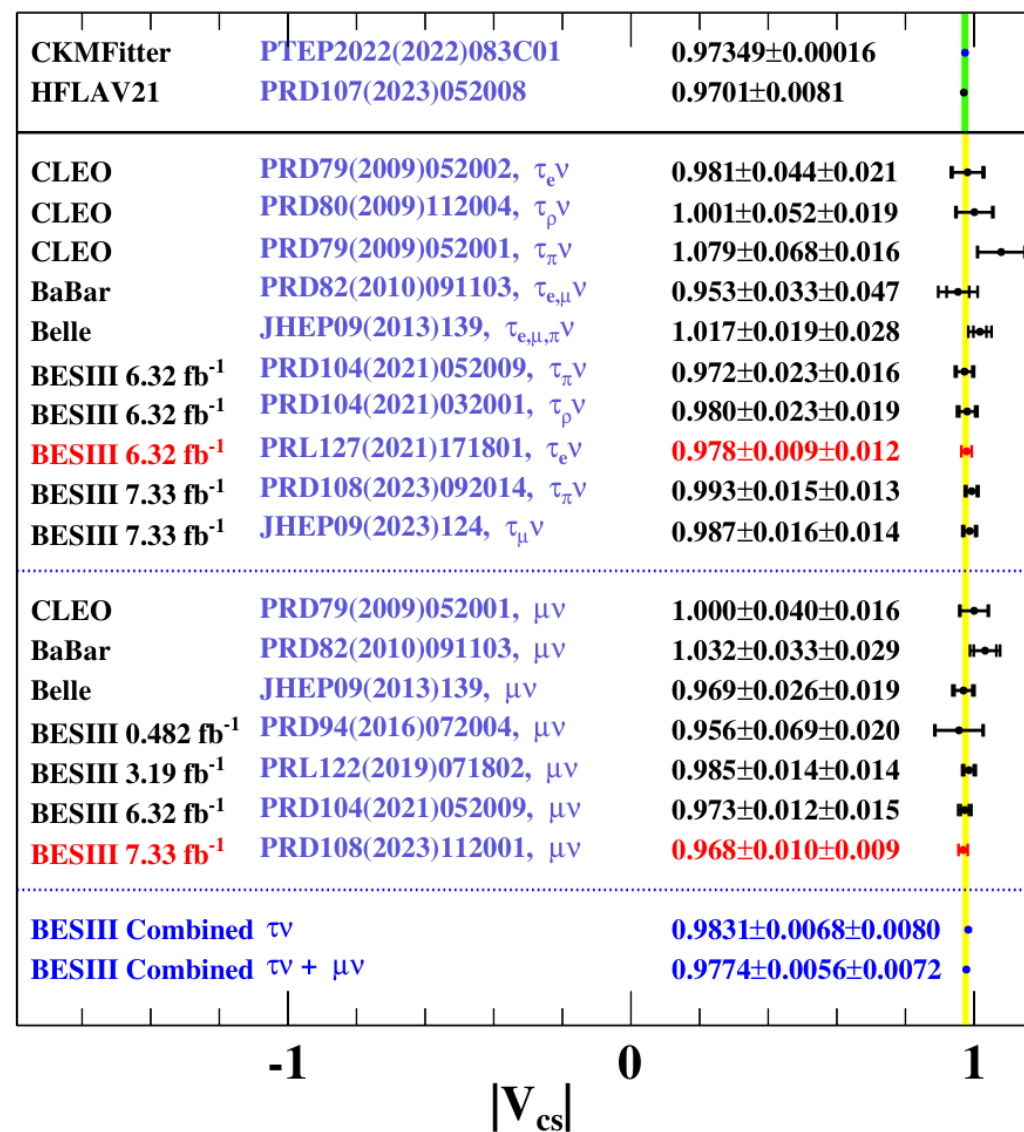
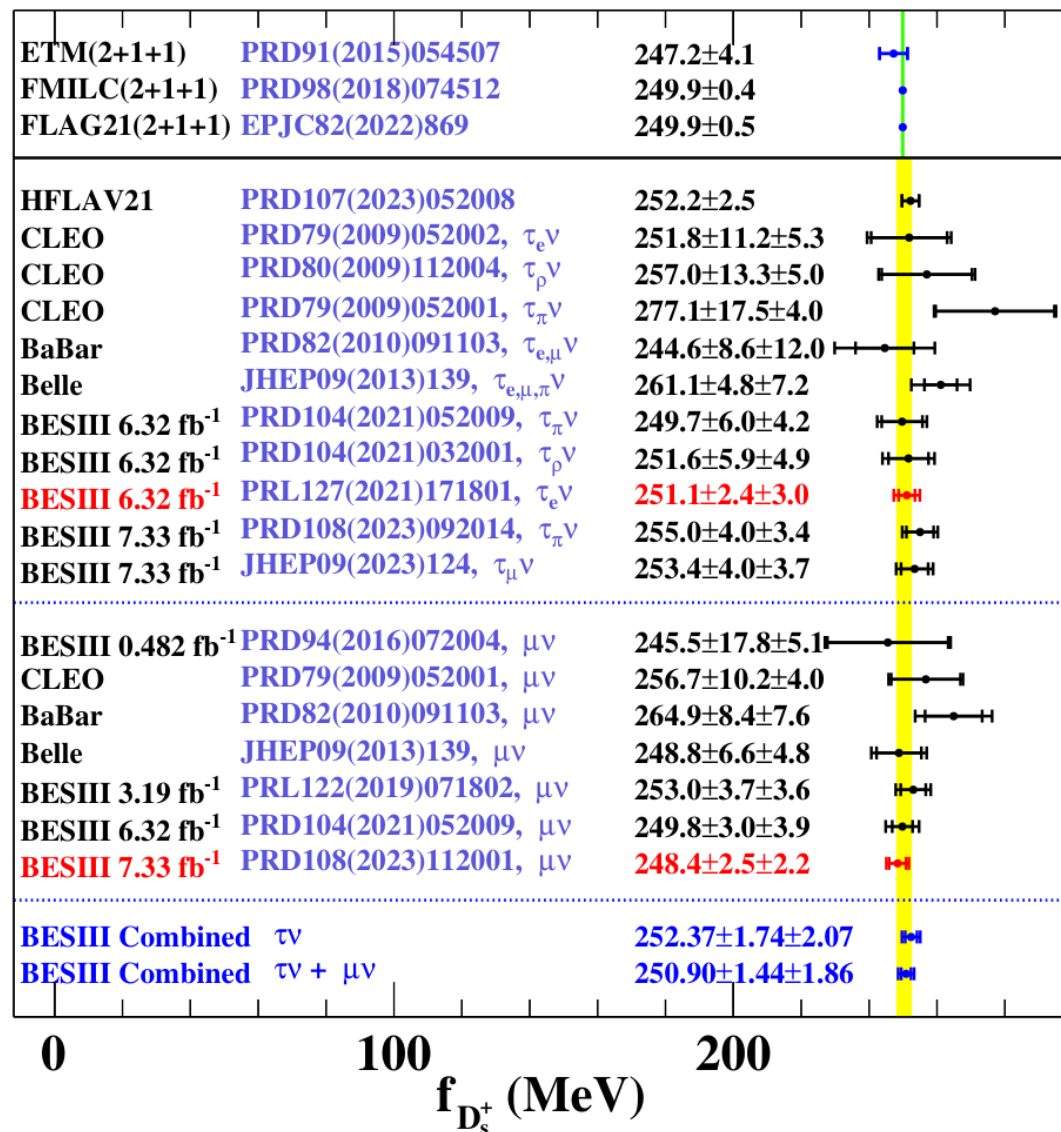
$$f_{D_s^+} = (255.0 \pm 4.0_{\text{stat}} \pm 3.2_{\text{syst}} \pm 1.0_{\text{input}})\text{MeV}$$

$$|V_{CS}| = 0.993 \pm 0.015_{\text{stat}} \pm 0.012_{\text{syst}} \pm 0.004_{\text{input}}$$

• $|V_{CS}|$ 精度: $\sim 2.1\%$

$D_s^+ \rightarrow \ell^+ \nu_\ell$: 结果比较

根据来自PDG2022的 G_F , $m_{D_s^+}$, m_τ , m_μ , 输入 $|V_{cs}|$, $f_{D_s^+}$



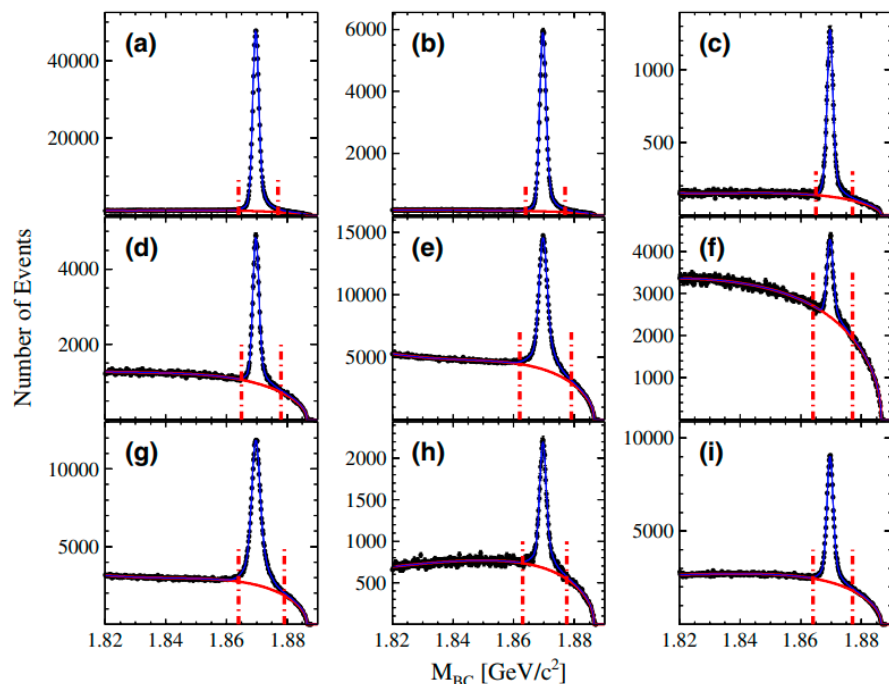
$$D^+ \rightarrow \ell^+ \nu_\ell : \ell = \mu^+$$

数据: $2.93 \text{ fb}^{-1} \psi(3770) @ 3.773 \text{ GeV}$

PRD 89, 051104 (2014)

$N_{\text{ST}}: 1703045 \pm 3405$

$N_{\text{DT}}: 409.0 \pm 21.3$

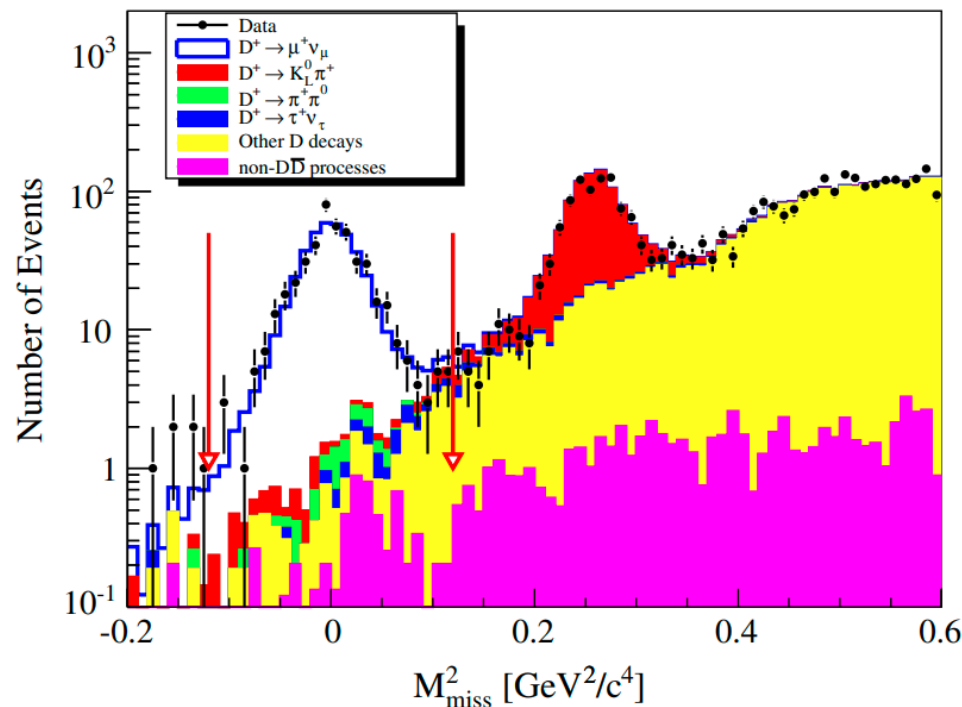


单标记: 拟合 D^- 事例

$$M_{\text{BC}} = \sqrt{E_{\text{beam}}^2 - |\vec{p}_{mK\eta\pi}|^2}$$

$$\Delta E = E_{\text{beam}} - E_{\bar{D}}$$

$$M_{\text{miss}}^2 = (E_{\text{cm}} - E_{\text{tag}} - E_{\mu^+})^2 - |-\vec{p}_{\text{tag}} - \vec{p}_{\mu^+}|^2$$



D^+ 信号事例
数据蒙卡比较

$|V_{cd}|$ 精度: $\sim 2.8\%$

$$\mathcal{B} = (3.71 \pm 0.19_{\text{stat}} \pm 0.06_{\text{syst}}) \times 10^{-4}$$

$$f_{D^+} |V_{cd}| = (45.75 \pm 1.20_{\text{stat}} \pm 0.39_{\text{syst}}) \text{ MeV}$$

$$f_{D^+} = (203.2 \pm 5.3_{\text{stat}} \pm 1.8_{\text{syst}}) \text{ MeV}$$

$$|V_{cd}| = 0.2210 \pm 0.0058_{\text{stat}} \pm 0.0047_{\text{syst}}$$

$$D^+ \rightarrow \ell^+ \nu_\ell : \ell = \tau^+$$

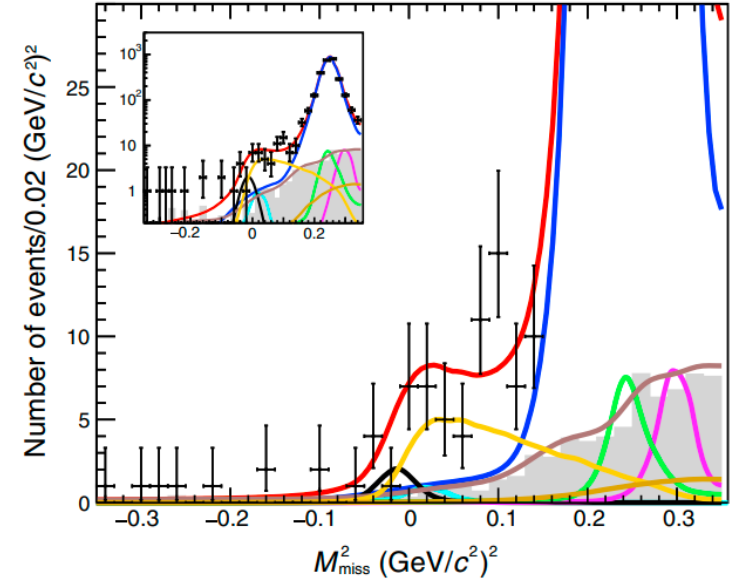
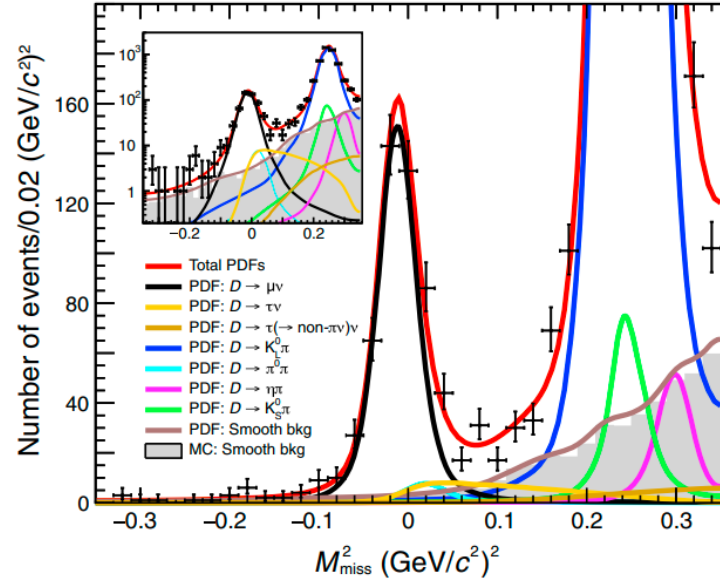
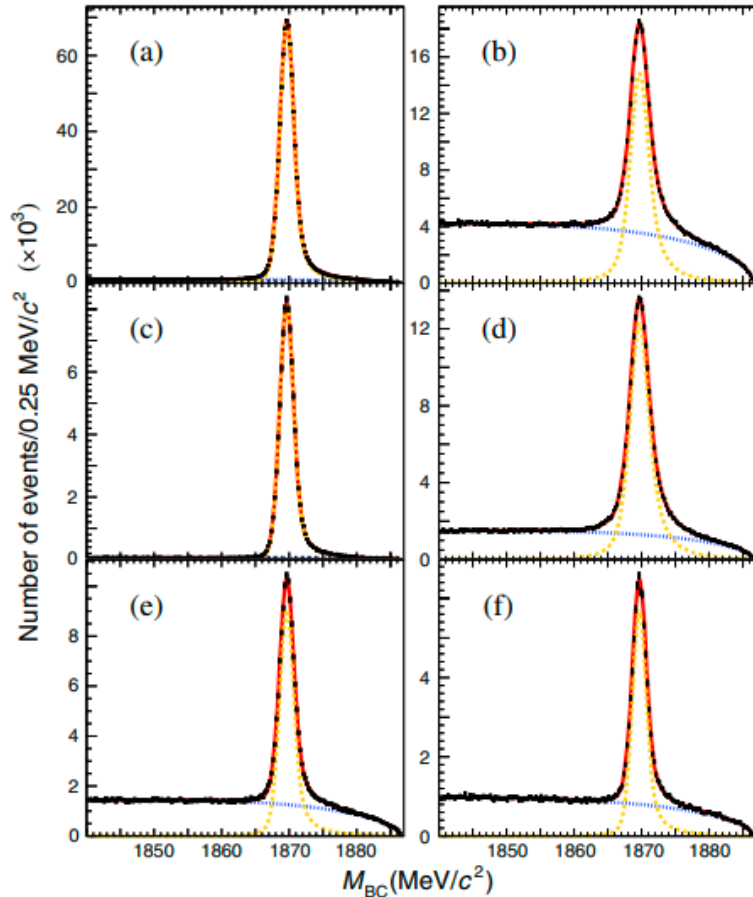
数据: $2.93 \text{ fb}^{-1} \psi(3770) @ 3.773 \text{ GeV}$

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单标记: 拟合 D^- 事例

联合拟合: μ^+ -like (左)

π^+ -like (右)



$$N_{D^+ \rightarrow \tau^+ \nu} = 137 \pm 27$$

$$B = (1.20 \pm 0.24_{\text{stat}} \pm 0.12_{\text{syst}}) \times 10^{-3}$$

$$f_{D^+} = (224.5 \pm 22.8_{\text{stat}} \pm 11.3_{\text{syst}} \pm 0.9_{\text{ex-syst}}) \text{ MeV}$$

$$|V_{cd}| = 0.237 \pm 0.024_{\text{stat}} \pm 0.012_{\text{syst}} \pm 0.001_{\text{ex-syst}}$$

$$R_{\tau/\mu} = 3.21 \pm 0.64_{\text{stat}} \pm 0.43_{\text{syst}}$$

实验上首次观测:

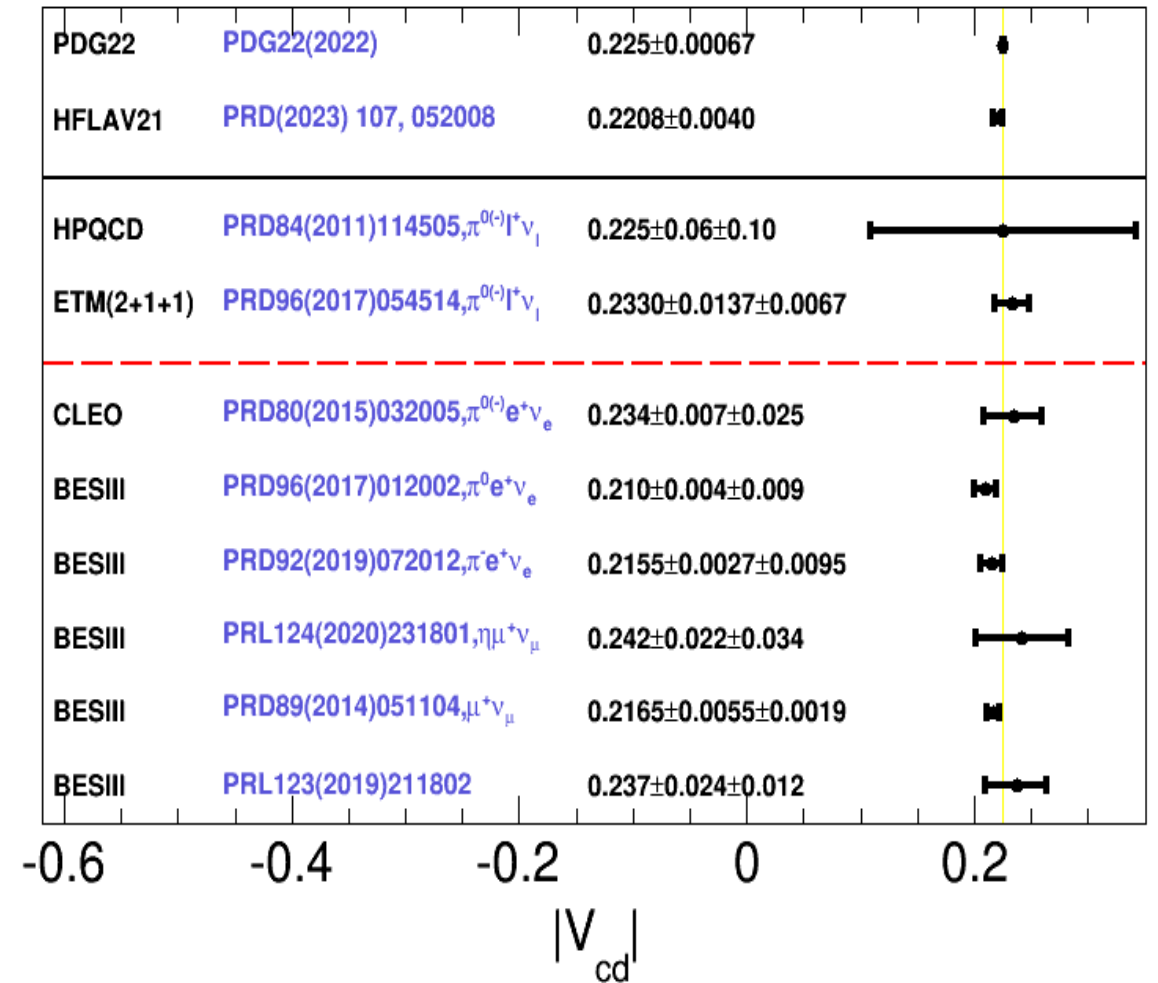
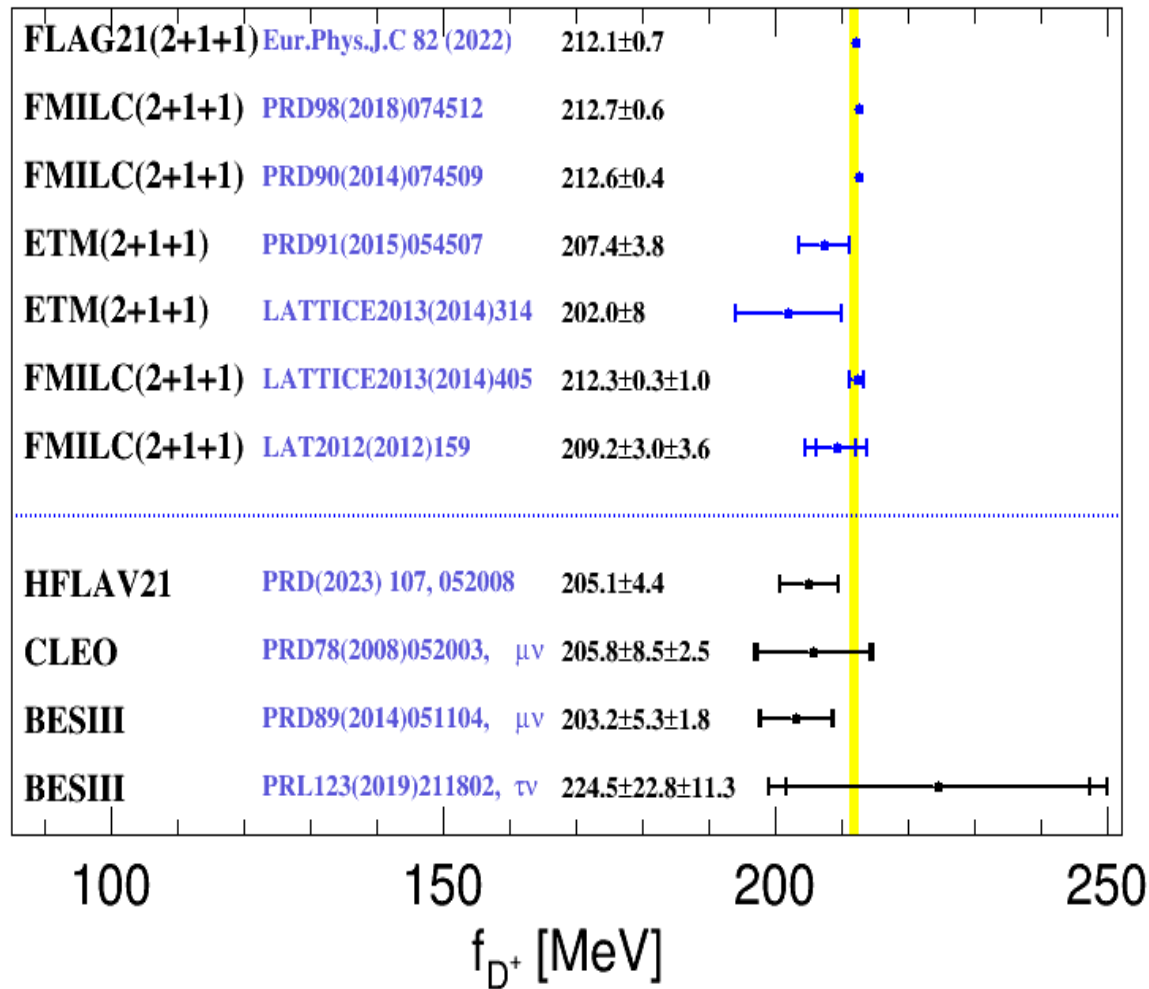
5.1σ

$|V_{cd}|$ 精度:

11.3%

$D^+ \rightarrow \ell^+ \nu_\ell$: 结果比较

根据来自PDG2022的 G_F , m_D , m_τ , m_μ , 输入 $|V_{cd}|$, f_D



总结与展望

- 精确测量 $|V_{cs}|$, $|V_{cd}|$ 和 $B(D_{(s)}^+ \rightarrow \ell^+ \nu_\ell)$ 在更高精度下检验标准模型CKM矩阵幺正性以及轻子普适性十分重要
- 通过BESIII采集的 7.33 fb^{-1} @4.128-4.226 GeV 和 2.93 fb^{-1} @3.773 GeV 数据样本对纯轻过程 $D_s^+ \rightarrow \ell^+ \nu_\ell$ 和 $D^+ \rightarrow \ell^+ \nu_\ell$ ($\ell = \mu^+, \tau^+$) 分别进行分析, 精确了测量 $|V_{cs}|$ ($\sim 1.5\%$), $|V_{cd}|$ ($\sim 2.8\%$)。

数据: @ 3.773 GeV :

$D^+ \rightarrow \mu^+ \nu_\mu$, $D^+ \rightarrow \tau^+ \nu_\tau$ 和 $D^+ \rightarrow e^+ \nu_e$

	数据 (fb^{-1})	$ V_{cd} $ 精度 (%)	状态
2010-2011	2.93	2.8	已发表
2021-2022	7.9	1.8	已发布
2023-2024	20	1.3	重建中

数据: @ 4.237 – 4.700 GeV: 10.64 fb^{-1}

$D_s^+ \rightarrow \mu^+ \nu_\mu$ 和 $D_s^+ \rightarrow \tau^+ \nu_\tau$

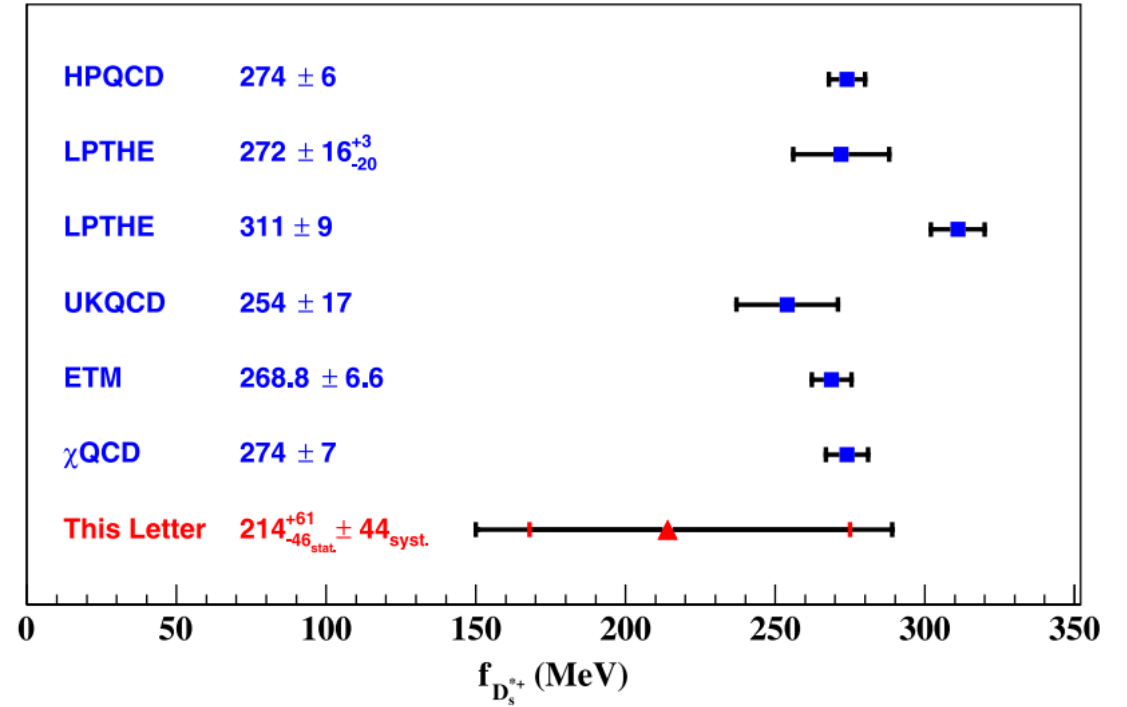
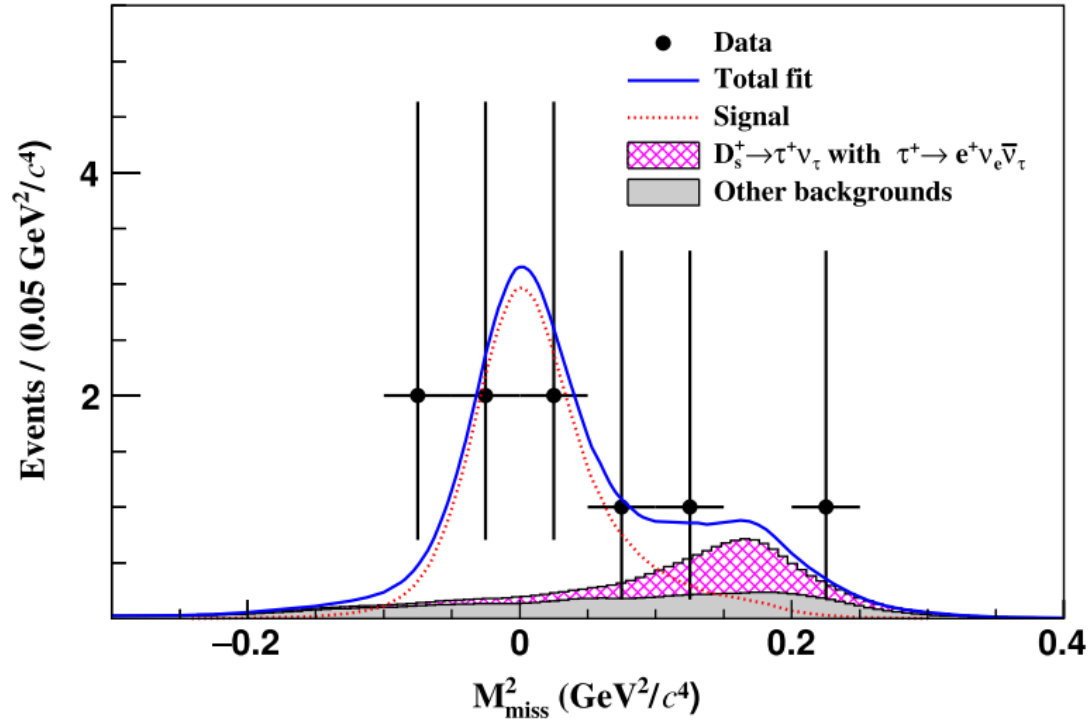
- 不同衰变链: $e^+ e^- \rightarrow D_s^{*+} D_s^{*-}$
- 测量: $D_s^+ \rightarrow \ell^+ \nu_\ell$ ($\ell = \mu^+, \tau^+$)
- 能量点更高, 检验补充
- BF ($D_s^+ \rightarrow \mu^+ \nu_\mu$) 精度: $\sim 8.5\%$
- BF ($D_s^+ \rightarrow \tau^+ \nu_\tau$) 精度: $\sim 4.0\%$

数据: @更多更高能量点: $D_s^+ \rightarrow \mu^+ \nu_\mu$ 和 $D_s^+ \rightarrow \tau^+ \nu_\tau$

谢谢!

Backup : $D_s^{*+} \rightarrow e^+ \nu_e$

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$$N_{\text{DT}} = 6.2_{-2.7}^{+3.4}$$

$$\mathcal{B} = (2.1_{-0.9}^{+1.2} \text{stat} \pm 0.2_{\text{syst}}) \times 10^{-5}$$

$$\Gamma_{D_s^{*+}}^{\text{total}} = (122_{-52}^{+70} \pm 12) \text{ eV}$$

$$f_{D_s^{*+}} = (214_{-46}^{+61} \pm 44_{\text{syst}}) \text{ MeV}$$

$$\Gamma(D_s^{*+} \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2}{12\pi} |V_{cs}|^2 f_{D_s^{*+}}^2 m_{D_s^{*+}}^3 \left(1 - \frac{m_{\ell^+}^2}{m_{D_s^{*+}}^2}\right)^2 \times \left(1 + \frac{m_{\ell^+}^2}{2m_{D_s^{*+}}^2}\right),$$