



Institute of High Energy Physics
Chinese Academy of Sciences



BESIII上粲介子强子衰变振幅分析

报告人：卢泽辉

2024.5.11

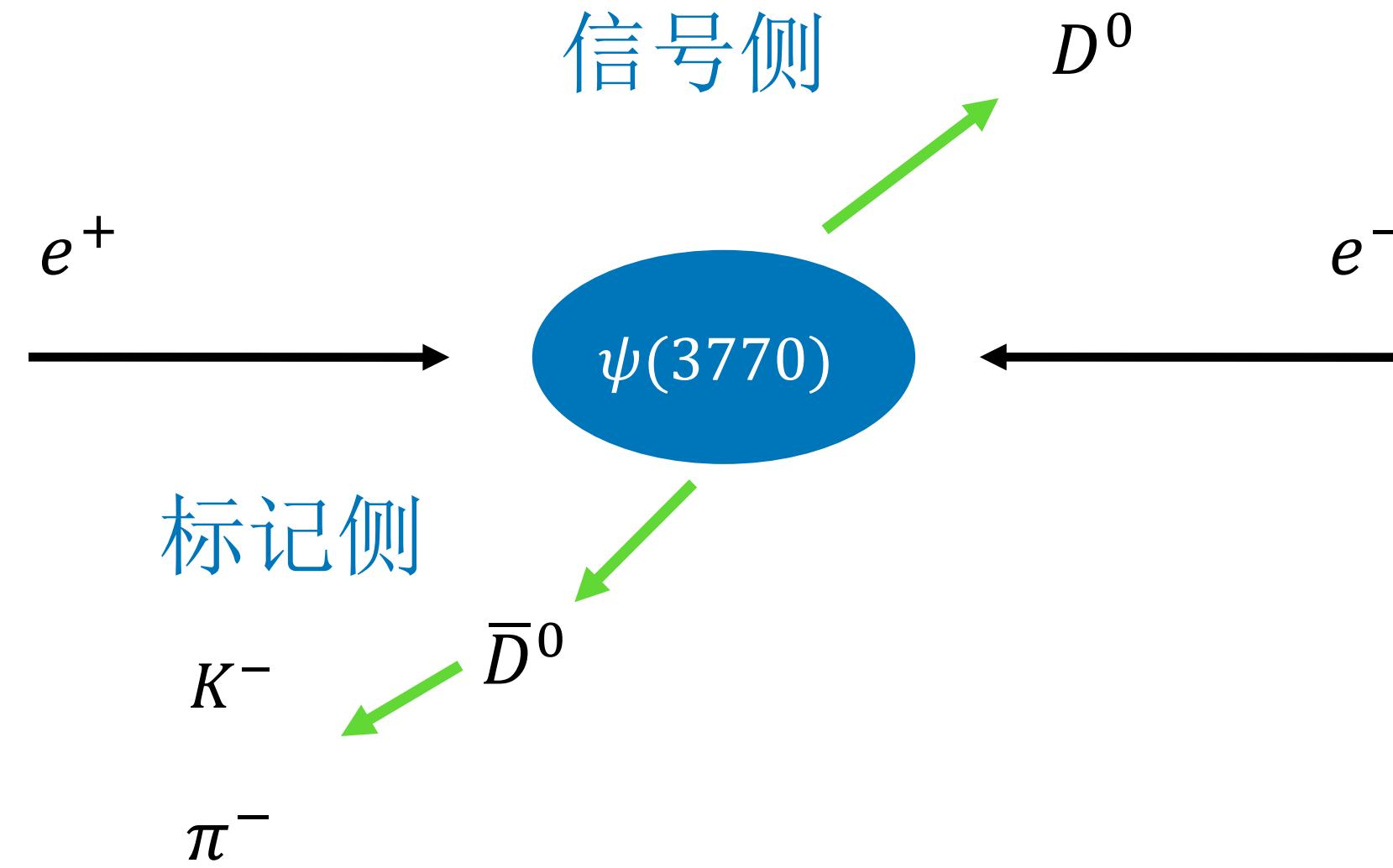
目录

- ✓ 引言
- ✓ 分析策略和数据集
- ✓ 振幅分析
- ✓ 总结

粲介子强子衰变振幅分析

- ✓ BESIII上的粲介子强子衰变振幅分析
 - 成果丰富：二十余篇已发表
 - 覆盖面广：三、四与五体分析
 - 意义重要
 - $D^+ \rightarrow K_S^0 \pi^+ \eta$
 - $D^0 \rightarrow \pi^+ \pi^- \eta$ & $D^0 \rightarrow \pi^+ \pi^0 \eta$
 - $D_s^+ \rightarrow \pi^+ \pi^0 \eta$
 - $D_s^+ \rightarrow K_S^0 K_S^0 \pi^+$
 - $D_s^+ \rightarrow K_S^0 K^+ \pi^0$
 - ...
- ✓ 检验非微扰QCD理论
 - 测量两体衰变 PP, VP, VV, SP, AP 等分支比
 - 研究 CP 破坏和 $SU(3)$ 味道对称性破缺
- ✓ 理解强子谱
 - 为深入探讨轻标量介子提供实验支持
- ✓ 为其他测量提供重要衰变模型
 - 强相位测量等

分析策略和数据集



数据集:

$D^{0/+}$: 收集于2011, 2022-2024, 共 20 fb^{-1} @ $E_{\text{cm}} = 3.773 \text{ GeV}$

D_s^+ : 收集于2013-2017, 共 7.33 fb^{-1} @ $E_{\text{cm}} = 4.128 - 4.226 \text{ GeV}$

单标记方法 (ST): 仅重建标记侧 D 介子

— 相对高的本底

— 更高的 效率

振幅构造:

$$A(p_j) = F_{D_s}^L F_r^L P^L S^L$$

$F_{D_s,r}^L$: Blatt-Weisskopf barrier factors

P^L : Propagator

S^L : Spin-dependent angular term

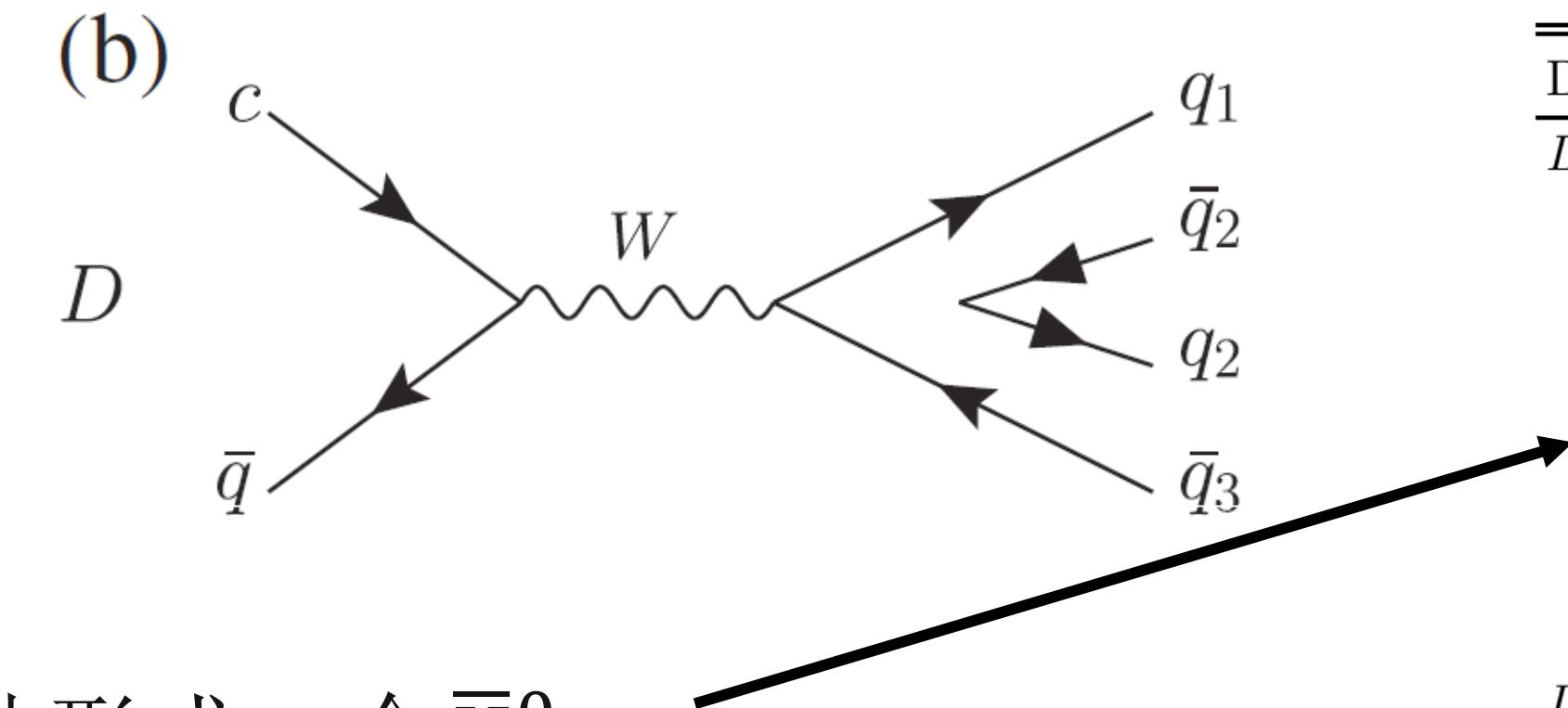
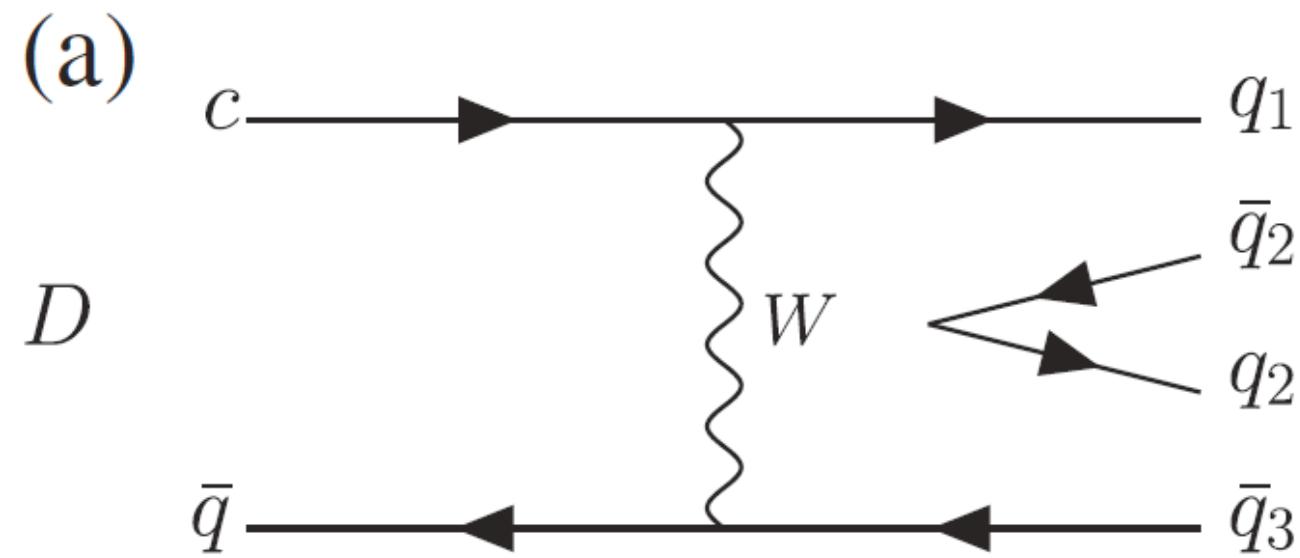
双标记方法 (DT): 同时重建信号和标记侧 D 介子

— 低本底 以研究不同衰变过程

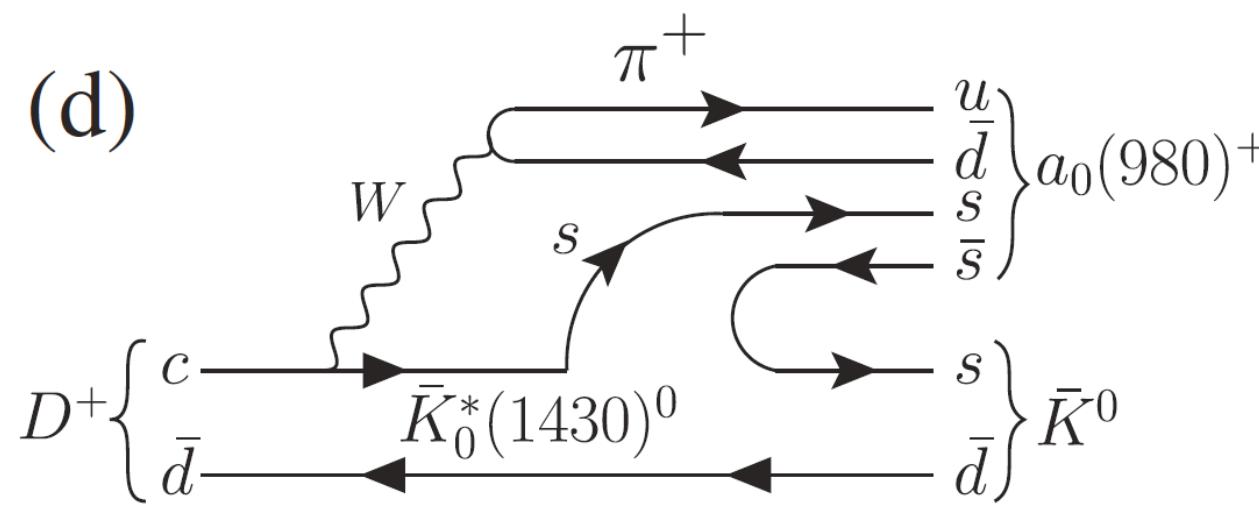
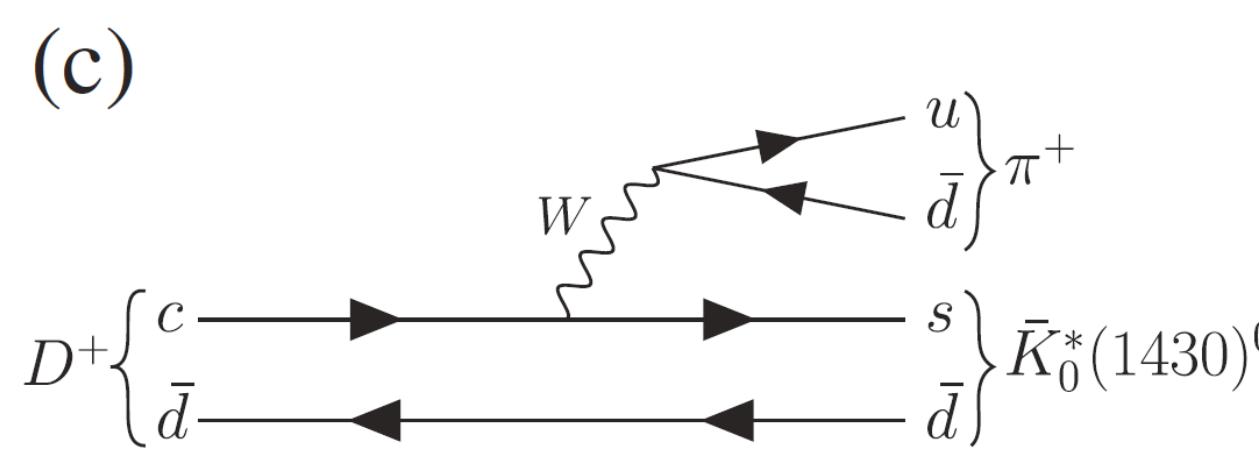
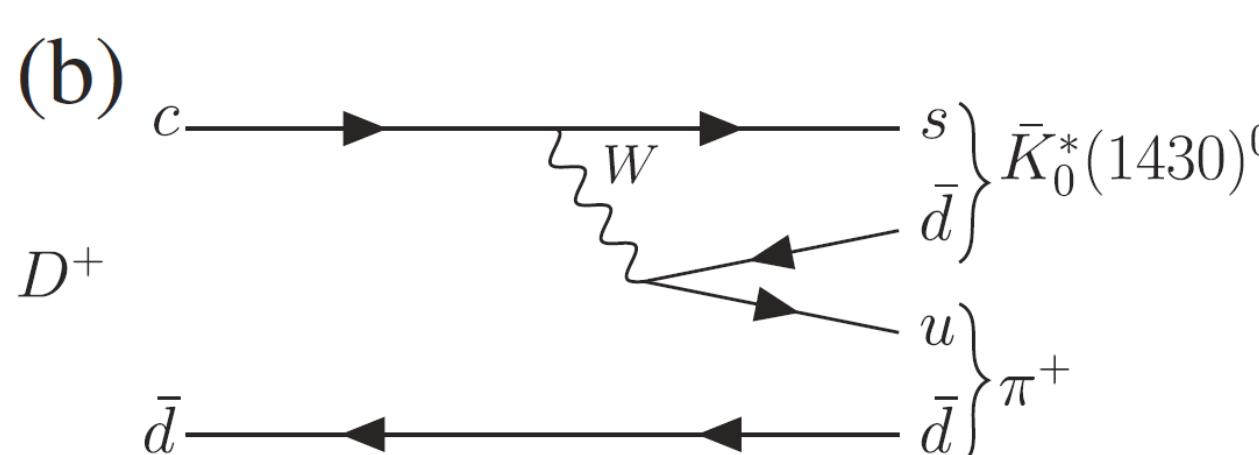
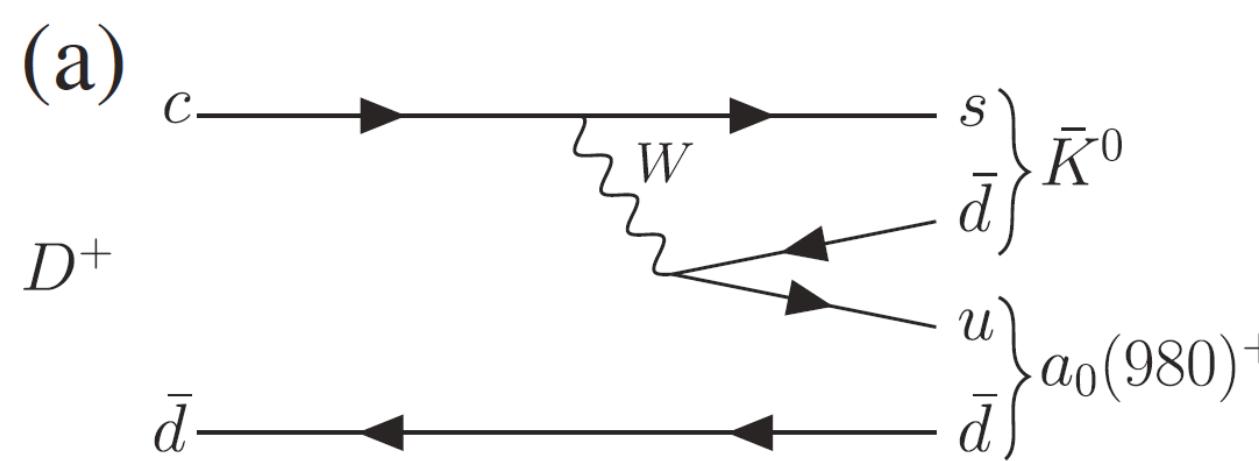
— 标记侧系统误差 几乎被抵消

Observation of $D^+ \rightarrow K_S^0 a_0(980)^+$

[Phys. Rev. Lett. 132, 131903 \(2024\)](#)



Decay	Amplitude	$D \rightarrow SP$
$D^+ \rightarrow f_0\pi^+$	$\frac{1}{\sqrt{2}}\alpha V_{cd}^* V_{ud}(T + C' + A + A')$	$\frac{1}{\sqrt{2}}\alpha V_{cd}^* V_{ud}(T + C' + A + A')$
$\rightarrow f_0 K^+$	$V_{cd}^* V_{us} \left[\frac{1}{\sqrt{2}}\alpha(T + A') + \beta A \right]$	$V_{cd}^* V_{us} \left[\frac{1}{\sqrt{2}}\alpha(T + A') + \beta A \right]$
$\rightarrow a_0^+ \bar{K}^0$	$V_{cs}^* V_{ud}(T' + C)$	无E和A贡献
$\rightarrow a_0^0 \pi^+$	$\frac{1}{\sqrt{2}}V_{cd}^* V_{ud}(-T - C' - A + A')$	$\frac{1}{\sqrt{2}}V_{cd}^* V_{ud}(-T - C' - A + A')$
$\rightarrow \sigma \pi^+$	$\frac{1}{\sqrt{2}}\beta V_{cd}^* V_{ud}(T + C' + A + A') - \alpha V_{cs}^* V_{us} C'$	$\frac{1}{\sqrt{2}}\beta V_{cd}^* V_{ud}(T + C' + A + A') - \alpha V_{cs}^* V_{us} C'$
$\rightarrow \bar{\kappa}^0 \pi^+$	$V_{cs}^* V_{ud}(T + C')$	$V_{cs}^* V_{ud}(T + C')$
$\rightarrow \bar{\kappa}^0 K^+$	$V_{cs}^* V_{us} T + V_{cd}^* V_{ud} A$	$V_{cs}^* V_{us} T + V_{cd}^* V_{ud} A$
$D^0 \rightarrow f_0\pi^0$	$\frac{1}{2}\alpha V_{cd}^* V_{ud}(-C + C' - E - E') + \frac{1}{\sqrt{2}}\beta V_{cs}^* V_{us} C'$	$\frac{1}{2}\alpha V_{cd}^* V_{ud}(-C + C' - E - E') + \frac{1}{\sqrt{2}}\beta V_{cs}^* V_{us} C'$
$\rightarrow f_0 \bar{K}^0$	$V_{cs}^* V_{ud} \left[\frac{1}{\sqrt{2}}\alpha(C + E) + \beta E' \right]$	$V_{cs}^* V_{ud} \left[\frac{1}{\sqrt{2}}\alpha(C + E) + \beta E' \right]$
$\rightarrow a_0^+ \pi^-$	$V_{cd}^* V_{ud}(T' + E)$	$V_{cd}^* V_{ud}(T' + E)$
$\rightarrow a_0^- \pi^+$	$V_{cd}^* V_{ud}(T + E')$	$V_{cd}^* V_{ud}(T + E')$
$\rightarrow a_0^0 K^-$	$V_{cs}^* V_{ud}(T' + E)$	$V_{cs}^* V_{ud}(T' + E)$
$\rightarrow a_0^0 \bar{K}^0$	$V_{cs}^* V_{ud}(C - E)/\sqrt{2}$	$V_{cs}^* V_{ud}(C - E)/\sqrt{2}$
$\rightarrow a_0^- K^+$	$V_{cd}^* V_{us}(T + E')$	$V_{cd}^* V_{us}(T + E')$
$\rightarrow \sigma \pi^0$	$\frac{1}{2}V_{cd}^* V_{ud}\beta(-C + C' - E - E') - \frac{1}{\sqrt{2}}\alpha V_{cs}^* V_{us} C'$	$\frac{1}{2}V_{cd}^* V_{ud}\beta(-C + C' - E - E') - \frac{1}{\sqrt{2}}\alpha V_{cs}^* V_{us} C'$



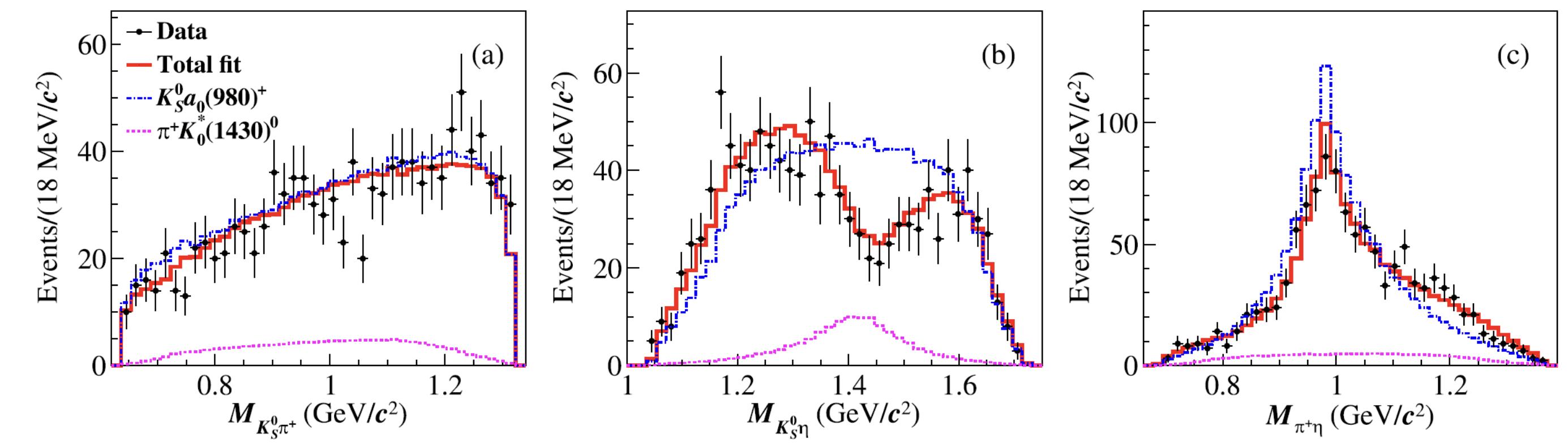
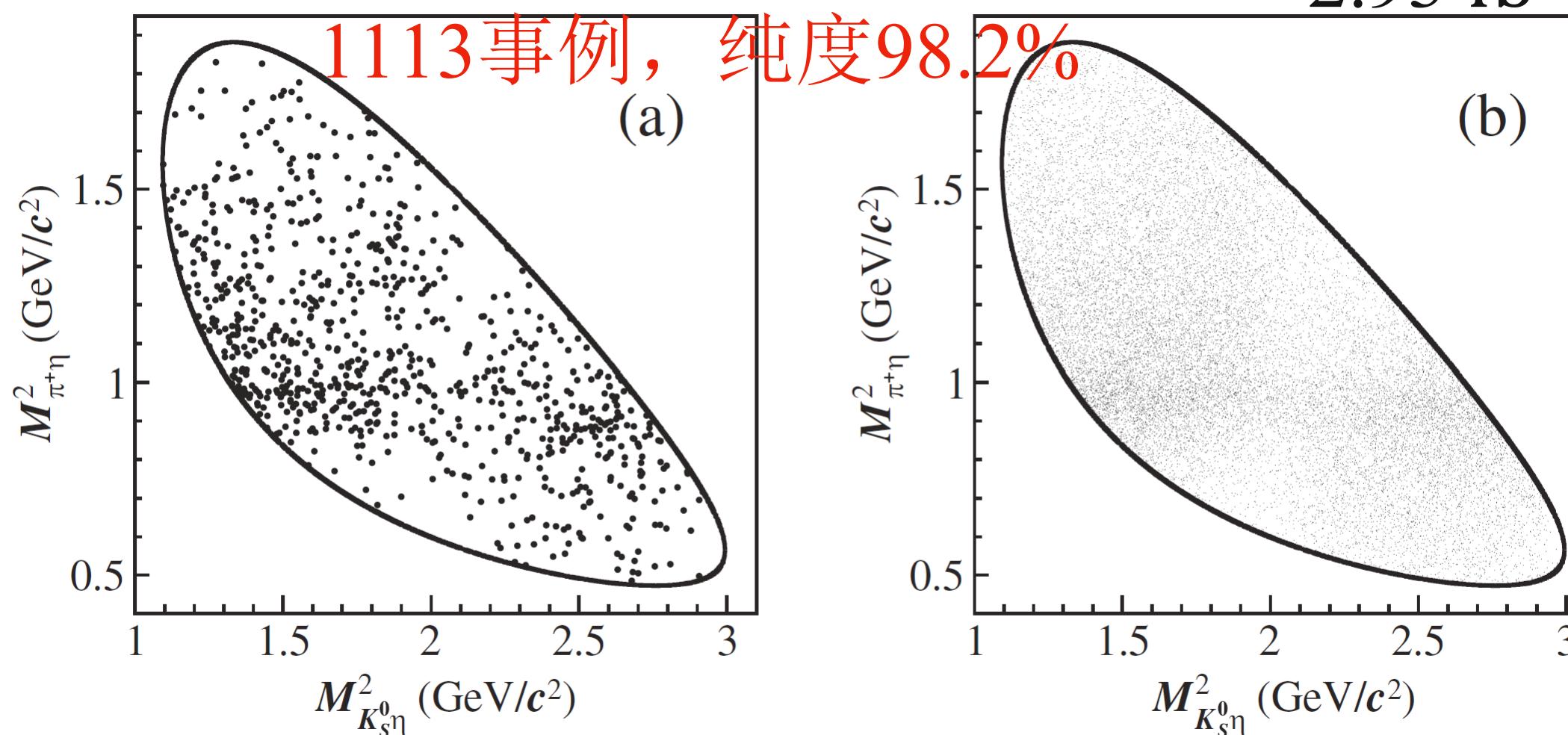
四夸克态下的 $a_0(980)^+$

- 为图方法研究 $D \rightarrow SP$ 过程提供重要参考
- 理解 $D \rightarrow a_0(980)P$ 理论与实验不一致
- 帮助研究 $a_0(980)$ 性质

Observation of $D^+ \rightarrow K_S^0 a_0(980)^+$

[Phys. Rev. Lett. 132, 131903 \(2024\)](#)

2.93 fb⁻¹



	Amplitude	Phase ϕ (rad)	FF (%)	Significance
首次观测	$D^+ \rightarrow K_S^0 a_0(980)^+$	0.0 (fixed)	$105.00 \pm 0.94 \pm 1.04 \pm 0.07$	$>10\sigma$ 考虑了干涉效应
	$D^+ \rightarrow \bar{K}_0^*(1430)^0 \pi^+$	$2.58 \pm 0.06 \pm 0.09 \pm 0.01$	$10.83 \pm 1.50 \pm 1.27 \pm 0.08$	$>10\sigma$

相消干涉: $(15.83 \pm 1.53 \pm 1.65)\%$

$$\mathcal{B}(D^+ \rightarrow K_S^0 \pi^+ \eta) = (1.27 \pm 0.04 \pm 0.03)\%$$

$$\left. \begin{aligned} &\mathcal{B}(D^+ \rightarrow K_S^0 a_0(980)^+, a_0(980)^+ \rightarrow \pi^+ \eta) = (1.33 \pm 0.05 \pm 0.04)\% \\ &\mathcal{B}(D^+ \rightarrow \bar{K}_0^*(1430) \pi^+, \bar{K}_0^*(1430) \rightarrow K_S^0 \eta) = (0.14 \pm 0.02 \pm 0.02)\% \end{aligned} \right\}$$

Observation of $D \rightarrow a_0(980)\pi$

[arXiv:2404.09219](#)

$$A(D_s^+ \rightarrow a_0(980)^+\pi^0) = -A(D_s^+ \rightarrow a_0(980)^0\pi^+)$$
$$\mathcal{B}(D_s^+ \rightarrow a_0(980)^{+(0)}\pi^{0(+)}, a_0(980)^{+(0)} \rightarrow \pi^{+(0)}\eta)$$
$$= (1.46 \pm 0.15 \pm 0.23)\% \quad \text{Phys. Rev. Lett. 123, 112001 (2019)}$$

比其它WA过程大
一个数量级



末态相互作用

Observation of a a_0 -like state $a_0(1817)^+$ in D_s^+
 $\rightarrow K_S^0 K^+ \pi^0$ [Phys. Rev. Lett. 129, 182001 \(2022\)](#)

可能是 $a_0(980)^{+(0)}$ 的激发态

$a_0(980)^{+(0)}$ 性质研究:

- 四夸克态
- 分子态
- $K^{(*)} \bar{K}^{(*)}$ 分子态

?

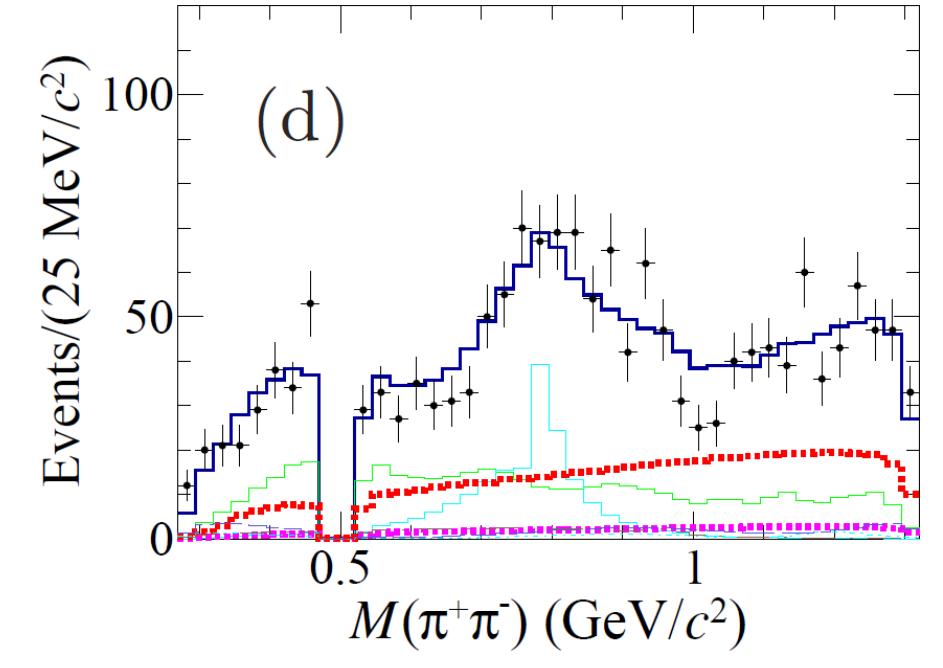
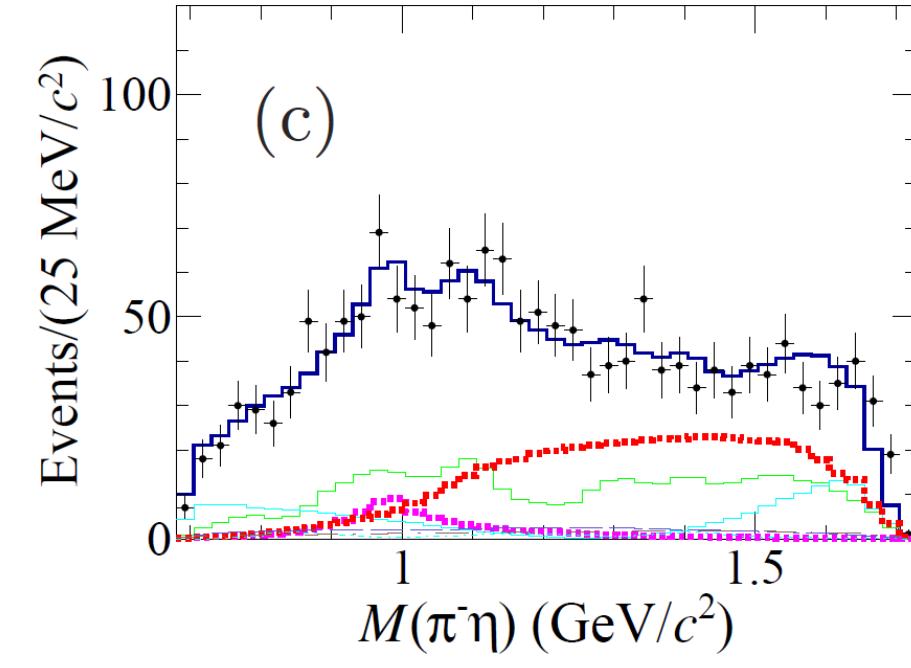
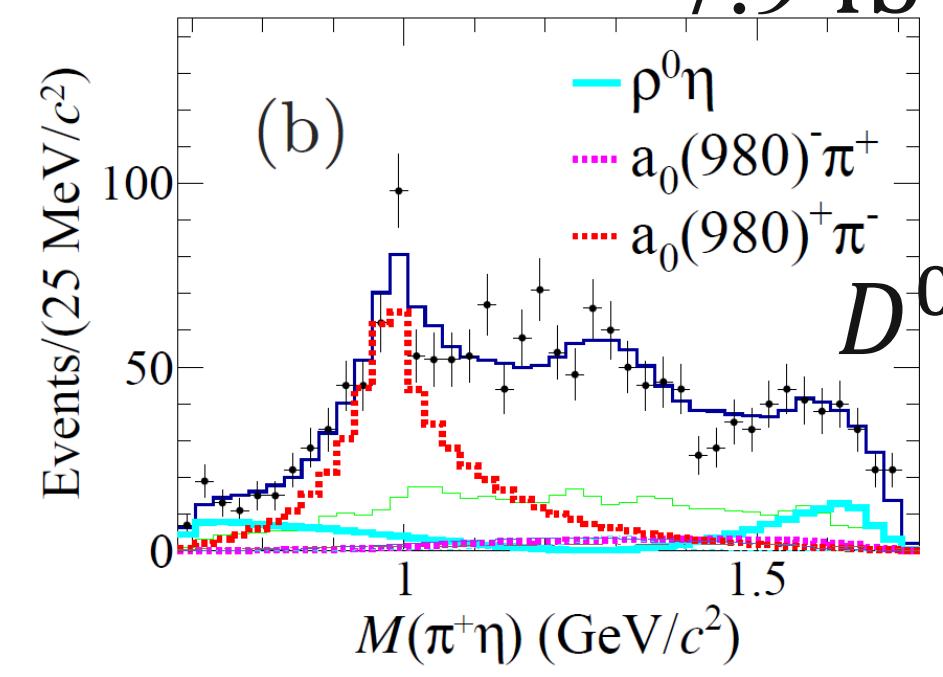
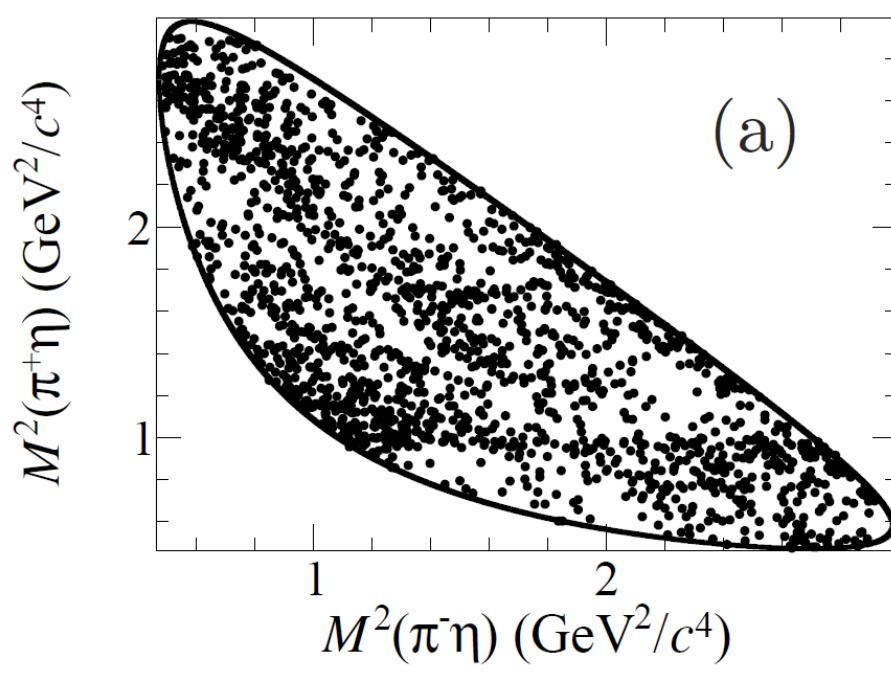
$$r_{+-} = \mathcal{B}(D^0 \rightarrow a_0(980)^+\pi^-) / \mathcal{B}(D^0 \rightarrow a_0(980)^-\pi^+) < 0.05$$

- $a_0(980)^\pm \pi^\mp$ 在 $D^0 \rightarrow K_S^0 K^\pm \pi^\mp$ 中误差巨大
- 在 $D^0 \rightarrow \pi^+\pi^-\eta$ 中仅观察到峰状结构无法确定其存在

$$r_{+/0} = \mathcal{B}(D^+ \rightarrow a_0(980)^+\pi^0) / \mathcal{B}(D^+ \rightarrow a_0(980)^0\pi^+)$$

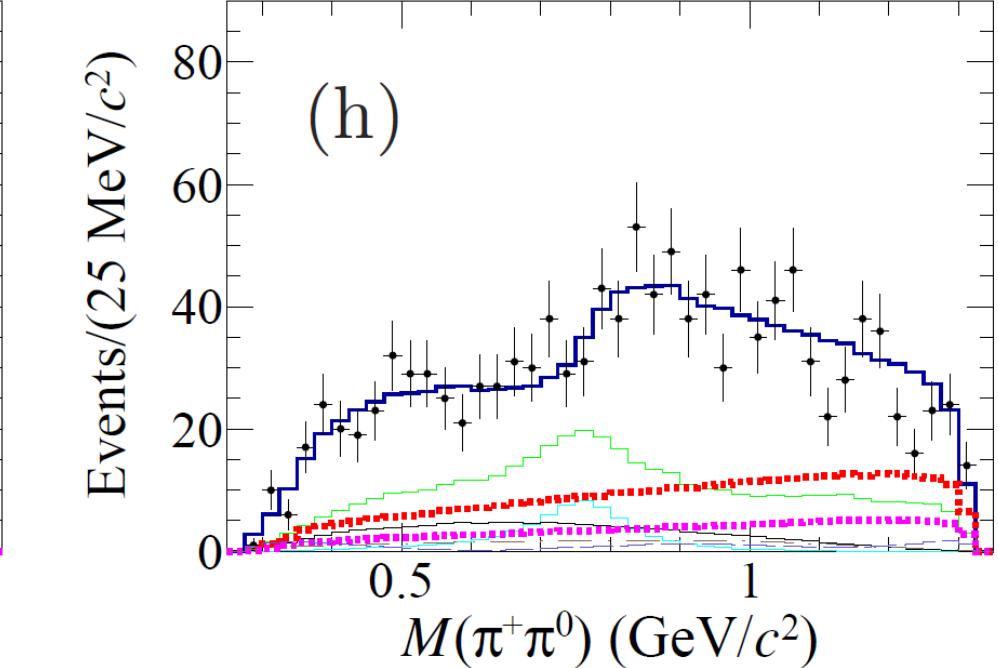
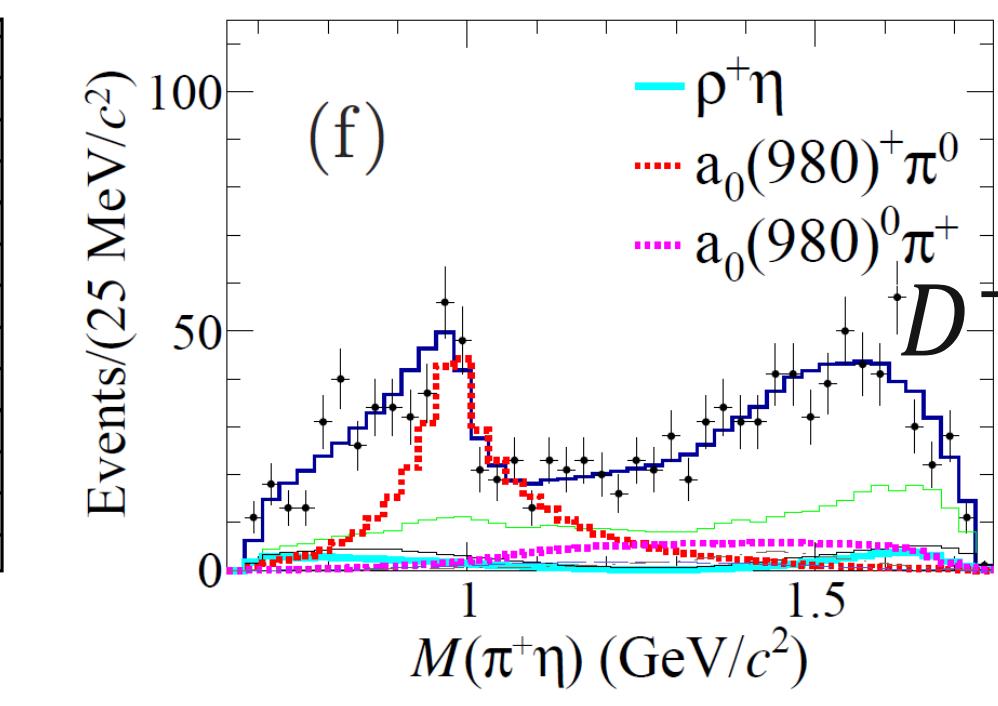
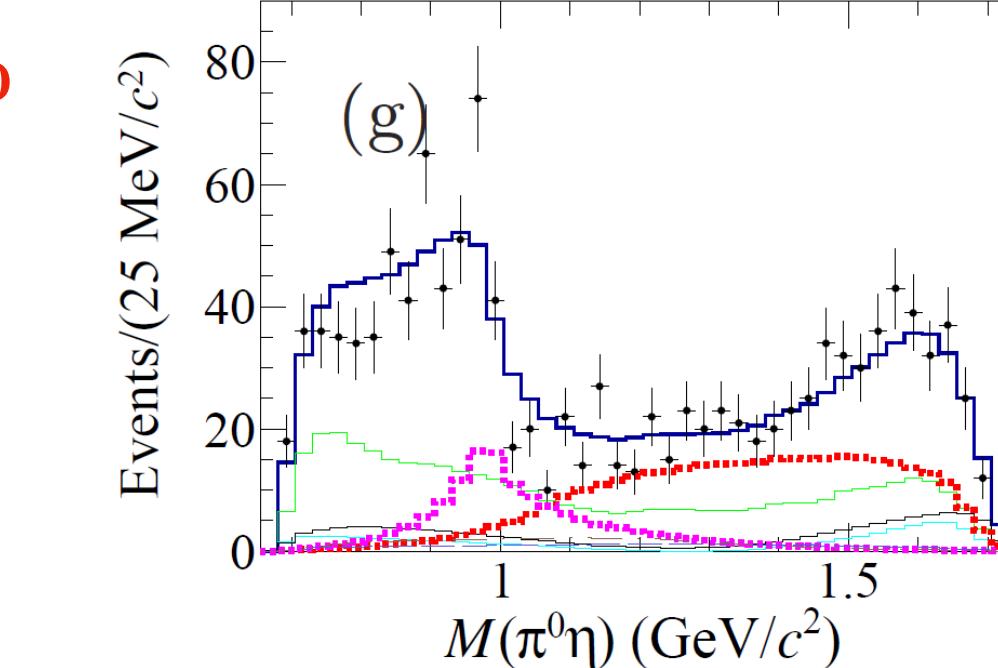
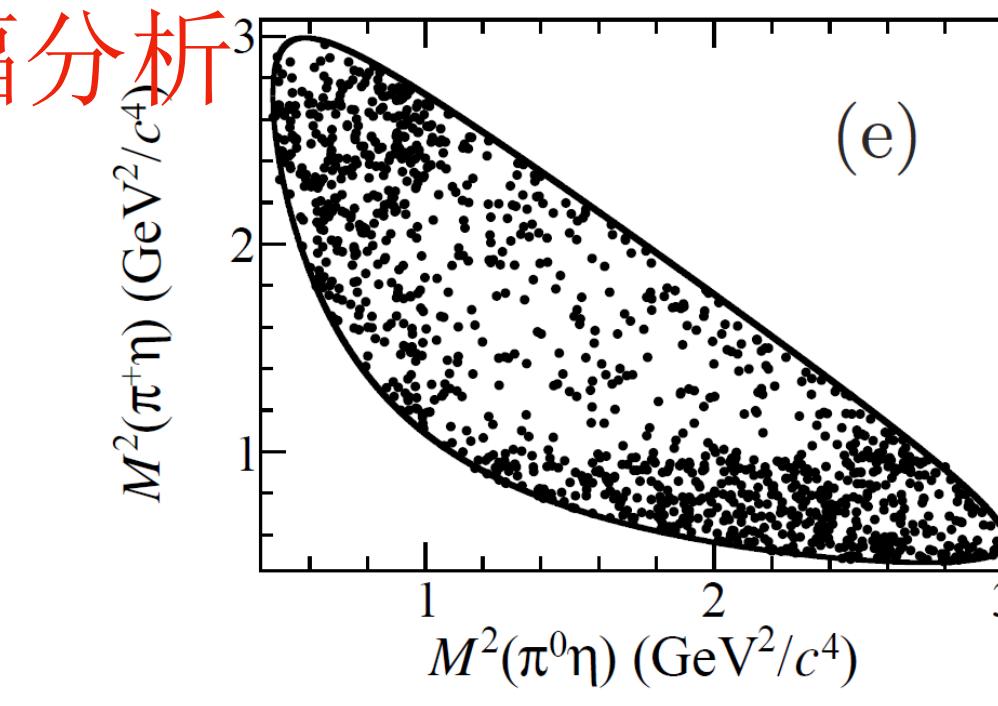
- 对于 D^+ 存在来自外/内发射树图贡献

Observation of $D \rightarrow a_0(980)\pi$



首次振幅分析
7.9 fb⁻¹

1678 事例
纯度 74.1%



arXiv:2404.09219

1226 事例
纯度 65.7%

Amplitude	Phase (in unit rad)	FF (%)	Significance (σ)	BF ($\times 10^{-3}$)
$D^0 \rightarrow \rho^0\eta$	0 (fixed)	$15.2 \pm 1.7 \pm 1.0$	> 10	$0.19 \pm 0.02 \pm 0.01$
$D^0 \rightarrow a_0(980)^-\pi^+$	$0.06 \pm 0.16 \pm 0.12$	$5.9 \pm 1.3 \pm 1.0$	8.9	$0.07 \pm 0.02 \pm 0.01$
$D^0 \rightarrow a_0(980)^+\pi^-$	$-1.06 \pm 0.12 \pm 0.10$	$44.0 \pm 4.0 \pm 5.3$	> 10	$0.55 \pm 0.05 \pm 0.07$
$D^0 \rightarrow a_2(1320)^+\pi^-$	$-1.16 \pm 0.25 \pm 0.23$	$2.1 \pm 0.9 \pm 0.8$	4.5	$0.03 \pm 0.01 \pm 0.01$
$D^0 \rightarrow a_2(1700)^+\pi^-$	$0.08 \pm 0.17 \pm 0.23$	$5.5 \pm 1.8 \pm 2.7$	6.1	$0.07 \pm 0.02 \pm 0.03$
$D^0 \rightarrow (\pi^+\pi^-)_{S\text{-wave}}\eta$	$-0.92 \pm 0.29 \pm 0.14$	$3.9 \pm 1.8 \pm 2.1$	5.3	$0.05 \pm 0.02 \pm 0.03$
r_{+-}		$7.5^{+2.5}_{-0.8} \pm 1.7$	7.7*	-
$D^+ \rightarrow \rho^+\eta$	$-4.03 \pm 0.19 \pm 0.13$	$9.3 \pm 3.0 \pm 2.1$	6.0	$0.20 \pm 0.07 \pm 0.05$
$D^+ \rightarrow (\pi^+\pi^0)_V\eta$	$-0.64 \pm 0.22 \pm 0.19$	$15.8 \pm 4.8 \pm 5.2$	4.7	$0.34 \pm 0.11 \pm 0.11$
$D^+ \rightarrow a_0(980)^+\pi^0$	0 (fixed)	$43.7 \pm 5.6 \pm 1.9$	9.1	$0.95 \pm 0.12 \pm 0.05$
$D^+ \rightarrow a_0(980)^0\pi^+$	$2.44 \pm 0.20 \pm 0.10$	$17.0 \pm 4.4 \pm 1.7$	7.9	$0.37 \pm 0.10 \pm 0.04$
$D^+ \rightarrow a_2(1700)^+\pi^0$	$0.92 \pm 0.20 \pm 0.14$	$4.2 \pm 2.1 \pm 0.7$	3.6	$0.09 \pm 0.05 \pm 0.02$
$D^+ \rightarrow a_0(1450)^+\pi^0$	$0.63 \pm 0.41 \pm 0.30$	$7.0 \pm 2.8 \pm 0.7$	4.7	$0.15 \pm 0.06 \pm 0.02$
$r_{+/0}$		$2.6 \pm 0.6 \pm 0.3$	4.0*	-

* The significance is for the test hypothesis $r = 1.0$.

$K^*K \rightarrow a_0(980)\pi$
重散射的重要性

$$r_{+-} = 7.5^{+2.5}_{-0.8} \pm 1.7$$

$$r_{+/0} = 2.6 \pm 0.6 \pm 0.3$$

$$\mathcal{B}(a_0(980)^+) > \mathcal{B}(a_0(980)^{-(0)})$$

$r_{+-} \rightarrow E\text{图} > T\text{图}$

$$\mathcal{B}(D^0 \rightarrow \pi^+\pi^-\eta) = (1.24 \pm 0.04 \pm 0.03) \times 10^{-3}$$

$$\mathcal{B}(D^+ \rightarrow \pi^+\pi^0\eta) = (2.18 \pm 0.12 \pm 0.05) \times 10^{-3}$$

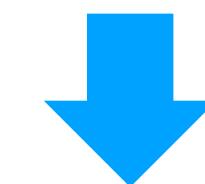
Amplitude analysis of $D^+ \rightarrow K_S^0 \pi^+ \pi^0 \pi^0$

[JHEP09\(2023\)077](#)

BESIII 2022: $\mathcal{B}(D^+ \rightarrow K_S^0 \pi^+ \pi^0 \pi^0) = (2.904 \pm 0.062 \pm 0.087)\%$

Source	1	2	3	4	5	6	7	8
$N_{\text{ST}}^{\text{tot}}$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$(K/\pi)^{\pm}$ tracking	–	0.4	0.4	0.2	0.6	0.2	0.6	–
$(K/\pi)^{\pm}$ PID	–	0.4	0.4	0.2	0.6	0.2	0.6	–
K_S^0 reconstruction	1.6	1.6	–	1.6	1.6	1.6	–	3.2
π^0 reconstruction	2.1	2.1	1.4	1.4	0.7	2.1	1.4	0.7
2D fit	1.8	2.0	1.9	0.8	1.6	4.9	3.5	3.9
ΔE_{sig} requirement	0.3	0.6	0.5	0.7	0.4	0.8	0.6	0.6
Quoted \mathcal{B}	0.12	0.10	0.10	0.10	0.08	0.12	0.07	0.14
MC modeling	1.8	0.6	1.2	1.6	2.9	0.8	0.3	–
MC statistics	0.7	0.8	0.9	0.6	0.8	0.9	0.6	0.5
QC effect	0.7	0.6	0.6	–	–	–	–	0.7
Total	3.8	3.3	3.4	3.0	4.0	5.8	4.0	5.2

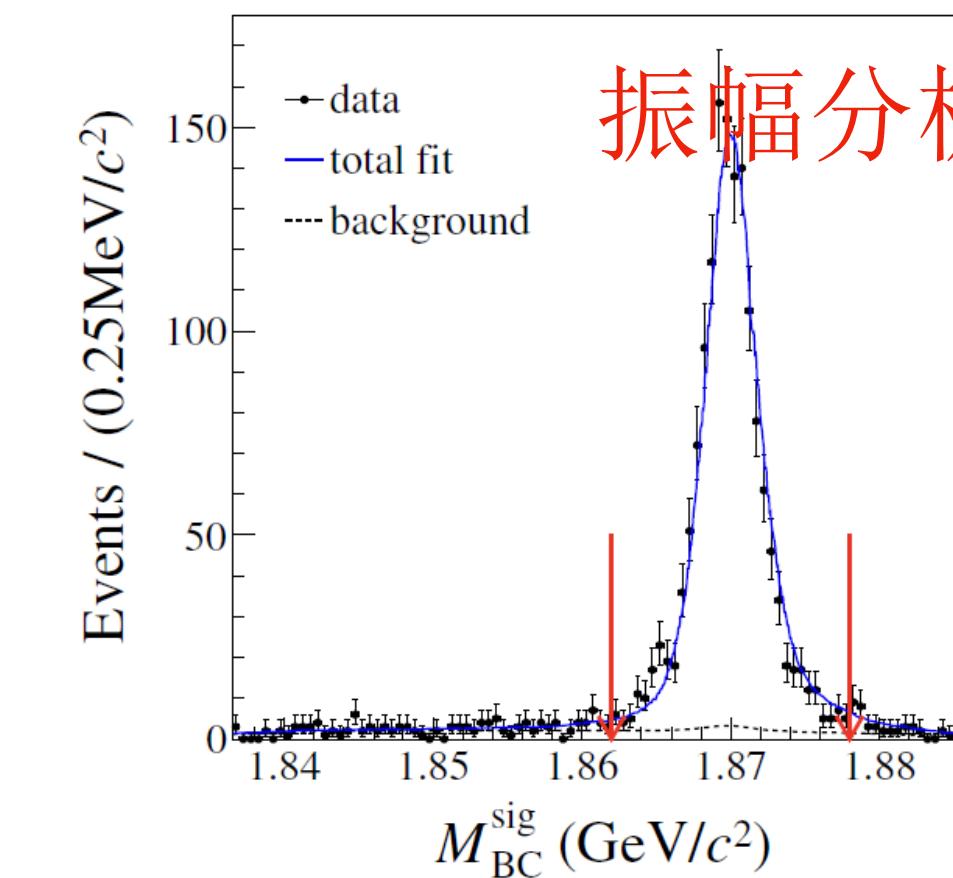
丰富的中间过程:
 $\bar{K}^{*0}, \rho^+, a_1(1260), K_1(1270), K_1(1400) \dots$



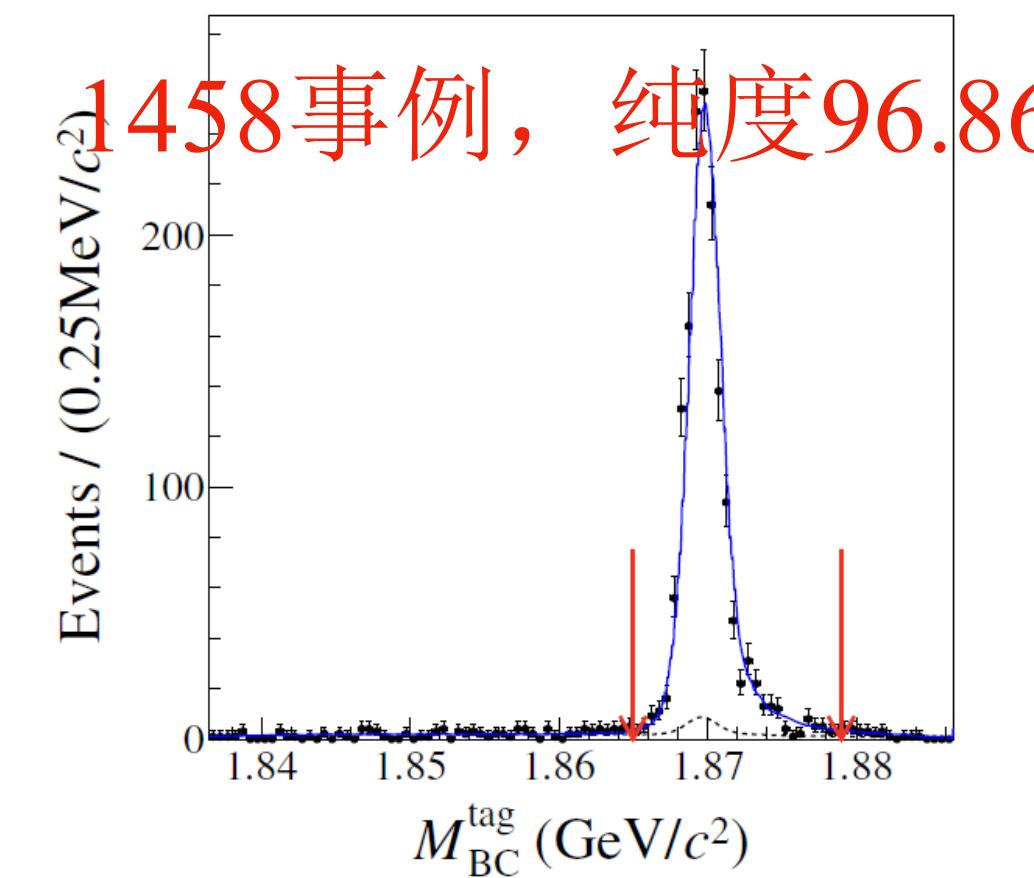
- 极化测量
- 轴矢量介子研究
- ...

MC 模型是系统误差的主要来源

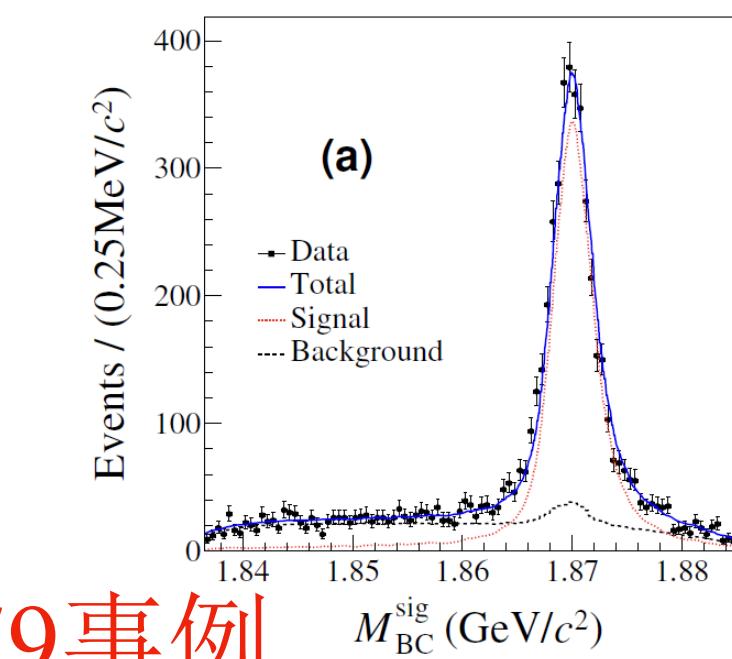
2.93 fb⁻¹



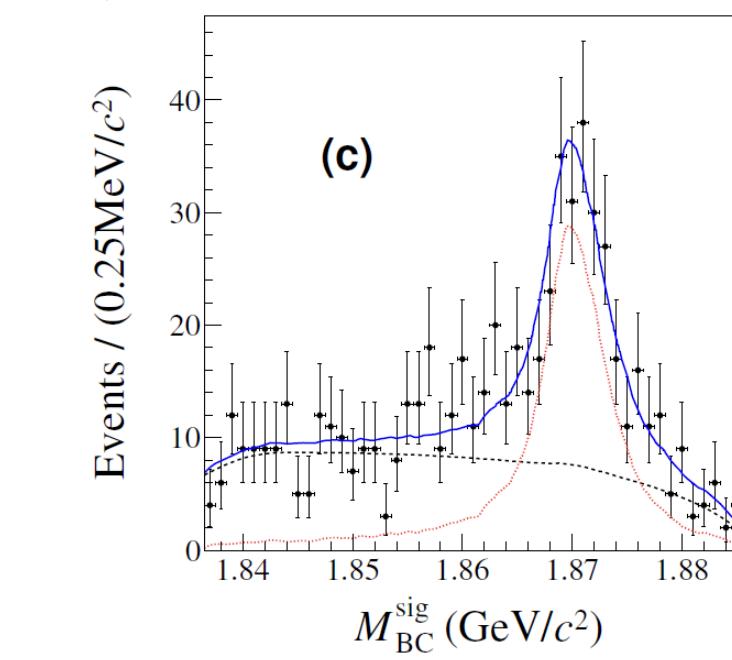
振幅分析:



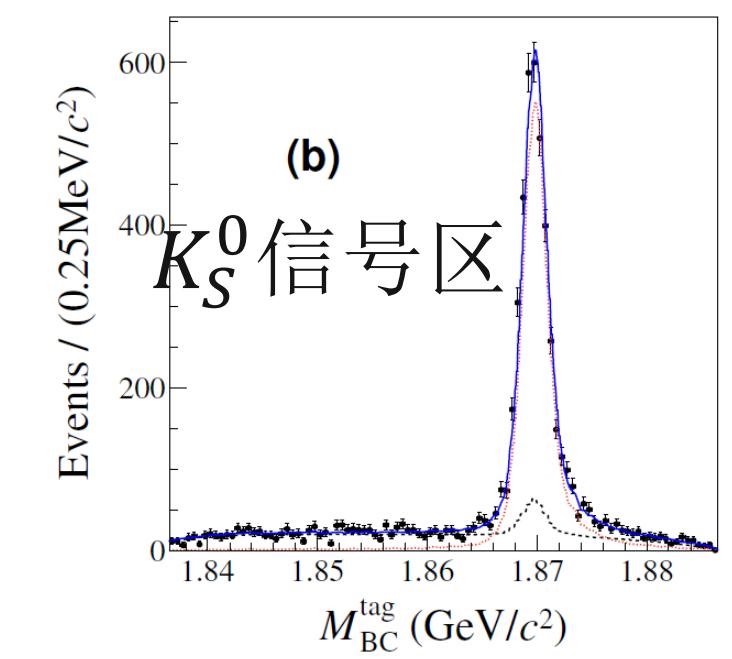
$M_{\text{BC}}^{\text{tag}} (\text{GeV}/c^2)$



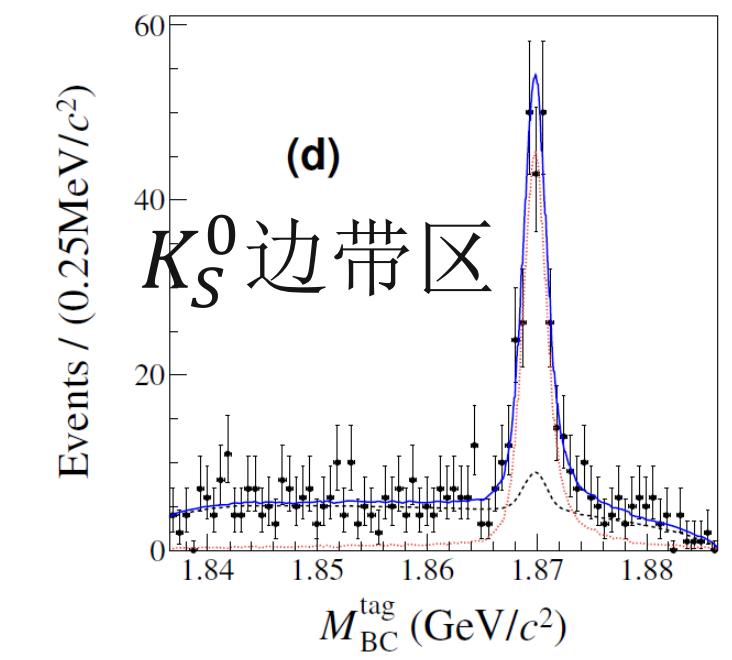
(a)



(c)



(b)



(d)

K_S^0 信号区

K_S^0 边带区

Amplitude analysis of $D^+ \rightarrow K_S^0 \pi^+ \pi^0 \pi^0$

[JHEP09\(2023\)077](#)

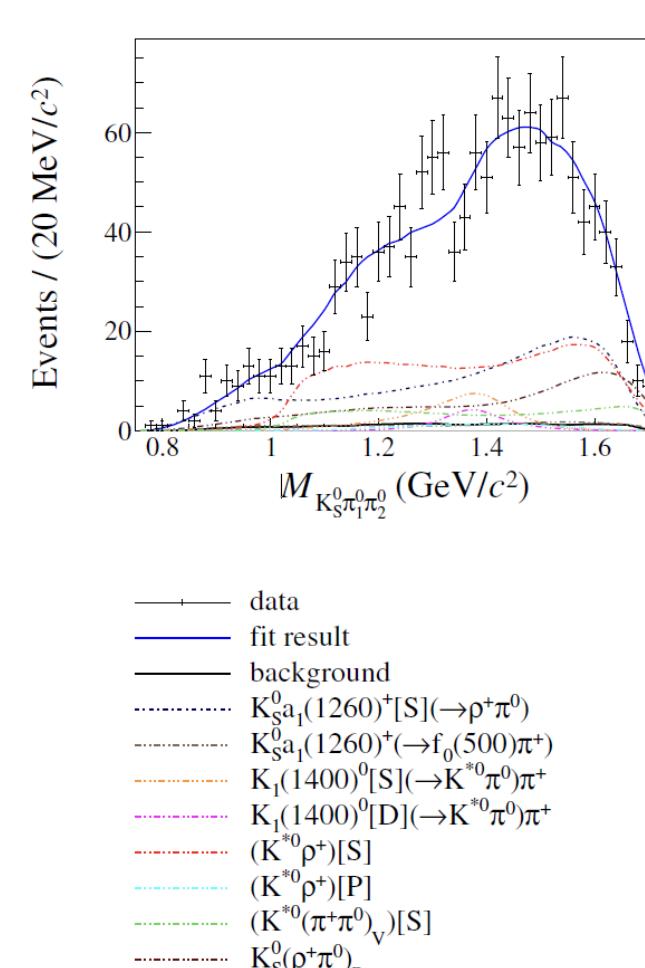
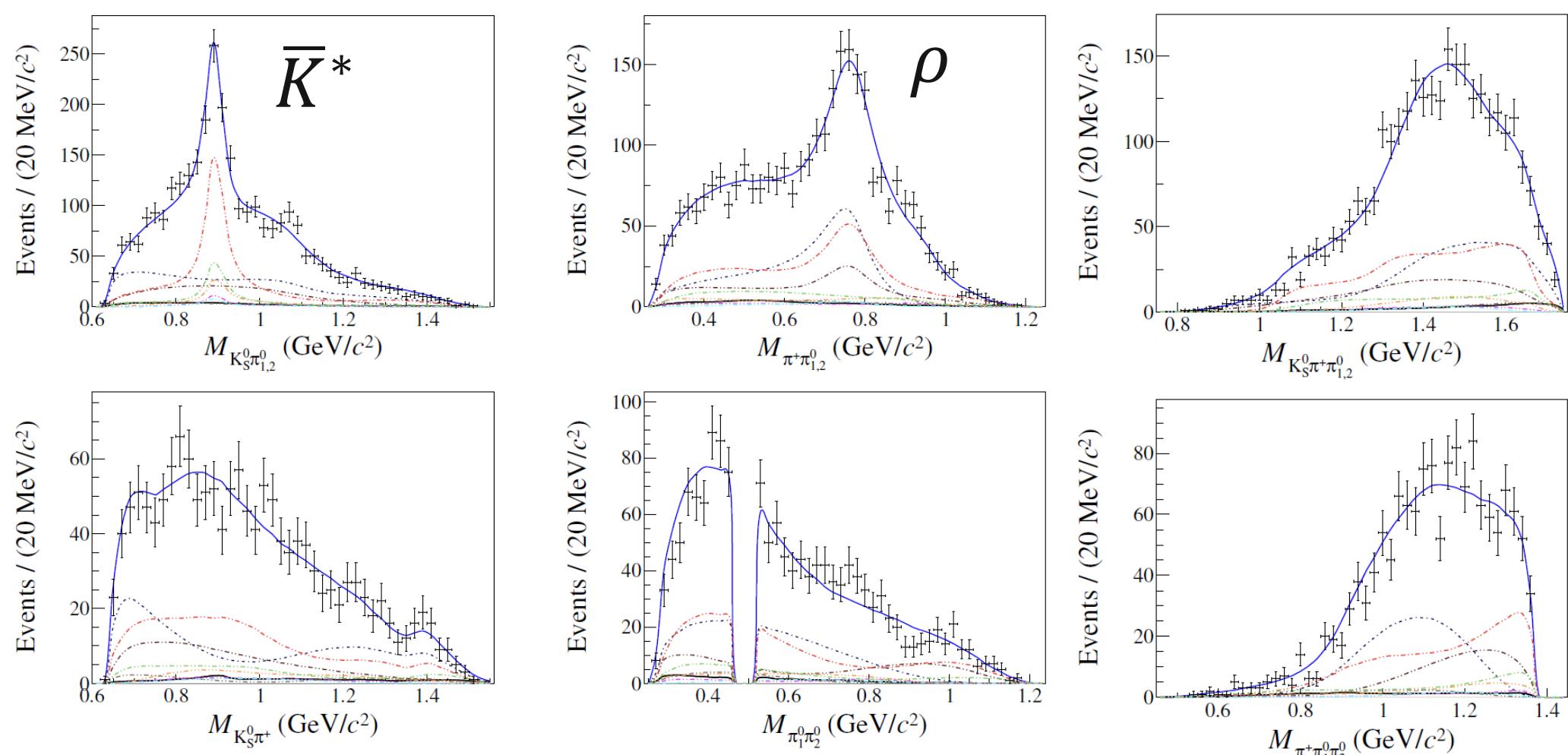
首次振幅分析

Amplitude	Phase ϕ_n (rad)	FF (%)	Significance (σ)
$D^+ \rightarrow K_S^0 a_1(1260)^+[S] (\rightarrow \rho^+ \pi^0)$	0.0 (fixed)	$30.0 \pm 3.6 \pm 4.2$	>10
$D^+ \rightarrow K_S^0 a_1(1260)^+ (\rightarrow f_0(500)\pi^+)$	$4.78 \pm 0.22 \pm 0.20$	$3.5 \pm 1.1 \pm 1.9$	6.9
$D^+ \rightarrow \bar{K}_1(1400)^0 [S] (\rightarrow \bar{K}^{*0} \pi^0) \pi^+$	$-3.01 \pm 0.12 \pm 0.16$	$6.0 \pm 1.2 \pm 0.3$	9.6
$D^+ \rightarrow \bar{K}_1(1400)^0 [D] (\rightarrow \bar{K}^{*0} \pi^0) \pi^+$	$4.29 \pm 0.16 \pm 0.20$	$2.4 \pm 0.6 \pm 0.2$	6.7
$D^+ \rightarrow \bar{K}_1(1400)^0 (\rightarrow \bar{K}^{*0} \pi^0) \pi^+$	—	$8.0 \pm 1.2 \pm 0.4$	—
$D^+ [S] \rightarrow \bar{K}^{*0} \rho^+$	—	$31.8 \pm 2.7 \pm 1.3$	>10
$D^+ [P] \rightarrow \bar{K}^{*0} \rho^+$	—	$1.7 \pm 0.6 \pm 0.1$	5.0
$D^+ \rightarrow \bar{K}^{*0} \rho^+$	—	$33.6 \pm 2.7 \pm 1.4$	—
$D^+ [S] \rightarrow \bar{K}^{*0} (\pi^+ \pi^0)_V$	—	$9.1 \pm 2.0 \pm 1.0$	9.4
$D^+ \rightarrow K_S^0 (\rho^+ \pi^0)_P$	$0.76 \pm 0.11 \pm 0.24$	$16.5 \pm 1.6 \pm 0.3$	>10

	This work	
$\bar{K}^{*0} \rho^+$	$(5.82 \pm 0.49 \pm 0.28)\%$	$(4.8 \pm 1.2 \pm 1.4)\%^{[1]}$
$K_S^0 a_1 [S], a_1 \rightarrow \rho^+ \pi^0$	$(8.66 \pm 1.04 \pm 1.24) \times 10^{-3}$	$(11.97 \pm 0.62 \pm 1.20 \pm 0.44) \times 10^{-3}^{[2]}$

[1] [Phys. Rev.D 45 \(1992\) 2196](#) [2] [Phys. Rev. D 100 \(2019\) 072008](#)

- 存在 $\bar{K}_1(1400)^0 (\rightarrow \bar{K}^{*0} \pi^0) \pi^+$, 无 $K_1(1270)^+ \pi^0$, 与理论预期一致
- 与 $D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-$ 一致

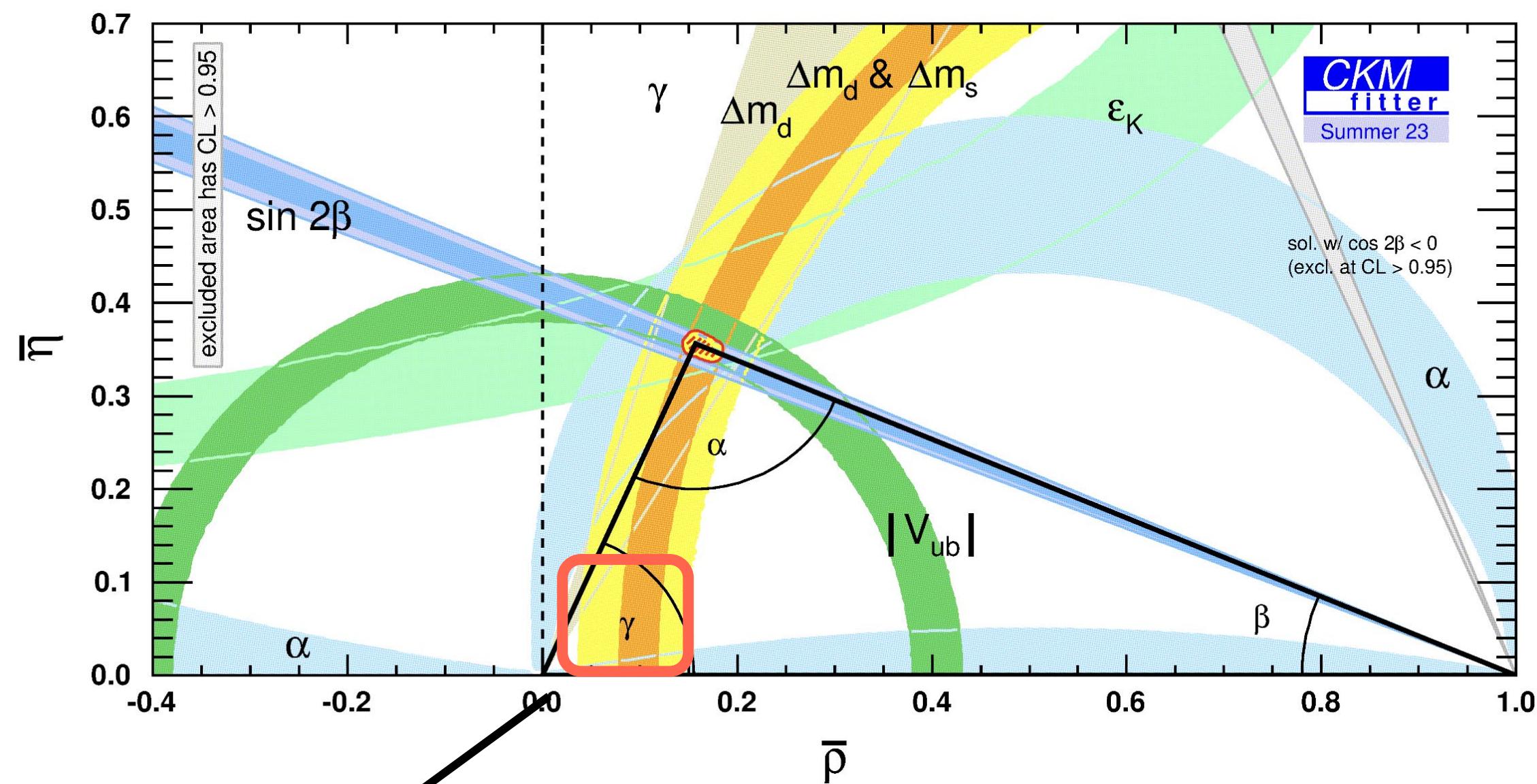


Source	Uncertainty (%)
ST yield	0.5
Tracking efficiency	0.1
PID efficiency	0.1
K_S^0 reconstruction	1.6
π^0 reconstruction	1.4
MC sample size	0.6
Quoted BFs	0.1
Amplitude model	0.6
2D fit	0.5
ΔE_{sig} requirement	0.4
Total	2.4

$$\mathcal{B}(D^+ \rightarrow K_S^0 \pi^+ \pi^0 \pi^0) = (2.888 \pm 0.058 \pm 0.069)\%$$

$$D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^- \quad \& \quad D^0 \rightarrow \pi^+ \pi^- \pi^0 \pi^0$$

Accepted by CPC



通过 $B \rightarrow DK$ 测量CKM角 γ 检验CKM三角形幺正性 $\rightarrow D^0$ 介子自共轭过程的 CP -even比例等参数是重要输入 $\rightarrow D^0 \rightarrow 4\pi$ 是提取CKM角 γ 的重要过程

需要可靠的 $D^0 \rightarrow 4\pi$ 振幅分析
模型为CKM角 γ 测量提供保障

$$D^0 \rightarrow 4\pi$$

- CP -even比例测量
- CP 破坏寻找
- $D \rightarrow VV, AP$ 过程提取

$$D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$$

- FOCUS, ~ 6000 事例, 本底 $\sim 10\%$
- CLEO, ~ 7000 事例, 本底 $\sim 20\%$

$$D^0 \rightarrow \pi^+ \pi^- \pi^0 \pi^0$$

- 无振幅分析结果

$D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ & $D^0 \rightarrow \pi^+ \pi^- \pi^0 \pi^0$

Accepted by CPC

Tag mode	$\bar{D}^0 \rightarrow K^+ \pi^-$	$\bar{D}^0 \rightarrow K^+ \pi^- \pi^0$	$\bar{D}^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$
N^{ST}	549586 ± 778	914531 ± 1321	600316 ± 856
ϵ^{ST}	0.6762 ± 0.0004	0.2953 ± 0.0001	0.3365 ± 0.0002
$N_{\pi^+ \pi^- \pi^+ \pi^-}^{DT}$	1719 ± 42	2560 ± 56	1520 ± 43
$\epsilon_{\pi^+ \pi^- \pi^+ \pi^-}^{DT}$	0.2792 ± 0.0006	0.1187 ± 0.0002	0.1221 ± 0.0003
$N_{\pi^+ \pi^- \pi^0 \pi^0}^{DT}$ (non- η)	721 ± 32	917 ± 39	562 ± 29
$\epsilon_{\pi^+ \pi^- \pi^0 \pi^0}^{DT}$ (non- η)	0.0818 ± 0.0003	0.0319 ± 0.0001	0.0334 ± 0.0001
$\frac{2rR\cos\delta}{1+r^2}$	-0.114 ± 0.004 [31]	-0.067 ± 0.005 [32]	$-0.046^{+0.013}_{-0.011}$ [32]

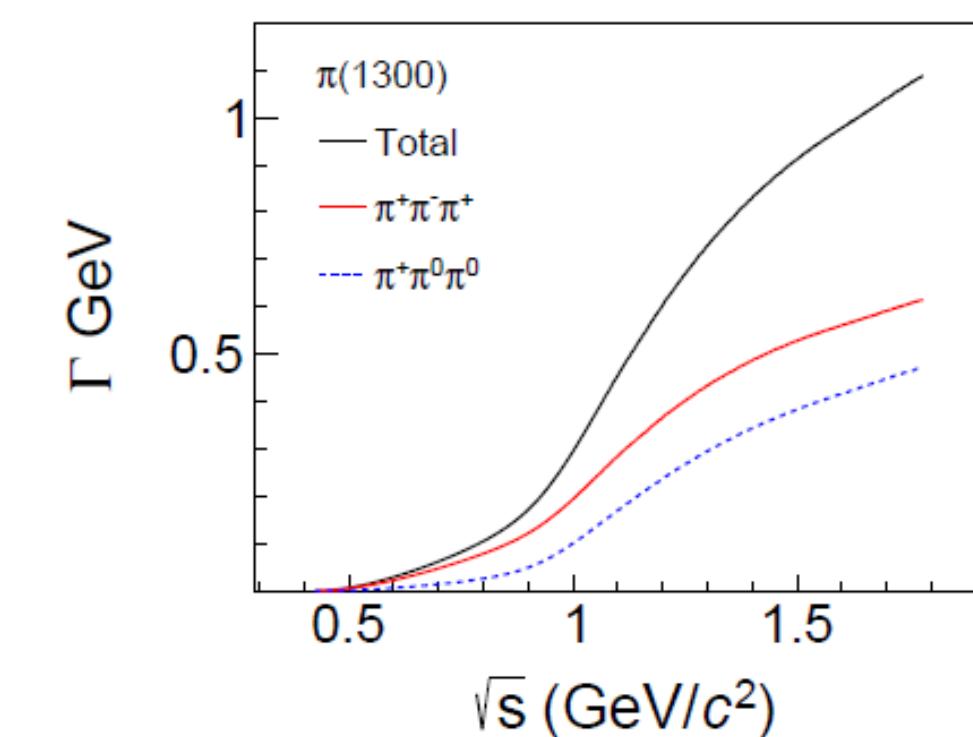
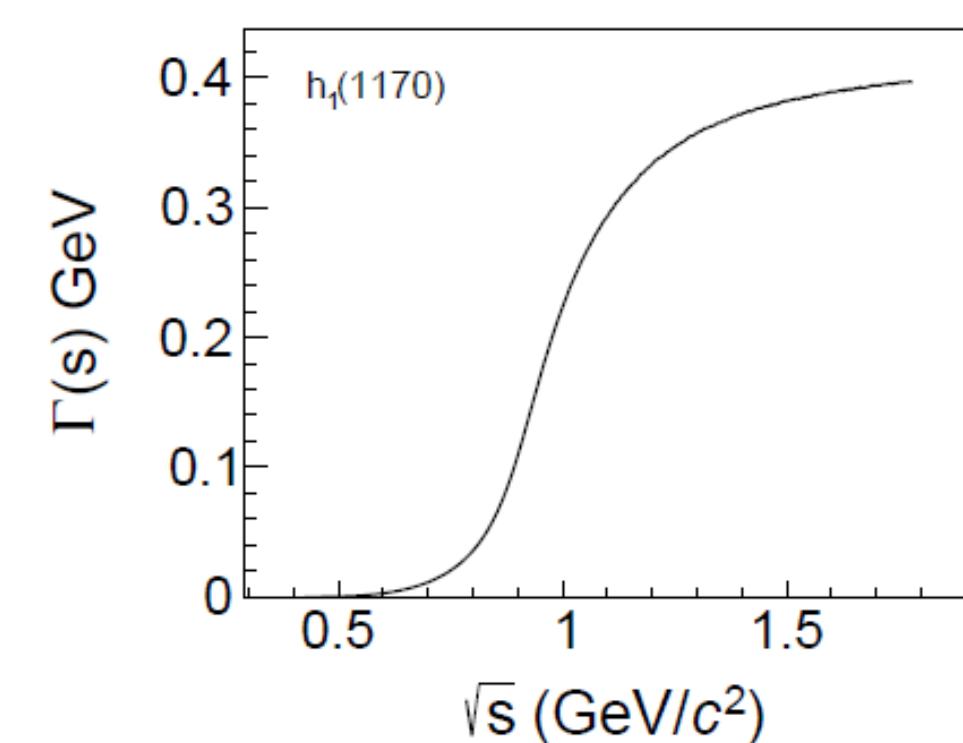
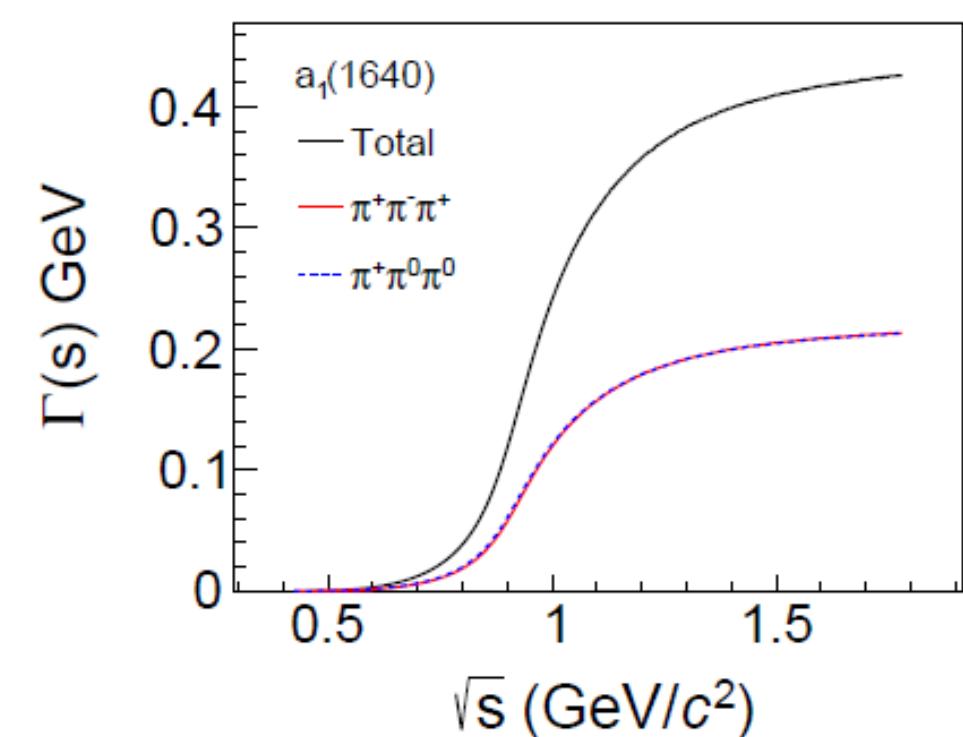
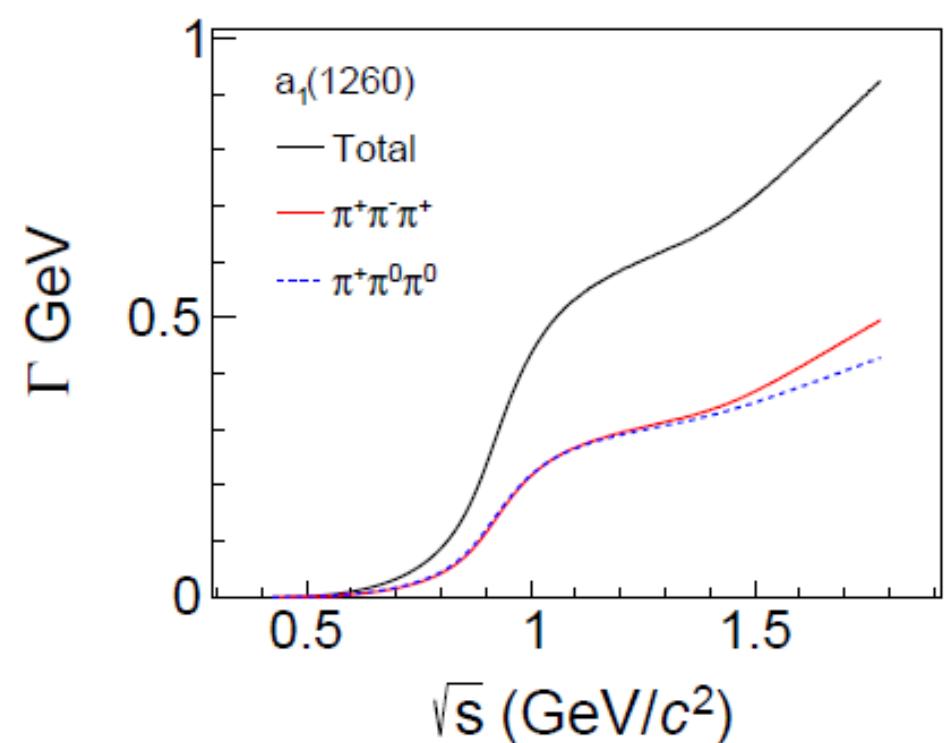
2.93 fb^{-1}

$D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$: ~5800事例, 纯度~92%

$D^0 \rightarrow \pi^+ \pi^- \pi^0 \pi^0$: ~2200事例, 纯度~63%

能量依赖宽度:

$$\Gamma_{a \rightarrow bcd}(s) \propto \frac{m_0}{\sqrt{s}} \int \sum_{\text{spin}} |A_{a \rightarrow bcd}|^2 d\Phi_3$$

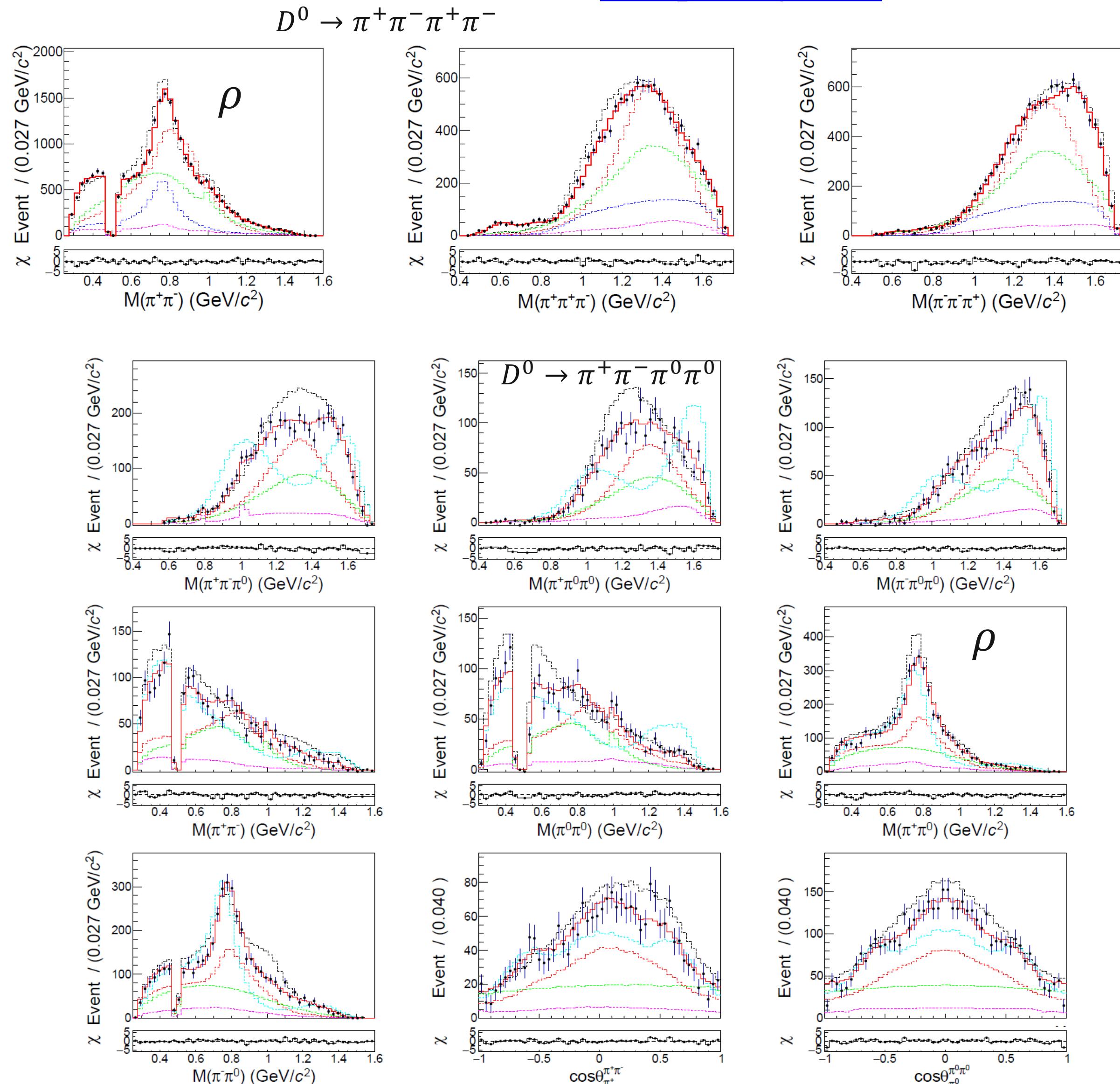


基于振幅分析模型结果

$D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ & $D^0 \rightarrow \pi^+ \pi^- \pi^0 \pi^0$

Accepted by CPC

Component	Branching fraction (%)	
	$\pi^+ \pi^- \pi^+ \pi^-$	$\pi^+ \pi^- \pi^0 \pi^0$
$D^0 \rightarrow a_1(1260)^+ \pi^-$	$0.566 \pm 0.024 \pm 0.008 \pm 0.110$	$0.546 \pm 0.027 \pm 0.011 \pm 0.069$
$D^0 \rightarrow a_1(1260)^- \pi^+$	$0.071 \pm 0.010 \pm 0.001 \pm 0.017$	$0.068 \pm 0.011 \pm 0.001 \pm 0.021$
$D^0 \rightarrow a_1(1260)^0 \pi^0$	-	$0.313 \pm 0.031 \pm 0.007 \pm 0.082$
$D^0 \rightarrow a_1(1640)^+ \pi^-$	$0.012 \pm 0.003 \pm 0.000 \pm 0.006$	$0.010 \pm 0.003 \pm 0.000 \pm 0.007$
$D^0 \rightarrow h_1(1170)^0 \pi^0$	-	$0.012 \pm 0.006 \pm 0.000 \pm 0.010$
$D^0 \rightarrow \pi(1300)^+ \pi^-$	$0.222 \pm 0.018 \pm 0.003 \pm 0.031$	$0.148 \pm 0.014 \pm 0.003 \pm 0.025$
$D^0 \rightarrow \pi(1300)^- \pi^+$	$0.162 \pm 0.016 \pm 0.002 \pm 0.028$	$0.108 \pm 0.011 \pm 0.002 \pm 0.021$
$D^0 \rightarrow \pi(1300)^0 \pi^0$	-	$0.221 \pm 0.027 \pm 0.005 \pm 0.033$
$D^0 \rightarrow \pi_2(1670)^0 \pi^0$	-	$0.010 \pm 0.002 \pm 0.000 \pm 0.004$
$D^0 \rightarrow \rho(770)^0 \rho(770)^0$	$0.193 \pm 0.013 \pm 0.003 \pm 0.022$	-
$D^0 \rightarrow \rho(770)^0 \rho(770)^0 [S]$	$0.012 \pm 0.004 \pm 0.000 \pm 0.003$	-
$D^0 \rightarrow \rho(770)^0 \rho(770)^0 [P]$	$0.067 \pm 0.007 \pm 0.001 \pm 0.006$	-
$D^0 \rightarrow \rho(770)^0 \rho(770)^0 [D]$	$0.159 \pm 0.015 \pm 0.002 \pm 0.017$	-
$D^0 \rightarrow \rho(770)^0 \rho(1450)^0$	$0.017 \pm 0.006 \pm 0.000 \pm 0.008$	-
$D^0 \rightarrow \rho(770)^0 \rho(1450)^0 [P]$	$0.007 \pm 0.003 \pm 0.000 \pm 0.003$	-
$D^0 \rightarrow \rho(770)^0 \rho(1450)^0 [D]$	$0.010 \pm 0.006 \pm 0.000 \pm 0.008$	-
$D^0 \rightarrow \rho(770)^+ \rho(770)^-$	-	$0.864 \pm 0.040 \pm 0.018 \pm 0.075$
$D^0 \rightarrow \rho(770)^+ \rho(770)^- [S]$	-	$0.124 \pm 0.019 \pm 0.003 \pm 0.033$
$D^0 \rightarrow \rho(770)^+ \rho(770)^- [P]$	-	$0.186 \pm 0.013 \pm 0.004 \pm 0.019$
$D^0 \rightarrow \rho(770)^+ \rho(770)^- [D]$	-	$0.342 \pm 0.029 \pm 0.007 \pm 0.024$
$D^0 \rightarrow \rho(770)^+ \rho(1450)^- [D]$	-	$0.016 \pm 0.008 \pm 0.000 \pm 0.016$
$D^0 \rightarrow \rho(770)^0 (\pi\pi)_S$	$0.019 \pm 0.004 \pm 0.000 \pm 0.012$	$0.010 \pm 0.002 \pm 0.000 \pm 0.004$
$D^0 \rightarrow (\pi^+ \pi^-)_S (\pi\pi)_S$	$0.432 \pm 0.032 \pm 0.006 \pm 0.066$	$0.356 \pm 0.029 \pm 0.007 \pm 0.049$
$D^0 \rightarrow f_2(1270)^0 (\pi\pi)_S$	$0.012 \pm 0.003 \pm 0.000 \pm 0.008$	$0.010 \pm 0.002 \pm 0.000 \pm 0.008$
$D^0 \rightarrow \omega(782) \pi^0$	-	$0.009 \pm 0.004 \pm 0.000 \pm 0.002$
$D^0 \rightarrow \phi(1020) \pi^0$	-	$0.014 \pm 0.004 \pm 0.000 \pm 0.003$



$D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ & $D^0 \rightarrow \pi^+ \pi^- \pi^0 \pi^0$

Accepted by CPC

FF(%)	9	13
1. $D^0 \rightarrow a_1(1260)^+ \pi^-$	-27.3 ± 2.1	5.2 ± 1.1
2. $D^0 \rightarrow a_1(1260)^- \pi^+$	-0.1 ± 0.1	3.4 ± 0.5
3. $D^0 \rightarrow a_1(1420)^+ \pi^-$	0.1 ± 0.1	1.0 ± 0.2
4. $D^0 \rightarrow a_1(1640)^+ \pi^-$	4.9 ± 0.7	-1.6 ± 0.6
5. $D^0 \rightarrow a_1(1640)^- \pi^+$	0.3 ± 0.1	-1.9 ± 0.5
6. $D^0 \rightarrow a_2(1320)^+ \pi^-$	0.0 ± 0.0	-0.0 ± 0.0
7. $D^0 \rightarrow a_2(1320)^- \pi^+$	-0.0 ± 0.0	0.0 ± 0.0
8. $D^0 \rightarrow \pi(1300)^+ \pi^-$	9.2 ± 1.5	-49.2 ± 4.1
9. $D^0 \rightarrow \pi(1300)^- \pi^+$	23.5 ± 2.3	-39.8 ± 3.9
10. $D^0 \rightarrow \rho(770)^0 \rho(770)^0$		2.0 ± 0.5
11. $D^0 \rightarrow \rho(770)^0 \rho(1450)^0$		-0.6 ± 0.3
12. $D^0 \rightarrow \rho(770)^0 (\pi\pi)_S$		0.0 ± 0.0
13. $D^0 \rightarrow (\pi^+ \pi^-)_S (\pi\pi)_S$		62.8 ± 4.6
14. $D^0 \rightarrow f_2(1270)^0 (\pi\pi)_S$		

FF(%)	12	14	17
1. $D^0 \rightarrow a_1(1260)^+ \pi^-$	-18.6 ± 1.8	-33.5 ± 1.8	2.4 ± 0.8
2. $D^0 \rightarrow a_1(1260)^- \pi^+$	-7.6 ± 1.0	-12.1 ± 1.1	2.0 ± 0.4
3. $D^0 \rightarrow a_1(1260)^0 \pi^0$	0.7 ± 0.1	-34.3 ± 2.6	4.8 ± 1.1
4. $D^0 \rightarrow a_1(1420)^+ \pi^-$	-0.3 ± 0.1	-0.4 ± 0.1	0.7 ± 0.1
5. $D^0 \rightarrow a_1(1640)^+ \pi^-$	3.7 ± 0.6	3.0 ± 0.5	-1.3 ± 0.4
6. $D^0 \rightarrow a_1(1640)^- \pi^+$	1.7 ± 0.5	1.1 ± 0.4	-1.4 ± 0.4
7. $D^0 \rightarrow a_2(1320)^+ \pi^-$	0.0 ± 0.0	-0.6 ± 0.2	-0.0 ± 0.0
8. $D^0 \rightarrow a_2(1320)^- \pi^+$	-0.0 ± 0.0	1.0 ± 0.2	-0.0 ± 0.0
9. $D^0 \rightarrow h_1(1170)^0 \pi^0$	-0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0
10. $D^0 \rightarrow \pi(1300)^+ \pi^-$	-3.7 ± 0.4	-5.3 ± 0.7	-14.6 ± 1.2
11. $D^0 \rightarrow \pi(1300)^- \pi^+$	-3.1 ± 0.4	-1.9 ± 0.6	-11.1 ± 1.1
12. $D^0 \rightarrow \pi(1300)^0 \pi^0$	23.2 ± 2.8	-5.3 ± 1.2	-21.5 ± 2.2
13. $D^0 \rightarrow \pi_2(1670)^0 \pi^0$		-0.6 ± 0.2	-0.0 ± 0.0
14. $D^0 \rightarrow \rho(770)^+ \rho(770)^-$		90.9 ± 3.9	-2.1 ± 1.0
15. $D^0 \rightarrow \rho(770)^+ \rho(1450)^- [D]$			-1.0 ± 0.3
16. $D^0 \rightarrow \rho(770)^0 (\pi\pi)_S$			0.0 ± 0.0
17. $D^0 \rightarrow (\pi^+ \pi^-)_S (\pi\pi)_S$			37.4 ± 3.0
18. $D^0 \rightarrow f_2(1270)^0 (\pi\pi)_S$			
19. $D^0 \rightarrow \omega(782) \pi^0$			
20. $D^0 \rightarrow \phi(1020) \pi^0$			

$D^0 \rightarrow a_1(1260)\pi, \pi(1300)\pi, \rho(770)\rho(770)$
及 $2(\pi\pi)_S$ 间存在巨大干涉

	$F_+^{\pi^+ \pi^- \pi^+ \pi^-}$	$F_+^{\pi^+ \pi^- \pi^0 \pi^0} (\text{non-}\eta)$
This work (model-dependent)	$(75.2 \pm 1.1_{\text{stat.}} \pm 1.5_{\text{syst.}})\%$	$(68.9 \pm 1.5_{\text{stat.}} \pm 2.4_{\text{syst.}})\%$
CLEO-c (model-dependent)	$(72.9 \pm 0.9_{\text{stat.}} \pm 1.5_{\text{syst.}} \pm 1.0_{\text{model}})\% [13]$	-
CLEO-c (model-independent, global)	$(73.7 \pm 2.8)\% [52]$	-
CLEO-c (model-independent, binned)	$(76.9 \pm 2.1_{\text{stat.}} \pm 1.0_{\text{syst.}} \pm 0.2_{K_S \text{ veto}})\% [6]$	-
BESIII (model-independent, global)	$(73.4 \pm 1.5_{\text{stat.}} \pm 0.8_{\text{syst.}})\% [53]$	$(68.2 \pm 7.7)\% [54]$

$$\mathcal{B}(D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-) = (0.688 \pm 0.010 \pm 0.010)\%$$

$$\mathcal{B}(D^0 \rightarrow \pi^+ \pi^- \pi^0 \pi^0) = (0.951 \pm 0.025 \pm 0.021)\%$$

其他粲介子强子衰变振幅分析测量

✓ $D_s^+ \rightarrow \pi^+ \pi^0 \pi^0$: [**JHEP01\(2022\)052**](#)

✓ $D_s^+ \rightarrow K^+ \pi^+ \pi^-$: [**JHEP08 \(2022\) 196**](#)

✓ $D_s^+ \rightarrow \pi^+ \pi^+ \pi^- \pi^0$: 待发表

✓ $D_s^+ \rightarrow \pi^+ \pi^+ \pi^- \pi^0 \pi^0$: 待发表

✓ $D^0 \rightarrow K_S^0 \pi^0 \eta$: 待发表

✓ $D^+ \rightarrow K_S^0 K_S^0 \pi^+$: 待发表

✓ $D^+ \rightarrow \pi^+ \pi^0 \pi^0$: 待发表

✓ $D^+ \rightarrow K^+ \pi^+ \pi^- \pi^0$: 待发表

.....

总结

✓ BESIII上粲介子强子衰变振幅分析研究成果丰富

- 三、四与五体振幅分析的成功研究
- $D \rightarrow VP, SP, AP, VV \dots$ 等中间过程的精确测量
- $a_0(980)$ 性质研究

✓ 未来会有更多的重要结果

- 20 fb^{-1} 数据结果
- DCS振幅分析研究

谢谢各位老师、同学的聆听！