

# Electronic semi-inclusive charm decays

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## Semi-inclusive charm decays

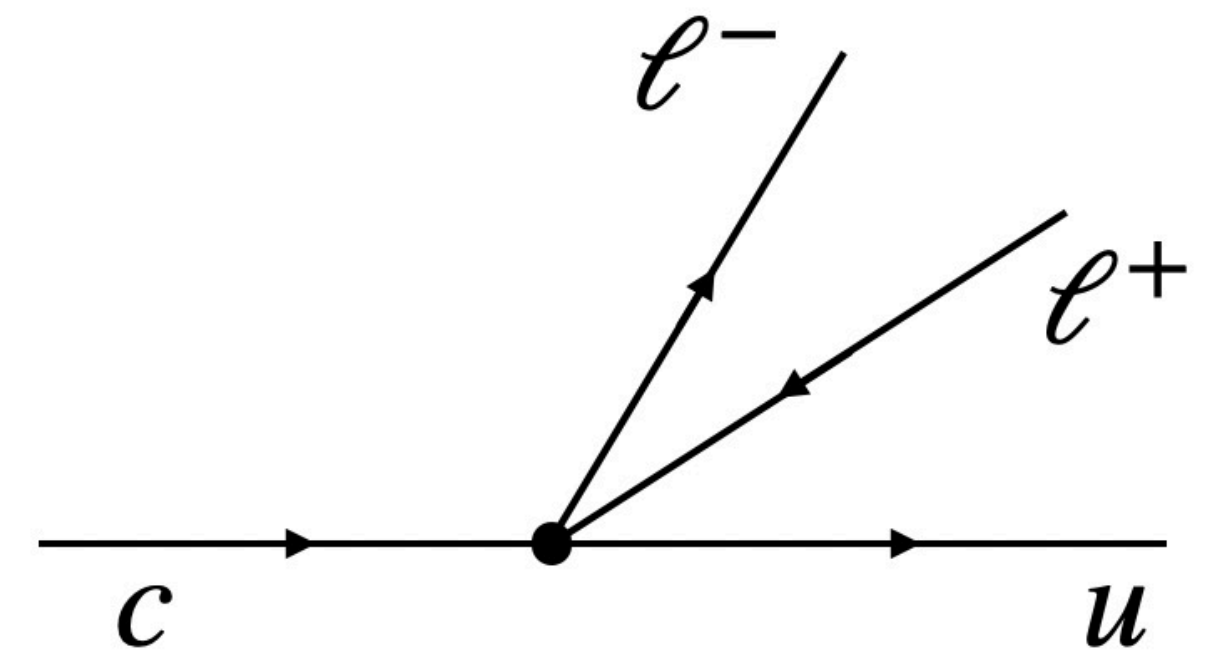
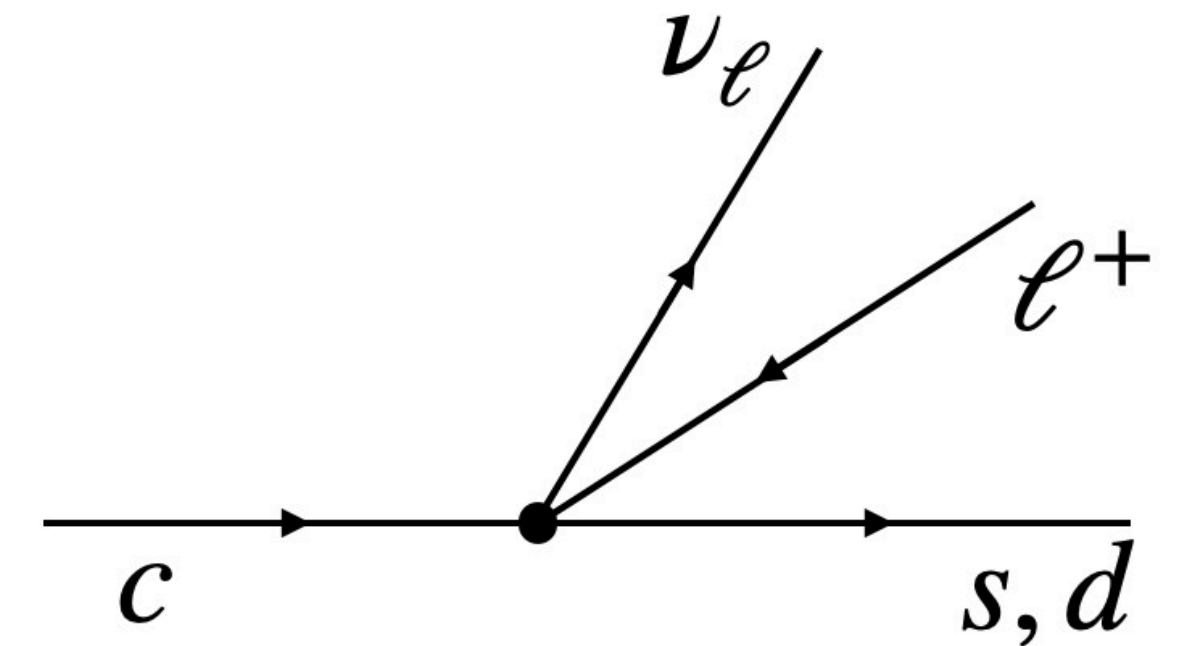
- **Experimental detection of partial final state particles**

➔  $D \rightarrow e^+ X$  ( $D \rightarrow e^+ \nu_e X$ , only  $e^+$  is detected)

- **Sum of a group of exclusive channels**

➔  $D^0 \rightarrow e^+ X_s = D \rightarrow e^+ \nu_e K^-, e^+ \nu_e K^- \pi^0, e^+ \nu_e \bar{K}^0 \pi^-, \dots$

➔  $D^0 \rightarrow e^+ X_d = D \rightarrow e^+ \nu_e \pi^-, e^+ \nu_e \pi^- \pi^0, e^+ \nu_e \pi^- \pi^+ \pi^-, \dots$



## Why inclusive charm decays?

- **As weak decays of heavy hadrons**

- ➡ Probe new physics

- ➡ Understand QCD

- **Compared to exclusive decays**

- ➡ **Better theoretical control**

- **Compared to beauty decays**

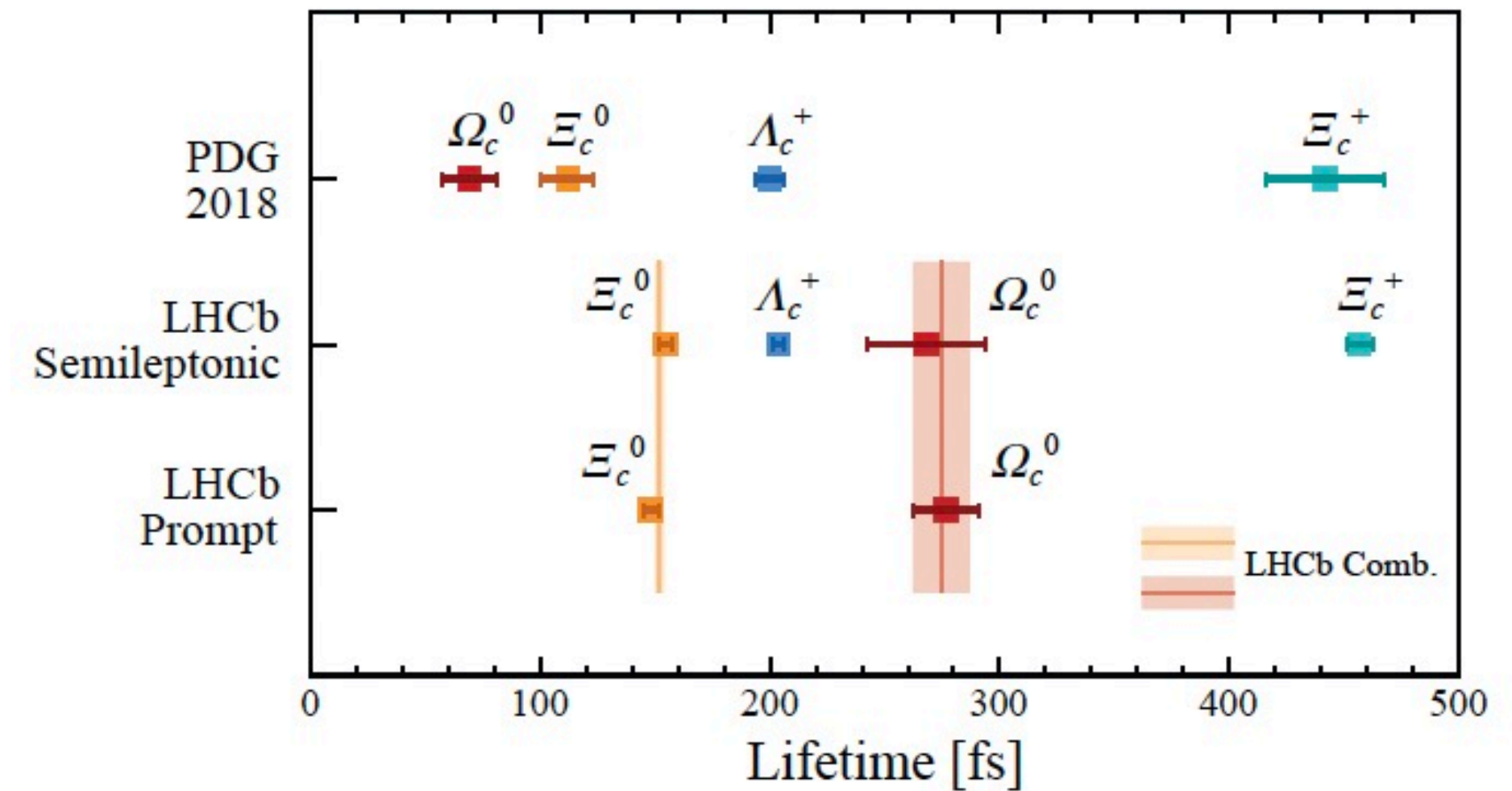
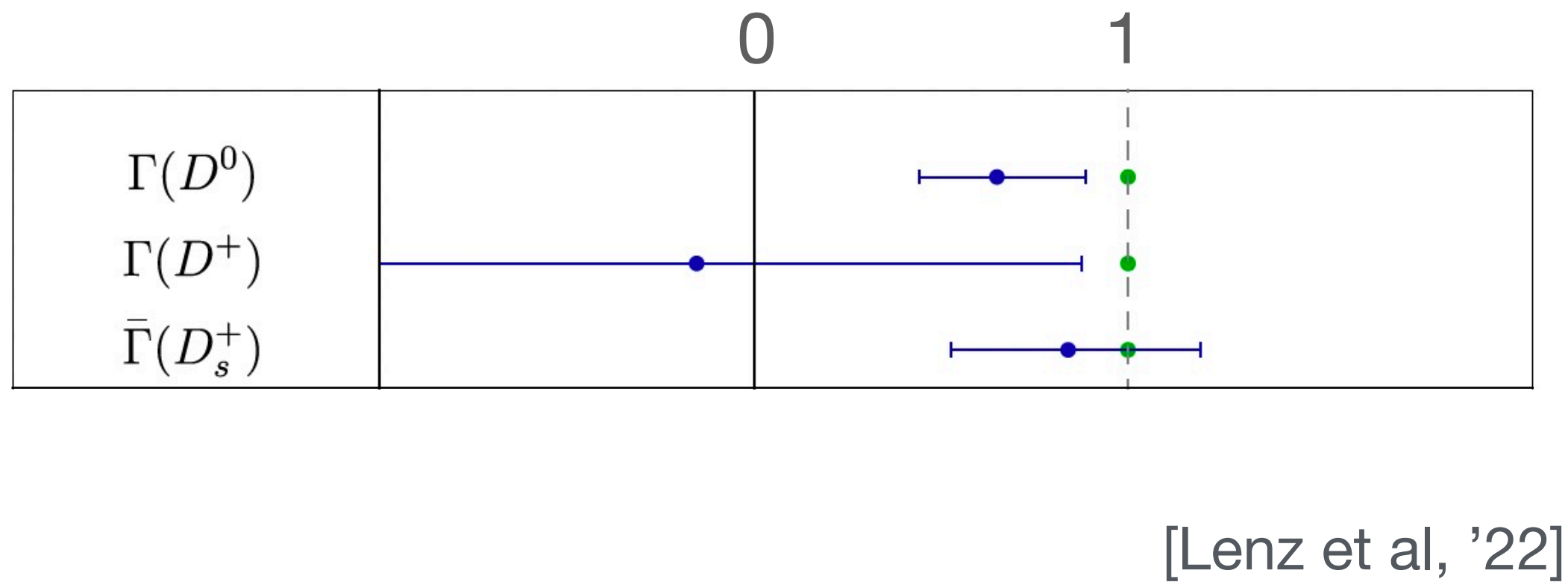
- ➡ **Special to new dynamics attached with up-type quarks**

- ➡ **More sensitive to power corrections**

★ Determination by charm, application in beauty.

# Why inclusive charm decays?

## Flavor puzzles.



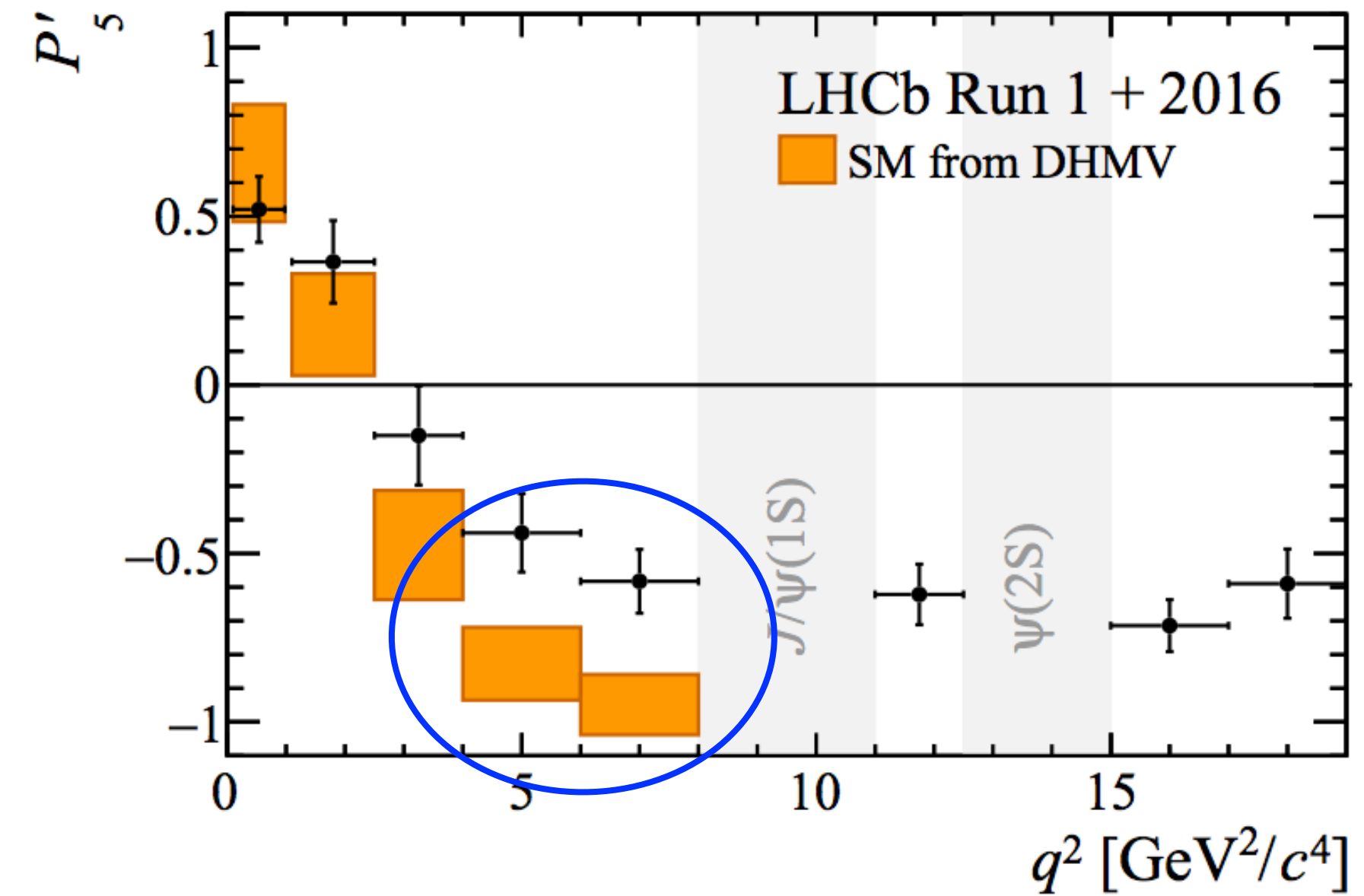
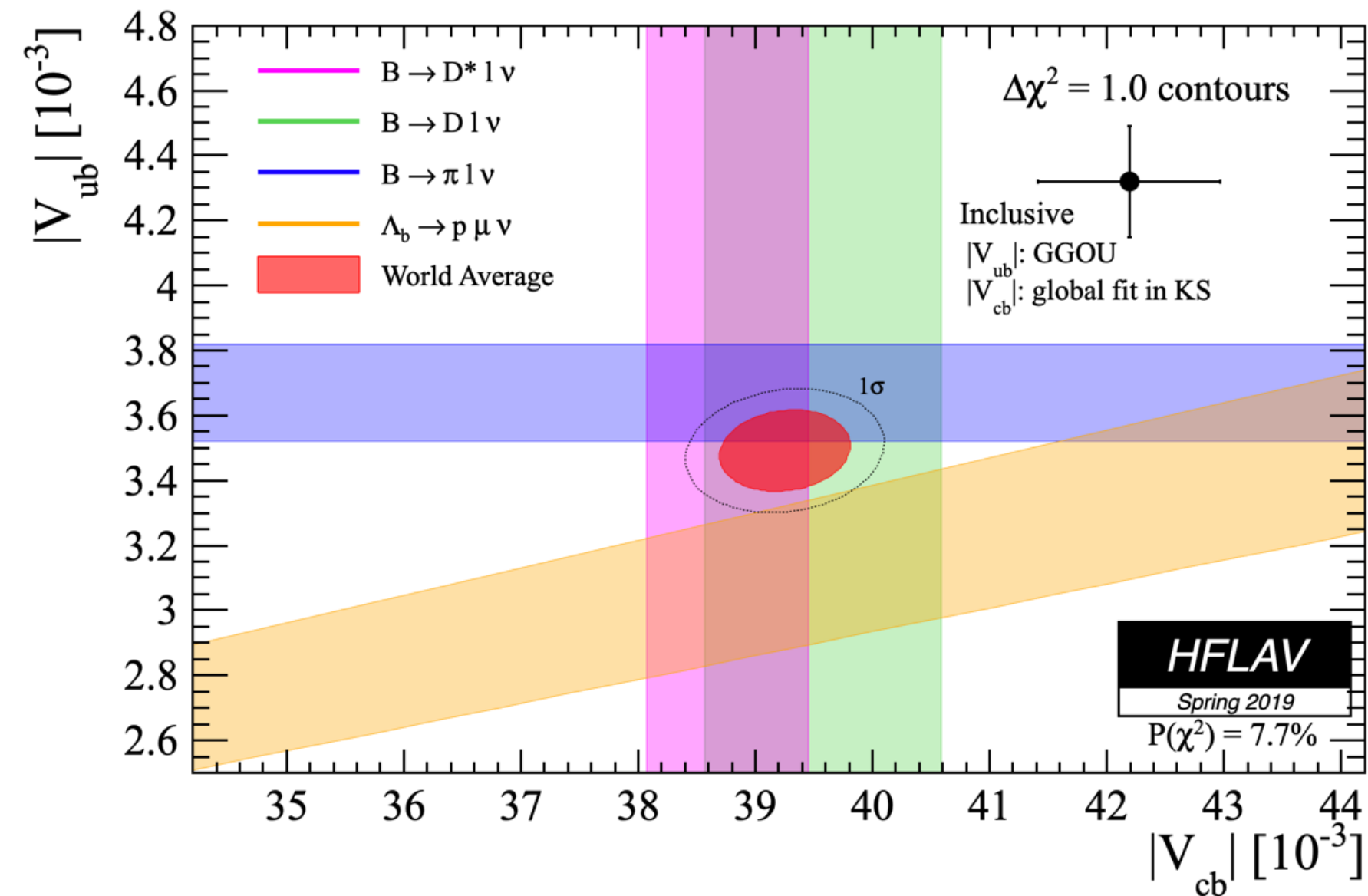
**Charmed hadron lifetimes:  
theory vs experiment**

$$\begin{aligned} \mathcal{O}(1/m_c^3) &\Rightarrow \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0), \\ \mathcal{O}(1/m_c^4) &\Rightarrow \tau(\Omega_c^0) > \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0), \\ \mathcal{O}(1/m_c^4) \text{ with } \alpha &\Rightarrow \tau(\Xi_c^+) > \tau(\Omega_c^0) > \tau(\Lambda_c^+) > \tau(\Xi_c^0). \end{aligned}$$

[Cheng, '21]

# Why inclusive charm decays?

## Flavor puzzles.



$V_{cb}$ ,  $V_{ub}$  puzzles:

inclusive vs exclusive

$b \rightarrow s$  anomalies:

$P'_5$  in  $B \rightarrow K^* \ell \ell$

# Why inclusive charm decays?

## Solutions/hints to flavor puzzles.

- **Charmed hadron lifetimes: theory vs experiment**

- ➔ Dependence on identical hadronic parameters in HQET,  $\langle H_c | O_i | H_c \rangle$

- ➔ Extraction in the inclusive decay spectrum and application to lifetime

Again a more precise experimental determination of  $\mu_\pi^2$  from fits to semileptonic  $D^+$ ,  $D^0$  and  $D_s^+$  meson decays – as it was done for the  $B^+$  and  $B^0$  decays – would be very desirable.

[Lenz et al, '22]

- $V_{cb}$ ,  $V_{ub}$  **puzzles: inclusive vs exclusive**

- ➔  $V_{cd}$ ,  $V_{cs}$  test: inclusive vs exclusive

- ➔ Inclusive still missing

- $b \rightarrow s$  **anomalies:  $P'_5$  in  $B \rightarrow K^* \ell \ell$**

- ➔ Test the  $c \rightarrow u$  transition, by angular distribution in  $D \rightarrow X_u \ell \ell$

# Theoretical framework

- Optical theorem

$$\sum \langle D | H | X \rangle \langle X | H | D \rangle \propto \text{Im} \int d^4x \langle D | T \{ H(x) H(0) \} | D \rangle$$

- Operator product expansion (OPE)

➡ Short distance  $x \sim 1/m_c$

➡ Fluctuation in D meson  $\sim \Lambda_{\text{QCD}}$

$$T \{ H(x) H(0) \} = \sum_n C_n(x) O_n(0) \rightarrow 1 + \frac{\Lambda_{\text{QCD}}}{m_c} + \frac{\Lambda_{\text{QCD}}^2}{m_c^2} + \dots$$

**Systematic OPE in HQET.**

# Theoretical framework

- Heavy quark effective theory

$$h_v(x) \equiv e^{-im_c v \cdot x} \frac{1 + \gamma \cdot v}{2} c(x) \quad v = (1, 0, 0, 0)$$

**Subtract the big intrinsic momentum,  
Leave only  $\sim \Lambda_{\text{QCD}}$  degrees of freedom.**

$$L \ni \bar{h}_v i v \cdot D h_v$$

$$-\bar{h}_v \frac{D_{\perp}^2}{2m_c} h_v - a(\mu) g \bar{h}_v \frac{\sigma \cdot G}{4m_c} h_v + \dots$$

**Similar to**  $\frac{m}{\sqrt{1-v^2}} = m + \frac{1}{2}mv^2 + \dots$



# Theoretical framework

- **OPE**

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0)$$

- ➔ Dim-3:  $\bar{h}_\nu h_\nu$  ( $\bar{c}\gamma^\mu c$ )  $\rightarrow$  partonic decay rate.

- ➔ Dim-5:  $\bar{h}_\nu D_\perp^2 h_\nu$ ,  $g\bar{h}_\nu \sigma \cdot G h_\nu$ .

- ➔ Dim-6:  $\bar{h}_\nu D_\mu (\nu \cdot D) D^\mu h_\nu$ ,  $(\bar{h}_\nu \Gamma_1 q)(\bar{q} \Gamma_2 h_\nu), \dots$

- Contribute to inclusive decay rate and also lifetime

- ➔ Matrix elements of the same operators

$$\lambda_1 \equiv \frac{1}{4m_D} \langle D | \bar{h}_\nu (iD)^2 h_\nu | D \rangle = -\mu_\pi^2$$

- ➔ Only different short-distance coefficients

$$\lambda_2 \equiv \frac{1}{16(s_c \cdot s_q)} \frac{1}{2m_D} \langle D | \bar{h}_\nu g \sigma \cdot G h_\nu | D \rangle = \frac{\mu_G^2}{3}$$

# Theoretical framework

- Analytical results

$$\frac{1}{\Gamma_0} \frac{d\Gamma}{dy} = \left\{ 2(3 - 2y)y^2\theta(1 - y) - \frac{2\lambda_1}{m_b^2} \left[ -\frac{5}{3}y^3\theta(1 - y) + \frac{1}{6}\delta(1 - y)\theta(1^+ - y) + \frac{1}{6}\delta'(1 - y)\theta(1^+ - y) \right] - \frac{2\lambda_2}{m_b^2} \left[ -y^2(6 + 5y)\theta(1 - y) + \frac{11}{2}\delta(1 - y)\theta(1^+ - y) \right] \right\} \quad y \equiv 2E_e/m_c$$

- Up to finite power, the obtained differential decay rate is **NOT** the experimental spectrum

➡ Observables require integration over final states

$$\Rightarrow \Gamma = \int \frac{d\Gamma}{dy} dy, \langle E_\ell \rangle = \frac{1}{\Gamma} \int \frac{d\Gamma}{dy} E_\ell dy, \langle E_\ell^2 \rangle = \frac{1}{\Gamma} \int \frac{d\Gamma}{dy} E_\ell^2 dy, \dots$$

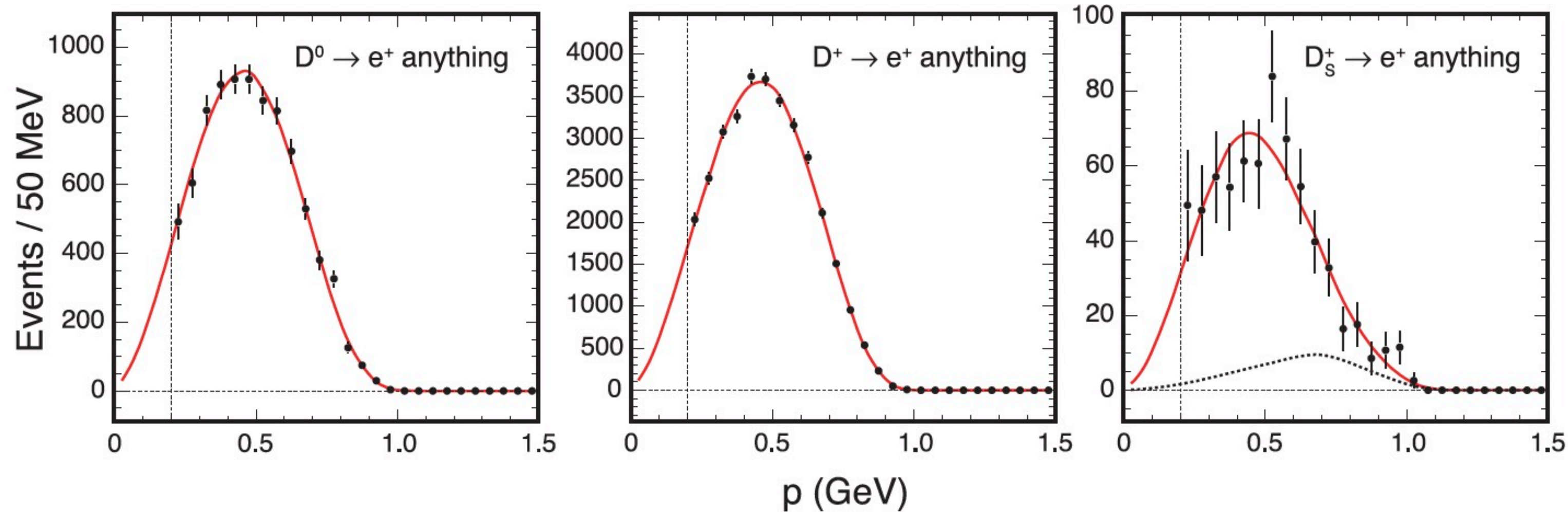
# Experimental status

## CLEO measurements

$$D^0 \rightarrow e^+ X$$

$$D^+ \rightarrow e^+ X$$

$$D_s^+ \rightarrow e^+ X$$



$$\mathcal{B}(D^0 \rightarrow X e^+ \nu_e) = (6.46 \pm 0.09 \pm 0.11)\%$$

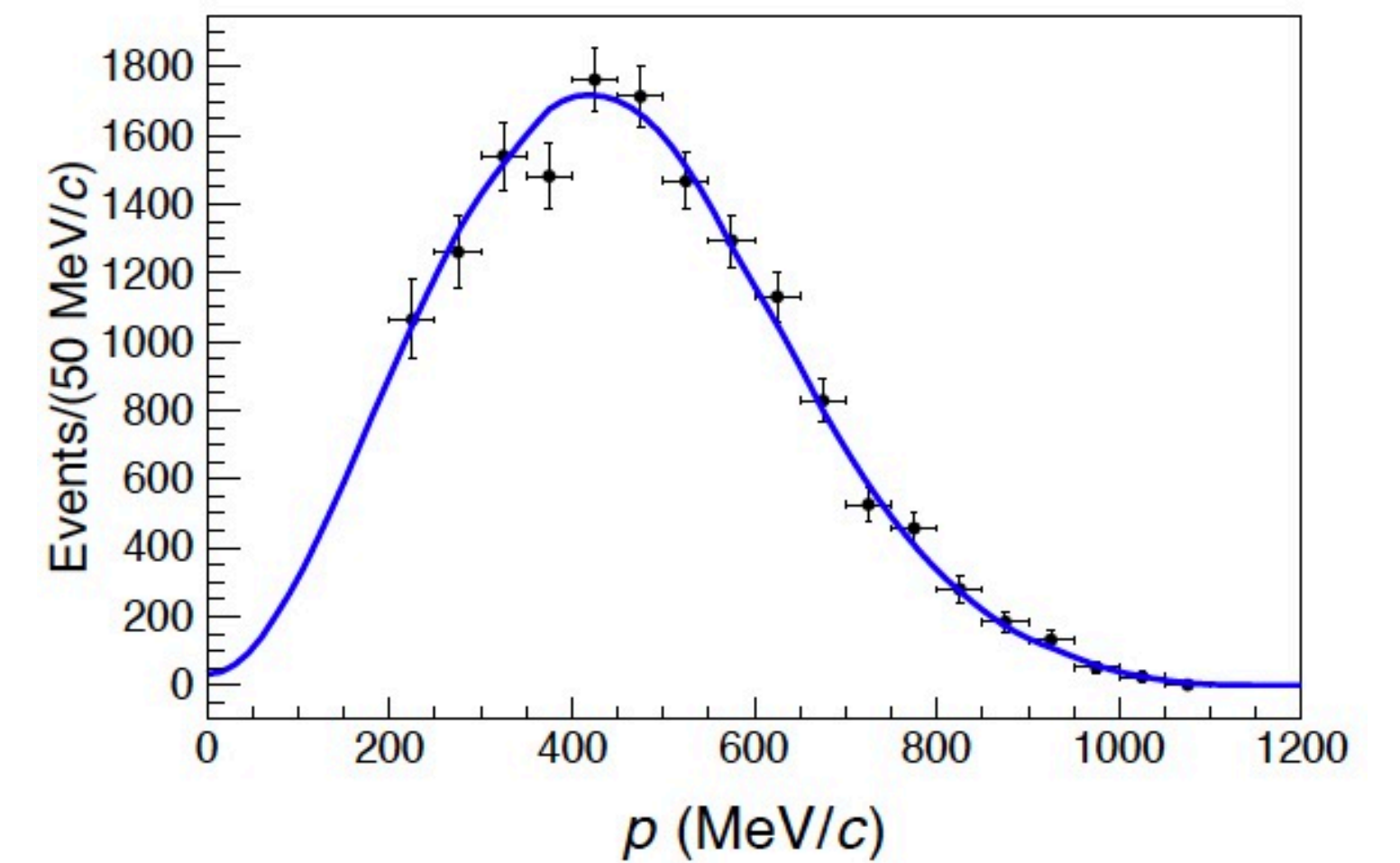
$$\mathcal{B}(D^+ \rightarrow X e^+ \nu_e) = (16.13 \pm 0.10 \pm 0.29)\%$$

$$\mathcal{B}(D_s^+ \rightarrow X e^+ \nu_e) = (6.52 \pm 0.39 \pm 0.15)\%$$

[CLEO, '09]

## BESIII measurements

$$D_s^+ \rightarrow e^+ X$$



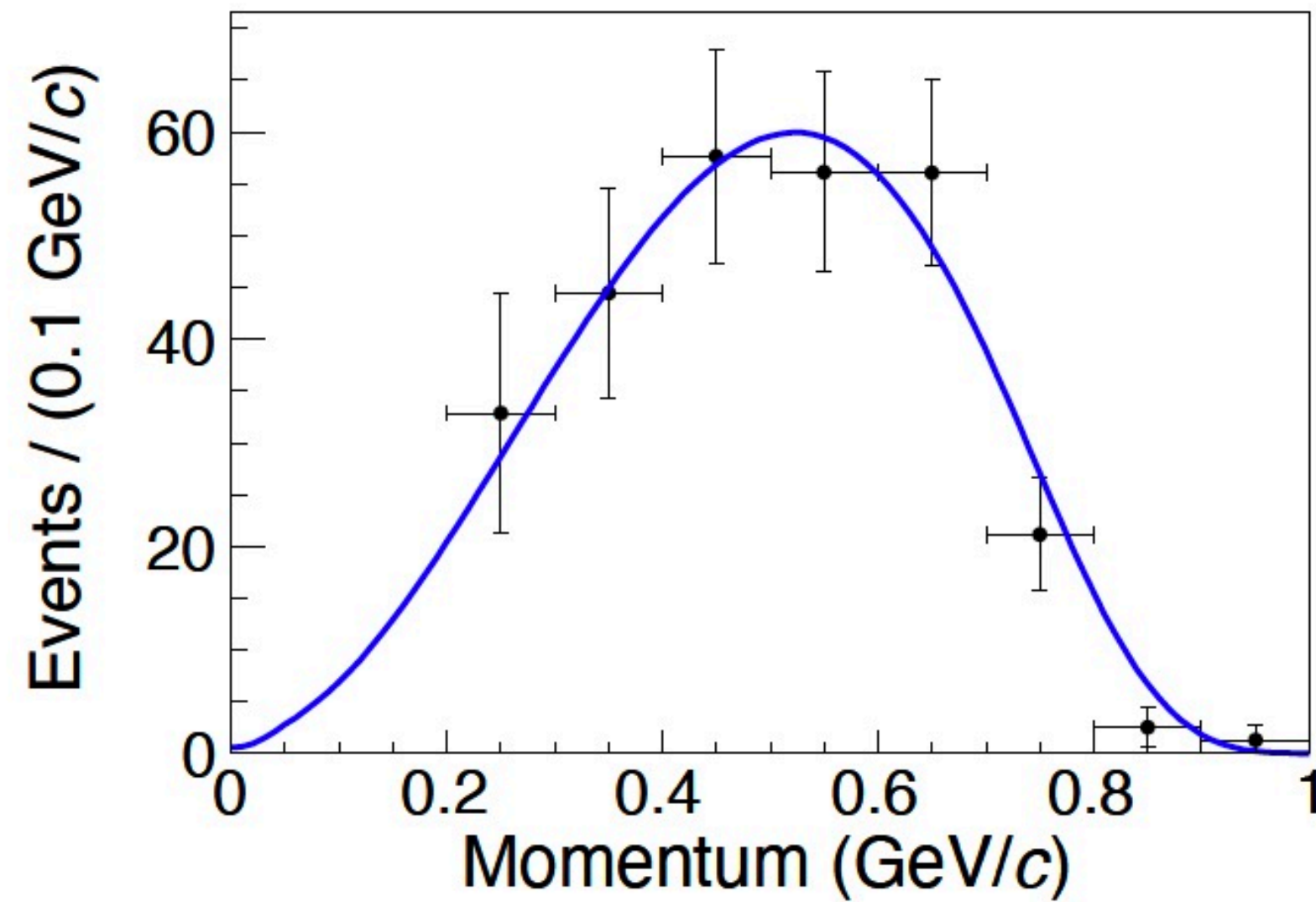
$$\mathcal{B}(D_s^+ \rightarrow X e^+ \nu_e) = (6.30 \pm 0.13 \pm 0.10)\%$$

[BESIII, '21]

# Experimental status

## BESIII measurements

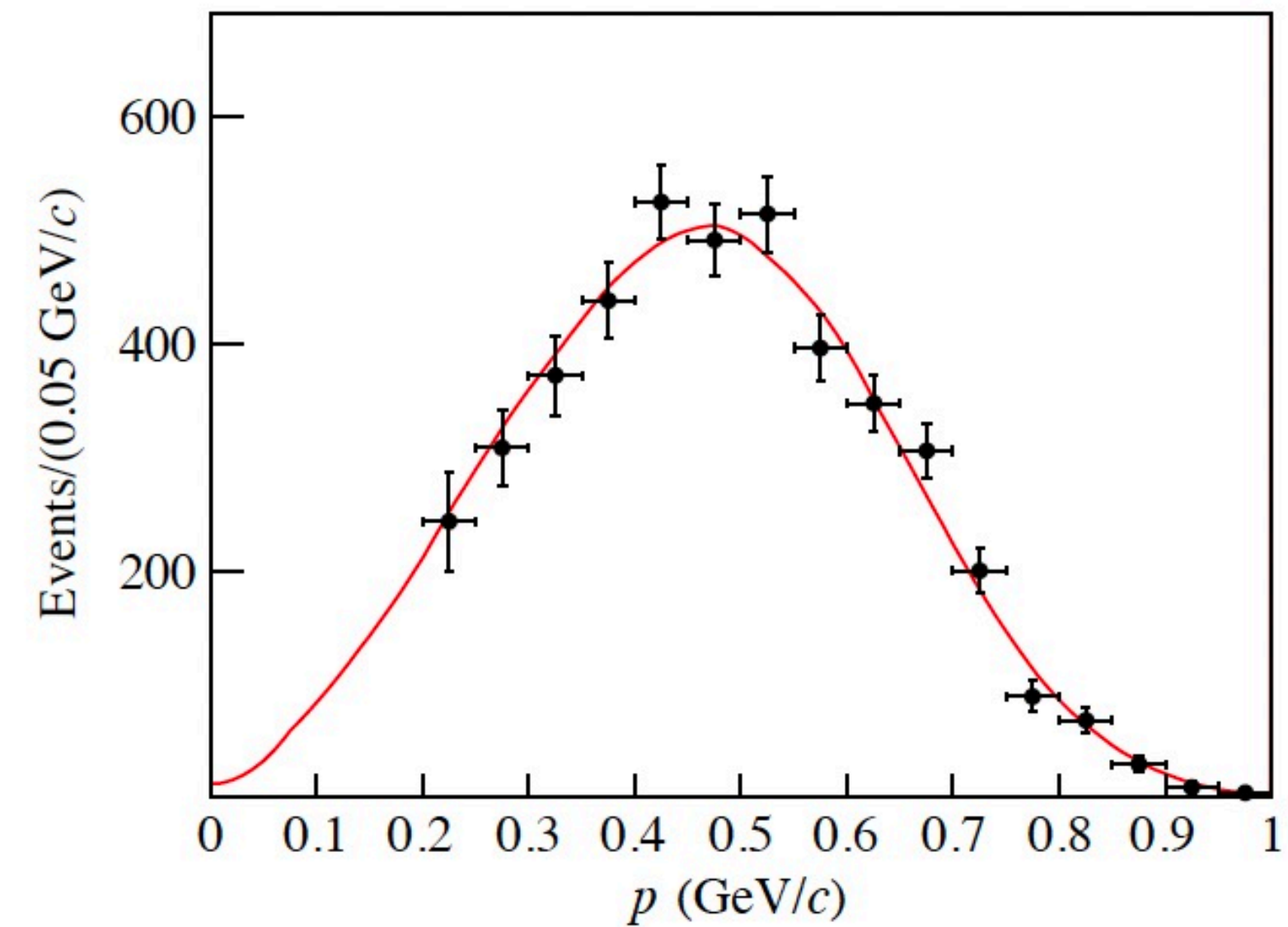
$$\Lambda_c \rightarrow e^+ X$$



$$\mathcal{B}(\Lambda_c^+ \rightarrow X e^+ \nu_e) = (3.95 \pm 0.34 \pm 0.09) \%$$

[BESIII (567 pb<sup>-1</sup>), '18]

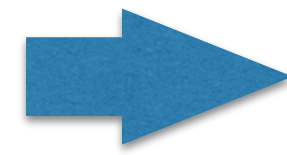
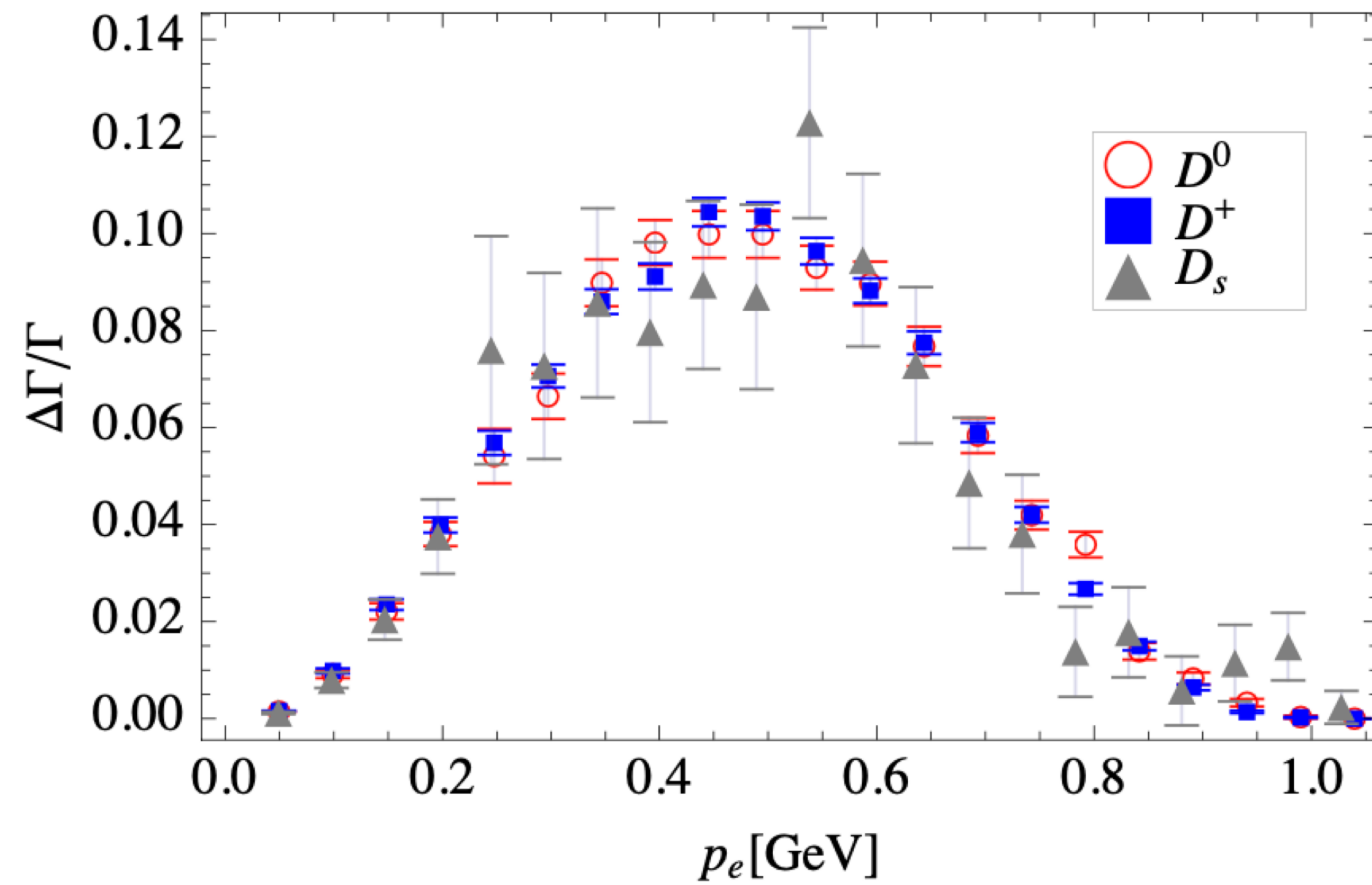
$$\Lambda_c \rightarrow e^+ X$$



$$\mathcal{B}(\Lambda_c^+ \rightarrow X e^+ \nu_e) = (4.06 \pm 0.10_{\text{stat.}} \pm 0.09_{\text{syst}}) \%$$

[BESIII (4.5 fb<sup>-1</sup>), '23]

# Experimental status



$$\langle E_e \rangle_{lab}^{D^0} = 0.465(3) \text{ GeV},$$

$$\langle E_e \rangle_{lab}^{D^+} = 0.459(1) \text{ GeV},$$

$$\langle E_e \rangle_{lab}^{D_s} = 0.466(12) \text{ GeV},$$

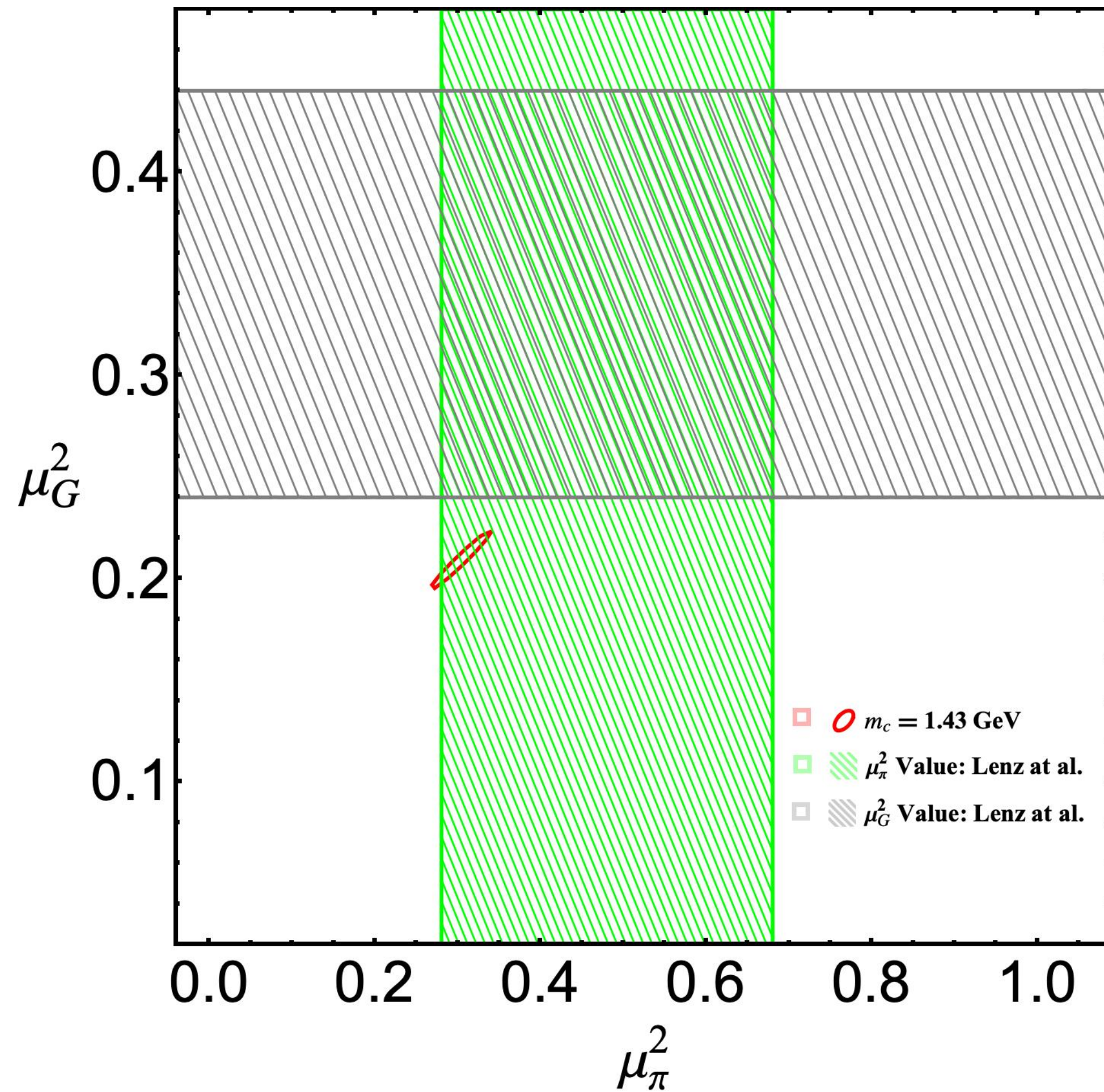
$$\langle E_e^2 \rangle_{lab}^{D^0} = 0.248(2) \text{ GeV}^2,$$

$$\langle E_e^2 \rangle_{lab}^{D^+} = 0.242(1) \text{ GeV}^2,$$

$$\langle E_e^2 \rangle_{lab}^{D_s} = 0.254(13) \text{ GeV}^2.$$

[Gambino, Kamenik, '10]

## Global fit (preliminary)



$$\mu_\pi^2(D) = (0.48 \pm 0.20)\text{GeV}^2$$

$$\mu_G^2(D) = (0.34 \pm 0.10)\text{GeV}^2$$

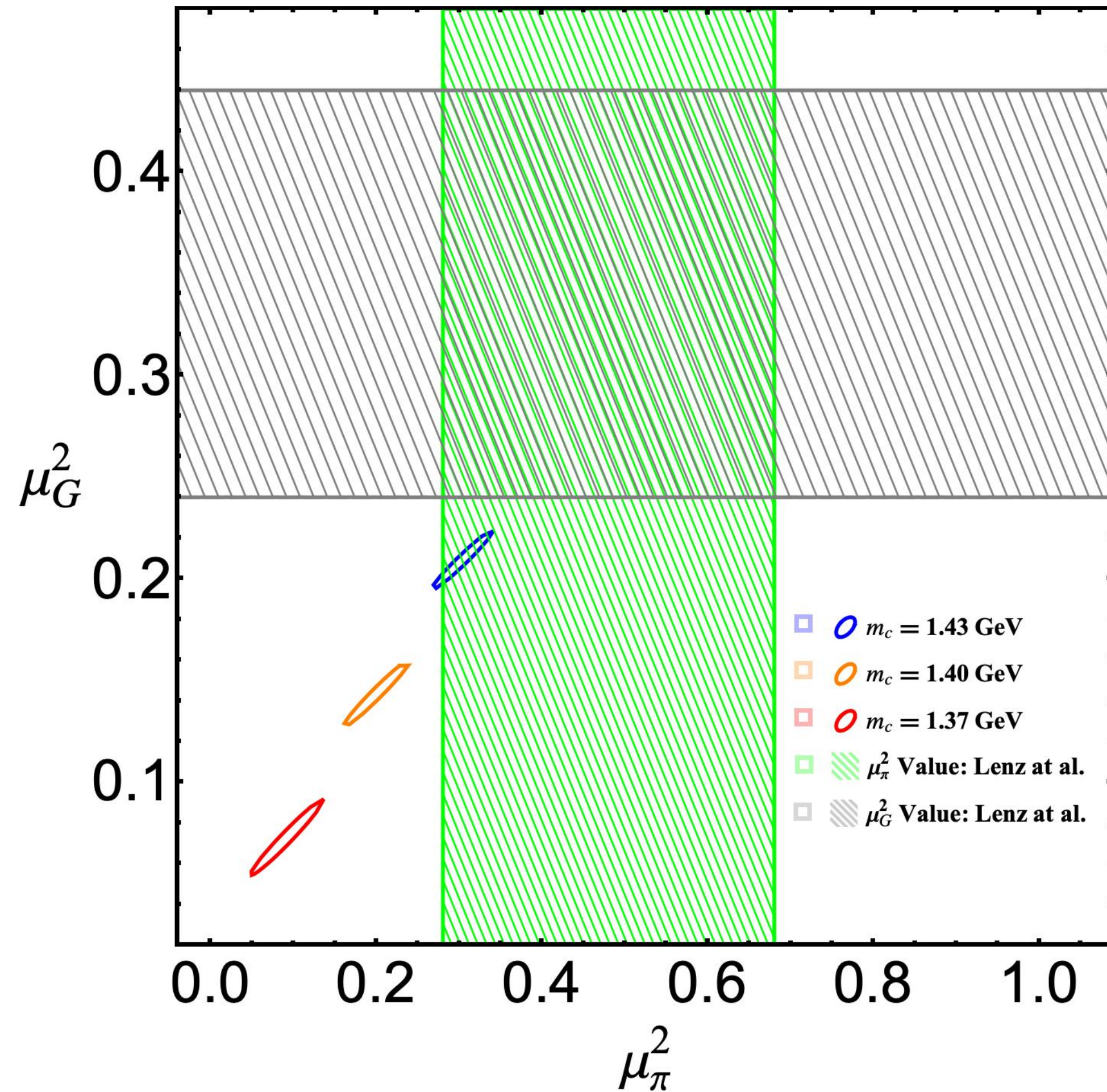
[Lenz et al, '22]

$$\mu_\pi^2(D) = (0.304 \pm 0.037)\text{GeV}^2$$

$$\mu_G^2(D) = (0.209 \pm 0.015)\text{GeV}^2$$

[Our results]

## Global fit (preliminary)



Pole mass scheme:  $m_c = 1.48 \text{ GeV}$

$\overline{MS}$  bar mass scheme:

$$m_c^{Pole} = \bar{m}_c(\bar{m}_c) \left[ 1 + \frac{4}{3} \frac{\alpha_s(\bar{m}_c)}{\pi} \right]$$

$$\bar{m}_c(\bar{m}_c) = 1.27 \text{ GeV}$$

Kinetic mass scheme:

$$m_c^{Pole} = m_c^{Kin} \left[ 1 + \frac{4\alpha_s}{3\pi} \left( \frac{4}{3} \frac{\mu^{cut}}{m_c^{Kin}} + \frac{1}{2} \left( \frac{\mu^{cut}}{m_c^{Kin}} \right)^2 \right) \right]$$

$$m_c^{kin}(0.5 \text{ GeV}) = 1.363 \text{ GeV}$$

$1S$  mass scheme:

$$m_c^{Pole} = m_c^{1S} \left( 1 + \frac{(C_F \alpha_s)^2}{8} \right)$$

$$m_c^{1S} \approx 1.44 \text{ GeV}$$

## Further Plans

- Include NLO corrections
- Include dimension-6 operator contributions
- Take into account the observable correlations
- Extract the hadronic parameters (and the charm mass)



# Wishlist

- Precision measurements of leptonic energy spectrum in the rest frame of charmed hadrons
- $q^2$  spectrum, good for higher-dimensional operators
- Separate  $X_d$ ,  $X_s$ , to give first measurements of  $V_{cd}$ ,  $V_{cs}$
- Rare decays:  $D \rightarrow X_u \ell \ell$ , STCF?

**Thank you!**

# Backup

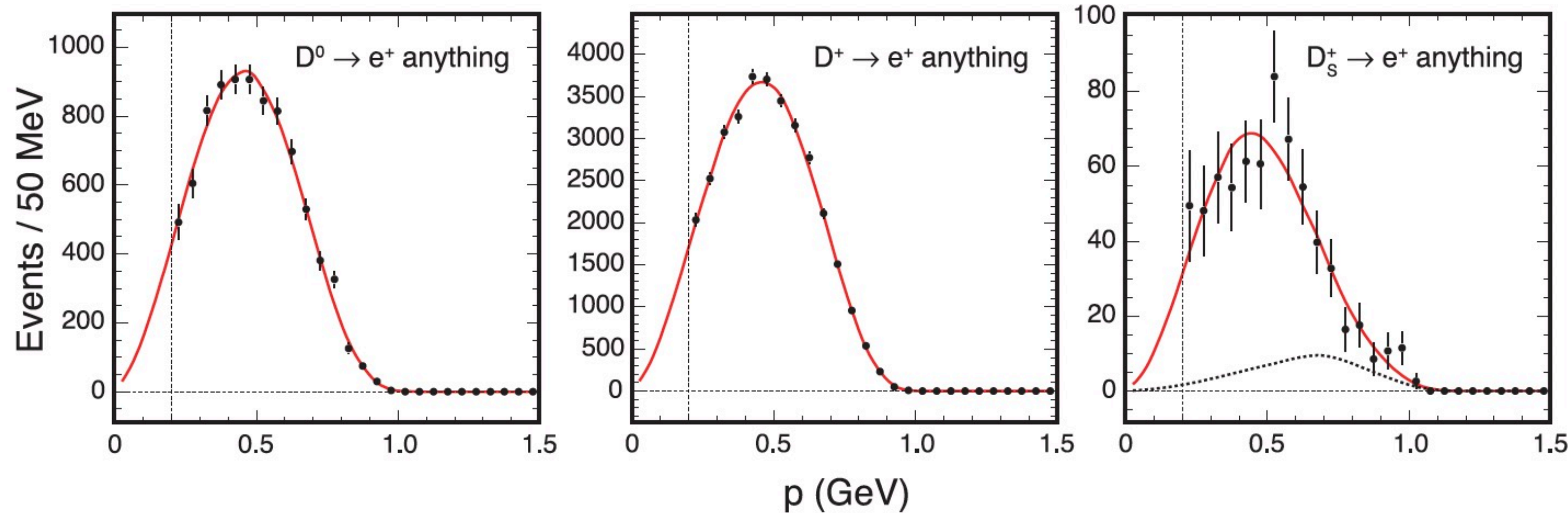
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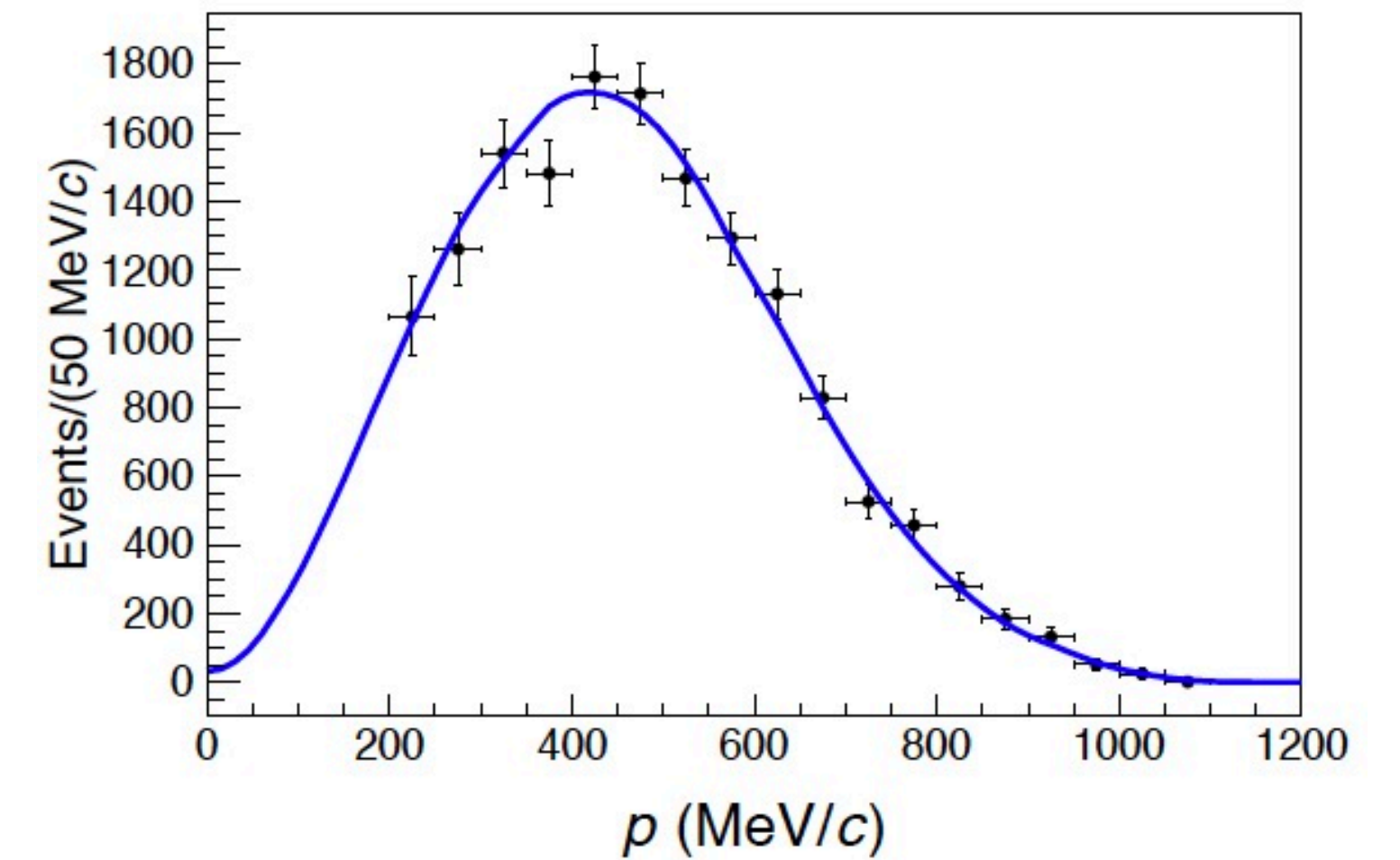


with  $3.0 \times 10^6$   $D^0 \bar{D}^0$  and  $2.4 \times 10^6$   $D^+ D^-$  pairs, and is used to study  $D^0 \rightarrow e^+ X$  decays. The latter data set contains  $0.6 \times 10^6$   $D_s^{*\pm} D_s^\mp$  pairs,

[CLEO ( $818 \text{pb}^{-1}(D^{0,\pm})$ ,  $602 \text{pb}^{-1}(D_s^\pm)$ ), '09]

## BESIII measurements

$$D_s^+ \rightarrow e^+ X$$



$E_{\text{cm}}$ (MeV)	$\int \mathcal{L} dt$ ( $\text{pb}^{-1}$ )	$N_{D_s} (\times 10^6)$
4178	$3189.0 \pm 0.9 \pm 31.9$	6.4
4189	$526.7 \pm 0.1 \pm 2.2$	1.0
4199	$526.0 \pm 0.1 \pm 2.1$	1.0
4209	$517.1 \pm 0.1 \pm 1.8$	0.9
4219	$514.6 \pm 0.1 \pm 1.8$	0.8
4225 – 4230 [32]	$1047.3 \pm 0.1 \pm 10.2$ [33]	1.3