

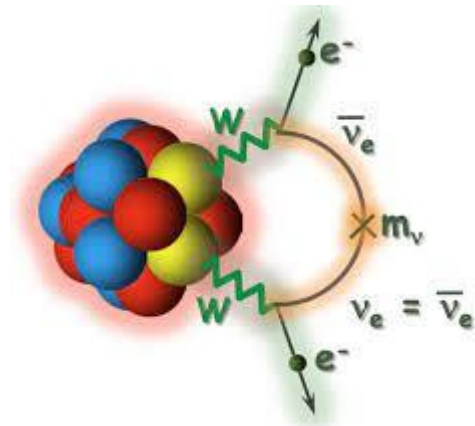
# Neutrinoless double beta decay and classification of the mechanisms

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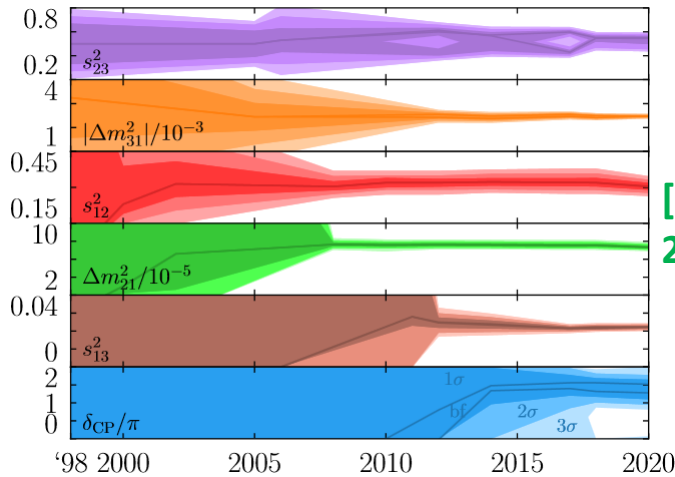
Seminar at experimental physics division, institute of high energy physics, CAS, April 19, 2024

Based on arXiv:2110.15347, JHEP 12 (2021) 169; arXiv:2301.02503, JHEP 03 (2023) 138; arXiv:2405.xxxxx, in collaboration with Ping-Tao Chen, Shi-Yu Li, Chang-Yuan Yao



# The state of art of neutrino mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



[Snowmass, 2212.00809]

## Global fit-Normal hierarchy

$$\Delta m_{21}^2 = 7.41^{+0.21}_{-0.20} \times 10^{-5} \text{eV}^2$$

$$\Delta m_{31}^2 = 2.507^{+0.026}_{-0.027} \times 10^{-3} \text{eV}^2$$

$$\theta_{12} = 33.41^{+0.75}_{-0.72} (^\circ)$$

$$\theta_{23} = 42.2^{+1.1}_{-0.9} (^\circ)$$

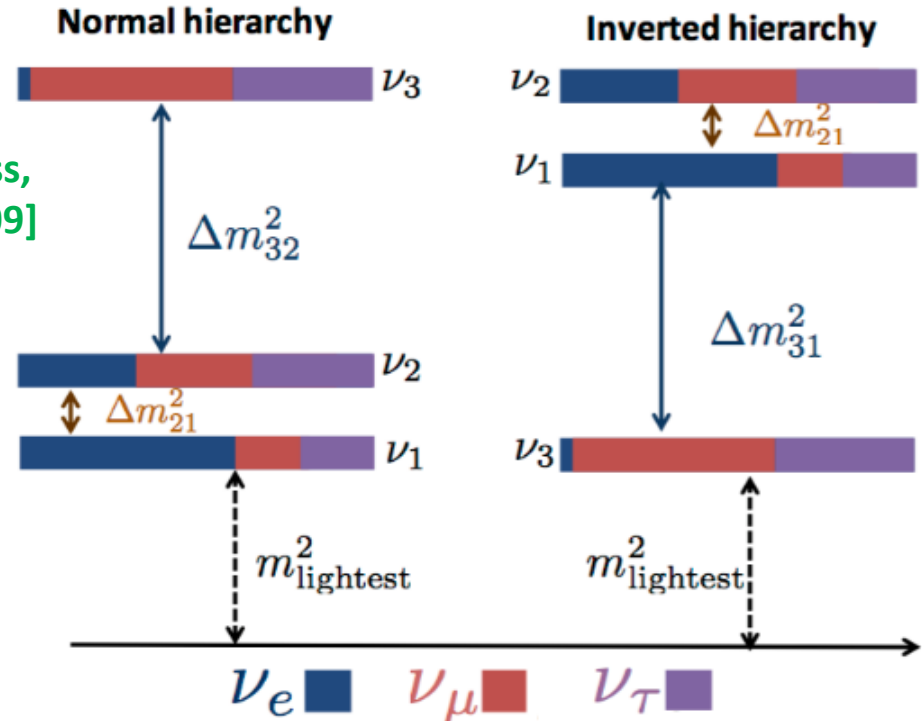
$$\theta_{13} = 8.58^{+0.11}_{-0.11} (^\circ)$$

$$\text{sign}(\Delta m_{32}^2) = ?$$

$$\theta_{23} < 45^\circ \text{ or } \theta_{23} > 45^\circ ?$$

$$\delta_{CP} = ?$$

$$m_{\text{lightest}} = ?$$



$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$$

[Gonzalez-Garcia et al., NuFIT5.2 (2022)]

# Massive neutrinos: Dirac or Majorana?

$$\nu \neq \nu^c$$



$$\nu = \nu^c$$



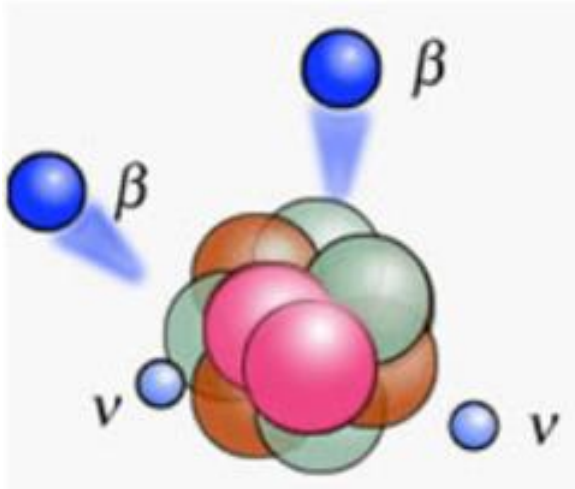
**VS.**

- **neutrinoless double beta decay**
- lepton number violation at collider
- cosmology

# The 2 $\beta$ -decays

## 2 $\nu\beta\beta$ decays

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\bar{\nu}_e$$



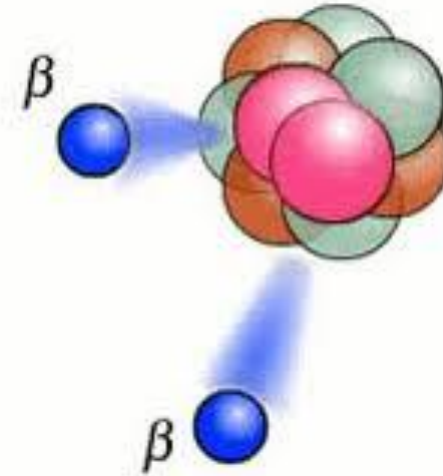
[Goeppert-Mayer, Phys. Rev.48,512(1935)]

- Allowed in SM
- second order in weak interaction
- Natural background for decay



## 0 $\nu\beta\beta$ decays

$$(A, Z) \rightarrow (A, Z + 2) + 2e^-$$



[Furry, Phys.Rev.56,1184(1939)]

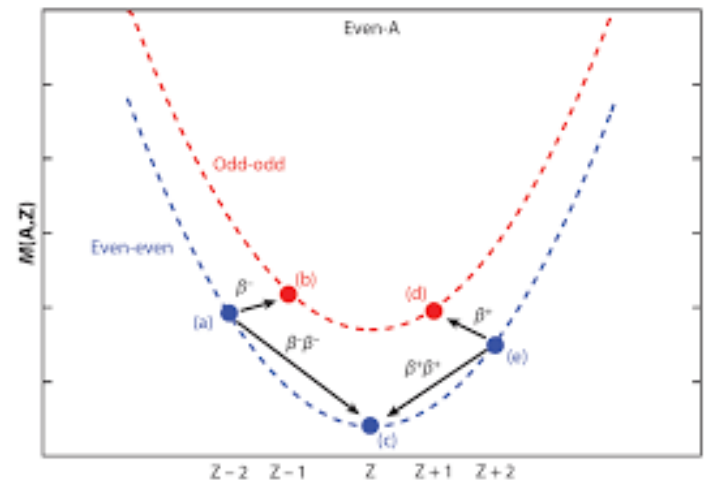
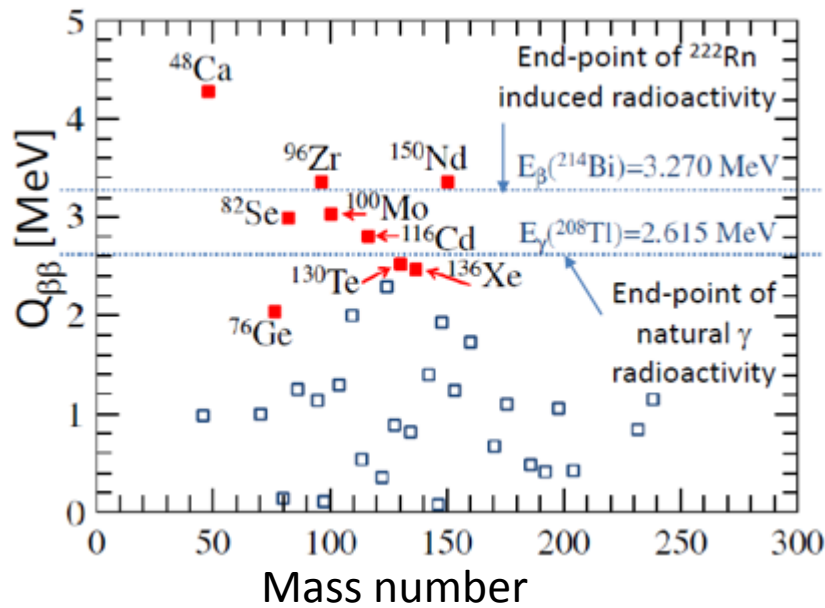
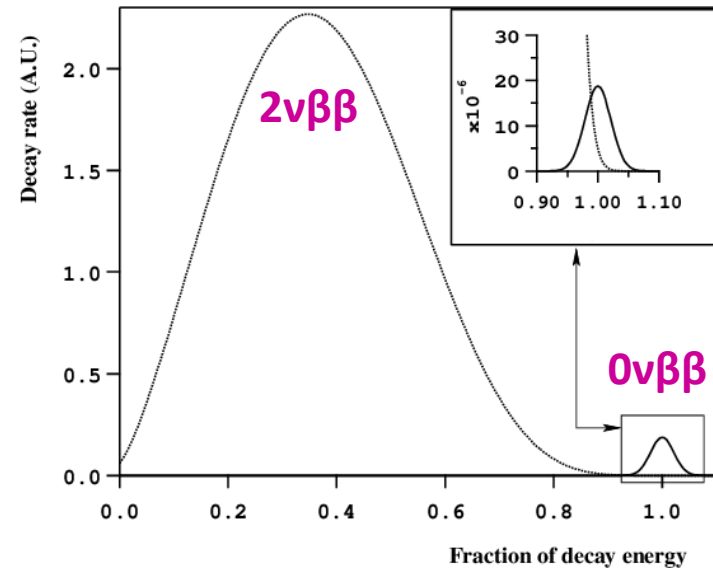
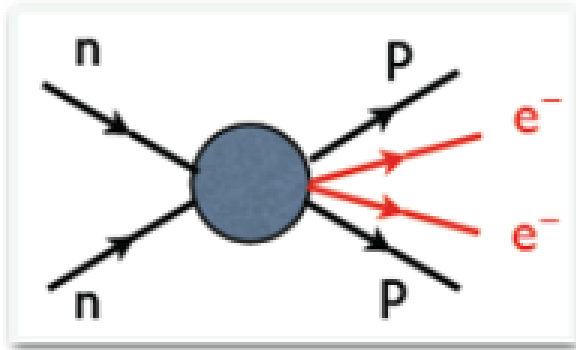
- **NOT** allowed in SM  $\rightarrow$  rare
- Lepton number violation  $\rightarrow$  neutrinos are **Majorana** fermions



# The $0\nu\beta\beta$ -decays: signature and candidate nuclei

$0\nu\beta\beta$  is potentially observable in certain even-even nuclei (9 isotopes including  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{100}\text{Mo}$ ,  $^{130}\text{Te}$ ,  $^{136}\text{Xe}$ ) for which single beta decay is energetically forbidden. The decay rate is less than **1 event per ton and year**.

$$T_{1/2} > 10^{25} \text{ yr}$$

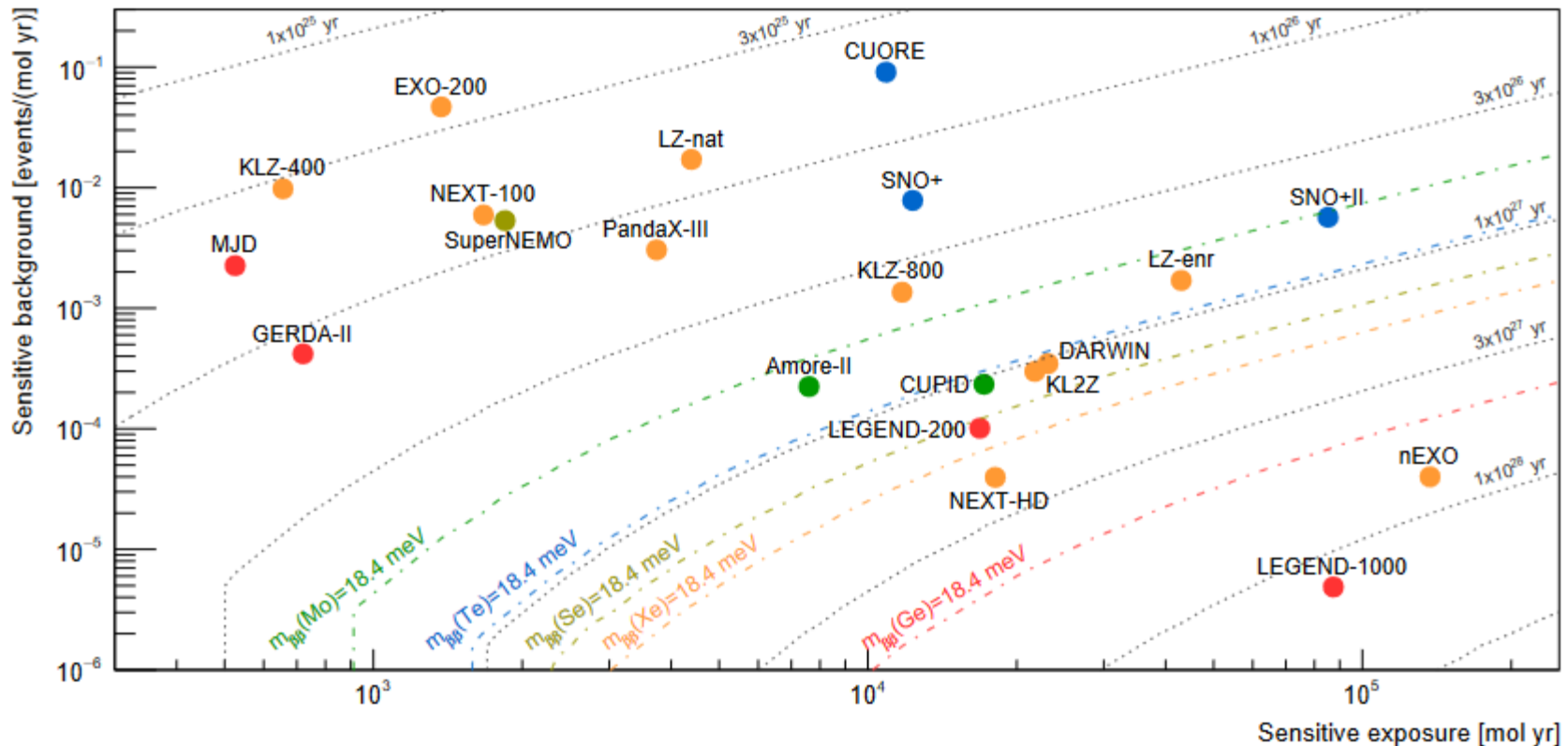


# Current and future experiments

Most stringent constraints on the half life:

- $^{136}\text{Xe}$  (KamLAND-Zen):  $T_{1/2} > 2.3 \times 10^{26}$  yrs [KamLAND-Zen Collaboration, 2203.02139]
- $^{76}\text{Ge}$  (GERDA):  $T_{1/2} > 1.8 \times 10^{26}$  yrs [GERDA collaboration, 2009.06079]
- $^{130}\text{Te}$  (CUORE):  $T_{1/2} > 2.2 \times 10^{25}$  yrs [CUORE collaboration, 2104.06906]

There are many  $0\nu\beta\beta$  decay experiments in plan and construction



[Agostini, Benato, Detwiler, Menendez, Vissani, 2202.01787]

# $0\nu\beta\beta$ experiments in China

CJPL-II (China Jin Ping underground Lab-II)



JUNO- $0\nu\beta\beta$

Te-LS or Xe-LS



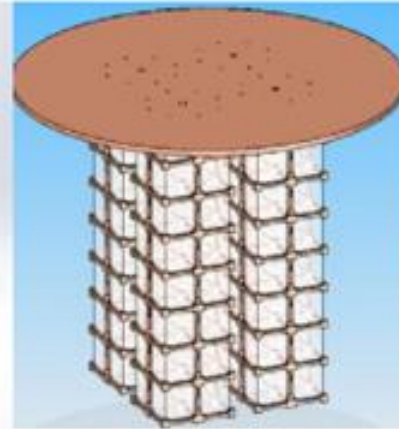
PandaX

Xe TPC



NvDEx

Ion TPC



CUPID-CJPL

Bolometer



CDEX

HPGe

$T_{1/2} > 1.8 \times 10^{28}$  yrs  $T_{1/2} > 2.7 \times 10^{26}$  yrs

$T_{1/2} > 4 \times 10^{26}$  yrs

CDEX-1T:

$T_{1/2} > 10^{28}$  yrs

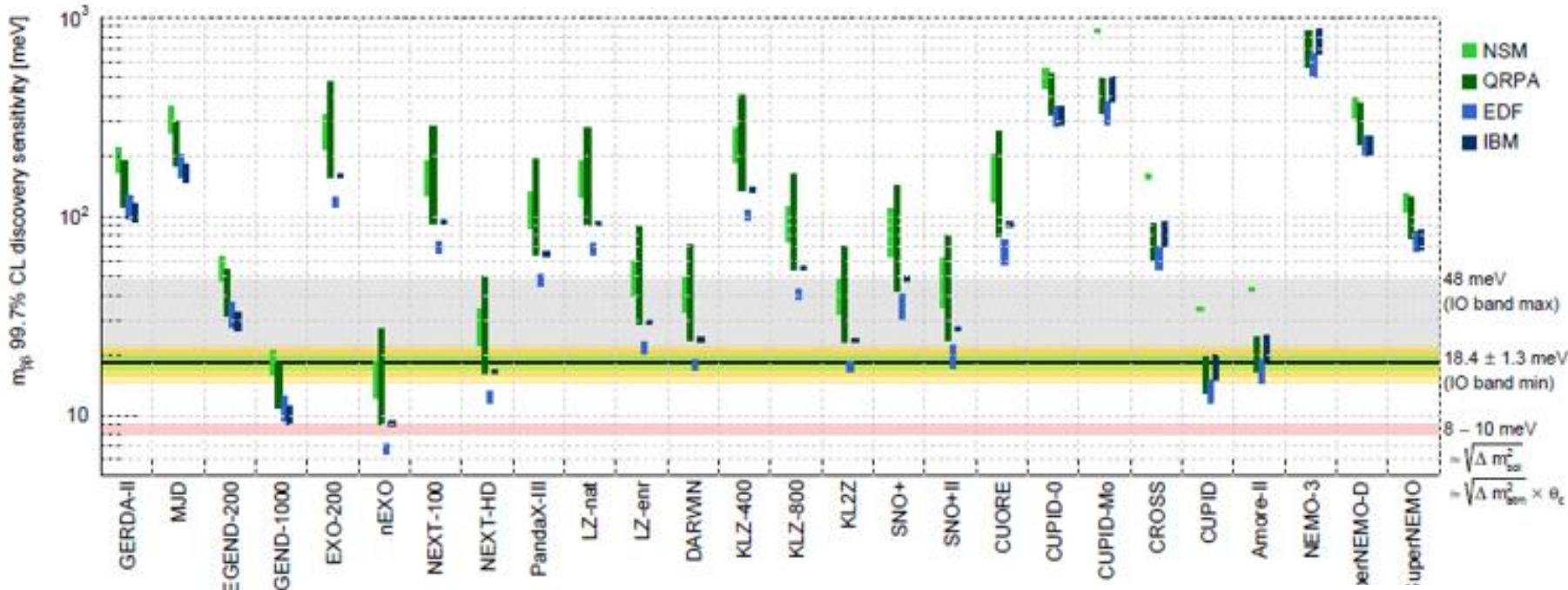
[Snowmass, 2209.03340]

[2102.08221,  
2211.14992]

[NvDEx-100 CDR,  
2304.08362]

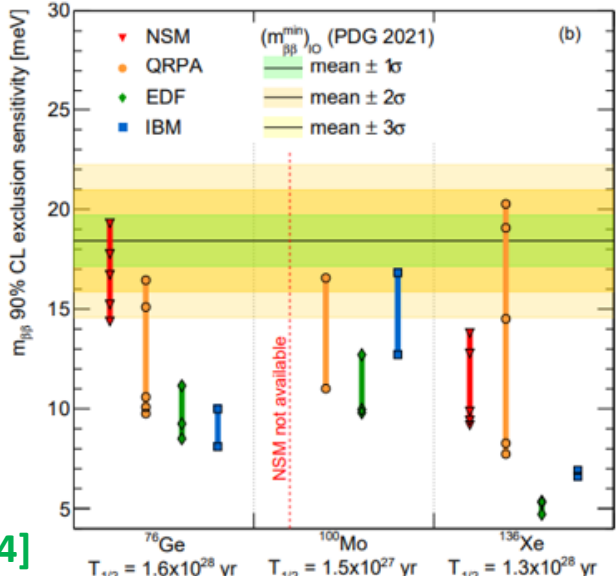
[<http://cdex.ep.tsinghua.edu.cn/column/Experiment#cdex1t>]

# Discovery sensitivities of current and next-generation $0\nu\beta\beta$ experiments



- tonne-scale to multi-tonne scale detectors
- multiple isotopes, multiple techniques: bolometers, scintillators, trackers, TPC, semiconductors
- the discovery sensitivity depends on the matrix elements
- will cover all IO region

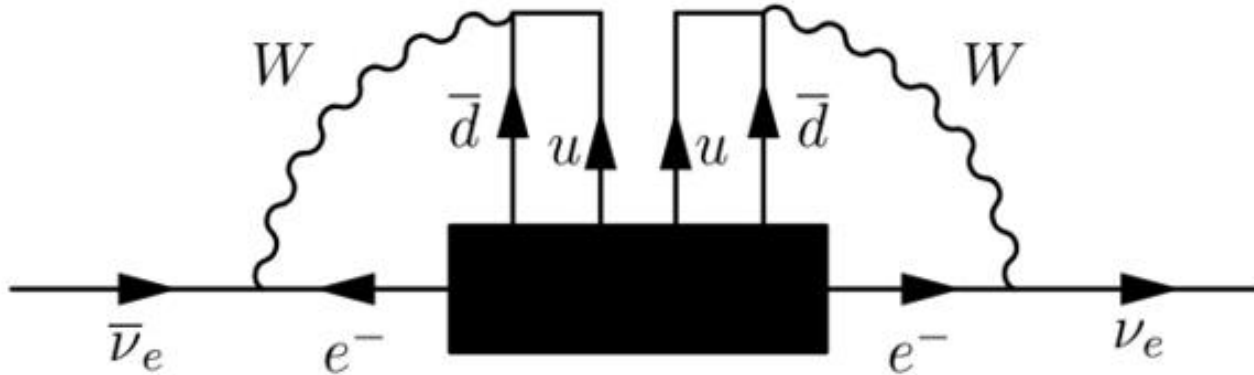
[Agostini, et al, 2107.09104]





# Black Box theorem (Schechter-Valle theorem)

Whatever is the mechanism of  $0\nu\beta\beta$  decay, if  $0\nu\beta\beta$  decay is observed, it is possible to construct a 4-loop diagram that contributes to the Majorana neutrino mass matrix



[Schechter, Valle, 1982']

Only very tiny Majorana mass is induced

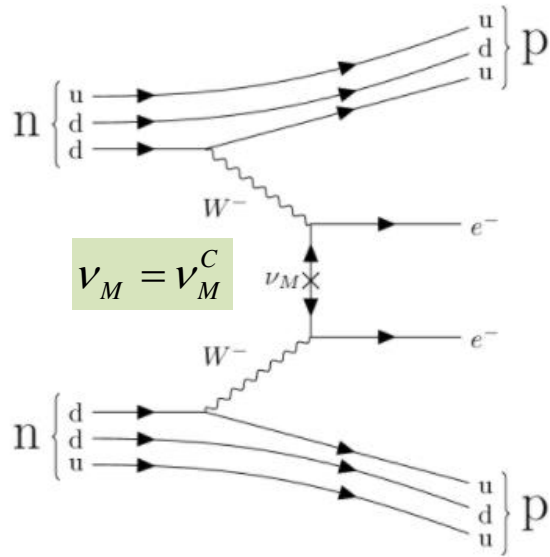
$$\delta m_\nu \approx 10^{-25} \text{ eV} \quad [\text{Duerr, Lindner, Merle, 1105.0901; Liu,Zhang,Zhou,1606.04886}]$$

- It is too small to explain observed neutrino masses and splittings.
- In a concrete model of  $0\nu\beta\beta$  decay, the leading order contribution to neutrino mass can appear at lower order and it can be larger than  $10^{-25} \text{ eV}$ .



# Majorana neutrino interpretation of $0\nu\beta\beta$

**Mass mechanism:**  $0\nu\beta\beta$  decay is usually assumed to be dominantly mediated by light and massive Majorana neutrinos.



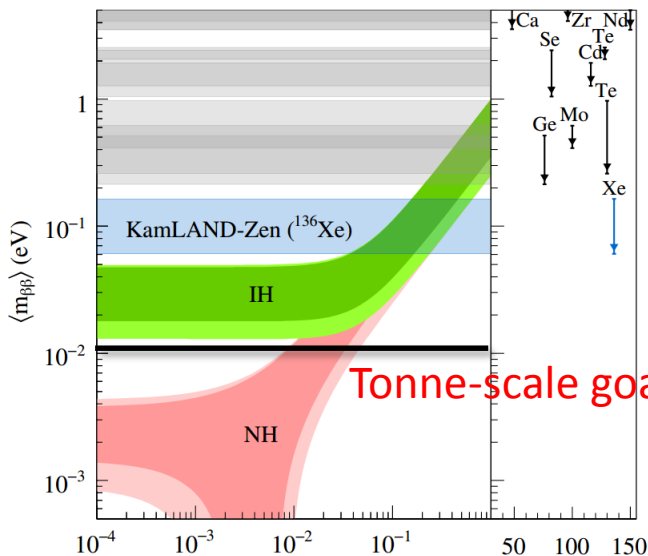
➤ The decay amplitude is proportional to the effective mass

$$|m_{\beta\beta}| = \left| \sum_k U_{ek}^2 m_k \right|$$

PMNS      neutrino masses      Majorana phases

$$= \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_{21}} m_2 + s_{13}^2 e^{i\alpha_{31}} m_3 \right|$$

cosines and sines of the mixing angles



- Unknown: lightest mass, hierarchy and Majorana phases
- Large theoretical uncertainties in the nuclear matrix elements

➤  $0\nu\beta\beta$  decay, single  $\beta$  decay and cosmology measure different combination of the neutrino masses, they are complementary probes of neutrino mass.

$$m_{\beta\beta} = \left| \sum_i U_{ei}^2 m_i \right|$$

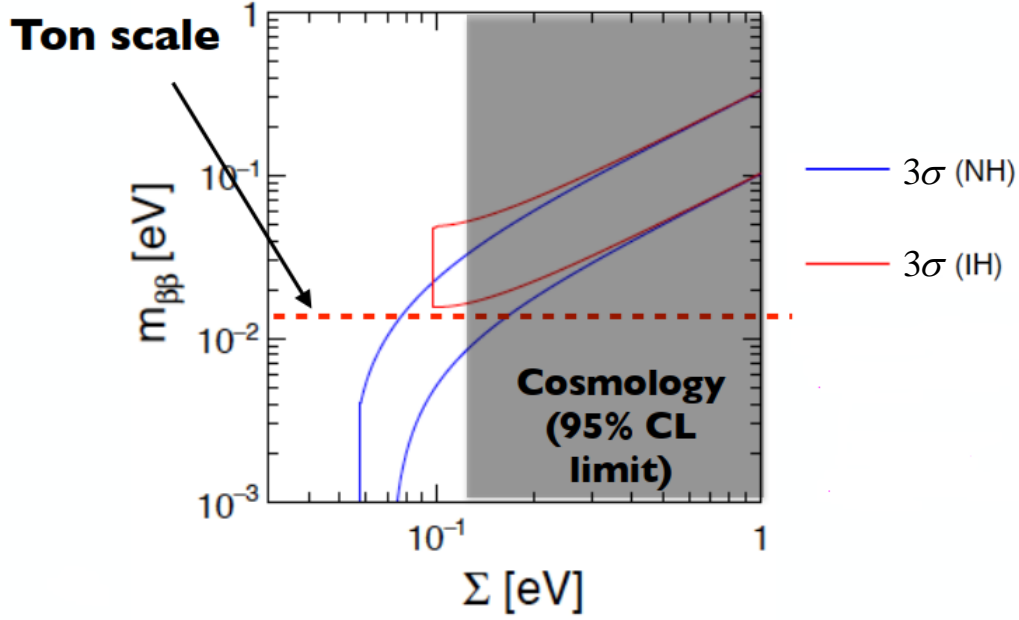
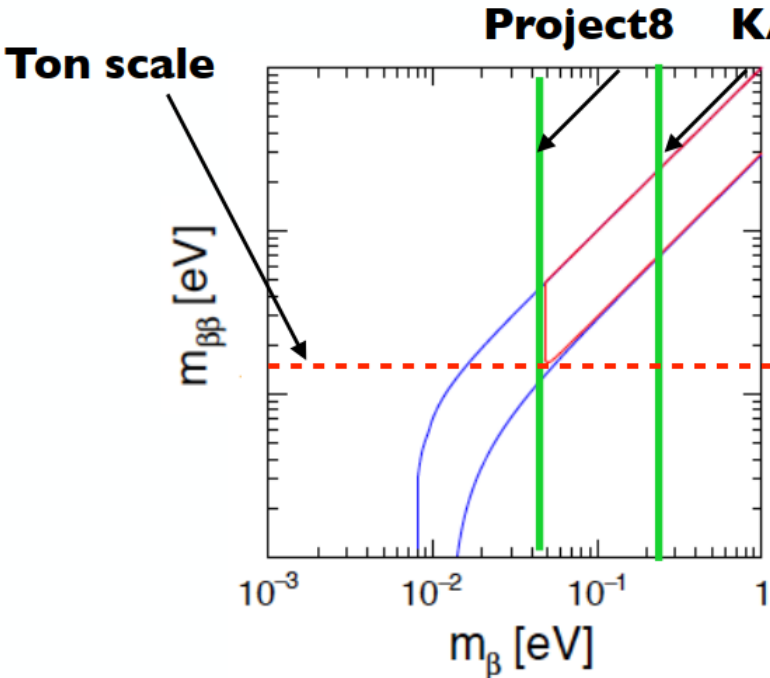
$0\nu\beta\beta$  decay

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

Tritium  $\beta$  decay

$$\Sigma = \sum_i m_i$$

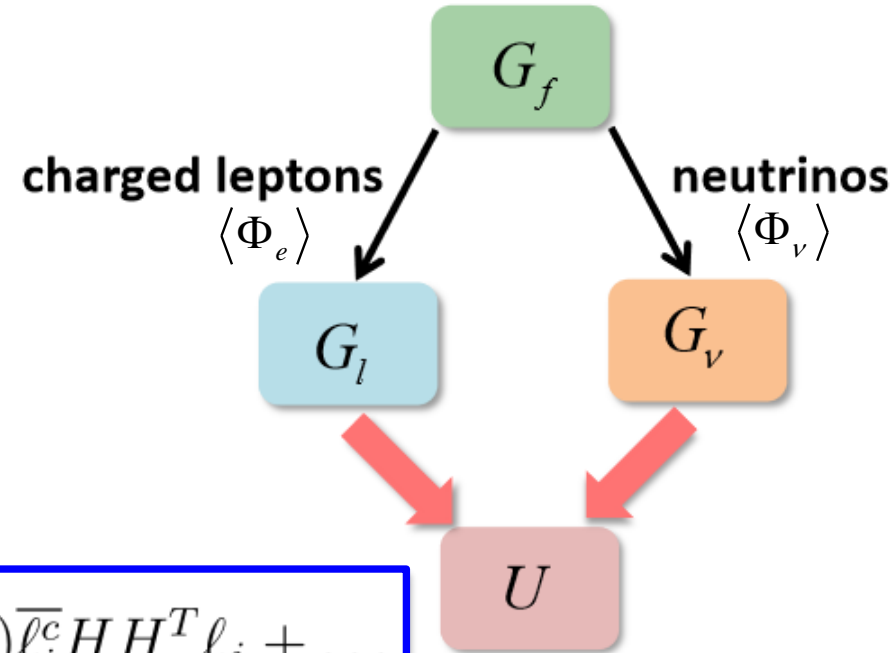
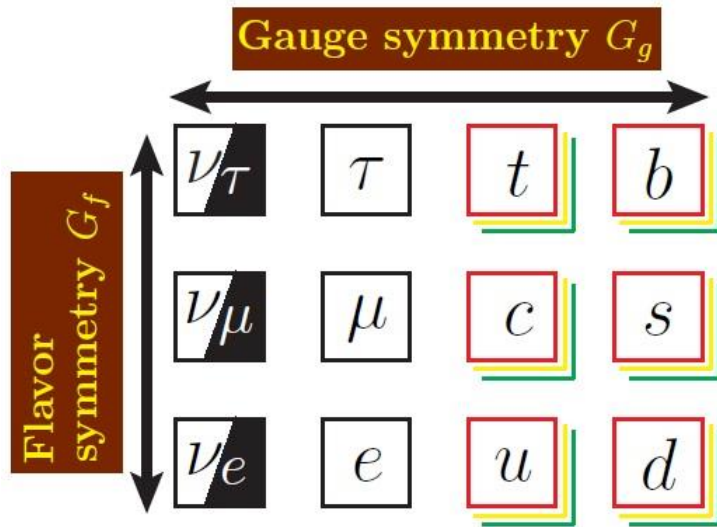
Cosmology



[Capozzi, Valentino, Lisi, Marrone, Melchiorri, Palazzo, 2003.08511]

# Flavor symmetry in $0\nu\beta\beta$ decay

**Flavor symmetry:** relate three generations of fundamental fermions to explain the origin of fermion of mass hierarchies and mixings.



$$\mathcal{L}_m = -Y_{ij}^e(\langle\Phi_e\rangle)\bar{\ell}_i H e_{Rj} - \frac{1}{2}Y_{ij}^\nu(\langle\Phi_\nu\rangle)\bar{\ell}_i^c H H^T \ell_j + \dots$$

Flavor symmetry

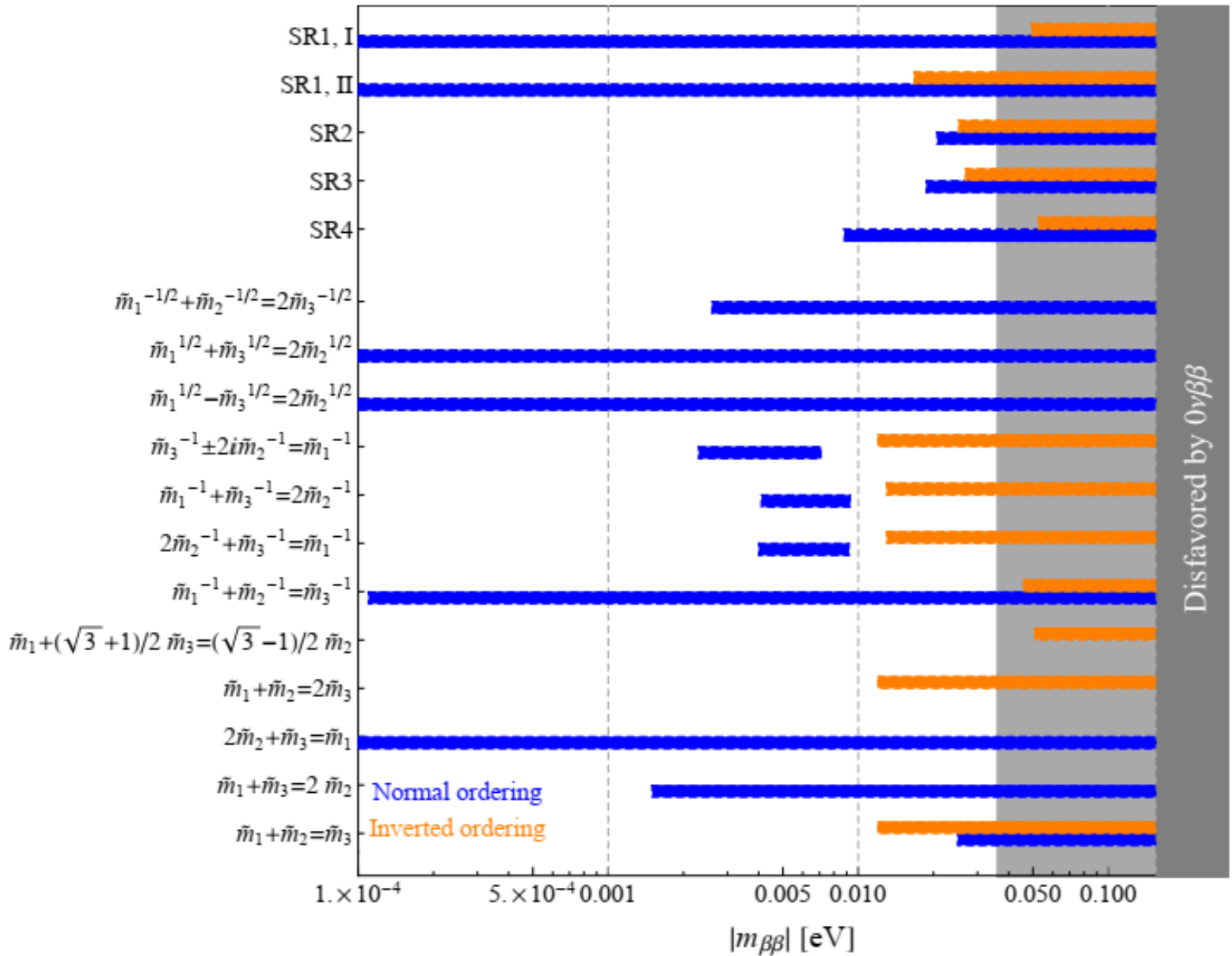


specific structure  
of mass matrices

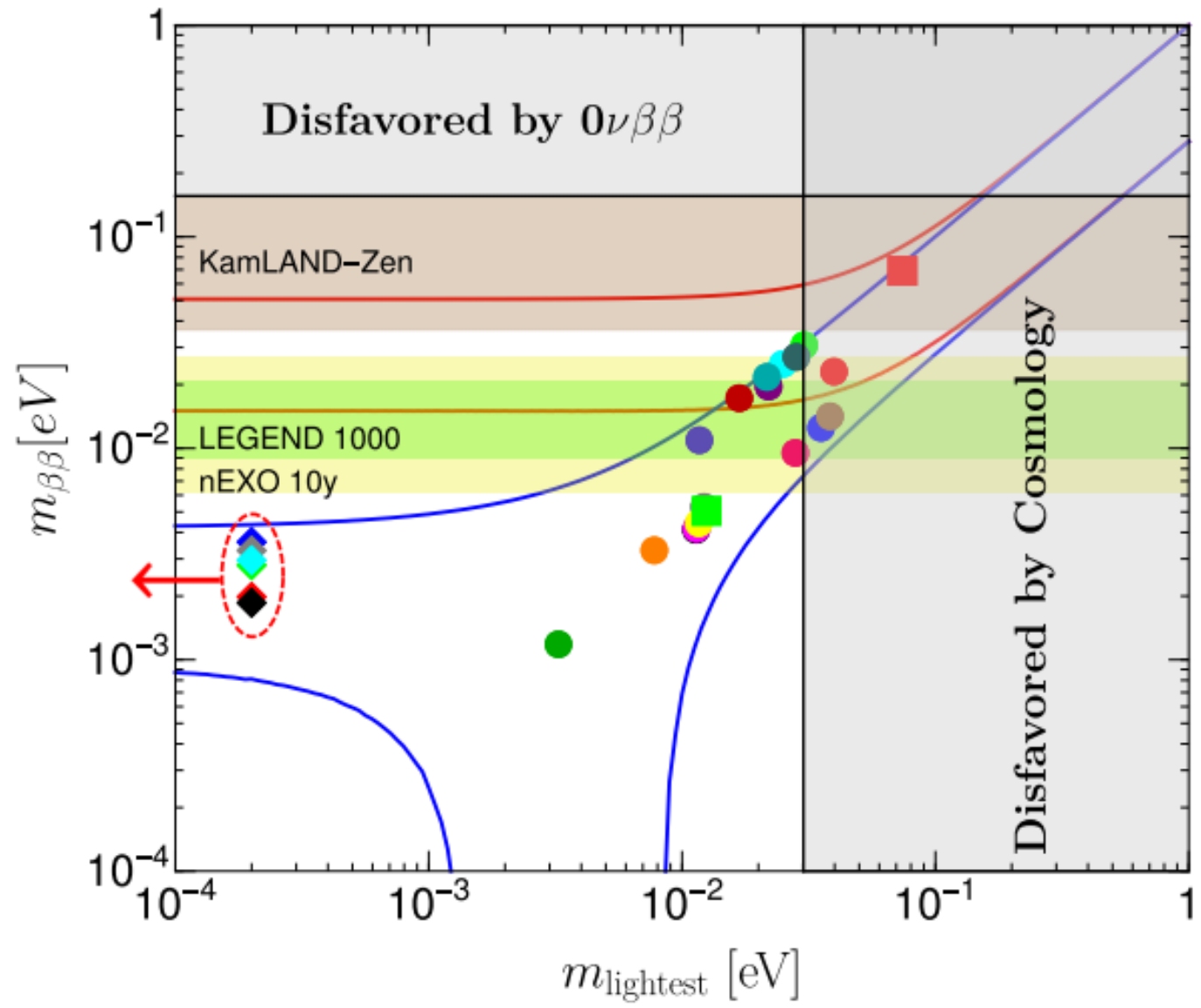


correlations  
among  $\nu$  masses  
and mixing  
parameters

➤ Test **neutrino mass sum rules** of flavor symmetry at  $0\nu\beta\beta$  decay



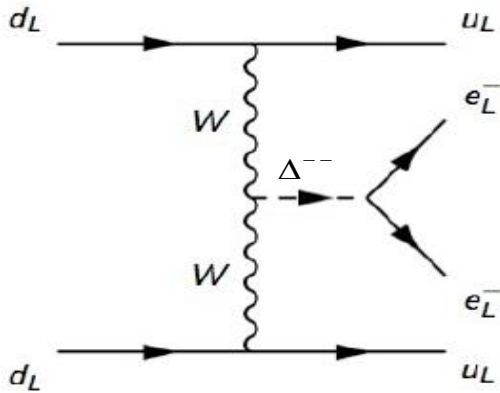
➤ Test modular invariant models at  $0\nu\beta\beta$



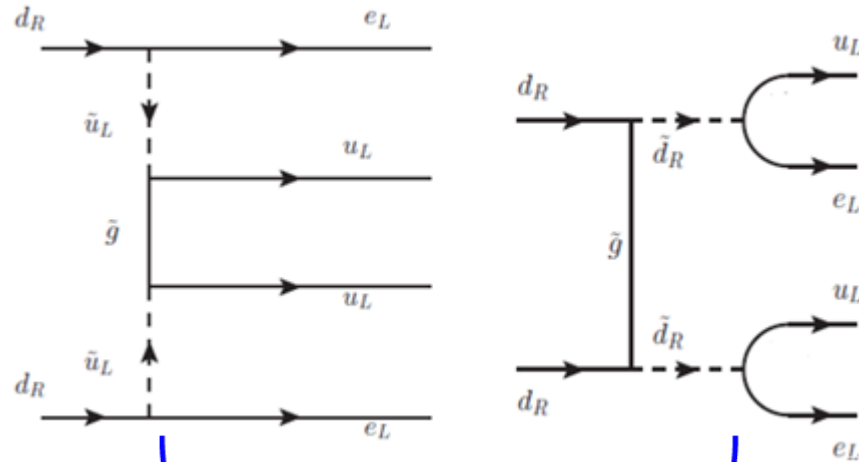
[Chen,Ding,King,2101.12724]

# Possible BSM physics in $0\nu\beta\beta$ decay

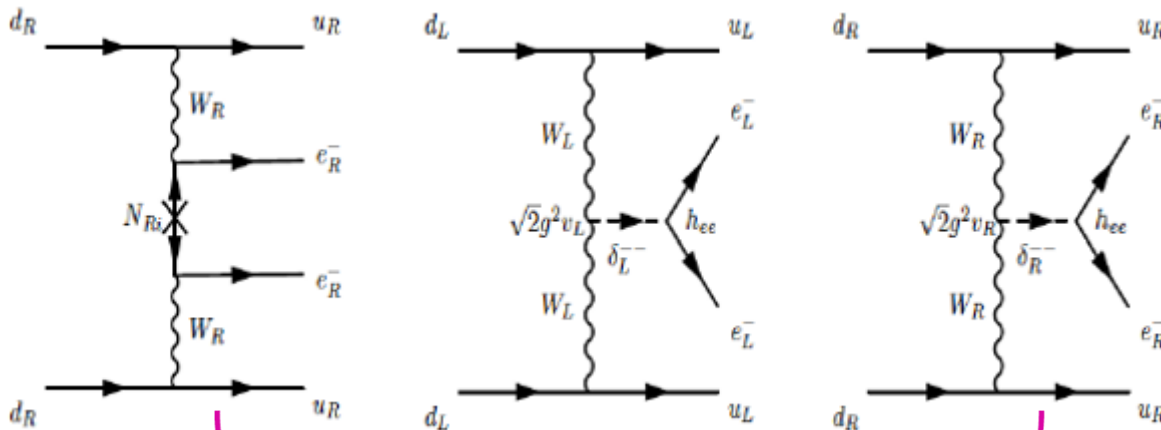
The  $0\nu\beta\beta$  decay can also be induced by other  $\Delta L=2$  physics besides the Majorana neutrino mass. There are many possible scenarios:



Type II seesaw



R-parity violating SUSY

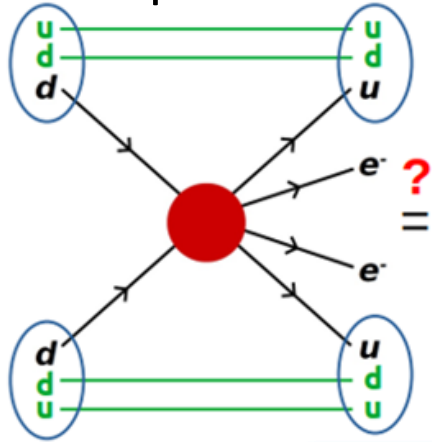


Left-right model

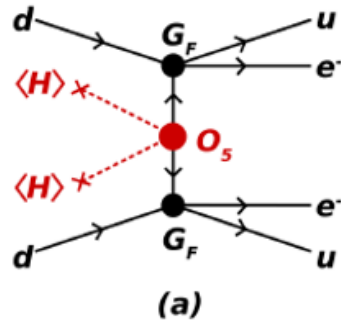
- $0\nu\beta\beta$  decay is connected to TeV scale physics and LFV.
- **A systematical classification is necessary!**

# Classification of $0\nu\beta\beta$ mechanisms from SMEFT

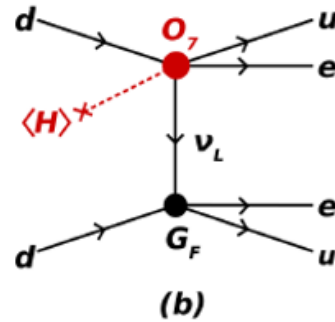
The amplitude of  $0\nu\beta\beta$  decay can be generally divided into:



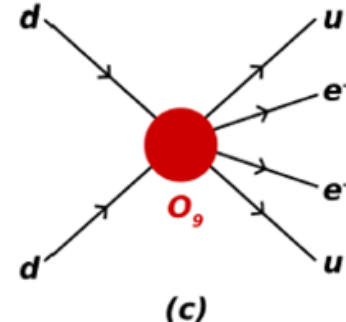
Mass mechanism



"long-range"



"short-range"



EFT:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C_i^{(5)}}{\Lambda} \mathcal{O}_i^{MM} + \frac{C_i^{(7)}}{\Lambda^3} \mathcal{O}_i^{LR} + \frac{C_i^{(9)}}{\Lambda^5} \mathcal{O}_i^{SR} + \dots$$

$$\mathcal{O}_1^{MM} = \epsilon^{ik} \epsilon^{jl} (\bar{\ell}_i^c \ell_j) H_k H_l$$

$$\mathcal{O}_1^{LR} = \epsilon^{ik} \epsilon^{jl} (\bar{\ell}_i^c \ell_j) (\bar{d}_R Q_k) H_l,$$

$$\mathcal{O}_2^{LR} = \epsilon^{ik} \epsilon^{jl} (\bar{\ell}_i^c \gamma^{\mu\nu} \ell_j) (\bar{d}_R \gamma_{\mu\nu} Q_k) H_l,$$

$$\mathcal{O}_3^{LR} = \epsilon^{jk} (\bar{\ell}_i^c \ell_j) (\bar{Q}^i u_R) H_k,$$

$$\mathcal{O}_4^{LR} = (\bar{\ell}_i^c \gamma^\mu e_R) (\bar{d}_R \gamma_\mu u_R) \epsilon^{ij} H_j$$

$$\mathcal{O}_1^{SR} = \epsilon_{ij} (\bar{Q}_i \gamma^\mu Q_m) (\bar{u}_R \gamma_\mu d_R) (\bar{\ell}_j \ell_m^c),$$

$$\mathcal{O}_2^{SR} = \epsilon_{ij} (\bar{Q}_i \gamma^\mu \lambda^A Q_m) (\bar{u}_R \gamma_\mu \lambda^A d_R) (\bar{\ell}_j \ell_m^c),$$

$$\mathcal{O}_3^{SR} = (\bar{u}_R Q_i) (\bar{u}_R Q_j) (\bar{\ell}_i \ell_j^c),$$

$$\mathcal{O}_4^{SR} = (\bar{u}_R \lambda^A Q_i) (\bar{u}_R \lambda^A Q_j) (\bar{\ell}_i \ell_j^c),$$

$$\mathcal{O}_5^{SR} = \epsilon_{ij} \epsilon_{mn} (\bar{Q}_i d_R) (\bar{Q}_m d_R) (\bar{\ell}_j \ell_n^c),$$

$$\mathcal{O}_6^{SR} = \epsilon_{ij} \epsilon_{mn} (\bar{Q}_i \lambda^A d_R) (\bar{Q}_m \lambda^A d_R) (\bar{\ell}_j \ell_n^c),$$

$$\mathcal{O}_7^{SR} = (\bar{u}_R \gamma^\mu d_R) (\bar{u}_R \gamma_\mu d_R) (\bar{e}_R e_R^c),$$

$$\mathcal{O}_8^{SR} = \epsilon_{ij} (\bar{u}_R \gamma^\mu d_R) (\bar{Q}_i d_R) (\bar{\ell}_j \gamma_\mu e_R^c),$$

$$\mathcal{O}_9^{SR} = \epsilon_{ij} (\bar{u}_R \gamma^\mu \lambda^A d_R) (\bar{Q}_i \lambda^A d_R) (\bar{\ell}_j \gamma_\mu e_R^c),$$

$$\mathcal{O}_{10}^{SR} = (\bar{u}_R \gamma^\mu d_R) (\bar{u}_R Q_i) (\bar{\ell}_i \gamma_\mu e_R^c),$$

$$\mathcal{O}_{11}^{SR} = (\bar{u}_R \gamma^\mu \lambda^A d_R) (\bar{u}_R \lambda^A Q_i) (\bar{\ell}_i \gamma_\mu e_R^c)$$

[Pas, Hirsch, Klapdor, Kovalenko, hep-ph/0008182, hep-ph/9804374; Graesser,1606.04549]

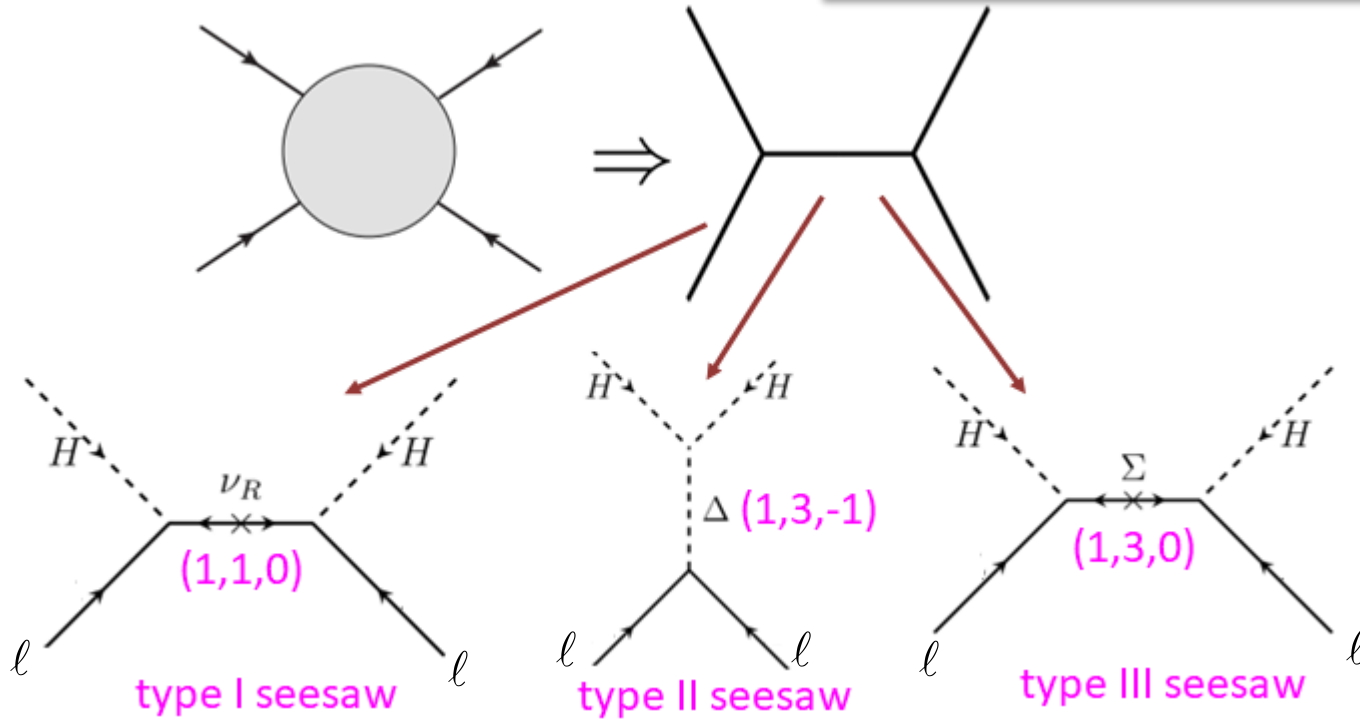


# Decomposing the short-range $0\nu\beta\beta$ operators

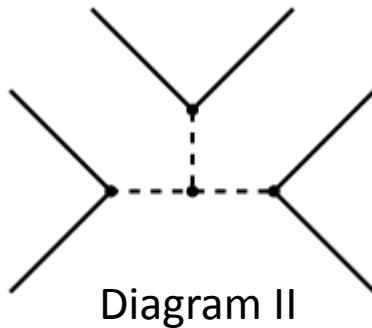
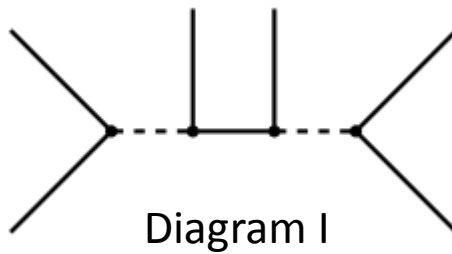
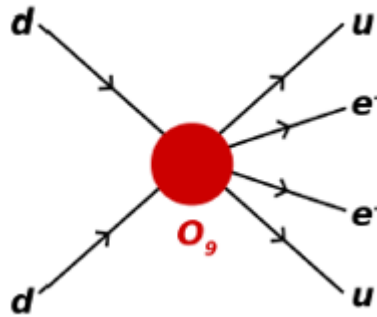
“Recipe” for a systematic classification of the possible realizations

- ① **Topologies**: identify the  $L$ -loop connected topologies with 6 external legs
- ② **Diagrams**: assign the fields of  $0\nu\beta\beta$  operators to external lines, and specify the Lorentz nature (spinor or scalar) of each internal line.
- ③ **Models**: fix the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  quantum numbers of the internal fields by gauge invariance of each interaction vertex

➤ Example: decompose Weinberg operator  $\mathcal{O}_W = \frac{c_{\alpha\beta}}{\Lambda} \epsilon^{ik} \epsilon^{jl} (\overline{\ell_{\alpha i}^c} \ell_{\beta j}) H_k H_l$



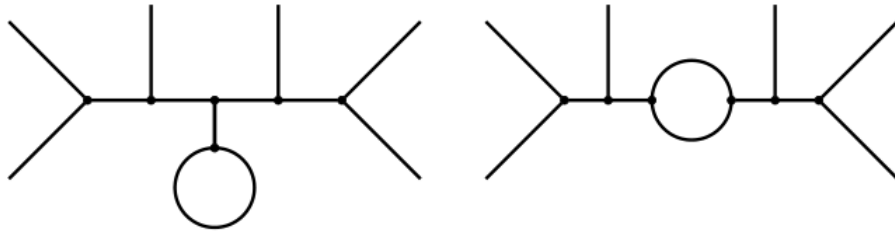
# Tree-level decomposition



#	Decomposition	Long Range?	Mediator ( $U(1)_{em}, SU(3)_c$ )			Models/Refs./Comments
			$S$ or $V_\rho$	$\psi$	$S'$ or $V'_\rho$	
1-i	$(\bar{u}d)(\bar{e})(\bar{e})(\bar{u}d)$	(a)	$(+1, \mathbf{1})$	$(0, \mathbf{1})$	$(-1, \mathbf{1})$	Mass mechan., RPV [58,60], LR-symmetric models [39], Mass mechanism with $\nu_S$ [61], TeV scale seesaw, e.g., [62,63,64]
1-ii-a	$(\bar{u}d)(\bar{u})(d)(\bar{e}\bar{e})$		$(+1, \mathbf{8})$ $(+1, \mathbf{1})$	$(0, \mathbf{8})$ $(+5/3, \mathbf{3})$	$(-1, \mathbf{8})$ $(+2, \mathbf{1})$	
1-ii-b	$(\bar{u}d)(d)(\bar{u})(\bar{e}\bar{e})$		$(+1, \mathbf{1})$ $(+1, \mathbf{8})$	$(+4/3, \mathbf{\bar{3}})$ $(+4/3, \mathbf{\bar{3}})$	$(+2, \mathbf{1})$ $(+2, \mathbf{1})$	
2-i-a	$(\bar{u}d)(d)(\bar{e})(\bar{u}\bar{e})$		$(+1, \mathbf{1})$ $(+1, \mathbf{8})$	$(+4/3, \mathbf{\bar{3}})$ $(+4/3, \mathbf{\bar{3}})$	$(+1/3, \mathbf{\bar{3}})$ $(+1/3, \mathbf{\bar{3}})$	
2-i-b	$(\bar{u}d)(\bar{e})(d)(\bar{u}\bar{e})$	(b)	$(+1, \mathbf{1})$ $(+1, \mathbf{8})$	$(0, \mathbf{1})$ $(0, \mathbf{8})$	$(+1/3, \mathbf{\bar{3}})$ $(+1/3, \mathbf{\bar{3}})$	RPV [58,60], LQ [65,66]
2-ii-a	$(\bar{u}d)(\bar{u})(\bar{e})(d\bar{e})$		$(+1, \mathbf{1})$ $(+1, \mathbf{8})$	$(+5/3, \mathbf{3})$ $(+5/3, \mathbf{3})$	$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{3})$	
2-ii-b	$(\bar{u}d)(\bar{e})(\bar{u})(d\bar{e})$	(b)	$(+1, \mathbf{1})$ $(+1, \mathbf{8})$	$(0, \mathbf{1})$ $(0, \mathbf{8})$	$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{3})$	RPV [58,60], LQ [65,66]
2-iii-a	$(d\bar{e})(\bar{u})(d)(\bar{u}\bar{e})$	(c)	$(-2/3, \mathbf{\bar{3}})$ $(-2/3, \mathbf{\bar{3}})$	$(0, \mathbf{1})$ $(0, \mathbf{8})$	$(+1/3, \mathbf{\bar{3}})$ $(+1/3, \mathbf{\bar{3}})$	RPV [58,60] RPV [58,60]
2-iii-b	$(d\bar{e})(d)(\bar{u})(\bar{u}\bar{e})$		$(-2/3, \mathbf{\bar{3}})$ $(-2/3, \mathbf{\bar{3}})$	$(-1/3, \mathbf{3})$ $(-1/3, \mathbf{\bar{6}})$	$(+1/3, \mathbf{\bar{3}})$ $(+1/3, \mathbf{\bar{3}})$	
3-i	$(\bar{u}\bar{u})(\bar{e})(\bar{e})(d\bar{d})$		$(+4/3, \mathbf{\bar{3}})$ $(+4/3, \mathbf{6})$	$(+1/3, \mathbf{\bar{3}})$ $(+1/3, \mathbf{6})$	$(-2/3, \mathbf{3})$ $(-2/3, \mathbf{6})$	only with $V_\rho$ and $V'_\rho$
3-ii	$(\bar{u}\bar{u})(d)(d)(\bar{e}\bar{e})$		$(+4/3, \mathbf{\bar{3}})$ $(+4/3, \mathbf{6})$	$(+5/3, \mathbf{3})$ $(+5/3, \mathbf{3})$	$(+2, \mathbf{1})$ $(+2, \mathbf{1})$	only with $V_\rho$
3-iii	$(d\bar{d})(\bar{u})(\bar{u})(\bar{e}\bar{e})$		$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{\bar{6}})$	$(+4/3, \mathbf{\bar{3}})$ $(+4/3, \mathbf{\bar{3}})$	$(+2, \mathbf{1})$ $(+2, \mathbf{1})$	only with $V_\rho$
4-i	$(d\bar{e})(\bar{u})(\bar{u})(d\bar{e})$	(c)	$(-2/3, \mathbf{\bar{3}})$ $(-2/3, \mathbf{\bar{3}})$	$(0, \mathbf{1})$ $(0, \mathbf{8})$	$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{3})$	RPV [58,60] RPV [58,60]
4-ii-a	$(\bar{u}\bar{u})(d)(\bar{e})(d\bar{e})$		$(+4/3, \mathbf{\bar{3}})$ $(+4/3, \mathbf{6})$	$(+5/3, \mathbf{3})$ $(+5/3, \mathbf{3})$	$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{3})$	only with $V_\rho$ see Sec. 4 (this work)
4-ii-b	$(\bar{u}\bar{u})(\bar{e})(d)(d\bar{e})$		$(+4/3, \mathbf{\bar{3}})$ $(+4/3, \mathbf{6})$	$(+1/3, \mathbf{\bar{3}})$ $(+1/3, \mathbf{6})$	$(+2/3, \mathbf{3})$ $(+2/3, \mathbf{3})$	only with $V_\rho$
5-i	$(\bar{u}\bar{e})(d)(d)(\bar{u}\bar{e})$	(c)	$(-1/3, \mathbf{3})$ $(-1/3, \mathbf{3})$	$(0, \mathbf{1})$ $(0, \mathbf{8})$	$(+1/3, \mathbf{\bar{3}})$ $(+1/3, \mathbf{\bar{3}})$	RPV [58,60] RPV [58,60]
5-ii-a	$(\bar{u}\bar{e})(\bar{u})(\bar{e})(d\bar{d})$		$(-1/3, \mathbf{3})$ $(-1/3, \mathbf{3})$	$(+1/3, \mathbf{\bar{3}})$ $(+1/3, \mathbf{6})$	$(-2/3, \mathbf{\bar{3}})$ $(-2/3, \mathbf{6})$	only with $V'_\rho$
5-ii-b	$(\bar{u}\bar{e})(\bar{e})(\bar{u})(d\bar{d})$		$(-1/3, \mathbf{3})$ $(-1/3, \mathbf{3})$	$(-4/3, \mathbf{3})$ $(-4/3, \mathbf{3})$	$(-2/3, \mathbf{\bar{3}})$ $(-2/3, \mathbf{6})$	only with $V'_\rho$

# Topologies for short-range $0\nu\beta\beta$

- Topologies:** Feynman diagrams where no property of fields is considered
- (i) All connected topologies with 3- and 4- point vertices and 6 external legs
  - (ii) Remove tadpoles and self-energies (**divergent**)

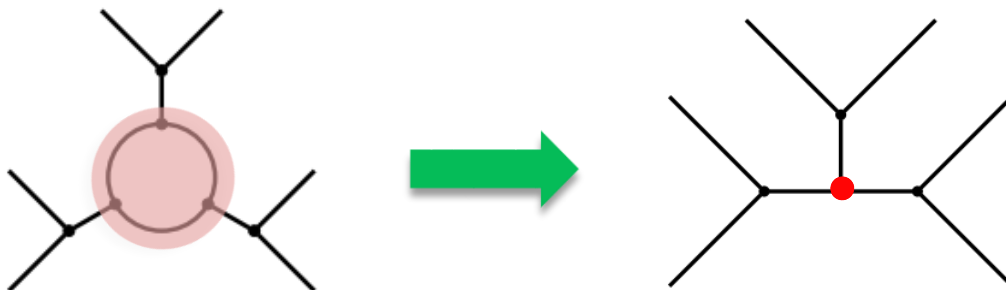


- (iii) Exclude non-renormalizable topologies



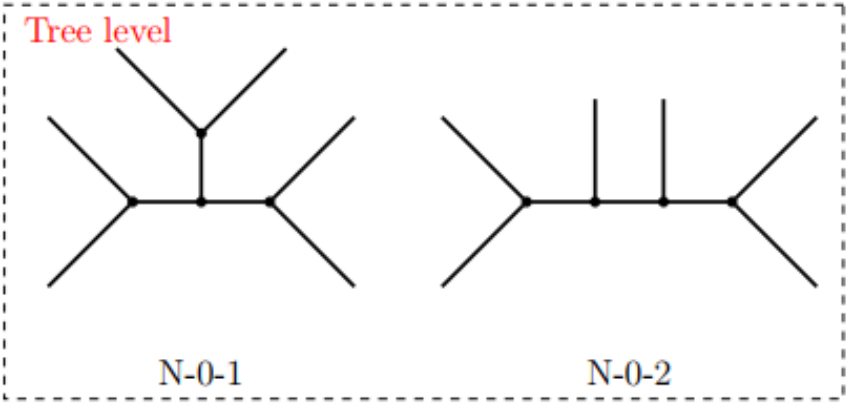
The 6 external legs are quark and lepton fields for short-range  $0\nu\beta\beta$

- (iv) Discard topologies with 3-point loop vertices

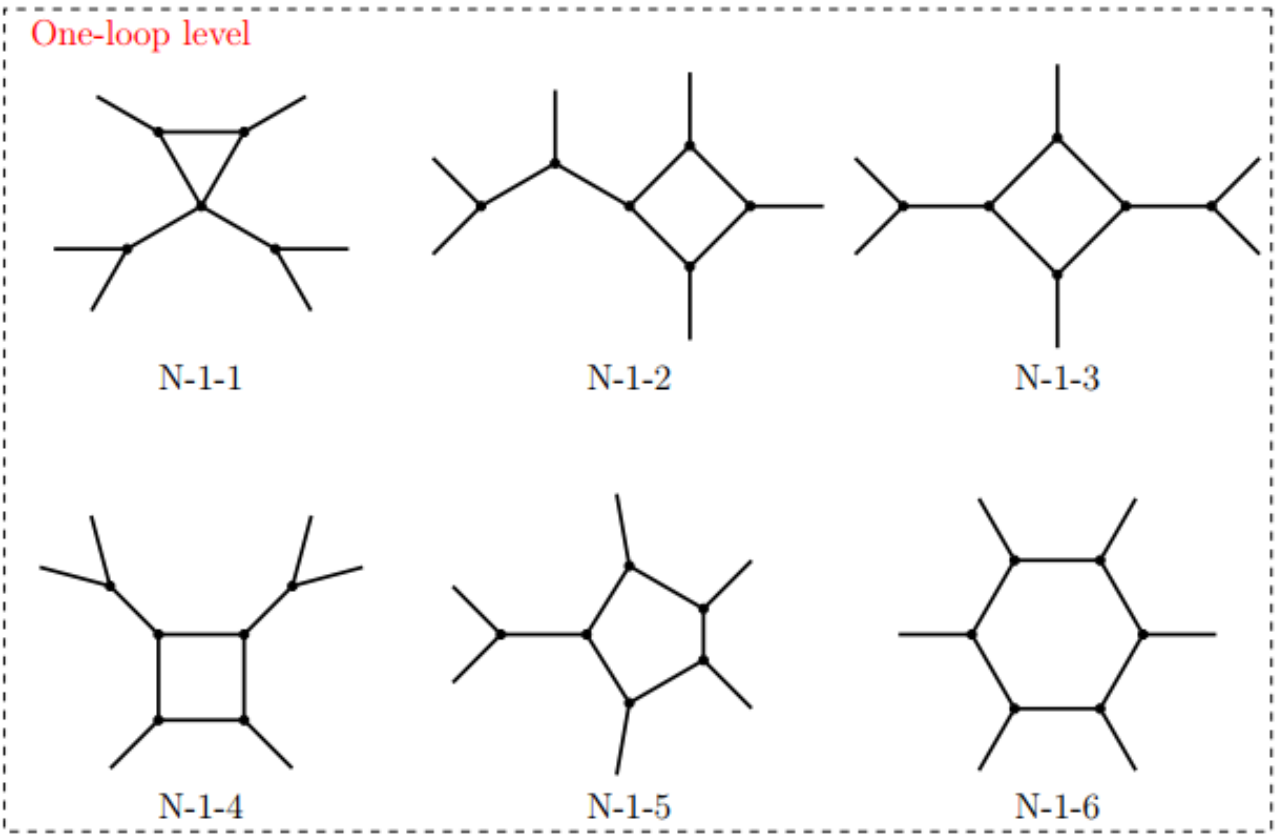


any loop with 3 external legs can be compressible to a renormalizable vertex<sub>19</sub>

# ➤ 2 tree + 6 one-loop renormalizable topologies

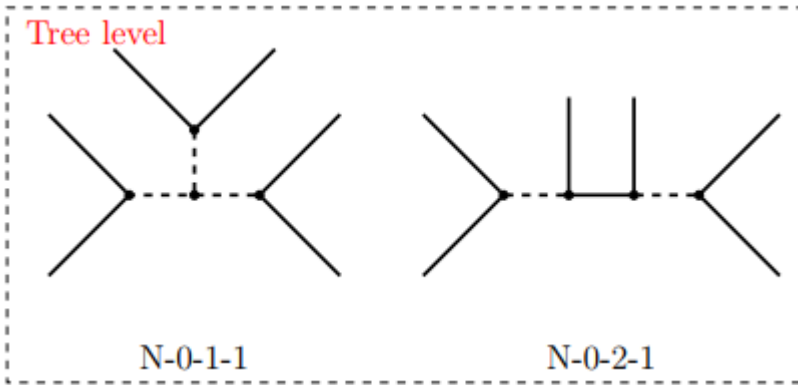


[Chen, Ding, Yao, 2110.15347]



# field insertions: topologies $\rightarrow$ diagrams

Focusing only on fermions and scalar bosons [Not considering gauge bosons]



➤ Three kinds of renormalizable vertices

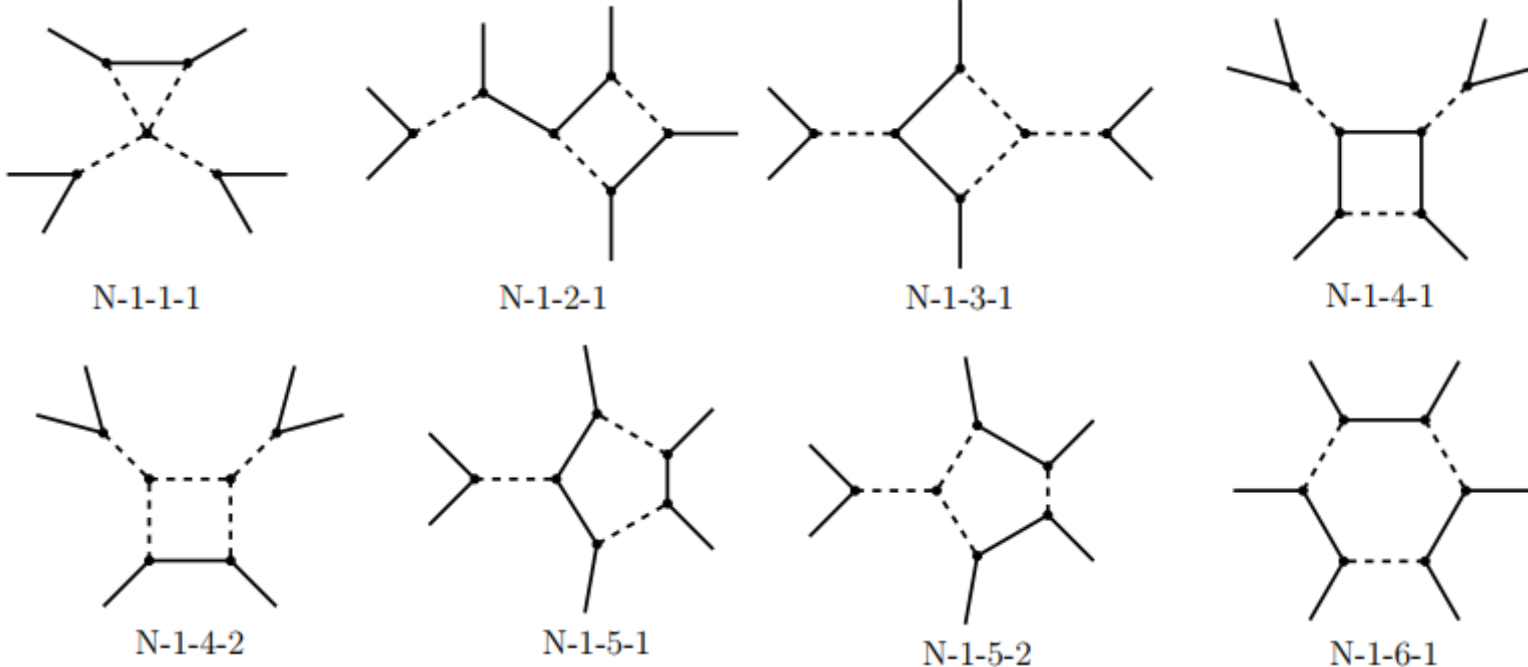
① fermion-fermion-scalar (**FFS**)

② scalar-scalar-scalar (**SSS**)

③ scalar-scalar-scalar-scalar (**SSSS**)

[Chen, Ding, Yao, 2110.15347]

One-loop level



# Determine quantum numbers: diagrams → models

The  $SU(3)_C \times SU(2)_L \times U(1)_Y$  quantum numbers of the mediators fields are fixed by gauge invariance of each interaction vertex

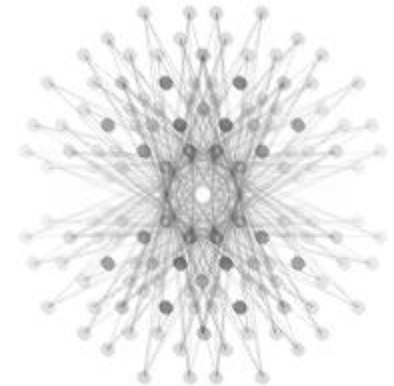
- **3-point vertex:**  $\bar{F}_1 F_2 S, S_1 S_2 S_3$

$$\begin{aligned} n_{\bar{F}_1} \otimes n_{F_2} \otimes n_S \supset \mathbf{1}, \quad Y_{\bar{F}_1} + Y_{F_2} + Y_S = 0 \\ n_{S_1} \otimes n_{S_2} \otimes n_{S_3} \supset \mathbf{1}, \quad Y_{S_1} + Y_{S_2} + Y_{S_3} = 0 \end{aligned}$$

$n_X$  denotes the  $SU(2)_L$  or  $SU(3)_C$  representation of the field  $X$

- **4-point vertex:**  $S_1 S_2 S_3 S_4$

$$n_{S_1} \otimes n_{S_2} \otimes n_{S_3} \otimes n_{S_4} \supset \mathbf{1}, \quad \sum_i Y_{S_i} = 0$$



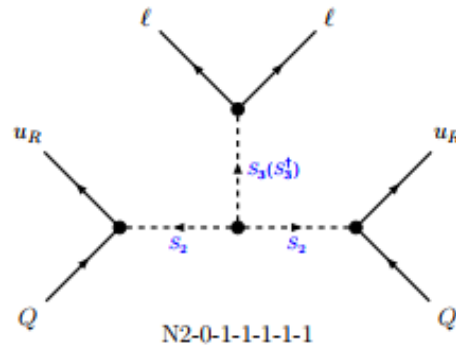
GROUPMATH

Mathematica package **GroupMath** can help to determine the SM quantum numbers.

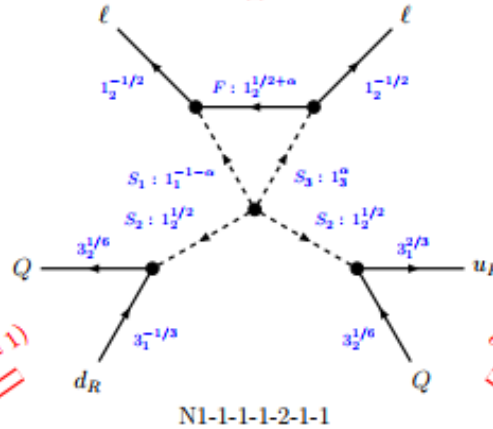
# Genuine models

“genuine” means:

- ① **leading** contribution to  $0\nu\beta\beta$  decay
- ② **no extra symmetries** beyond these of the SM



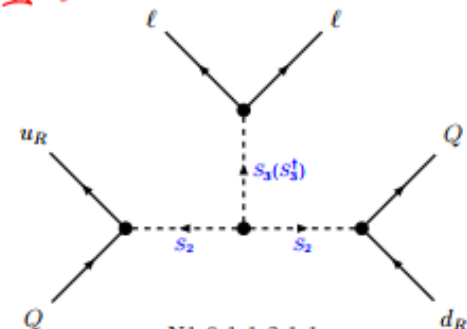
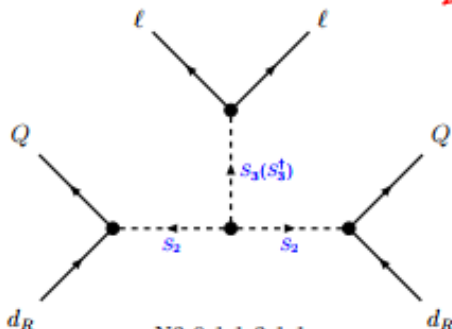
$\alpha = -1$  ( $\alpha = 1$ ) **X**



$\alpha = -1$  ( $\alpha = 1$ ) **X**

$\alpha = -1$  ( $\alpha = 1$ ) **X**

Filter models



# Determine quantum numbers: diagrams $\rightarrow$ models

Model Implementations of Neutrinoless Double  $\beta$  Decay up to one-loop

Loop: 0  1

Class:  N1  N2  N3  N4  N5  N6

Topology

Diagram

External Legs &  $U_Y(1)$

$SU_L(2)$

$SU_C(3)$

N-1-1

N-1-1-1

N1-1-1-1-1

N1-1-1-1-1-1

N1-1-1-1-1-1-1

N-1-2

N1-1-1-1-2

N1-1-1-1-1-2

N1-1-1-1-1-1-2

N-1-3

N1-1-1-1-1-2

N1-1-1-1-1-2

N1-1-1-1-1-1-2

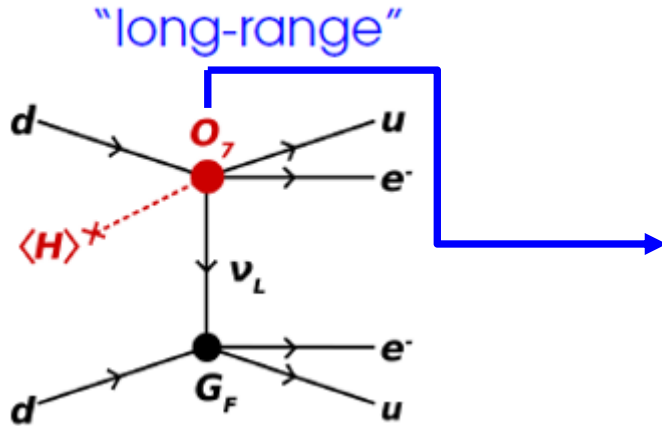
**A large number of possible diagrams.** For detail, see the attachment [http://staff.ustc.edu.cn/~dinggj/supplementary\\_materials/0nbb.zip](http://staff.ustc.edu.cn/~dinggj/supplementary_materials/0nbb.zip)



# Decomposing the long-range $0\nu\beta\beta$ operators

- Long-range mechanism is **not subject to helicity suppression!**

$$\Delta L = 2$$



$$\begin{aligned} \mathcal{O}_1^{LR} &= \epsilon^{ik} \epsilon^{jl} (\bar{\ell}_i^c \ell_j) (\bar{d}_R Q_k) H_l, \\ \mathcal{O}_2^{LR} &= \epsilon^{ik} \epsilon^{jl} (\bar{\ell}_i^c \gamma^{\mu\nu} \ell_j) (\bar{d}_R \gamma_{\mu\nu} Q_k) H_l, \\ \mathcal{O}_3^{LR} &= \epsilon^{jk} (\bar{\ell}_i^c \ell_j) (\bar{Q}^i u_R) H_k, \\ \mathcal{O}_4^{LR} &= (\bar{\ell}_i^c \gamma^\mu e_R) (\bar{d}_R \gamma_\mu u_R) \epsilon^{ij} H_j \end{aligned}$$

topologies



diagrams

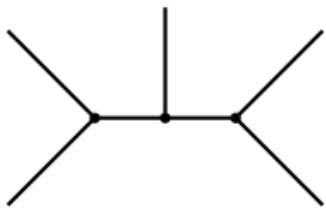


models

[Babu, Leung, hep-ph/0106054; Helo, Hirsch, Ota, 1602.03362; Lehman, 1410.4193]

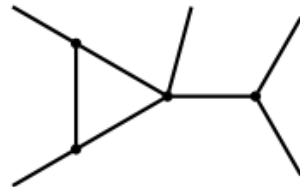
- Topologies

Tree level

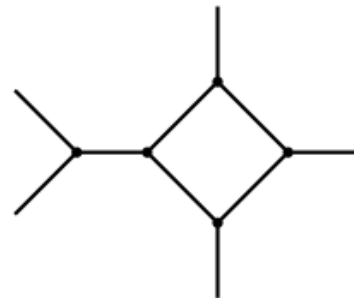


NL-0-1

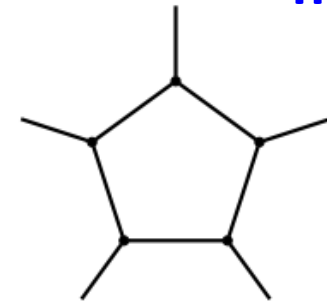
One loop



NL-1-1



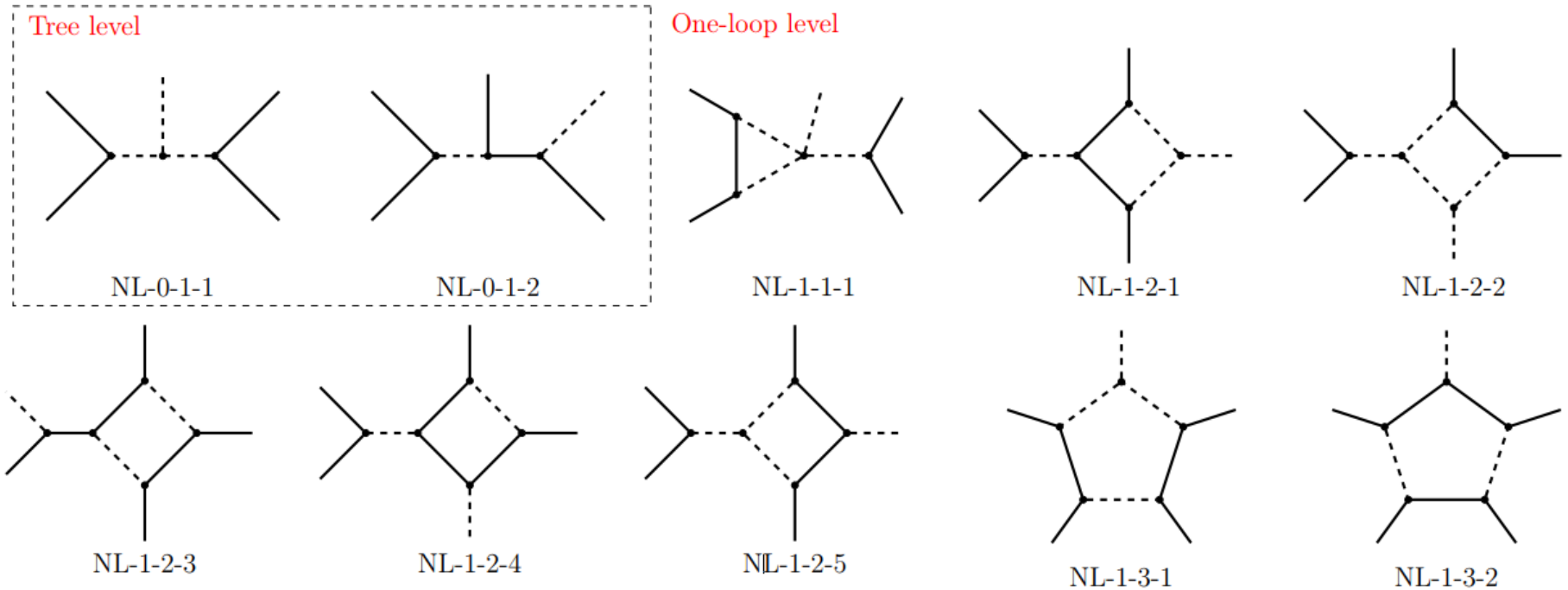
NL-1-2



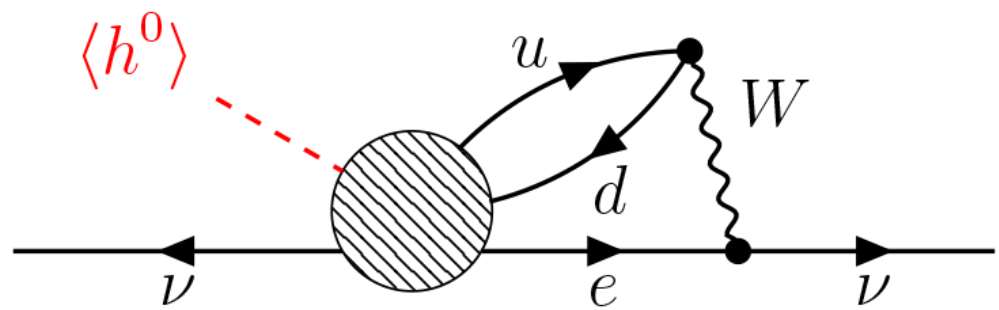
NL-1-3

[Chen, Ding, Yao, 2301.02503]

➤ Diagrams



- Models: large variety of possible realizations accessible at high-energy colliders and high-intensity facilities, all genuine long-range  $0\nu\beta\beta$  models up to 1-loop in the file [http://staff.ustc.edu.cn/~dinggj/supplementary\\_materials/Long\\_range\\_0nbb.zip](http://staff.ustc.edu.cn/~dinggj/supplementary_materials/Long_range_0nbb.zip)
- Black box theorem in long-range  $0\nu\beta\beta$ :  $\Delta L = 2$  operators  $\rightarrow 0\nu\beta\beta$  &  $\nu$  mass



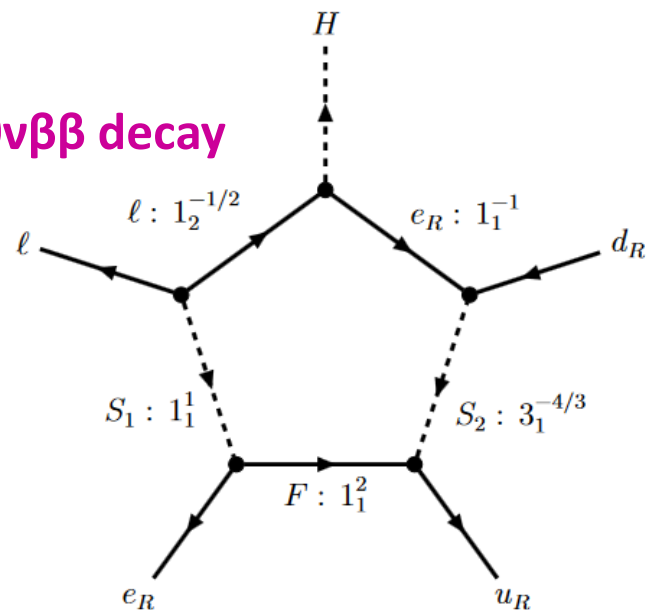
Majorana neutrino masses are generated at least at the **2-loop order**, regardless of long-range  $0\nu\beta\beta$  operators

# An example model of long-range $0\nu\beta\beta$ decay

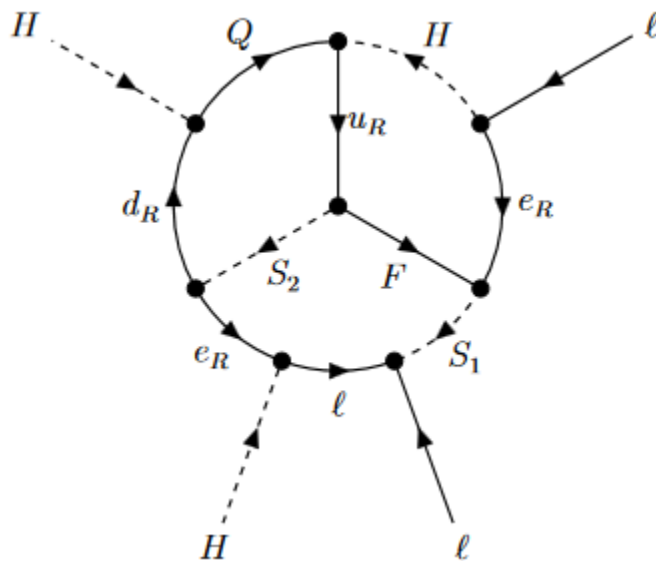
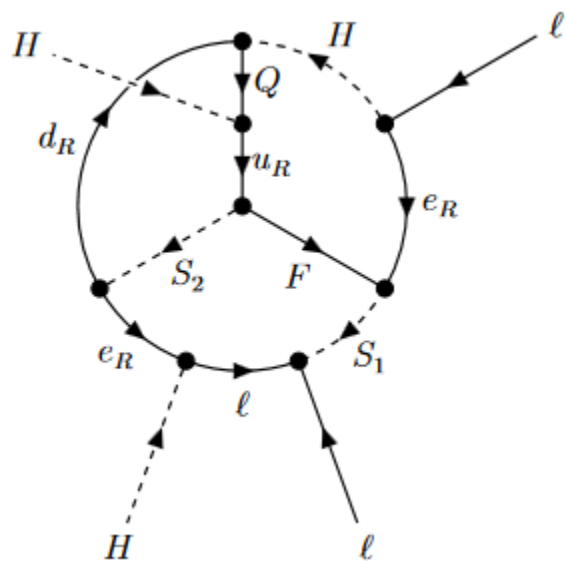
- **3** new fields: two scalars  $S_1, S_2$  and a vector-like fermion  $F$

Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$S_1$	<b>1</b>	<b>1</b>	1
$S_2$	<b>3</b>	<b>1</b>	$-4/3$
$F$	<b>1</b>	<b>1</b>	2

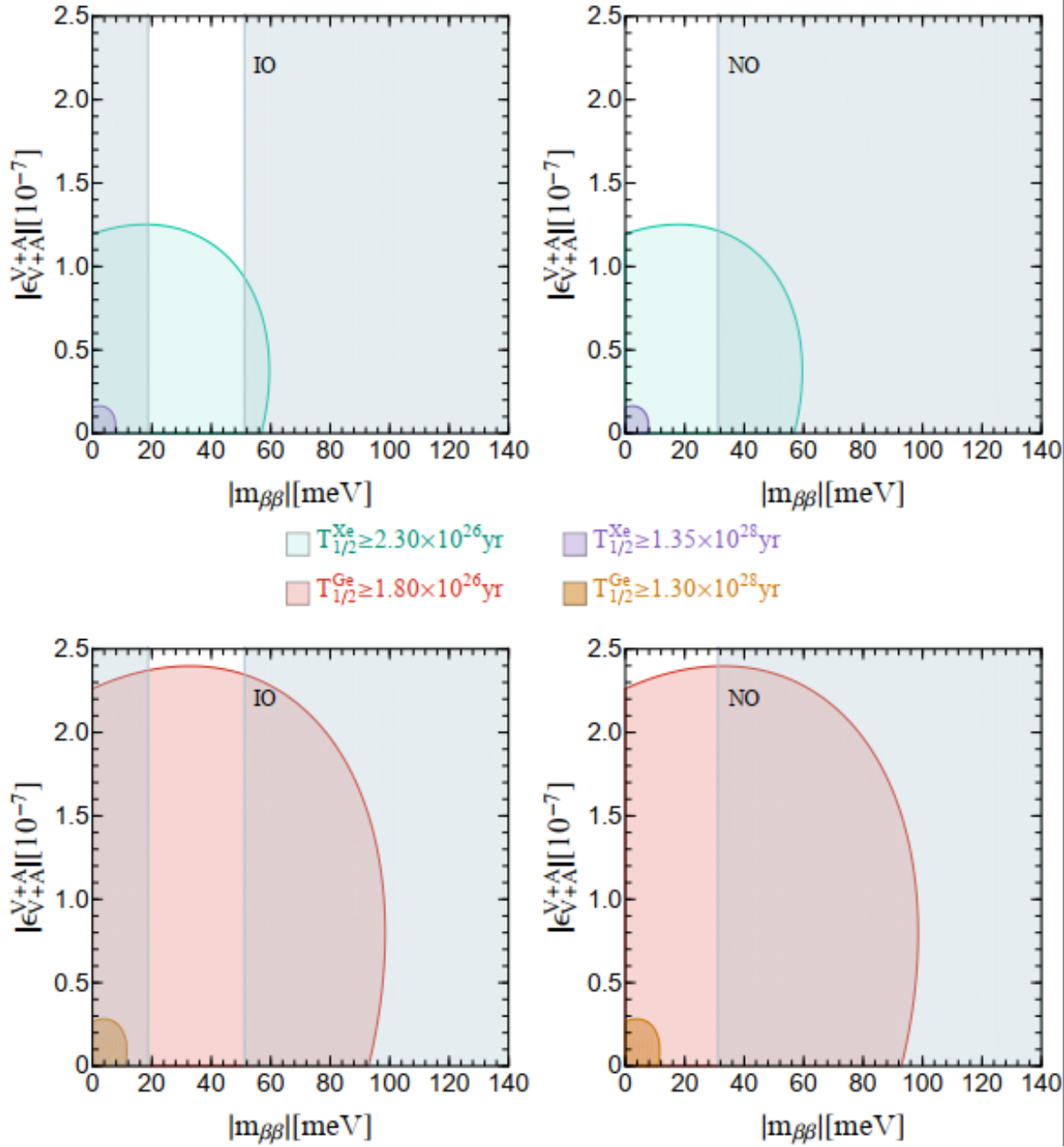
1-loop  $0\nu\beta\beta$  decay

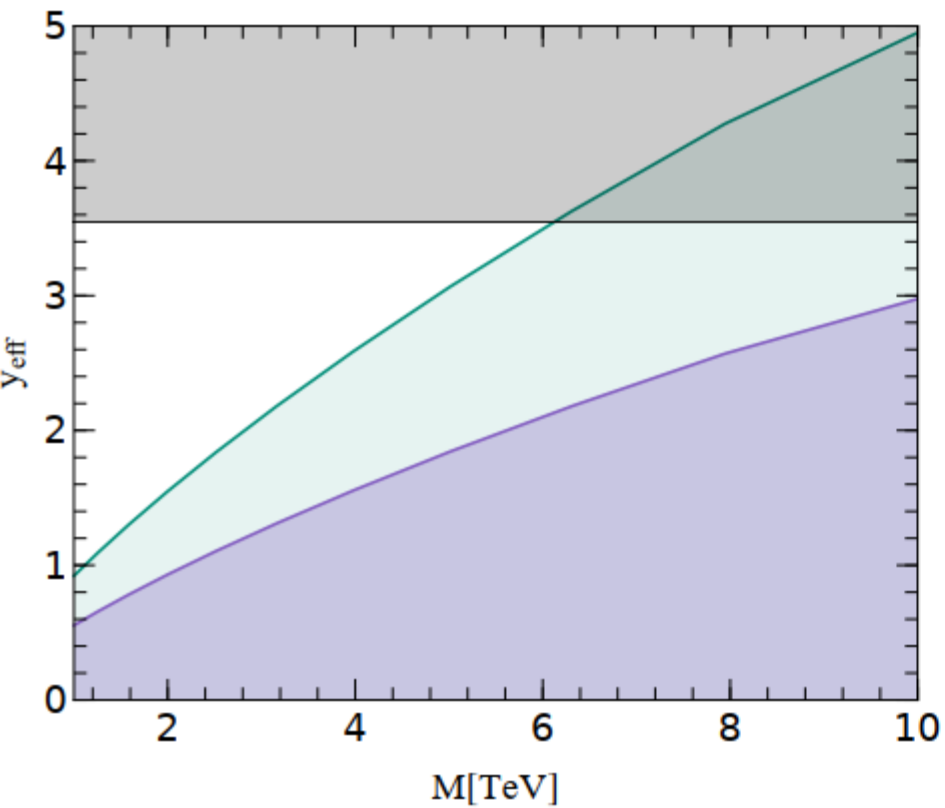


- Majorana neutrino masses generated at **3-loop**



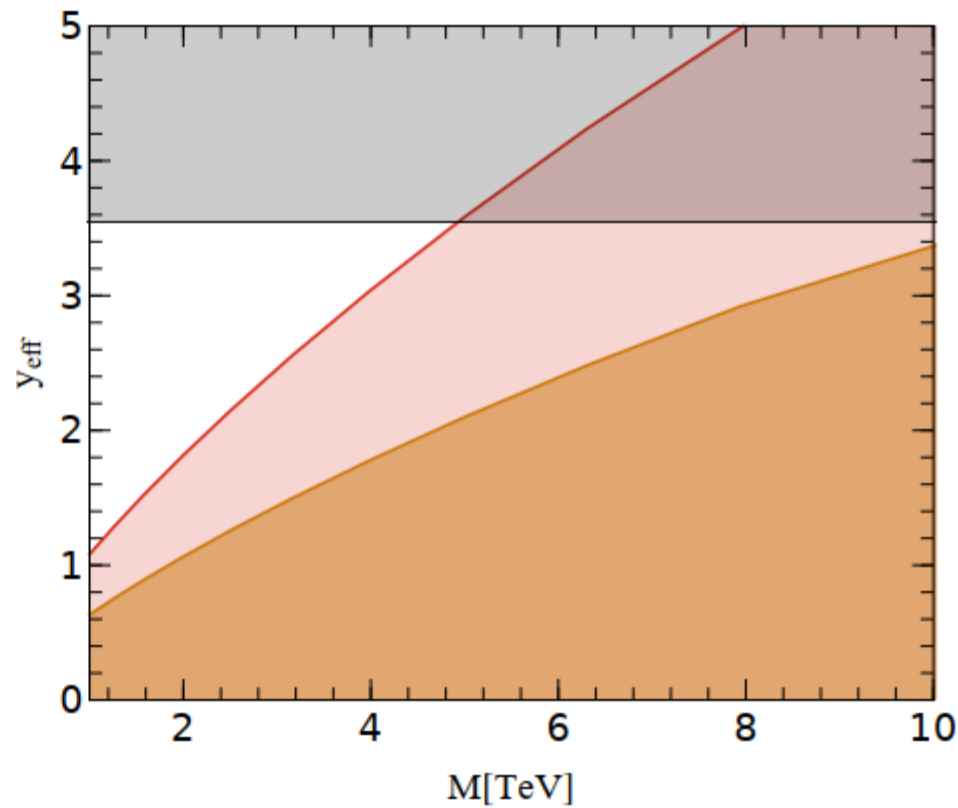
➤ Future ton-scale experiments impose strong constraint on the model and new physics contribution





$T_{1/2}^{\text{Xe}} \geq 2.30 \times 10^{26} \text{ yr}$

$T_{1/2}^{\text{Xe}} \geq 1.35 \times 10^{28} \text{ yr}$



$T_{1/2}^{\text{Ge}} \geq 1.80 \times 10^{26} \text{ yr}$

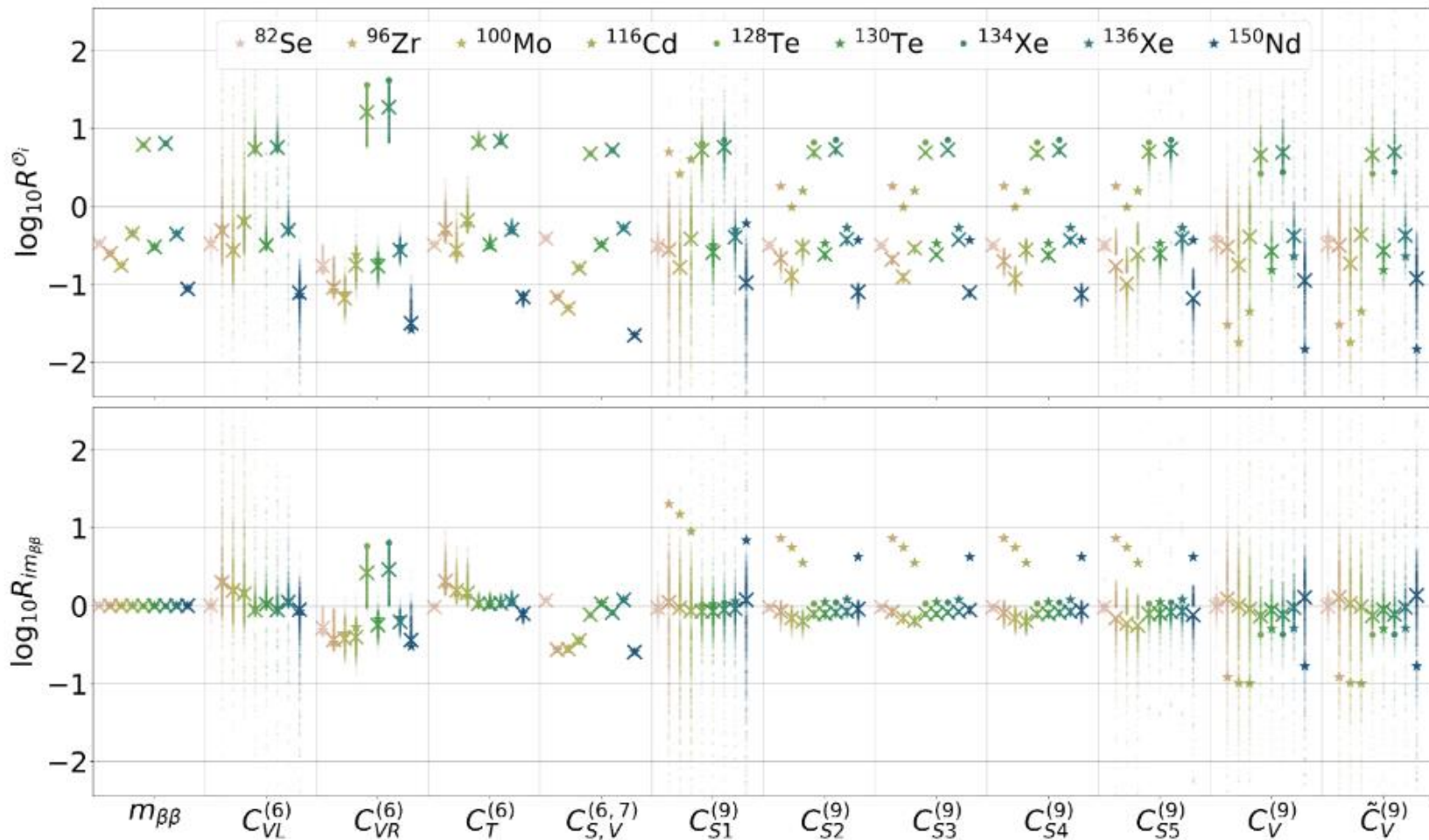
$T_{1/2}^{\text{Ge}} \geq 1.30 \times 10^{28} \text{ yr}$

# Distinguishing different $0\nu\beta\beta$ mechanisms

- Comparison of the decay rates obtained using different isotopes

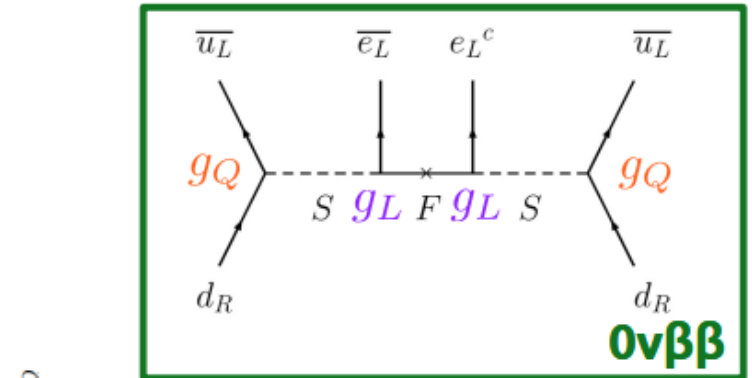
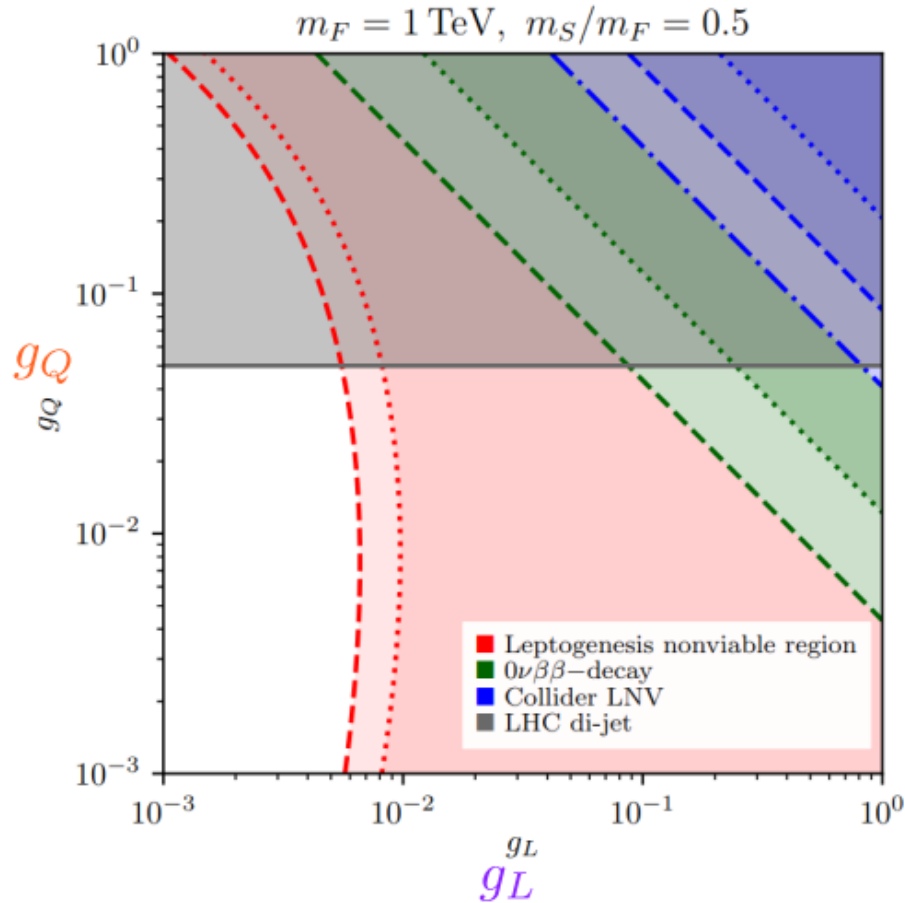
$$R^{O_i(A X)} \equiv \frac{T_{1/2}^{O_i(A X)}}{T_{1/2}^{O_i(^{76}\text{Ge})}} = \frac{\sum_j |\mathcal{M}_j^{O_i(^{76}\text{Ge})}|^2 G_j^{O_i(^{76}\text{Ge})}}{\sum_k |\mathcal{M}_k^{O_i(A X)}|^2 G_k^{O_i(A X)}}$$

[Graf,Lindner,Scholer,2204.10845]

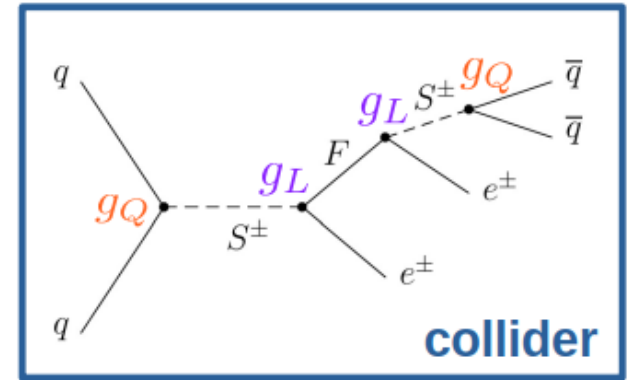


➤ Combining  $0\nu\beta\beta$  decay, collider measurements and cosmology

$$\mathcal{L} \supset g_Q \bar{Q} S d_R + g_L \bar{L} (i\tau^2) S^* F + \lambda_{HS} (S^\dagger H)^2 + \text{h.c.}$$



$g_Q$



[Harz, Ramsey-Musolf, Shen, Urrutia-Quiroga, 2106.10838;  
 Graesser, Li, Ramsey-Musolf, Shen, Urrutia-Quiroga, 2202.01237]

# Summary

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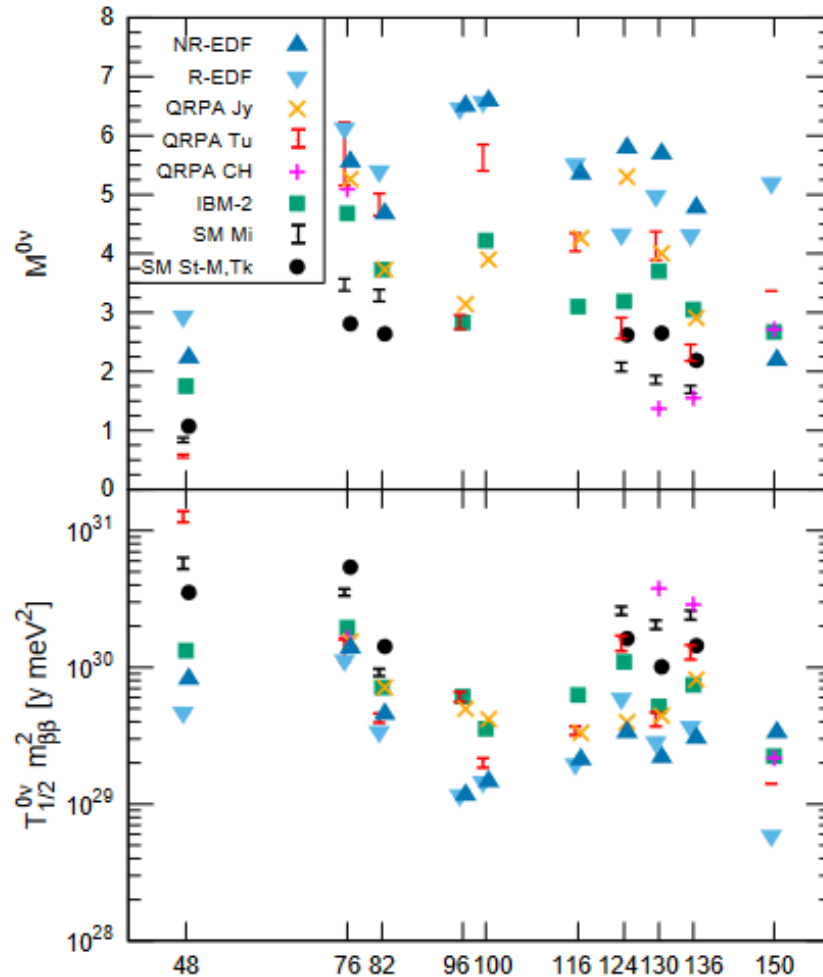
- $0\nu\beta\beta$  is the most sensitive probe to the Majorana nature of neutrinos. However, there are many possible physics mechanisms divided into mass mechanism, short-range mechanism and long-range mechanism.
- Systematic decomposition  $0\nu\beta\beta$  operators: **topologies** → **diagrams** → **models**
- Many open questions: the  $0\nu\beta\beta$  models in future colliders and LFV searches, implications in cosmology and leptogenesis....
- Both **theoretical and experimental efforts** are needed to fix the  $0\nu\beta\beta$  signal and underlying mechanism.



***Thank you for your attention!***

Backup

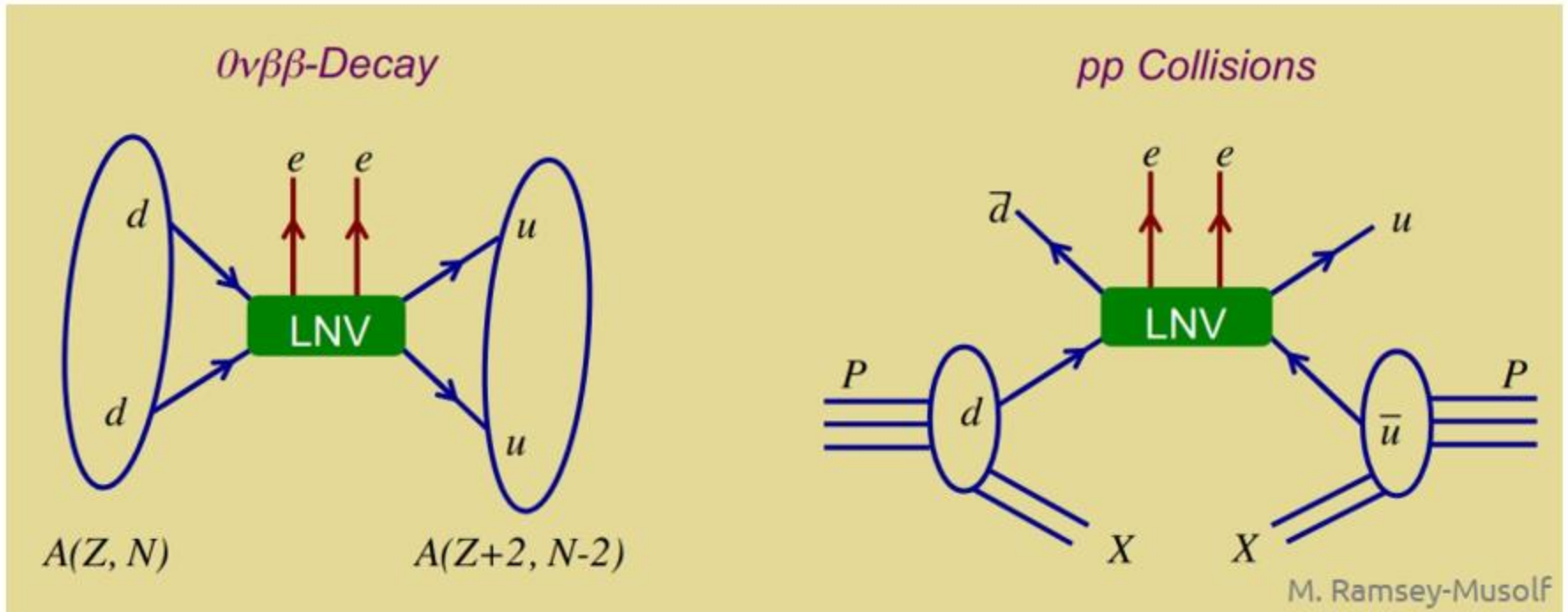
# Nuclear matrix elements



[Engel, Menendez, 1610.06548]

Unfortunately, nuclear matrix elements can be calculated only based on some models which describe many-body interactions of nucleons in nuclei. Since different models focus on different aspects of nuclear physics, **large uncertainties (a factor of 2 or 3)** are unavoidable.

# $0\nu\beta\beta$ at LHC

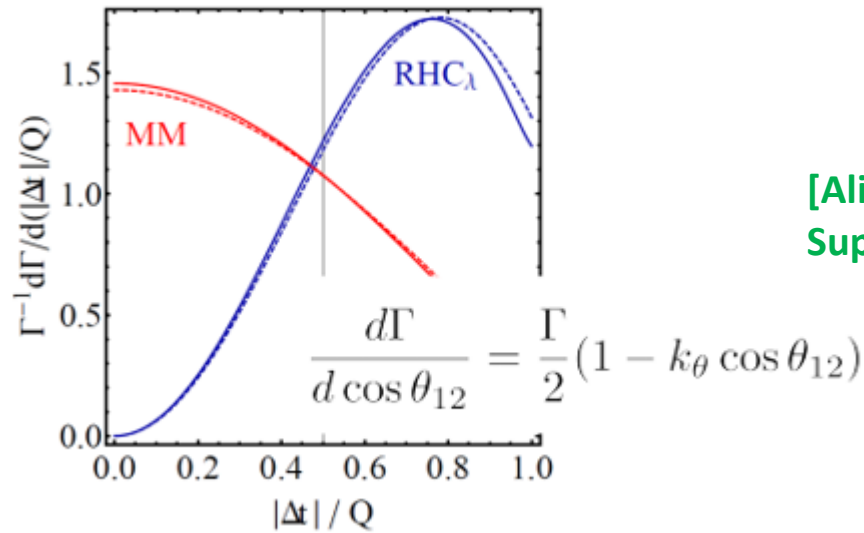


**Collider signature:** same-sign dilepton + 2 jets + no  $E_T^{\text{miss}}$

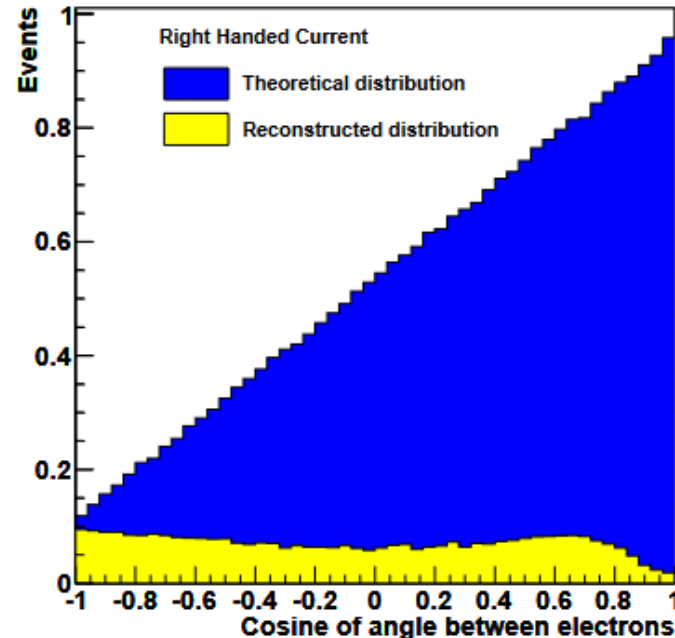
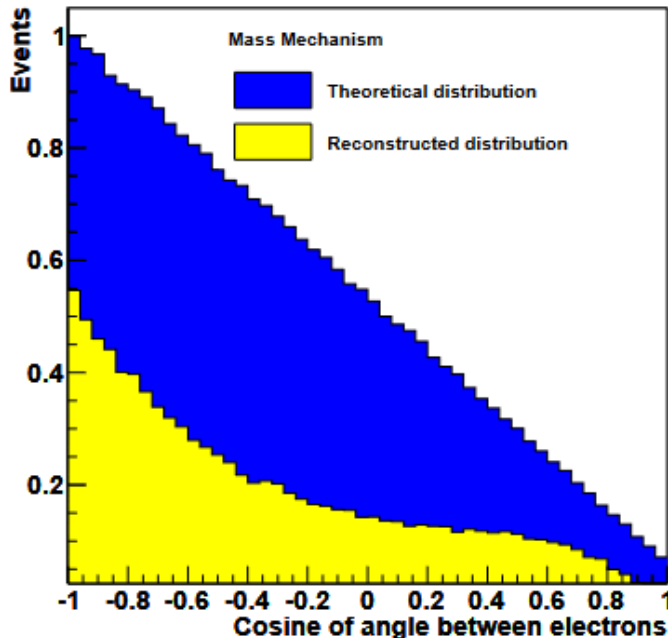
$$p + p \rightarrow 2e^{\pm} + 2\text{jets}$$

# determine $0\nu\beta\beta$ mechanisms

- measure the angular and energy distributions of electron



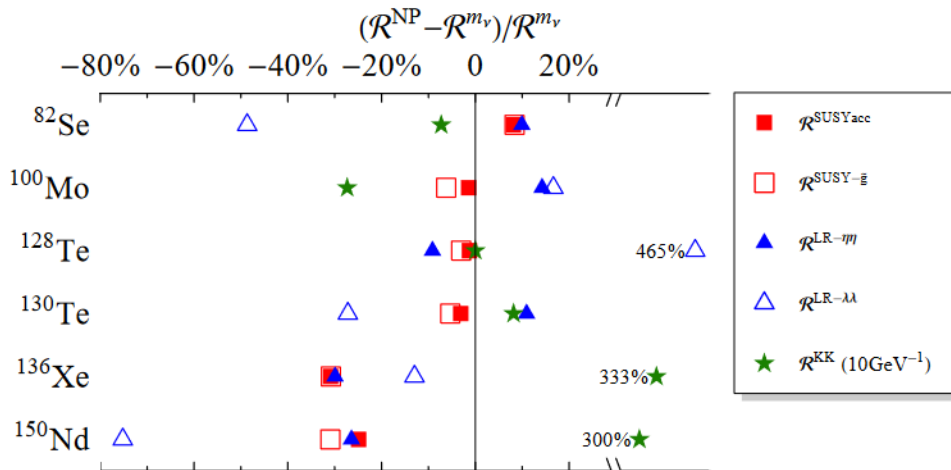
[Ali, Borisov, Zhuridov, 0706.4165;  
SuperNEMO Collaborarion,1005.1241]



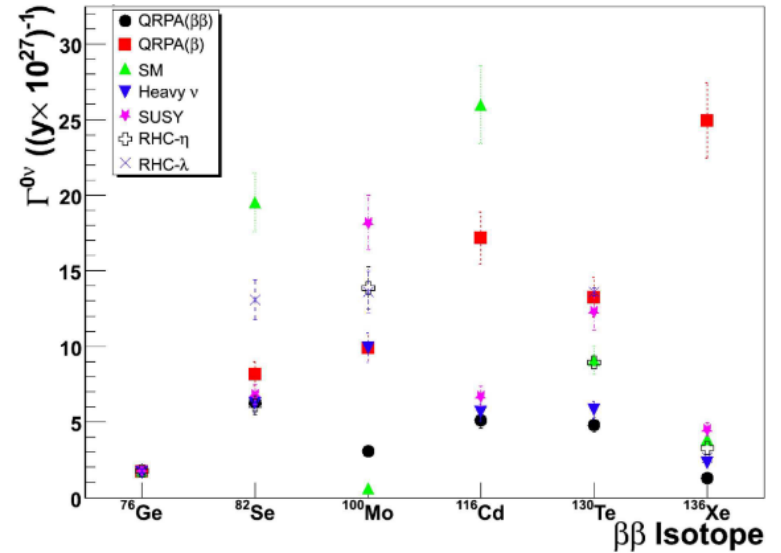
➤ Distinguish different mechanisms via measurements in different isotopes

$$[T_{1/2}^{NP}]^{-1} = \epsilon_{NP}^2 G_{NP} |\mathcal{M}^{NP}|^2$$

$$\frac{T_{1/2}(^A X)}{T_{1/2}(^{76}\text{Ge})} = \frac{|\mathcal{M}(^{76}\text{Ge})|^2 G(^{76}\text{Ge})}{|\mathcal{M}(^A X)|^2 G(^A X)}$$



[Deppisch, Pas, hep-ph/0612165]



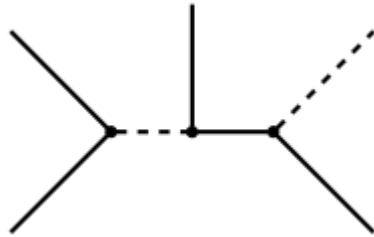
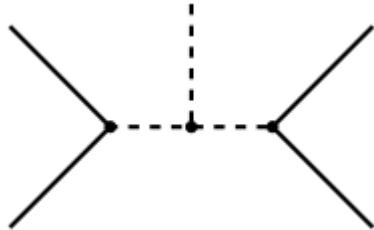
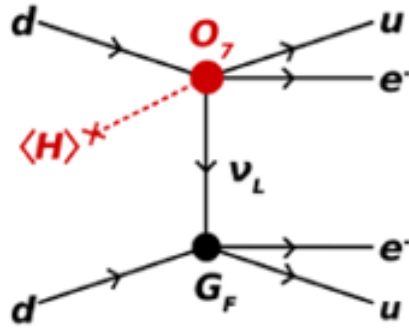
Isotope Ordering	Confidence Level	Number of Isotopes				
		2	3	4	5	6
Atomic Number	90%	<2%	8%	16%	23%	24%
	68%	<2%	19%	36%	45%	48%
$\Gamma^{0\nu}$ Spread	90%	6%	18%	27%	27%	24%
	68%	13%	29%	41%	42%	47%
Experimental Readiness	90%	3%	11%	24%	24%	24%
	68%	7%	18%	46%	47%	47%
Alternative Ordering	90%	3%	11%	17%	15%	24%
	68%	7%	18%	34%	32%	47%
Experimental Readiness (All 7 models, no $^{116}\text{Cd}$ )	90%	< 2%	6%	14%	16%	
	68%	< 2%	12%	22%	24%	

[Gehman, Elliott, hep-ph/0701099]

# determine $0\nu\beta\beta$ mechanisms

mechanism	amplitude and particle physics parameter	current limit	test
light neutrino exchange	$\frac{G_F^2}{q^2}  U_{ei}^2 m_i $	0.5 eV	oscillations, cosmology, neutrino mass
heavy neutrino exchange	$G_F^2 \frac{S_{ei}^2}{M_i}$	$2 \times 10^{-8} \text{ GeV}^{-1}$	LFV, collider
heavy neutrino and RHC	$G_F^2 m_W^4 \frac{V_{ei}^2}{M_i M_{WR}^4}$	$4 \times 10^{-16} \text{ GeV}^{-5}$	flavor, collider
Higgs triplet and RHC	$G_F^2 m_W^4 \left  \frac{(M_R)_{ee}}{m_{\Delta_R}^2 M_{WR}^4} \right $	$10^{-15} \text{ GeV}^{-1}$	flavor, collider $e^-$ distribution
$\lambda$ -mechanism with RHC	$G_F^2 \frac{m_W^2}{q} \left  \frac{U_{ei} \tilde{S}_{ei}}{M_{WR}^2} \right $	$1.4 \times 10^{-10} \text{ GeV}^{-2}$	flavor, collider, $e^-$ distribution
$\eta$ -mechanism with RHC	$G_F^2 \frac{1}{q} \tan \zeta \left  U_{ei} \tilde{S}_{ei} \right $	$6 \times 10^{-9}$	flavor, collider, $e^-$ distribution
short-range $\dot{R}$	$\frac{ \lambda'_{111} }{\Lambda_{\text{SUSY}}^5}$ $\Lambda_{\text{SUSY}} = f(m_{\tilde{g}}, m_{\tilde{u}_L}, m_{\tilde{d}_R}, m_{\chi_i})$	$7 \times 10^{-18} \text{ GeV}^{-5}$	collider, flavor
long-range $\dot{R}$	$\frac{G_F}{q} \left  \sin 2\theta^b \lambda'_{131} \lambda'_{113} \left( \frac{1}{m_{\tilde{b}_1}^2} - \frac{1}{m_{\tilde{b}_2}^2} \right) \right $ $\sim \frac{G_F}{q} m_b \frac{ \lambda'_{131} \lambda'_{113} }{\Lambda_{\text{SUSY}}^3}$	$2 \times 10^{-13} \text{ GeV}^{-2}$ $1 \times 10^{-14} \text{ GeV}^{-3}$	flavor, collider
Majorons	$\propto  \langle g_\chi \rangle  \text{ or }  \langle g_\chi \rangle ^2$	$10^{-4} \dots 1$	spectrum, cosmology

# tree-level decomposition of long-range $0\nu\beta\beta$ operators



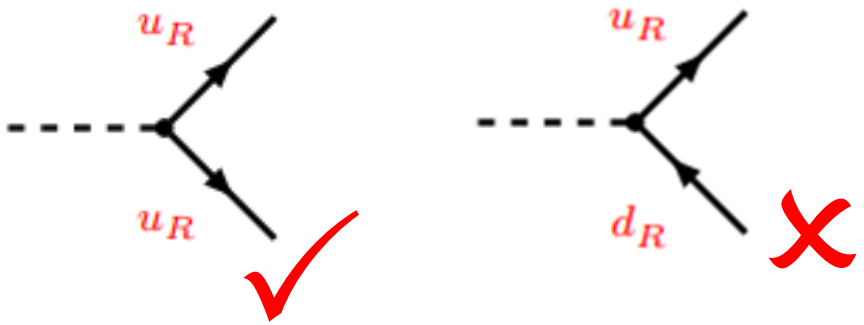
#	Decompositions	Mediators	Projection to the basis ops.	$m_\nu$ @tree	$m_\nu$ @1loop	$m_\nu$ @2loop	
#1	$(L_\alpha L_\beta)(H)(\overline{d}_R Q)$	$S(1,1)_{+1}$	$S'(1,2)_{+\frac{1}{2}}$	$-\mathcal{O}_{3a}(\alpha, \beta)$	—	$T\nu$ I-ii w. $\overline{\ell}_R L S^\dagger$	$T2_4^B(\alpha \neq \beta)$ $\mathcal{O}_3^7$ in [38]
		$S(1,3)_{+1}$	$S'(1,2)_{+\frac{1}{2}}$	$-\mathcal{O}_{3b}(\alpha, \beta) - \mathcal{O}_{3b}(\beta, \alpha)$	type II		
#2	$(L_\alpha Q)(H)(\overline{d}_R L_\beta)$	$S(\overline{3},1)_{+\frac{1}{3}}$	$S'(\overline{3},2)_{-\frac{1}{6}}$	$\frac{1}{2}\mathcal{O}_{3b}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha, \beta)$	—	$T\nu$ I-ii [53] $\mathcal{O}_3^8$ in [38]	[14, 68]
		$S(\overline{3},3)_{+\frac{1}{3}}$	$S'(\overline{3},2)_{-\frac{1}{6}}$	$\frac{1}{2}\mathcal{O}_{3a}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha, \beta)$ $-\frac{1}{2}\mathcal{O}_{3b}(\beta, \alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta, \alpha)$	—	$T\nu$ I-ii [53] $\mathcal{O}_3^9$ in [38]	[14]
#3	$(L_\alpha L_\beta)(Q)(\overline{d}_R H)$	$S(1,1)_{+1}$	$\psi_{L,R}(3,2)_{-\frac{5}{6}}$	$-\mathcal{O}_{3a}(\alpha, \beta)$	—	$T\nu$ I-ii [53] w. $S^\dagger H H'$	$T2_1^B(\alpha \neq \beta)$ $\mathcal{O}_3^1$ in [38]
		$S(1,3)_{+1}$	$\psi_{L,R}(3,2)_{-\frac{5}{6}}$	$-\mathcal{O}_{3b}(\alpha, \beta) - \mathcal{O}_{3b}(\beta, \alpha)$	type II		
#4	$(L_\alpha H)(Q)(\overline{d}_R L_\beta)$	$\psi_R(1,1)_0$	$S(\overline{3},2)_{-\frac{1}{6}}$	$\frac{1}{2}\mathcal{O}_{3b}(\beta, \alpha) + \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta, \alpha)$	type I		
		$\psi_R(1,3)_0$	$S(\overline{3},2)_{-\frac{1}{6}}$	$-\frac{1}{2}\mathcal{O}_{3a}(\alpha, \beta) + \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha, \beta)$ $-\frac{1}{2}\mathcal{O}_{3b}(\alpha, \beta) + \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha, \beta)$	type III		
#5	$(L_\alpha L_\beta)(\overline{d}_R)(QH)$	$S(1,1)_{+1}$	$\psi_{L,R}(3,1)_{+\frac{2}{3}}$	$\mathcal{O}_{3a}(\alpha, \beta)$	—	$T\nu$ I-ii [53] w. $S^\dagger H H'$	$T2_2^B(\alpha \neq \beta)$ $\mathcal{O}_3^2$ in [38]
		$S(1,3)_{+1}$	$\psi_{L,R}(3,3)_{+\frac{2}{3}}$	$-\mathcal{O}_{3b}(\alpha, \beta) - \mathcal{O}_{3b}(\beta, \alpha)$	type II		
#6	$(L_\alpha Q)(\overline{d}_R)(L_\beta H)$	$S(\overline{3},1)_{+\frac{1}{3}}$	$\psi_R(1,1)_0$	$-\frac{1}{2}\mathcal{O}_{3b}(\alpha, \beta) + \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha, \beta)$	type I		
		$S(\overline{3},3)_{+\frac{1}{3}}$	$\psi_R(1,3)_0$	$\frac{1}{2}\mathcal{O}_{3a}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha, \beta)$ $-\frac{1}{2}\mathcal{O}_{3b}(\beta, \alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta, \alpha)$	type III		
#7	$(L_\alpha Q)(L_\beta)(\overline{d}_R H)$	$S(\overline{3},1)_{+\frac{1}{3}}$	$\psi_{L,R}(3,2)_{-\frac{5}{6}}$	$\frac{1}{2}\mathcal{O}_{3b}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha, \beta)$	—	$T\nu$ I-iii $\mathcal{O}_3^4$ in [38]	
		$S(\overline{3},3)_{+\frac{1}{3}}$	$\psi_{L,R}(3,2)_{-\frac{5}{6}}$	$\frac{1}{2}\mathcal{O}_{3a}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha, \beta)$ $-\frac{1}{2}\mathcal{O}_{3b}(\beta, \alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta, \alpha)$	—	$T\nu$ I-iii $\mathcal{O}_3^5$ in [38]	
#8	$(\overline{d}_R L_\alpha)(L_\beta)(QH)$	$S(3,2)_{+\frac{1}{6}}$	$\psi_{L,R}(3,1)_{+\frac{2}{3}}$	$-\frac{1}{2}\mathcal{O}_{3a}(\alpha, \beta) - \frac{1}{2}\mathcal{O}_{3a}^{\text{ten.}}(\alpha, \beta)$	—	—	$T2_2^B(m_\nu)_{\alpha \neq \beta}$ $\mathcal{O}_3^3$ in [38], [43]
		$S(3,2)_{+\frac{1}{6}}$	$\psi_{L,R}(3,3)_{+\frac{2}{3}}$	$\frac{1}{2}\mathcal{O}_{3b}(\alpha, \beta) + \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\alpha, \beta)$ $+\frac{1}{2}\mathcal{O}_{3b}(\beta, \alpha) - \frac{1}{2}\mathcal{O}_{3b}^{\text{ten.}}(\beta, \alpha)$	—	$T\nu$ I-iii $\mathcal{O}_3^6$ in [38]	
#9	$(L_\alpha H)(L_\beta)(\overline{d}_R Q)$	$\psi_R(1,1)_0$	$S(1,2)_{+\frac{1}{2}}$	$\mathcal{O}_{3b}(\beta, \alpha)$	type I		
		$\psi_R(1,3)_0$	$S(1,2)_{+\frac{1}{2}}$	$\mathcal{O}_{3a}(\alpha, \beta) + \mathcal{O}_{3b}(\alpha, \beta)$	type III		



# ➤ Diagrams continuum: attach external fields

Lorentz invariance fixes the mediator to be **scalar or vector** by chirality of external fermions

## Classify $0\nu\beta\beta$ operators



Notation	$0\nu\beta\beta$ decay operators	External fields
N1	$\mathcal{O}_1^{SR}, \mathcal{O}_2^{SR}$	$\bar{Q}, Q, \bar{u}_R, d_R, \bar{\ell}, \ell^c$
N2	$\mathcal{O}_3^{SR}, \mathcal{O}_4^{SR}$	$Q, Q, \bar{u}_R, \bar{u}_R, \bar{\ell}, \ell^c$
N3	$\mathcal{O}_5^{SR}, \mathcal{O}_6^{SR}$	$\bar{Q}, \bar{Q}, d_R, d_R, \bar{\ell}, \ell^c$
N4	$\mathcal{O}_7^{SR}$	$\bar{u}_R, \bar{u}_R, d_R, d_R, \bar{e}_R, e_R^c$
N5	$\mathcal{O}_8^{SR}, \mathcal{O}_9^{SR}$	$\bar{u}_R, \bar{Q}, d_R, d_R, \bar{\ell}, e_R^c$
N6	$\mathcal{O}_{10}^{SR}, \mathcal{O}_{11}^{SR}$	$\bar{u}_R, \bar{u}_R, Q, d_R, \bar{\ell}, e_R^c$

### • A large number of possible diagrams

		$\mathcal{O}_i^{SR}$	N1	N2	N3	N4	N5	N6
Tree	TOPO							
		N-0-1-1	2	2	5	2	2	2
		N-0-2-1	11	6	18	6	11	11
One-loop		N-1-1-1	6	5	12	5	6	6
		N-1-2-1	96	30	54	30	96	96
		N-1-3-1	11	9	21	9	12	12
		N-1-4-1	11	6	18	6	11	11
		N-1-4-2	11	6	18	6	11	11
		N-1-5-1	48	18	30	18	48	48
		N-1-5-2	48	18	30	18	48	48
		N-1-6-1	60	18	18	18	60	60

The redundant diagrams should be removed.