华南师范大学
SOUTH CHINA NORMAL UNIVERSITY

Lattice Parton Collaboration

轻介子LCDA研究的现状与挑战

Based on PRL127(2021)062002, PRL129(2022)132001, JHEP 08 (2023) 172, arXiv2302.09961

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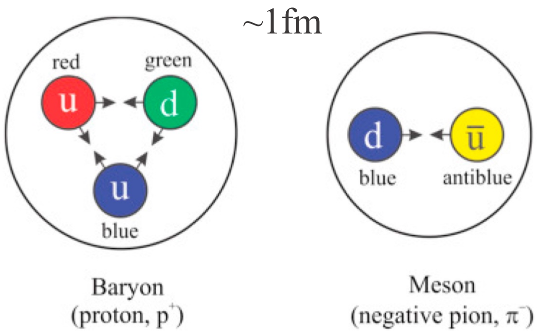
Apr. 17, 2024 @ CCNU

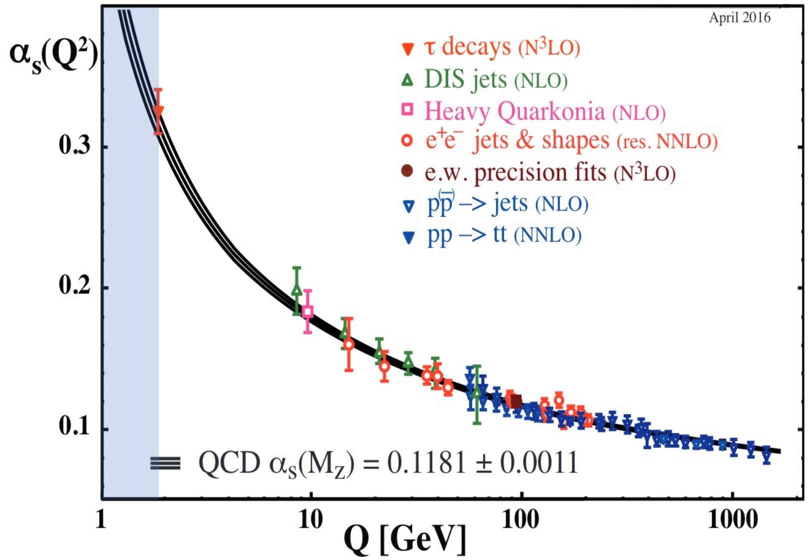
Outline

- ➢ Motivation
- ➢ Light meson LCDAs
  - Research journey
  - Recent progresses with lattice QCD
  - Pseudoscalar and vector meson LCDAs by LaMET
  - Reflections on current issues
- ➢ Three dimensional TMDWF
- ➢ Outlook and Summary

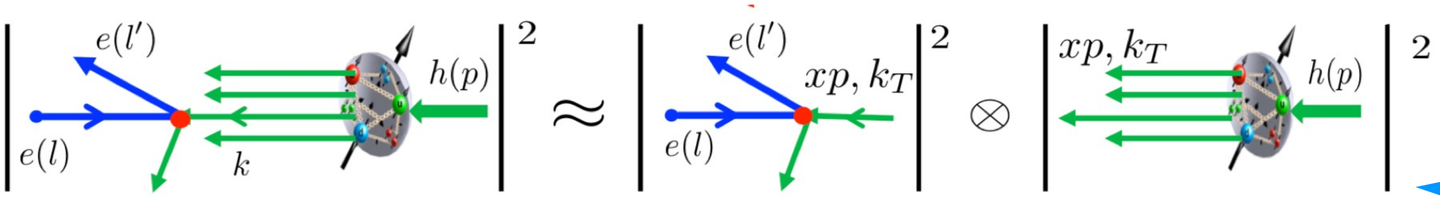
Motivation

Limited by QCD confinement





➢ **QCD factorization (1982)**



parton density amplitude (PDF, LCDA)

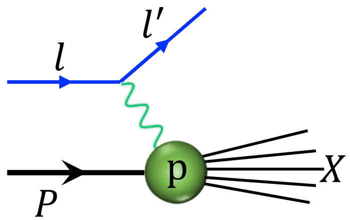
Cross section

Asymptotic freedom (hard kernel)

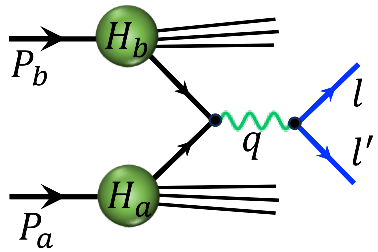
Color confinement (Parton in hadron)

- **PDFs: the probability distribution of partons (quarks and gluons) within a hadron —— Inclusive process**

DIS

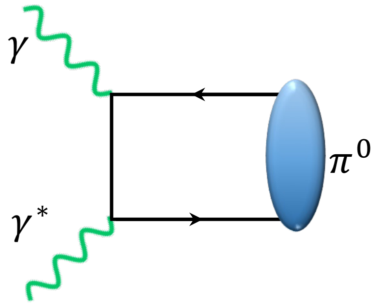


Drell-Yan

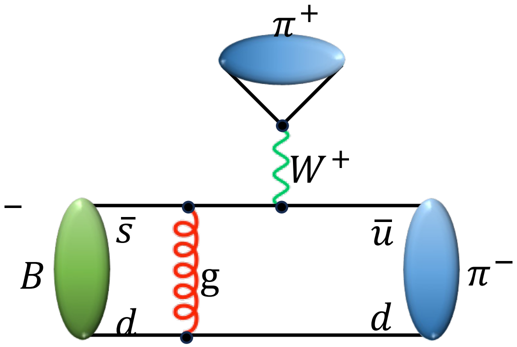


- **LCDAs: the probability amplitude for partons within a hadron ——Exclusive process**

$\gamma\gamma^* \to \pi$



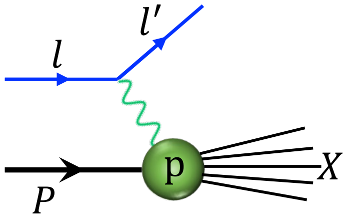
$B \to \pi^+\pi^-$



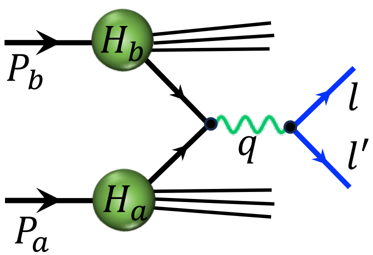
Motivation

➢ PDF

DIS

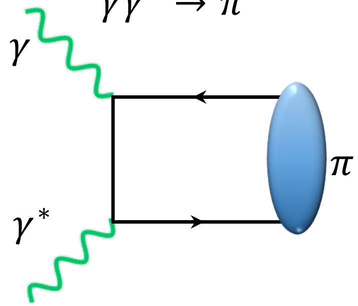


Drell-Yan

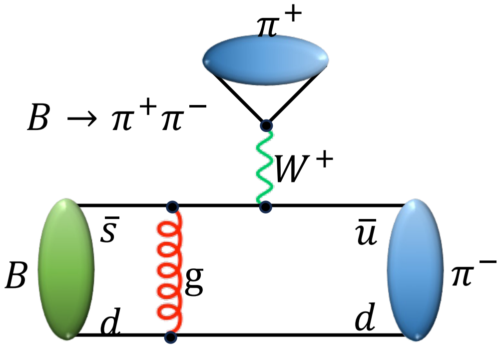


➢ LCDA

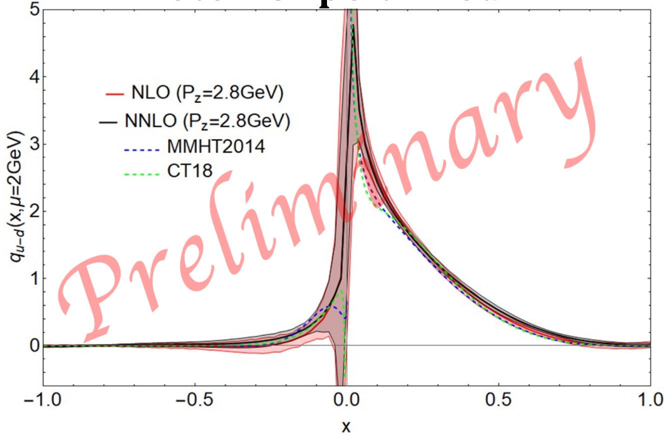
$\gamma\gamma^* \to \pi$



$B \to \pi^+\pi^-$

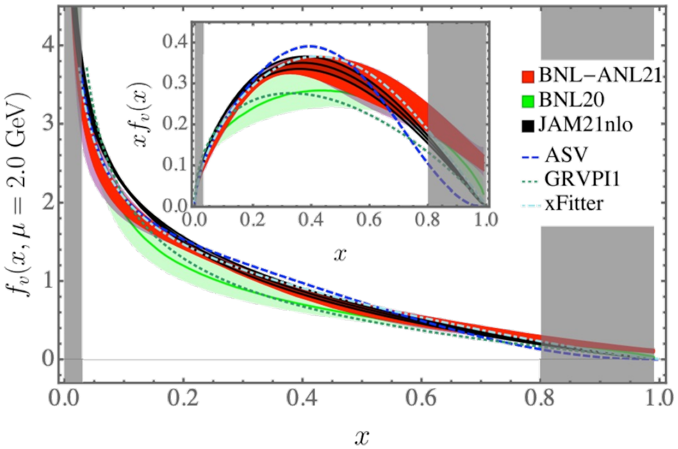


Proton Unpolarized PDF



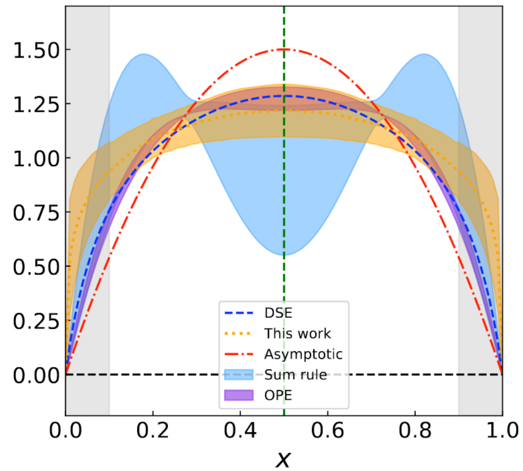
Preliminary from LPC

Pion valancePDF



X.Gao, et.al. PRL.128.142003 (2022)

Pion LCDA



(LPC) J.Hua et.al. PRL.129,132001 (2022)

Motivation

- Heavy flavor exclusive processes are important:
  - **Precise tests of SM**
  - **Searching for NP**
  - **Understanding the origins of CPV**
  - **……**

- $B \to \pi\pi$: *Beneke, Buchalla, Neubert, Sachrajda, 1999; 1422 citations*
- $B \to \pi K$: *Beneke, Buchalla, Neubert, Sachrajda, 2001; 1177 citations*
- $B \to \pi \ell v$: *Becher, Hill, 2005; 215 citations*
  Khodjamirian, Mannel, Offen, Wang, 2011; 192 citations
- $B \to K^{(*)}\ell\ell$: *Khodjamirian, Mannel, Pivavorov, Wang, 2010; 486 citations*
- $B \to D\ell v$: *HPQCD Collaboration, 2015; 387 citations*

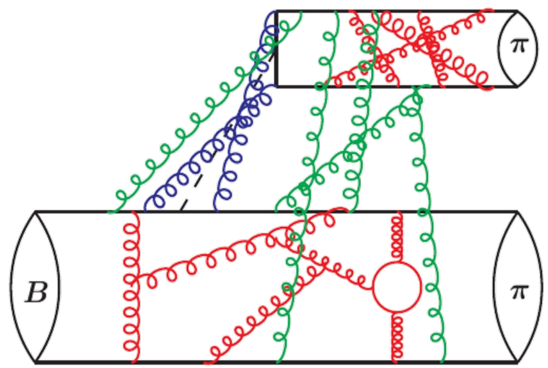
- Factorization: categories by different characteristic scales

$$\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

Form factor =
Hard kernel + LCDAs

Hard kernel
(Perturbative)

Meson LCDAs
(Nonperturbative)



Research journey and challenges

Research journey

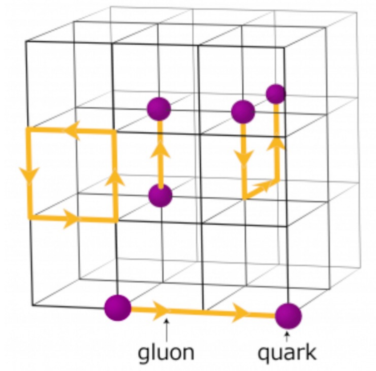
➢ Light meson LCDAs have been extensively pursued: (1970s - now)

- **Asymptotic LCDAs**
  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; Efremov, Radyushkin, 1980
- **Dyson-Schwinger Equation**
  Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, 2013; Gao, Chang, Liu, Roberts, Schmidt, 2014; Roberts, Richards, Chang, 2021
- **Sum rules**
  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; Ball, Braun, Koike, Tanaka, 1998; Ball, Braun, 1998; Khodjamirian, Mannel, Melcher, 2004; Ball,, Lenz, 2007
- **Inverse Problem**
  Li, 2022
- **Models**
  Arriola, Broniowski, 2002, 2006; Zhong, Zhu, Fu, Wu, Huang, 2021;
- **Global Fits**
  Stefanis, 2020; Cheng, Khodjamirian, Rusov, 2020; Hua, Li, Lu, Wang, Xing, 2021
- **Lattice with current-current correlation**
  Bali, Braun, Gläßle, Göckeler, Gruber, 2017, 2018;
- **Lattice with OPE**
  Martinelli, Sachrajda, 1987; Braun, Bruns, et al., 2016; RQCD collaboration, 2019, 2020
- **Lattice with LaMET**
  Zhang, Chen, Ji, Jin, Lin, 2017; LP3 Collaboration, 2019; Zhang, Honkala, Lin, Chen, 2020; Lin, Chen, Fan, Zhang^2, 2021; LPC Collaboration, 2021, 2022
- **Quantum Computing**
  QuNu Collaboration, 2023, 2024

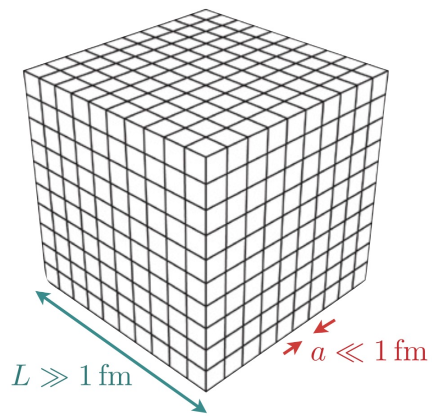
Recent progresses with lattice QCD

Recent progresses with lattice QCD

LQCD is formulated as a Feynman path integral on a 4D Euclidean grid.



- Gluon fields on links of a hypercube;
- Quark fields on sites: approaches to fermion discretization – Wilson, Staggered, Overlap, ……;



- **Discrete:** lattice spacing a → UV regulator; box length L → IR regulator;
- Derivatives: **discretization errors** ($a \to 0$); $\mathcal{O}(a)$ improved actions; ……
- Finite volume ($M_\pi L \to \infty$): **FV errors** exponentially small for $M_\pi L > 4$;
- **Chiral extrapolation** ($M_\pi \to 135\text{MeV}$);
- Numerical importance sampling of path integral: statistical errors.

Recent progresses with lattice QCD

➢ **Recover to continuum physics**

Lattice v.s. Continuum	
We simulate:	**We want:**
😀 At finite lattice spacing a	🤔 $a \to 0$
😀 In finite volume L^3	🤔 $L \to \infty$
😀 Euclidean space	🤔 Minkowski space ⇨ **Lost the real time information!**
😀 Lattice regularization	🤔 Some continuum scheme
😀 Some bare input quark masses: am_l, am_s, am_c, am_b In general, $m_\pi^{\mathrm{lat}} \neq m_\pi^{\mathrm{phy}}$	🤔 $m_q^{\mathrm{lat}} = m_q^{\mathrm{phy}}$

- **Need to control all limits:** particularly simultaneously control FV and discretization
- **Universality:** different input parameters **must** give converge results.

Recent progresses with lattice QCD

- **Light-like correlators cannot be simulated on Euclidean lattice directly**

Recent progresses with lattice QCD

- **Light-like correlators cannot be simulated on Euclidean lattice directly $\Rightarrow$ Local correlators can!**

- Lattice with OPE: **OPE moments $\Rightarrow$ Gegenbauer moments**

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \phi_\pi(x)$$

- The nonlocal operator can be defined as a generating function for renormalized local operators:

$$\bar{d}(z_2 n) \not{n}\gamma_5 [z_2 n, z_1 n] u(z_1 n) = \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k! l!} n^\rho n^{\mu_1} \dots n^{\mu_{k+l}} \mathcal{M}^{(k,l)}_{\rho\mu_1\dots\mu_{k+l}}$$

$$\mathcal{M}^{(k,l)}_{\rho\mu_1\dots\mu_{k+l}} = \bar{d}(0) \overleftarrow{D}_{(\mu_1} \dots \overleftarrow{D}_{\mu_k} \vec{D}_{\mu_{k+1}} \dots \vec{D}_{\mu_{k+l}} \gamma_{\rho)} \gamma_5 u(0)$$

- Moments of the pion DA are given by matrix elements of local operators:

$$i^{k+l} \left\langle 0 \left| \mathcal{M}^{(k,l)}_{\rho\mu_1\dots\mu_{k+l}} \right| \pi(p) \right\rangle = i f_\pi p_{(\rho} p_{\mu_1} \dots p_{\mu_{k+l}} \left\langle x^l (1-x)^k \right\rangle$$

Recent progresses with lattice QCD

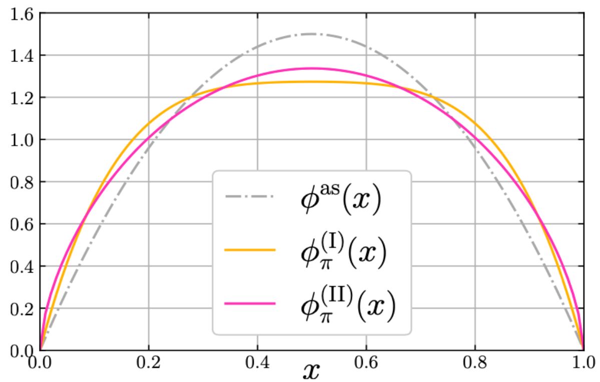
- The nonlocal operator can be defined as a generating function for renormalized local operators:

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \phi_\pi(x)$$

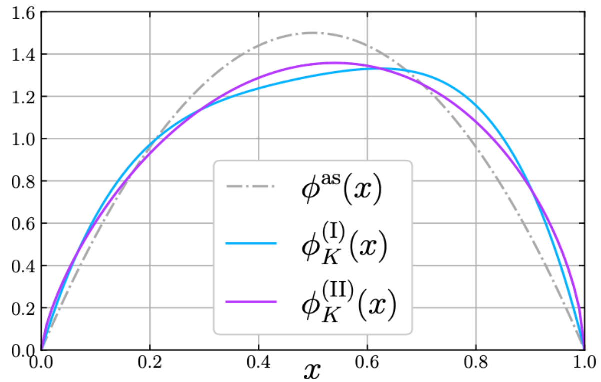
OPE moments ⇒ Gegenbauer moments

$$\mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} = \bar{d}(0) \overleftarrow{D}_{(\mu_1} \ldots \overleftarrow{D}_{\mu_k} \vec{D}_{\mu_{k+1}} \ldots \vec{D}_{\mu_{k+l}} \gamma_\rho)\, \gamma_5 u(0)$$

[RQCD collaboration, 2019]



$a_2^\pi = 0.101^{+24}_{-24}$ $\quad a_4^\pi = 0.002^{+71}_{-71}$



$a_1^K = 0.0533^{+34}_{-35}$ $\quad a_2^K = 0.090^{+19}_{-20}$ ⟵ $\phi_\pi(x) = 6x(1-x) \sum_{n=1,2,\cdots} a^\pi_{2n-2} C^{(3/2)}_{2n-2}(2x-1)$

😀 **Precise at low order moments**

🤔 **Operator mixing**

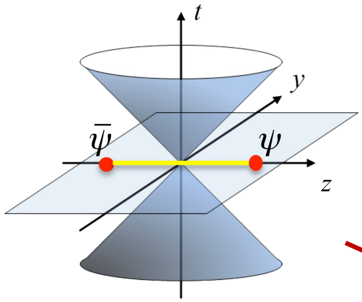
Convergence problem

Computational complexity

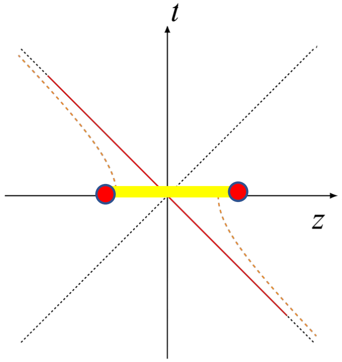
Pseudoscalar and vector meson LCDAs by LaMET

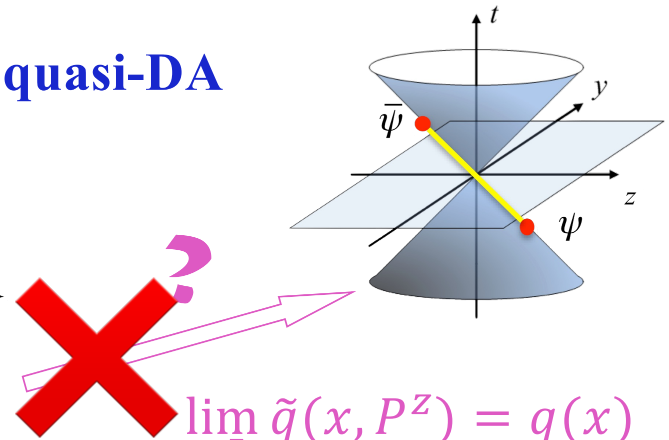
Pseudoscalar and vector meson LCDAs by LaMET

- Define a lattice calculable, equal-time correlation: **quasi-DA**



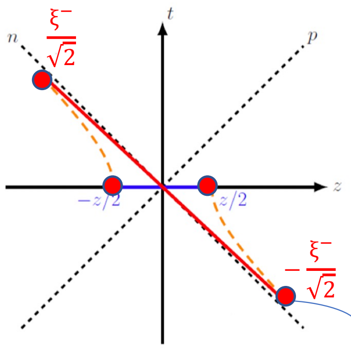
Boosting P^z





$$\lim_{P^z} \tilde{q}(x, P^z) = q(x)$$

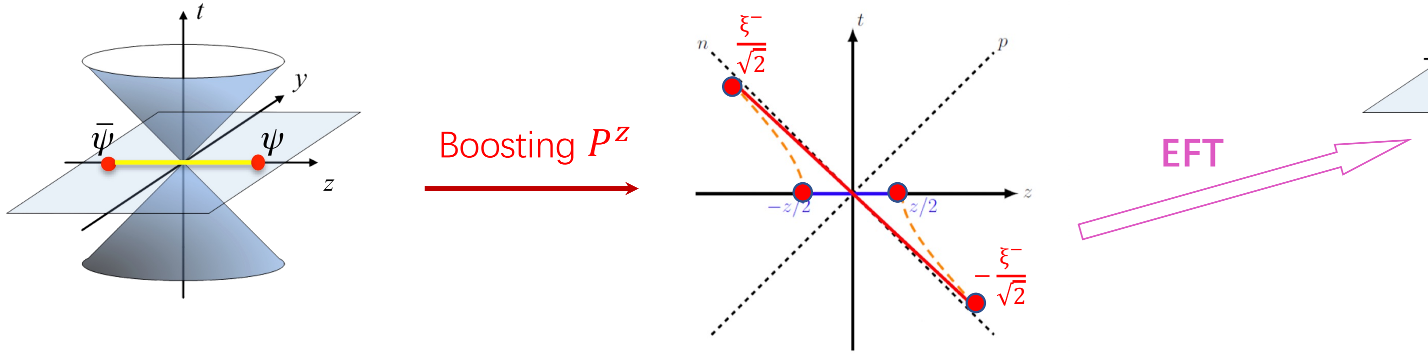
- z^2 is Lorentz invariant, time-like correlator cannot become to light-like;
- In the large P^z limit, the deviation from light-cone is power suppressed by $m^2/(P^z)^2$ and $\Lambda^2/(P^z)^2$.

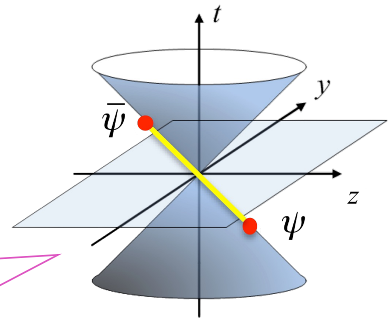


Power suppressed by $m^2/(P^z)$^2, $\Lambda^2/(P^z)$^2

Pseudoscalar and vector meson LCDAs by LaMET

- Define a lattice calculable, equal-time correlation: **quasi-DA**





- Effective field theory:
  - Instead of taking $P^z \to \infty$ calcuation, one can perform an expansion for large but finite P^z:

$$\underbrace{\tilde{q}(x, P^z, \mu)}_{\text{Quasi-DA}} = \int \frac{dy}{|y|} \underbrace{\boxed{C(x, y, P^z, \mu)}}_{\text{Matching kernel}} \underbrace{q(y, \mu)}_{\text{LCDA}} + \underbrace{\mathcal{O}\left(\frac{\Lambda^2, M^2}{(P^z)^2}\right)}_{\text{High power correction}}$$

Pseudoscalar and vector meson LCDAs by LaMET

- **Pion LCDA:**

I. Calculate the bare quasi-DA correlation

$$\tilde{h}(z, a, P_z) = \langle 0 | \bar{\psi}_1(0) n_z \cdot \gamma\gamma_5 U(0,z) \psi_2(z) | \pi(P) \rangle$$

II. Non-perturbative renormalization

$$\tilde{h}(z, a, P_z) = Z(z, a) \tilde{h}_R(z, a, P_z)$$

III. Fourier transform (Extrapolation)

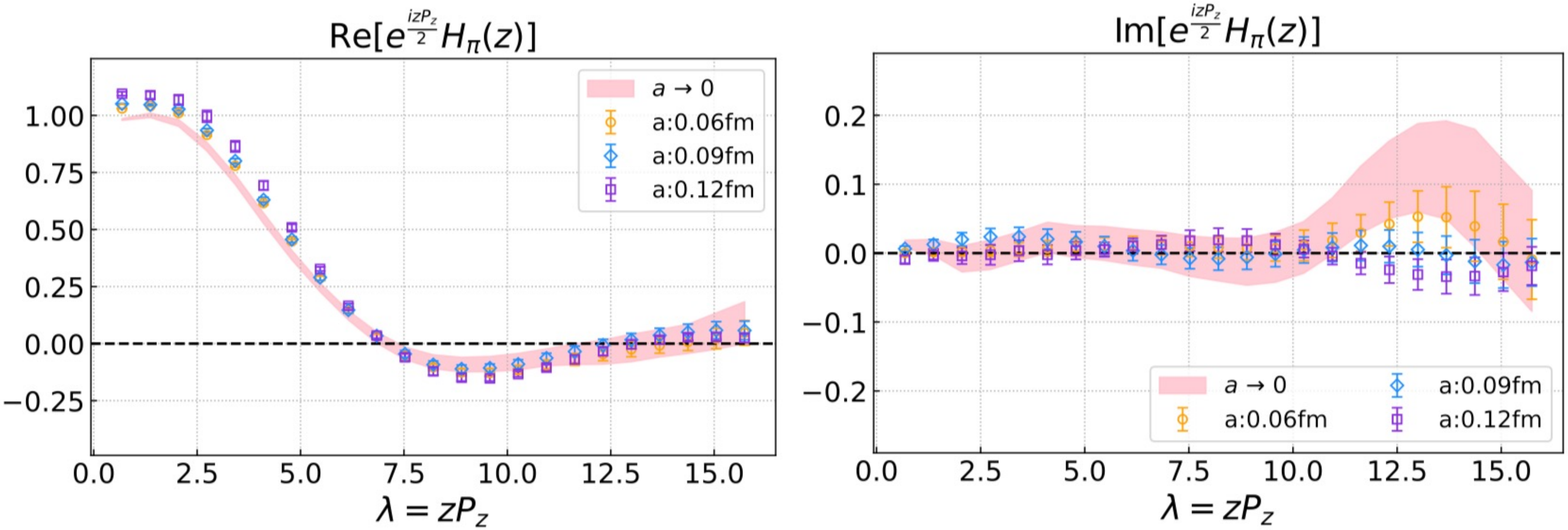
$$i f_\pi \tilde{\phi}_\pi(x, P_z) = \int \frac{d_z}{2\pi} e^{-ixzP_z} \tilde{h}_R(z, a \to 0, P_z)$$

IV. Matching to light clone

$$\tilde{\phi}_\pi(x, P_z) = \int dy\, Z(x, y, Pz, \mu) \phi(x, \mu) + p.c.$$

Pseudoscalar and vector meson LCDAs by LaMET

➢ Renormalized quasi-DA in coordinate space:

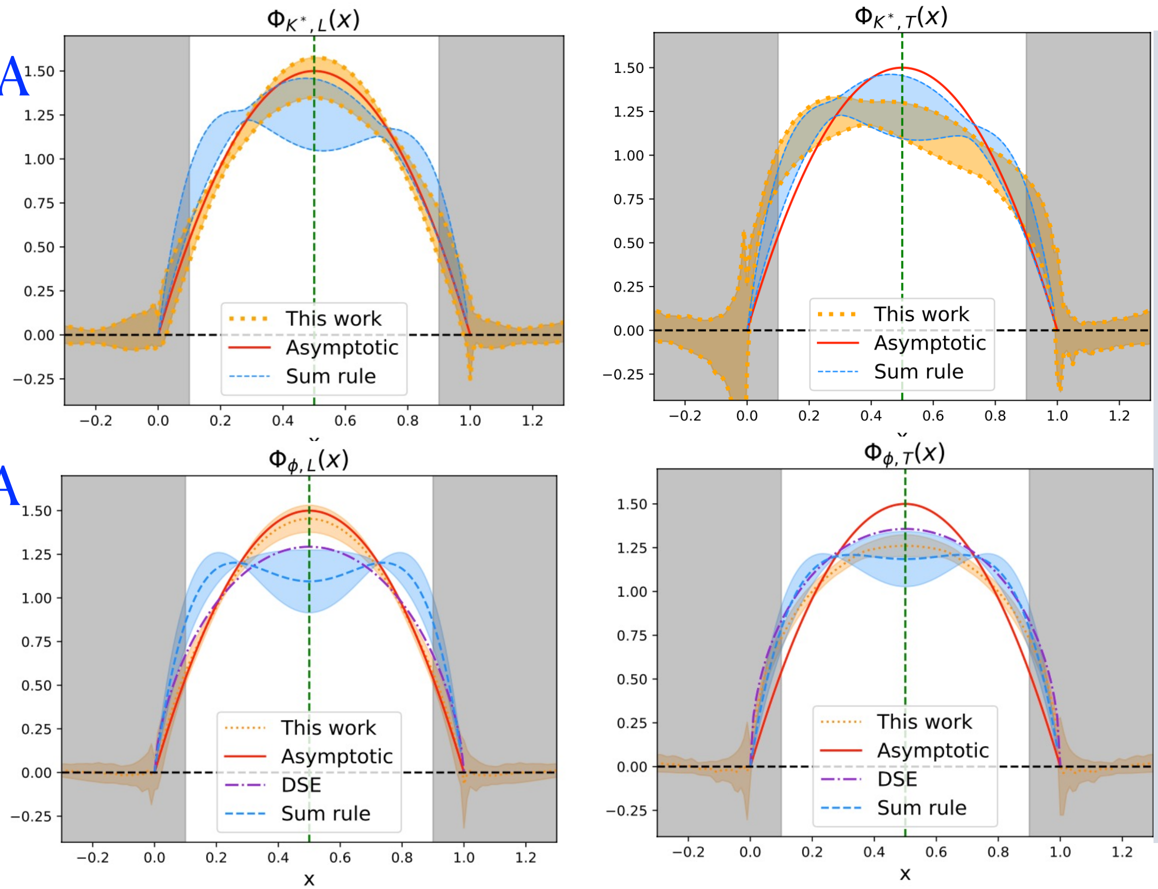


$P_z = 2.15GeV$ (LPC)J.Hua et.al. PRL129. 132001(2022)

Pseudoscalar and vector meson LCDAs by LaMET

K^* LCDA

ϕ LCDA

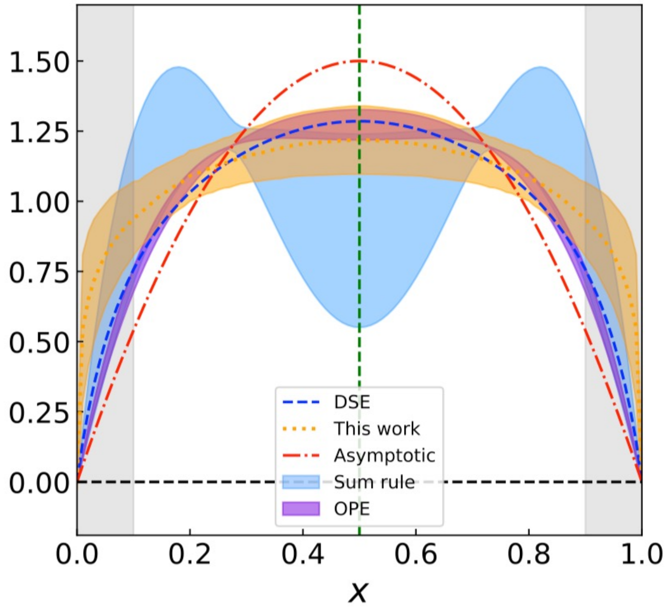


(LPC)J.Hua et.al. PRL127. 062002(2021)

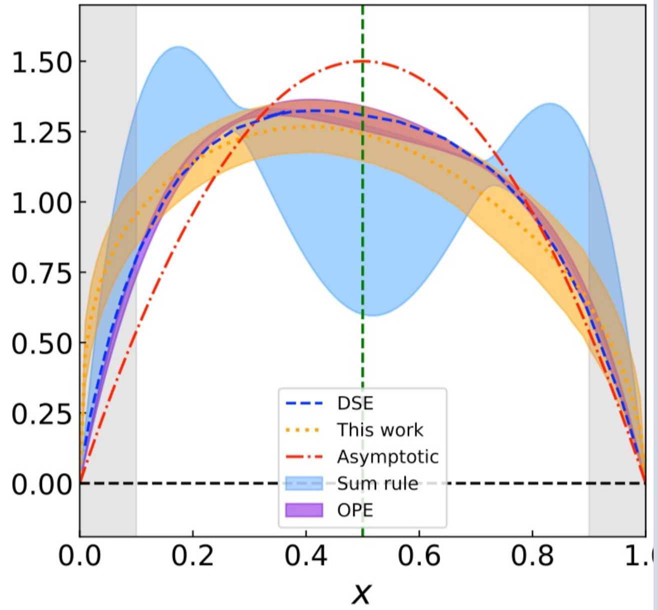
- 3 lattice spacings: (0.12, 0.09, 0.06) fm, largest volume ($96^3 \times 192$)
- 3 momentum: (1.29, 1.72, 2.15) GeV
- mass: K^*: 0.89GeV, ϕ: 1.02GeV
- Hybrid scheme (based on RI/MOM)

Pseudoscalar and vector meson LCDAs by LaMET

π LCDA:



K LCDA:



(LPC)J.Hua et.al. PRL129. 132001(2022)

- 3 lattice spacings: (0.12, 0.09, 0.06) fm, largest volume ($96^3 \times 192$)
- 3 momentum: (1.29, 1.72, 2.15) GeV
- mass: π: 0.13GeV, K: 0.49GeV
- Hybrid scheme (Self renormalization)

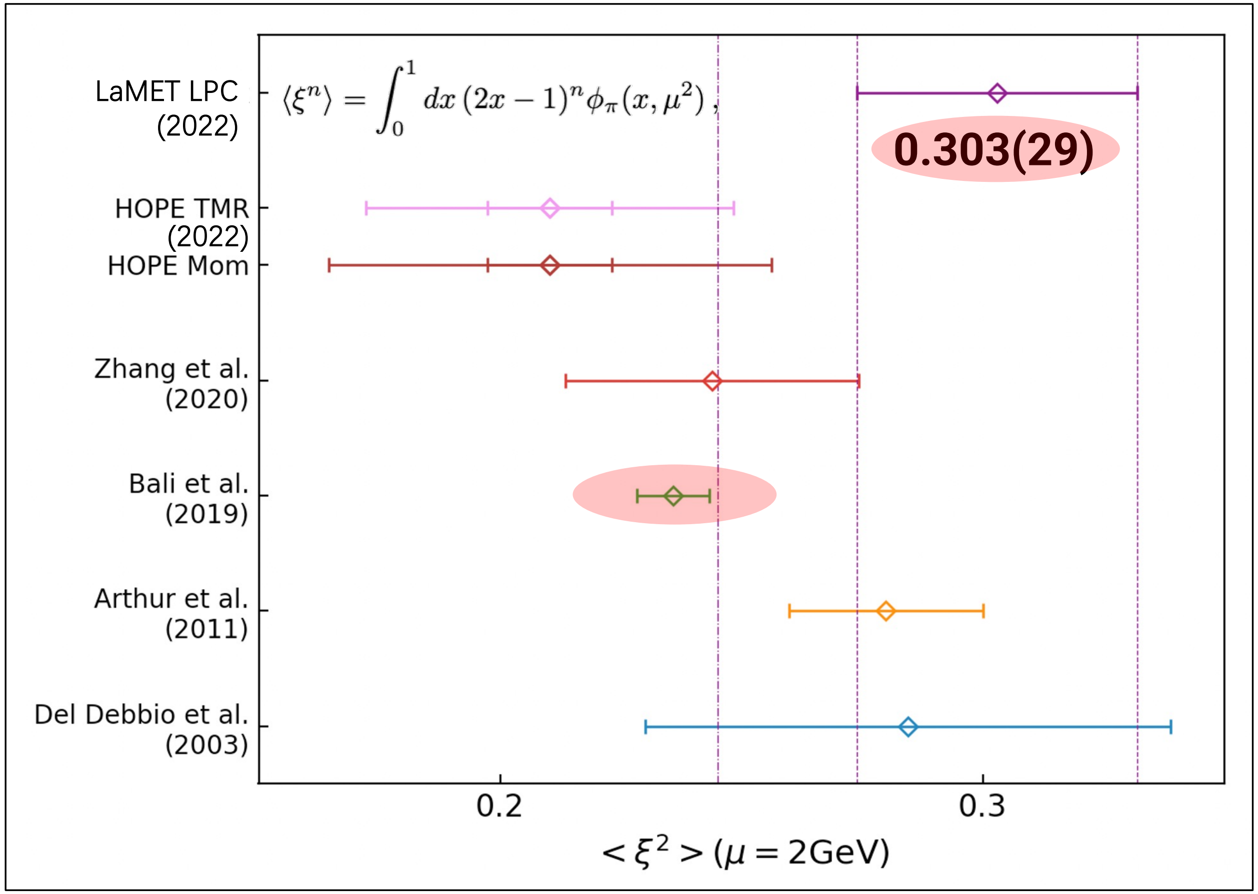
There are uncontrolled systematic uncertainty in the endpoint region

Pseudoscalar and vector meson LCDAs by LaMET

A comparison for moments calculated by different approaches on lattice:

$$\langle \xi^n \rangle = \int_0^1 dx(2x-1)^n \phi_\pi(x)$$

Moments by different approaches are **inconsistent** !



Reflections on current issues

Reflections on current issues

- **Gegenbauer moments:**

$$\phi_\pi(x) = 6x(1-x)\sum_{n=1,2,\cdots} a^\pi_{2n-2} C^{(3/2)}_{2n-2}(2x-1)$$

- **OPE moments:**

$$\langle \xi^n \rangle \equiv \int_0^1 dx (2x-1)^n \phi_\pi(x)$$

Limitations of OPE !

$$\begin{aligned}
a_0^\pi &= \langle \xi^0 \rangle, \\
a_2^\pi &= \frac{7}{12}\left(5\langle \xi^2\rangle - \langle \xi^0\rangle\right), \\
a_4^\pi &= \frac{11}{24}\left(21\langle \xi^4\rangle - 14\langle \xi^2\rangle + \langle \xi^0\rangle\right), \\
a_6^\pi &= \frac{5}{64}\left(429\langle \xi^6\rangle - 495\langle \xi^4\rangle + 135\langle \xi^2\rangle - 5\langle \xi^0\rangle\right), \\
a_8^\pi &= \frac{19}{384}\left(2431\langle \xi^8\rangle - 4004\langle \xi^6\rangle + 2002\langle \xi^4\rangle - 308\langle \xi^2\rangle + 7\langle \xi^0\rangle\right), \\
a_{10}^\pi &= \frac{23}{1536}\left(29393\langle \xi^{10}\rangle - 62985\langle \xi^8\rangle + 46410\langle \xi^6\rangle - 13650\langle \xi^4\rangle + 1365\langle \xi^2\rangle - 21\langle \xi^0\rangle\right)
\end{aligned}$$

Huge coefficients !

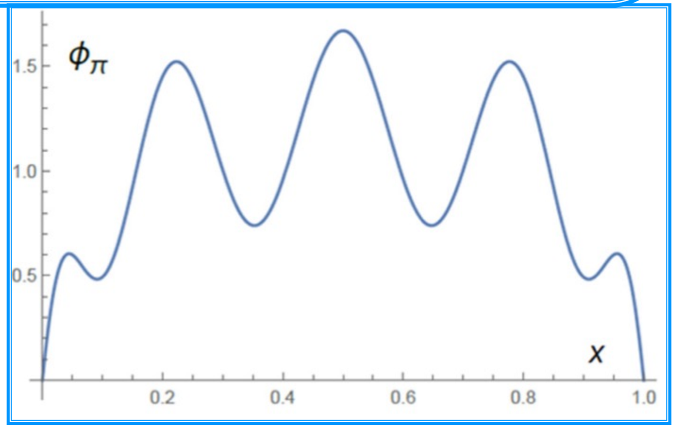
Good convergence

$$(\langle \xi^0\rangle, \langle \xi^2\rangle, \langle \xi^4\rangle, \langle \xi^6\rangle, \langle \xi^8\rangle, \langle \xi^{10}\rangle)|_{\mu=2\,\mathrm{GeV}} = (1, 0.254, 0.125, 0.077, 0.054, 0.041)$$

T.Zhong PRD104. 016021(2021)

Bad convergence

$$(a_0^\pi, a_2^\pi, a_4^\pi, a_6^\pi, a_8^\pi, a_{10}^\pi)|_{\mu=2\,\mathrm{GeV}} = (1, 0.157, 0.032, 0.035, 0.098, -0.046)$$



Hsiang-nan Li PRD106. 034015(2022)

Reflections on current issues

➢ **Consider the form of Gegenbauer expansion:**

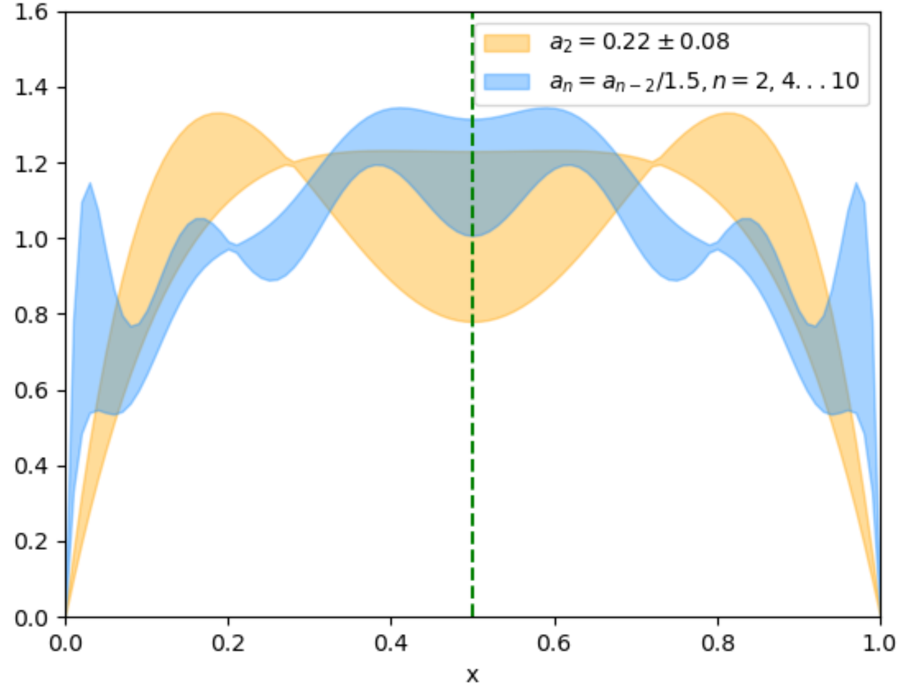
$$\phi_\pi(x) = 6x(1-x) \sum_{n=1,2,\cdots} a_{2n-2}^{\pi} C_{2n-2}^{(3/2)}(2x-1)$$

$$C_2 = \frac{3}{2}(5*(2x-1)^2 - 1)$$

$$C_4 = \frac{15}{8}(1 - 14*(2x-1)^2 + 21*(2x-1)^4)$$

$$C_6 = \cdots$$

The endpoint region of LCDA is more sensitive to the high order moments.

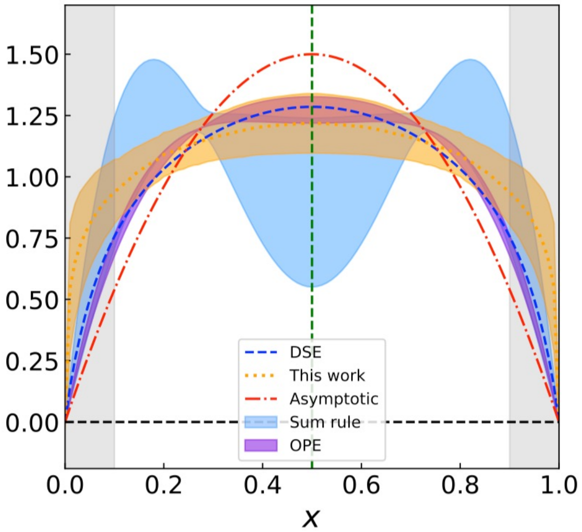


➢ The finite-order moment can not give a correct prediction for the endpoint region, but it does give a portion of the global constraints

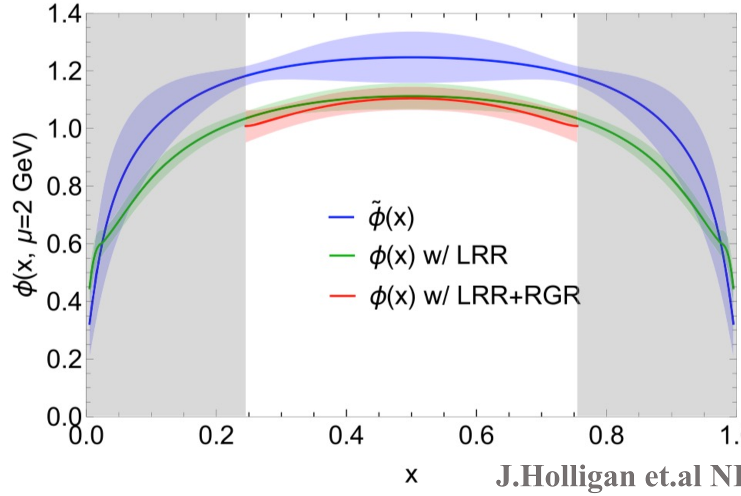
Reflections on current issues

➢ LaMET factorization

$$q(y,P^z,\mu)=\int dx C^{-1}(x,y,P^z,\mu)\tilde{q}(x,\mu)+\boxed{\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(yP^z)^2},\frac{\Lambda_{QCD}^2}{((1-y)P^z)^2}\right)}$$



Some resummation improvement:



J.Holligan et.al NPB 993. 116282(2023)

➢ LaMET does give x-dependent reliable results in the middle region, but is currently incapable for the endpoint region.

Reflections on current issues

We are facing:

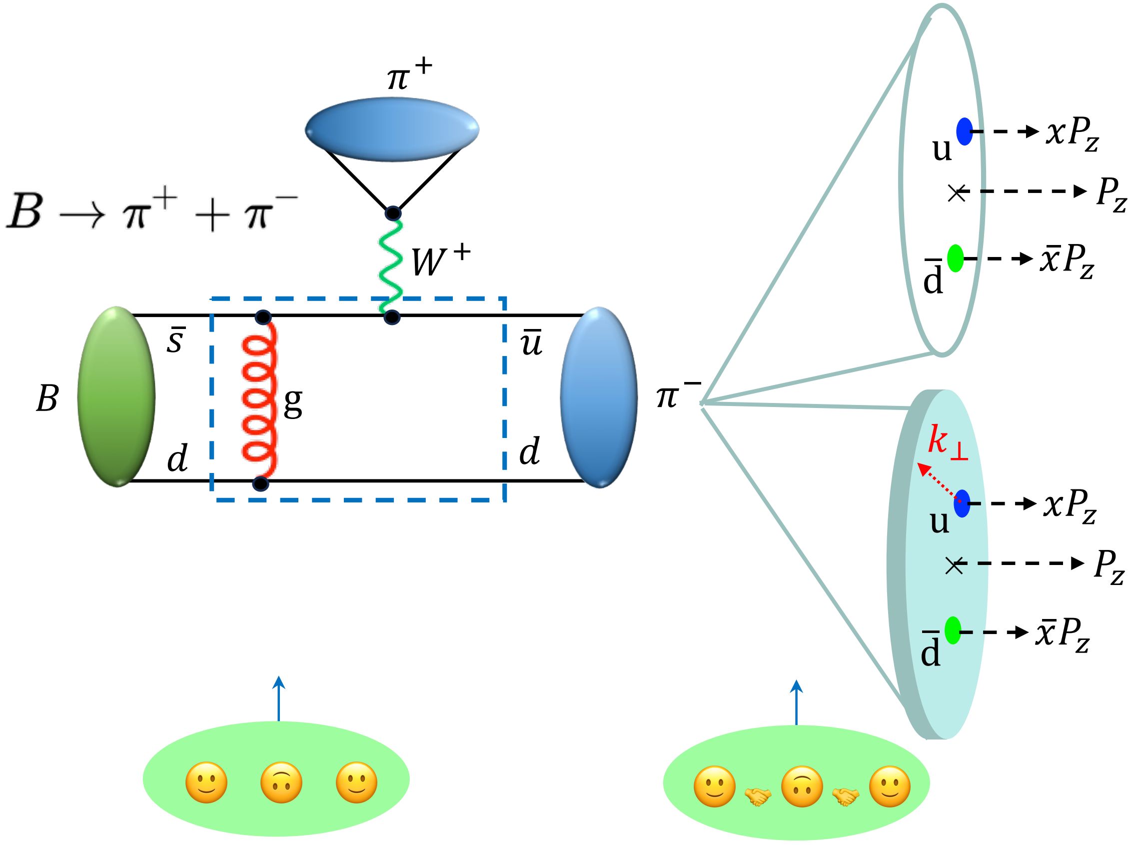
I. Explain why different approaches get different moments

II. How to combine different results to a accurate and reliable results

Three dimensional TMDWF

Three dimensional TMDWF

- Three dimensional / TMD Structures

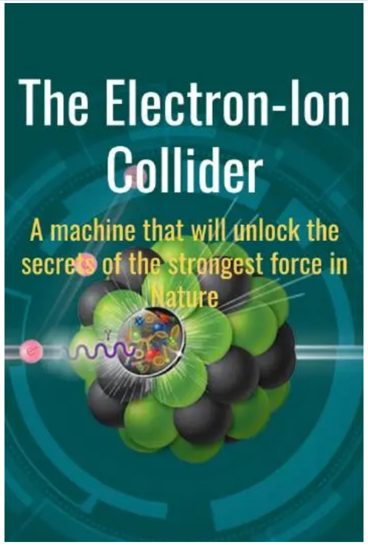


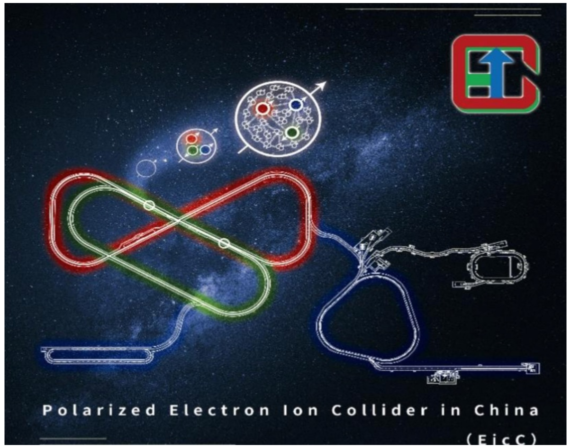
Collinear Factorization

TMD Factorization

EIC &

EICC





One dimensional LCDA

Three dimensional TMDWF

Three dimensional TMDWF

- **Multiplicative factorization of quasi-TMDWF in LaMET**

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z)\, S_I^{\frac{1}{2}}(b_\perp, \mu)$$
$$= H^{\pm}(x, \zeta^z, \mu) \exp\left[\frac{1}{2} K(b_\perp, \mu) \ln \frac{\pm\zeta^z + i\epsilon}{\zeta}\right] \Psi^{\pm}(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

X.D.Ji et.al. Rev.Mod.Phys. 93, 035005 (2021)

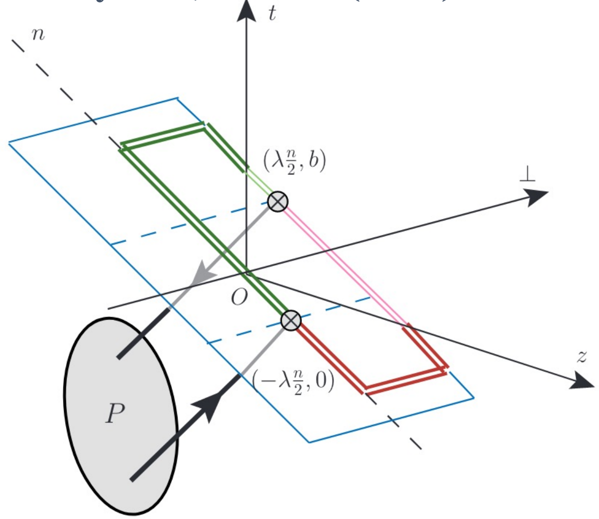
$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta_z)$: Quasi-TMDWF,

$S_r(b_\perp, \mu)$: Intrinsic soft function,

$H^{\pm}(\zeta_z, \bar{\zeta}_z, \mu^2)$: Matching coefficient,

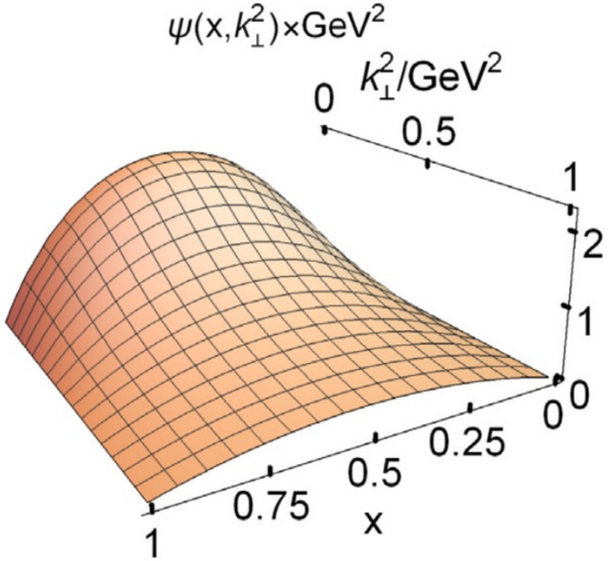
$K(b_\perp, \mu)$: Collins-Soper kernel,

$\Psi^{\pm}(x, b_\perp, \mu, \zeta)$: TMDWF.



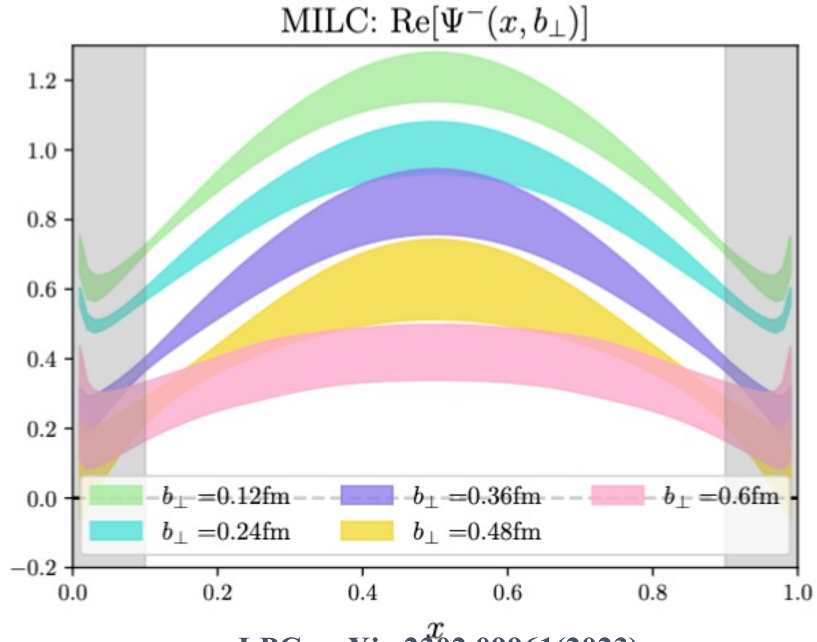
Three dimensional TMDWF

- 3-D curved surface（expected）with $x \to P_z$ and $k_\perp$



C.D.Roberts et.al. PPNP.120, 138883 (2021)

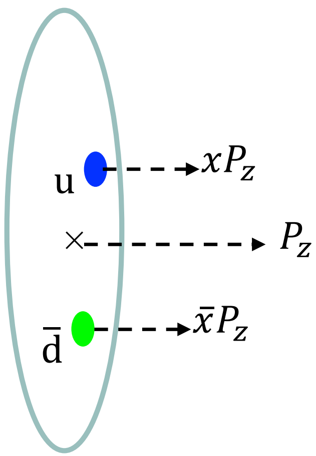
- $b_\perp$ (FT $\to$ $k_\perp$)dependent TMDWF (currently available)

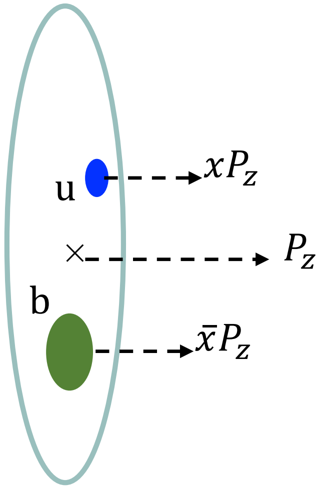


LPC, arXiv 2302.09961(2023)

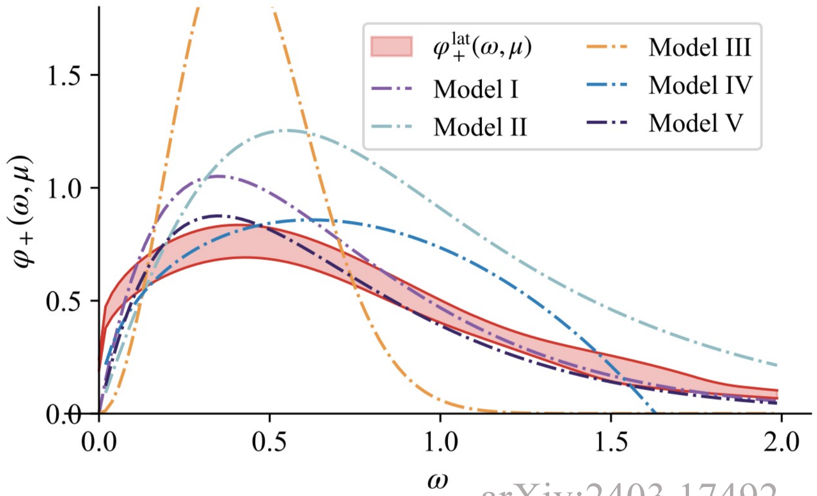
Outlook and Summary

Light meson π/K

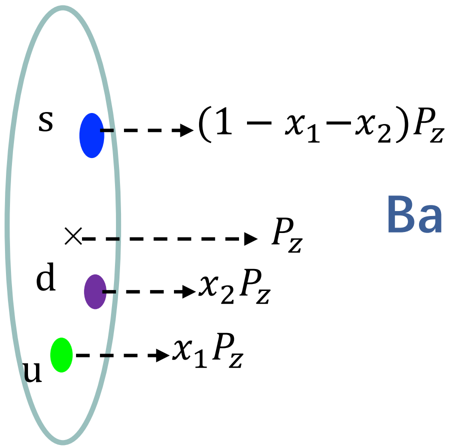




Heavy meson D/B



arXiv:2403.17492



Baryon Λ, proton

Coming soon !

Outlook and Summary

Light meson LCDAs:

- The disagree between lattice OPE and LaMET lattice calcuation

LaMET:

- Large P^z to suppress the power corrections;
- Some resummation skills ……

OPE:

- High order moment

- More generalized distributions, as TMD-DA, ……

More gengeralized distribution, as TMD-WFs …

More hadrons as heavy mesons, baryons …

Thanks!