Tensor Interaction in Nuclear and Hadron Physics
November 1 – 3, Beihang University, Beijing, China

Self-consistent study of spin-isospin resonances and its application in astrophysics

Haozhao Liang

School of Physics, Peking University, China

November 3, 2011

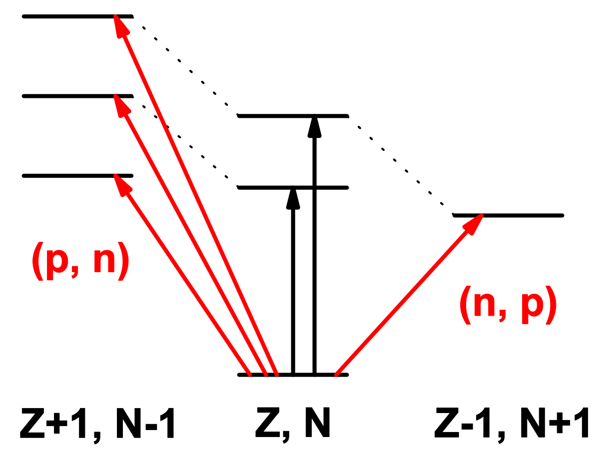
In collaboration with:

Jie Meng, Yifei Niu, Zhongming Niu, Nguyen Van Giai, Pengwei Zhao

Outline

Outline

Nuclear spin-isospin resonances



- Nuclear charge-exchange excitations
  - ⋆ β-decay
  - ⋆ charge-exchange reactions
- These excitations play important roles
  - ⋆ spin and isospin properties of the in-medium nuclear interaction
  - ⋆ neutron skin thickness Krasznahorkay:1999, Vretenar:2003, Yako:2006
  - ⋆ β-decay rates of nuclei in r-process path Engel:1999, Borzov:2006
  - ⋆ inclusive neutrino-nucleus cross sections Kolbe:2003, Vogel:2006
  - ⋆ isospin corrections for superallowed β-deacys Towner and Hardy:2010
  - ⋆ $\beta\beta$-decay rates Ejiri:2000, Avignone:2008

- Nuclear spin-isospin resonances become one of the central topics in nuclear physics and astrophysics.

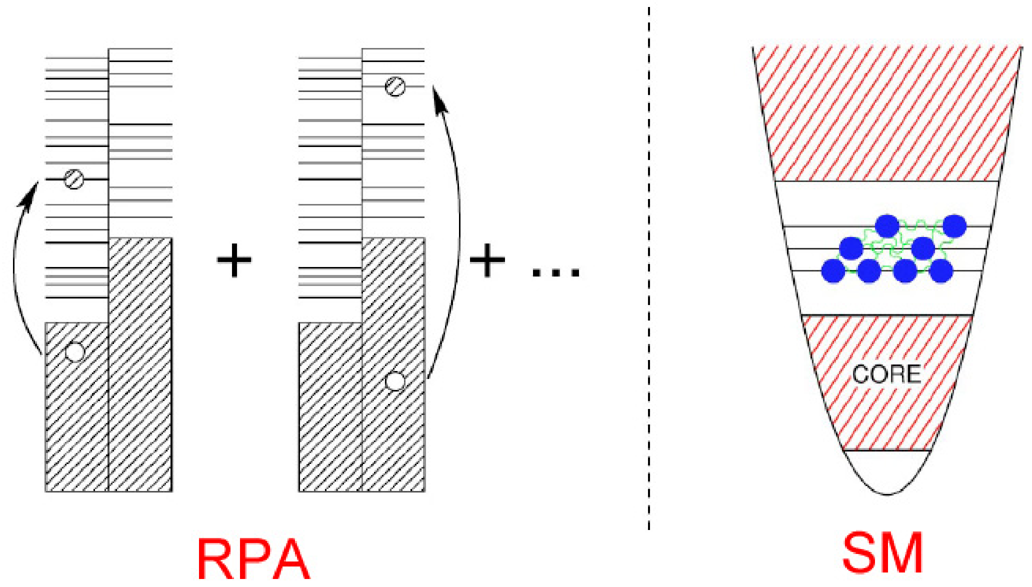
Microscopic theories for spin-isospin resonances

- Shell models ($A \sim 60$)

  Radha:1997, Caurier:1999,2005

- Random Phase Approximation (RPA) based on density functional theories
  - ⋆ traditional (non-relativistic) density functional

    Halbleib:1967, Auerbach:1981, Colò:1994, Engel:1999, Bender:2002, Fracasso:2005,2007
  - ⋆ covariant (relativistic) density functional: RH+RPA, RHF+RPA



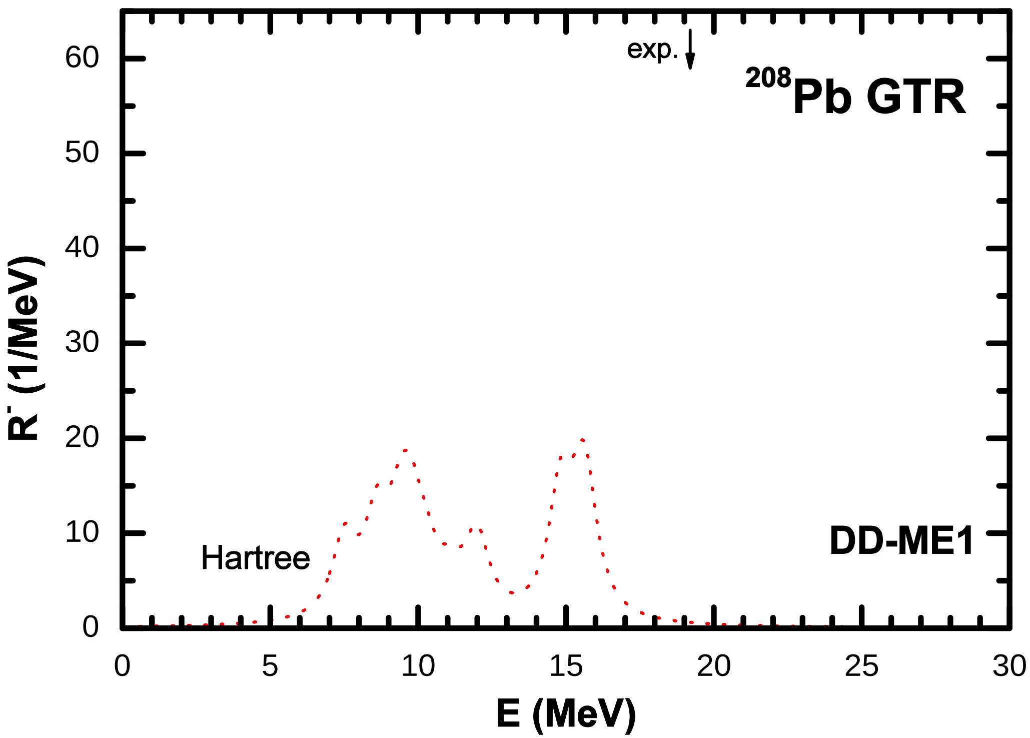
Covariant density functional theory – RH theory

- Covariant density functional theory in Hartree level (RH/RMF theory) has received wide attention due to its successful description of lots of nuclear phenomena.
  Serot:1986, Ring:1996, Vretenar:2005, Meng:2006
  - ⋆ spin-orbit splittings
  - ⋆ nuclear saturation properties (the Coester line) Brockmann:1990,1992
  - ⋆ binding energy per nucleon E/A Reinhard:1989, Ring:1996
  - ⋆ isotopic shifts in the Pb region Sharma:1993
  - ⋆ halo and giant halo in exotic nuclei Meng:1996,1998,2002
  - ⋆ pseudospin symmetry in nucleon spectrum Ginocchio:1997,2005, HL:2011
  - ⋆ spin symmetry in anti-nucleon spectrum Zhou:2003
  - ⋆ ......

RH+RPA for spin-isospin resonances

- RH+RPA for spin-isospin resonances
  De Conti:1998, 2000, Vretenar: 2003, Ma:2004, Paar:2004, Nikšić:2005

example: Gamow-Teller resonance (GTR) in ^{208}Pb ($\Delta S = 1$, $\Delta L = 0$, $J^{\pi} = 1^{+}$)

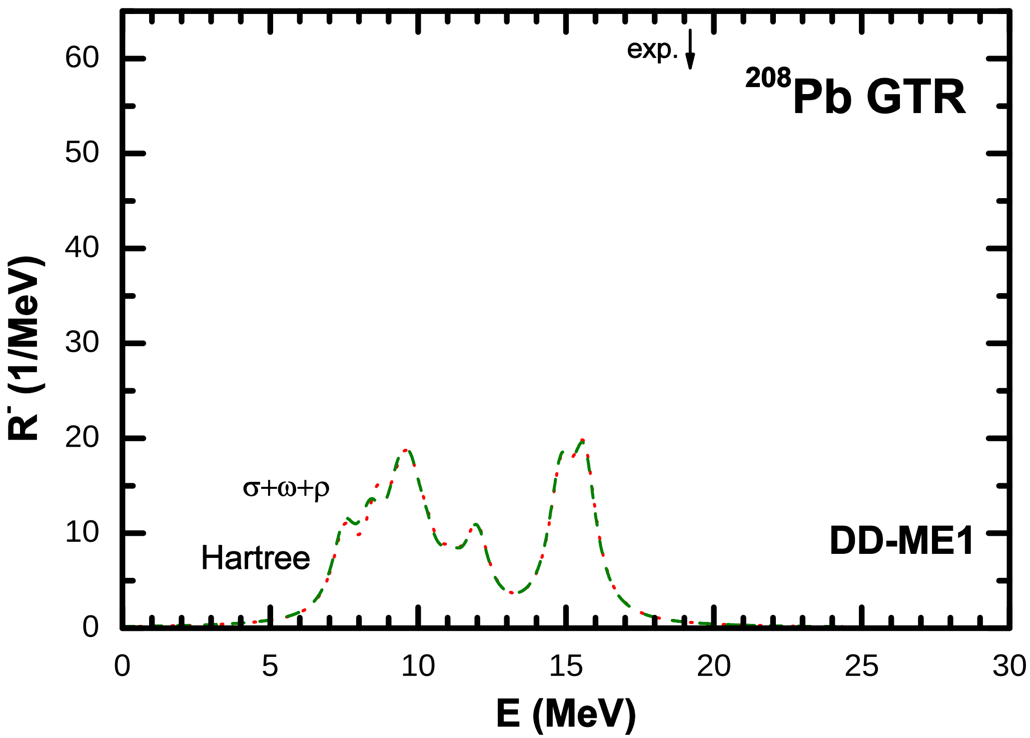


RH+RPA for spin-isospin resonances

- RH+RPA for spin-isospin resonances

  De Conti:1998, 2000, Vretenar: 2003, Ma:2004, Paar:2004, Nikšić:2005

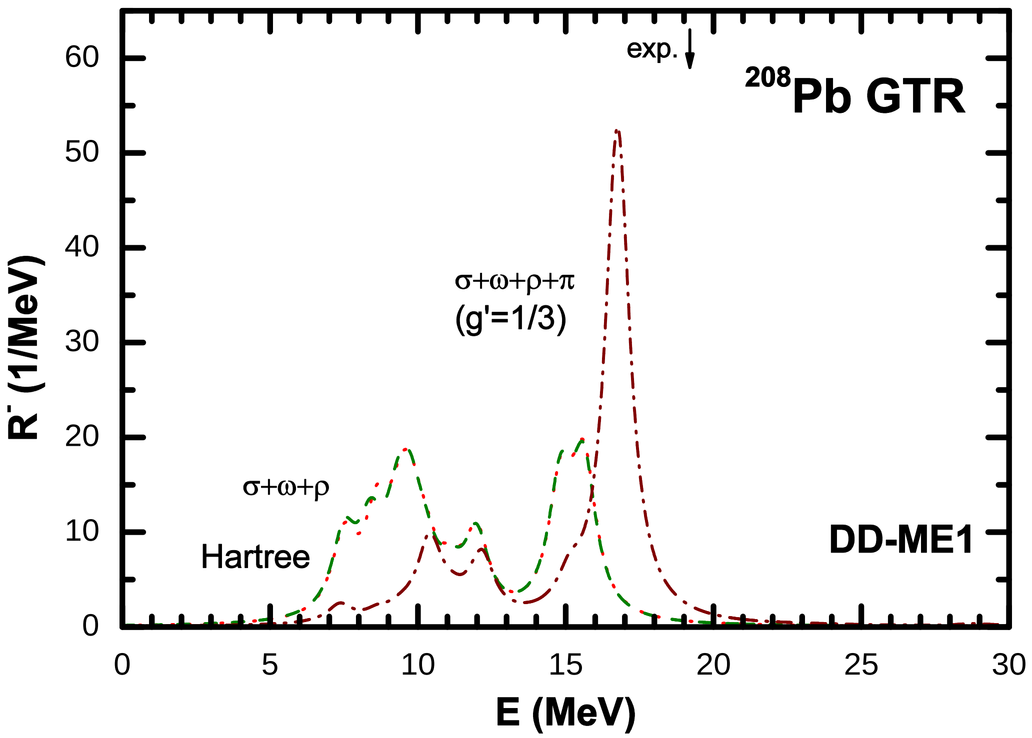
example: Gamow-Teller resonance (GTR) in ^{208}Pb ($\Delta S = 1$, $\Delta L = 0$, $J^{\pi} = 1^{+}$)



RH+RPA for spin-isospin resonances

- RH+RPA for spin-isospin resonances
  De Conti:1998, 2000, Vretenar: 2003, Ma:2004, Paar:2004, Nikšić:2005

example: Gamow-Teller resonance (GTR) in ^{208}Pb ($\Delta S = 1$, $\Delta L = 0$, $J^{\pi} = 1^{+}$)



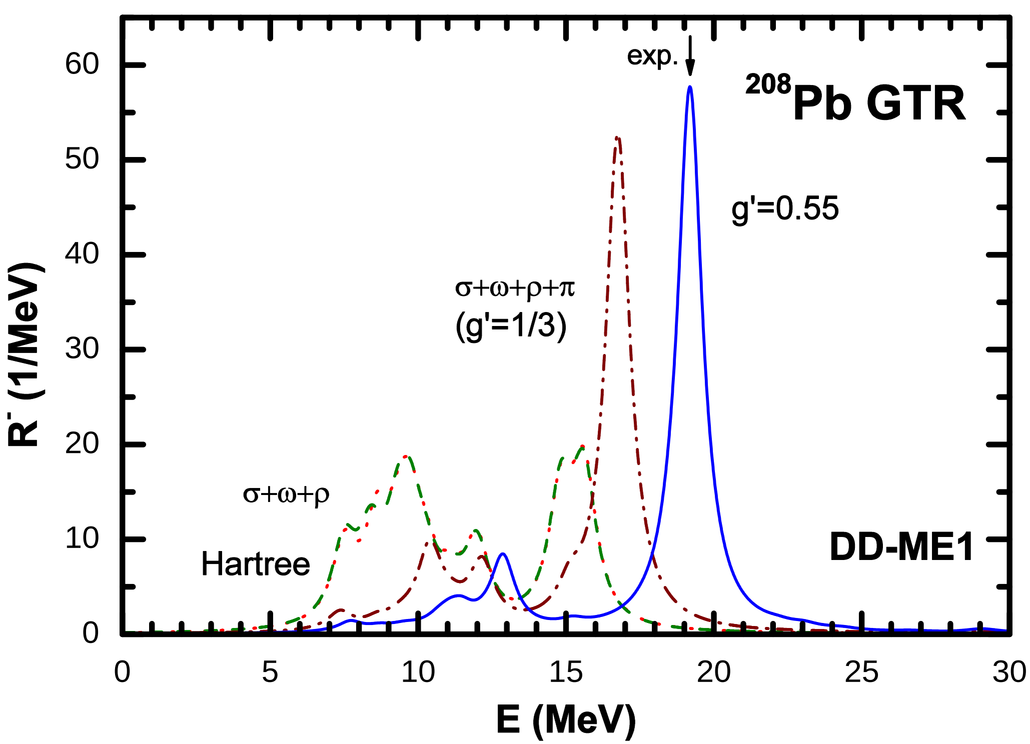
a. add π-meson

RH+RPA for spin-isospin resonances

- RH+RPA for spin-isospin resonances

  De Conti:1998, 2000, Vretenar: 2003, Ma:2004, Paar:2004, Nikšić:2005

example: Gamow-Teller resonance (GTR) in ^{208}Pb ($\Delta S = 1$, $\Delta L = 0$, $J^{\pi} = 1^{+}$)



a. add π-meson b. fit g' self-consistency is missing

Covariant density functional theory – RHF theory

- Covariant density functional theory in Hartree-Fock level (RHF theory) for nuclear ground-state properties
  - ⋆ several attempts to include the Fock term in the relativistic framework

    Bouyssy:1985,1987, Bernardos:1993, Marcos:2004
  - ⋆ DDRHF theory achieved quantitative descriptions of binding energies and radii on the same level as RH theory

    Long, Giai, Meng, *PLB* **640**, 150 (2006), Long, Sagawa, Giai, Meng, *PRC* **76**, 034314 (2007)
  - ⋆ improvement on the descriptions of the nuclear shell structures and their evolutions

    Long, Sagawa, Giai, Meng, *PRC* **76**, 034314 (2007), Long, Sagawa, Meng, Giai, *EPL* **82**, 12001 (2008)
  - ⋆ isospin properties of nuclear matter and neutron stars at high densities

    Sun, Long, Meng, Lombardo, *PRC* **78**, 065805 (2008)
  - ⋆ spin and pseudospin symmetries in nucleon spectra

    Long, Sagawa, Meng, Giai, *PLB* **639**, 242 (2006), HL, Long, Meng, Giai, *EPJA* **44**, 119 (2010)

RHF+RPA for nuclear spin-isospin resonances

- Can spin-isospin resonances and related weak interaction processes be described fully self-consistently in covariant density functional theory?

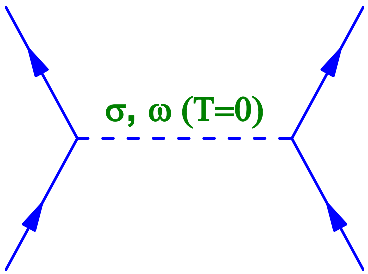
In this work

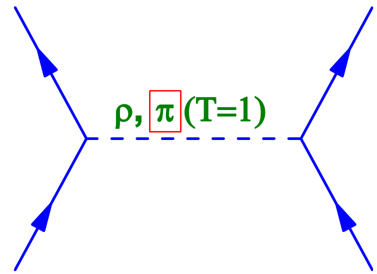
- Fully self-consistent RPA approach is established based on the RHF theory.
- Applications
  - ⋆ nuclear spin-isospin resonances
  - ⋆ β-decay rates of nuclei in r-process path
  - ⋆ inclusive charged-current neutrino-nucleus cross sections
  - ⋆ isospin symmetry-breaking corrections for the superallowed β-decays
  - ⋆ ...

Outline

Covariant density functional theory – RHF theory

- Effective Lagrangian density Bouyssy:1987, Long:2006





$$\mathscr{L} = \bar{\psi}\left[i\gamma^\mu\partial_\mu - M - g_\sigma\sigma - \gamma^\mu\left(g_\omega\omega_\mu + g_\rho\vec{\tau}\cdot\vec{\rho}_\mu + e\frac{1-\tau_3}{2}A_\mu\right) - \frac{f_\pi}{m_\pi}\gamma_5\gamma^\mu\partial_\mu\vec{\pi}\cdot\vec{\tau}\right]\psi$$
$$+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\cdot\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu$$
$$+\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}\cdot\vec{\pi} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \qquad (1)$$

- Energy functional of the system

$$E = \langle\Phi_0|\, H\, |\Phi_0\rangle = E_k + E_\sigma^D + E_\omega^D + E_\rho^D + E_A^D + E_\sigma^E + E_\omega^E + E_\rho^E + E_\pi^E + E_A^E \qquad (2)$$

Random Phase Approximation

- RPA equations

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega_\nu \begin{pmatrix} X \\ Y \end{pmatrix} \tag{3}$$

where the matrix elements of particle-hole residual interactions read

$$\mathcal{A} = \begin{pmatrix} (E_A - E_a)\delta_{AB}\delta_{ab} & \\ & (E_\alpha - E_a)\delta_{\alpha\beta}\delta_{ab} \end{pmatrix} + \begin{pmatrix} \langle f_A f_b \,|V|\, f_B f_a - f_a f_B \rangle & \langle f_A f_b \,|V|\, f_\beta f_a - f_a f_\beta \rangle \\ \langle f_\alpha f_b \,|V|\, f_B f_a - f_a f_B \rangle & \langle f_\alpha f_b \,|V|\, f_\beta f_a - f_a f_\beta \rangle \end{pmatrix}, \tag{4a}$$

$$\mathcal{B} = \begin{pmatrix} \langle f_A f_B \,|V|\, f_b f_a - f_a f_b \rangle & \langle f_A f_\beta \,|V|\, f_b f_a - f_a f_b \rangle \\ \langle f_\alpha f_B \,|V|\, f_b f_a - f_a f_b \rangle & \langle f_\alpha f_\beta \,|V|\, f_b f_a - f_a f_b \rangle \end{pmatrix} \tag{4b}$$

- Particle-hole residual interactions in self-consistent RPA
  - ⋆ derived from the second derivative of the energy functional
  - ⋆ with rearrangement terms, if the meson-nucleon couplings are density-dependent

RHF+RPA in charge-exchange channel

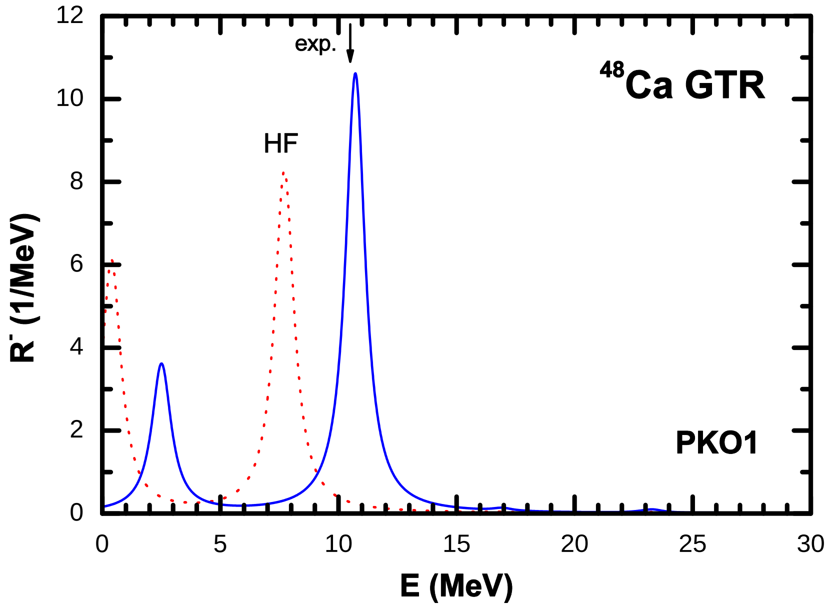
- Particle-hole residual interactions
  - ⋆ σ-meson: $$V_\sigma(1,2) = -[g_\sigma\gamma_0]_1[g_\sigma\gamma_0]_2 D_\sigma(1,2) \tag{5a}$$
  - ⋆ ω-meson: $$V_\omega(1,2) = [g_\omega\gamma_0\gamma^\mu]_1[g_\omega\gamma_0\gamma_\mu]_2 D_\omega(1,2) \tag{5b}$$
  - ⋆ ρ-meson: $$V_\rho(1,2) = [g_\rho\gamma_0\gamma^\mu\vec{\tau}]_1 \cdot [g_\rho\gamma_0\gamma_\mu\vec{\tau}]_2 D_\rho(1,2) \tag{5c}$$
  - ⋆ pseudovector π-N coupling: $$V_\pi(1,2) = -[\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\gamma^k\partial_k]_1 \cdot [\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\gamma^l\partial_l]_2 D_\pi(1,2) \tag{5d}$$
  - ⋆ zero-range counter-term of π-meson: $$V_{\pi\delta}(1,2) = g'[\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\boldsymbol{\gamma}]_1 \cdot [\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\boldsymbol{\gamma}]_2 \delta(\mathbf{r}_1 - \mathbf{r}_2), \quad g' = 1/3 \tag{5e}$$

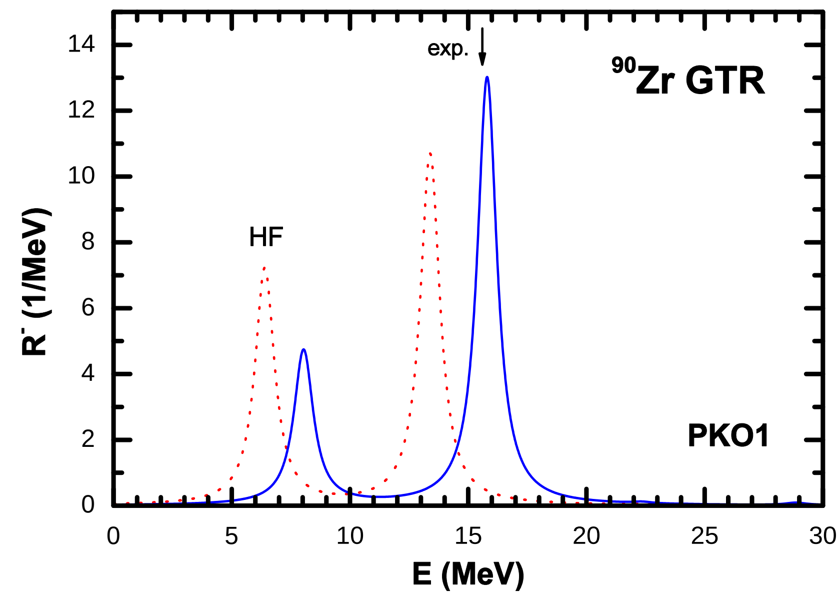
- π-meson is included naturally.
- $g' = 1/3$ in the zero-range counter-term of π-meson is maintained for the sake of self-consistency.

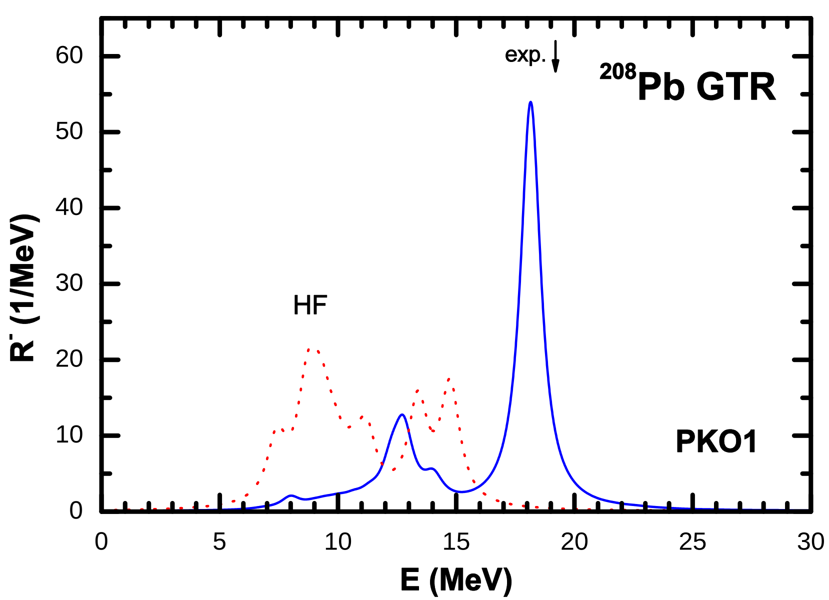
Outline

RHF+RPA for Gamow-Teller resonances

★ Gamow-Teller resonances in ^{48}Ca, ^{90}Zr, and ^{208}Pb







✓ GTR excitation energies can be reproduced in a fully self-consistent way.

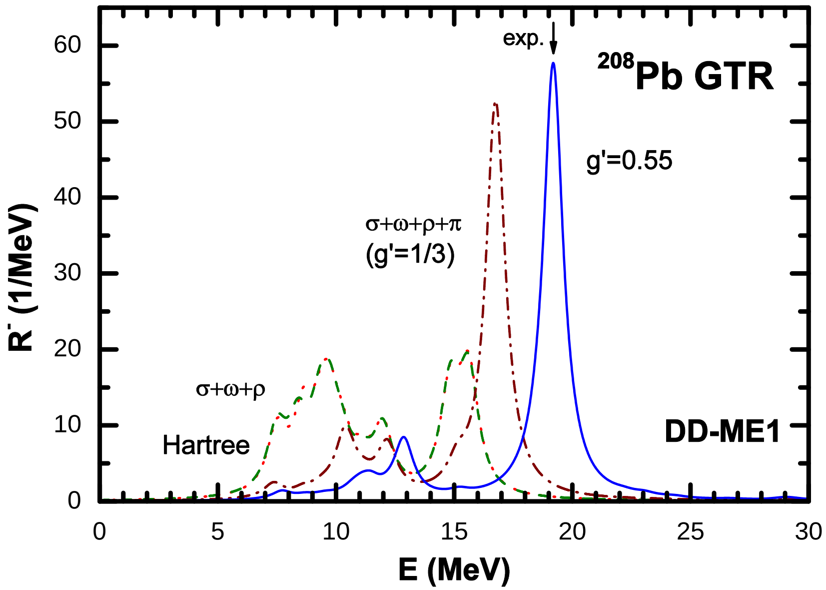
GTR excitation energies and strength

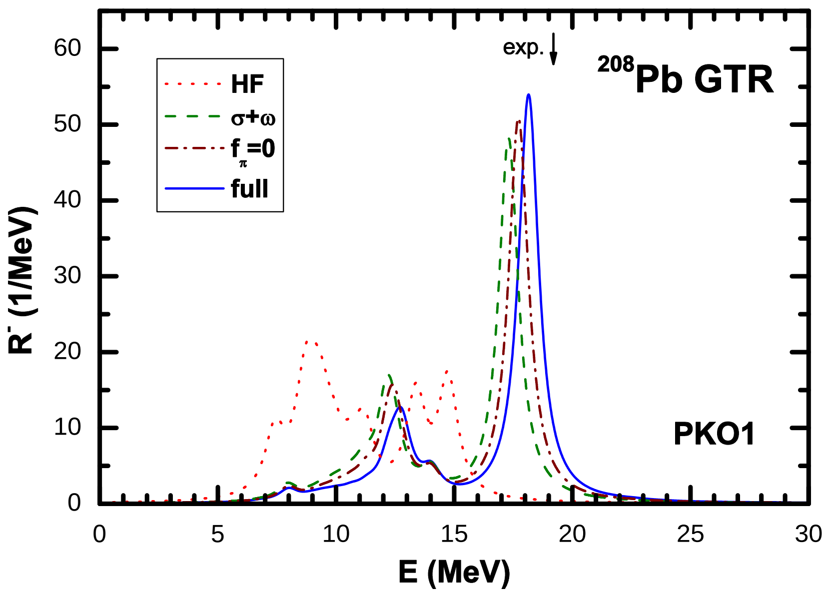
★ GTR excitation energies in MeV and strength in percentage of the $3(N-Z)$ sum rule within the RHF+RPA framework. Experimental and the RH+RPA results are given for comparison.
HL, Giai, Meng, *PRL* 101, 122502 (2008)

		^{48}Ca		^{90}Zr		^{208}Pb	
		energy	strength	energy	strength	energy	strength
experiment		~ 10.5		15.6 ± 0.3		19.2 ± 0.2	60-70
RHF+RPA	PKO1	10.72	69.4	15.80	68.1	18.15	65.6
	PKO2	10.83	66.7	15.99	66.3	18.20	60.5
	PKO3	10.42	70.7	15.71	68.9	18.14	67.7
RH+RPA	DD-ME1	10.28	72.5	15.81	71.0	19.19	70.6

- The pion is not included in PKO2.

Physical mechanisms of GTR

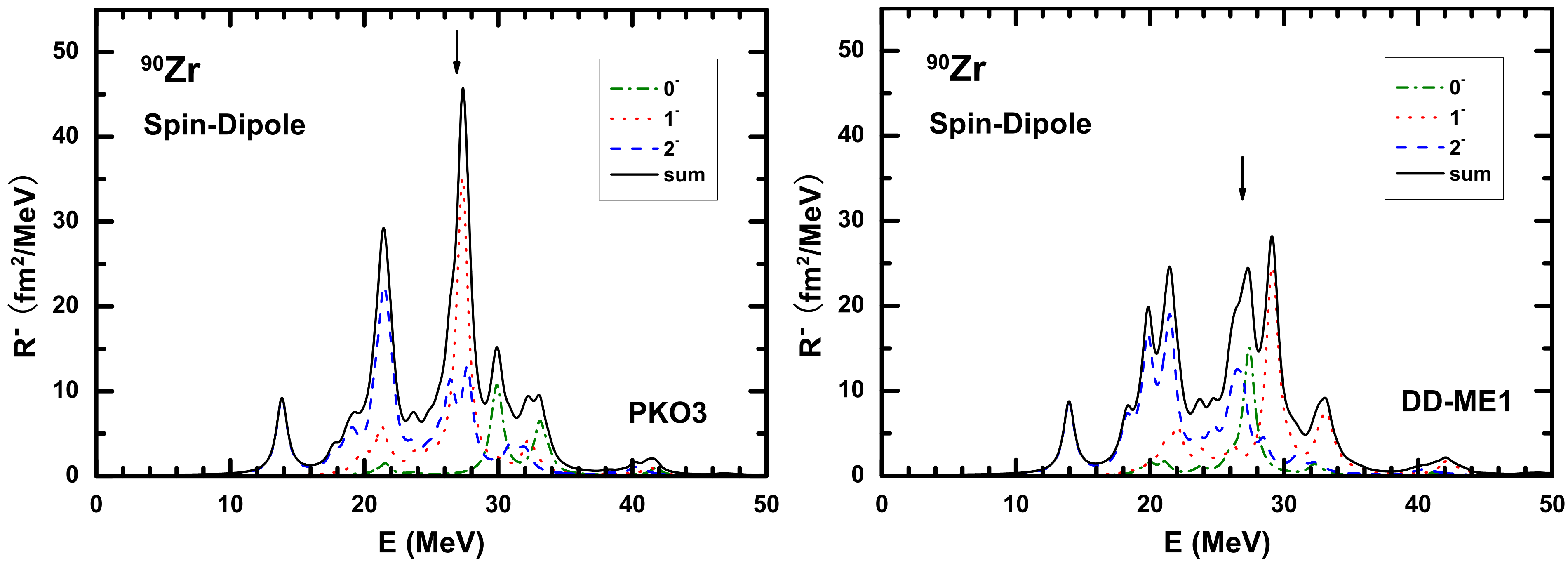




- RH+RPA
  - ⋆ no contribution from isoscalar mesons (σ, ω), because exchange terms are missing.
  - ⋆ π-meson is dominant in this resonance.
  - ⋆ g' has to be refitted to reproduce the experimental data.
- RHF+RPA
  - ⋆ isoscalar mesons (σ, ω) play an essential role via the exchange terms.
  - ⋆ π-meson plays a minor role.
  - ⋆ $g' = 1/3$ is kept for self-consistency.

HL, Giai, Meng, *PRL* 101, 122502 (2008)

Spin-dipole resonances

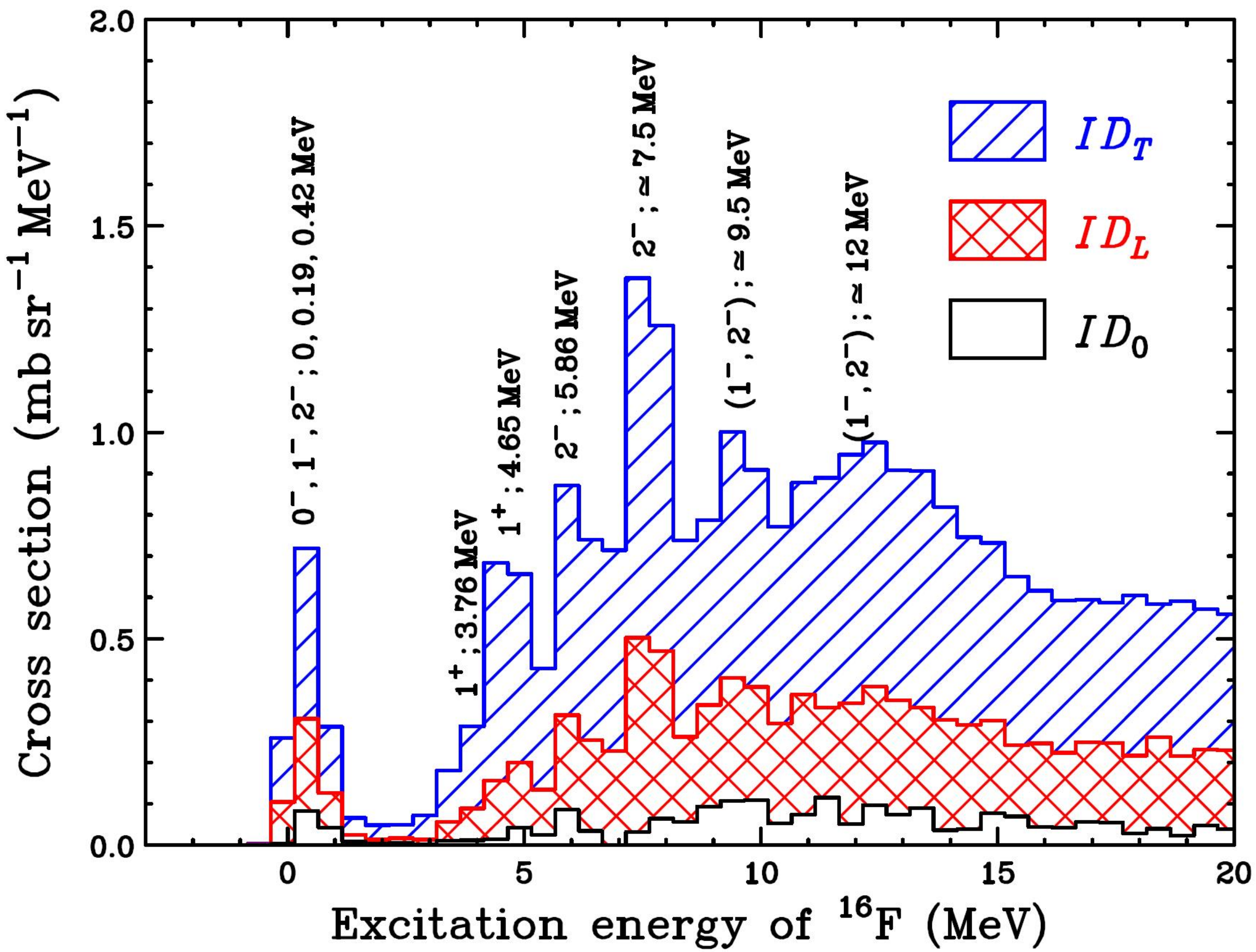


- Main peak can be reproduced by RHF+RPA exp. Yako:2006
- Energy hierarchy
  - ⋆ RHF+RPA: $E(2^-) < E(1^-) < E(0^-)$ agree with SHF+RPA Fracasso:2007
  - ⋆ RH+RPA: $E(2^-) < E(0^-) < E(1^-)$

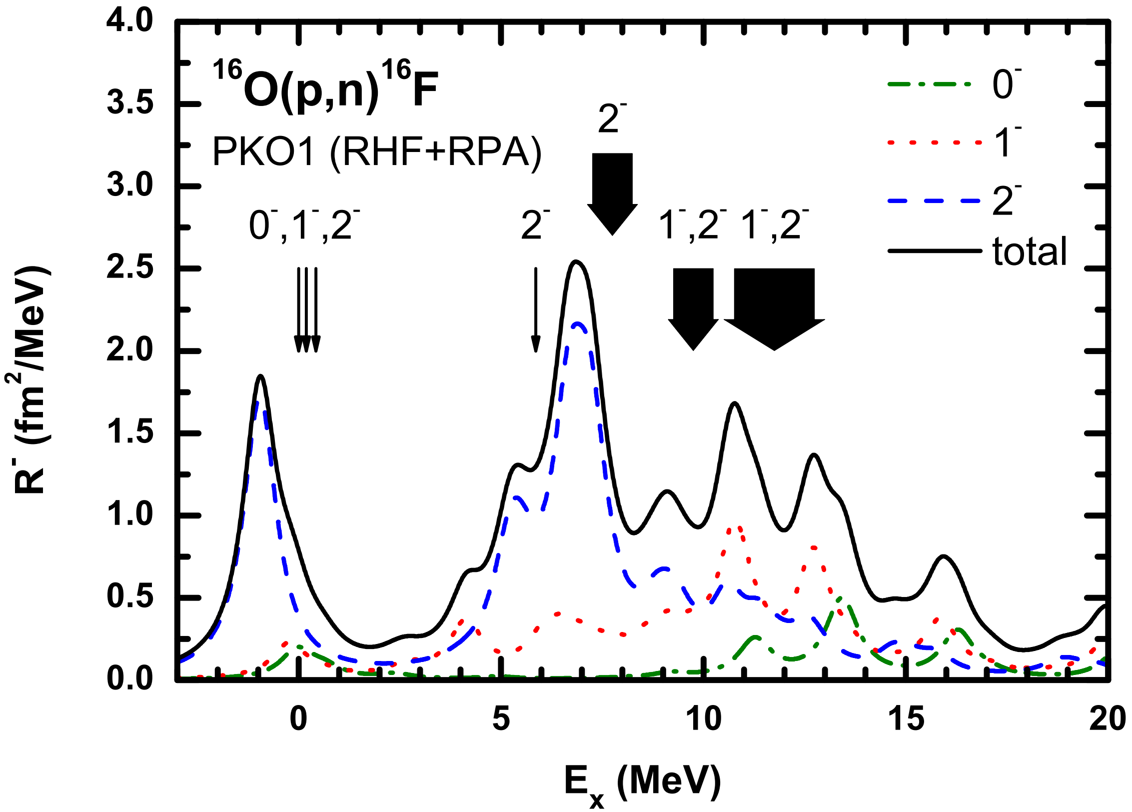
- Separating experimentally the different components from the total transition strength would be helpful to evaluate the theoretical predictive power, e.g.,
  - ⋆ SDR in ^{208}Pb Wakasa:2010 arXiv and ^{16}O Wakasa:2011

Fine structure of GT and SD excitations in ^{16}O

- A most recent $^{16}O(\vec{p},\vec{n})^{16}F$ experiment Wakasa *et al.*, *PRC* **84**, 014614 (2011)



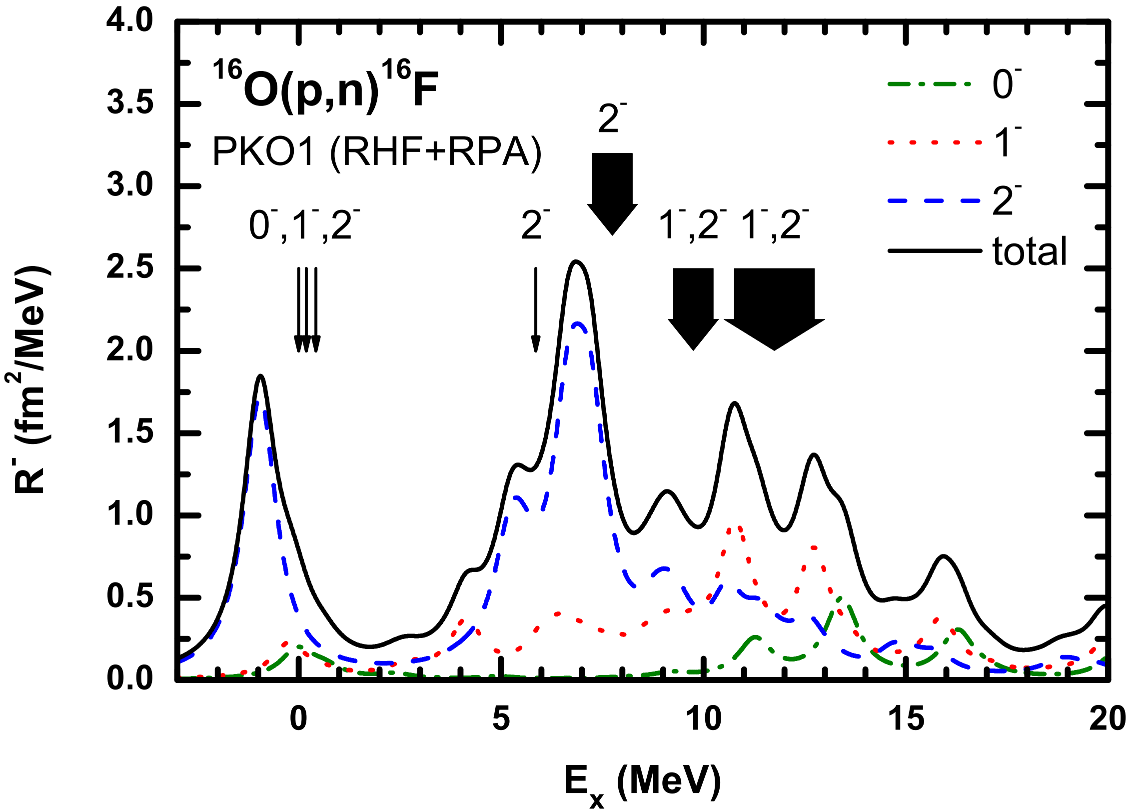
Spin-dipole excitations by RHF+RPA



⋆ SDR in T_- channel by RHF+RPA, where for E_x the 0_1^- state is taken as reference. exp. Wasaka:2011

- In general, the 0^-, 1^-, and 2^- excited states are well reproduced within 1 MeV.
- The 0_1^-, 1_1^-, and 2_1^- triplets are found at $E_x \simeq 0$ MeV.
- The shoulder at $E_x = 5.86$ MeV and giant resonance at $E_x \simeq 7.5$ MeV are nicely reproduced. In particular, the shoulder state cannot be described by shell model calculations.

Spin-dipole excitations by RHF+RPA



⋆ SDR in T_- channel by RHF+RPA, where for E_x the 0^-_1 state is taken as reference. exp. Wasaka:2011

- The mixtures of 1^- and 2^- states at $E_x \simeq 9.5$ MeV and $E_x \simeq 12$ MeV are reproduced, and the former one is dominant by 2^- component, whereas the latter one is dominant by 1^- component.
- The 0^- resonances are predicted to be fragmented at $11 \sim 17$ MeV with the peak at $E_x \simeq 13.5$ MeV.

2^- states at $E_x \simeq 5.5$ MeV

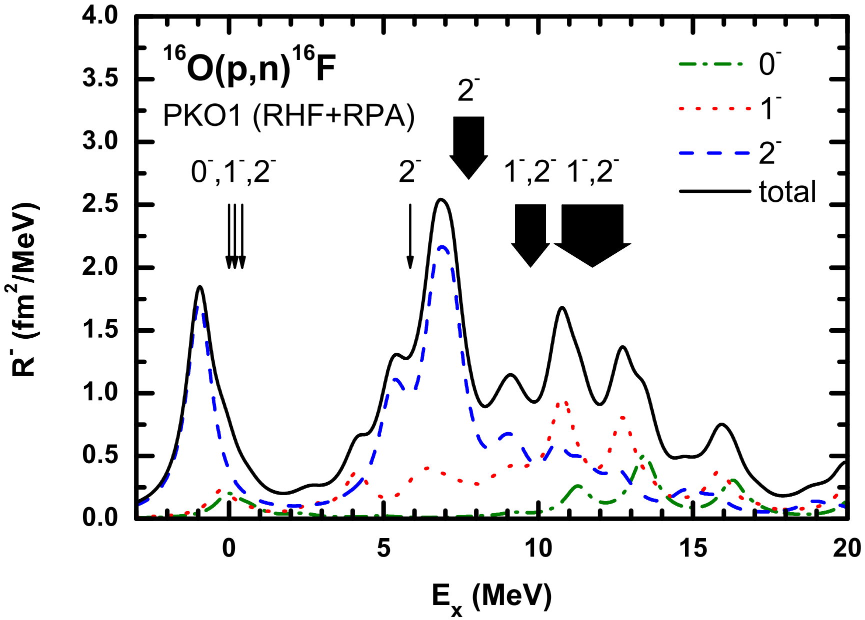
⋆ Particle-hole amplitudes X_{ph}^2 for the 2^- states at $E_x \simeq 5.5$ MeV.

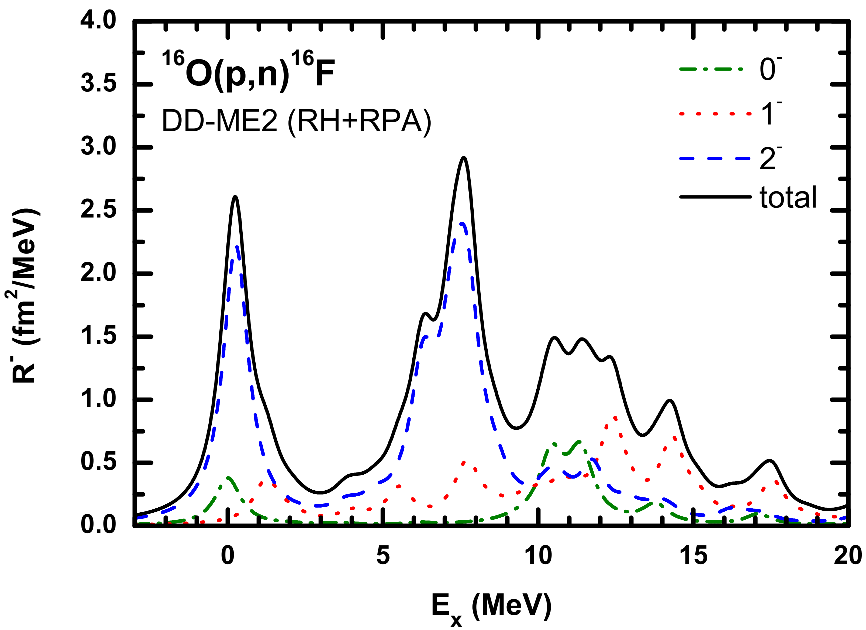
p-h configuration	$E_x = 5.23$ MeV	$E_x = 5.54$ MeV
$\nu 1p_{1/2} \rightarrow \pi 4d_{5/2}$	0.42	0.57
$\nu 1p_{3/2} \rightarrow \pi 1d_{5/2}$	0.37	0.20
$\nu 1p_{1/2} \rightarrow \pi 4d_{3/2}$	0.09	0.11
$\nu 1p_{3/2} \rightarrow \pi 2s_{1/2}$	0.07	0.09
$\nu 1p_{1/2} \rightarrow \pi 3d_{3/2}$	0.03	0.01
$\nu 1p_{1/2} \rightarrow \pi 1d_{5/2}$	0.01	0.01

$\varepsilon(\nu 1p_{3/2}) = -20.98$ MeV, $\varepsilon(\nu 1p_{1/2}) = -14.56$ MeV
$\varepsilon(\pi 1d_{5/2}) = -1.89$ MeV, $\varepsilon(\pi 4d_{5/2}) = 6.23$ MeV

- They are collective excitations.
- The most dominant particle-hole configuration $\nu 1p_{1/2} \rightarrow \pi 4d_{5/2}$ is not considered in shell model calculations [Wakasa:2011].

SD excitations by RHF+RPA and RH+RPA





RH+RPA results

- The general pattern of 2^- excitations are similar to that of RHF+RPA calculations, except the peak at $E_x \simeq 9.5$ MeV is missing.
- The 1^- resonances are predicted at $12 \sim 15$ MeV, somehow too high in energy by comparing to data.
- The 0^- resonances are predicted to be centralized at $10 \sim 12$ MeV, but not seen in experiments yet.

Conclusion

- By comparing with the experimental date, it is found that the self-consistent RHF+RPA calculations are more favored.

Outline

Nuclear β-decay half-lives

- Nuclear β-decay half-life in the allowed Gamow-Teller approximation

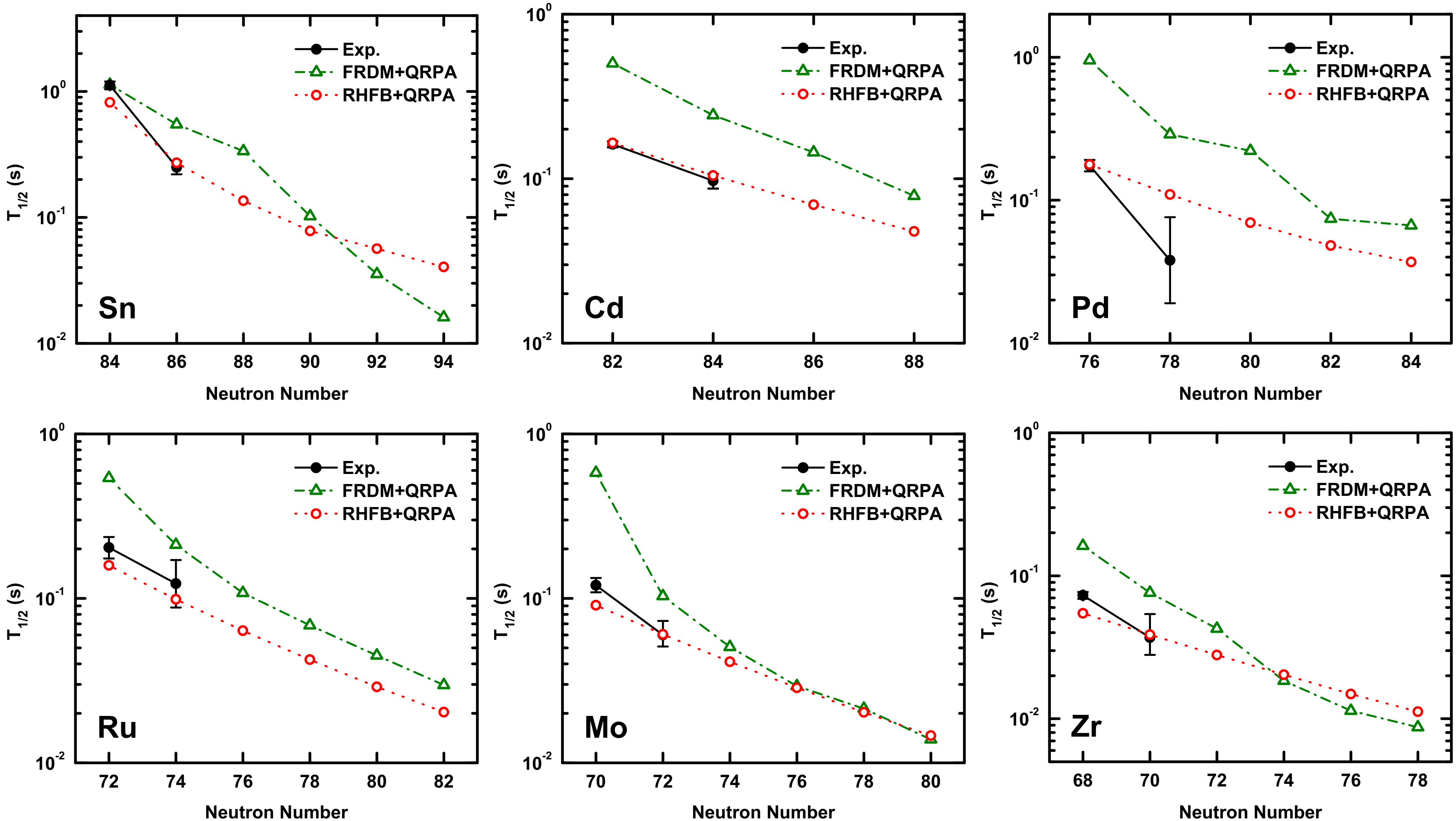
$$T_{1/2} = \frac{\ln 2}{\lambda_\beta} = \frac{D}{g_A^2 \sum_m |\langle 1_m^+ |\sigma\tau| 0^+\rangle|^2 f(Z, A, E_m)} \tag{6}$$

  - ⋆ constants $D = \frac{\hbar^7 2\pi^3 \ln 2}{g^2 m_e^5 c^4} = 6163.4$ s and $g_A = 1$
  - ⋆ integrated $(e, \bar{\nu}_e)$ phase volume

$$f(Z, A, E_m) = \frac{1}{m_e^5} \int_{m_e}^{E_m} p_e E_e (E_m - E_e)^2 F_0(Z, A, E_e) dE_e \tag{7}$$

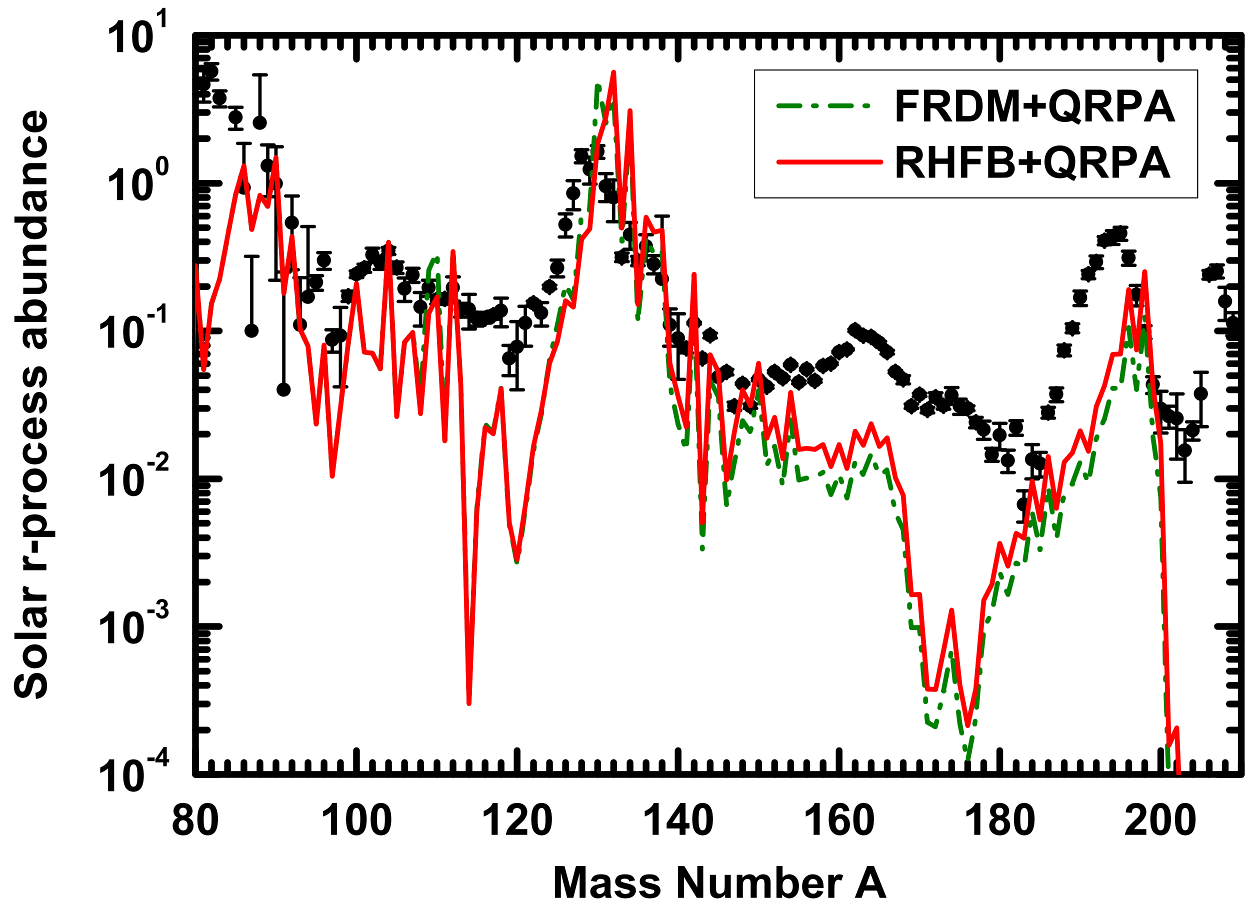
  - ⋆ transition probabilities $|\langle 1_m^+ |\sigma\tau| 0^+\rangle|^2$ determined by the QRPA calculations

β-decay half-lives of Sn, Cd, Pd, Ru, Mo, Zr isotopes



⋆ β-decay half-lives of Sn, Cd, Pd, Ru, Mo, and Zr isotopes.

Solar *r*-process abundance



- A speeding-up of the *r*-matter flow is obtained with the β-decay rates predicted by RHFB+QRPA.
- This leads to a larger *r*-process abundance distribution in $A \gtrsim 140$ region.

Outline

Summary

★ A fully self-consistent charge-exchange relativistic RPA approach based on the RHF approach is established.

- Spin-isospin resonances
  - ✓ GTR excitation energies can be described in a fully self-consistent way.
  - ✓ Isoscalar mesons (σ, ω) are found to play an essential role via the exchange terms in GTR and SDR, while π-meson plays a minor role.
  - ✓ Fine structure of SD excitations in ^{16}O is well reproduced.
- β-decay half-lives
  - ✓ β-decay half-lives of neutron-rich Sn, Cd, Pd, Ru, Mo, and Zr isotopes are well reproduced by RHFB+QRPA calculations.
  - ✓ A speeding-up of the r-matter flow through $N = 82$ waiting points is obtained, which leads to a larger r-process abundance distribution in $A \gtrsim 140$ region.

Thank you!