Tensor correlations in light nuclei

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Tensor interaction in nuclear and hadron physics
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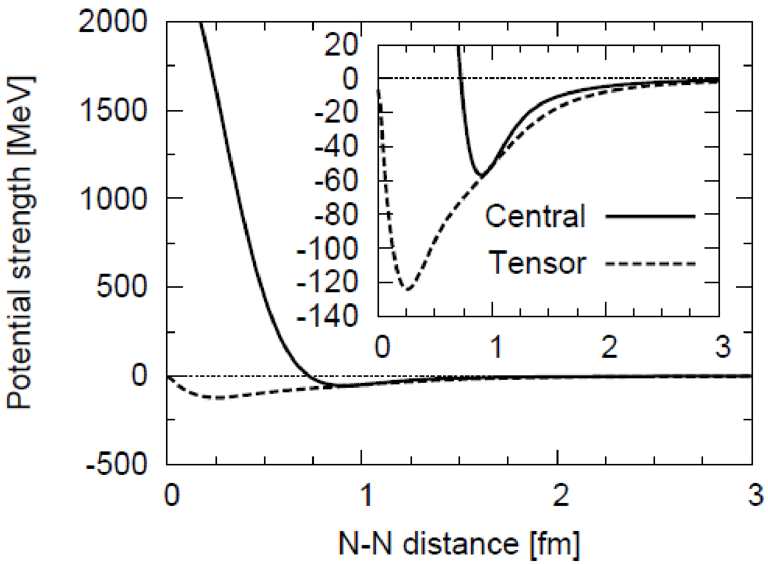
Introduction

- "Realistic" nucleon-nucleon force
  - Nucleon-nucleon scattering
  - Deuteron properties
- Coordinate space
  - Short-range repulsion
  - Strong tensor component
- Momentum space
  - High momentum
  - Off diagonal matrix element

Argonne potential (V8')



Calculation becomes complicated but a role of the short-range correlations is important.

Outline

- Study of short-range and tensor correlations
  - Impact on nuclear structures and reactions
  - Low-momentum effective interaction
  - Saturation properties of nuclear matter
  - (e, e') experiment at Jlab, R. Subedi, Science 320, 1476(2008).
- Approach: Variational calculation with explicitly correlated basis
  - Correlated Gaussian and global vectors
  - Stochastic variational method

    → "Exact" many-body states
- Results
  - Spectrum of ^{4}He
  - One-body densities
  - Two-body densities for different spin-isospin channels
  - Comparison with the Unitary Correlation Operator Method (UCOM)

    R. Roth, T. Neff, H. Feldmeier, Prog. Part. Nucl. Phys. 65, 50 (2010).
- Summary and outlook

Variational calculation for many-body systems

Hamiltonian

$$H = \sum_{i=1}^{A} T_i - T_{\rm cm} + \sum_{i<j}^{A} v_{ij} + \sum_{i<j<k}^{A} v_{ijk}$$

$$v_{12} = V_c(r) + V_{\rm Coul.}(r) P_{1\pi} P_{2\pi} + V_t(r) S_{12} + V_b(r) \boldsymbol{L} \cdot \boldsymbol{S}$$

Argonne V8' interaction: central, tensor, spin-orbit

Generalized eigenvalue problem

$$\Psi_{JM_J} = \sum_{i=1}^{K} c_i \Psi(\alpha_i)$$

$$\sum_{j=1}^{K} (H_{ij} - E B_{ij}) c_j = 0 \quad (i = 1, \ldots, K)$$

$$\begin{pmatrix} H_{ij} \\ B_{ij} \end{pmatrix} = \langle \Psi(\alpha_i) | \begin{pmatrix} H \\ 1 \end{pmatrix} | \Psi(\alpha_j) \rangle$$

Basis function

$$\Psi_{(LS)JM_J TM_T} = \mathcal{A} \left\{ \left[\psi_L^{\rm (space)} \psi_S^{\rm (spin)} \right]_{JM_J} \psi_{TM_T}^{\rm (isospin)} \right\}$$

$$\psi_{SM_S}^{\rm (spin)} = \left| [\cdots [[[\tfrac{1}{2} \tfrac{1}{2}] S_{12} \tfrac{1}{2}] S_{123}] \cdots] SM_S \right\rangle$$

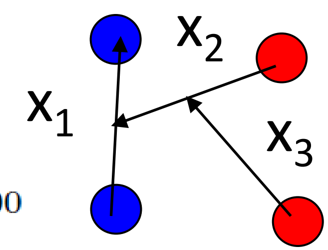
Explicitly correlated basis function

Correlated Gaussian

Explicit correlations among particles

$$\exp\left(-\frac{1}{2}ar^2\right) \to \exp\left(-\frac{1}{2}\tilde{x}Ax\right) = \exp\left(-\frac{1}{2}\sum_{i,j=1}^{A-1} A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j\right)$$

$$\exp\left(A_{ij}\boldsymbol{x}_i\cdot\boldsymbol{x}_j\right) \sim \sum_n (\boldsymbol{x}_i\cdot\boldsymbol{x}_j)^n \sim \sum_{\ell=n,n-2,\ldots} [\mathcal{Y}_\ell(\boldsymbol{x}_i)\mathcal{Y}_\ell(\boldsymbol{x}_j)]_{00}$$



Global vector

Rotational motion with total angular momentum L

$$r^l Y_{lm}(\hat{\boldsymbol{r}}) \equiv \mathcal{Y}_{lm}(\boldsymbol{r}) \to \mathcal{Y}_{LM_L}(\tilde{u}\boldsymbol{x}) = \mathcal{Y}_{LM_L}\left(\sum_{i=1}^{A-1} u_i\boldsymbol{x}_i\right)$$

$$\mathcal{Y}_{LM_L}(u_1x_1+u_2x_2) = \sum_{\ell=0}^{L}\sqrt{\frac{4\pi(2L+1)!}{(2\ell+1)!(2L-2\ell+1)!}}\; u_1^\ell u_2^{L-\ell}[\mathcal{Y}_\ell(x_1)\mathcal{Y}_{L-\ell}(x_2)]_{LM_L}$$

Correlated Gaussian with two global vectors

Parity $(-1)^{L_1+L_2}$

$$F_{(L_1L_2)LM}(u_1,u_2,A,\boldsymbol{x}) = \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)[\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})]_{LM}$$

Correlated basis approach

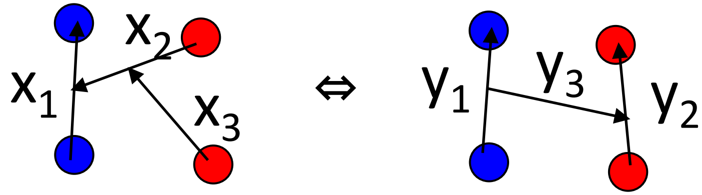
Double Global Vector Representation

Y. Suzuki, W.H., M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$F_{(L_1 L_2)LM}(u_1, u_2, A, \boldsymbol{x}) = \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}\right)[\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})]_{LM}$$

- Formulation for N-particle system
- Matrix elements can analytically be obtained
- Functional form does not change under any coordinate transformation

$$\boldsymbol{y} = T\boldsymbol{x} \implies \tilde{\boldsymbol{y}}B\boldsymbol{y} = \tilde{\boldsymbol{x}}\tilde{T}BT\boldsymbol{x} \qquad \tilde{v}\boldsymbol{y} = \widetilde{\tilde{T}v}\boldsymbol{x}$$



Stochastic variational method

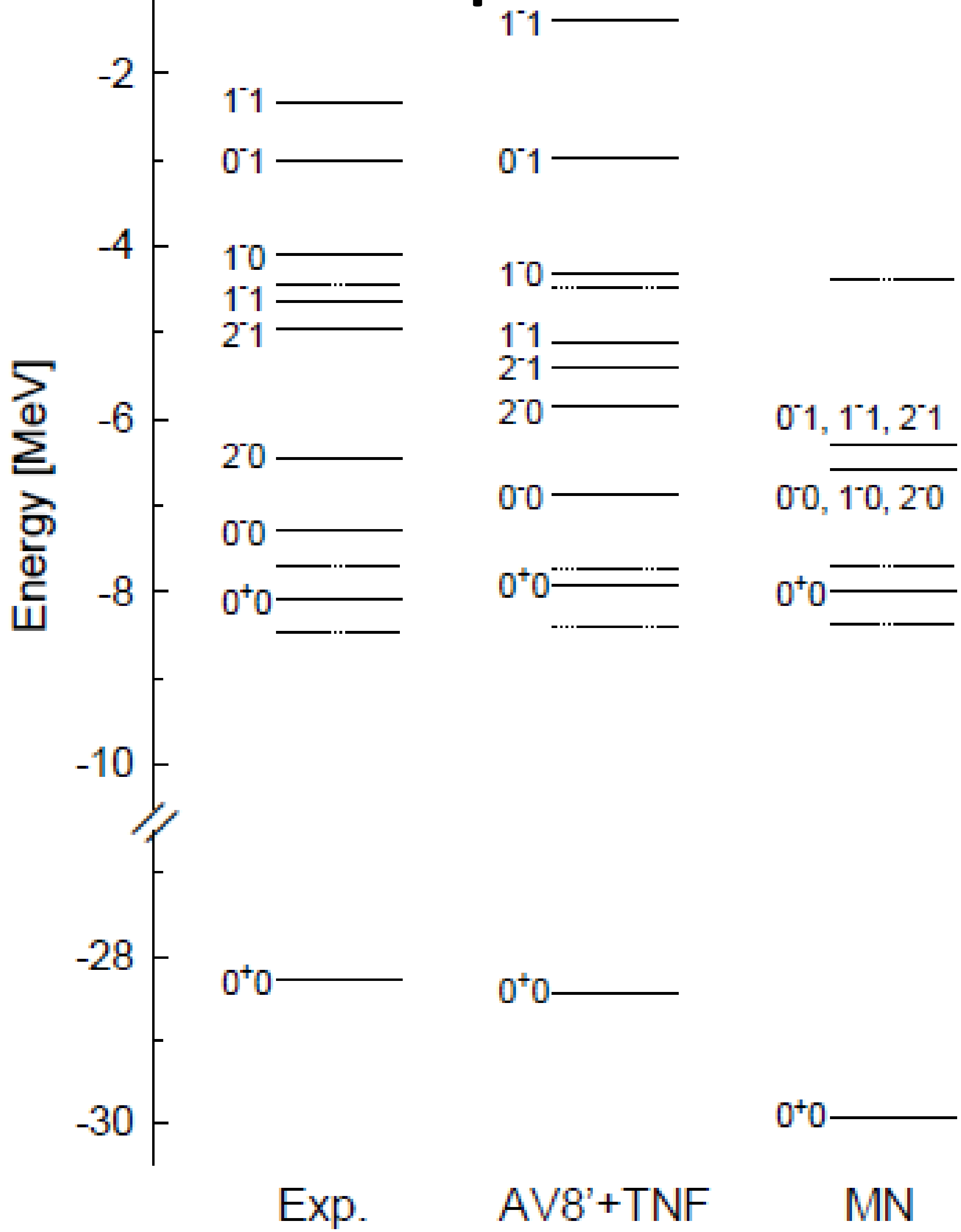
K. Varga and Y. Suzuki, PRC52, 2885 (1995).

- Examine randomly generated basis and increase (or replace) the number of basis until convergence is reached

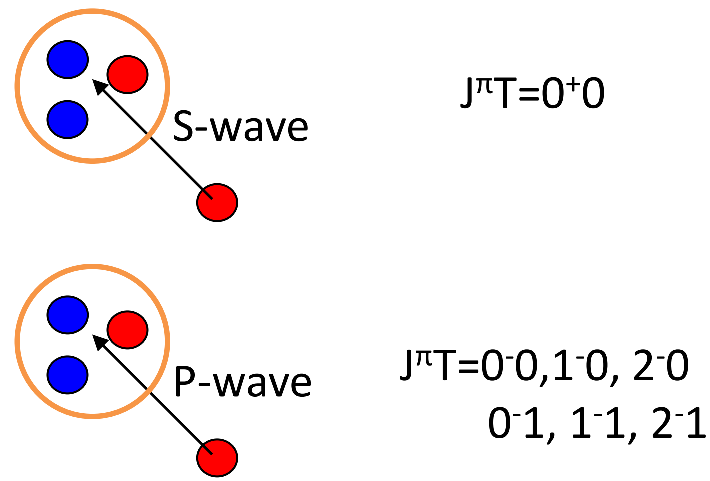
^{4}He energy agrees with the benchmark calculation

H. Kamada et al., PRC64, 044001 (2001)

^{4}He spectrum



- AV8'+phenomenological three-body force
- All excited levels below the four-particle threshold are reproduced
- Degenerate levels with Minnesota force (central)
- *3N+N* cluster structure appear

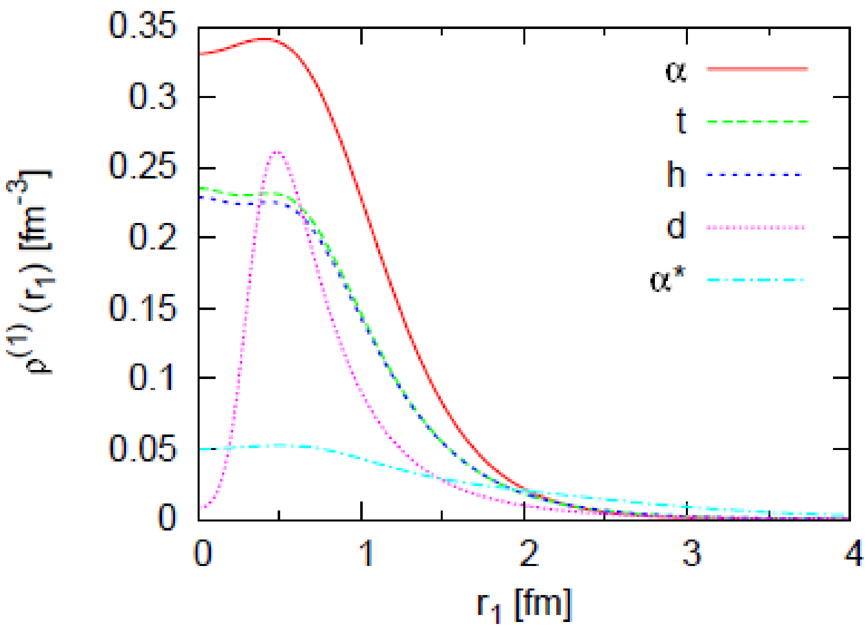


E. Hiyama et al. PRC70, 031001(R) (2002)
W.H. and Y. Suzuki, PRC78, 034305 (2008)

One-body densities (AV8')

Coordinate space

$$\rho^{(1)}(\boldsymbol{r}_1) = \langle\Phi|\sum_{i=1}^{A}\delta^3(\boldsymbol{r}_i - \boldsymbol{r}_1)|\Phi\rangle$$

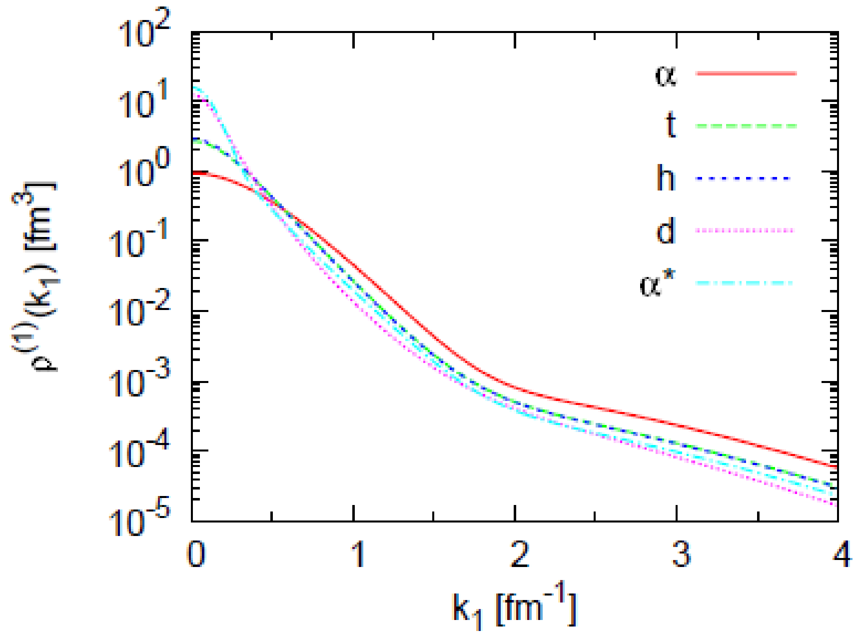


Strongly depends on the system
Dilute 3N+N structure in α*

W.H. and Y. Suzuki, PRC78, 034305 (2008)

Momentum space

$$\rho^{(1)}(\boldsymbol{k}_1) = \langle\Phi|\sum_{i=1}^{A}\delta^3(\boldsymbol{k}_i - \boldsymbol{k}_1)|\Phi\rangle$$



Size of the system
High momentum component

How to extract correlated information

- Antisymmetrized many-body states Φ
  - Two-body: ^{2}H(d)
  - Three-body: ^{3}H(t), ^{3}He(h)
  - Four-body: ^{4}He(α), ^{4}He(0_2^+) (α*)

- *A*-body density: all information on correlation
  - Too much information
    - Position or momentum vectors: *A*
    - Spin-isospin possibilities: 4**A*
  - Two-body correlation

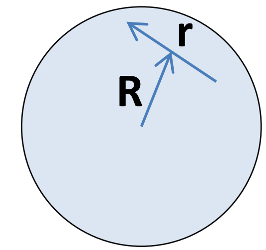
    -> integrate over A-2 particle degrees of freedom

Two-body densities

Two-body density

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_1)\delta^3(\boldsymbol{r}_j-\boldsymbol{r}_2)\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

Spin (isospin) projector



Two-body density in relative coordinate

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r},\boldsymbol{R}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\delta^3(\frac{1}{2}(\boldsymbol{r}_i+\boldsymbol{r}_j)-\boldsymbol{R})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

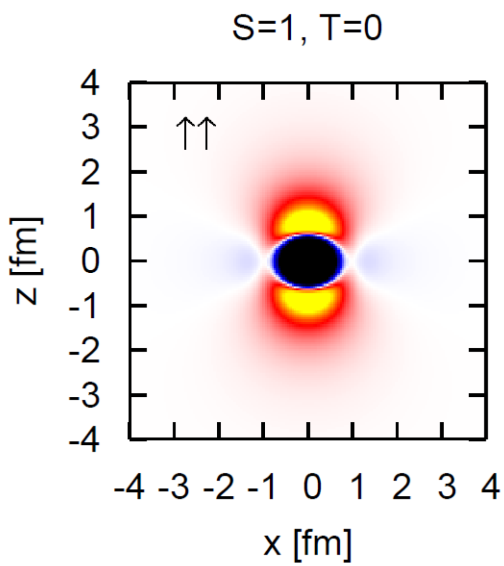
$$\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{r}) = \int d\boldsymbol{R}\,\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r},\boldsymbol{R})$$
$$= \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

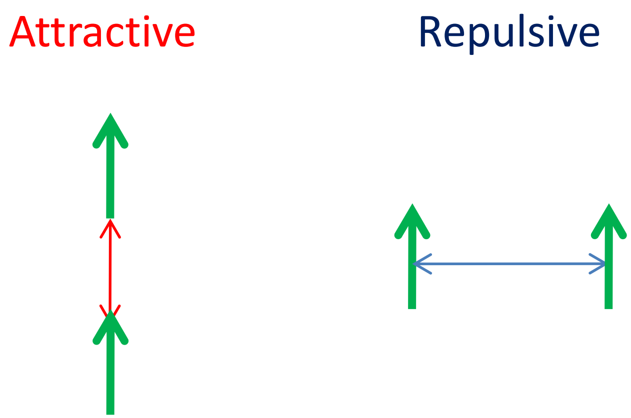
Momentum space

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{k},\boldsymbol{K}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\frac{1}{2}(\boldsymbol{k}_i-\boldsymbol{k}_j)-\boldsymbol{k})\delta^3(\boldsymbol{k}_i+\boldsymbol{k}_j-\boldsymbol{K})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

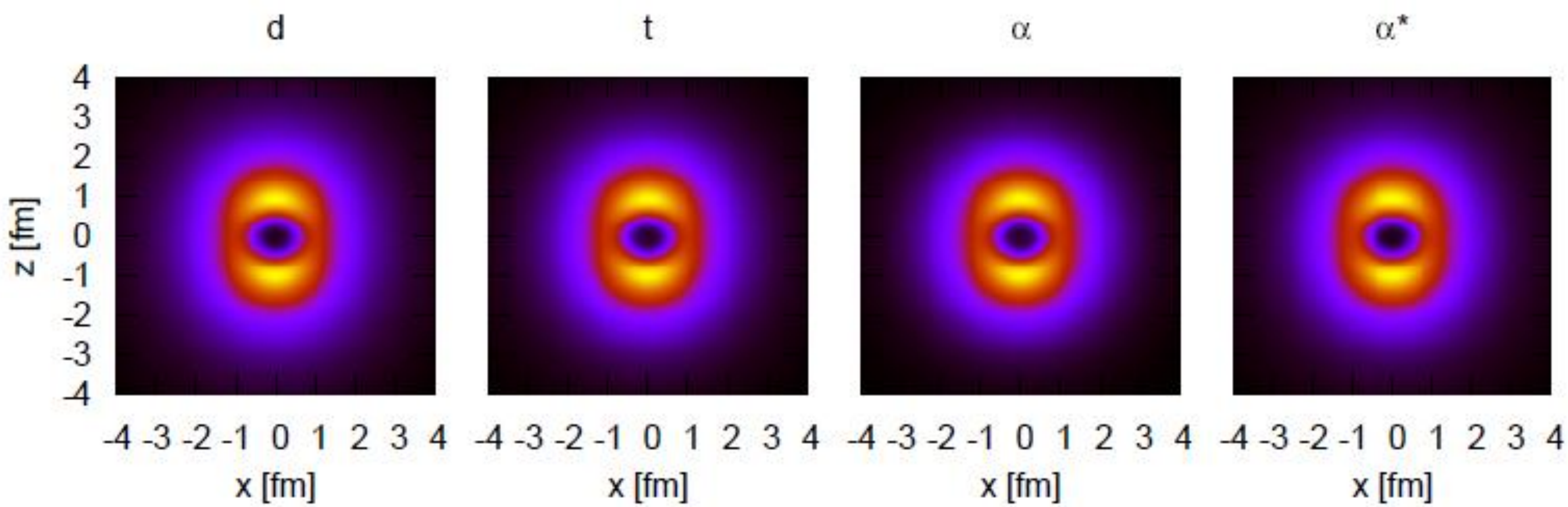
$$\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{k}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\frac{1}{2}(\boldsymbol{k}_i-\boldsymbol{k}_j)-\boldsymbol{k})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

Two-body density (SM_S=11, T=0)

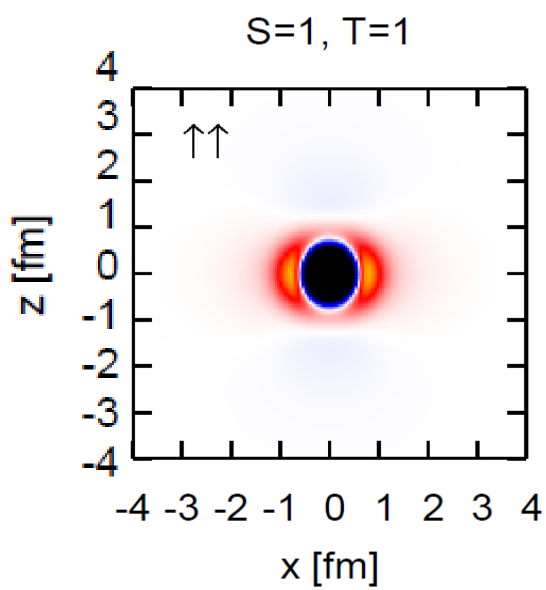


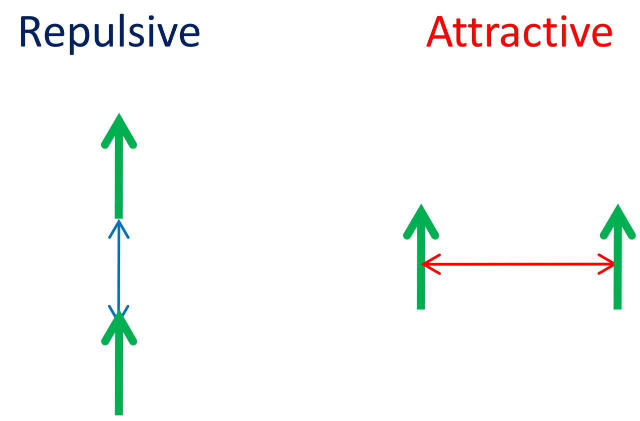


$$\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{r}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$

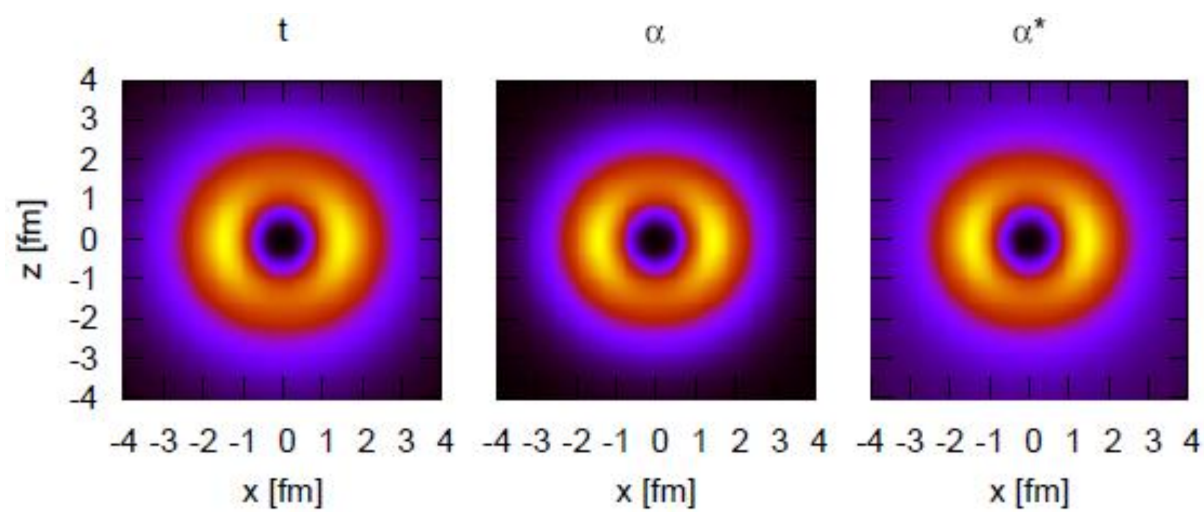


Two-body density (SM_S=11, T=1)

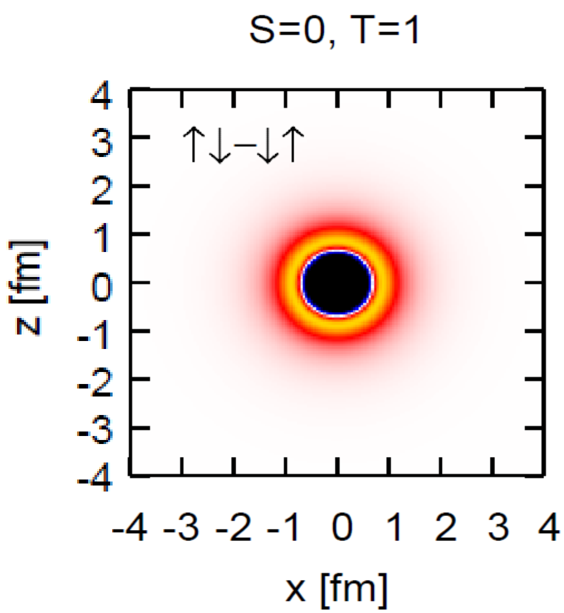


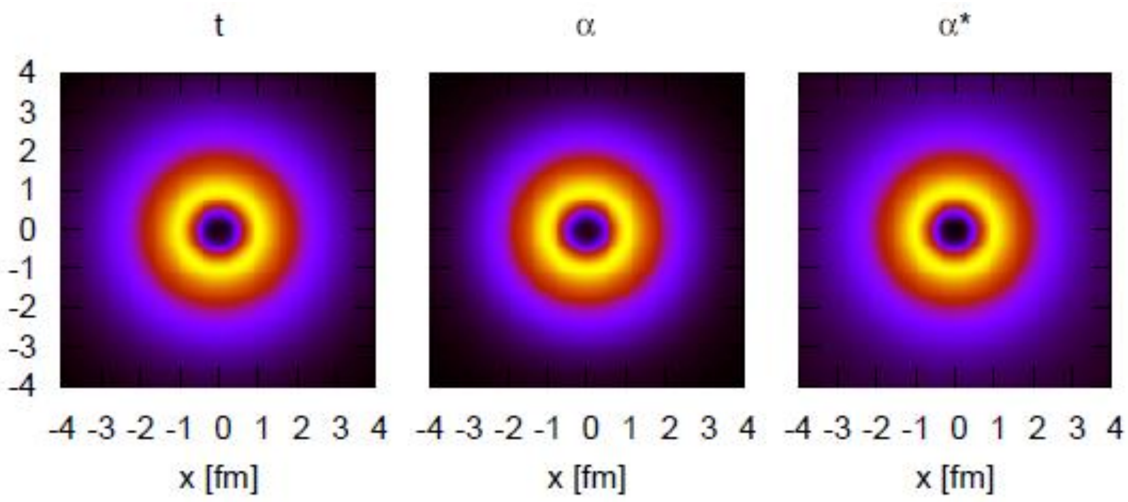


$$\rho^{(\mathrm{rel})}_{SM_S,TM_T}(\boldsymbol{r}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i - \boldsymbol{r}_j - \boldsymbol{r})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$



Two-body density (S=0, T=1)





S=0, T=0 channel -> small components in w. f.

One-to-one correspondence between the density and potential for all ST channels

Relevance to density functional theory

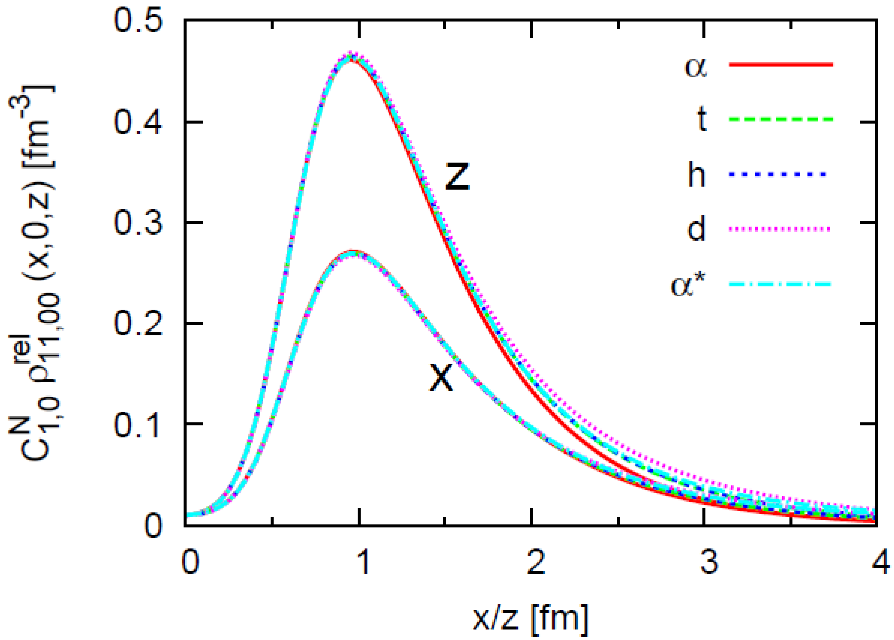
Y. Suzuki, W.H., Nucl. Phys. A 818, 188 (2009).

Universality of short-range correlations Two-body density in coordinate space

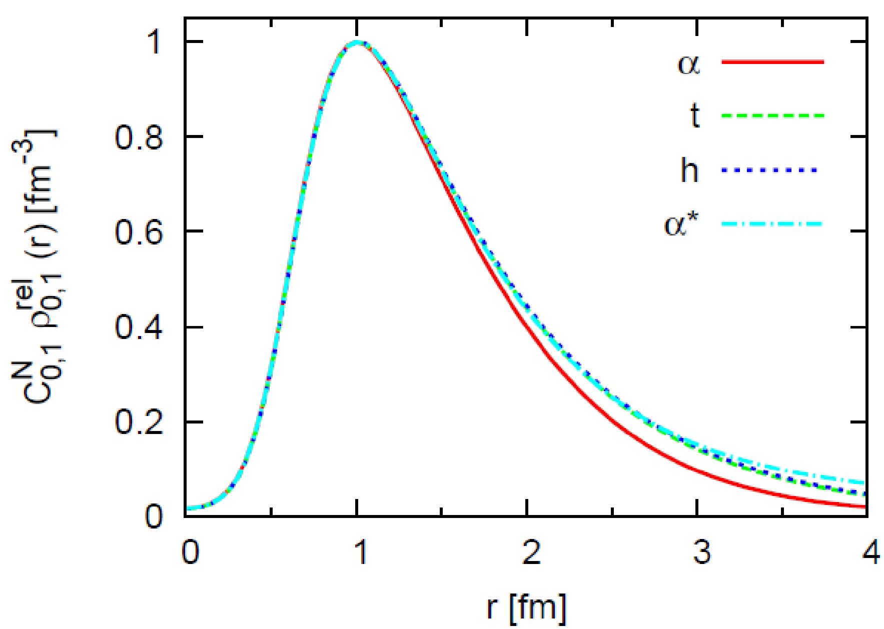
Density cut along z- and x-direction

Normalized at 1 fm of z axis

S=1 M_S=1, T=0



S=0, T=1

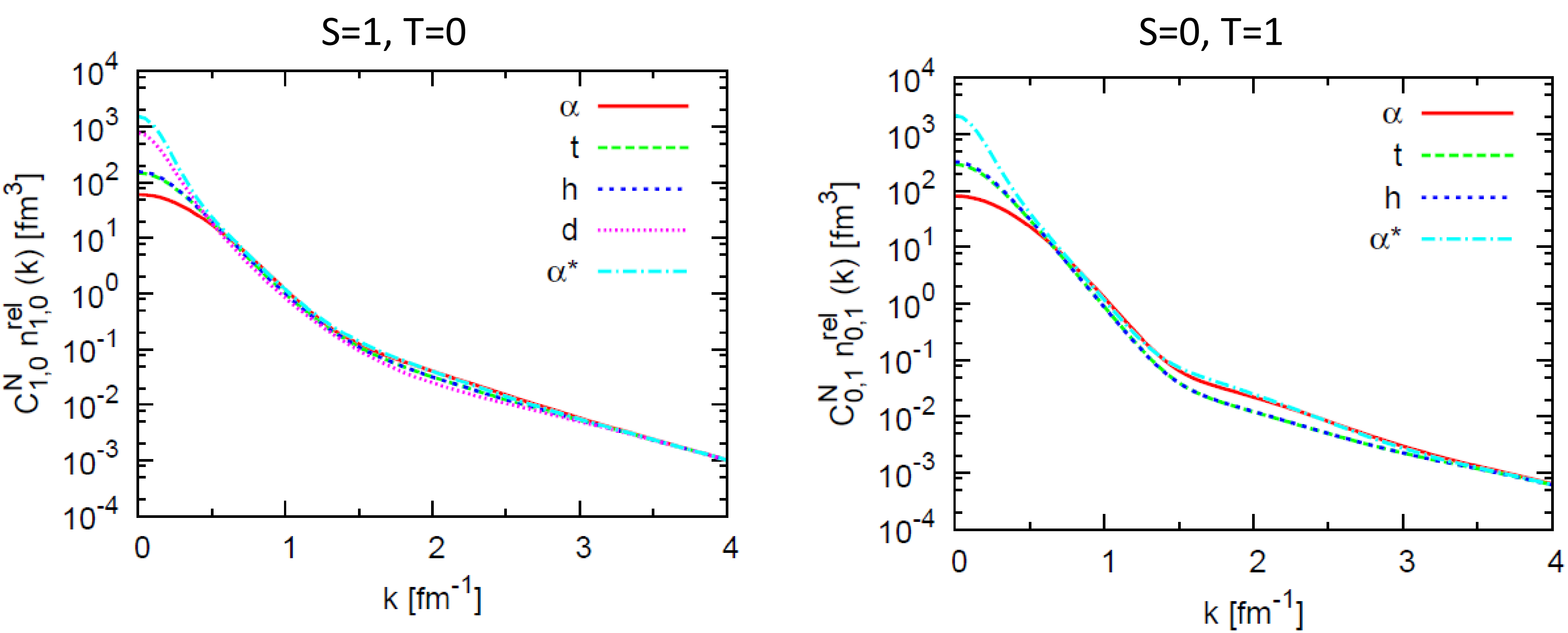


Universal behavior at short distances

Universality of short-range correlation Two-body density in momentum space

Two-body densities in momentum space

The same normalization factors are used as these in coordinate space.



Universal behavior at high momenta.

Comparison with UCOM

Unitary Correlation Operator Method (UCOM)

Unitary transformation $|\Psi\rangle = \hat{C}\,|\Phi\rangle$

Short-range correlations
-> central and tensor correlation functions

Effective Hamiltonian $\hat{H}_{\rm UCOM} = \hat{C}^\dagger \hat{H} \hat{C}$ "Correlated" Hamiltonian

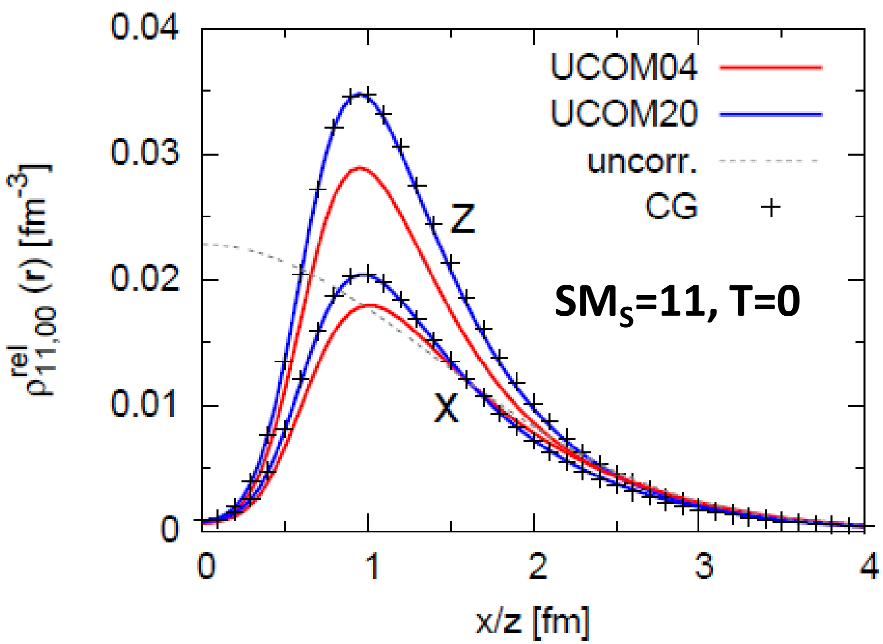
$\hat{H}_{\rm UCOM}\,|\Phi\rangle = E\,|\Phi\rangle$ Trial w.f. can be simple

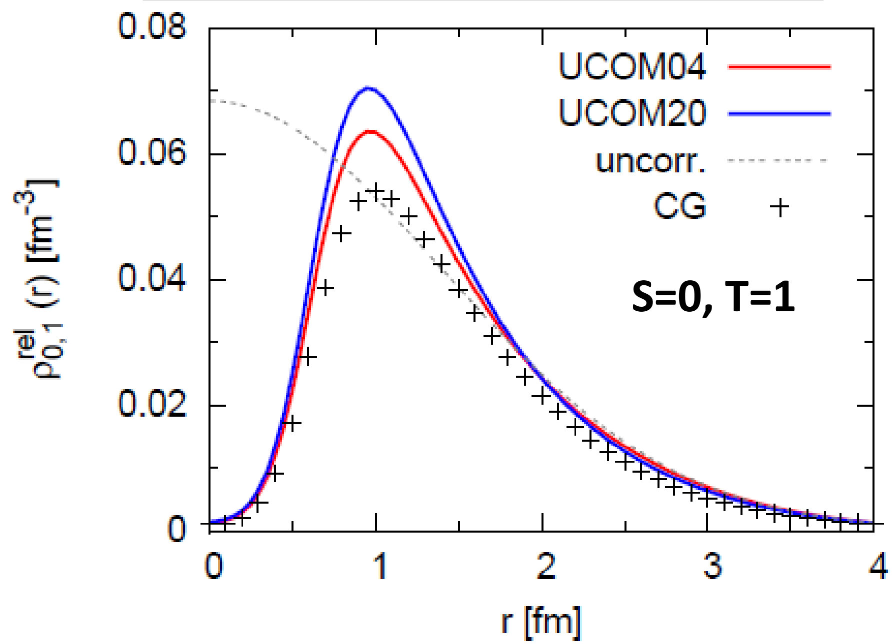
Simple trial w.f. $|\Phi\rangle = |(0s)^4\rangle$

Correlation functions: SRG evolved AV8'

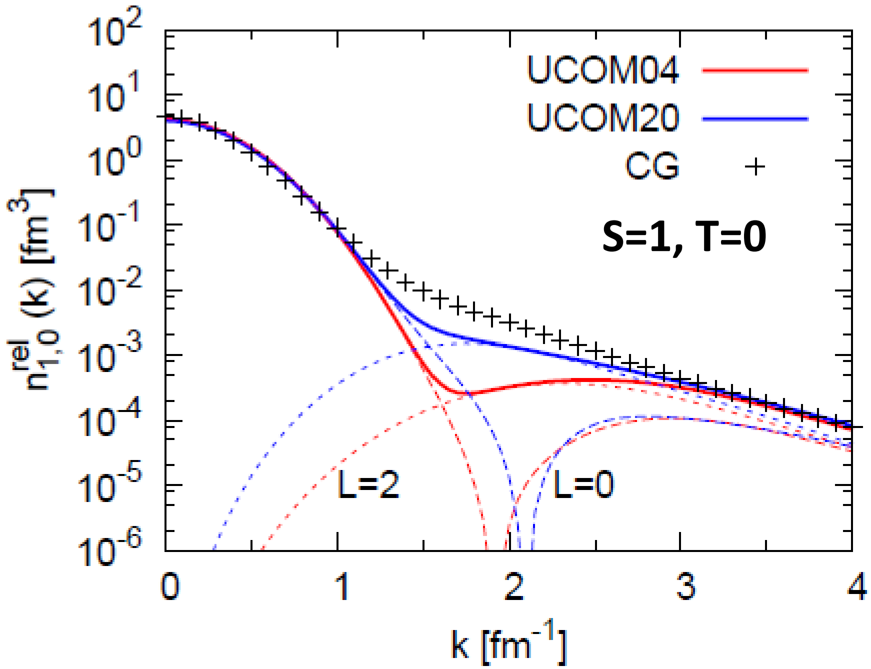
R. Roth, T. Neff, H. Feldmeier, Prog. Part. Nucl. Phys. 65, 50 (2010).

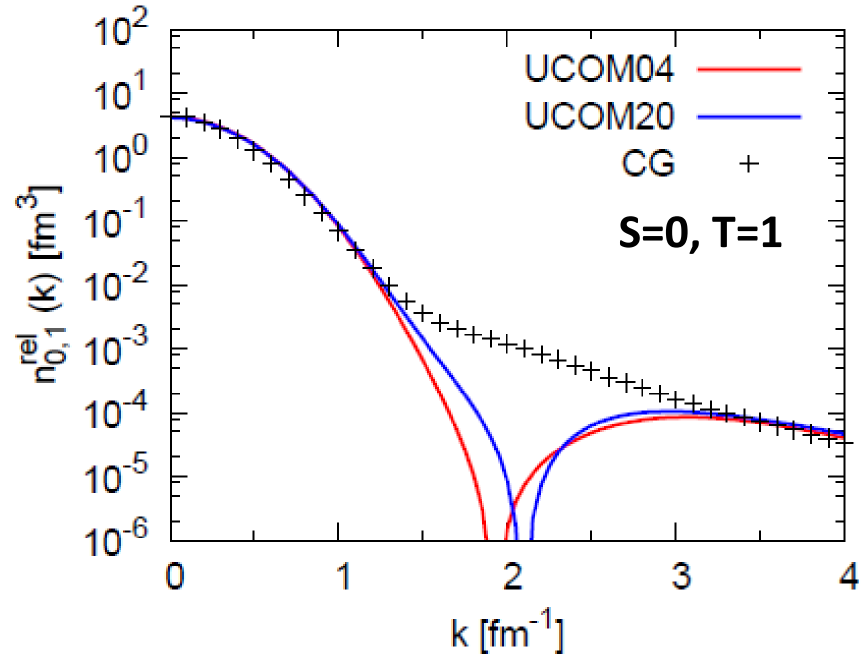
UCOM04(soft): -18.50 MeV
UCOM20(very soft): -25.10 MeV





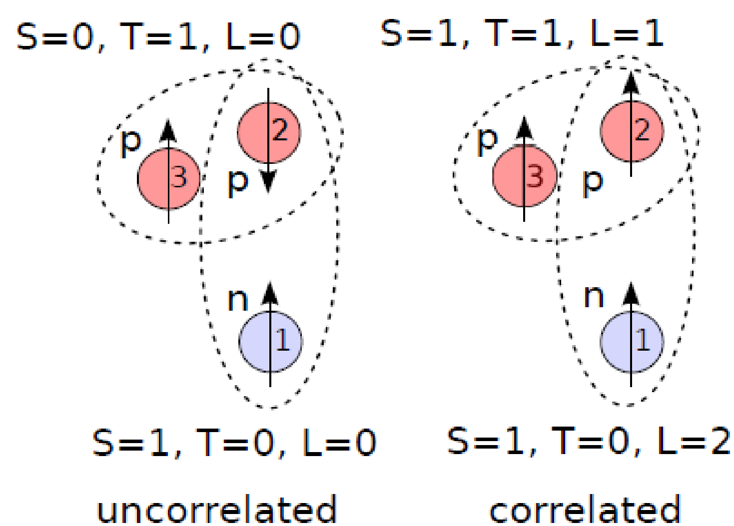
Comparison with UCOM





Lack of the three-body correlations

	$(0s)^4$	correlated
(ST)=(10) pair:	3.0	2.992
(00) pair:	0.0	0.008
(ST)=(01) pair:	3.0	2.572
(11) pair:	0.0	0.438



The deviation can partially be restored by a more elaborated trial wave function (NCSM, in progress).

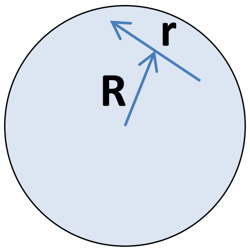
Summary and outlook

- Highly correlated many-body states (d, t, h, α, α*)
  - *Ab initio* type calculation with the Argonne V8' interaction
    - Correlated Gaussian with global vectors
    - Stochastic variational method
  - Tensor force is important for reproducing the spectrum of ^{4}He
- Two-body densities
  - One-to-one correspondence between two-body potential and density
  - Universality at short distances (< 1 fm) and high momenta (> 3 fm^{-1})
- Comparison with the Unitary Correlation Operator Method (UCOM)
  - Success of low-momentum interaction
  - Lack of three-body correlation
    - Too simple trial wave function (-> no-core shell model, etc)
    - Correlation operator determined in two-body level

Accepted for publication in Phys. Rev. C (2011.9.20).

- Outlook
  - Two-body density with two variables

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r},\boldsymbol{R}) = \langle\Phi|\sum_{i<j}^{A}\delta^3(\boldsymbol{r}_i-\boldsymbol{r}_j-\boldsymbol{r})\delta^3(\frac{1}{2}(\boldsymbol{r}_i+\boldsymbol{r}_j)-\boldsymbol{R})\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}|\Phi\rangle$$



  - More particle systems (A>4)