Quantum scattering of three particles in stars: new understanding of the formation of ^{12}C

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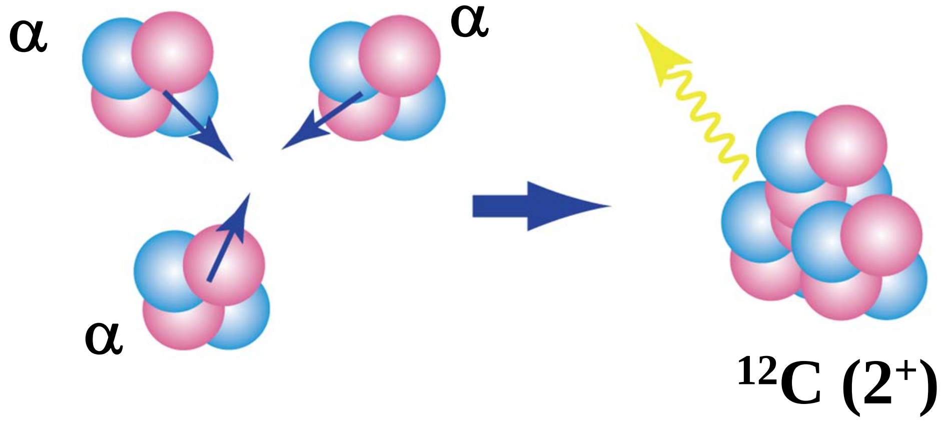
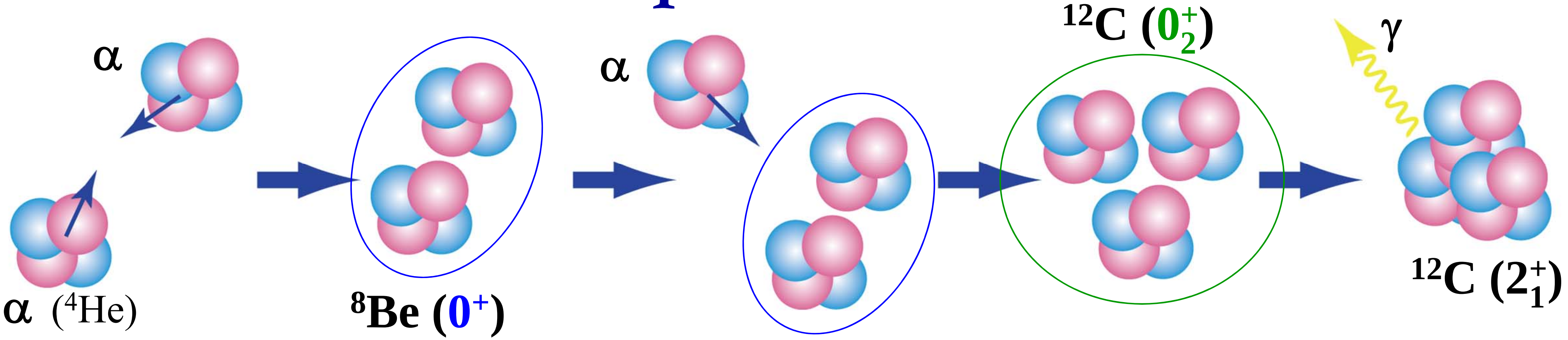


The triple α reaction



$(\tau \sim 10^{-17}$ sec)

α + α

0.0918 **0^+**

^{8}Be

7.3666

^{8}Be + α

9.641 **3_1^-**

7.6542 **0_2^+**

Hoyle state

4.4389 **2_1^+**

0_1^+

^{12}C

I think, therefore it is!

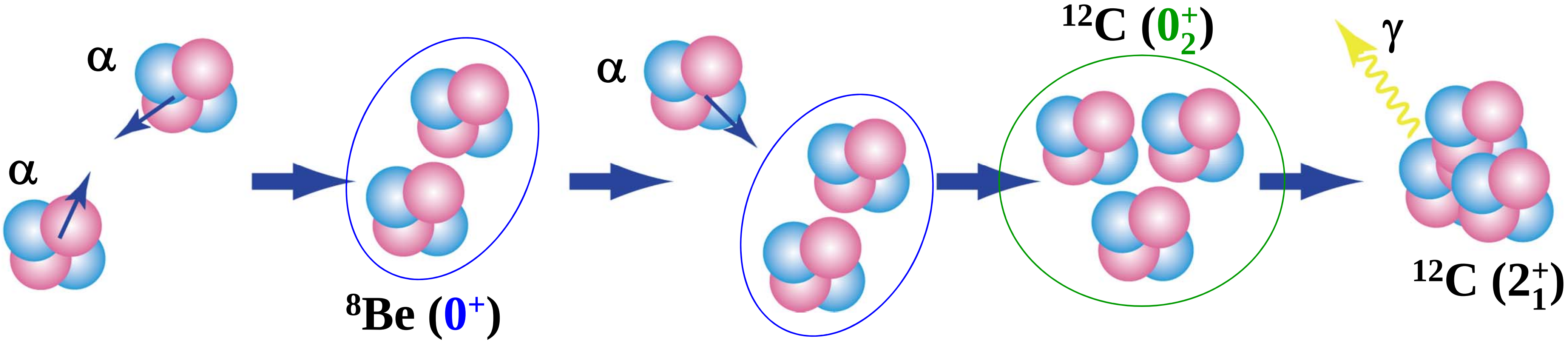
Fred Hoyle

Question:

How is this picture of the triple alpha reaction accurate?

The resonant and nonresonant 3α process

☐ **$T >$ a few 10^8 K: resonant capture**



α

α

^{8}Be (0^+)

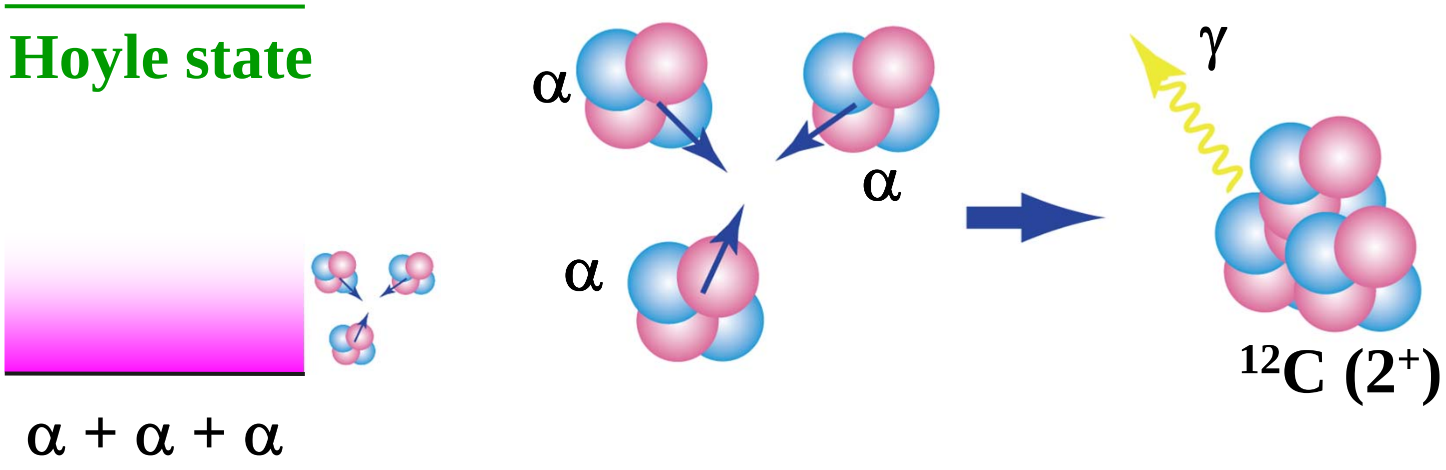
α

^{12}C (0_2^+)

γ

^{12}C (2_1^+)

☐ **$T < 10^8$ K: nonresonant capture (Ternary Fusion Process)**



Hoyle state

α + α + α

α

α

α

γ

^{12}C (2^+)

The nonresonant 3α reaction: now and past

☐ Preceding studies

- ✓ Pioneering study by Nomoto (*Resonance Shift Method; RSM*)

  — *K. Nomoto, Astrophys. J. 253, 798 (1982).*

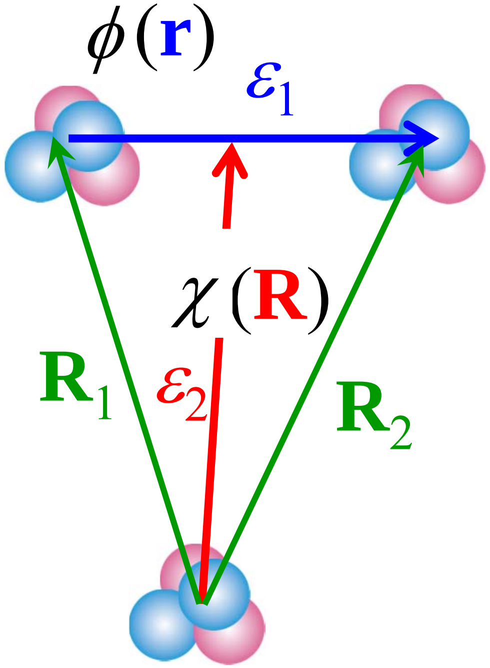
- ✓ Potential model by Langanke

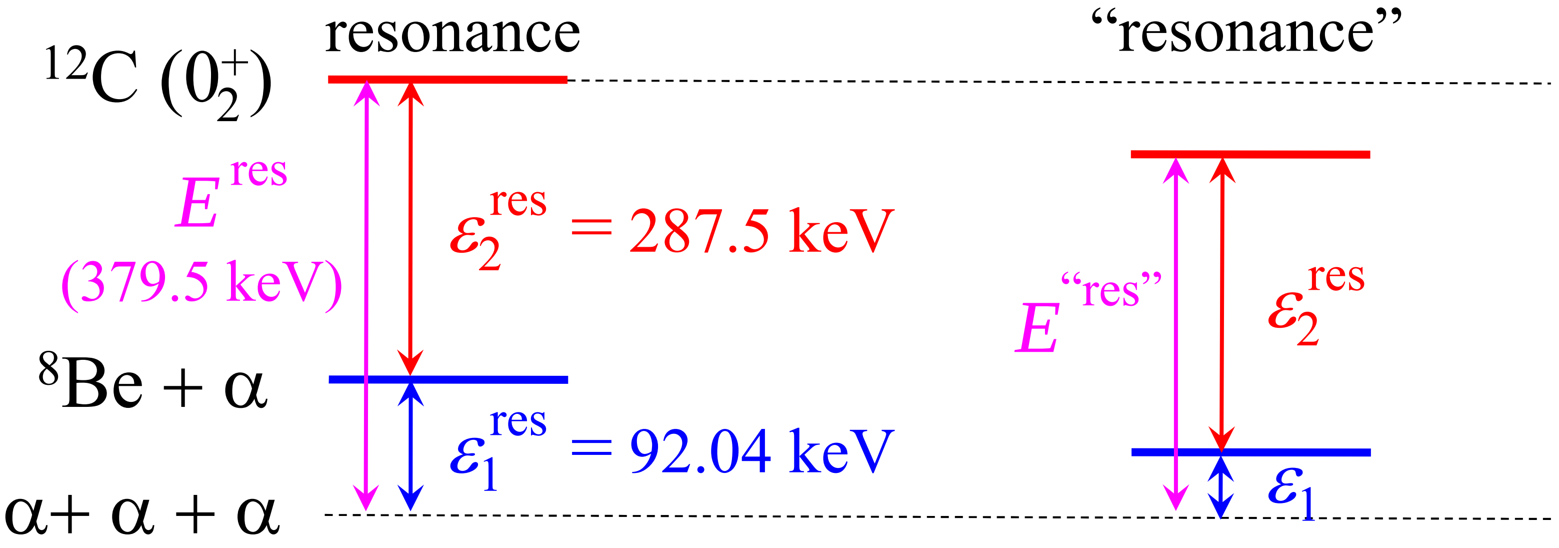
  — *K. Langanke et al., Z. Phys. A 324, 147 (1986).*

⇨ Still based on the resonance picture with an “energy shift” of the Hoyle state as a correction

RSM for “nonresonant” capture

—— *K. Nomoto, Astrophys. J. 253, 798 (1982).*




resonance

“resonance”

$^{12}\mathrm{C}\ (0_2^+)$

E^{res} (379.5 keV)

$\varepsilon_2^{\mathrm{res}} = 287.5\ \mathrm{keV}$

$^{8}\mathrm{Be} + \alpha$

$\varepsilon_1^{\mathrm{res}} = 92.04\ \mathrm{keV}$

$\alpha + \alpha + \alpha$

$E^{\text{“res”}}$

$\varepsilon_2^{\mathrm{res}}$

ε_1

☐ Schroedinger Eq. (1ch)

$$\int \phi^*(\mathbf{r})\left[T_R + V_{\alpha\text{-}\alpha}(R_1) + V_{\alpha\text{-}\alpha}(R_2) + h_{\alpha\alpha} - E\right]\phi(\mathbf{r})\chi(\mathbf{R})\,d\mathbf{r} = 0$$

$$\Downarrow$$

$$\left[T_R + \boxed{V_{\alpha\alpha\text{-}\alpha}(R)} + \cancel{\varepsilon_1} - (\cancel{\varepsilon_1} + \varepsilon_2)\right]\chi(\mathbf{R}) = 0$$

Accurate only if αα-α interaction is independent of the αα states!

The nonresonant 3α reaction: now and past

☐ Preceding studies

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— *K. Nomoto, Astrophys. J. 253, 798 (1982).*

✓ Potential model by Langanke

— *K. Langanke et al., Z. Phys. A 324, 147 (1986).*

⇨ Still based on the resonance picture with an "energy shift" of the Hoyle state as a correction

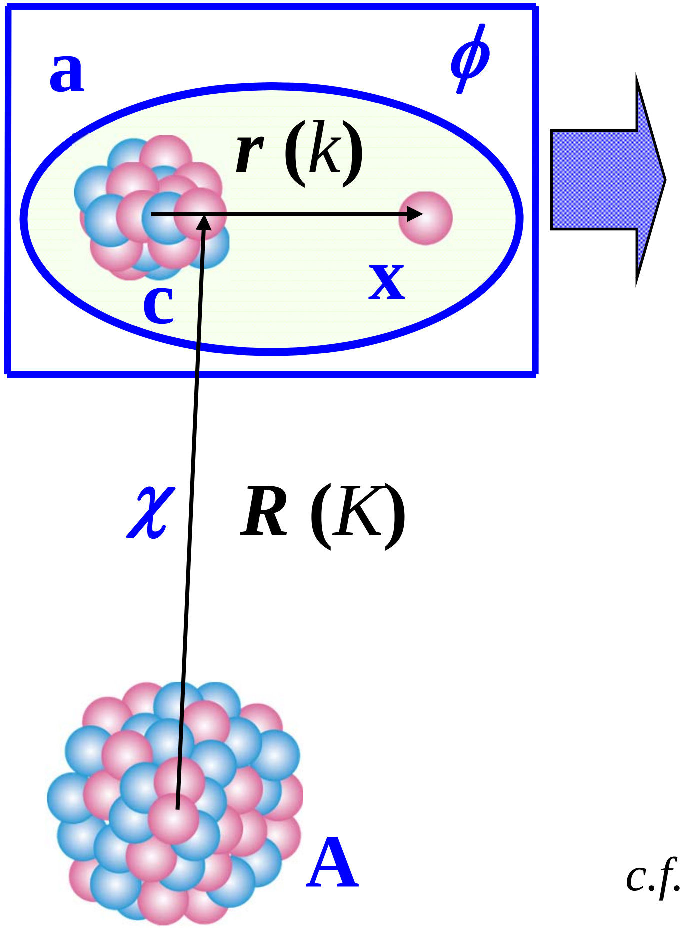
☐ This work

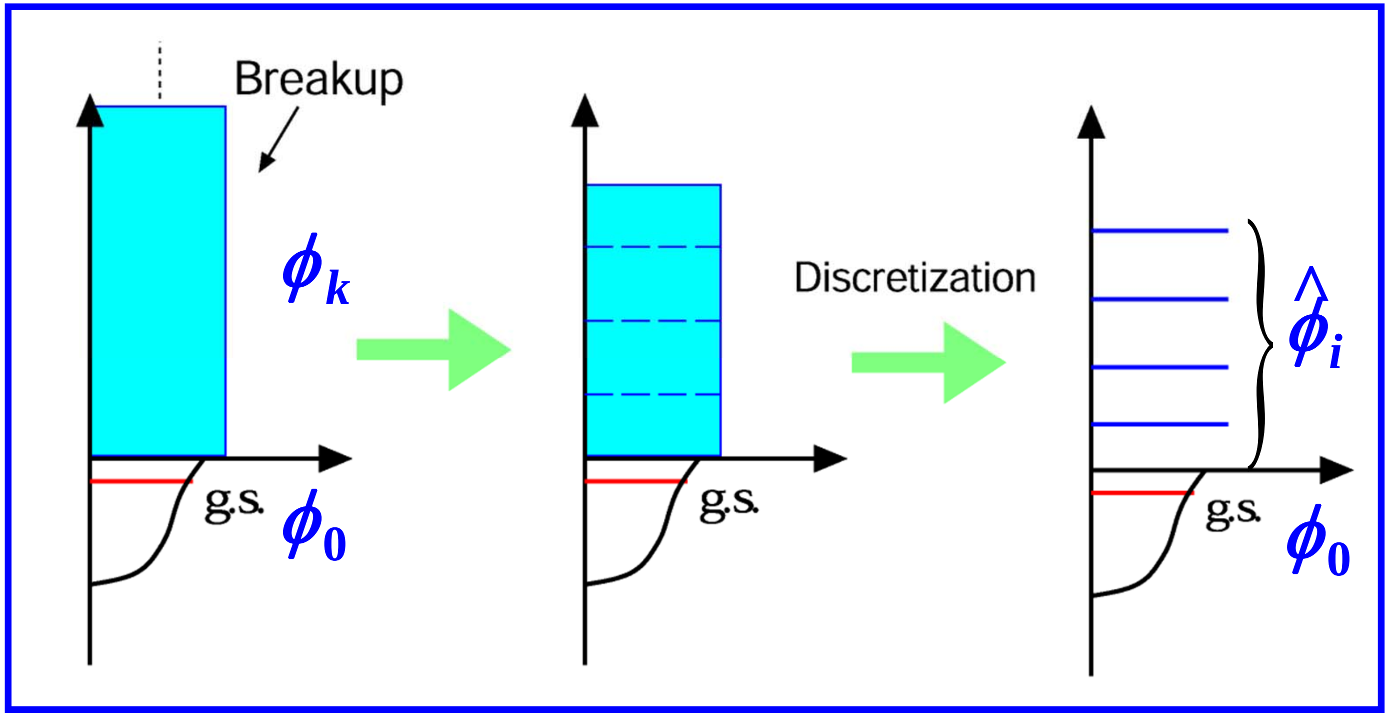
Accurate description of the three-body reaction treating the resonant and nonresonant processes on the same footing.

c.f. M. Kamimura and Y. Fukushima, Proceedings of the INS International Symposium on Nuclear Direct Reaction Mechanism, Shikanoshima, Fukuoka, Japan, 1978, p. 409.
P. Descouvemont and D. Baye, Phys. Rev. C ***36****, 54 (1987).*

The Continuum-Discretized Coupled-Channels method: CDCC (conventional CDCC)

—— *M. Kamimura, Yahiro, Iseri, Sakuragi, Kameyama and Kawai, PTP Suppl.* ***89****, 1 (1986);*
N. Austern, Iseri, Kamimura, Kawai, Rawitscher and Yahiro, Phys. Rep. ***154*** *(1987) 126.*

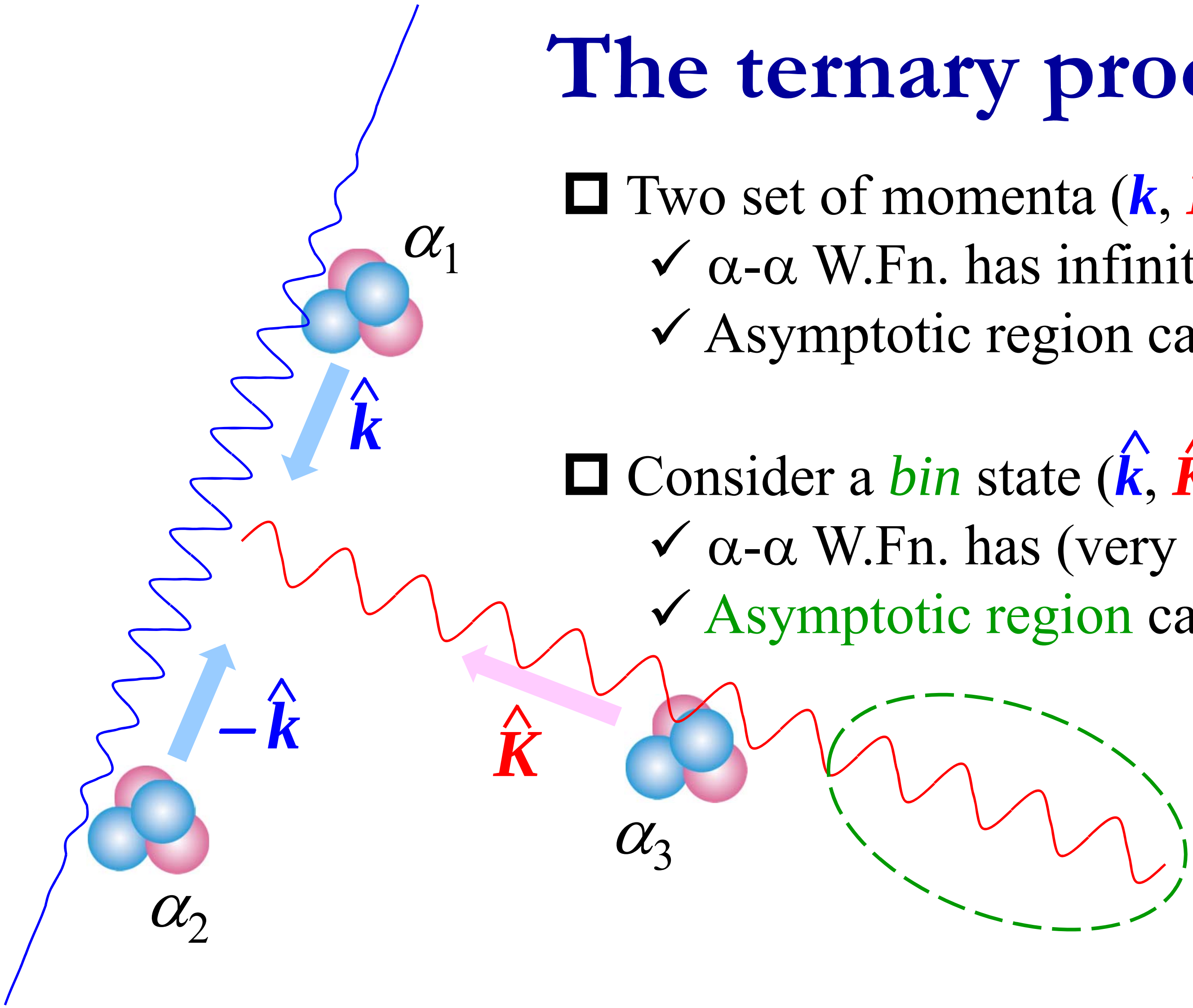




$$\psi = \phi_0\chi_0 + \int_0^\infty \phi_k\chi_k dk \Rightarrow \psi^{\mathrm{CDCC}} = \sum_i^{i_{\max}} \hat{\phi}_i\hat{\chi}_i$$

c.f. N. Austern, M. Yahiro, and M. Kawai, Phys. Rev. Lett. ***63****, 2649 (1989);*
*N. Austern, M. Kawai, and M. Yahiro, Phys. Rev. C****53****, 314 (1996).*

The ternary process

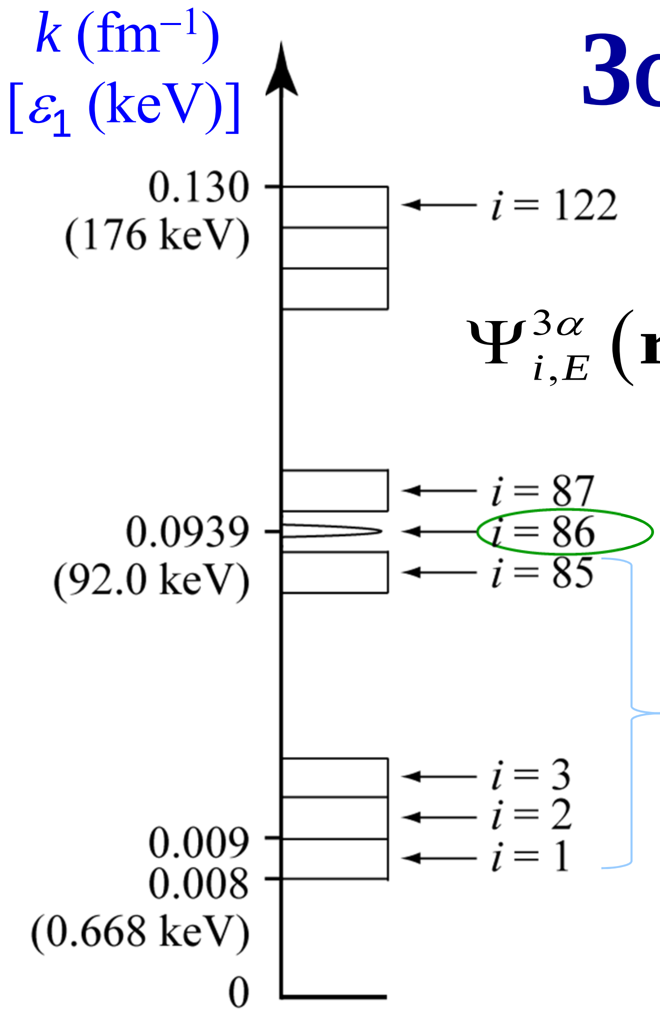


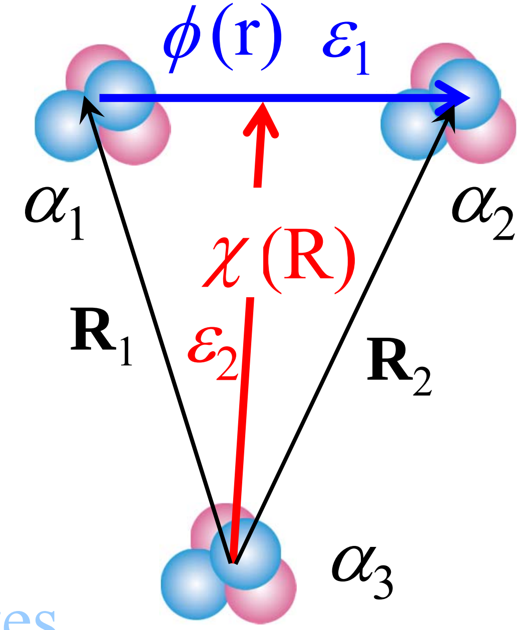
- ☐ Two set of momenta ($\boldsymbol{k}$, $\boldsymbol{K}$) define the incident channel.
  - ✓ α-α W.Fn. has infinite range.
  - ✓ Asymptotic region cannot be defined.
- ☐ Consider a *bin* state ($\hat{\boldsymbol{k}}$, $\hat{\boldsymbol{K}}$) as an incident channel.
  - ✓ α-α W.Fn. has (very long but) finite range.
  - ✓ Asymptotic region can be defined.

3α wave function

$$\Psi^{3\alpha}_{i,E}(\mathbf{r},\mathbf{R}) = \sum_j \phi_{j,i}(\mathbf{r})\,\chi_{j,i,E}(\mathbf{R})$$

α_1-α_2 states: ϕ





resonance

low-energy nonresonant states

play essential roles, but were naively neglected in the previous calculations.*

*M. Kamimura and Y. Fukushima, Proc. INS Int. Symp. on Nuclear Direct Reaction Mechanism, p. 409; P. Descouvemont and D. Baye, PR C**36**, 54.*

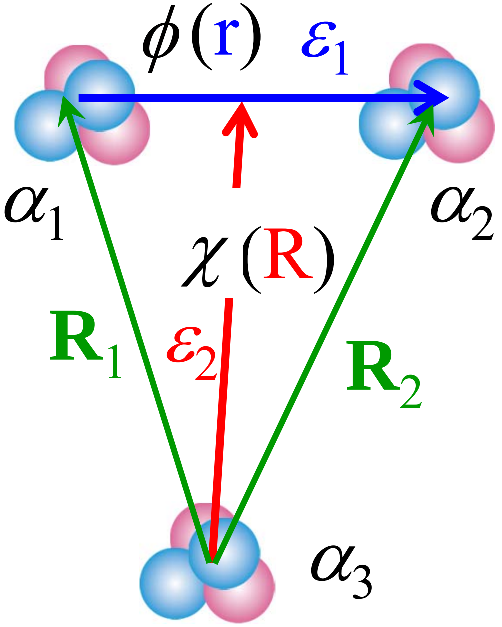
(α_1-α_2)-α_3 states: χ

$$\left[T_R + V^{i}_{\alpha 1\alpha 2\text{-}\alpha 3}(R) + \varepsilon_2\right]\chi_{i,E}(\mathbf{R}) = 0,$$

$$V^{i}_{\alpha 1\alpha 2\text{-}\alpha 3}(R) = \left\langle \phi_i(\mathbf{r}) \middle| V^{\rm N+C}_{\alpha 1\alpha 3}(R_1) + V^{\rm N+C}_{\alpha 2\alpha 3}(R_2) \middle| \phi_i(\mathbf{r}) \right\rangle_{\mathbf{r}}.$$

Constraints on $V_{\alpha\alpha}^{\mathrm{N}}$

$V_{\alpha\alpha}^{\mathrm{N}}$: 2-range Gaussian (with repulsive part simulating the Orthogonal Condition Model; OCM)



1. ^{8}Be resonance properties

$\varepsilon_{1\mathrm{res}} =$ 92.0 keV, $\Gamma = 4.8$ eV

exp. 92.04+/–0.05 5.57+/–0.25

2. Hoyle resonance properties (for *i* = 86)

$\varepsilon_{2\mathrm{res}} = 287.5$ keV, $\Gamma = 4.0$ eV

exp. 287.5 8.5+/–1.0

Achieved by reducing $V_{\alpha\alpha}^{\mathrm{N}}$ by only 1.5% in

$$V_{\alpha 1\alpha 2\text{-}\alpha 3}^{ij}(R) = \left\langle \phi_i(\mathbf{r}) \middle| V_{\alpha 1\alpha 3}^{\mathrm{N+C}}(R_1) + V_{\alpha 2\alpha 3}^{\mathrm{N+C}}(R_2) \middle| \phi_j(\mathbf{r}) \right\rangle_{\mathbf{r}} .$$

Reaction rate of the 3α reaction

W.Fn obtained by Gaussian Expansion Method (GEM) with rearrangement

- **E2 transition from 3α scattering state**

$$(\sigma v)_{\hat{k}_{i_0},E} = \frac{2(2\pi)^7}{75\hbar}\left(\frac{\hbar\omega}{\hbar c}\right)^5 \sum_M \left|\left\langle \Psi_M^{2^+} \middle| O_M^{\mathrm{E2}} \middle| \Psi_{i,E}^{3\alpha} \right\rangle\right|^2$$

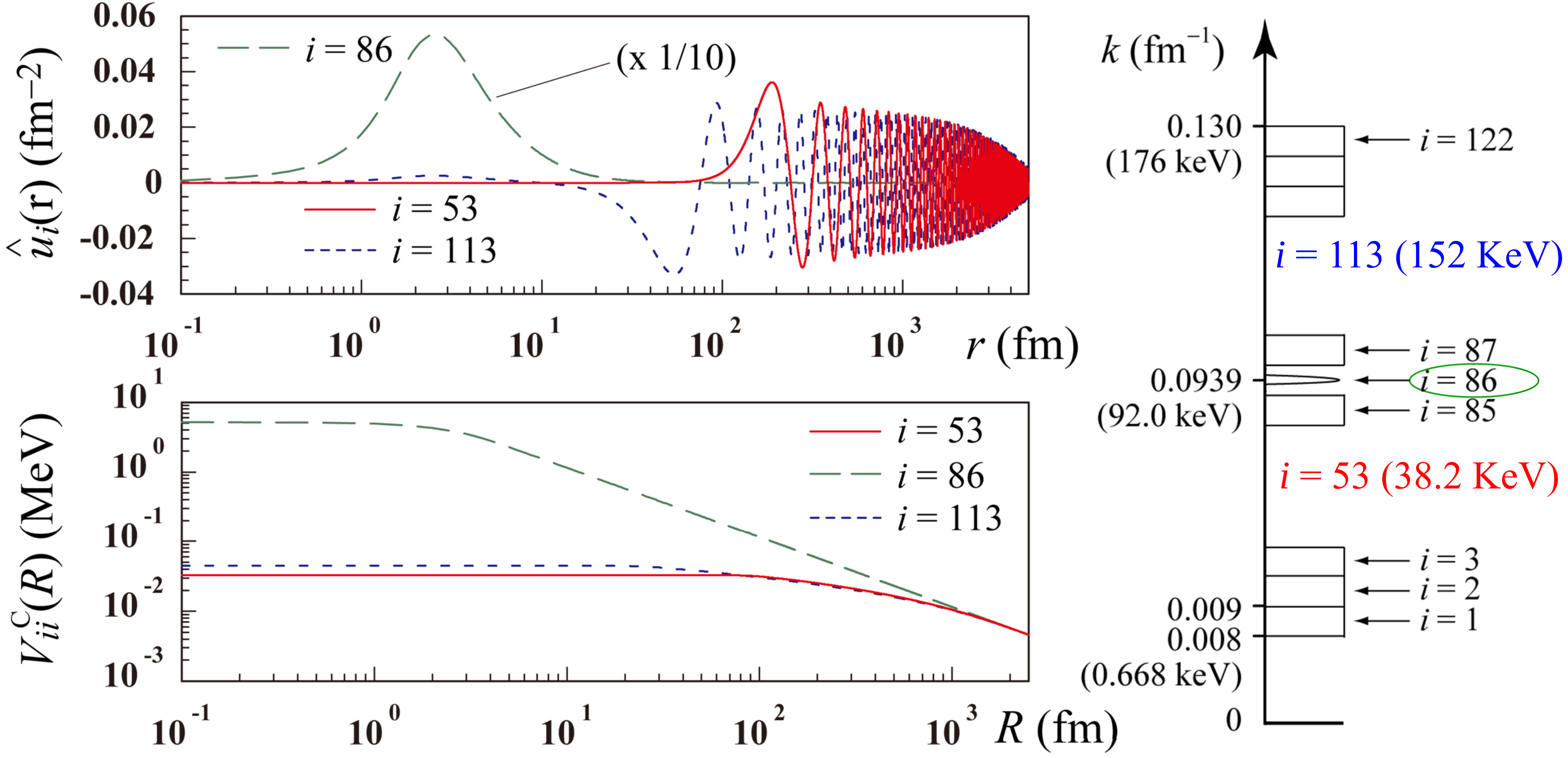
- **Reaction rate**

$$\langle\alpha\alpha\alpha\rangle(T) = 3N_{\mathrm{A}}^2 \frac{4}{\pi (k_{\mathrm{B}}T)^3} \int \left\{ \sum_{i_0=1}^{122} w_{i_0} (\sigma v)_{\hat{k}_{i_0},E} \right\} \exp\left(-\frac{E}{k_{\mathrm{B}}T}\right) dE$$

$$w_{i_0} = \frac{2\hat{\epsilon}_{12,i_0}}{\hat{k}_{i_0}} \sqrt{\hat{\epsilon}_{12,i_0}(E - \hat{\epsilon}_{12,i_0})}$$

- **Correction with effective charge δe to reproduce Γ_γ**
  - ✓ We include $\delta e = 0.77\ e$ so that the B(E2) value obtained by the normalized 0_2^+ W.Fn. and the 2_1^+ W.Fn. reproduces the exp. value of 13.4 e^2 fm^4.

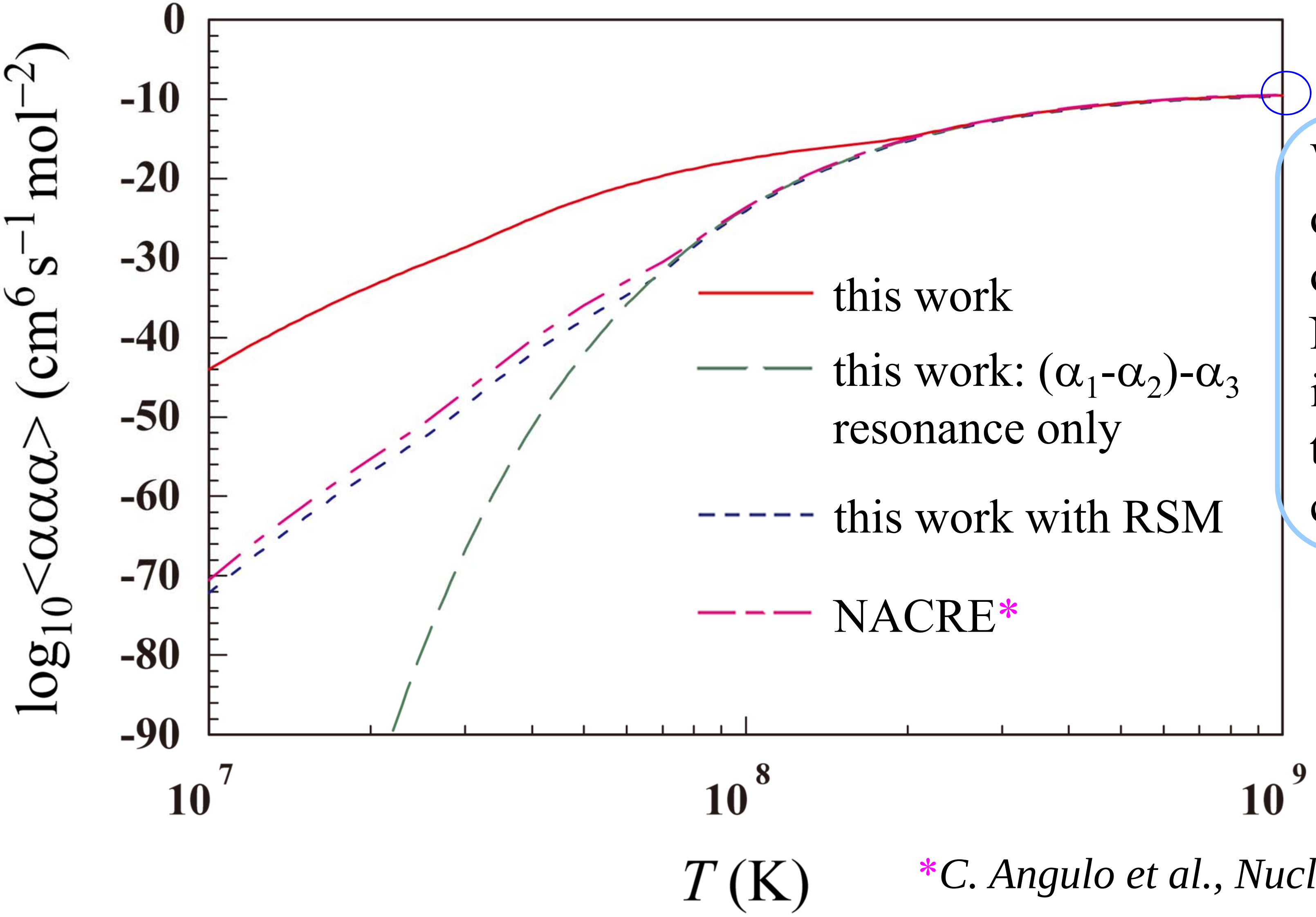
α_1-α_2 W.Fn. and (α_1-α_2)-α_3 Coulomb pot.



Resonant and nonresonant Coulomb potentials are completely different.
The RSM neglects this difference and is a very crude approximation.

The reaction rate

—— *K.O., M. Kan, and M. Kamimura, Prog. Theor. Phys.* ***122****,*
1055 (2009); arXiv:0905.0007 [astro-ph.SR].

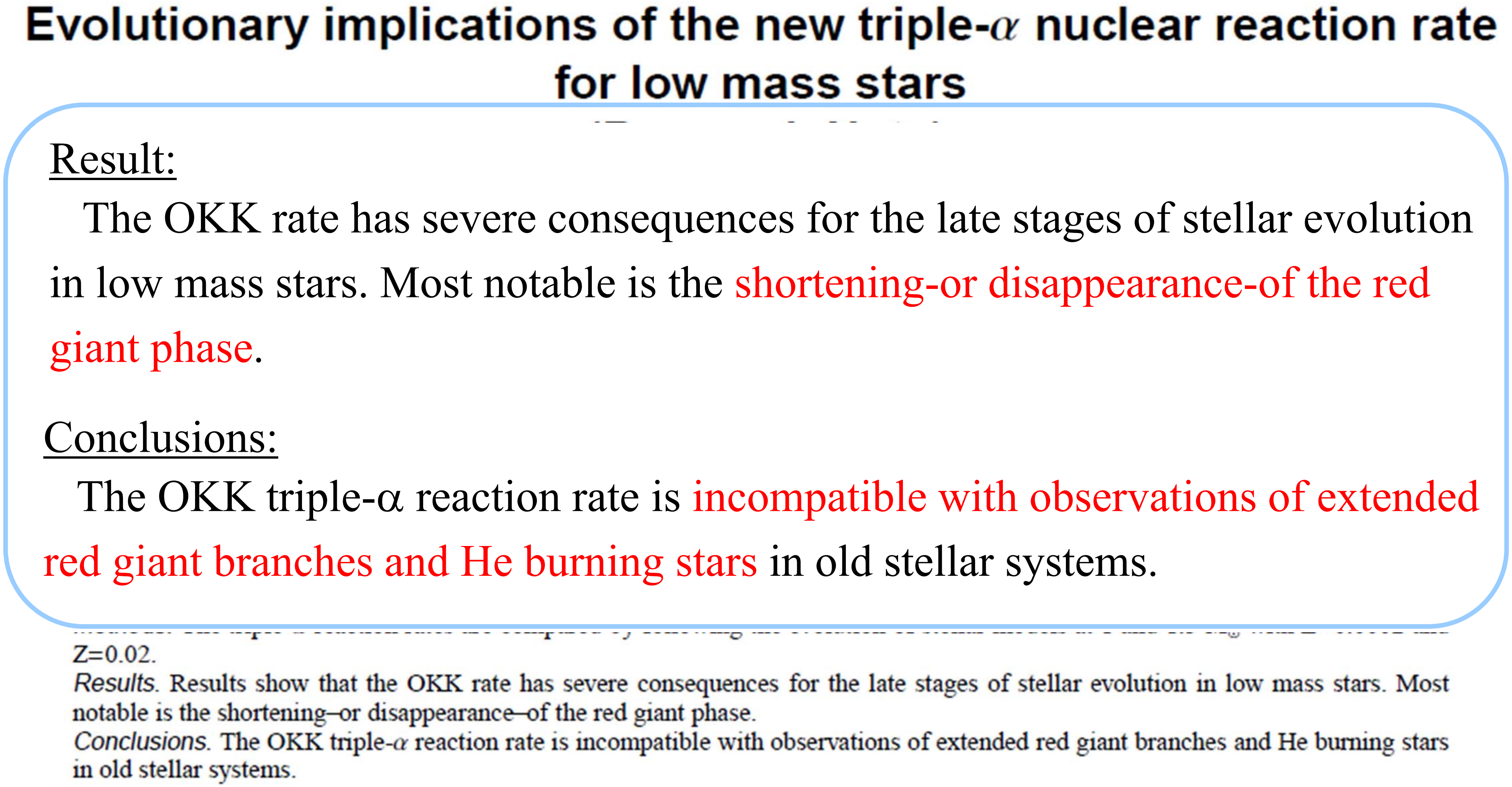


We have normalized our results to the rate of NACRE at 10^9 K. Normalization factor is 1.5 that indicates the uncertainty of our calculation.

C. Angulo et al., Nucl. Phys.* ***A656**, 3 (1999).*

Implication of the new reaction rate

—— *A. Dotter and B. Paxton, Astron. Astrophys. 520 (2010) A41.*



Evolutionary implications of the new triple-α nuclear reaction rate for low mass stars

<u>Result:</u>

The OKK rate has severe consequences for the late stages of stellar evolution in low mass stars. Most notable is the shortening-or disappearance-of the red giant phase.

<u>Conclusions:</u>

The OKK triple-α reaction rate is incompatible with observations of extended red giant branches and He burning stars in old stellar systems.

Z=0.02.

Results. Results show that the OKK rate has severe consequences for the late stages of stellar evolution in low mass stars. Most notable is the shortening–or disappearance–of the red giant phase.

Conclusions. The OKK triple-α reaction rate is incompatible with observations of extended red giant branches and He burning stars in old stellar systems.

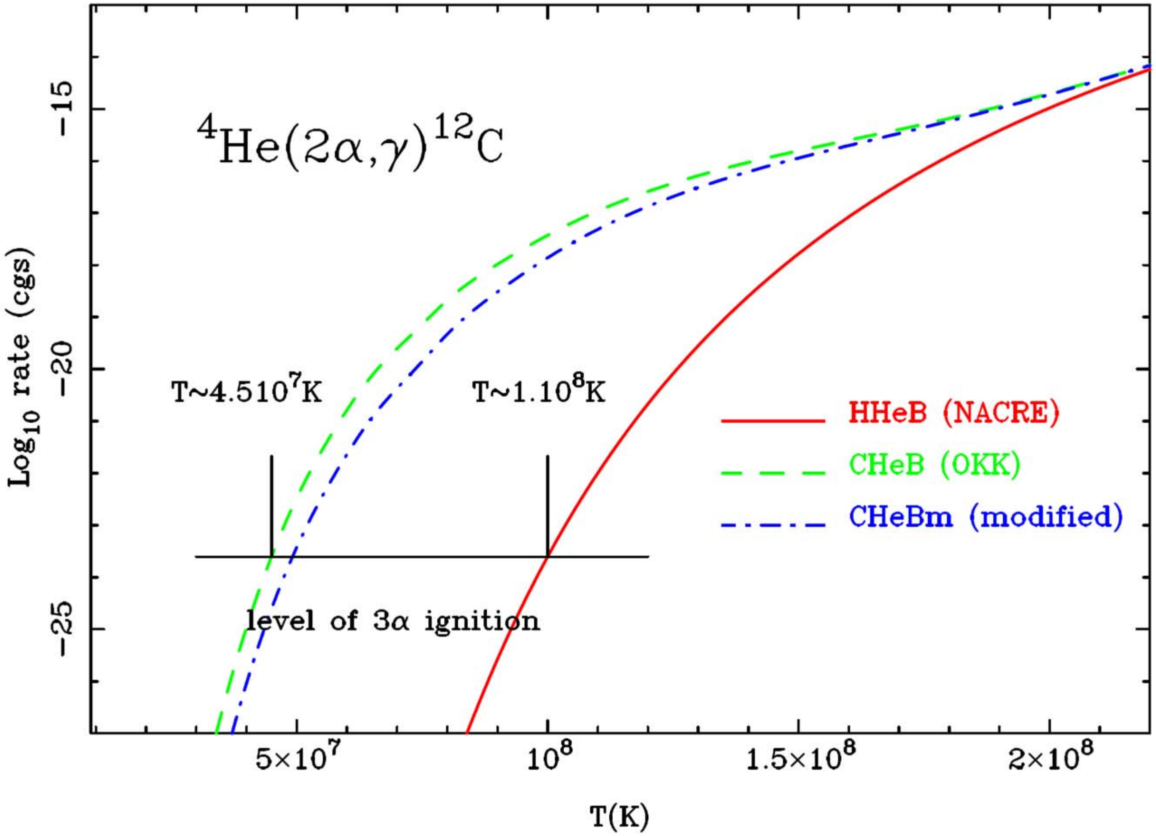
Implications of a new triple-α nuclear reaction rate.

Consequences for Cepheids

P.Morel[1], J.Provost[1], B.Pichon[1], Y.Lebreton[2,3], and F.Thévenin[1]

Conclusions. This preliminary work indicates that the new rate may solve some of the long-lasting unresolved theoretical problems of Cepheids. …

Astronomy and Astrophysics 507 (2009) 1617.



Summary

☐ **The triple-α reaction rate is reevaluated.**

- ✓ The ternary fusion process (TFP) is formulated by CDCC.
- ✓ The resonant and nonresonant processes are described on the same footing.
- ✓ The α_1-α_2 nonresonant states below the resonance are essentially important.
- ✓ The (α_1-α_2)-α_3 Coulomb barrier in the nonresonant capture process is much lower than that in the resonant process.
- ✓ We obtain a markedly larger reaction rate than NACRE below 10^8 K.

☐ **The previous method for the triple-α reaction is examined**

- ✓ The Resonance Shift Method (used in many studies including NACRE) is shown *a very crude approximation* to the present three-body calculation.

Future Perspective

- **Rearrangement Channels**
  - ✓ Differential method will be more appropriate.
  - ✓ Inclusion of closed channels (compact W. Fn.) in the framework.
  - ✓ Important also for nonperturbative transfer calculation.

- **Understanding of the TFP nucleosynthesis**
  - ✓ $\alpha(\alpha n,\gamma)^{9}\mathrm{Be}$, $n(p\alpha,{}^{6}\mathrm{Li})$ etc.
  - ✓ 2p processes.
  - ✓ Experimental verification of TFP