Relativistic EOS for Supernova Simulations

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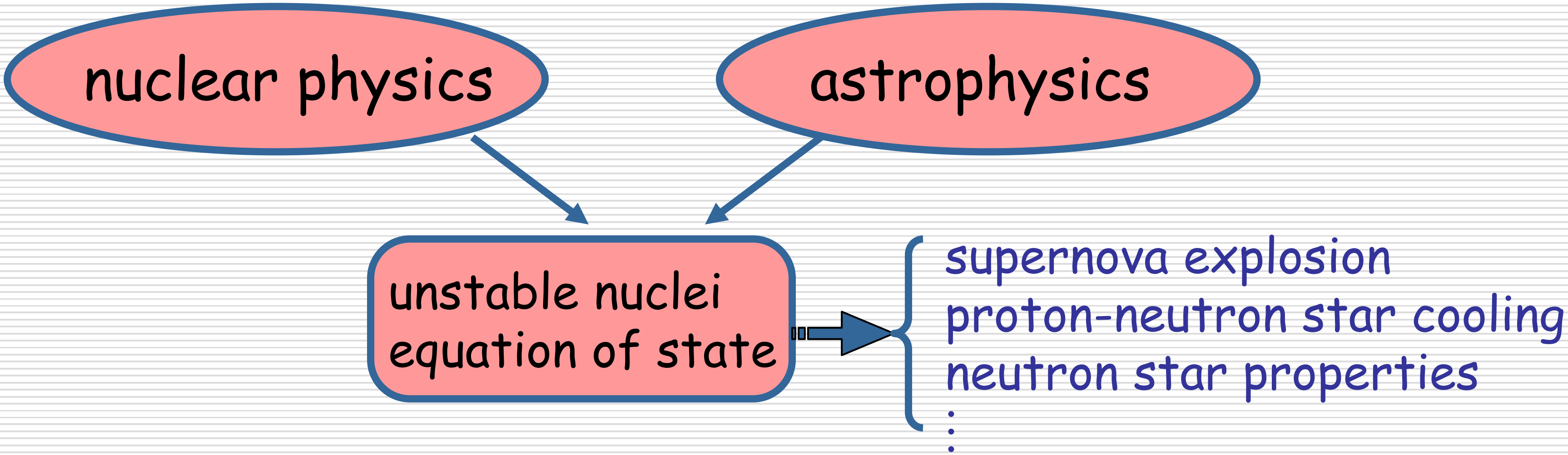


nuclear physics

astrophysics

unstable nuclei
equation of state

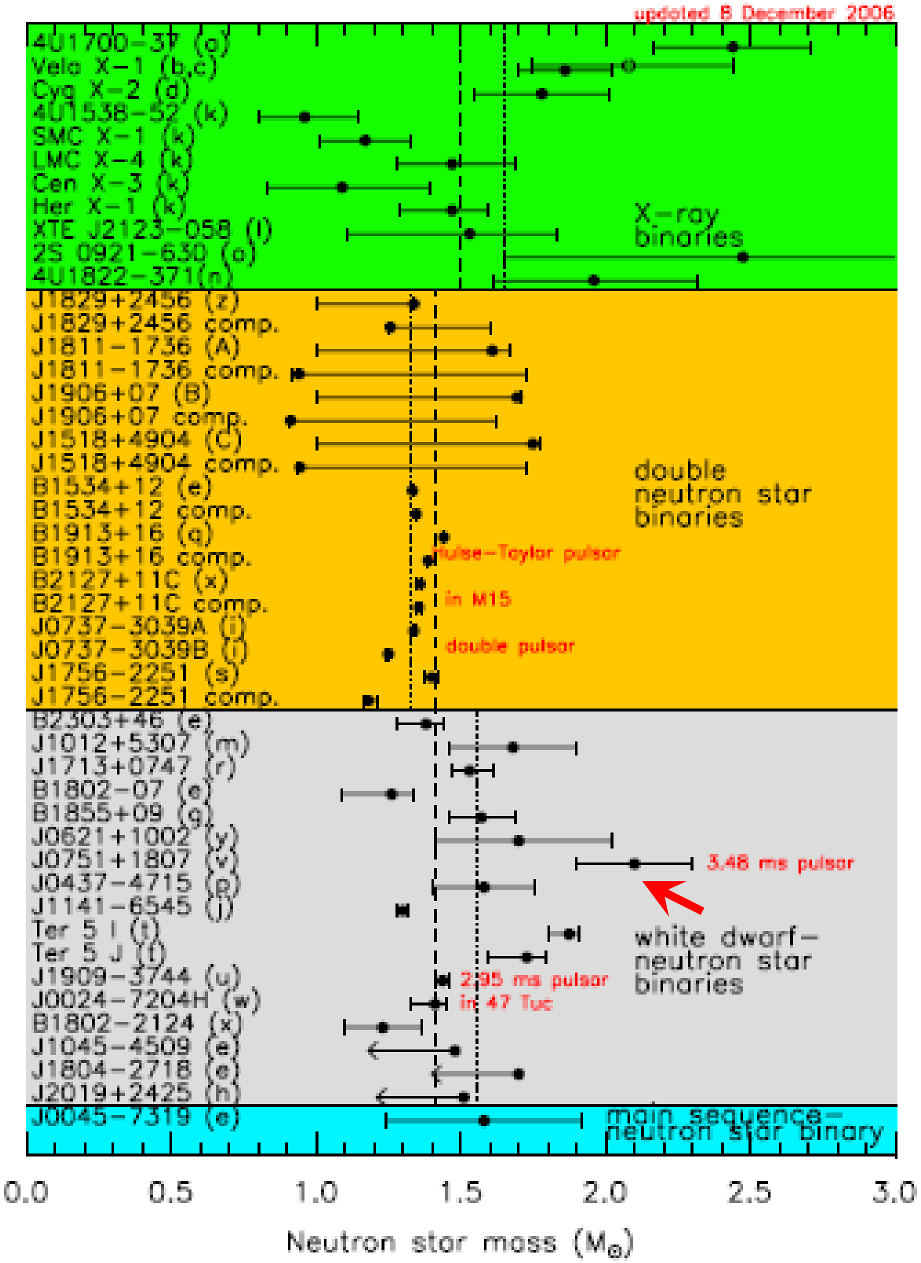
supernova explosion
proton-neutron star cooling
neutron star properties
⋮

neutron star matter: charge neutrality; β equilibrium; T~0

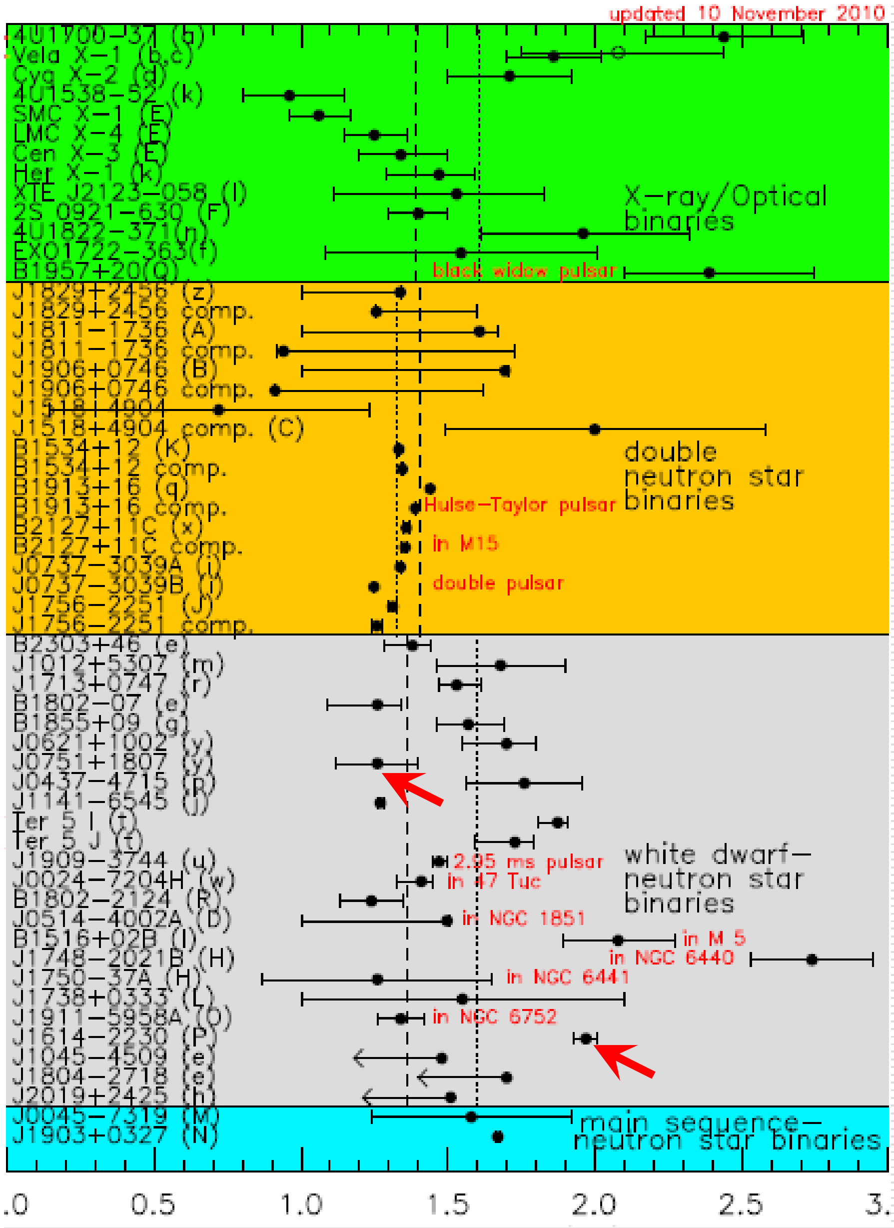
supernova matter: charge neutrality; fixed fractions; T≠0

What is the situation about EOS ?

2006.12.8



2010.11.10



EOS for supernova simulations

★ nonrelativistic EOS (compressible liquid-drop)

Lattimer, Swesty, Nucl. Phys. A 535 (1991) 331

★ relativistic EOS (RMF + Thomas-Fermi)

Shen, Toki, Oyamatsu, Sumiyoshi, Nucl. Phys. A 637 (1998) 435

★ relativistic EOS (RMF)

G. Shen, Horowitz, Teige, Phys. Rev. C 82 (2010) 015806

★ EOS (nuclear statistical equilibrium)

Hempel, Schaffner-Bielich, Nucl. Phys. A 837 (2010) 210

EOS for supernovae

wide range

temperature (T):

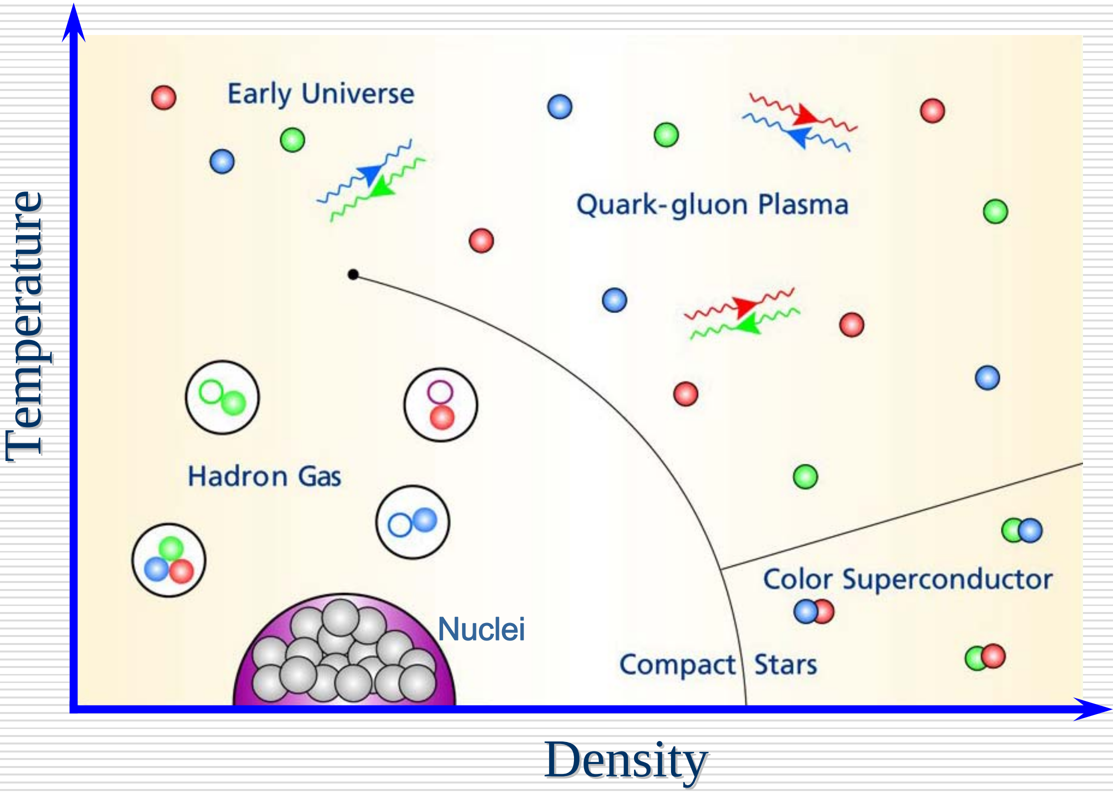
0 ~ 100 MeV

proton fraction (Yp):

0 ~ 0.6

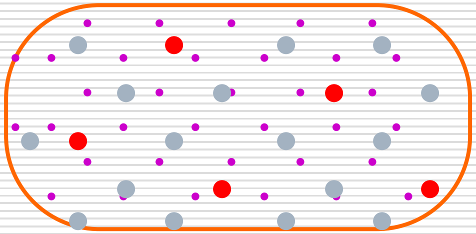
density (ρ_B):

$10^5 \sim 10^{16}$ g/cm^3



Models used for EOS

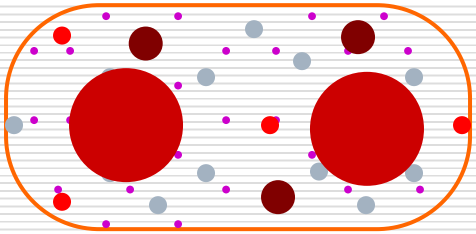
uniform matter
at high density



● proton
● neutron
• electron

RMF (relativistic Mean Field)

non-uniform matter
at low density



● nuclei
● alpha
● proton
● neutron
• electron

RMF + Thomas-Fermi approximation

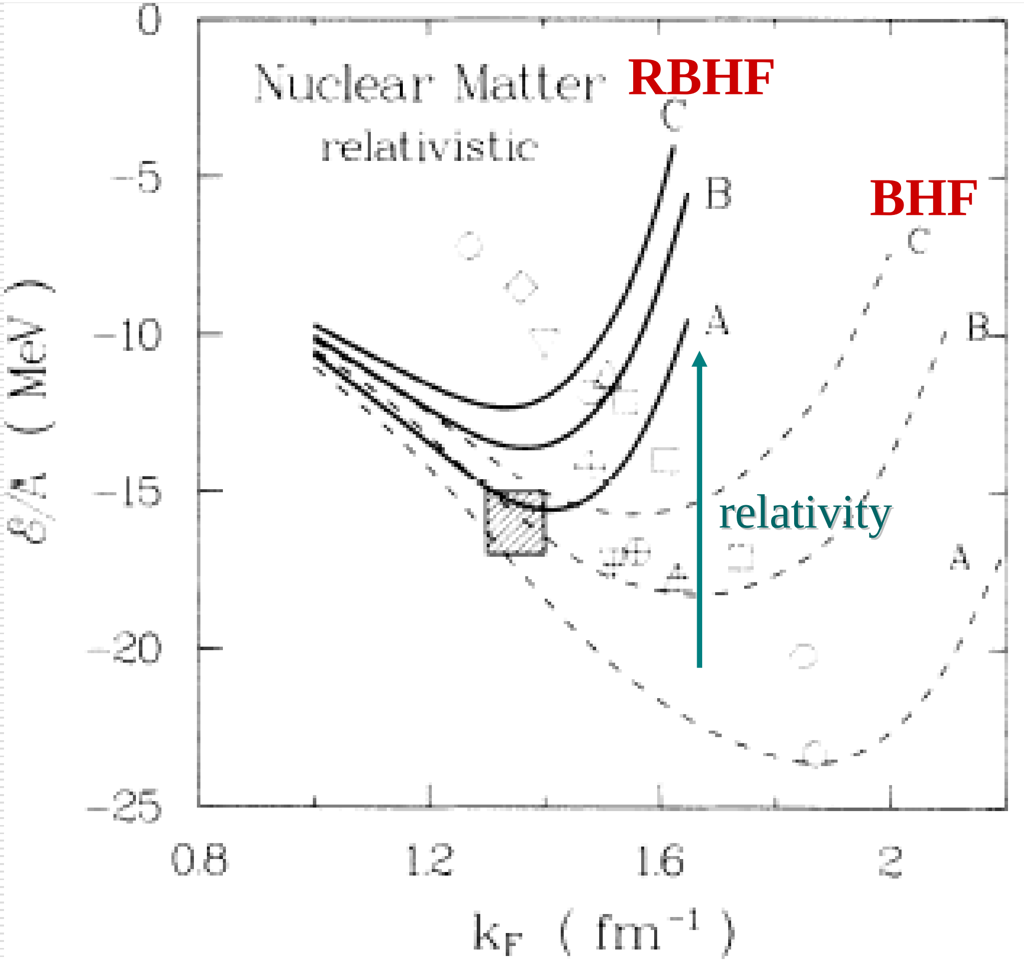
Why prefer the RMF theory ?

nuclear many-body methods

nonrelativistic	relativistic
Shell Model	Relativistic Mean-Field (**RMF**)
Skyrme-Hartree-Fock (**SHF**)	Relativistic Hartree-Fock (**RHF**)
Brueckner-Hartree-Fock (**BHF**)	Relativistic Brueckner-Hartree-Fock (**RBHF**)
…	…

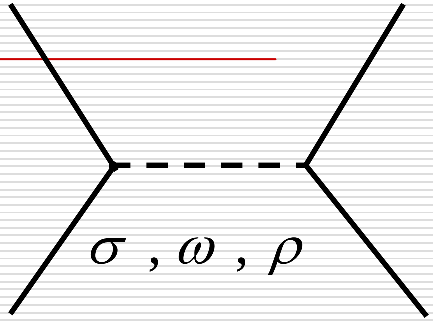
Relativity is important !

- natural explanation of spin-orbit force
- natural explanation of three-body force
- good saturation of nuclear matter



Brockmann, Machleidt, Phys. Rev. C 42 (1990) 1965

What is the RMF theory ?



Relativistic Mean Field Theory (RMF)

mean-field approximation: *meson field operators are replaced by their expectation values*

no-sea approximation: *contributions from the negative-energy Dirac sea are ignored*

Applications

	flavor SU(2)	flavor SU(3)
infinite matter:	*nuclear matter*	*strange hadronic matter*
finite system:	*nuclei*	*hypernuclei*

Comparison with nuclear data

2157 nuclei

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(M^{i}_{\text{theo}} - M^{i}_{\text{expt}}\right)^{2}}{n}}$$

$$= 2.1$$

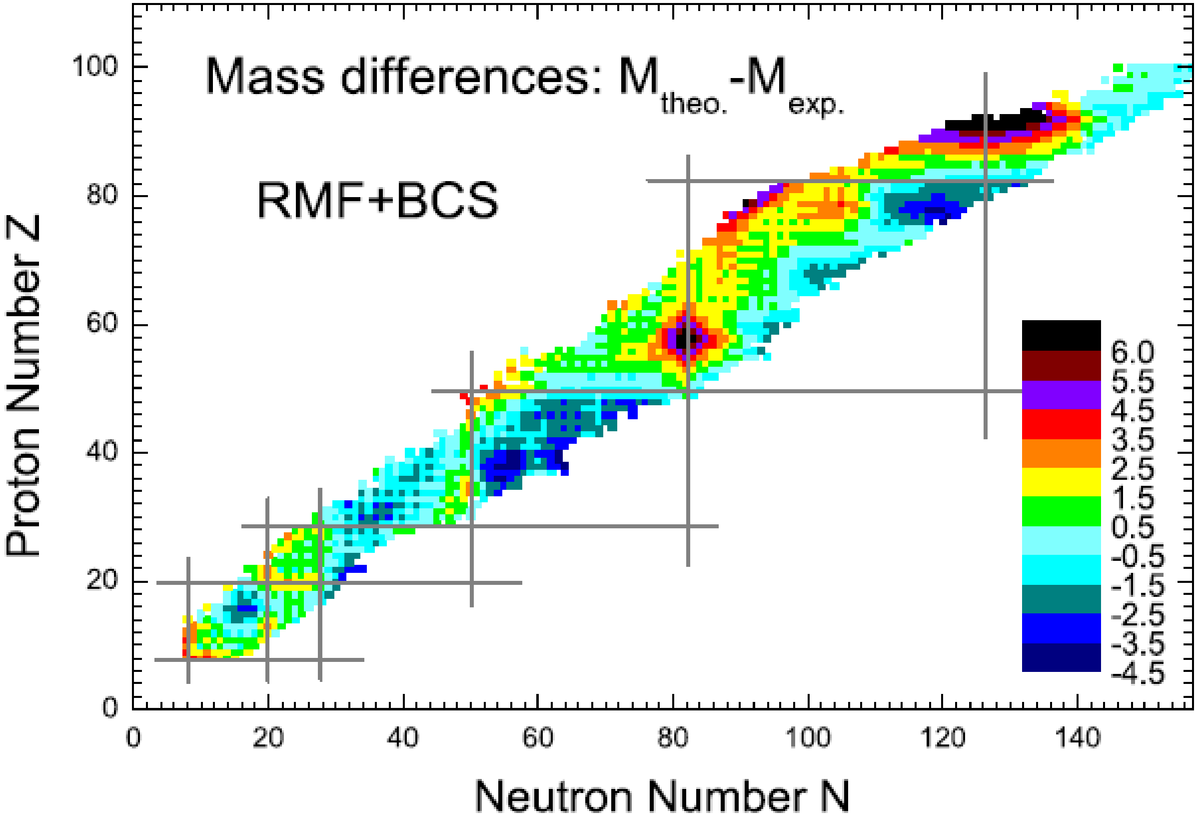


Fig. 2. Mass differences between the predictions of the present work and the experimental data for 2157 nuclei whose measured uncertainties for the masses are less than 0.2 MeV.[34)]

L.S.Geng, H.Toki, J.Meng, Prog. Theor. Phys. 113 (2005) 785

Relativistic Mean Field Theory

Lagrangian

$$
\begin{aligned}
L = \bar{\psi}[i\gamma_{\mu}\partial^{\mu} - M - g_{\sigma}\sigma - g_{\omega}\gamma_{\mu}\omega^{\mu} - g_{\rho}\gamma_{\mu}\tau_{a}\rho^{a\mu}]\psi \\
+ \frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma - \frac{1}{2}m_{\sigma}^{2}\sigma^{2} - \underline{\frac{1}{3}g_{2}\sigma^{3}} - \underline{\frac{1}{4}g_{3}\sigma^{4}} \\
- \frac{1}{4}W_{\mu\upsilon}W^{\mu\upsilon} + \frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu} + \underline{\frac{1}{4}c_{3}(\omega_{\mu}\omega^{\mu})^{2}} \\
- \frac{1}{4}R_{\mu\upsilon}^{a}R^{a\mu\upsilon} + \frac{1}{2}m_{\rho}^{2}\rho_{\mu}^{a}\rho^{a\mu}
\end{aligned}
$$

TM1 parameter set

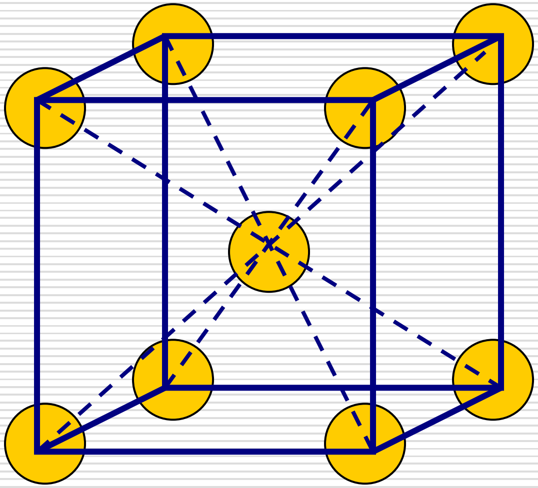
Lagrangian ⇨ Equations ⇨ Mean-Field Approximation

⇩

Calculate everything such as ε, p, S…

Thomas-Fermi approximation

- * body-centered cubic lattice
- * parameterized nucleon distribution
- * RMF input



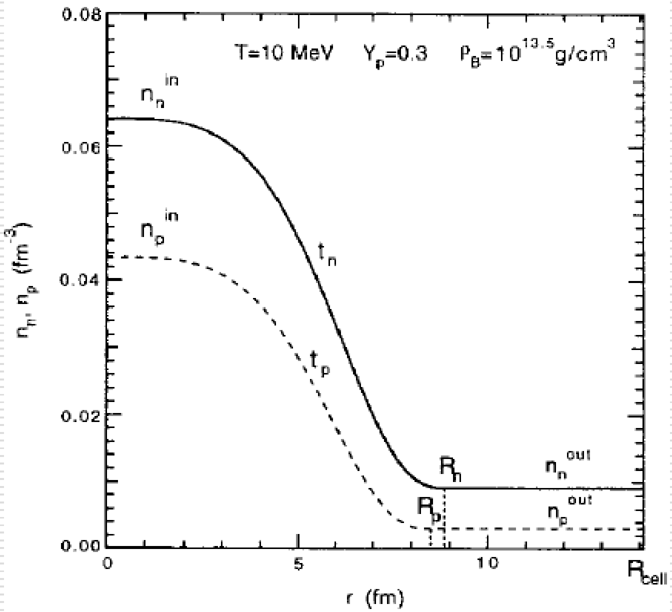
$$E = E_{bulk} + E_{surface} + E_{Coulomb} + E_{Lattice} + E_{electron}$$

assume states $\xrightarrow{\textit{minimize free energy}}$ favorable state

Thomas-Fermi approximation

parameterized nucleon distribution

$$n_i(r) = \begin{cases} \left(n_i^{in} - n_i^{out}\right)\left[1-\left(\frac{r}{R_i}\right)^{t_i}\right]^3 + n_i^{out}, & 0 \le r \le R_i \\ n_i^{out}, & R_i \le r \le R_{cell} \end{cases}$$



H.Shen, H.Toki, K.Oyamatsu, K.Sumiyoshi, Nucl. Phys. A637 (1998) 435

Check the parameterization

Self-consistent Thomas-Fermi approximation

Lagrangian

$$L_{RMF} = \bar{\psi}\left[i\gamma_{\mu}\partial^{\mu} - \left(M + g_{\sigma}\sigma\right) - \left(g_{\omega}\omega + g_{\rho}\tau_{3}\rho + e\frac{\tau_{3}+1}{2}A \right)\gamma^{0} \right]\psi$$

$$-\frac{1}{2}(\nabla\sigma)^{2} - \frac{1}{2}m_{\sigma}^{2}\sigma^{2} - \frac{1}{3}g_{2}\sigma^{3} - \frac{1}{4}g_{3}\sigma^{4}$$

$$+\frac{1}{2}(\nabla\omega)^{2} + \frac{1}{2}m_{\omega}^{2}\omega^{2} + \frac{1}{4}c_{3}\omega^{4}$$

$$+\frac{1}{2}(\nabla\rho)^{2} + \frac{1}{2}m_{\rho}^{2}\rho^{2} + \frac{1}{2}(\nabla A)^{2}$$

$$+\sum_{l}\bar{\psi}_{l}\left(i\gamma_{\mu}\partial^{\mu} - m_{l} + eA\gamma^{0}\right)\psi_{l}$$

Equations

$$-\Delta\sigma + m_{\sigma}^{2}\sigma = -g_{\sigma}\rho_{s} - g_{2}\sigma^{2} - g_{3}\sigma^{3},$$

$$-\Delta\omega + m_{\omega}^{2}\omega = g_{\omega}\rho_{v} - c_{3}\omega^{3},$$

$$-\Delta\rho + m_{\rho}^{2}\rho = g_{\rho}\left(\rho_{v}^{p} - \rho_{v}^{n}\right),$$

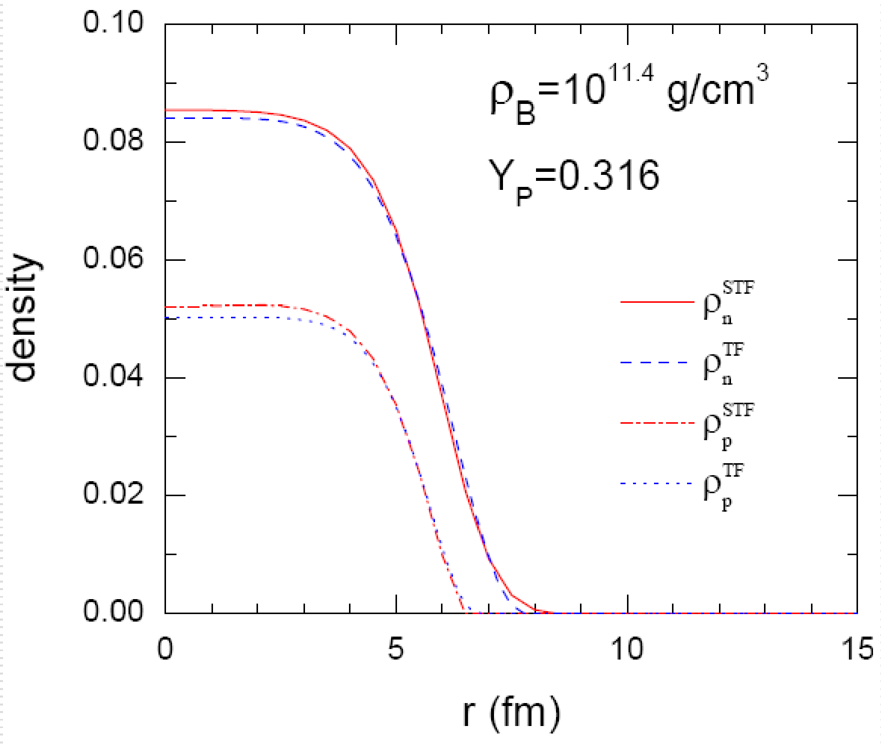
$$-\Delta A = e\left(\rho_{v}^{p} - \rho_{v}^{l}\right).$$

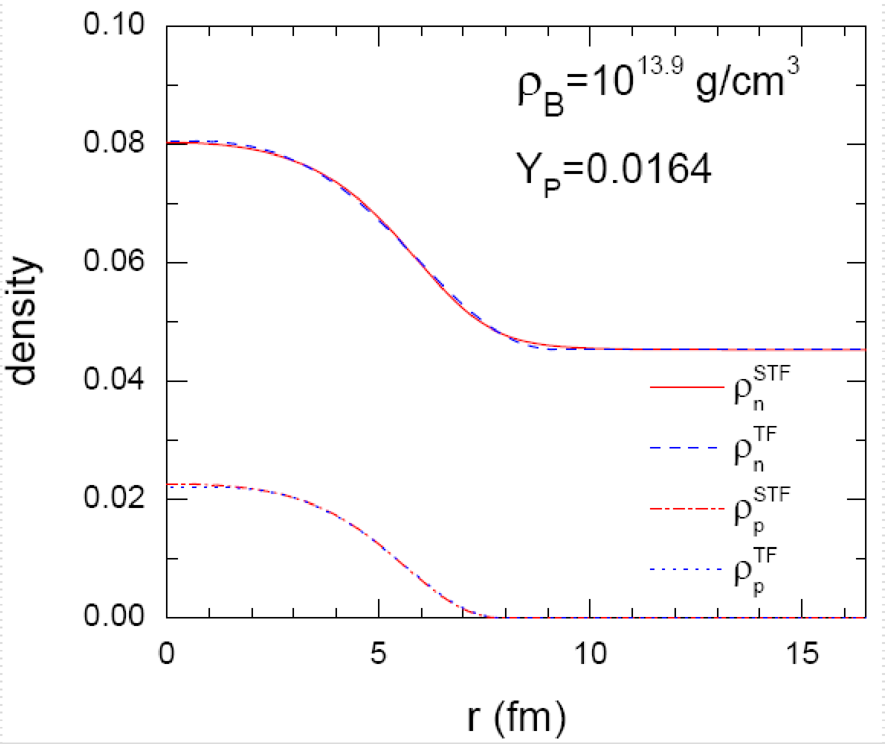
$$\left(k_{F}^{b}\right)^{2} = \left(\mu_{b} - U_{v}^{b}\right)^{2} - M^{*2}$$

$$M^{*} = M + g_{\sigma}\sigma$$

$$U_{v}^{b} = g_{\omega}\omega + g_{\rho}\tau_{3}\rho + e\frac{\tau_{3}+1}{2}A$$

Self-consistent Thomas-Fermi approximation





New version of EOS tables

EOS1 **(1998-version, nucleon)**

Shen, Toki, Oyamatsu, Sumiyoshi, Prog. Theor. Phys. 100 (1998) 1013

EOS2 **(2010-version, nucleon)**

Shen, Toki, Oyamatsu, Sumiyoshi, Astrophys. J. Suppl. (2011) in press

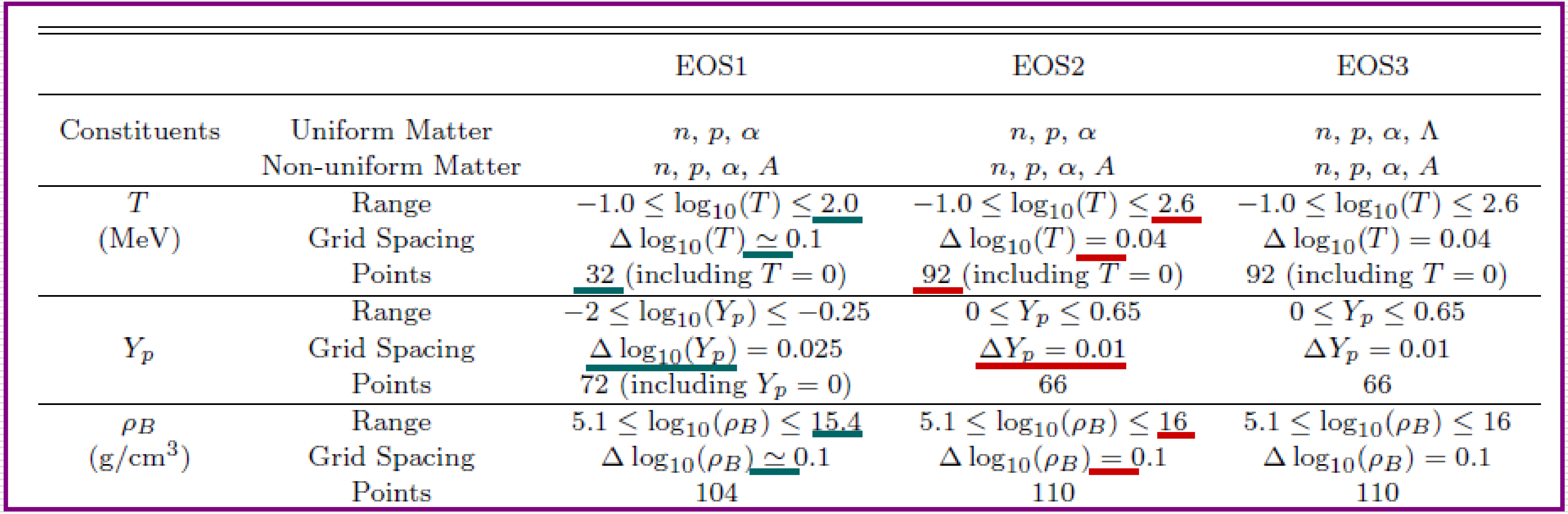
EOS3 **(2010-version, nucleon+Λ)**

Shen, Toki, Oyamatsu, Sumiyoshi, Astrophys. J. Suppl. (2011) in press

http://physics.nankai.edu.cn/grzy/shenhong/EOS/index.html

http://user.numazu-ct.ac.jp/~sumi/eos/index.html

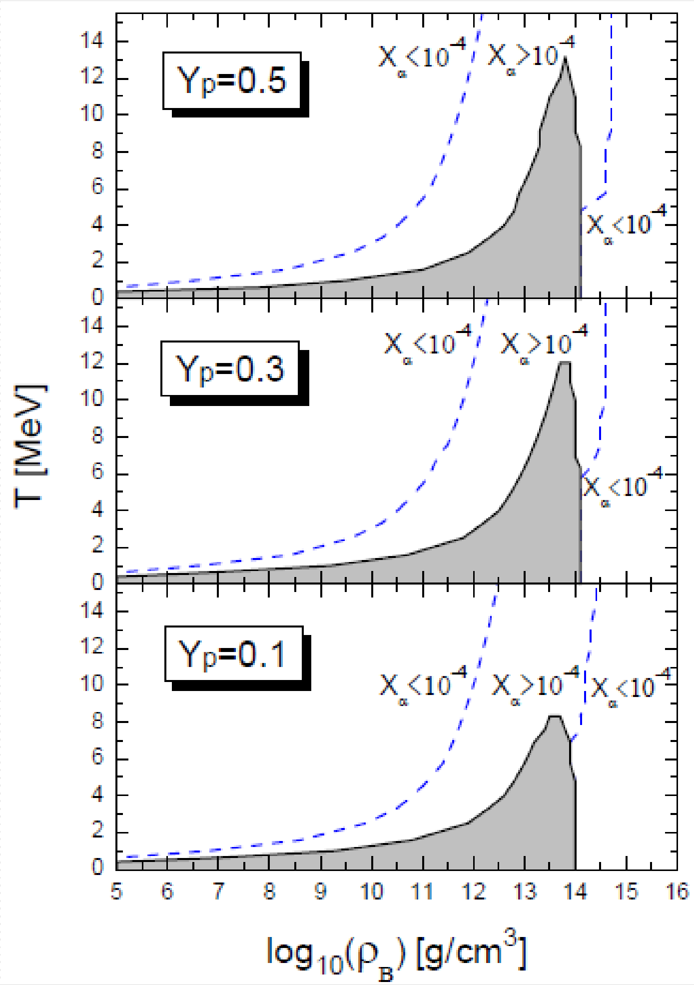
Comparison between EOS tables

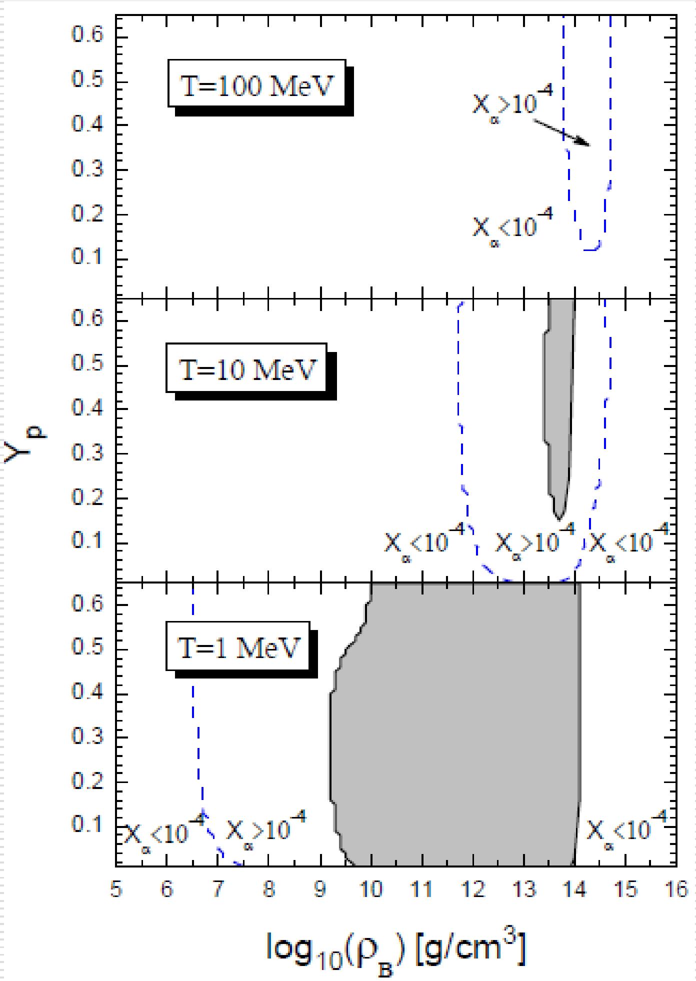


		EOS1	EOS2	EOS3
Constituents	Uniform Matter	$n,\ p,\ \alpha$	$n,\ p,\ \alpha$	$n,\ p,\ \alpha,\ \Lambda$
	Non-uniform Matter	$n,\ p,\ \alpha,\ A$	$n,\ p,\ \alpha,\ A$	$n,\ p,\ \alpha,\ A$
T (MeV)	Range	$-1.0 \leq \log_{10}(T) \leq 2.0$	$-1.0 \leq \log_{10}(T) \leq 2.6$	$-1.0 \leq \log_{10}(T) \leq 2.6$
	Grid Spacing	$\Delta \log_{10}(T) \simeq 0.1$	$\Delta \log_{10}(T) = 0.04$	$\Delta \log_{10}(T) = 0.04$
	Points	32 (including $T = 0$)	92 (including $T = 0$)	92 (including $T = 0$)
Y_p	Range	$-2 \leq \log_{10}(Y_p) \leq -0.25$	$0 \leq Y_p \leq 0.65$	$0 \leq Y_p \leq 0.65$
	Grid Spacing	$\Delta \log_{10}(Y_p) = 0.025$	$\Delta Y_p = 0.01$	$\Delta Y_p = 0.01$
	Points	72 (including $Y_p = 0$)	66	66
ρ_B (g/cm^3)	Range	$5.1 \leq \log_{10}(\rho_B) \leq 15.4$	$5.1 \leq \log_{10}(\rho_B) \leq 16$	$5.1 \leq \log_{10}(\rho_B) \leq 16$
	Grid Spacing	$\Delta \log_{10}(\rho_B) \simeq 0.1$	$\Delta \log_{10}(\rho_B) = 0.1$	$\Delta \log_{10}(\rho_B) = 0.1$
	Points	104	110	110

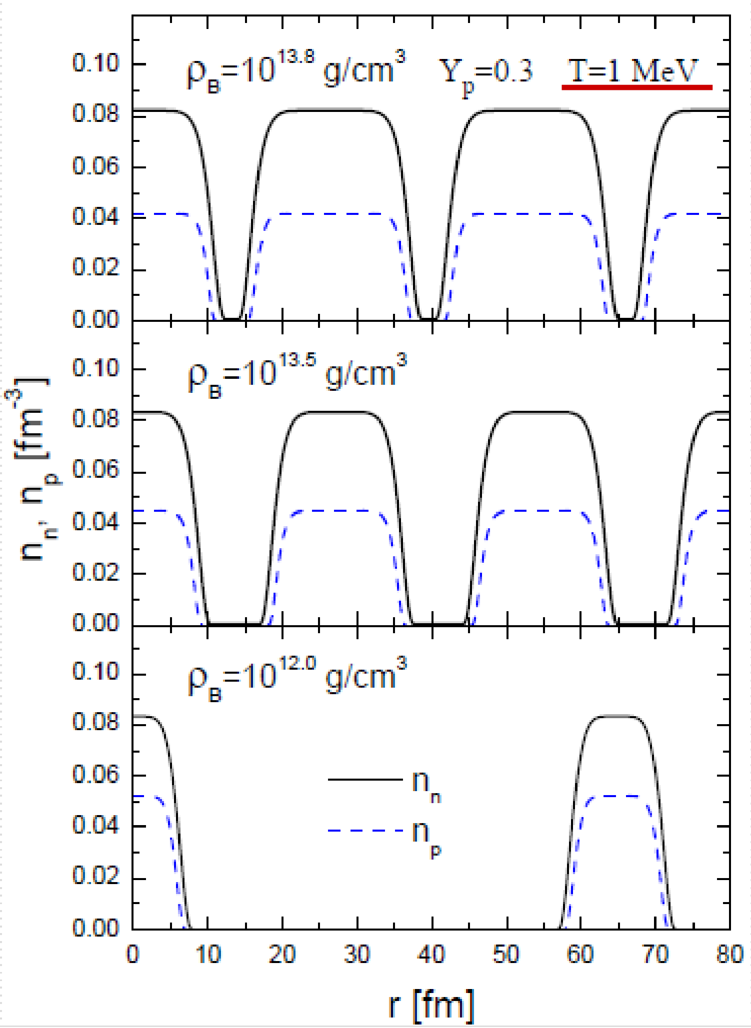
- T number of points is increased; upper limit is extended; equal grid is used
- Y_p linear grid is used; upper limit is extended
- ρ_B upper limit is extended; equal grid is used

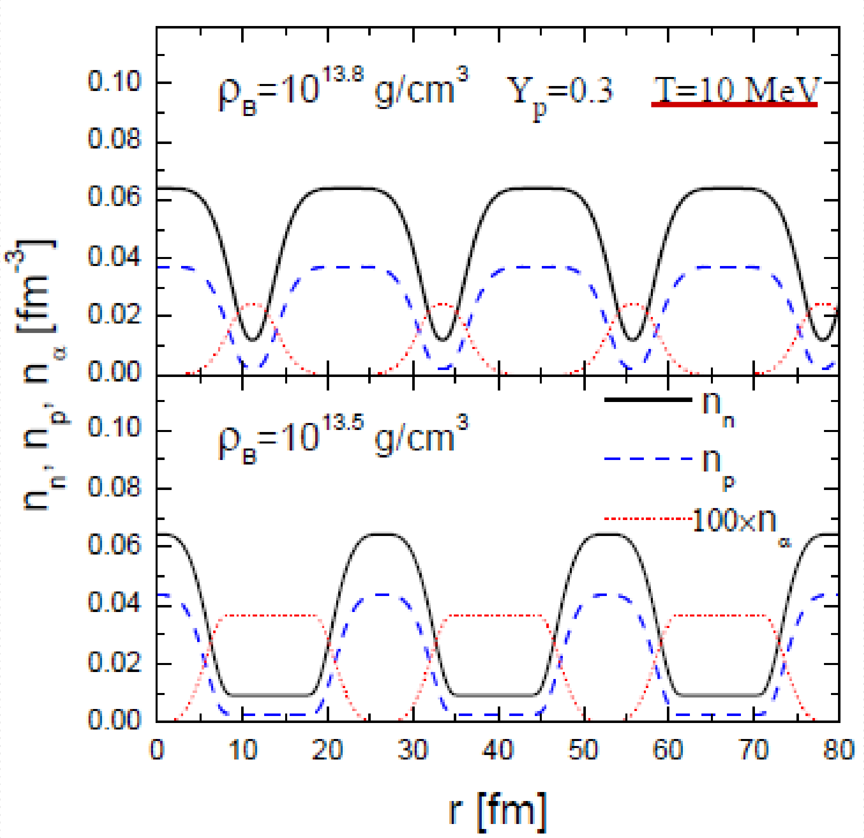
Phase diagrams



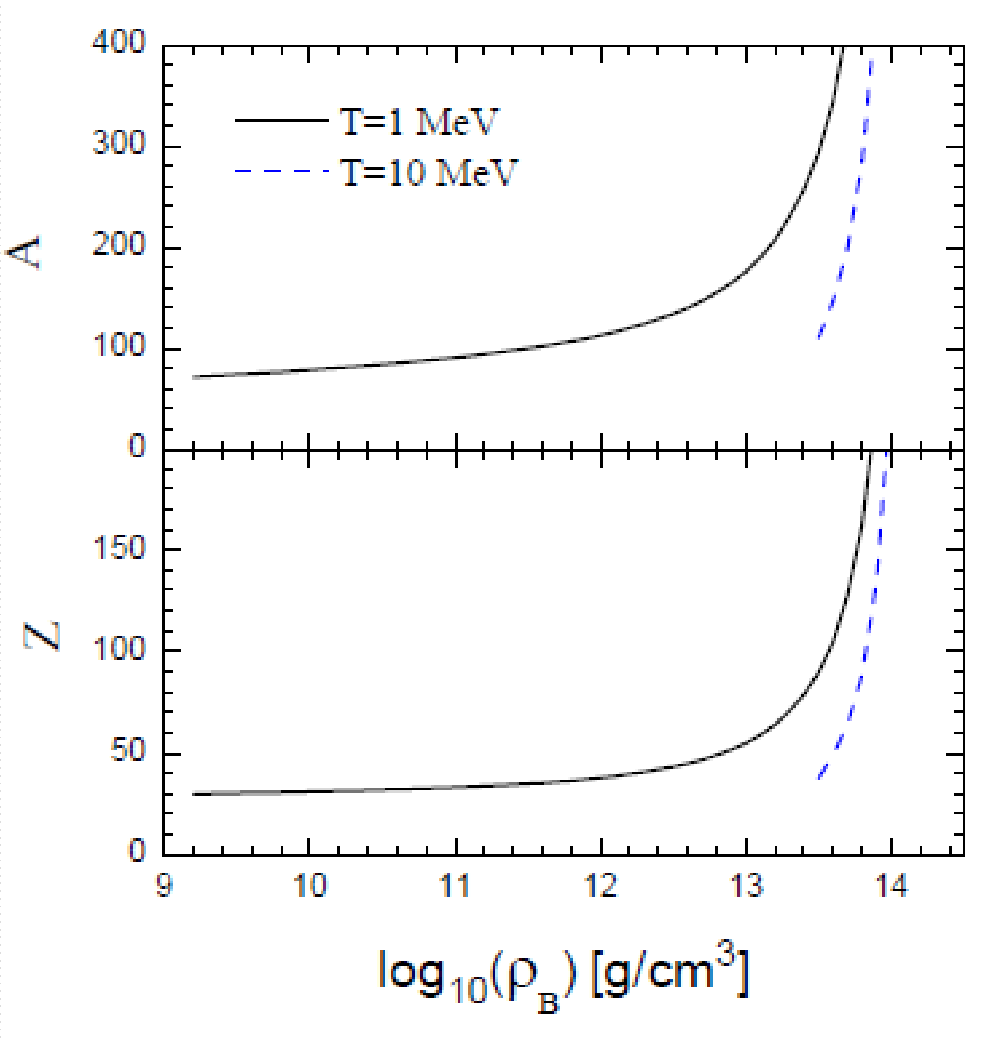


Distributions in non-uniform matter

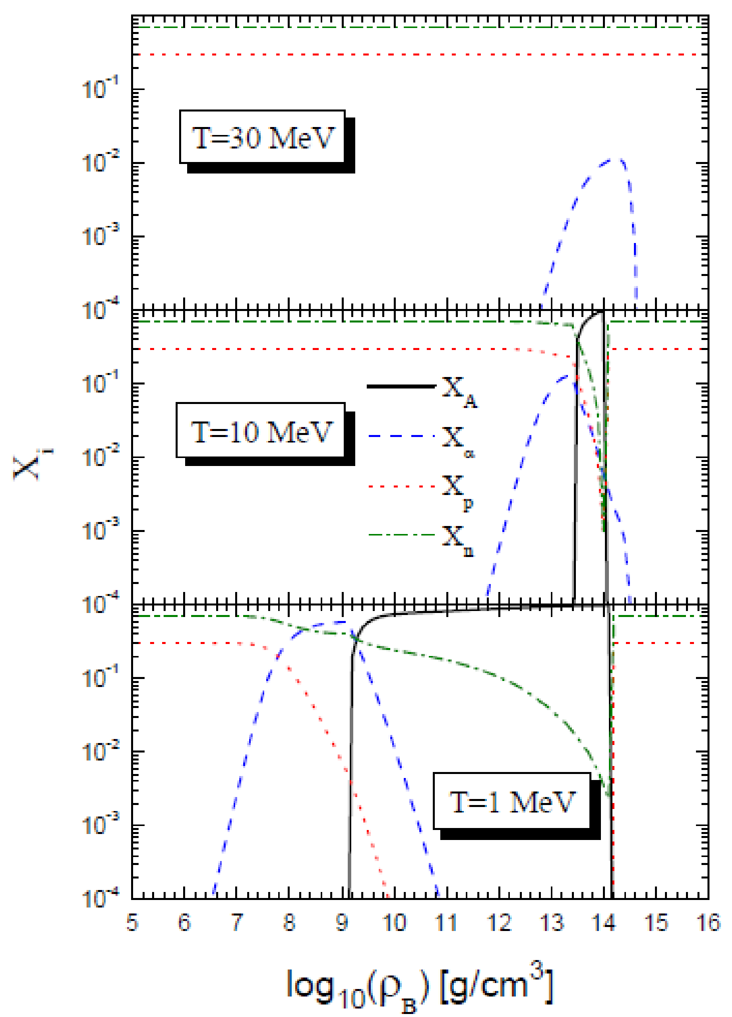




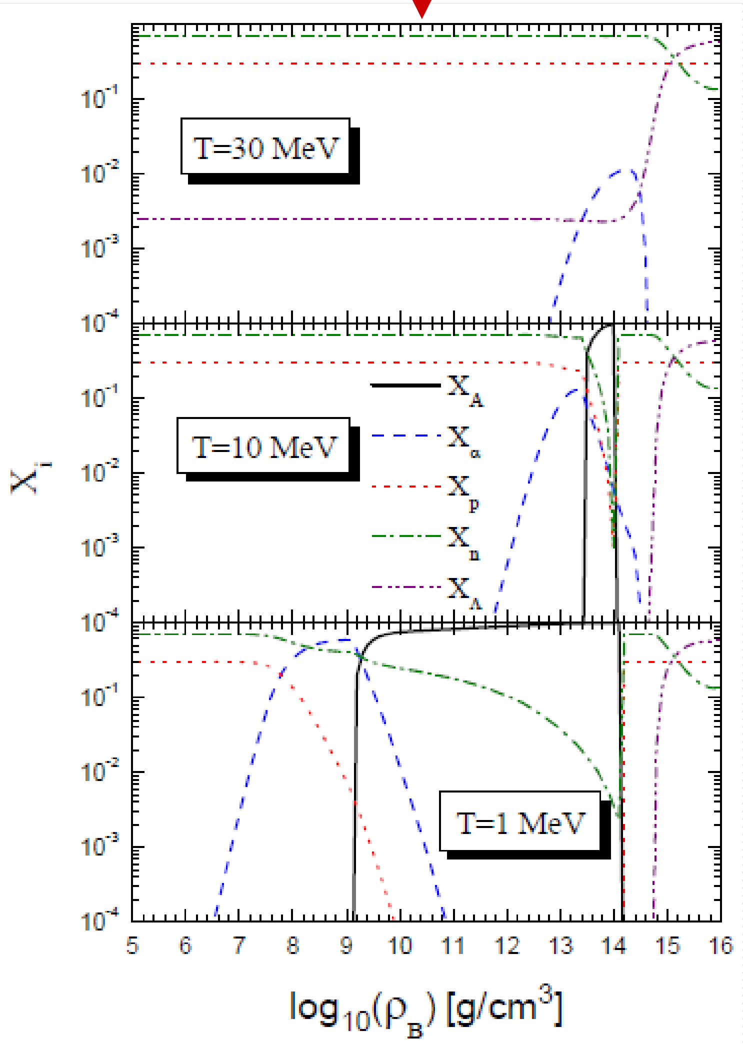
Heavy nuclei in non-uniform matter



Fractions of components



with Λ hyperons

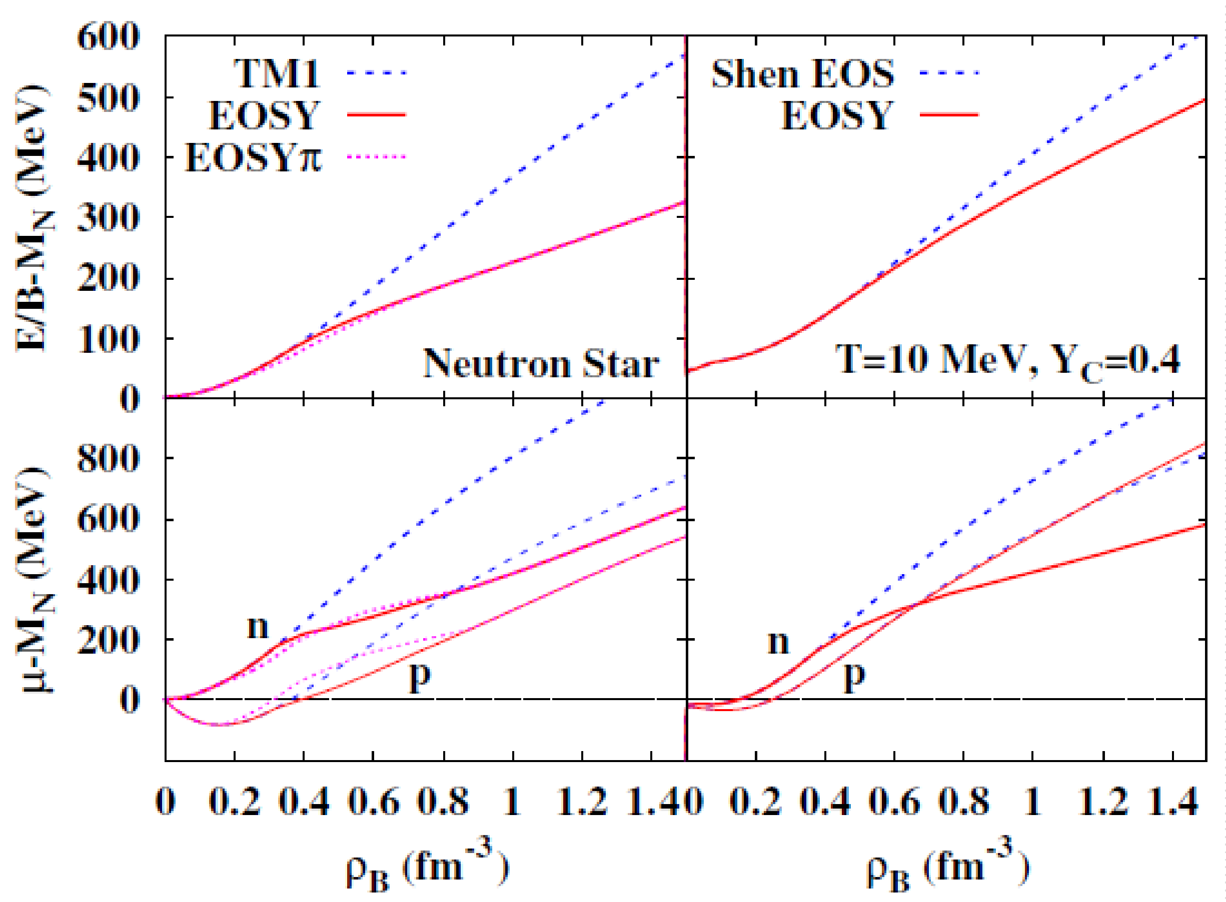


Effects of Λ hyperons

non-nucleonic degrees of freedom

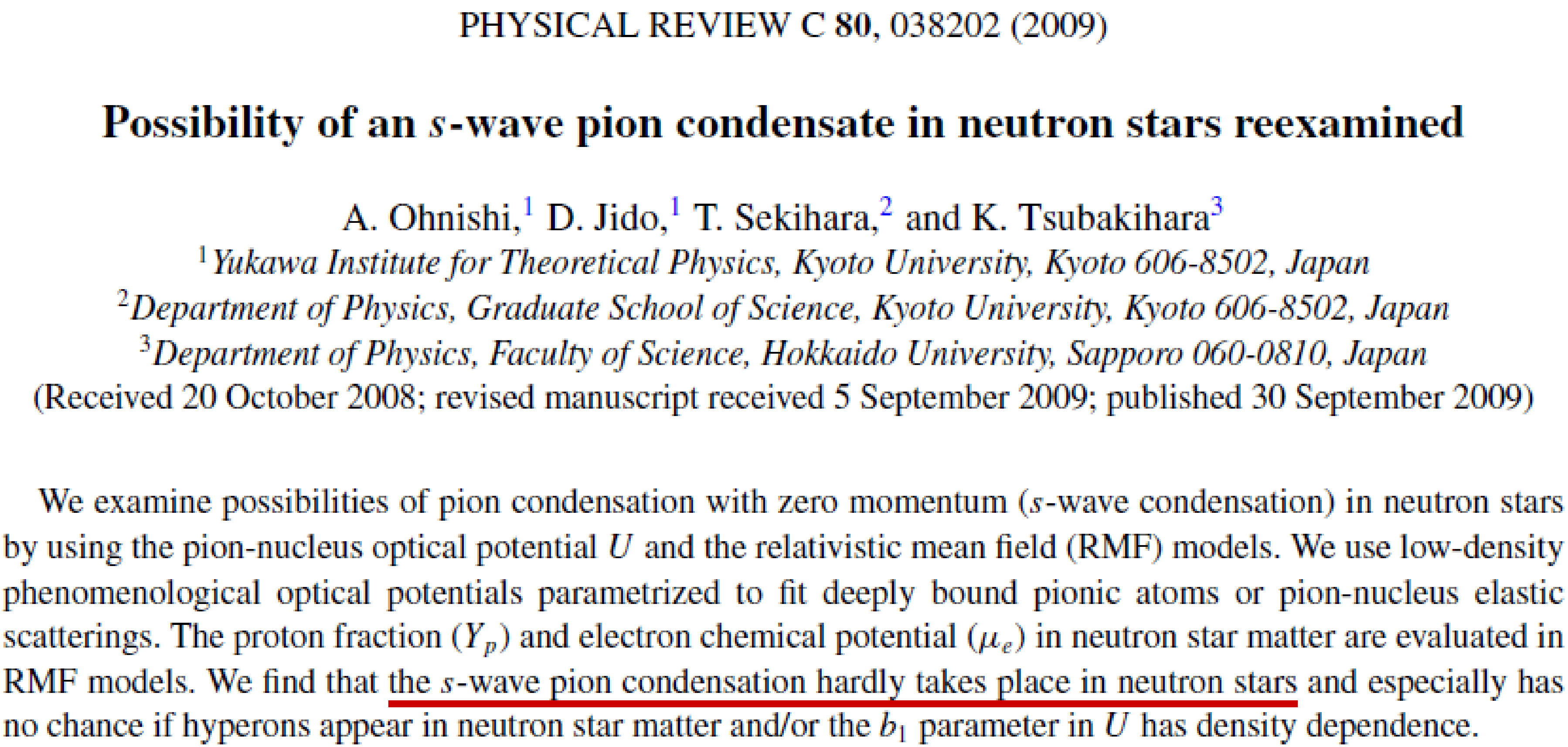
- hyperons: $\underline{\Lambda,}$ $\Sigma,$ Ξ
- boson condensates: $\pi,$ K
- quarks: $u,$ $d,$ s

EOS for supernovae with hyperons



C. Ishizuka, A. Ohnishi, K. Tsubakihara, K. Sumiyoshi, S. Yamada,
J. Phys. G 35 (2008) 085201

Pion condensate



PHYSICAL REVIEW C **80**, 038202 (2009)

Possibility of an *s*-wave pion condensate in neutron stars reexamined

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[2] *Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan*
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We examine possibilities of pion condensation with zero momentum (*s*-wave condensation) in neutron stars by using the pion-nucleus optical potential U and the relativistic mean field (RMF) models. We use low-density phenomenological optical potentials parametrized to fit deeply bound pionic atoms or pion-nucleus elastic scatterings. The proton fraction (Y_p) and electron chemical potential (μ_e) in neutron star matter are evaluated in RMF models. We find that the *s*-wave pion condensation hardly takes place in neutron stars and especially has no chance if hyperons appear in neutron star matter and/or the b_1 parameter in U has density dependence.

Experimental information

scattering experiments

NN scattering data > 4000

YN scattering data ~ 40

no YY scattering data

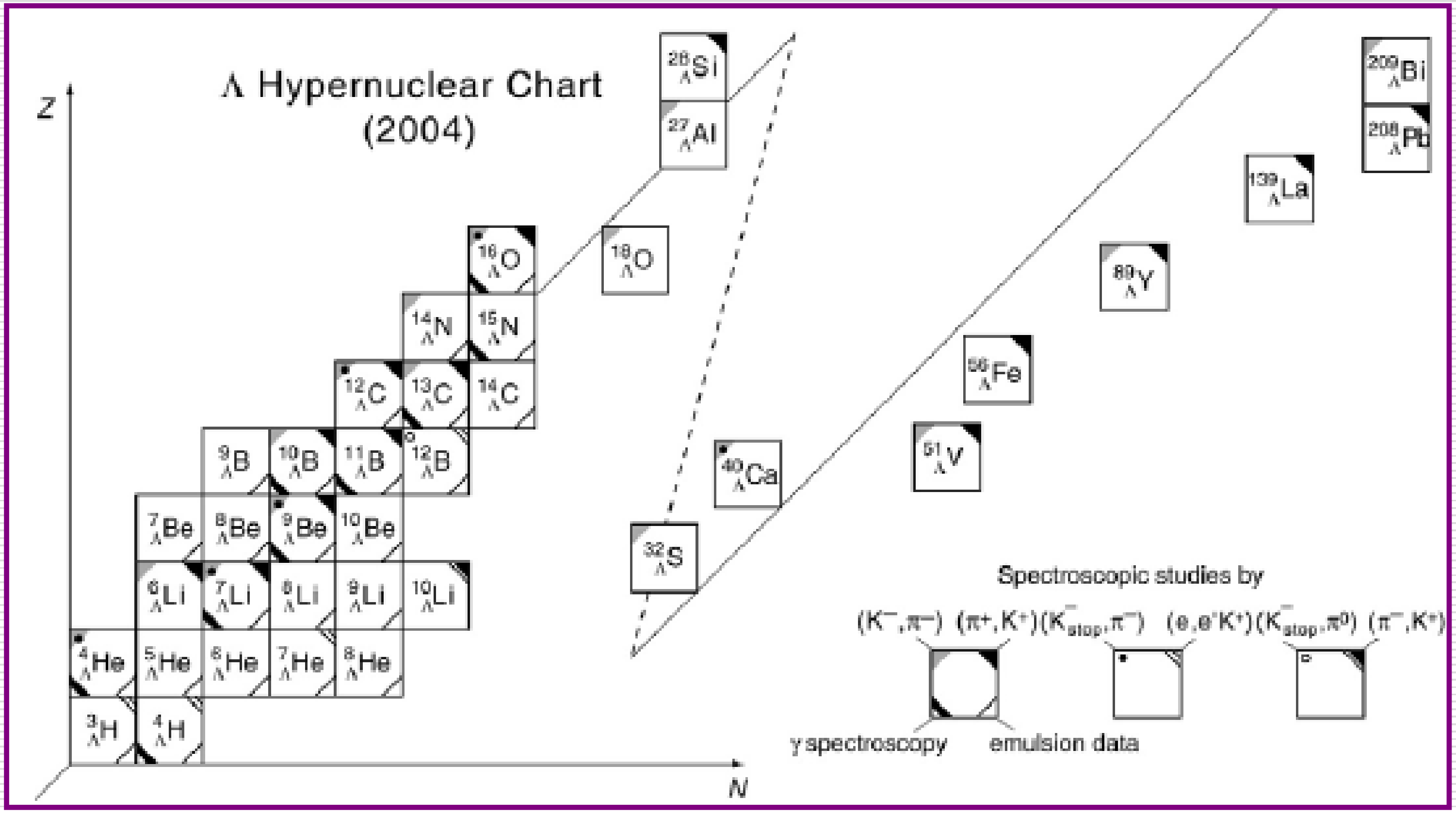
hypernuclear data

single-Λ hypernuclei > 30

double-Λ hypernuclei ~ 4

single-Σ hypernuclei ~ 1

Hypernuclear Chart



O. Hashimoto, H. Tamura, Prog. Part. Nucl. Phys. 57 (2006) 564

Neutron star matter with hyperons

include baryon octet

$n,\ p,\ \Lambda,\ \Sigma^-,$

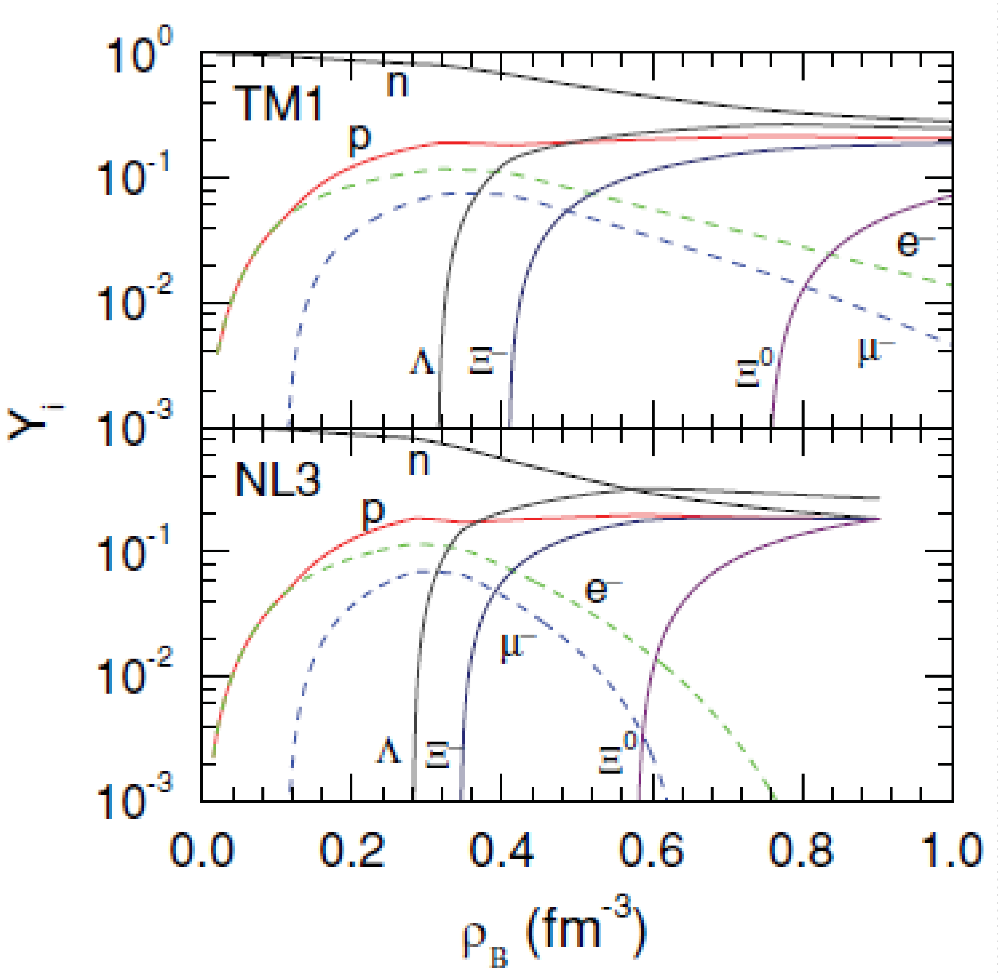
$\Sigma^0,\ \Sigma^+,\ \Xi^-,\ \Xi^0$

$U_\Lambda^{(N)} = -30\ \text{MeV}$ ✓

$U_\Sigma^{(N)} = +30\ \text{MeV}$?

$U_\Xi^{(N)} = -15\ \text{MeV}$?

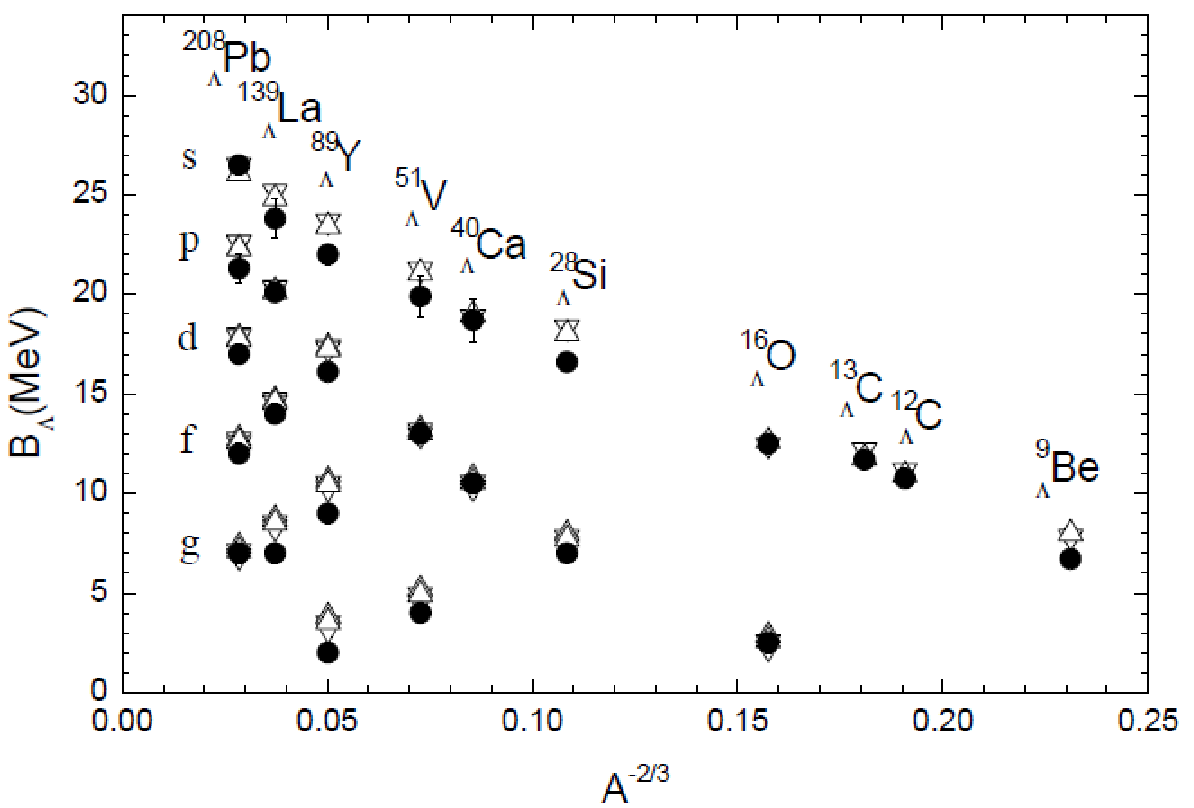
$U_\Lambda^{(\Lambda)} = -5\ \text{MeV}$?



Y.N.Wang, H.Shen, Phys. Rev. C 81 (2010) 025801

Hypernuclei in the RMF model

Single-Λ hypernuclei



H.Shen, F.Yang, H.Toki, Prog. Theor. Phys. 115 (2006) 325

Hypernuclei in the RMF model

Double-Λ hypernuclei

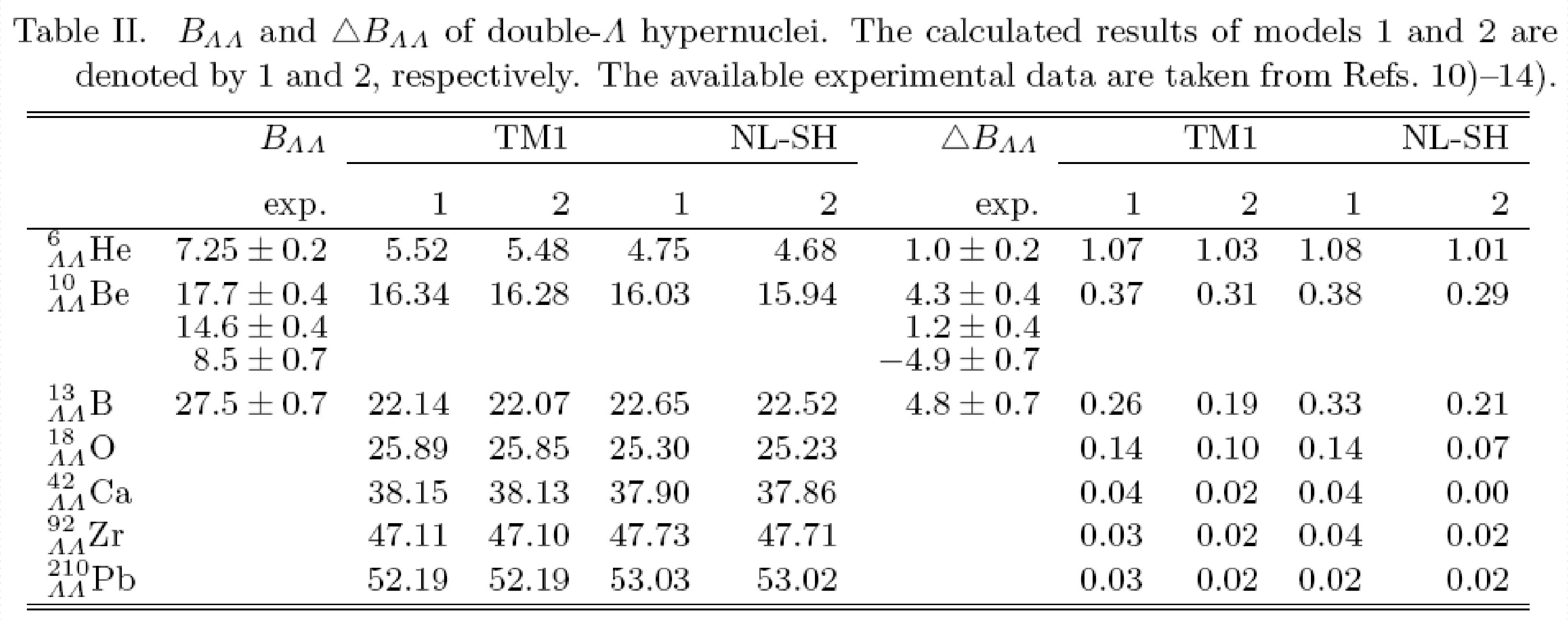


Table II. $B_{\Lambda\Lambda}$ and $\triangle B_{\Lambda\Lambda}$ of double-Λ hypernuclei. The calculated results of models 1 and 2 are denoted by 1 and 2, respectively. The available experimental data are taken from Refs. 10)–14).

	$B_{\Lambda\Lambda}$	TM1		NL-SH		$\triangle B_{\Lambda\Lambda}$	TM1		NL-SH	
	exp.	1	2	1	2	exp.	1	2	1	2
$^{6}_{\Lambda\Lambda}$He	7.25 ± 0.2	5.52	5.48	4.75	4.68	1.0 ± 0.2	1.07	1.03	1.08	1.01
$^{10}_{\Lambda\Lambda}$Be	17.7 ± 0.4 14.6 ± 0.4 8.5 ± 0.7	16.34	16.28	16.03	15.94	4.3 ± 0.4 1.2 ± 0.4 -4.9 ± 0.7	0.37	0.31	0.38	0.29
$^{13}_{\Lambda\Lambda}$B	27.5 ± 0.7	22.14	22.07	22.65	22.52	4.8 ± 0.7	0.26	0.19	0.33	0.21
$^{18}_{\Lambda\Lambda}$O		25.89	25.85	25.30	25.23		0.14	0.10	0.14	0.07
$^{42}_{\Lambda\Lambda}$Ca		38.15	38.13	37.90	37.86		0.04	0.02	0.04	0.00
$^{92}_{\Lambda\Lambda}$Zr		47.11	47.10	47.73	47.71		0.03	0.02	0.04	0.02
$^{210}_{\Lambda\Lambda}$Pb		52.19	52.19	53.03	53.02		0.03	0.02	0.02	0.02

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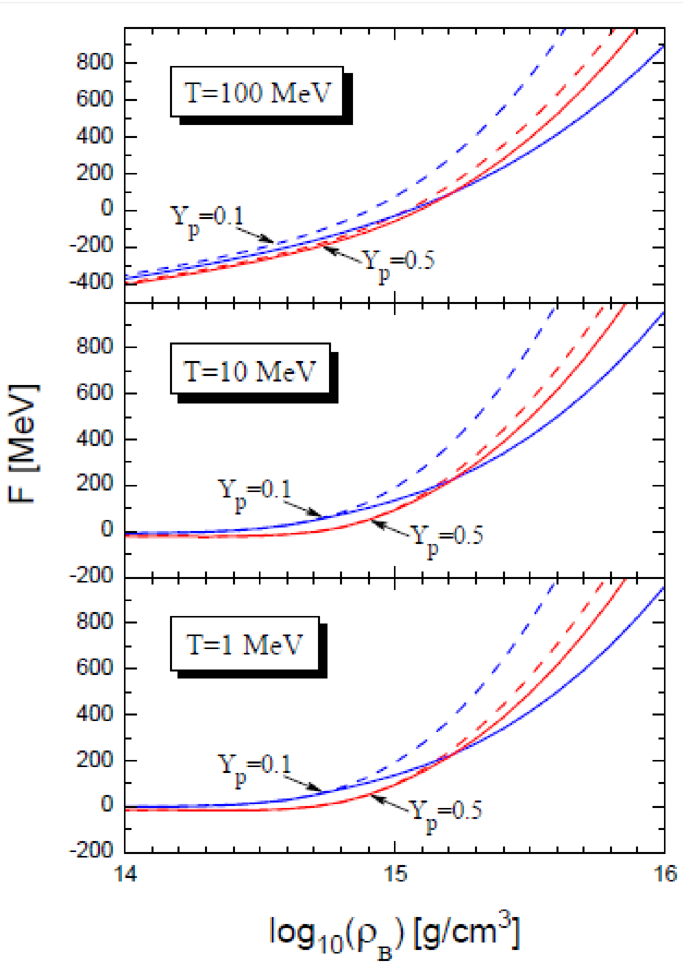
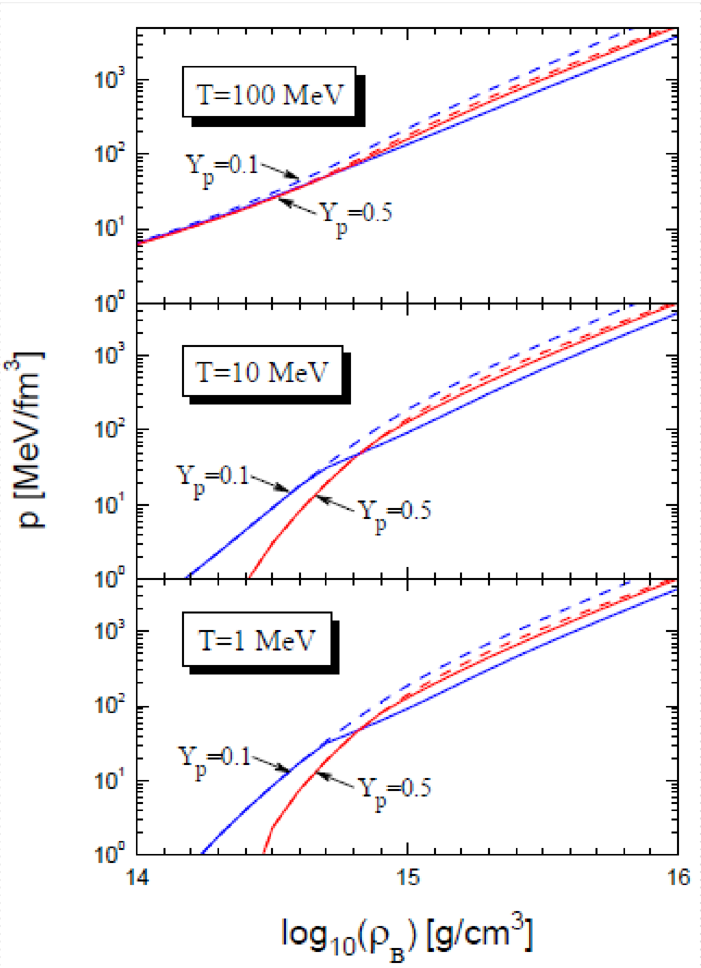
Effects of Λ hyperons

- - - - EOS2

—— EOS3





Summary

- Relativity is important at high density
- New versions of EOS tables are available
  EOS2, EOS3
- Λ hyperon can soften EOS at high density
- Exotic phases are quite uncertain