Tensor properties and transverse Spin Structures of the pion and Nucleon

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Nucleon pion & kaon

Tensor form factors and the spin structure

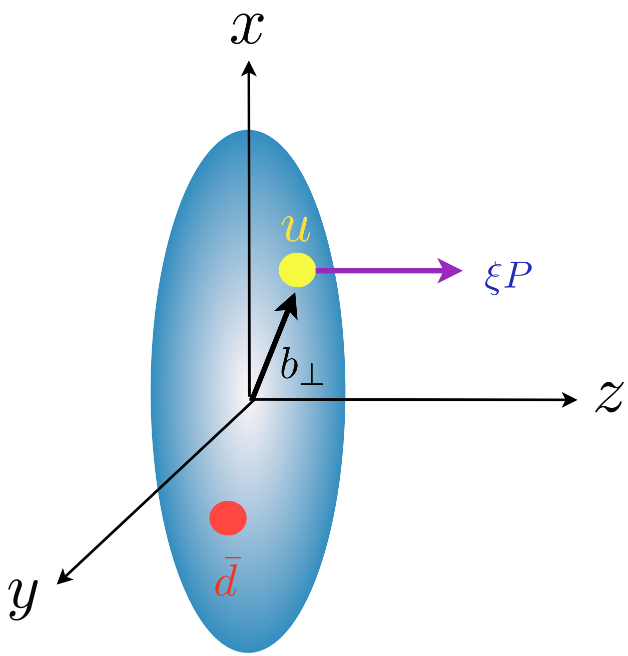
of

the Pion

S.i. Nam & HChK, Phys. Lett. B 700, 305 (2011)

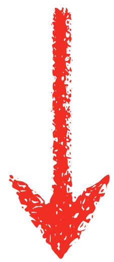
The spin structure of the Pion

Vector & Tensor Form factors of the pion



Pion: Spin S=0

What is the spin distribution of the quark inside the nucleon?



Transversity of the pion

The spin distribution of the quark

$$\rho_n(b_\perp, s_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp) = \frac{1}{2}\left[A_{n0}(b_\perp^2) - \frac{s_\perp^i \epsilon^{ij} b_\perp^j}{m_\pi} \frac{\partial B_{n0}(b_\perp^2)}{\partial b_\perp^2}\right]$$

Spin probability density function in impact parameter space

A_{n0}: Vector form factor of the pion, B_{n0}: Tensor form factor of the pion

$$\int_{-1}^{1} dx\, x^{n-1} H(x, \xi=0, b_\perp^2) = A_{n0}(b_\perp^2), \quad \int_{-1}^{1} dx\, x^{n-1} E(x, \xi=0, b_\perp^2) = B_{n0}(b_\perp^2)$$

Vector and Tensor form factors of the pion

$$\langle \pi(p_f) | \psi^\dagger \gamma_\mu \hat{Q} | \pi(p_i) \rangle = (p_i + p_f) A_{10}(q^2)$$

$$\langle \pi^+(p_f) | \mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} | \pi^+(p_i) \rangle = \mathcal{AS}\left[\frac{(p^\mu q^\nu - q^\mu p^\nu)}{m_\pi} \sum_{i=\text{even}}^{n-1} q^{\mu_1} \cdots q^{\mu_i} p^{\mu_{i+1}} \cdots p^{\mu_{n-1}} B_{ni}(Q^2)\right]$$

Nonlocal chiral quark model

Effective Chiral Action for the tensor current

$$S_{\rm eff}[m,\pi] = -{\rm Spln}\left[i\partial\!\!\!/ + im + i\sqrt{M(i\partial)}U^{\gamma_5}(\pi)\sqrt{M(i\partial)} + \sigma\cdot T\right]$$

The chiral quark model from the instanton vacuum

- Fully relativistically field theoretic model.
- Related to QCD via the Instanton vacuum.
- Renormalization scale is naturally given.
- No free parameter

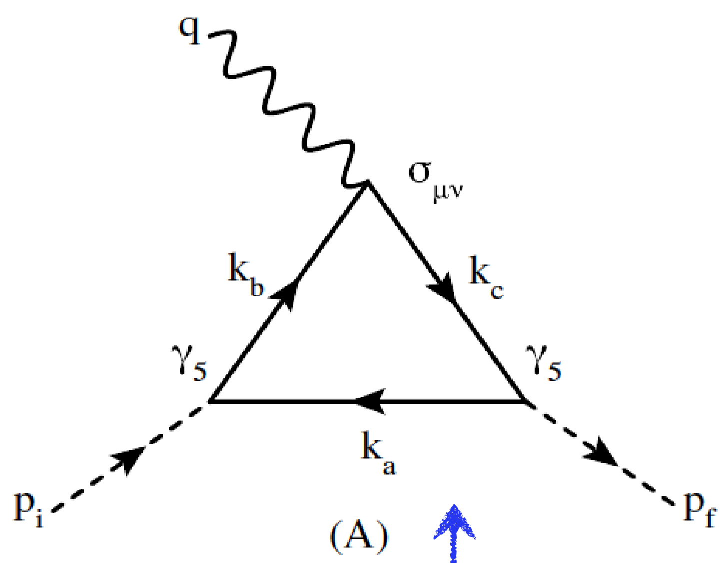
$\rho \approx 0.3\,{\rm fm},\;\; R \approx 1\,{\rm fm}$

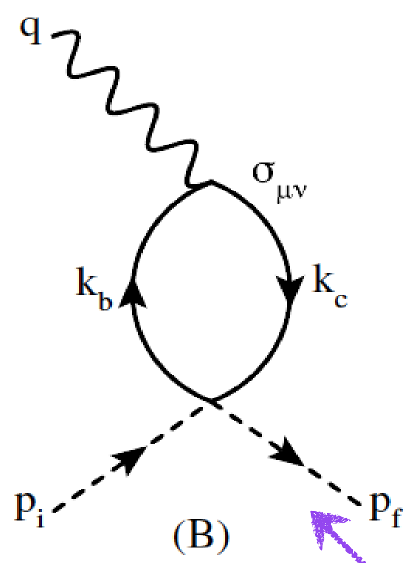
↓

$\mu \approx 600\,{\rm MeV}$

HChK et al. Prog. Part. Nucl. Phys. Vol.37, 91 (1996)

Tensor form factor of the pion

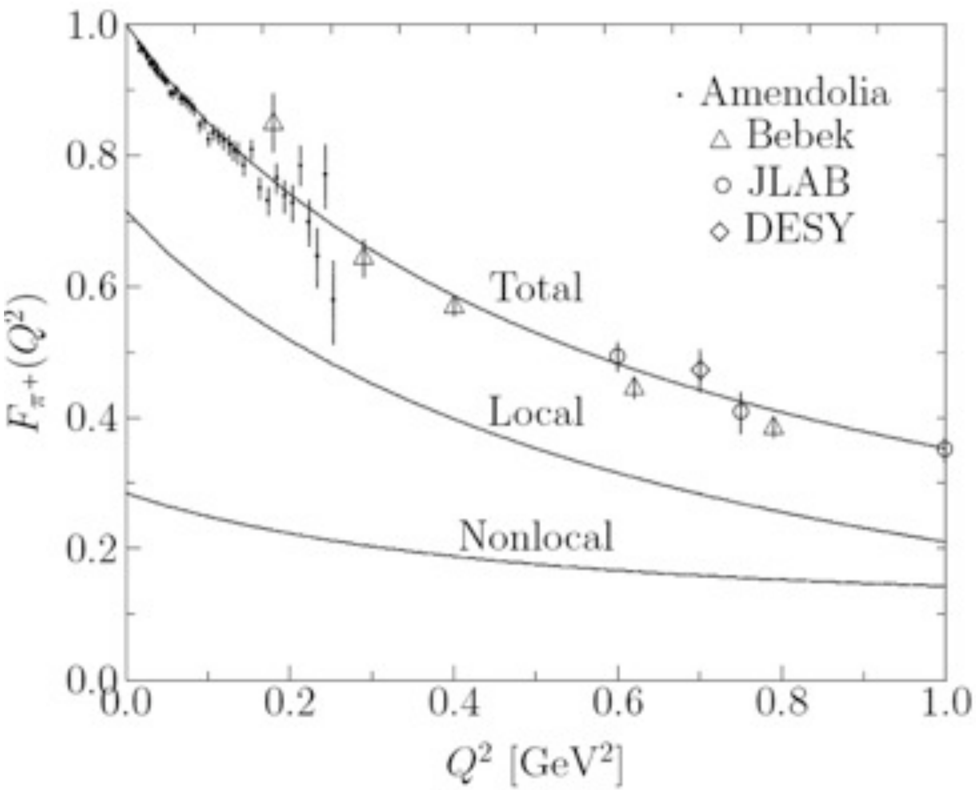




$$\frac{\delta^3 \mathcal{S}_{\mathrm{eff}}[m,\pi,T]}{\delta T\,\delta\pi^a\delta\pi^b}\bigg|_{T=0} = -\frac{1}{F_\pi^2}\mathrm{Sp}\left[\frac{1}{i\not{D}}\sqrt{M}\gamma_5\tau^a\sqrt{M}\frac{1}{i\not{D}}\sqrt{M}\gamma_5\tau^b\sqrt{M}\frac{1}{i\not{D}}\sigma_{\mu\nu}\right] - \frac{i}{2F_\pi^2}\mathrm{Sp}\left[\frac{1}{i\not{D}}\sqrt{M}\tau^a\tau^b\sqrt{M}\frac{1}{i\not{D}}\sigma_{\mu\nu}\right]$$

EM Form factor of the pion

EM form factor (A_{10}) has been already studied.



$$\sqrt{\langle r^2 \rangle} = 0.675\,\mathrm{fm}$$

$$\sqrt{\langle r^2 \rangle} = 0.672 \pm 0.008\,\mathrm{fm}\ (\mathrm{Exp})$$

$$F_\pi(Q^2) = A_{10}(Q^2) = \frac{1}{1 + Q^2/M^2}$$

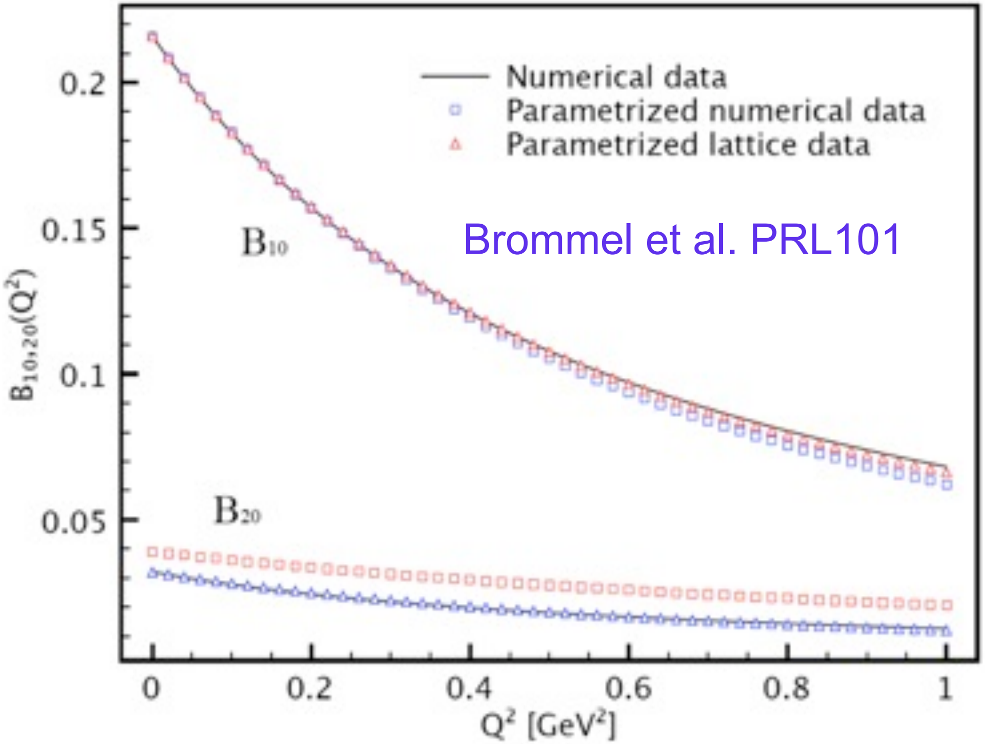
M(Phen.): 0.714 GeV

M(Lattice): 0.727 GeV

M(This work): 0.738 GeV

S.i. Nam & HChK, Phys. Rev. D77 (2008) 094014

Tensor Form factor of the pion



$$B_{10}(Q^2,\mu) = B_{10}(Q^2,\mu_0)\left[\frac{\alpha(\mu)}{\alpha(\mu_0)}\right]^{\gamma/2\beta_0}$$

RG evolution between lattice and model

$$B_{10}(Q^2) = B_{10}(0)\left[1+\frac{Q^2}{p\,m_p^2}\right]^{-p}$$

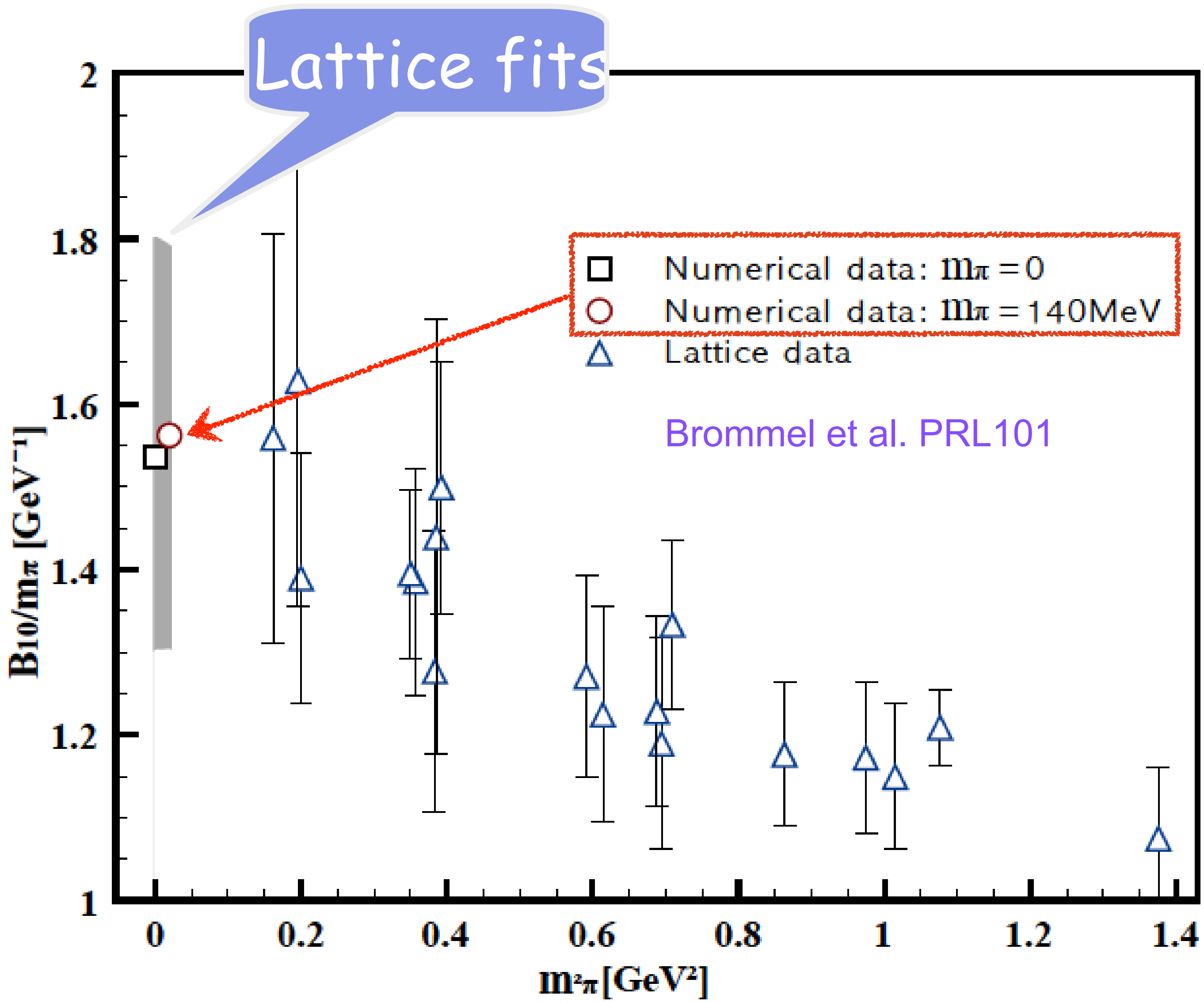
$$B_{n0}(Q^2,\mu) = B_{n0}(Q^2,\mu_0)\left[\frac{\alpha(\mu)}{\alpha(\mu_0)}\right]^{\gamma_n/(2\beta_0)}$$

$$\gamma_1 = 8/3, \quad \gamma_2 = 8,$$

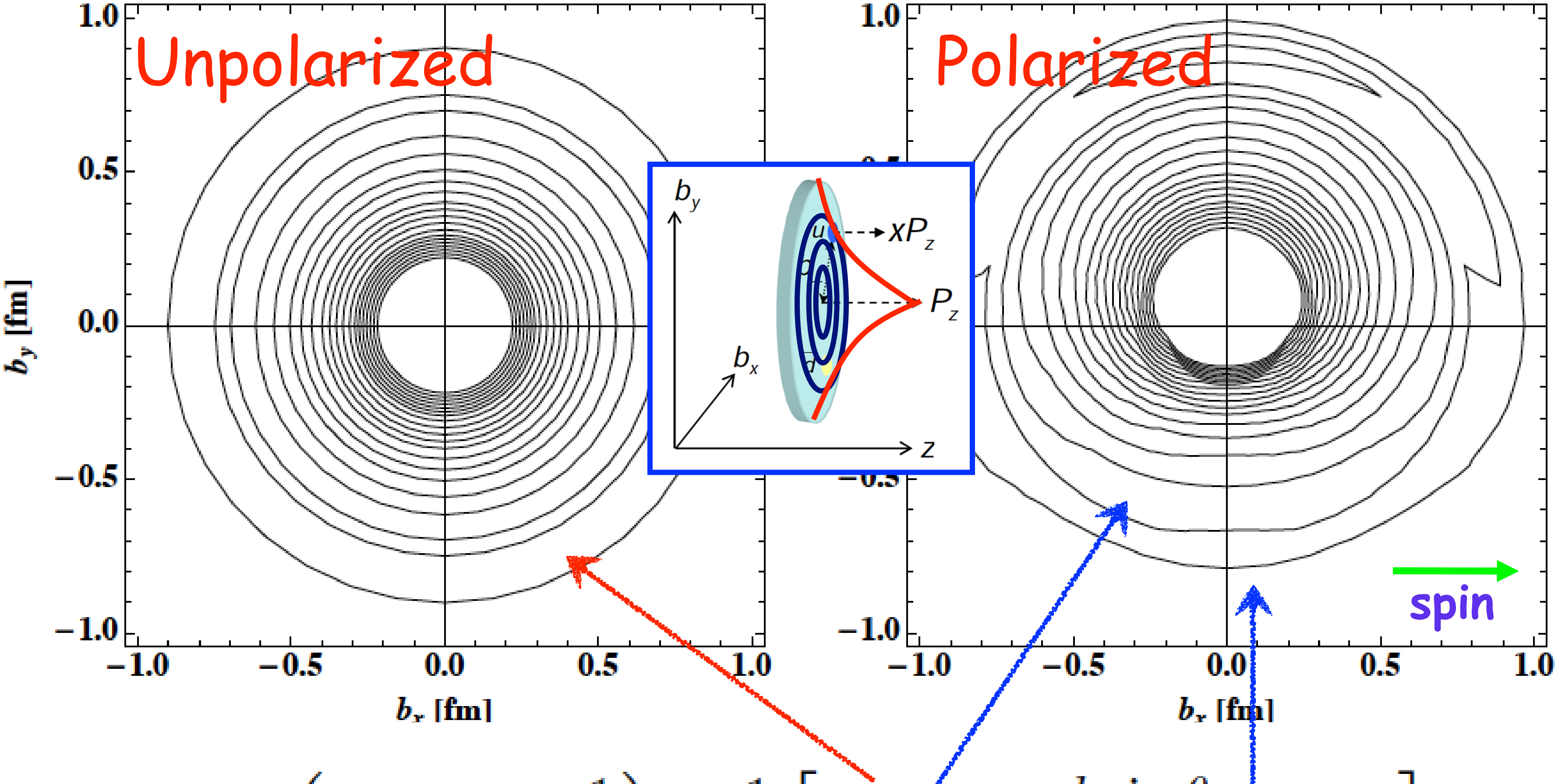
$$\beta_0 = 11N_c/3 - 2N_f/3$$

S.i. Nam & HChK, Phys. Lett. B 700, 305 (2011)

Tensor Form factor of the pion



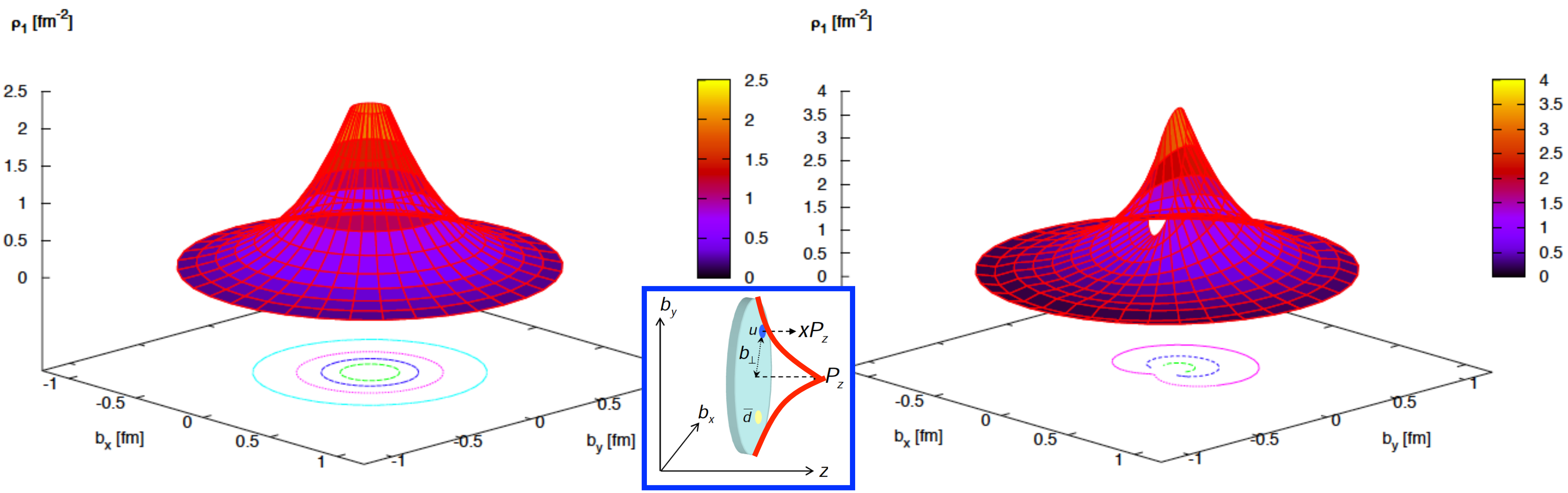
Spin density of the quark



$$\rho_1\left(b_\perp, s_x = \pm\frac{1}{2}\right) = \frac{1}{2}\left[A_{10}(b^2) \mp \frac{b\sin\theta}{m_\pi}B'_{10}(b^2)\right]$$

Transverse polarization

Spin density of the quark

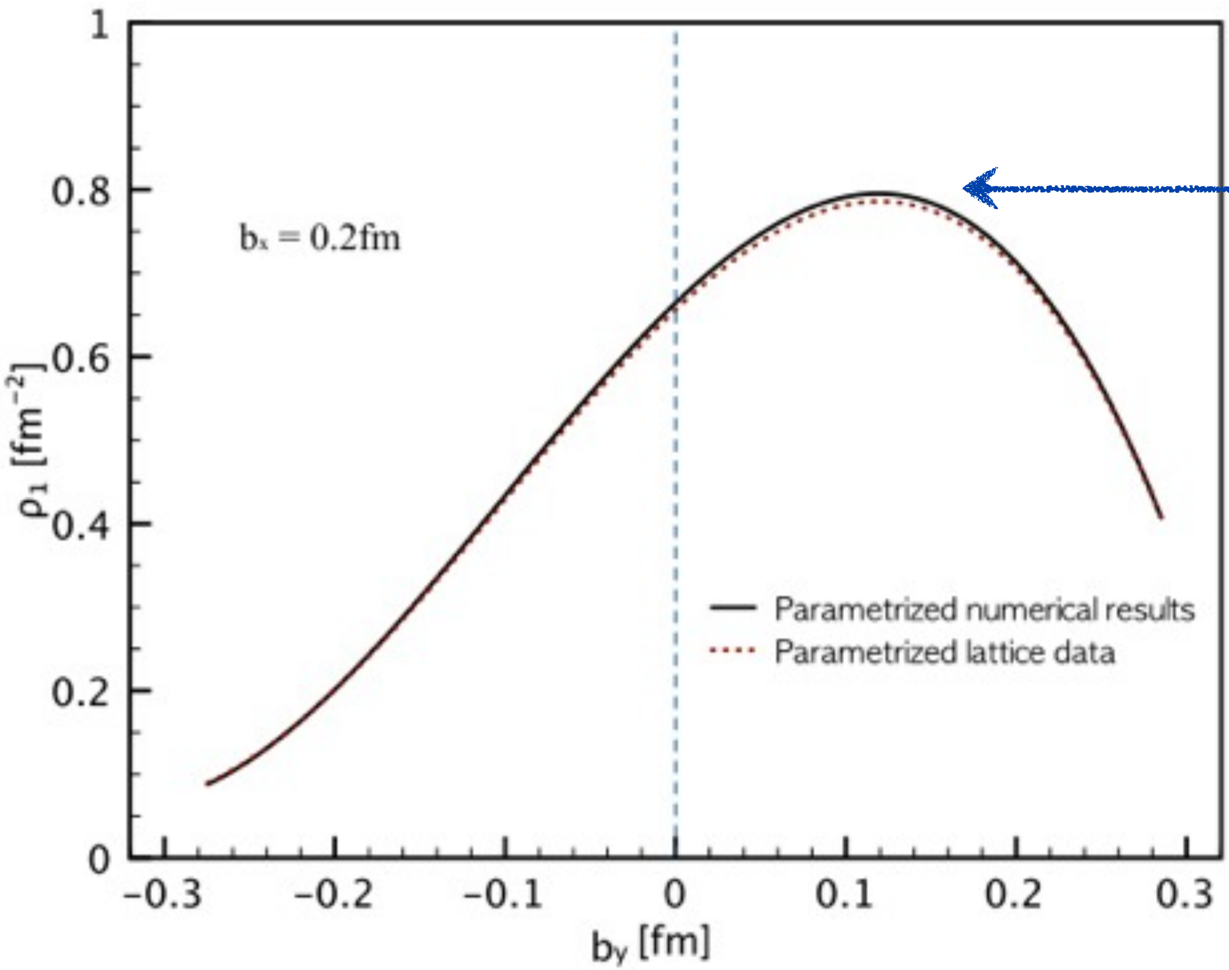


Significant distortion appears for the polarized: a hint for spin structure of the pion!!!

$m_\pi = 140$ MeV	$B_{10}(0)$	m_{p_1} [GeV]	$\langle b_y \rangle$ [fm]	$B_{20}(0)$	m_{p_2} [GeV]
Present work	0.216	0.762	0.152	0.032	0.864
Lattice QCD [7]	0.216 ± 0.034	0.756 ± 0.095	0.151	0.039 ± 0.099	1.130 ± 0.265

Results are in a good agreement with lattice!!!

Spin density of the quark



Distorted spin distribution of the quark inside the pion

Tensor form factors and spin structure

of

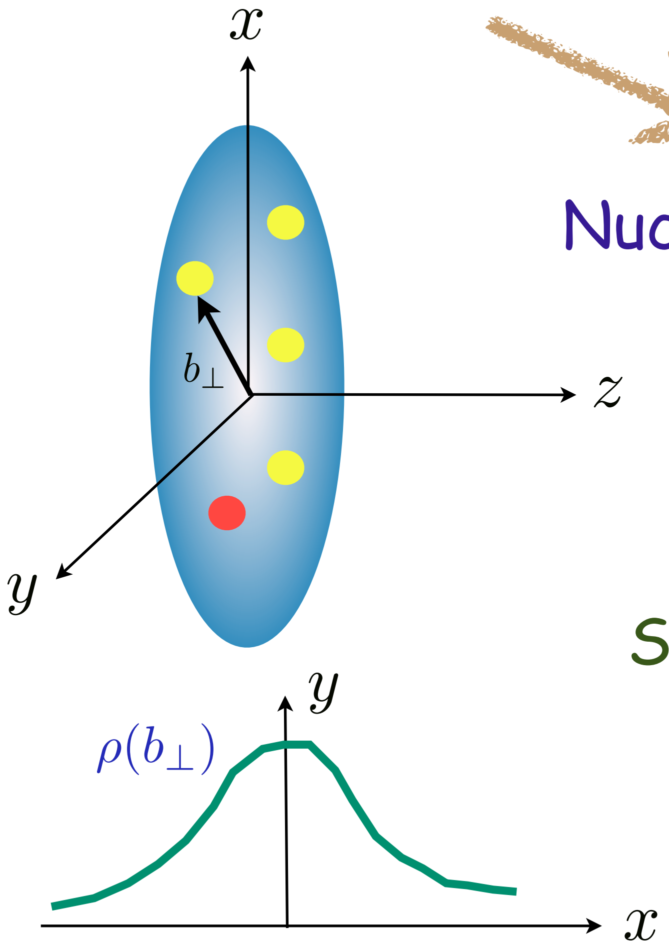
the Nucleon

T. Ledwig & HChK, arXiv:1107.4952

The spin structure of the Nucleon

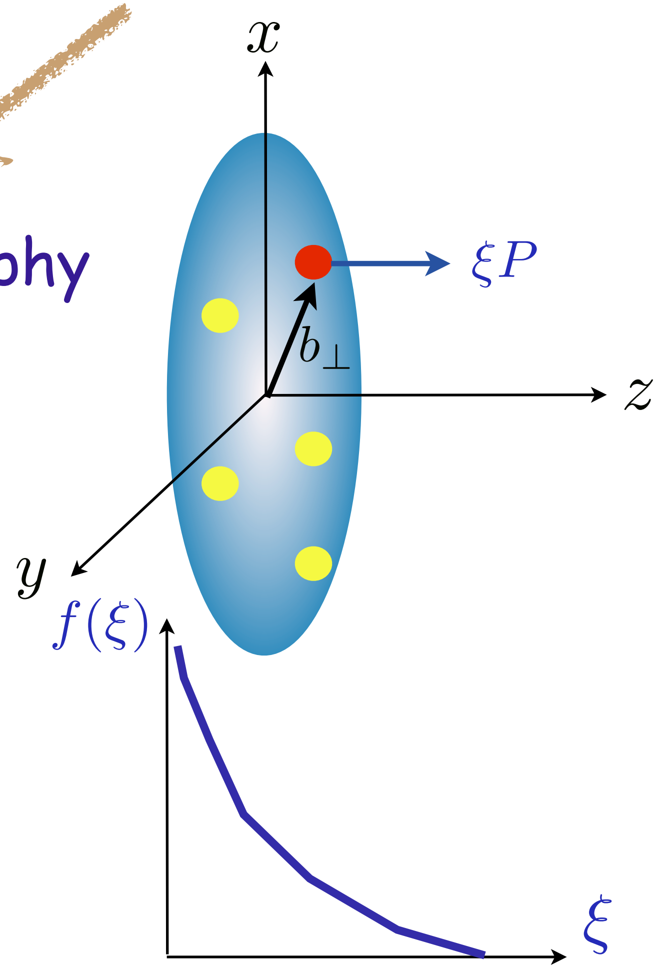
Axial & Tensor Form factors, Axial-vector charges, Tensor charges

Structure functions

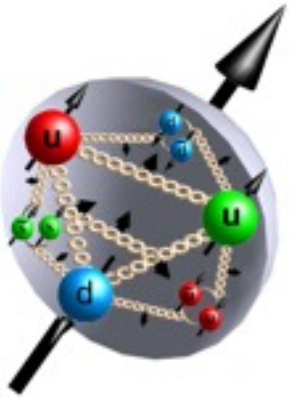


Nucleon Tomography

Spin Structure



The Spin of the nucleon



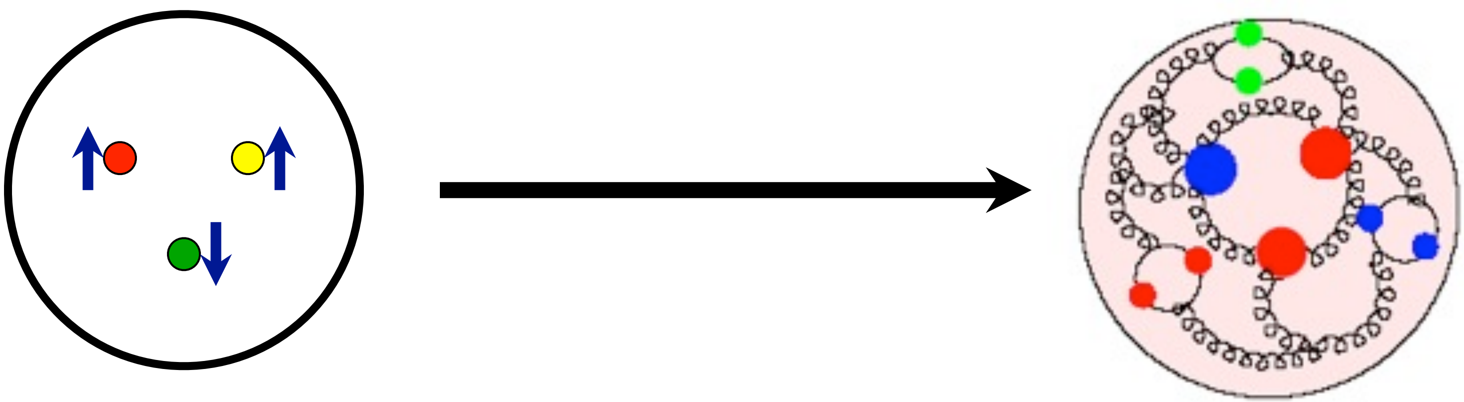
The spin of the proton must be ½!

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

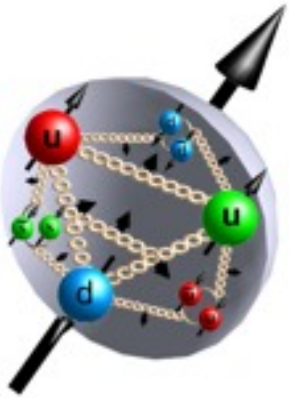
0.2-0.4 → Spin Crisis

Quark Model

QCD



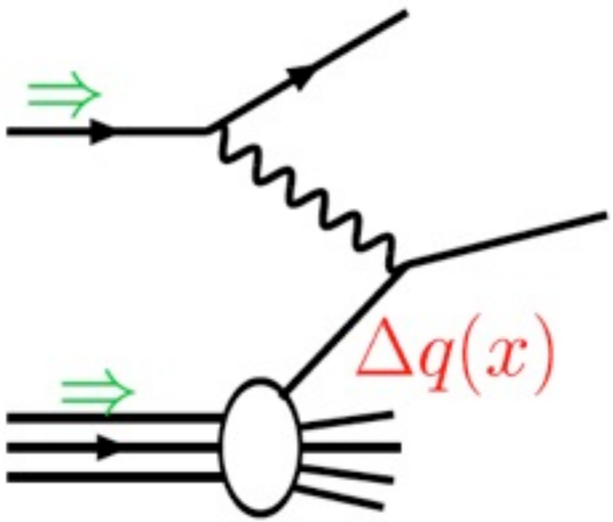
The Spin of the nucleon



$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

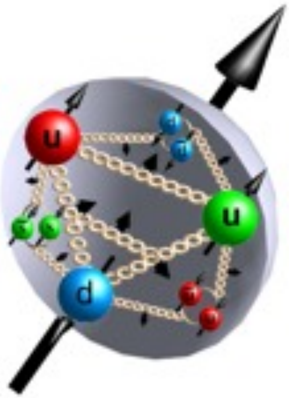
Longitudinally polarized quark spin

$$\mathbf{\Delta q(x)} =$$ 



Polarized DIS

The Spin of the nucleon



$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

$$\langle N | \bar{\psi}\gamma_\mu \gamma^5 \lambda^\chi \psi | N \rangle \sim \text{Axial-vector charges}$$

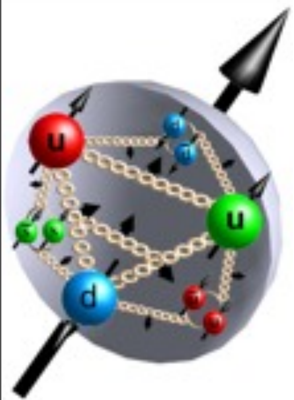
Singlet Axial vector constant ⟶ Quark spin content

$$\Delta s = -0.10 \pm 0.04$$

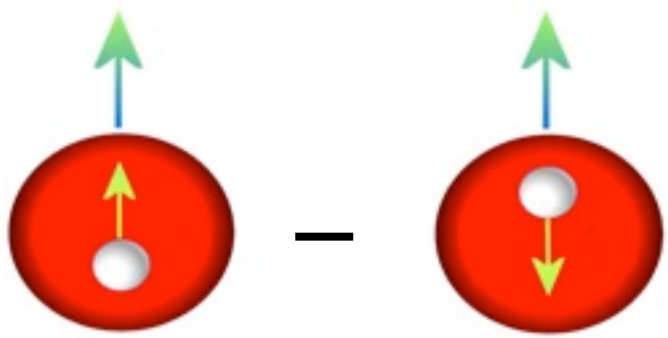
$$g_A^{(0)}|_{\text{pDIS}} = 0.15 - 0.35$$

S.D. Bass, RMP 77, 1257 (2005)

Transversity: Tensor Charges



$\delta \mathbf{q}(\mathbf{x}) =$



$$\langle N \left| \bar{\psi} \sigma_{\mu\nu} \lambda^{\chi} \psi \right| N \rangle \sim \text{Tensor charges}$$

- No explicit probe for the tensor charge! Difficult to be measured.
- Chiral odd Parton Distribution Function can get accessed via the SSA of SIDIS (HERMES and COMPASS).

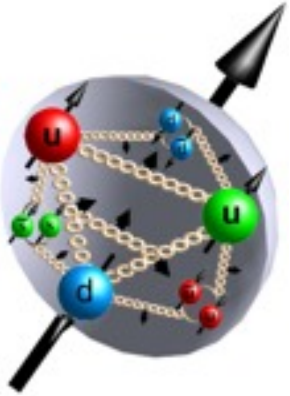
A. Airapetian et al. (HERMES Coll.), PRL 94, 012002 (2005).

E.S. Ageev et al. (COMPASS Coll.), NPB 765, 31 (2007).

CLAS & CLAS12 Coll. (Talk by H. Avakian)

ppbar Drell-Yan process (PAX Coll.): Technically too difficult for the moment (polarized antiproton: hep-ex/0505054).

Transversity: Tensor Charges



$$\delta u = 0.60^{+0.10}_{-0.24}\,, \quad \delta d = -0.26^{+0.1}_{-0.18} \text{ at } 0.36\,\mathrm{GeV}^2$$

Based on SIDIS (HERMES) data:
M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

Tensor form factors

$$\langle N_{s'}(p')|\overline{\psi}(0)i\sigma^{\mu\nu}\lambda^\chi\psi(0)|N_s(p)\rangle = \overline{u}_{s'}(p')\left[H_T^\chi(Q^2)i\sigma^{\mu\nu} + E_T^\chi(Q^2)\frac{\gamma^\mu q^\nu - q^\mu\gamma^\nu}{2M} + \tilde{H}_T^\chi(Q^2)\frac{(n^\mu q^\nu - q^\mu n^\nu)}{2M^2}\right]u_s(p)$$

$$\int_{-1}^{1} dx\, H_T^\chi(x,\xi,t) = H_T^\chi(q^2),$$

$$\int_{-1}^{1} dx\, E_T^\chi(x,\xi,t) = E_T^\chi(q^2),$$

$$\int_{-1}^{1} dx\, \tilde{H}_T^\chi(x,\xi,t) = \tilde{H}_T^\chi(q^2),$$

$$H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s$$

$$H_T^3(0) = g_T^3 = \delta u - \delta d$$

$$H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)$$

Anomalous tensor magnetic form factors

$$H_T^{*\chi}(Q^2) = \frac{2M}{\mathbf{q}^2}\int\frac{d\Omega}{4\pi}\langle N_{\frac{1}{2}}(p')|\psi^\dagger\gamma^k q^k\lambda^\chi\psi|N_{\frac{1}{2}}(p)\rangle$$

$$\kappa_T^\chi = -H_T^\chi(0) - H_T^{*\chi}(0)$$

Together with the anomalous magnetic moment, this will allow us to describe the transverse spin quark densities inside the nucleon.

Tensor form factors

Tensor charges and anomalous tensor magnetic moments are scale-dependent.

$$\delta q(\mu^2) = \left(\frac{\alpha_S(\mu^2)}{\alpha_S(\mu_i^2)}\right)^{4/27}\left[1 - \frac{337}{486\pi}\left(\alpha_S(\mu_i^2) - \alpha_S(\mu^2)\right)\right]\delta q(\mu_i^2)\,,$$

$$\alpha_S^{NLO}(\mu^2) = \frac{4\pi}{9\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})}\left[1 - \frac{64}{81}\frac{\ln\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})}{\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})}\right]$$

$\Lambda_{\mathrm{QCD}} = 0.248\,\mathrm{GeV}$

M. Gluck, E. Reya, and A. Vogt, Z.Phys. C 67, 433(1995).

Chiral quark-soliton model

$$\mathcal{Z}_{\chi\text{QSM}} = \int \mathcal{D}U \exp(-S_{\text{eff}})$$

$$H(U) = -i\gamma_4\gamma_i\partial_i + \gamma_4 M U^{\gamma_5}$$

$$S_{\text{eff}} = -N_c \text{Tr} \ln D(U)$$

$$D(U) = \partial_4 + H(U) + \hat{m}$$

$$\hat{m} = m_0\gamma_4\mathbf{1} + m_3\gamma_4\lambda^3 + m_8\gamma_4\lambda^8$$

Merits of the chiral quark-soliton model

- Fully relativistically field theoretic model.
- Related to QCD via the Instanton vacuum.
- Renormalization scale is naturally given.
- All parameters were fixed already.

HChK et al. Prog. Part. Nucl. Phys. Vol.37, 91 (1996)

Chiral quark-soliton model

Nucleon consisting of Nc quarks

$$\Pi_N = \langle 0 | J_N(0, T/2) J_N^\dagger(0, -T/2) | 0 \rangle$$

$$J_N(\vec{x}, t) = \frac{1}{N_c!} \epsilon^{\beta_1 \cdots \beta_{N_c}} \Gamma^{\{f\}}_{JJ_3Y', TT_3Y} \psi_{\beta_1 f_1}(\vec{x}, t) \cdots \psi(\vec{x}, t)$$

$$\lim_{T \to \infty} \Pi_N(T) \simeq e^{-M_N T}$$

$$\Pi_N(\vec{x}, t) = \Gamma_N^{\{f\}} \Gamma_N^{\{g\}*} \frac{1}{Z} \int dU \prod_{i=1}^{N_c} \left\langle 0, T/2 \left| \frac{1}{D(U)} \right| 0, -T/2 \right\rangle_{f,g} e^{-S_{\mathrm{eff}}}$$

$$\lim_{T \to \infty} \frac{1}{Z} \prod_{i=1}^{N_c} \left\langle 0, T/2 \left| \frac{1}{D(U)} \right| 0, -T/2 \right\rangle \sim e^{-(N_c E_{\mathrm{val}}(U) + E_{\mathrm{sea}}(U))T}$$

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

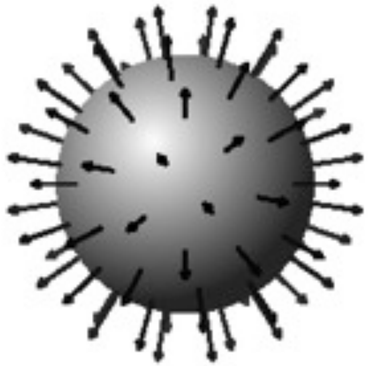
Chiral quark-soliton model

Classical solitons

$$\frac{\delta}{\delta U}(N_c E_{\rm val} + E_{\rm sea}) = 0 \rightarrow M_{\rm cl} = N_c E_{\rm val}(U_c) + E_{\rm sea}(U_c)$$

Hedgehog Ansatz:

$$U_{\rm SU(2)} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$$


hedgehog

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton model

Collective quantization

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

$$U(\boldsymbol{x},t) = R(t)U_c(\boldsymbol{x}-\boldsymbol{Z}(t))R^\dagger(t)$$

$$\int DU[\cdots] \rightarrow \int DAD\boldsymbol{Z}[\cdots]$$

$$\mathcal{L} = -M_{sol} + \frac{I_1}{2}\sum_{i=1}^{3}{\Omega_i}^2 + \frac{I_2}{2}\sum_{i=4}^{7}{\Omega_i}^2 + \frac{N_c}{2\sqrt{3}}\Omega_8$$

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Results

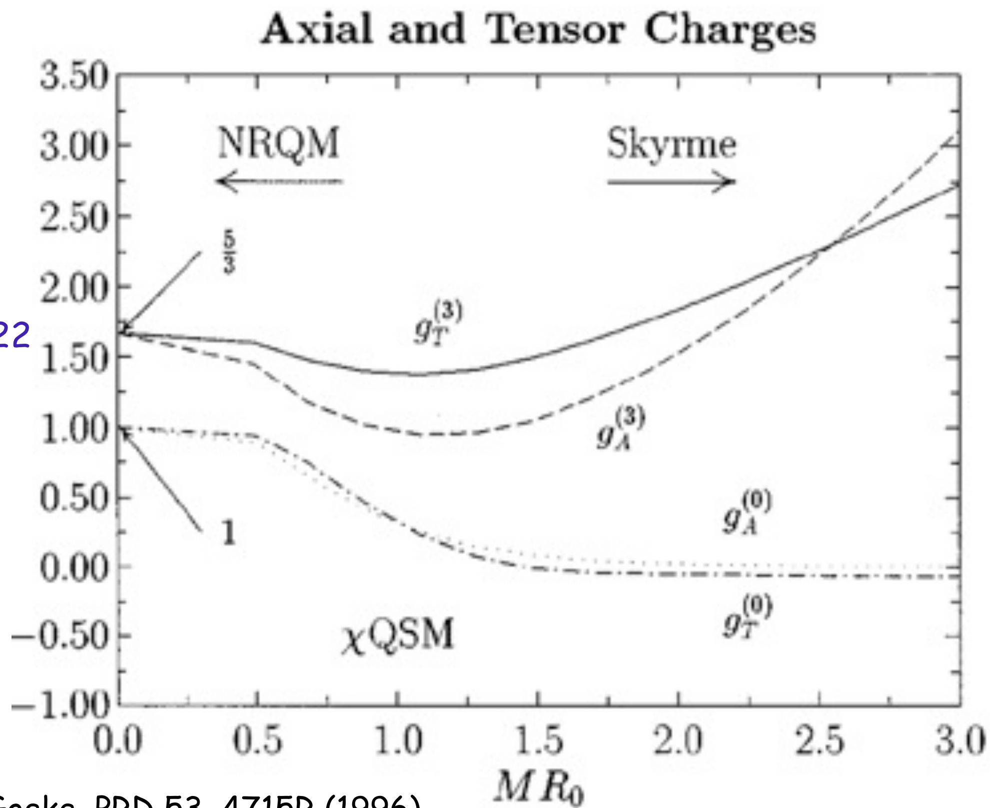
	g_T^0	g_T^3	g_T^8	g_A^0	g_A^3	g_A^8	Δu	δu	Δd	δd	Δs	δs
χQSM SU(3)	0.76	1.40	0.45	0.45	1.18	0.35	0.84	1.08	−0.34	−0.32	−0.05	−0.01
χQSM SU(2)	0.75	1.44	−−	0.45	1.21	−−	0.82	1.08	−0.37	−0.32	−−	−−
NRQM	1	5/3	−−	1	5/3	−−	$\frac{4}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	−−	−−

$$g_A^3 \sim (MR_0)^2 \qquad g_T^3 \sim MR_0$$

$$g_A^0 \sim \frac{1}{(MR_0)^4} \qquad g_T^0 \sim \frac{1}{MR_0}$$

T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

$$g_T^\chi > g_A^\chi$$



HChK, M. Polyakov, K. Goeke, PRD 53, 4715R (1996)

Results

Proton	This work	SU(2)	Lattice	SIDIS	NR
$\lvert\delta d/\delta u\rvert$	0.30	0.36	0.25	$0.42^{+0.0003}_{-0.20}$	0.25

SIDIS [16] ($0.80\,\mathrm{GeV}^2$):	$\delta u = 0.54^{+0.09}_{-0.22}$,	$\delta d = -0.231^{+0.09}_{-0.16}$,
SIDIS [16] ($0.36\,\mathrm{GeV}^2$):	$\delta u = 0.60^{+0.10}_{-0.24}$,	$\delta d = -0.26^{+0.1}_{-0.18}$,
Lattice [21] ($4.00\,\mathrm{GeV}^2$):	$\delta u = 0.86 \pm 0.13$,	$\delta d = -0.21 \pm 0.005$,
Lattice [21] ($0.36\,\mathrm{GeV}^2$):	$\delta u = 1.05 \pm 0.16$,	$\delta d = -0.26 \pm 0.01$,
χQSM ($0.36\,\mathrm{GeV}^2$):	$\delta u = 1.08$,	$\delta d = -0.32$,

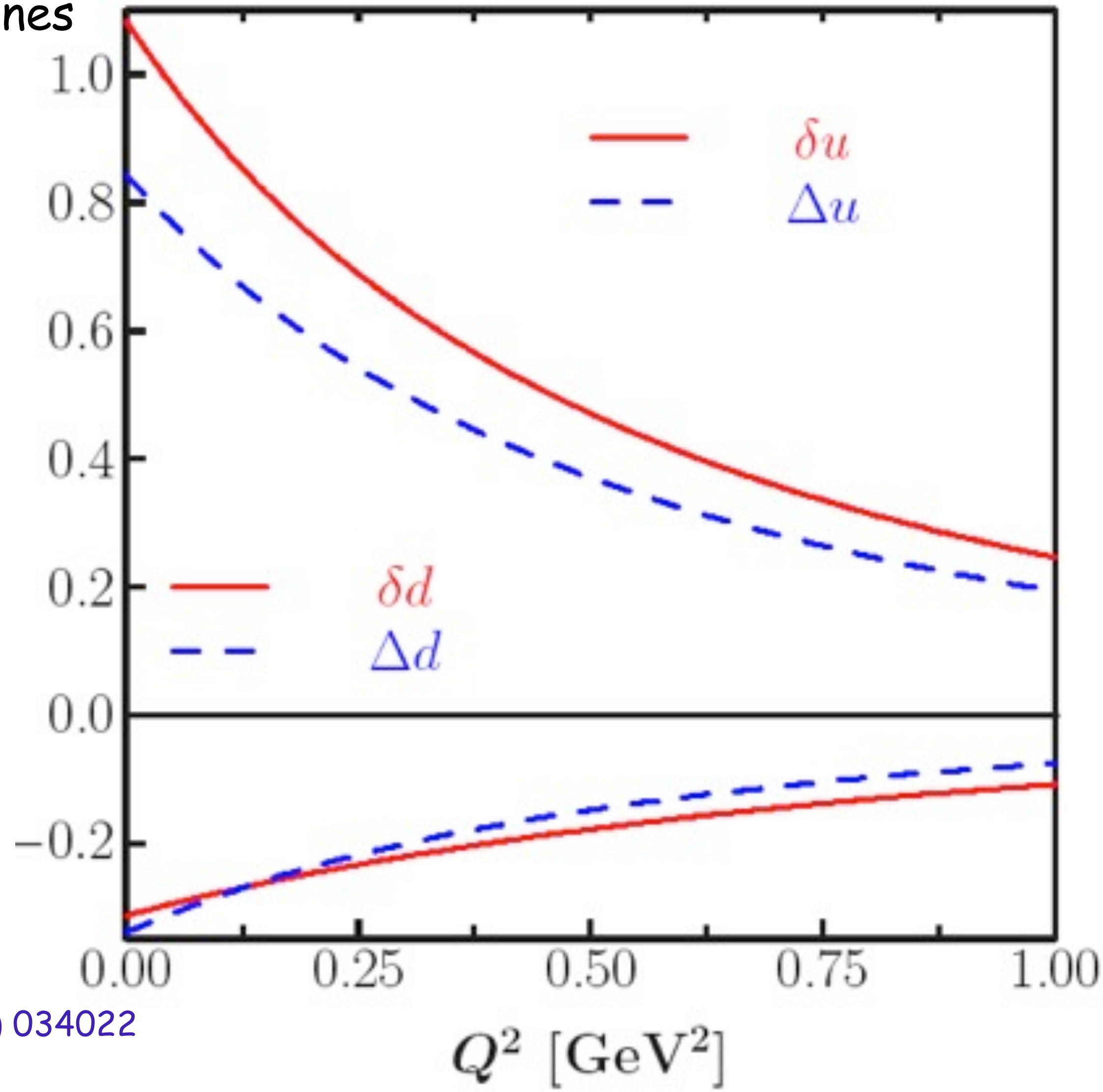
[16] M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

[21] M. Goeckeler et al., PLB 627, 113 (2005)

T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

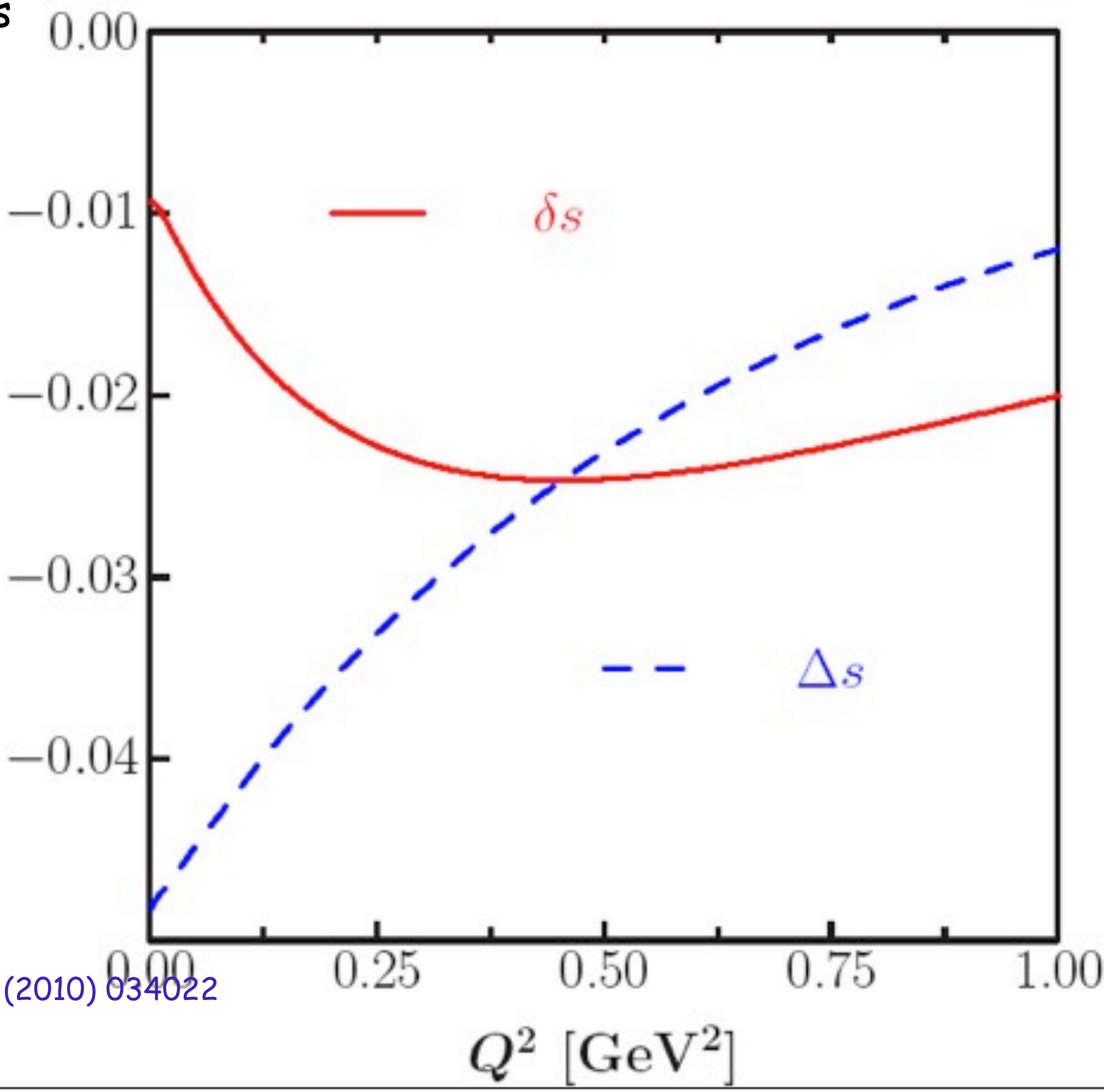
Up and down tensor form factors compared with the axial-vector ones



T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Strange tensor form factors compared with the axial-vector ones

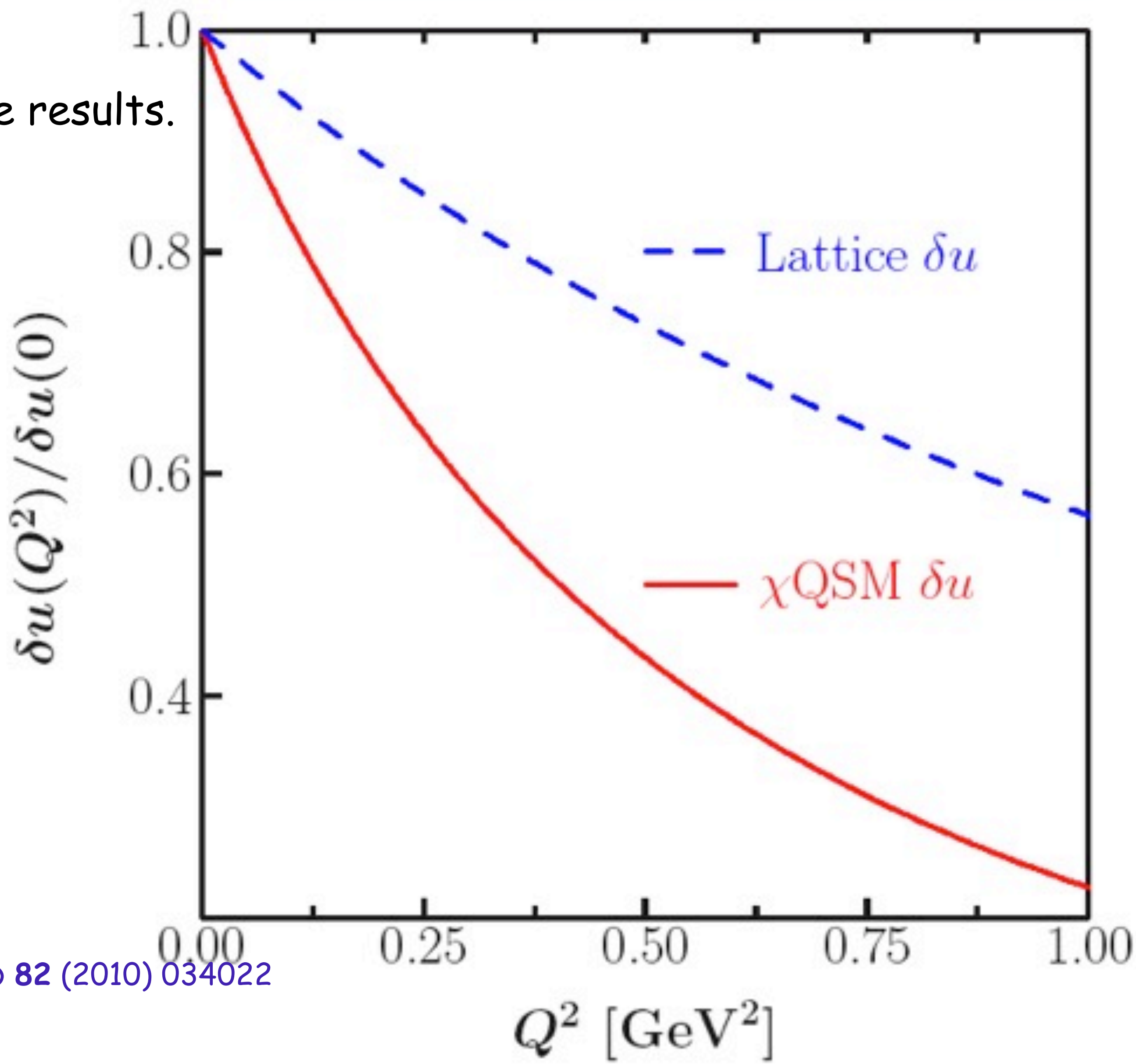


T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Comparison with the lattice results.

M. Goeckeler et al., PLB 627, 113 (2005)



T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Comparison with the lattice results.

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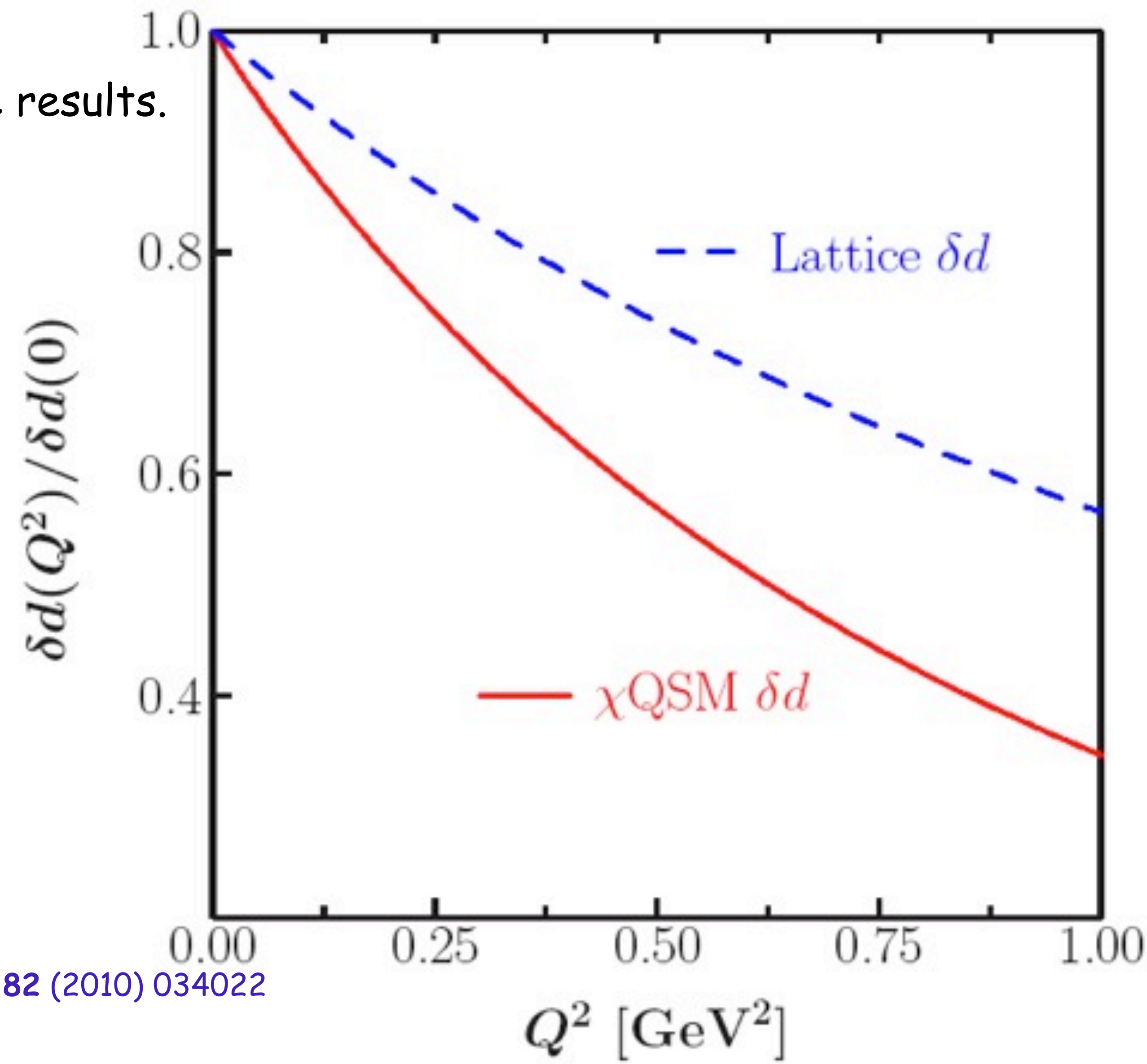


T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

	$p(uud)$	$n(ddu)$	$\Lambda(uds)$	$\Sigma^+(uus)$	$\Sigma^0(uds)$	$\Sigma^-(dds)$	$\Xi^0(uss)$	$\Xi^-(dss)$
δu	1.08	−0.32	−0.03	1.08	0.53	−0.02	−0.32	−0.02
δd	−0.32	1.08	−0.03	−0.02	0.53	1.08	−0.02	−0.32
δs	−0.01	−0.01	0.79	−0.29	−0.29	−0.29	1.06	1.06

Isospin relations

$$\delta u_p = \delta d_n, \quad \delta u_n = \delta d_p, \quad \delta u_\Lambda = \delta d_\Lambda, \quad \delta u_{\Sigma^+} = \delta d_{\Sigma^-},$$
$$\delta u_{\Sigma^0} = \delta d_{\Sigma^0}, \quad \delta u_{\Sigma^-} = \delta d_{\Sigma^+}, \quad \delta u_{\Xi^0} = \delta d_{\Xi^-}, \quad \delta u_{\Xi^-} = \delta d_{\Xi^0},$$
$$\delta s_p = \delta s_n, \quad \delta s_{\Sigma^\pm} = \delta s_{\Sigma^0}, \quad \delta s_{\Xi^0} = \delta s_{\Xi^-},$$

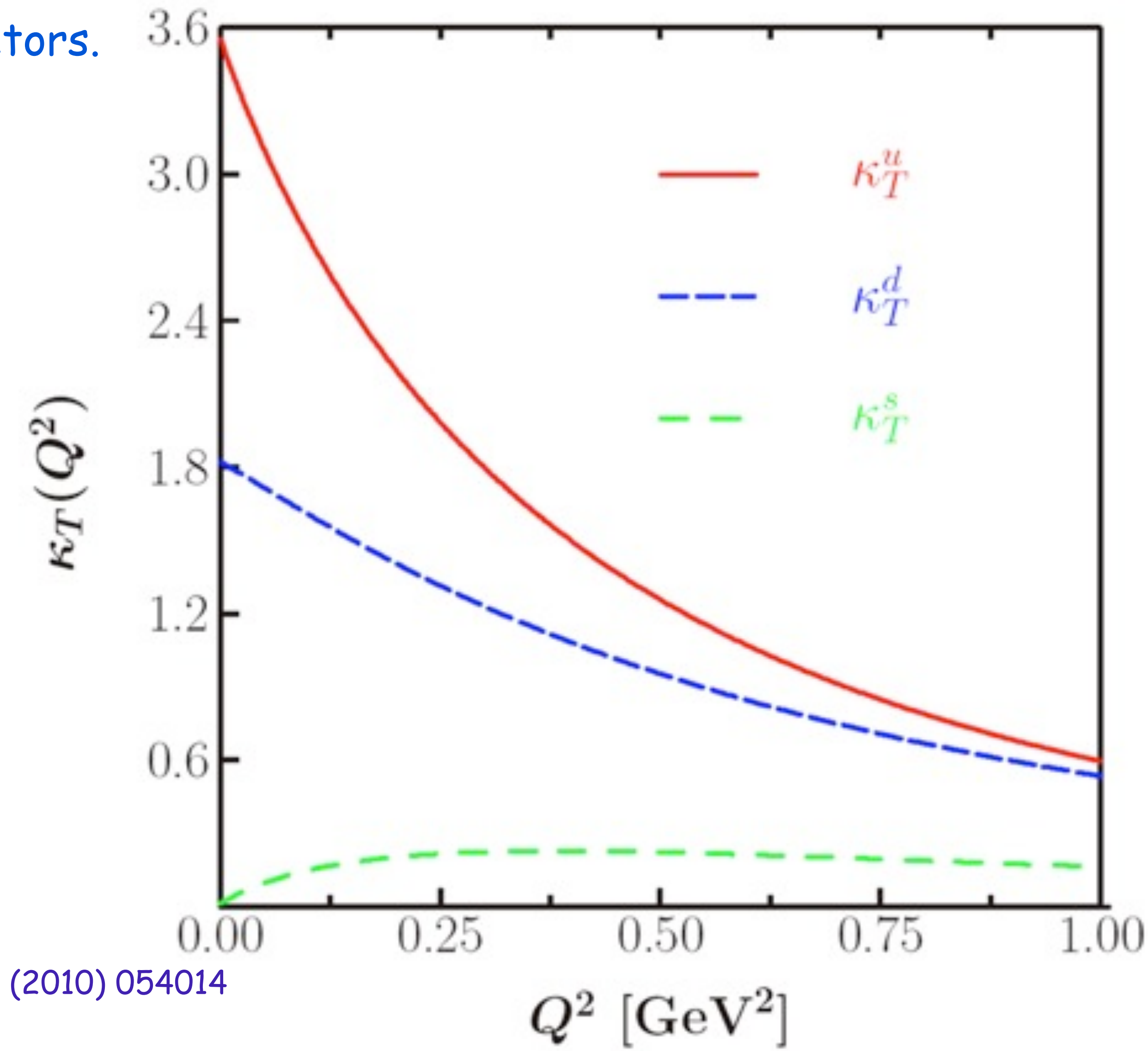
SU(3) relations → Effects of SU(3) symmetry breaking are almost negligible!

$$\delta u_p = \delta d_n = \delta u_{\Sigma^+} = \delta d_{\Sigma^-} = \delta s_{\Xi^0} = \delta s_{\Xi^-},$$
$$\delta u_n = \delta d_p = \delta u_{\Xi^0} = \delta d_{\Xi^-} = \delta s_{\Sigma^\pm} = \delta s_{\Sigma^0}.$$

T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Flavor decomposition of the anomalous tensor magnetic form factors.

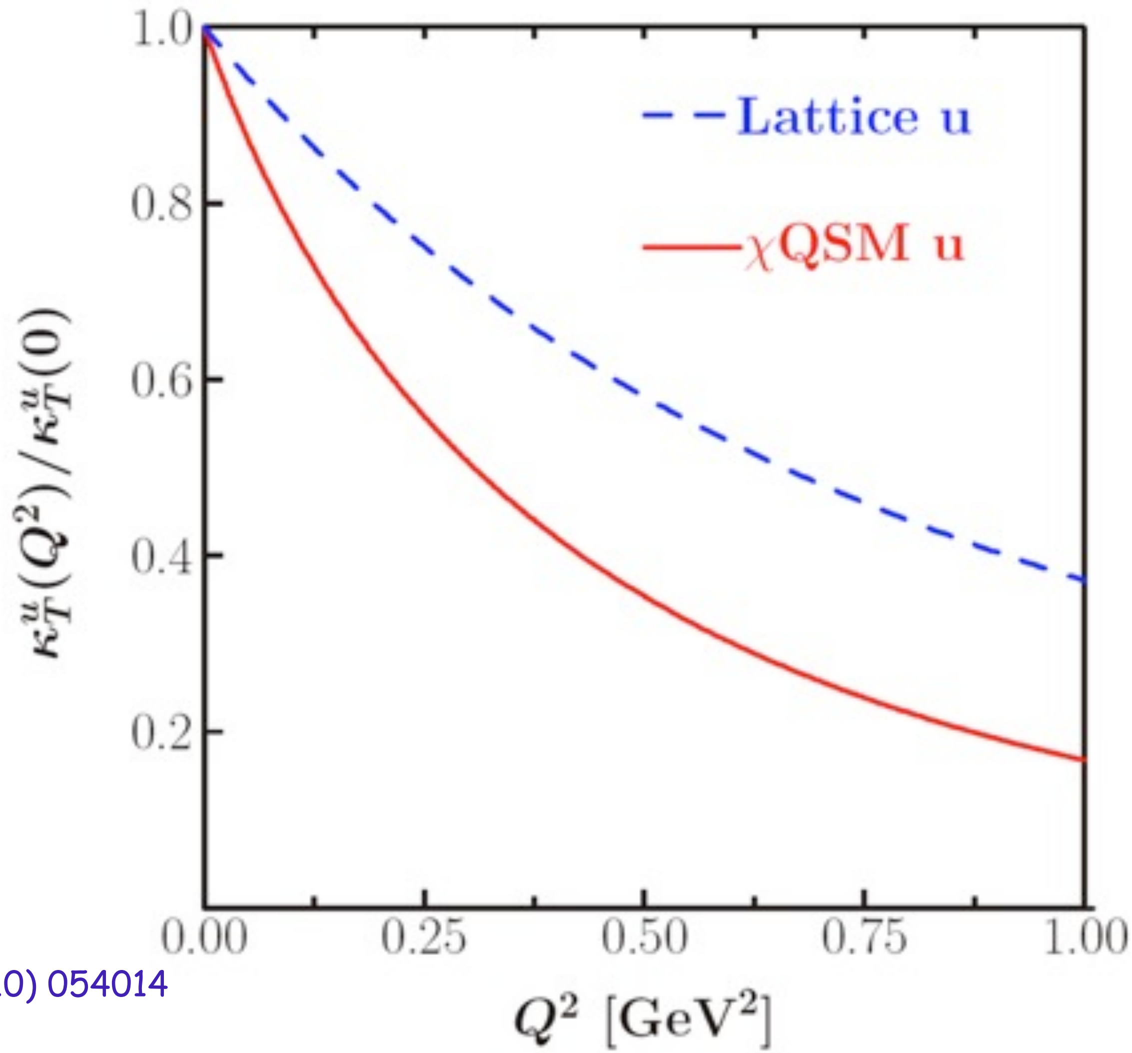


Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Results

Up anomalous tensor magnetic form factors compared with the lattice one.

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)

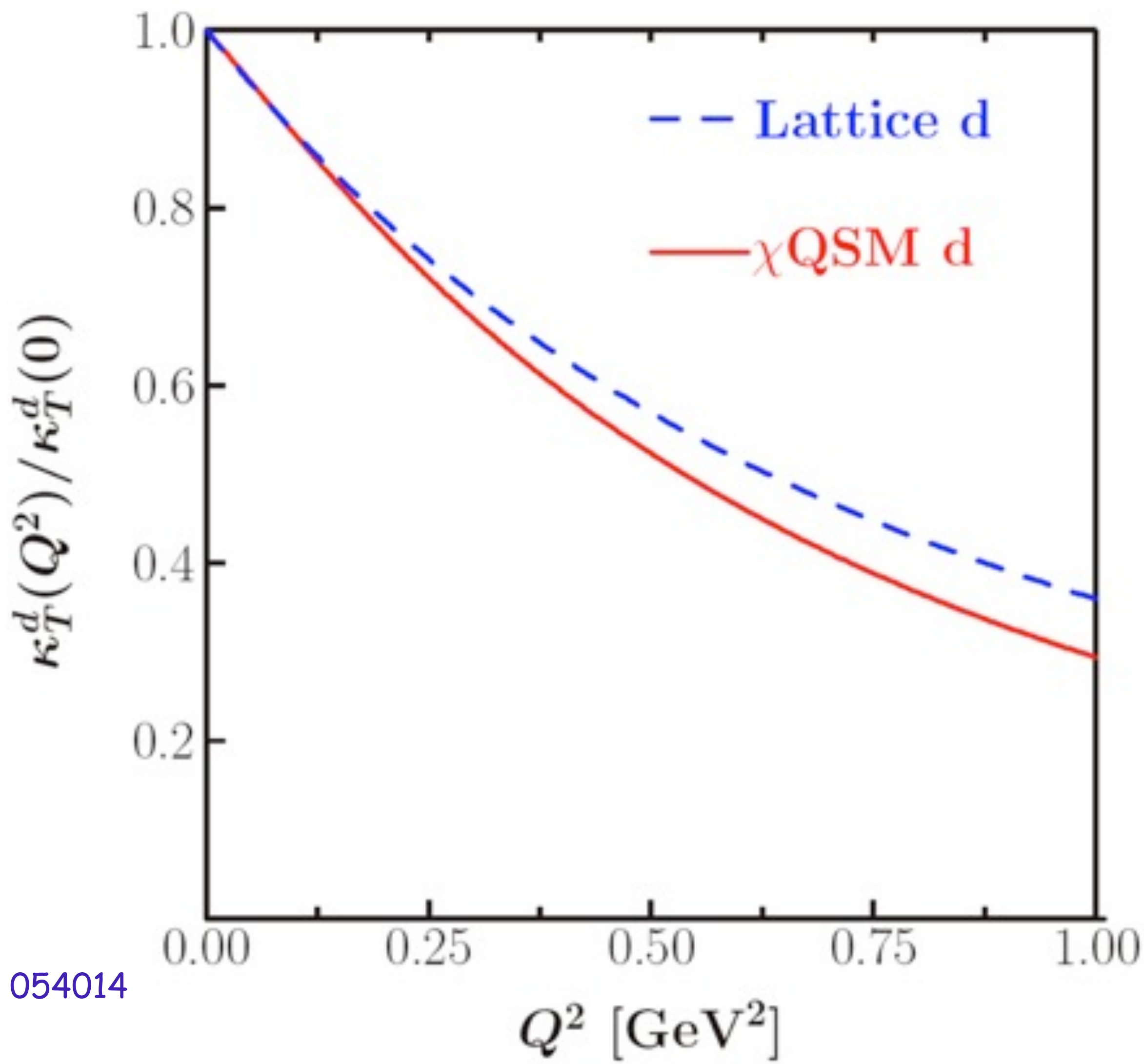


Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Results

Down anomalous tensor magnetic form factors compared with the lattice one.

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)



Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Results

$\mu^2 = 0.36\,\mathrm{GeV}^2$

	Present work SU(3)	Present work SU(2)	Lattice
κ_T^u	3.56	3.72	3.00 (3.70)
κ_T^d	1.83	1.83	1.90 (2.35)
κ_T^s	$0.2 \sim -0.2$		
κ_T^u/κ_T^d	1.95	2.02	1.58

The present results are in good agreement with the lattice data!

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Transverse spin density

$$\rho(\mathbf{b}, \mathbf{S}, \mathbf{s}) = \frac{1}{2}\left[H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial \kappa_T(b^2)}{\partial b^2} \right]$$

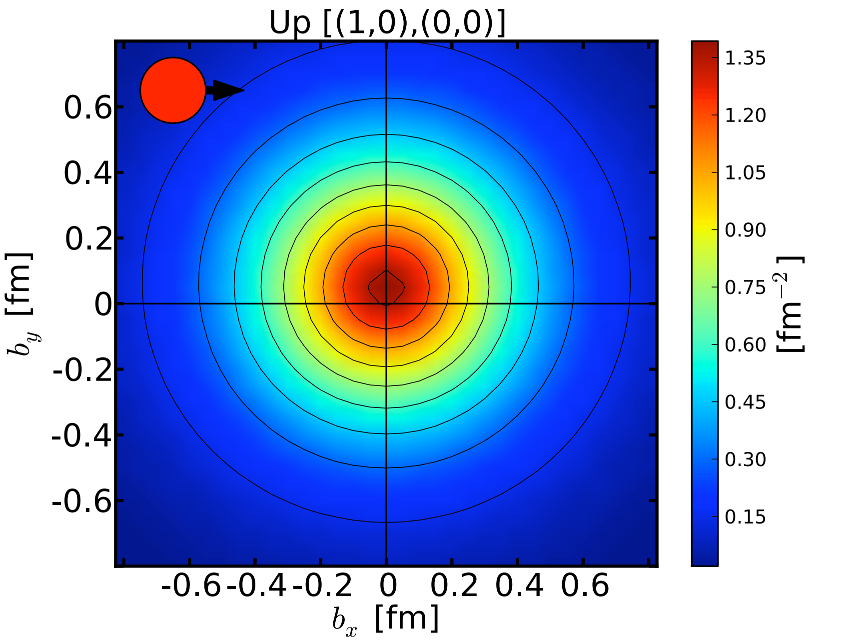
$$[\mathbf{S}, \mathbf{s}] = [(1,0),(0,0)],\ \ [\mathbf{S}, \mathbf{s}] = [(0,0),(1,0)]$$

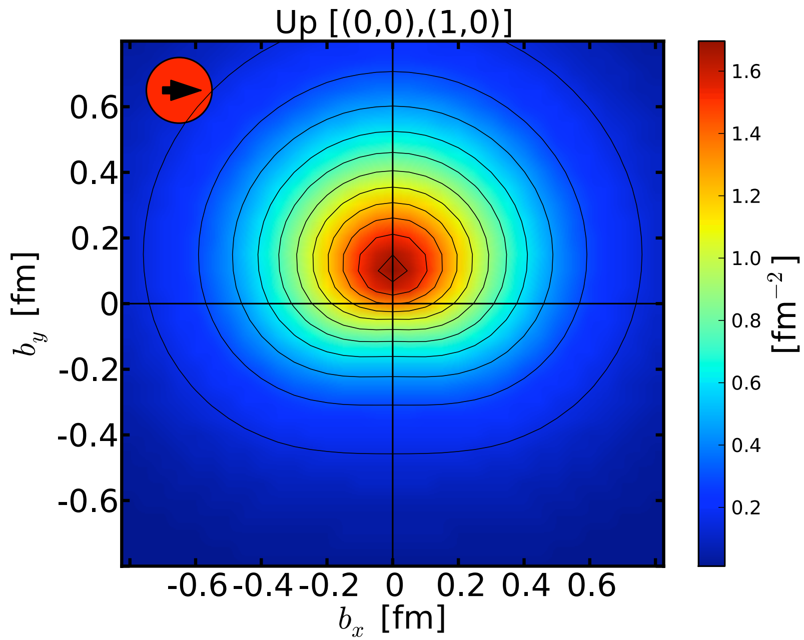
$$\mathcal{F}^\chi(b^2) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F^\chi(Q^2)$$

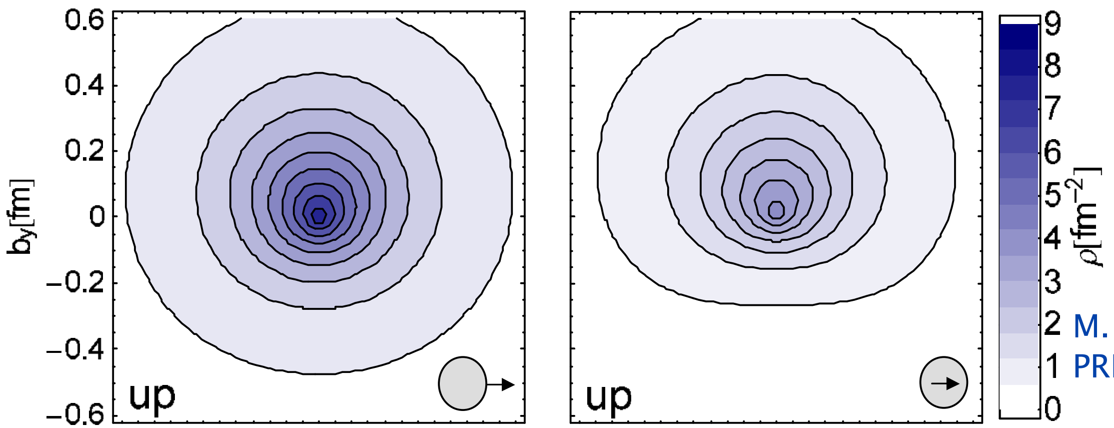
$$H(b^2) = F_1(b^2), \quad E(b^2) = F_2(b^2)$$

Results

Up quark transverse spin density inside a nucleon





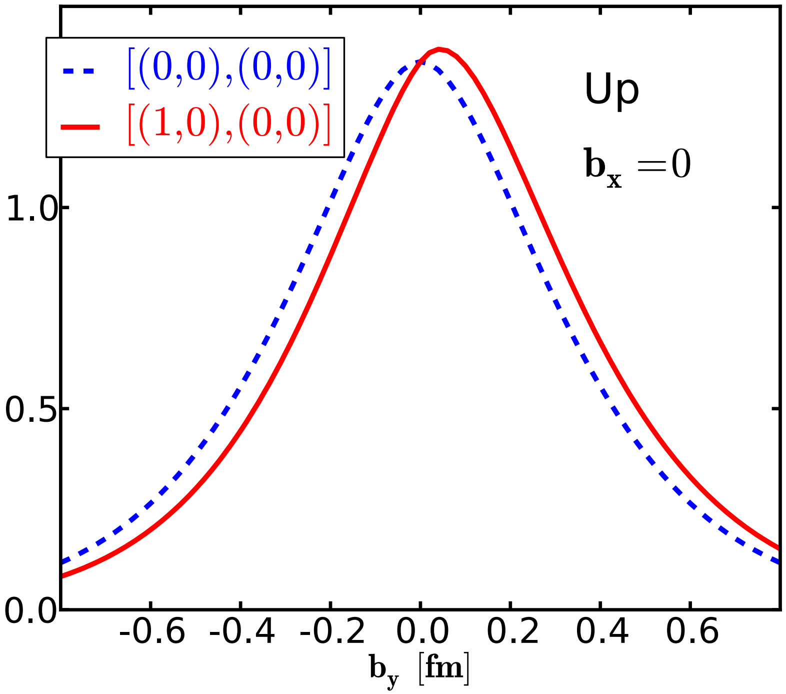


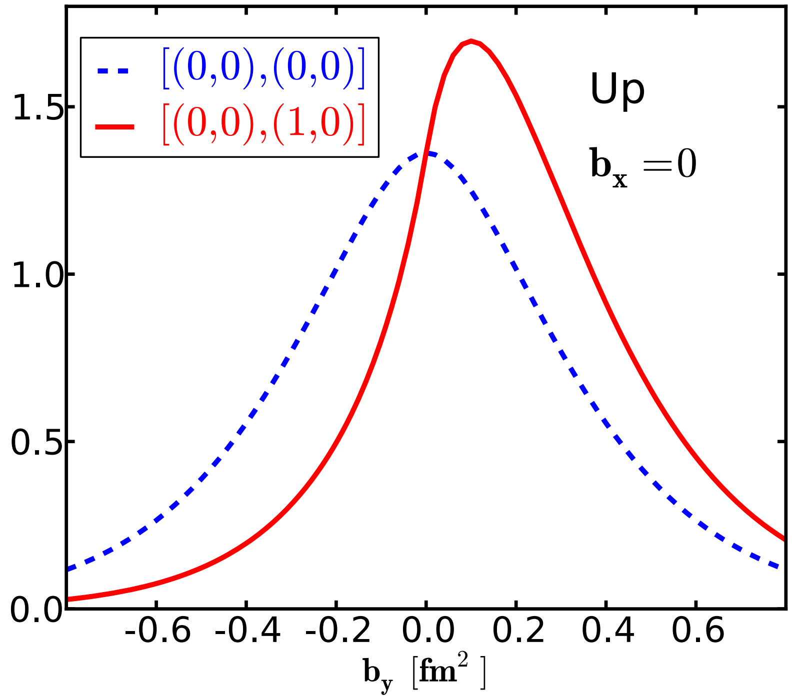
Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

Results

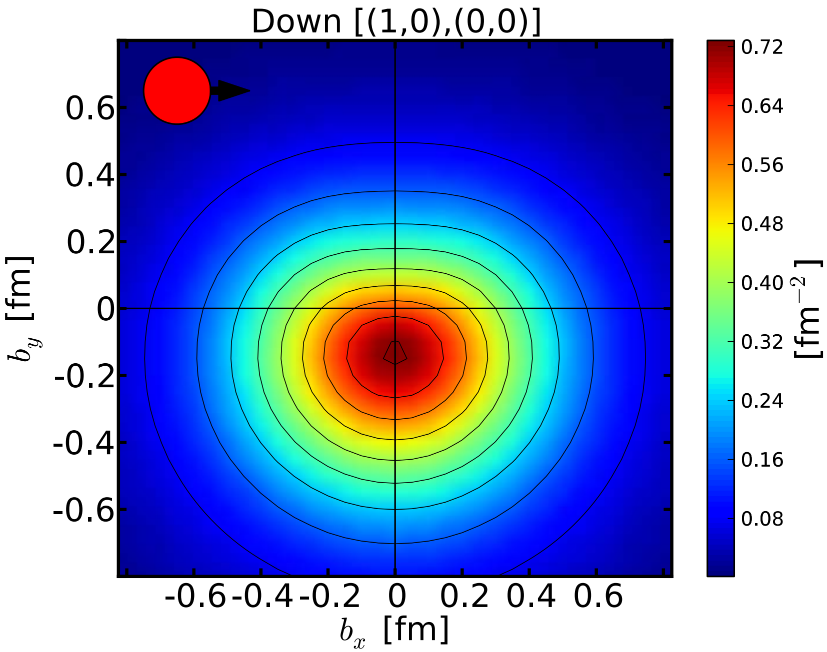
Up quark transverse spin density inside a nucleon

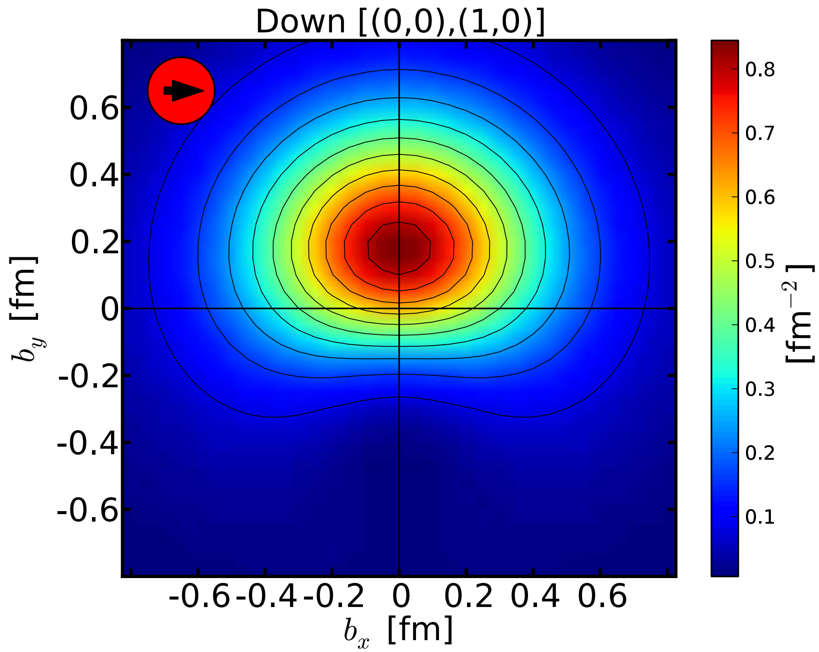


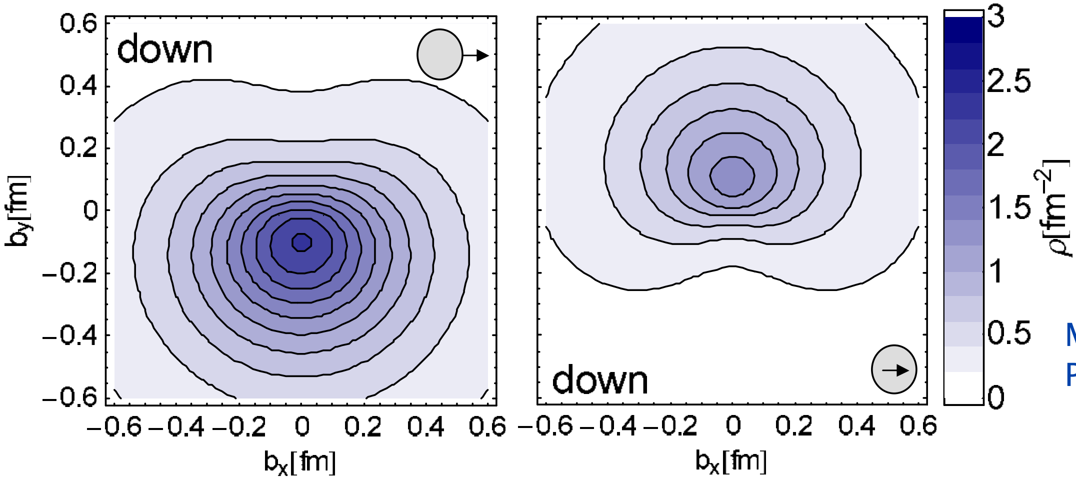


Results

Down quark transverse spin density inside a nucleon





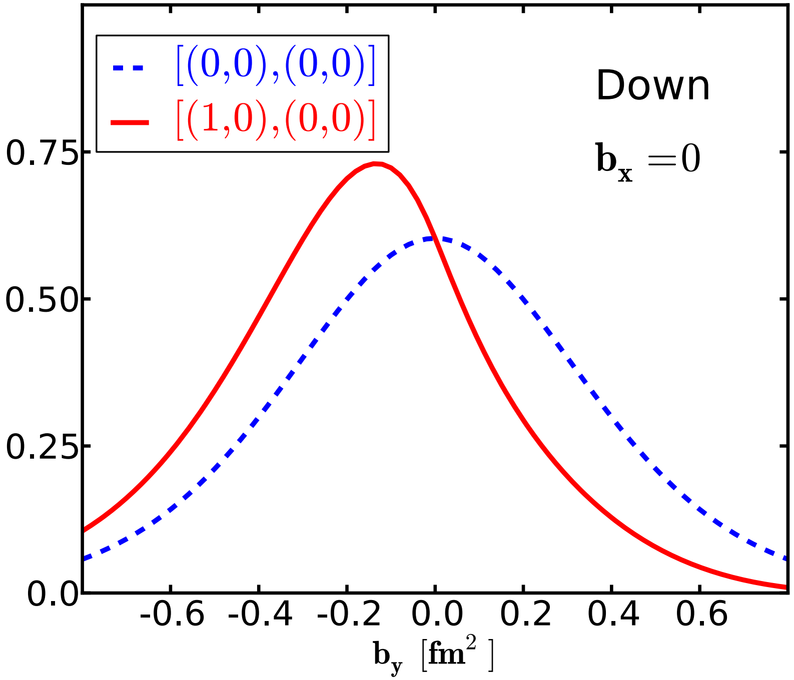


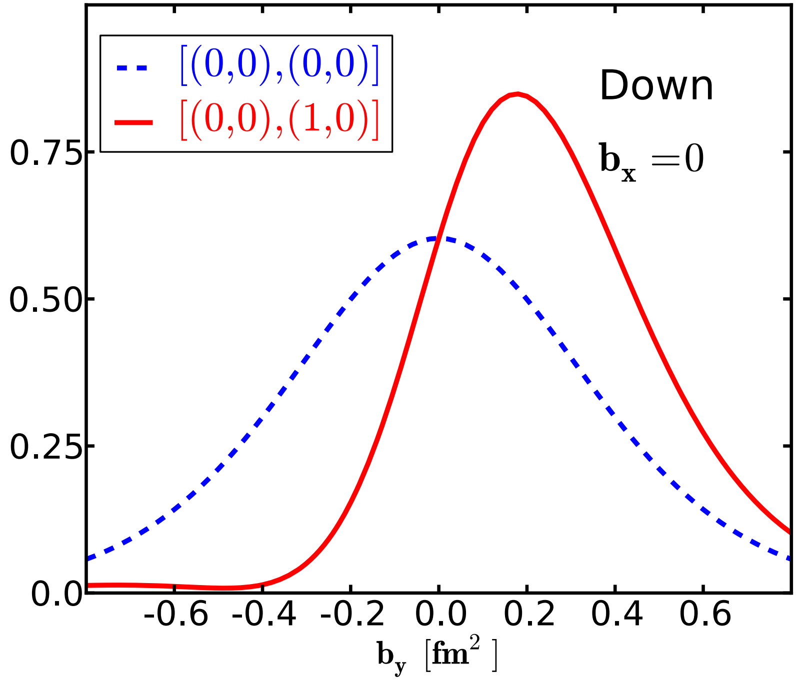
Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

Results

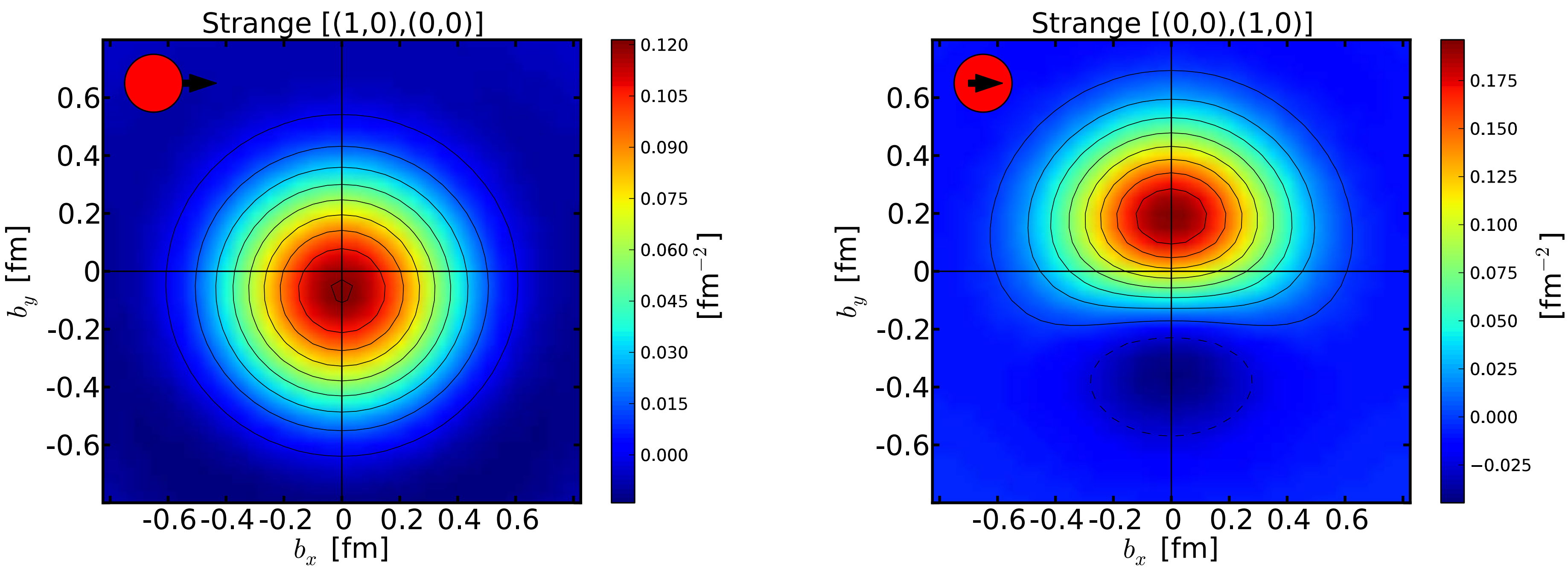
Down quark transverse spin density inside a nucleon





Results

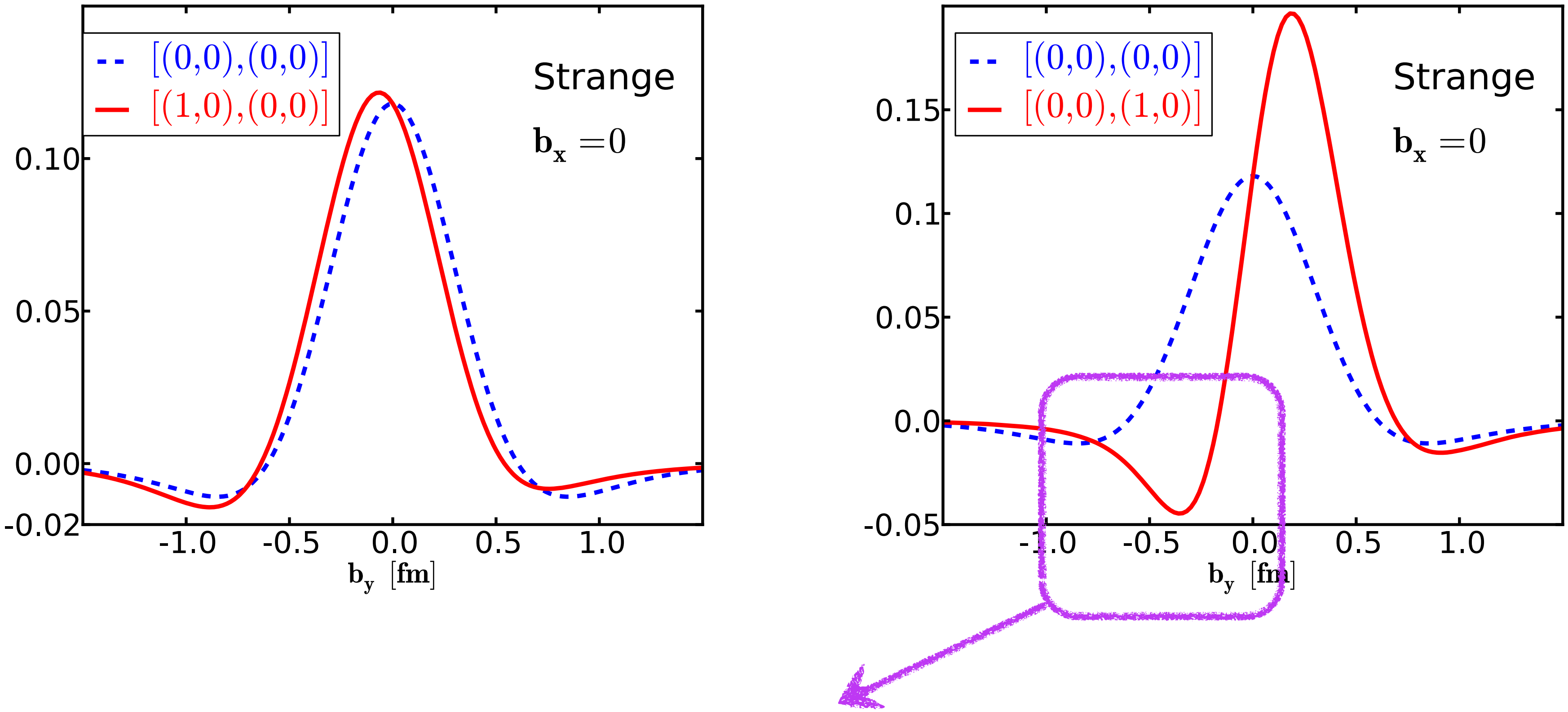
Strange quark transverse spin density inside a nucleon



This is the first result of the strange quark transverse spin density inside a nucleon

Results

Strange quark transverse spin density inside a nucleon



Polarized to the negative direction in the b plane.

Summary

- We have reviewed recent investigations on the spin structures of the pion and nucleon, based on the chiral quark-(soliton) model.
- The results were compared with those of the lattice QCD and turned out to be in good agreement with them.
- The first strange anomalous tensor magnetic moment was obtained, though it is compatible with zero.
- The transverse quark spin densities inside the proton were presented.
- The strange quark transverse spin density was first announced in this work.

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!