

# Selected baryon properties in covariant chiral perturbation theory

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In collaboration with

- L. Alvarez-Ruso, J. Martin Camalich, M. J. V. Vacas (Valencia, Spain)
- M. Altenbuchinger, N. Kaiser, and W. Weise (Munich, Germany)





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# Contents

QCD, chiral symmetry, and Chiral Perturbation Theory (ChPT)

#### Baryon ChPT

- ChPT in the one-baryon sector
- Power counting problem and solutions
- A covariant formulation of the baryon ChPT

Baryon masses and sigma terms in connection with lattice data

SU(3)-breaking corrections to the hyperon vector coupling  $f_1(0)$ and the determination of the  $V_{us}$ 

Baryon magnetic moments (MM)

- MMs of the Octet baryons
- MMs of the Decuplet baryon

#### Summary and Outlook

### QCD—non-perturbative at low energies

Quantum ChromoDynamics—the theory of the strong interaction



### QCD—non-perturbative at low energies

Quantum ChromoDynamics—the theory of the strong interaction



### Effective field theory: (Weinberg 1979)

• There exists a natural cutoff, which allows for a separation of scales such that:

✓ All high-energy dynamics can be integrated out  $\rightarrow$  contact interactions

✓ One only needs to care about low-energy dof's



• Expansion in powers of external energy/momenta Q over the large scale  $\Lambda$  (instead of expansion in terms of the running coupling constant)

$$\mathcal{M} = \sum_{i} \left(\frac{Q}{\Lambda}\right)^{i} C(Q/\mu, g_j)$$

 $\mu$ -regularization scale  $g_j$ -low energy constants

C, encoding short distance physics, should be of

 $\mathcal{O}(1)$  aturalness

### Chiral symmetry and its breaking (I)

Quark mass hierarchy:



### Chiral symmetry and its breaking (II)

• The vacuum is invariant under the vector charge:  $Q_V^a |0
angle = 0$ 

• The vacuum is not invariant under the axial charge:  $Q^a_A |0
angle 
eq 0$ 

- <u>Spontaneous symmetry breaking</u> (Nobel prize in physics, 2008)
- 8 massless Nambu-Goldstone bosons, i.e.,  $\pi^{\pm}, \pi^{0}, K^{\pm}, K^{0}, \bar{K}^{0}, \eta$

Real world:  $\overline{\pi, K, \eta}$  are not massless <u>explicit symmetry breaking</u>  $\mathcal{L}_{QCD} = \mathcal{L}_{QCD}^{0} - \overline{q_R} \mathcal{M} q_L - \overline{q_L} \mathcal{M}^{\dagger} q_R$  $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$ 

small: perturbative treatment

All these features combined with the idea of effective field theory leads to Chiral Perturbation Theory!

### Chiral Perturbation Theory (ChPT) in essence

Maps quark (u, d, s) dof's to those of the asymptotic states, hadrons

$$\mathcal{L}_{\text{QCD}}[q, \bar{q}; G] \to \mathcal{L}_{\text{ChPT}}[U, \partial U, \dots, \mathcal{M}, N]$$

- $\bullet~U$  parameterizes the Nambu-Goldstone bosons
- $\partial U$  vanishes at  $E = \vec{p} = 0$  (Nambu-Goldstone theorem)
- M parameterizes the explicit symmetry breaking
- N denotes interactions with matter fields
- Exact mapping via chiral Ward identities

• ChPT exploits the symmetry of the QCD Lagrangian and its ground state; in practice, one solves in a perturbative manner the constraints imposed by chiral symmetry and unitarity by expanding the Green functions in powers of the external momenta and of the quark masses. (J. Gasser, 2003)

For further details and reviews, look for the following names: S. Weinberg, H. Leutwyler, J. Gasser, Ulf-G Meissner, V. Vernard, N. Kaiser, A. Pich, S. Scherer, and many others....

#### Power-counting-breaking (PCB) in the one-baryon sector

- ChPT very successful in the study of Nanbu-Goldstone boson self-interactions.
   (at least in SU(2))
- In the one-baryon sector, things become problematic because of the nonzero (large) baryon mass in the chiral limit, which leads to the fact that high-order loops contribute to lower-order results, i.e., a systematic power counting is lost!



Chiral order =  $4L - 2N_M - N_B + \sum_k kV_k$ .



Naively  
(no PCB) 
$$M_N = M_0 + bm_\pi^2 + loop$$
  
 $loop(= cm_\pi^3 + \cdots)$ 

However  $loop = aM_0^3 + b'M_0m_{\pi}^2 + cm_{\pi}^3 + \cdots$ 

Chiral order =  $4L - 2N_M - N_B + \sum_k kV_k$ .



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However 
$$loop = aM_0^3 + b'M_0m_{\pi}^2 + cm_{\pi}^3 + \cdots$$

No need to calculate, simply recall that  $M_0 \sim O(p^0)$ 

## Power-counting-restoration methods

- Heavy Baryon ChPT: baryons are treated "semi-relativistically" by a simultaneous expansion in terms of external momenta and 1/M<sub>N</sub> (Jenkins e al., 1993). It converges slowly for certain observables!
- **Relativistic baryon ChPT**: removing power counting breaking terms but retaining higher-order relativistic corrections, thus, keeping relativity.
  - Infrared baryon ChPT (*T. Becher and H. Leutwyler, 1999*)
     Fully relativistic baryon ChPT–Extended On-Mass-Shell (EOMS) scheme (*J. Gegelia et al., 1999; T. Fuchs et al., 2003*)
- IR scheme separates the full integral into the Infrared and Regular parts:

$$H = \frac{1}{ab} = \int_0^1 dz \frac{1}{[(1-z)a + zb]^2} \equiv I + R = \int_0^\infty \dots dz - \int_1^\infty \dots dz$$

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- Heavy Baryon ChPT: baryons are treated "semi-relativistically" by a simultaneous expansion in terms of external momenta and 1/M<sub>N</sub> (Jenkins ε al., 1993). It converges slowly for certain observables!
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- IR scheme separates the full integral into the Infrared and Regular parts:

# H = Infrared

# Extended-on-Mass-Shell (EOMS)

tree = 
$$M_0 + bm_\pi^2$$
 + loop =  $aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \cdots$   
 $\downarrow a = 0; b' = 0$ 

$$M_N = M_0 + b \ m_\pi^2 + cm_\pi^3 + \cdots \ (\mathcal{O}(p^3))$$

# Extended-on-Mass-Shell (EOMS)

tree = 
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tree = 
$$M_0 + bm_\pi^2$$
 + loop =  $aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \cdots$   
 $\bigvee M_0^r = M_0(1 + aM_0^2); b^r = b^0 + b'M_0$   
 $M_N = M_0^r + b^r m_\pi^2 + cm_\pi^3 + \cdots (\mathcal{O}(p^3))$ 

# Extended-on-Mass-Shell (EOMS)

• "Throw away" the PCB terms

tree = 
$$M_0 + bm_\pi^2$$
 + loop =  $aM_0^3 + b'M_0m_\pi^2 + cm_\pi^3 + \cdots$   
 $\downarrow a = 0; b' = 0$ 

$$M_N = M_0 + b \ m_\pi^2 + cm_\pi^3 + \cdots \ (\mathcal{O}(p^3))$$

• Equivalent to redefinition of the LECs

ChPT contains all possible terms allowed by symmetries, therefore  
whatever analytical terms come out from a loop amplitude, they must  
have a corresponding LEC  
$$\mu \mu_{1} - \mu_{$$

# HB vs. Infrared vs. EOMS

- Heavy baryon ChPT
  - non-relativistic
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor space)
- Infrared ChPT
  - breaks analyticity of loop amplitudes
  - converges slowly (particularly in three-flavor space)
- EOMS ChPT
  - satisfies all symmetry and analyticity constraints
  - converges relatively faster--an appealing feature

#### The nucleon scalar form factor at $q^3$



S. Scherer, Prog.Part.Nucl.Phys.64:1-60,2010

#### Proton and neutron magnetic moments: chiral extrapolation



V. Pascalutsa et al., Phys.Lett.B600:239-247,2004.

### Successful applications of Covariant baryon ChPT

Octet (decuplet) baryon magnetic moments:

Phys.Rev.Lett. 101:222002,2008;

Phys.Lett.B676:63-68,2009; Phys.Rev.D80:034027,2009

• Hyperon vector coupling  $f_1(0)$ 

Phys.Rev.D79:094022,2009

Nucleon-Delta axial coupling

Phys.Rev.D78:014011,2008

Octet and Decuplet baryon masses

Phys.Rev.D82:074504,2010; Phys.Rev.D84:074024,2011

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#### Baryon masses, Sigma terms and lattice QCD calculations

Phys.Rev. D82 (2010) 074504

#### Origin of baryon masses

1) Mass of its constituents—quarks In SM, due to the Higgs mechanism → LHC@CERN



#### LHC@CERN

#### 2)Strong interaction—lattice QCD

mass of proton (940 MeV)  $\neq$  sum of current quark masses (~10 MeV).

#### The Weight of the World Is Quantum Chromodynamics

Andreas S. Kronfeld

he reason for excitement surrounding the start-up of the Large Hadron Collider (LHC) in Geneva, Switzerland, has often been conveyed to the general public as the quest for the origin of masswhich is true but incomplete. Almost all of the mass (or weight) of the world we live in comes from atomic nuclei, which are composed of neutrons and protons (collectively called "nucleons"). Nucleons, in turn, are composed of particles called quarks and gluons, and physicists have long believed that the nucleon's mass comes from the complicated way in which gluons bind the quarks to each other, according to the laws of quantum chromodynamics (QCD). A challenge since the introduction of QCD (1-3) has been to carry out an ab initio calculation of the



Ab initio calculations of the proton and neutron masses have now been achieved, a milestone in a 30-year effort of theoretical and computational physics.

> connected to physics and to computation. The first obstacle is describing the "vacuum." In classical physics, the vacuum has nothing in it (by definition), but in quantum field theories, such as QCD, the vacuum contains "virtual particles" that fiit in and out of existence. In particular, the QCD vacuum is a jumble of gluons and quark-antiquark pairs, so to compute accurately in lattice QCD, many snapshots of the vacuum are needed.

The second obstacle is the extremely high amount of computation needed to incorporate the influence of the quark-antiquark pairs on the gluon vacuum. The obstacle

#### Budapest-Marseille-Wuppertal Collaboration



ongoing projects on Blue Gene/P, total sustained performance for QCD: Jülich Supercomputing Centre: 82.5 Teraflops, IDRIS/CNRS: 51,5 Teraflops CPU and GPU clusters, Bergische Universität Wuppertal and at CNRS Marseille 31 Teraflops (sustained for QCD)

#### S. Durr et al., Science 322, 1224(2008).

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### Recent 2+1 f IQCD calculation of the baryon masses

#### Unphysical quark (u&d) masses

- BMW (S. Durr et al.), Science 322, 1224(2008).
  - -2+1 improved Wilson fermions,
  - −mPS ≥ 190 MeV
- <u>PACS-CS</u> (S. Aoki et al.), Phys.Rev.D79:034503, 2009.
  - -2+1 impr. Wilson fermions,
  - $-mPS \ge 160 \text{ MeV}$  no systematic error incl.
- HSC (Huey-Wen Lin et al.), Phys.Rev.D79:034502, 2009.
  - -2+1 anistropic Clover fermions,
  - $-mPS \ge 370 \text{ MeV}$  no systematic error incl.
- LHP (A. Walker-Loud et al.), Phys.Rev.D79:054502, 2009.
   −MA: 2+1 stagg. sea/DWF valence,
   −mPS ≥ 300 MeV
- ETM (C. Alexandrou et al.), Phys.Rev.D80:114503, 2009.
   2 twisted mass fermions, mPS ≥ 270 MeV

#### Physical-point simulations

PACS-CS (S. Aoki *et al.*), Phys.Rev.D81:074503,2010
 -2+1 improved Wilson quark actions

In the present work, we will concentrate on the 2009 PACS-CS data.

### LQCD: numerical implementation of QCD Most results so far have been obtained using unphysical quark masses and have to be extrapolated to the physical point using ChPT

#### ChPT: low-energy effective field theory of QCD

-Powerful but with (many) unknown low-energy-constants (LECs)

-One can use IQCD quark-mass dependent results to determine the values of the LECs and then make predictions for physical observables that are described by the same LECs

Main purpose: use the covariant ChPT to study the lattice masses, determine the relevant LECs and make predictions for the baryon Sigma terms.

#### Chiral extrapolations of baryon masses

#### Pioneering works in 2-flavor space—successful:

-Massimiliano Procura, Thomas R. Hemmert, Wolfram Weise, Phys.Rev.D69:034505,2004. -M. Procura, B.U. Musch, T. Wollenweber, T.R. Hemmert, W. Weise, Phys.Rev.D73:114510,2006.



✓ Recent studies of the latest 2+1 flavor IQCD results using NLO HBChPT have failed miserably, however!

#### • LHPC (A. Walker-Loud et al.), Phys.Rev.D79:054502, 2009.

TABLE XVII. Results from NLO bootstrap  $\chi$  extrapolations of the octet baryon masses, using mixed action (MA) and SU(3) heavy C=1.2(2), D=0.715(50), F=0.453(50)

FIT: NLO	Range	$M_0$ (GeV)	$\sigma_M \; ({\rm GeV}^{-1})$	$\alpha_M \; ({\rm GeV}^{-1})$	$\beta_M$ (GeV <sup>-1</sup> )	С	D	F	$\chi^2$	d.o.f.
$M_N, M_\Lambda,$	007–020: MA	1.087(51)	-0.03(5)	-0.72(8)	-0.62(4)	0.15(9)	0.33(4)	0.14(3)	6.0	5
$M_{\Sigma}, M_{\Xi}$	007–020: SU(3)	1.014(32)	-0.07(4)	-0.77(10)	-0.56(5)	0.18(9)	0.30(6)	0.19(4)	5.5	5
	007-030: MA	1.149(57)	0.01(4)	-0.79(11)	-0.67(7)	0.12(9)	0.38(6)	0.16(3)	14.4	9
	007–030: SU(3)	1.091(66)	-0.04(3)	-0.99(28)	-0.73(19)	0.1(1)	0.44(14)	0.24(7)	11.9	9
	007–040: MA	1.147(52)	0.01(3)	-0.78(10)	-0.68(6)	0.13(9)	0.39(6)	0.16(3)	14.9	13
	007–040: SU(3)	1.090(61)	-0.04(3)	-0.99(26)	-0.73(18)	0.1(1)	0.45(13)	0.25(6)	12.5	13

TABLE XX. Results from NLO bootstrap  $\chi$  extrapolations of the decuplet masses, using mixed action (MA) and SU(3) heavy baryon  $\chi$ PT.

FIT: NLO	Range	$M_{T,0}$ (GeV)	$\bar{\sigma}_M \; ({\rm GeV}^{-1})$	$\gamma_M \; ({\rm GeV}^{-1})$	С	Н	$\chi^2$	d.o.f.
$M_{\Delta}, M_{\Sigma^*},$	007–020: MA	1.68(10)	-0.04(3)	1.2(3)	0.00(07)	1.2(2)	18.9	7
$\stackrel{-}{M_{\Xi^*}}, \stackrel{-}{M_{\Omega^-}}$	007–020: SU(3)	1.52(05)	-0.20(4)	1.3(3)	0.00(15)	1.4(3)	20.3	7
	007-030: MA	1.64(08)	-0.05(2)	1.1(2)	0.00(07)	1.1(2)	21.0	11
	007–030: SU(3)	1.52(04)	-0.19(4)	1.3(3)	0.00(15)	1.4(3)	21.1	11
	007–040: MA	1.73(08)	-0.01(1)	1.2(2)	0.00(06)	1.2(2)	32.8	15
	007–040: <i>SU</i> (3)	1.57(04)	-0.18(4)	1.4(3)	0.00(14)	1.6(2)	34.8	15

mixed action heavy baryon chiral perturbation theory. Both the three-flavor and two-flavor functional forms describe our lattice results, although the low-energy constants from the next-to-leading order SU(3) fits are inconsistent with their phenomenological values. Next-to-next-to-leading order SU(2) continuum

#### Problems reported in SU(3) HBChPT (II)

# PACS-CS (K.-I. Ishikawa), Phys.Rev.D80:054502 (2009), PACS-CS (S. Aoki et al.), Phys.Rev.D79:034503, 2009.

obtained w	blamed with (case 2) and without (case 1) fixing $D, F$ , and C at the phenomenological estimate.										
		NLO									
	LO	Case 1	Case 2	Phenomenological							
$m_B$	0.410(14)	0.391(39)	-0.15(9)								
$\alpha_M$	-2.262(62)	-2.62(62)	-15.3(2.0)								
$\beta_M$	-1.740(58)	-2.6(1.5)	-21.3(3.0)								
$\sigma_M$	-0.53(12)	-0.71(34)	-9.6(1.4)								
D		$0.000(16) \times 10^{-8}$	0.80 fixed	0.80							
F		$0.000(9) \times 10^{-8}$	0.47 fixed	0.47							
$\mathcal{C}$		0.36(30)	1.5 fixed	1.5							
$\chi^2/dof$	1.10(63)	1.39(77)	153(82)								

TABLE VI. Fit results with the SU(3) HBChPT for the octet baryon masses. NLO results are obtained with (case 2) and without (case 1) fixing D, F, and C at the phenomenological estimate.

We investigate the quark mass dependence of baryon masses in 2 + 1 flavor lattice QCD using SU(3) heavy baryon chiral perturbation theory up to one-loop order. The baryon mass data used for the analyses are obtained for the degenerate up-down quark mass of 3 to 24 MeV and two choices of the strange quark mass around the physical value. We find that the SU(3) chiral expansion fails to describe both the octet and the decuplet baryon data if phenomenological values are employed for the meson-baryon couplings. The SU(2) case is also examined for the nucleon. We observe that higher order terms are controlled only around the physical point. We also evaluate finite size effects using SU(3) heavy baryon chiral perturbation theory, finding small values of order 1% even at the physical point.

#### The first covariant ChPT study



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#### Description of the PACS-CS data

TABLE III: Extrapolation in MeV and values of the LECs from the fits to the PACS-CS results [3] on the baryon masses using B $\chi$ PT up to NLO. The  $\chi^2$  is the estimator for the fits to the lQCD results whereas  $\bar{\chi}^2$  include also experimental data. See the text for details.

	GMO	HB	Covariant	Expt.
$M_N$	974(22)	767(21)	896(19)(39)	940(2)
$M_{\Lambda}$	1117(21)	1045(20)	1090(20)(14)	1116(1)
$M_{\Sigma}$	1168(23)	1213(22)	1180(24)(6)	1193(5)
$M_{\Xi}$	1286(22)	1394(21)	1325(24)(20)	1318(4)
$M_{\Delta}$	1321(28)	1267(22)	1224(24)(49)	1232(2)
$M_{\Sigma^*}$	1434(27)	1468(22)	1377(24)(29)	1385(4)
$M_{\Xi^*}$	1548(27)	1624(23)	1532(25)(8)	1533(4)
$M_{\Omega^{-}}$	1662(27)	1735(24)	1687(28)(13)	1672(1)
$M_{B0}$ [MeV]	906(40)	513(32)	760(32)	
$b_0  [{ m GeV}^{-1}]$	-0.261(24)	-1.654(19)	-0.979(38)	
$b_D  [\text{GeV}^{-1}]$	0.042(9)	0.368(9)	0.192(25)	
$b_F [\text{GeV}^{-1}]$	-0.173(7)	-0.824(6)	-0.520(20)	
$M_{D0}$ [MeV]	1250(48)	1122(32)	957(37)	
$t_0  [{\rm GeV}^{-1}]$	-0.11(5)	-0.705(37)	-1.05(8)	
$t_D  [\text{GeV}^{-1}]$	-0.253(10)	-0.738(10)	-0.683(20)	
$\chi^2_{ m d.o.f.}$	0.60	9.3	2.2	
$\bar{\chi}^2_{\rm d.o.f.}$	4.3	36.4	2.8	



- EOMS describes much better the PACS results, but not as well as the LO
- 2. However, EOMS describes better the physical masses.
- 3. HB performs badly, confirming previous studies

Chiral logs still play a role even for m as small as 160 MeV, only 20 MeV from the physical point.

#### Determination of the LECs

Seven LECs determined in the present work:

LEC	Physical masses	+ IQCD data
m <sub>B0</sub>		760(32)
b <sub>o</sub>		-0.979(38)
b <sub>D</sub>	0.199(4)	0.192(25)
b <sub>F</sub>	-0.530(2)	-0.520(20)
<i>m</i> <sub>D0</sub>		957(37)
t <sub>o</sub>		-1.05(8)
t <sub>D</sub>	-0.694(2)	-0.683(20)

- > 4 cannot be determined using only the physical masses:  $m_{B0}$ ,  $b_0$ ,  $m_{D0}$ ,  $t_0$ .
- > 3 can be determined either way,  $b_D$ ,  $b_F$ , and  $t_D$ , we obtained consistent values
- One can then use these LECs to make predictions

#### Predictions: baryon Sigma terms

$\sigma_{\pi B} = \frac{m}{2M_B} \langle B   \bar{u}u + \bar{d}d   B \rangle$
$\sigma_{sB}=\frac{m_s}{2M_B}\langle B \bar{s}s B\rangle$

$$\sigma_{\pi B} = \frac{m_{\pi}^2}{2} \left( \frac{1}{m_{\pi}} \frac{\partial M_B}{\partial m_{\pi}} + \frac{1}{2m_K} \frac{\partial M_B}{\partial m_K} + \frac{1}{3m_\eta} \frac{\partial M_B}{\partial m_\eta} \right)$$
$$\sigma_{sB} = \left( m_K^2 - \frac{m_{\pi}^2}{2} \right) \left( \frac{1}{2m_K} \frac{\partial M_B}{\partial m_K} + \frac{2}{3m_\eta} \frac{\partial M_B}{\partial m_\eta} \right)$$

TABLE IV: Predictions on the  $\sigma_{\pi}$  and  $\sigma_s$  terms (in MeV) in covariant SU(3)-B $\chi$ PT after fitting the LECs to the PACS-CS results.

	N	$\Lambda$	$\Sigma$	Ξ	$\Delta$	$\Sigma^*$	Ξ*	$\Omega^{-}$
$\sigma_{\pi}$	59(2)(17)	39(1)(10)	26(2)(5)	13(2)(1)	55(4)(18)	39(3)(12)	22(3)(6)	4(2)(1)
$\sigma_s$	-7(23)(25)	123(26)(35)	157(27)(44)	264(31)(50)	54(24)(1)	158(27)(7)	272(32)(15)	357(34)(26)



 $\sigma_s$  is a few tens not hundreds MeV—important consequences (see, e.g.,

Joel Giedt, Anthony W. Thomas, Ross D. Young, Phys.Rev.Lett.103:201802,2009.

# Inconsistent with scattering data?

 $\sigma_{\pi N}(\text{ours}) = 59 \pm 2 \pm 17 \,\text{MeV} > 45 \,\text{MeV}(\text{Gasser and Leutwyler})$ 

• A recent analysis showed that the more latest scattering data set prefers a larger  $\sigma_{\pi N}$ 

TABLE II: Values of the  $\mathcal{O}(p^2)$  LECs in units of GeV<sup>-1</sup> and of  $\sigma_{\pi N}$  in MeV obtained from the different  $\pi N$  PW analyses.

	$c_1$	$c_2$	$c_3$	$c_4$	$\sigma_{\pi N}$
KH	-0.80(6)	1.12(13)	-2.96(15)	2.00(7)	43(5)
GW	-1.00(4)	1.01(4)	-3.04(2)	2.02(1)	59(4)
EM	-1.00(1)	0.58(3)	-2.51(4)	1.77(2)	59(2)



FIG. 1: (Color on-line) Phase shifts given by the Lorentz covariant  $\mathcal{O}(p^3) \pi N$  scattering amplitude in the EOMS scheme fitted to the GW solution (circles) [9] up to  $W_{max} = 1.2$  GeV.

arXiv:1110.3797 [hep-ph]

# SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ and the determination of $V_{us}$ from hyperon decay data

Phys.Rev.D79:094022,2009.

### $V_{\mu s}$ , CKM unitarity, and the $f_1(0)$

Cabibbo-Kobayashi-Maskawa (CKM) matrix plays a very important role in our study and understanding of flavor physics

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Particularly, an accurate value of  $V_{us}$  is crucial in determinations of the other parameters and in tests of CKM unitarity, e.g., the 1st row unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.$$

 $4V_{\mu\nu}$ : small, can be neglected at the present precision

V<sub>ud</sub>: superallowed nuclear beta-decays, neutron decays, pion decays ↓ V<sub>us</sub>: kaon decays, tau decays, <u>hyperon decays</u>





### $V_{\mu s}$ from hyperon decays and the Ademollo-Gatto theorem

- To determine  $V_{\mu s}$  from hyperon decays, one must know the hyperon vector coupling  $f_1(0)$  since experimentally only  $|V_{us} f_1(0)|$  is accessible.
- Theoretically,  $f_1(0)$  is known up to SU(3) breaking effects due to the hypothesis of Conservation of Vector Current (CVC). To obtain an accurate  $f_1(0)$ , one then needs to know the size of SU(3) breaking, which could be (naively)  $\sim$ 30%.
- On the other hand, the Ademollo-Gatto theorem tells that \*

 $f_1(0) = g_V + \mathcal{O}((m_s - m)^2)$  PRL 13, 264 (1964).

M. Ademollo and R. Gatto.

which 1) implies that SU(3) breaking corrections are of  $\sim 10\%$ ;

2) has an important consequence for a ChPT study

#### Theoretical determination of SU(3) breaking corrections to $f_1(0)$

Theoretical methods used to calculate  $f_1(0)$ :

• Quark models: J. F. Donoghue et al., 1987; F. Schlumpf, 1995;

A. Faessler et al., 2008, etc.

- Large Nc : *R. Flores-Mendieta, 2004.*
- Lattice QCD : D. Guadagnoli et al., 2007; S. Sasaki et al., 2008.
- ChPT : A. Krause, 1990; J. Anderson et al., 1993; N. Kaiser, 2001;
   G. Villadoro, 2006; A. Lacour et al., 2007.

#### Purpose of the present study:

to calculate SU(3) breaking corrections to  $f_1(0)$  using covariant ChPT

Two improvements compared to earlier ChPT studies:

Removal of power-counting-restoration (PCR) dependence

Errst covariant, order 4, taking into account dynamical decuplet contributions

#### AG theorem's implication and the 1<sup>st</sup> novelty of the present work

**L**The first improvement is possible, because according to the Ademollo-Gatto theorem

$$f_1(0) = g_V + \mathcal{O}((m_s - m)^2)$$

No analytical terms breaking SU(3) symmetry with chiral order less than or equal to 4 in ChPT, both at tree- and loop- levels

No need to apply any power-counting restoration (PCR) procedures (e.g.,

HB or IR), which thus removes the PCR dependence.

2<sup>nd</sup> improvement: contributions of dynamical decuplet baryons



Second, we have taken into account the contributions of virtual decuplet baryons.

- ◆ They are important because  $m_D$ - $m_B$ ~0.231 GeV is similar to pion mass and smaller than kaon (eta) mass. Therefore, in SU(3) ChPT, the exclusion of decuplet baryons is not well justified.
- ♦As I will show, the decuplet baryons do provide sizable contributions that completely change the results obtained with only dynamical octet baryons

#### Definition of $f_1(0)$ and notations used in this work

Baryon vector form factors as probed by the charged  $\Delta S=1$  weak current

$$\langle B'|V^{\mu}|B\rangle = V_{us}\bar{u}(p')\left[\gamma^{\mu}f_1(q^2) + \frac{2i\sigma^{\mu\nu}q_{\nu}}{M_{B'} + M_B}f_2(q^2) + \frac{2q^{\mu}}{M_{B'} + M_B}f_3(q^2)\right]u(p),$$

We will parameterize the SU(3)-breaking corrections order-by-order in the covariant chiral expansion as follows:

$$f_1(0) = g_V \left( 1 + \delta^{(2)} + \delta^{(3)} + \cdots \right)$$

where and  $\delta^{(2)}$  are the LO and NLO SU(3)-breaking corrections induced by loops, corresponding to and  $O(p^3)$  chiral calculations.

Note :two order schemes, SU(3) breaking and chiral

#### Dynamical octet and decuplet contributions



WF normalization diagrams not shown

### Comparison with the results of other approaches

	Present work	Large $N_c$	quenched LQCD			
		Ref. [3]	Ref. [11]	Ref. [12]	Ref. [13]	
$\Lambda \to N$	$0.1^{+1.3}_{-1.0}$	$2\pm 2$	-1.3	-2.4	0.1	
$\Sigma \to N$	$8.7^{+4.2}_{-3.1}$	$4\pm3$	-1.3	-2.4	0.9	$-1.2 \pm 2.9 \pm 4.0$ [19]
$\Xi\to\Lambda$	$4.0^{+2.8}_{-2.1}$	$4\pm4$	-1.3	-2.4	2.2	
$\Xi \to \Sigma$	$1.7^{+2.2}_{-1.6}$	$8\pm5$	-1.3	-2.4	4.2	$-1.3 \pm 1.9$ [20]

TABLE V: SU(3)-breaking corrections (in percentage) to  $f_1(0)$  obtained in different approaches.

Consistent with the large Nc results and those of chiral quark models
Agree marginally with the quenched LQCD results

[3] R. Flores-Mendieta, PRD 70, 114036 (2004).
[11] J. F. Donoghue et al., P.RD 35, 934 (1987).
[12] F. Schlumpf, PRD 51, 2262 (1995).
[13] A. Faessler et al., PRD 78, 094005 (2008).
[19] D. Guadagnoli et al., NPB 761, 63 (2007)
[20] S. Sasaki et al., arXiv:0811.1406 [hep-ph].

### Implications for the value of $V_{us}$

- ◆Full analysis using all data sets is complicated.
- A simple (naive) calculation
  - •Using only experimental decay rates and  $g_1/f_1$  ratios
  - •Using SU(3) symmetric values of  $g_2=0$  and  $f_2$
  - •Using our calculated  $f_1(0)$
  - •We obtain

$$V_{us} = 0.2177 \pm 0.0030$$

Dr. Nicola Cabibbo et al. have performed a similar calculation but have used SU(3) symmetric  $f_1(0)$ , they obtained

$$V_{us} = 0.2250(27)$$
 PRL 92, 251803 (2004)

• Comparing these two numbers, one can easily see the importance of  $f_1(0)$ .

♦ Of course, both numbers should be taken with caution.

### Baryon magnetic moments in EOMS BChPT

*Phys.Rev.Lett.*101:222002,2008. *Phys.Lett.*B676:63-68,2009.

#### Why baryon magnetic moments?

Magnetic moment, a measure of the strength of a system's net magnetic source, is a fundamental concept in physics, but it is also widely applied in other fields, such as chemistry, electrical engineering, etc.

Elementary particles have intrinsic magnetic moments; All point-like charged Dirac particles should have a g-factor  $\sim 2$  and all neutral ones should have  $g_{s}\sim 0$ ; the contrary indicates that the particles have an internal structure.

For instance, the measured proton  $g_s = 5.58$  and neutron  $g_s = -3.82$  show that they are not point-like particles– a clear indication of their composite nature. Nowadays, we know they are composed of 3 (constituent) quarks.

Measuring and understanding magnetic moments of hadrons have played and will continue to play an important role in our understanding of their properties.

### Octet baryon magnetic moments

Phys.Rev.Lett.101:222002,2008.

#### Magnetic moments of the baryon octet and SU(3) flavor symmetry



- Coleman-Glashow (C-G) relations (1961):
  - Related to the measured proton and neutron MMs;

$$\mu_{\Sigma^+} = \mu_p \quad 2\mu_{\Lambda} = \mu_n \quad \mu_{\Sigma^-} + \mu_n = -\mu_p$$
$$\mu_{\Sigma^-} = \mu_{\Xi^-} \quad \mu_{\Xi^0} = \mu_n \quad 2\mu_{\Lambda\Sigma^0} = -\sqrt{3}\mu_n$$

• Nothing but the SU(3) flavor symmetry.

	р	п	٨	$\Sigma^{-}$	Σ0	$\Sigma^+$	Ξ	$\Xi^0$	$\Lambda\Sigma^0$
Exp.	2.79	-1.91	-0.61	-1.16		2.46	-0.65	-1.25	1.61
C-G	2.56	-1.60	-0.80	-0.97	0.80	2.56	-0.97	-1.60	1.38

• The SU(3) f-symmetry is **broken**, as indicated in the above comparison!

#### A natural question:

How to implement SU(3) breaking in a systematic and well-controlled way– Chiral Perturbation Theory.

#### ChPT and the baryon octet MMs—a puzzle of 30 years!

- The ChPT study of the leading SU(3) breaking effects on the MMs of the baryon octet has a long history, which spans over 30 years:
  - Pioneering works of Caldi and Pagels on leading chiral corrections (1974).
  - Systematic HBChPT calculations of Jenkins et al. (1993), Meissner et al. (1997), and others, up to NNLO.
  - Infrared relativistic BChPT by Kubis and Meissner (2001), up to NNLO.
- All the early attempts found: The leading SU(3) breaking effects induced by loops are too large and always worsen the SU(3) symmetric description.
- Many possibilities have been explored to understand this "apparent" failure of ChPT

But as we will show in this talk, analyticity and relativity play a very important role!

### MMs of the baryon octet in ChPT: formalism

- Definition:
  - The matrix element of the electromagnetic current can be parameterized by two form factors:

$$\langle \psi(p')|J^{\mu}|\psi(p)\rangle = |e|\overline{u}(p')\left\{\gamma^{\mu}F_{1}(t) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M}F_{2}(t)\right\}u(p).$$

- At  $q^2 = 0$ :  $F_1(0) = Q$  (charge),  $F_2(0) = \kappa$  (anomalous magnetic moment).
- The fermion (baryon) MM:  $\mu \equiv (Q + \kappa) \frac{|e|}{2M} \Longrightarrow \kappa$  is calculated!



• SU(3)-symmetric contribution *(a)* contains 2 low-energy constants (LECs).

• SU(3)-breaking induced by meson masses through loop diagrams (b) and (c).

#### EOMS BChPT results: Analytical

 SU(3)-symmetric description (order 2) depends on Clebsch-Gordan coeff., known MB couplings, and two LECs. b<sub>D</sub> and b<sub>F</sub>:

$$\kappa_B^{(2)} = \alpha_B \boldsymbol{b}^D + \beta_B \boldsymbol{b}^F$$

SU(3)-breaking is induced by loop-functions H<sup>(b)</sup>(m) and H<sup>(c)</sup>(m), which are convergent, do not contain unknown LEC's, and are genuine predictions of ChPT at this order!:

$$\kappa_B^{(3)} = \frac{1}{8\pi^2 F_{\phi}^2} \left( \sum_{r=\pi,K} \xi_{BM}^{(b)} H^{(b)}(m_r) + \sum_{r=\pi,K,\eta} \xi_{BM}^{(c)} H^{(c)}(m_r) \right)$$
  
$$H^{(b)}(m) = -M^2 + 2m^2 + \frac{2m(m^4 - 4m^2M^2 + 2M^4)}{M^2\sqrt{4M^2 - m^2}} \arccos\left(\frac{m}{2M}\right) + \frac{m^2}{M^2} (2M^2 - m^2) \log\left(\frac{m^2}{M^2}\right)$$
  
$$H^{(c)}(m) = M^2 + 2m^2 + \frac{2m^3(m^2 - 3M^2)}{M^2\sqrt{4M^2 - m^2}} \arccos\left(\frac{m}{2M}\right) + \frac{m^2}{M^2} (M^2 - m^2) \log\left(\frac{m^2}{M^2}\right).$$

The  $M^2$  terms break power counting, and have to be removed.

### EOMS vs. HBChPT and IR BChPT

• EOMS:  $H^{(b,c)} \equiv H^{(b,c)}_{\text{full}}$ , fulfills both relativity and analyticity!

$$b_6^D \longrightarrow \tilde{b}_6^D = b_6^D + \frac{3DFM_B^2}{2\pi^2 F_\phi^2}, \quad b_6^F \longrightarrow \tilde{b}_6^F = b_6^F$$

Heavy Baryon ChPT, breaks both!

$$H^{(b)}(m)\simeq \pi m M_B+\cdots, \quad H^{(c)}(m)\simeq 0+\cdots$$

• Infrared ChPT:  $H^{(b,c)} = H^{(b,c)}_{full} - R^{(b,c)}$ , breaks analyticity!

$$R^{(b)}(m) = -M_B^2 + \frac{19m^4}{6M_B^2} - \frac{2m^6}{5M_B^4} + \cdots ,$$
  

$$R^{(c)}(m) = M_B^2 + 2m^2 + \frac{5m^4}{2M_B^2} - \frac{m^6}{2M_B^4} + \cdots$$

#### EOMS BChPT results: Numerical

	р	п	٨	$\Sigma^{-}$	$\Sigma^0$	$\Sigma^+$	Ξ-	$\Xi^0$	$\Lambda\Sigma^0$	$\chi^2$
C-G	2.56	-1.60	-0.80	-0.97	0.80	2.56	-0.97	-1.60	1.38	0.46
HB	3.01	-2.62	-0.42	-1.35	0.42	2.18	-0.52	-0.70	1.68	1.01
IR	2.08	-2.74	-0.64	-1.13	0.64	2.41	-1.17	-1.45	1.89	1.83
EOMS	2.58	-2.10	-0.66	-1.10	0.66	2.43	-0.95	-1.27	1.58	0.18 ★
Exp.	2.79	-1.91	-0.61	-1.16		2.46	-0.65	-1.25	1.61	

 $\chi^2 = \sum (\mu_{th} - \mu_{exp})^2$ 

• Contribution of the chiral series [LO(1+NLO/LO)]:

$$\begin{split} \mu_p &= 3.47(1-0.257), \quad \mu_n = -2.55(1-0.175), \quad \mu_\Lambda = -1.27(1-0.482), \\ \mu_{\Sigma^-} &= -0.93(1+0.187), \quad \mu_{\Sigma^+} = 3.47(1-0.300), \quad \mu_{\Sigma^0} = 1.27(1-0.482), \\ \mu_{\Xi^-} &= -0.93(1+0.025), \quad \mu_{\Xi^0} = -2.55(1-0.501), \quad \mu_{\Lambda\Sigma^0} = 2.21(1-0.284). \end{split}$$

✓ The EOMS NLO-calculation improves the C-G relations!
 ✓ Better convergence: NLO/LO ~50%; vs. ~70% and ~300% in HB and IR!

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- The three approaches agree in the vicinity of the chiral limit.
- IR and EOMS coincide up to  $x \sim 0.4$ . IR description then gets worse.
- Shaded area(s) represent the variation 0.8GeV  $\leq M_0 \leq 1.1$ GeV.
- Only EOMS exhibits a proper behavior (at this order)!

### **Decuplet baryon magnetic moments**

Phys.Rev.D80:034027,2009.

Renewed interests in the MM's of the decuplet baryons

**\clubsuit** Renewed interests in the MDMs of  $\Delta$ ++ and  $\Delta$ +,

- Experiments: e.g., Kotulla (2008)
- Lattice QCD: Leinweber et al. (1992), Lee et al. (2005), Aubin et al. (2008), Alexandrou et al. (2009), Boinepalli et al. (2009), ...



There exist many theoretical predictions, such as Quark models, QCD sum rules, large Nc, HBChPT,

### **Definitions and Feynman diagrams**





Diagrams up to NLO

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#### Lagrangians and parameter values

In order to calculate the previously shown diagrams, one needs

$$\mathcal{L}_{\phi BD}^{(1)} = \frac{i \mathcal{C}}{M_D F_{\phi}} \varepsilon^{abc} \left( \partial_{\alpha} \bar{T}_{\mu}^{ade} \right) \gamma^{\alpha \mu \nu} B_c^e \, \partial_{\nu} \phi_b^d + \text{h.c.},$$

C is fixed to reproduce the  $\Delta$  width

$$\mathcal{L}^{(1)}_{\phi DD} = \frac{i\mathcal{H}}{M_D F_{\phi}} \bar{T}^{abc}_{\mu} \gamma^{\mu\nu\rho\sigma} \gamma_5 \left(\partial_{\rho} T^{abd}_{\nu}\right) \partial_{\sigma} \phi^c_d,$$

*H* is fixed by its large Nc relation with  $g_A$ 

$$\mathcal{L}_{\gamma DD}^{(2)} = -\frac{g_d}{8M_D} \bar{T}_{\mu}^{abc} \sigma^{\rho\sigma} g^{\mu\nu} (F_{\rho\sigma}^+, T_{\nu})^{abc},$$

$$g_d \text{ is fixed in such a way that the } \Omega^- \text{ MM is reproduced}$$

$$2.02 \pm 0.05 \mu_n$$

#### Numerical results

	1									
	$\Delta^{++}$	$\Delta^+$	$\Delta^0$	$\Delta^{-}$	$\Sigma^{*+}$	${\Sigma^{*0}}$	$\Sigma^{*-}$	$\Xi^{*0}$	[*-	$\Omega^{-}$
$\bigstar$ SU(3)-symm.	4.04	2.02	0	-2.02	2.02	0	-2.02	0	-2.02	-2.02
NQM $[10]$	5.56	2.73	-0.09	-2.92	3.09	0.27	-2.56	0.63	-2.2	-1.84
$\operatorname{RQM}[12]$	4.76	2.38	0	-2.38	1.82	-0.27	-2.36	-0.60	-2.41	-2.35
$\chi QM [14]$	6.93	3.47	0	-3.47	4.12	0.53	-3.06	1.10	-2.61	-2.13
$\chi \text{QSM} [16]$	4.85	2.35	-0.14	-2.63	2.47	-0.02	-2.52	0.09	-2.40	-2.29
QCD-SR [21]	4.1(1.3)	2.07(65)	0	-2.07(65)	2.13(82)	-0.32(15)	-1.66(73)	-0.69(29)	-1.51(52)	-1.49(45)
lQCD [34]	6.09(88)	3.05(44)	0	-3.05(44)	3.16(40)	0.329(67)	-2.50(29)	0.58(10)	-2.08(24)	-1.73(22)
lQCD [36]	5.24(18)	0.97(8)	-0.035(2)	-2.98(19)	1.27(6)	0.33(5)	-1.88(4)	0.16(4)	-0.62(1)	
large $N_c$ [25]	5.9(4)	2.9(2)		-2.9(2)	3.3(2)	0.3(1)	-2.8(3)	0.65(20)	-2.30(15)	-1.94
$\bigstar$ HB $\chi$ PT [28]	4.0(4)	2.1(2)	-0.17(4)	-2.25(19)	2.0(2)	-0.07(2)	-2.2(2)	0.10(4)	-2.0(2)	-1.94
This work	6.04(13)	2.84(2)	-0.36(9)	-3.56(20)	3.07(12)	0	-3.07(12)	0.36(9)	-2.56(6)	-2.02
Expt. $[4]$	$5.6 \pm 1.9$	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$								$-2.02\pm0.05$

Compared to 2<sup>nd</sup> order and HB results, our NLO EOMS results seem to be more consistent with current experimental data

- –Uncertainties solely due to varying  $\mu$  from 0.7 to 1.3 GeV
- -Higher-order contributions, (naively), can be as large as 50% of the NLO

# Covariant ChPT for heavy-light systems

- SU(3) breaking corrections to D(B) decay constants arXiv:1109.0460 [hep-ph]
- Light quark mass dependence of the D and Ds decay constants *Phys.Lett. B696 (2011) 390-395*
- Low-energy interactions of Nambu-Goldstone bosons with D mesons Phys.Rev. D82 (2010) 054022



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# Summary

- We have shown in a number of cases that covariant ChPT is superior to its non-relativistic counterpart
- We have studied some hot topics of current interests
  - Masses, hyperon vector couplings (Vus), magnetic moments, decay constants, scattering lengths, etc.

# Outlook

- Further applications of covariant ChPT to the one-baryon sector/heavy-light systems look quite promising:
  - baryon semileptonic decays-Vus
  - meson-baryon interactions
  - coupled-channel effects in heavy-light systems
  - precision B physics