Coexistence of Cluster and Mean-Field Dynamics and Duality of Many-Nucleon Wave Function

H.Horiuchi RCNP Osaka University, Osaka
Internat. Inst. for Adv. Studies, Kyoto

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---tensor interaction in nuclear and hadron physics---
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1. Introduction

Structure of cluster states is very different from that of mean-field-type states.

But there is an important link between two structures.

It is the duality of of mean-field-type structure and cluster structure possessed by the ground state and some excited states.

The purpose of this talk is to discuss this duality.

The duality is clearly seen in large magnitude of monopole transition between cluster states and the ground state.

The existence of cluster states can be said to be an inevitable consequence of the duality of the ground state.
We show this fact through the AMD reproduction of very many observables up to ^{44}Ti.

The duality of the ground state can be seen also in nuclear reactions.

The duality of the spatially-compact cluster state is important for understanding the cluster-gas state and liquid-gas phase transition, which is discussed by using AMD calculation of caloric curves.

This talk will be published in the paper with the same title as this talk which is to appear In *Romanian Journal of Physics*, special volume celebrating 80th birthday of Prof. A.E. Sandulescu.

Detailed explanation of the many parts of this talk will be given in the paper by H.Horiuchi, K. Ikeda, and K. Kato in *Progress of Theoreticla Physics*, supplement volume, next April.

Some parts of this talk were published in previous papers;

H. Horiuchi, *Lecture Notes in Physics* (Springer), vol.818, 57-108 (2010).

T. Yamada, et al., *Prog. Theor. Phys.* 120, 1139 (2008).

2. Large difference of structure but strong monopole transitions between cluster states and the ground state

Structure of cluster states is very different from that of mean-field-type states.

For example, the Hoyle state of ^{12}C Is a 3-alpha gas-like state whose density is about 1/3 of the ground-state density.

However, the observed strengths of the monopole transitions between the cluster states and the ground state are large and comparable with the single-nucleon strength.
This looks very contradictory because cluster states are described by superposed many-particle many-hole configurations.

***E0* transition of the Hoyle state and those of many cluster states in ^{16}O are good examples.**

Single-nucleon strength of *E0* transition

$M(E0) \sim \langle u_f \,|\, r^2 \,|\, u_i \rangle \sim (3/5)R^2 = 5.4$ fm^2 (for R = nuclear radius = 3 fm)

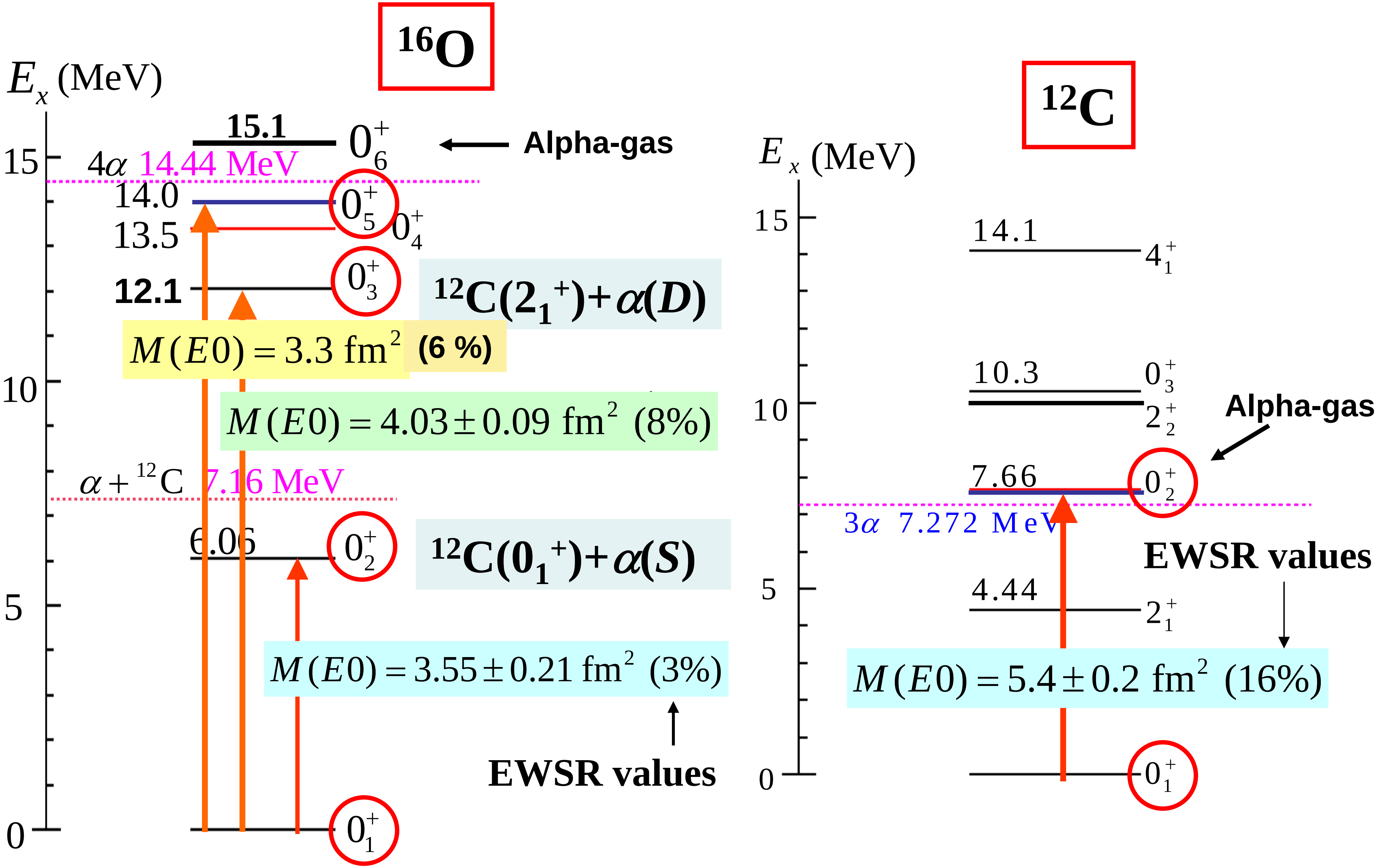
uniform-density approximation for $u_f(r)$ and $u_i(r)$

$u(r) = (3/R^3)^{1/2}$ for $0 \le r \le R$

$u(r) = 0$ for $R < r$

Observed *M(E0)* in ^{16}O and ^{12}C

Single particle *M(E0)*~5.4 fm²



Calculate *M(E0)* for 0_6^+ is 1.0 fm²

3. Duality of mean-field-type structure and cluster structure possessed by the ground state wave function

Large monopole transitions between cluster states and the ground state imply deep relation between cluster structure and ground-state structure.

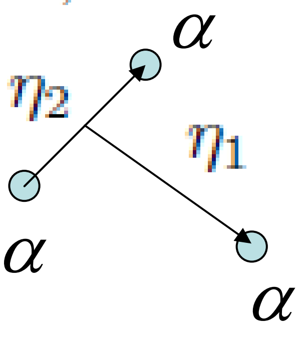
It is the duality of of mean-field-type structure and cluster structure possessed by the ground state.

<u>We explain this in the case of the *E0* transition of the Hoyle state to the ground state</u>:

Main component of the ground state of ^{12}C:

$$|(0s)^4(0p)^8, (\lambda,\mu) = (04)J = 0\rangle$$
$$= N_0 \mathcal{A}\left\{R_{4,0}(\eta_1, (8/3)\nu) R_{4,0}(\eta_2, 2\nu)\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\} g(X_G, 12\nu)$$

Duality of shell model and cluster model structures



Bayman-Bohr theorem

Hoyle state:

$$\mathcal{A}\left\{\exp\left[-\gamma\sum_{i=1}^{3}(\boldsymbol{X}_i-\boldsymbol{X}_G)^2\right]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\}$$
$$=\mathcal{A}\left\{\exp\left[-\gamma((8/3)\eta_1^2+2\eta_2^2)\right]\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\right\}$$

E0 **transition is the transition of relative motion**

$$R_{4,0}(\eta_1,(8/3)\nu)R_{4,0}(\eta_2,2\nu) \longleftrightarrow \exp[-\gamma((8/3)\eta_1^2+2\eta_2^2)]$$

E0 **transition**

Actually, in the ***E0*** **transition operator,**

$$O(E0,{}^{12}\mathrm{C})=\frac{1}{2}\sum_{i=1}^{12}(\boldsymbol{r}_i-\boldsymbol{X}_G)^2$$
$$=O(E0,\alpha_1)+O(E0,\alpha_2)+O(E0,\alpha_3)+\frac{1}{2}((8/3)\eta_1^2+2\eta_2^2))$$

only the relarive motion part, $((8/3)\eta_1^2+2\eta_2^2)$ **, contributes.**

Analytic formula for the *E0* matrix element

$$M(E0, 0_2^+ - 0_1^+) = \sqrt{\frac{7}{6}} \sqrt{\frac{\langle F_4 \rangle}{\langle F_5 \rangle}} \xi_5 \langle R_{40}(r, \nu_N) | r^2 | R_{60}(r, \nu_N) \rangle,$$

≈ 1.0

$$\langle F_n \rangle = \langle Q_n | \mathcal{A}\{Q_n\} \rangle, \quad Q_n = F_n(\xi_1, \xi_2) \phi(\alpha_1) \phi(\alpha_2) \phi(\alpha_3),$$

$$F_n(\xi_1, \xi_2) = \frac{1}{4\pi} \sum_{n_1+n_2=n} \sqrt{\frac{(2n_1+1)!!(2n_2+1)!!}{2n_1!!2n_2!!}} R_{2n_1,0}(\xi_1, (8/3)\nu) R_{2n_2,0}(\xi_2, 2\nu),$$

$$\Phi_{0_2^+} = \sum_{n=5}^{\infty} \xi_n (e_n \mathcal{A}\{Q_n\}), \quad ||e_n \mathcal{A}\{Q_n\}|| = 1.$$

<Fn> represent the antisymmetrization effect, but they appear only in the form of ratio <F4>/<F5>.

This formula shows clearly that the *E0* strength is comparable with the single-nucleon strength.

SU_3 shell model ground state ⟹ **$M(E0)$ = 1.3 fm^2**

Order of magnitude of the observed $M(E0)$ = 5.4 fm^2 is already obtained without G.S. correlation

With G.S. correlation due to 3-alpha motion, we get $M(E0)$ = 6.7 fm^2

Similar explanation applies to the case of ^{16}O.

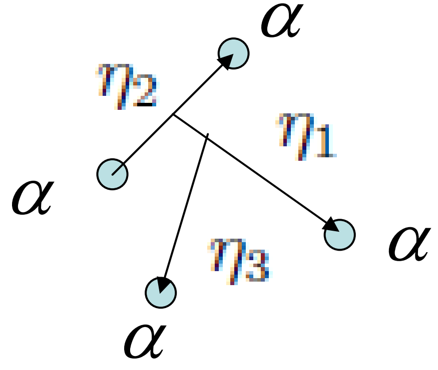
Main component of the ground state of ^{16}O is the doubly-closed-shell wave function:

$$\det|(0s)^4(0p)^{12}| = C_L \mathcal{A}\left[R_{4,L}(r_{C-\alpha}, 3\nu)[Y_L(\hat{r}_{C-\alpha})\phi_L(^{12}\mathrm{C})]_{J=0}\phi(\alpha)\right] g(\boldsymbol{X}_G, 16\nu)$$

$$= D_{L_1,L_2,L}\mathcal{A}\left[R^{12,J=0}_{L_1,L_2,L}(\eta_1,\eta_2,\eta_3)\phi(\alpha_1)\phi(\alpha_2)\phi(\alpha_3)\phi(\alpha_4)\right] g(\boldsymbol{X}_G, 16\nu)$$

$$R^{12,J=0}_{L_1,L_2,L}(\eta_1,\eta_2,\eta_3) = [[R_{4,L_1}(\eta_1,(8/3)\nu)R_{4,L_2}(\eta_2,2\nu)]_L R_{4,L}(\eta_3,3\nu)]_{J=0}$$

Doubly-closed-shell wave function
= ^{12}C + α wave function (most compact)
= 4α wave function (most compact)



Duality of mean-field-type wave function and cluster wave function

E0 **transitions between cluster states and the ground state are the** ***E0*** **transitions of inter-cluster relative motions** **of the cluster states and the ground state.**

0_2^+ **state: main component is** $^{12}\mathrm{C}(0_1^+)+\alpha(S)$

0_3^+ **state: main component is** $^{12}\mathrm{C}(2_1^+)+\alpha(D)$

$$\mathcal{A}\{\chi_L(r)[Y_L(\hat{r})\phi_L(^{12}\mathrm{C})]_0\phi(\alpha)\}$$

***E0* transition**

0+ Cluster states ⟷ **Ground state**

$$\mathcal{A}\{\chi_L(r)[Y_L(\hat{r})\phi_L(^{12}\mathrm{C})]_0\phi(\alpha)\} \qquad \mathcal{A}\{R_{4,L}(r)[Y_L(\hat{r})\phi_L(^{12}\mathrm{C})]_0\phi(\alpha)\}$$

***E0* transition is the transition of relative motion**

$\chi_L(r)$ ⟷ $R_{4,L}(r)$ **(H.O. function, *4=2n+L*)**

$$M(E0, 0_2^+ \to 0_1^+) = \frac{1}{2}\sqrt{\frac{\tau_{0,4}}{\tau_{0,6}}}\eta_6\langle R_{40}(r,\nu_N)\,|\,r^2\,|\,R_{60}(r,\nu_N)\rangle,$$

$$M(E0, 0_3^+ \to 0_1^+) = \frac{1}{2}\sqrt{\frac{\tau_{2,4}}{\tau_{2,6}}}\varsigma_6\langle R_{42}(r,\nu_N)\,|\,r^2\,|\,R_{62}(r,\nu_N)\rangle,$$

$$\sqrt{\frac{\tau_{0,4}}{\tau_{0,6}}} \approx \sqrt{\frac{\tau_{2,4}}{\tau_{2,6}}} \approx 1.0$$

$$M(E0) \equiv \langle n_{cl}\mathcal{A}\{\chi_L(r)[Y_L(\hat{r})\phi_L(^{12}\mathrm{C})]_0\}|\sum_{protons} r_p^2|n_0\mathcal{A}\{R_{4L}(r)[Y_L(\hat{r})\phi_L(^{12}\mathrm{C})]_0\}\rangle$$

$$\nu_N = \frac{m\omega}{2\hbar}$$

$$\tau_{L,N} = \langle \Psi_{L,N} | \mathcal{A}\{\Psi_{L,N}\} \rangle$$

$\tau_{L,N}$ **represent the antisymmetrization effect, but they appear only in the form of ratio** $\tau_{L,4}/\tau_{L,6}$

$$\Psi_{L,N} = R_{N,L}(r_{\alpha-\mathrm{C}}, 3\nu)[Y_L(\hat{r}_{\alpha-\mathrm{C}})\phi_L(^{12}\mathrm{C})]_0\phi(\alpha)$$

$$|0_2^+\rangle = \sum_{N=6}^{\infty} \eta_N (C_N \mathcal{A}\{\Psi_{0,N}\}), \quad ||C_N \mathcal{A}\{\Psi_{0,N}\}|| = 1,$$

$$|0_3^+\rangle = \sum_{N=6}^{\infty} \zeta_N (D_N \mathcal{A}\{\Psi_{2,N}\}), \quad ||D_N \mathcal{A}\{\Psi_{2,N}\}|| = 1,$$

No contribution from ^{12}C part and α part

***E0* strength comes only from relative motion**

relative motion part

$$O(E0, {}^{16}\mathrm{O}) = (1/2)\sum_{i=1}^{16}(\boldsymbol{r}_i - \boldsymbol{X}_G)^2$$

$$= (1/2)\sum_{i\in{}^{12}\mathrm{C}}(\boldsymbol{r}_i - \boldsymbol{r}_\mathrm{C})^2 + (1/2)\sum_{i\in\alpha}(\boldsymbol{r}_i - \boldsymbol{r}_\alpha)^2 + (1/2)\frac{12\times 4}{16}\boldsymbol{r}_{\mathrm{C}-\alpha}^2$$

With doubly-closed shell wave function, the order of magnitude of observed *E0* strengths are repoduced.

	Cal.	Obs.
$M(E0, 0_2^+ \to 0_1^+)$	1.97	3.55
$M(E0, 0_3^+ \to 0_1^+)$	3.89	4.03

4. Existence of cluster states as an inevitable consequence of the duality of the ground state

The ground states of ^{12}C and ^{16}O have dual characters of mean-field-type structure and cluster structure.

It implies that the ground states have both degrees of freedom of mean-field-type dynamics and clustering dynamics.

Therefore,
Excitation of mean-field degree of freedom ⟹ mean-field-type excited states
Excitation of clustering degree of freedom ⟹ cluster-type excited states

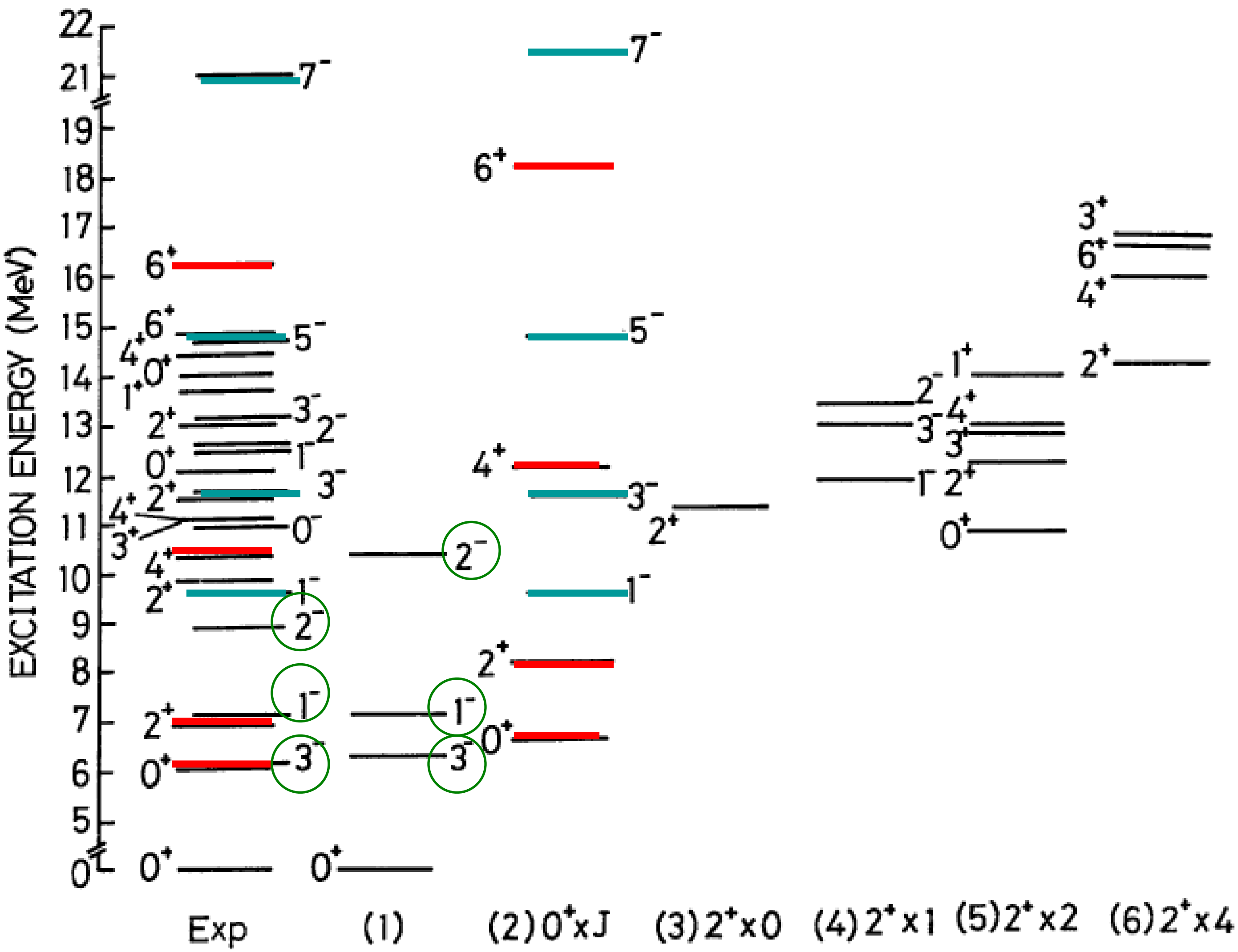
A typical example can be seen in ^{16}O:
excitation by mean-field dynamics ⟹ 1p-1h states; $(3^-)_1$, $(1^-)_1$, $(2^-)_1$, $(0^-)_1$,......
excitation by clustering dynamics ⟹ ^{12}C + α states; $(0^+)_2$, $(2^+)_1$, $(1^-)_2$, $(3^-)_2$, ...

$$\det |(0s)^4(0p)^{12}|$$

$$= C_L ' \mathcal{A}\{\underline{R_{4,L}(r)[Y_L(\hat{r})}\phi_L(^{12}\mathrm{C})]_0\phi(\alpha)\} \Longrightarrow \mathcal{A}\{\underline{\chi_\ell(r)[Y_\ell(\hat{r})}\phi_L(^{12}\mathrm{C})]_J\phi(\alpha)\}$$

$^{12}C + \alpha$ OCM calculation

Y.Suzuki, Prog. Theor. Phys. 56 (1976) 111.



Calculation (1) is for mean-field-type states, while calculations (2) – (6) are for cluster states with main components *Lx l* ($^{12}C(L) \times \alpha$ (*l*)).

The dual character of the ground state explained in ^{12}C and ^{16}O is common to all the N=Z=even light nuclei.

AMD studies have confirmed the existence of lots of clustering excited states by good reproduction of many experimental data.
These cluster states are due to the excitation of the clustering degree of freedom imbedded in the ground state with dualilty character.

Typical examples are as follows:

^{20}Ne **The ground band contains the ^{16}O + α component at most 70 % whis is mostly equivalent to *SU₃* shell model wave function due to Bayman-Bohr theorem:**

$$|(0s)^4(0p)^{12}(1s,0d)^4; SU_3(8,0)J\rangle = C_J \mathcal{A}\{R_{8,J}(\boldsymbol{r}_{\alpha-^{16}\mathrm{O}})\phi(\alpha)\phi(^{16}\mathrm{O})\}\phi_G(\boldsymbol{r}_G)$$

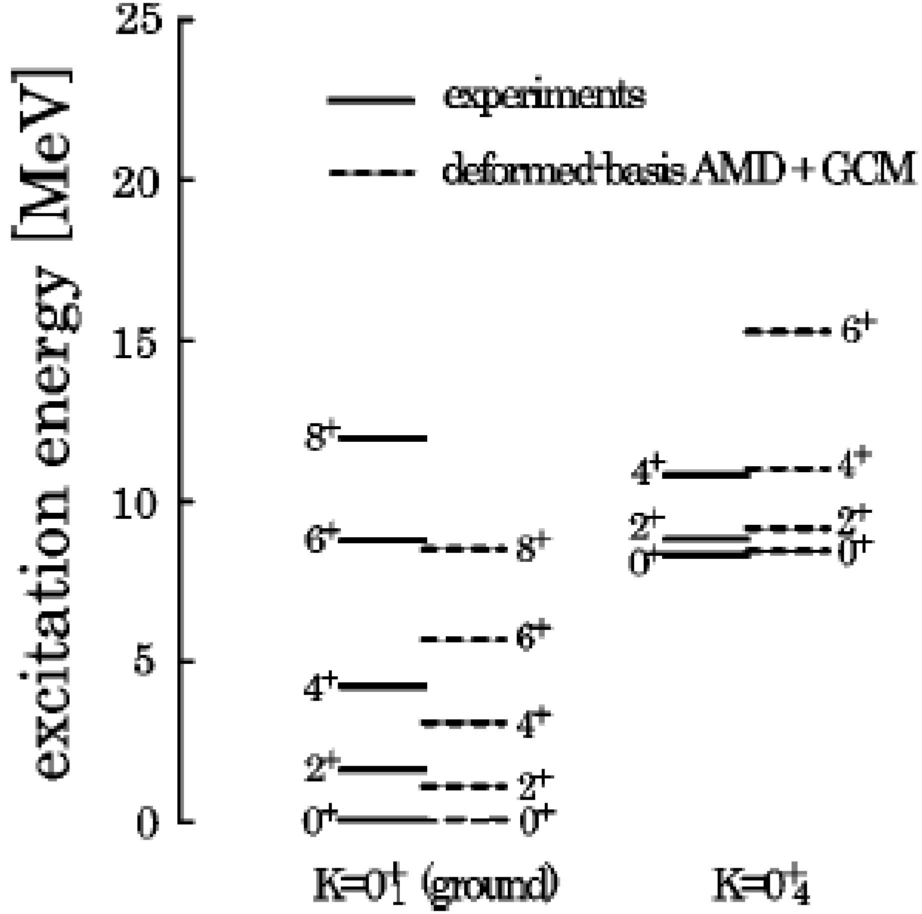
excitation by mean-field degree of freedom ⟹ $K^\pi = 2^-$ **band (5p-1h)**
excitation by clustering degree of freedom ⟹ **^{16}O + α states of** $K^\pi = 0^-$ **and** $K^\pi = 0_4^+$ **bands**

$K^\pi = 0^-$ **band: Almost pure ^{16}O+α clustering for low spins: *2n+L* = 9**
$K^\pi = 0_4^+$ **band: ^{16}O+α component is about 82% for low spins: *2n+L* = 10**

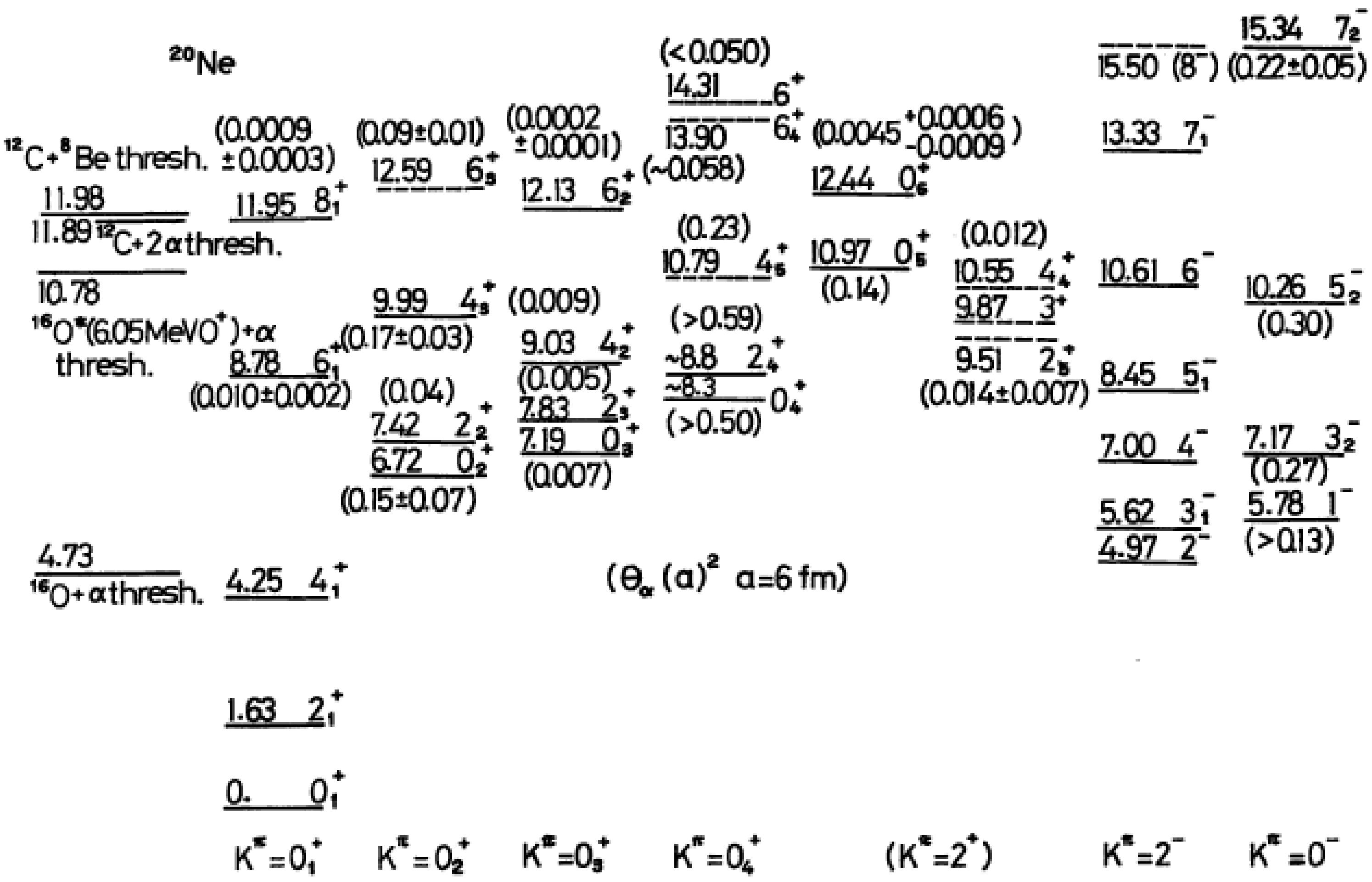
$$\mathcal{A}\{\chi_L(r)Y_L(\hat{r})\phi(^{16}\mathrm{O})\phi(\alpha)\}$$

AMD+GCM

M. Kimura
Phys. Rev. C 69, 044319 (2004)



Observed levels of ^{20}Ne



^{44}Ti

The ground band contains the ^{40}Ca + α component at most 40 % whis is mostly equivalent to *SU₃* shell model wave function due to Bayman-Bohr theorem:

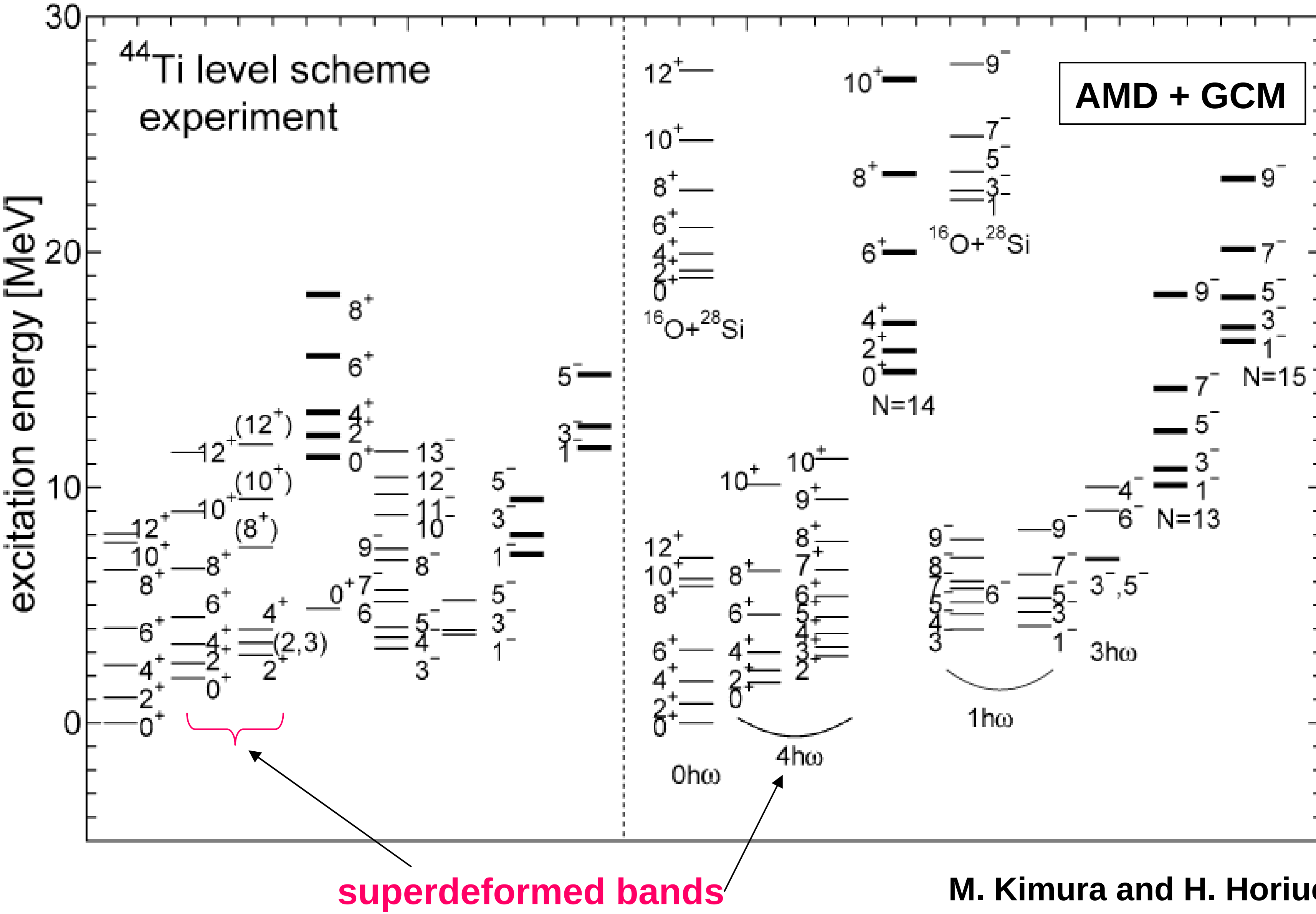
$$|^{40}\mathrm{Ca}\ (0f,1p)^4; SU_3(12,0)J\rangle = D_J \mathcal{A}\{R_{12,J}(\boldsymbol{r}_{\alpha-^{40}\mathrm{Ca}})\phi(\alpha)\phi(^{40}\mathrm{Ca})\}\,\phi_G(\boldsymbol{r}_G)$$

excitation by mean-field degree of freedom ⇨ $K^\pi = 3^-$ **band (5p-1h), superdeformed band (** $K^\pi = 0^+$ **), its side-band (**$K^\pi = 2_1^+$**)**

excitation by clustering degree of freedom ⇨ **^{40}Ca + α states of** $K^\pi = 0^-$ **,** ***N*** **= 14 (** $K^\pi = 0^+$ **), and** ***N*** **= 15 (** $K^\pi = 0^-$ **) bands**

$K^\pi = 0^-$ **band: ^{40}Ca+ α component is about 55 % for low spins: *2n*+*L* = 13,**
N **= 14 band: ^{40}Ca+ α component is about 46 % for low spins: 2*n*+*L* = 14,**
N **= 15 band: ^{40}Ca+ α component is about 63 % for low spins: 2*n*+*L* = 15**

$$\mathcal{A}\{\chi_L(r)Y_L(\hat{r})\phi(^{40}\mathrm{Ca})\phi(\alpha)\}$$



M. Kimura and H. Horiuchi
Nucl. Phys. A767, 58 (2006)

^{40}Ca + α component is contained much in bold line levels

We have seen that the cluster states are formed inevitably by the excitaion of the degrees of freedom of inter-cluster relative motion embedded in the ground state having the dual character.

Namely, the existence of cluster states is an inevitable consequence of the dual character of the ground state.

^{20}Ne Ground band : ^{16}O+ α component: **about 70 % for low spins,**
(duality component) **about 30 % for high spins.**

^{44}Ti Ground band : ^{40}Ca+ α component: **about 40 % for low spins,**
(duality component) **about 5 % for high spins.**

Cluster components (duality components) contained in the ground state (band) become increasingly minor in hevier nuclei.

It Is the decrease of the SU_3 components with dual character in heavier nuclei.

However, there may be excited mean-field-type states which have dual character.

Duality of of mean-field-type structure and cluster structure is possessd also by some excited states.

These excited states are formed by the mean-field dynamics from the ground state.

Two examples are as follows:

^{28}Si

The ground state is a band head of an oblate band, while there exists a band with prolate deformation upon 6.7 MeV 0^+ whose main component has an SU_3 symmetry (12,0). This SU_3 wave function has the following duality character:

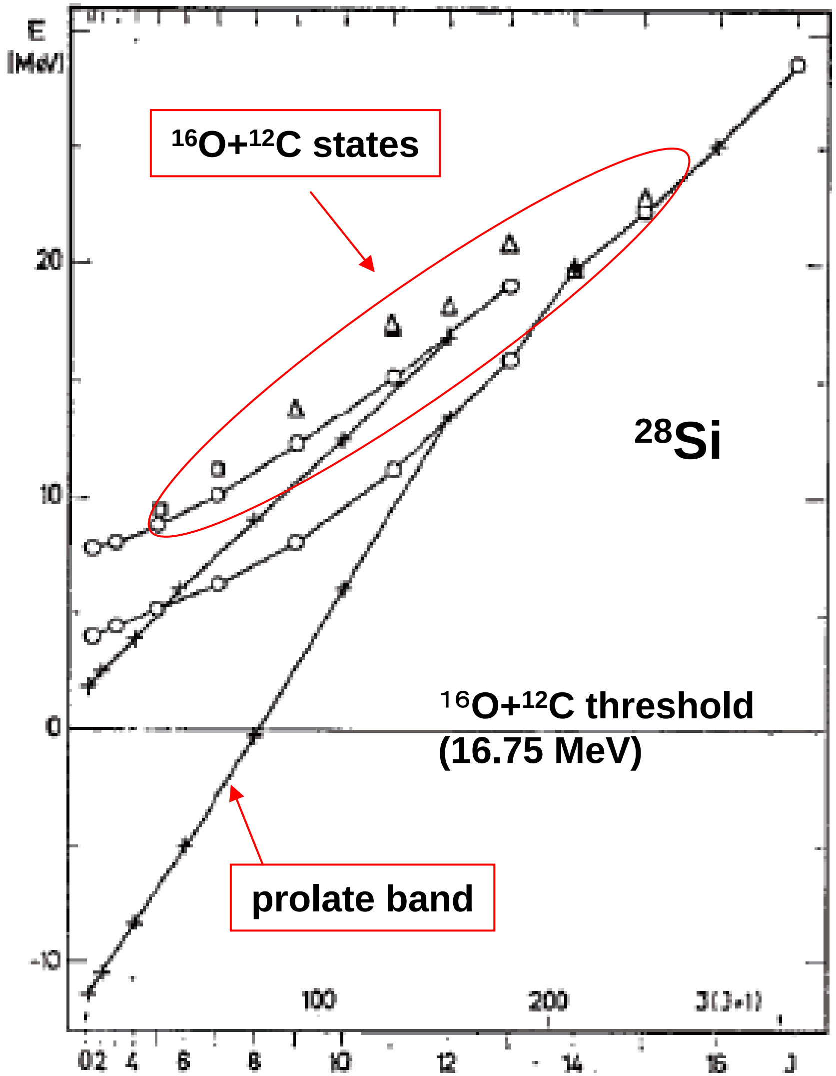
$$|(1s0d)^{12}[4](12,0)J, S=T=0\rangle$$
$$= b_J \mathcal{A}\left\{[R_{16}(r_{\mathrm{O-C}}, (48/7)\nu)\phi_{(0,4)}(^{12}\mathrm{C})]_{(12,0),J}\phi(^{16}\mathrm{O})\right\} g(X_G, 28\nu)$$

$$[R_{16}(r_{\mathrm{O-C}}, (48/7)\nu)\phi_{(0,4)}(^{12}\mathrm{C})]_{(12,0),J}$$
$$= \sum_i \langle (16,0)\ell_i, (0,4)L_i || (12,0)J\rangle R_{16,\ell_i}(r_{\mathrm{O-C}}, (48/7)\nu)[Y_{\ell_i}(\hat{r}_{\mathrm{O-C}})\phi_{L_i}(^{12}\mathrm{C})]_J$$

The excitation of degree of freedom of ^{16}O-^{12}C clustering gives rise to the formation of ^{16}O-^{12}C molecular states.

$^{16}O+^{12}C$ GCM
(Brink cluster model)

D.Baye Nucl.Phys.A272(1976),445.
D.Baye and P.H.Heenen,Nucl.Phys.A283(1977),176.

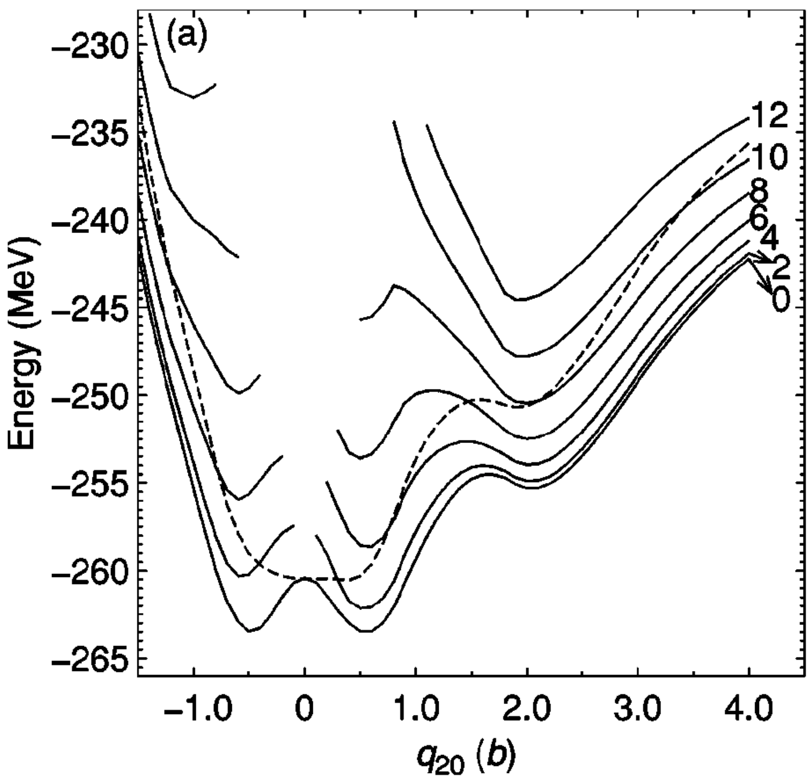


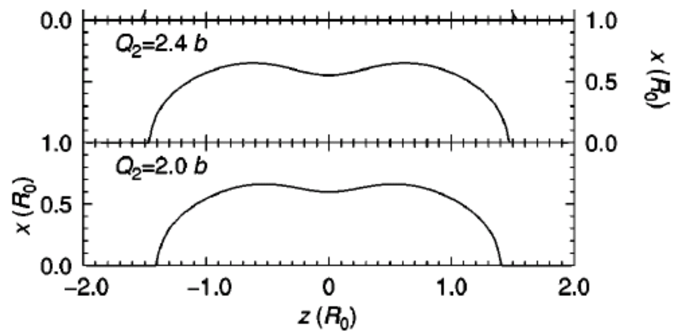
Calculations: lines with crosses and circles
Experiments: triangles and squares
$^{16}O+^{12}C$ resonances

32S

Any Hartree-Fock calculations and also AMD calculation predict the existence of the superdeformed band which starts from E_x ~ 9 MeV.

HF(B) calculations for ^{32}S

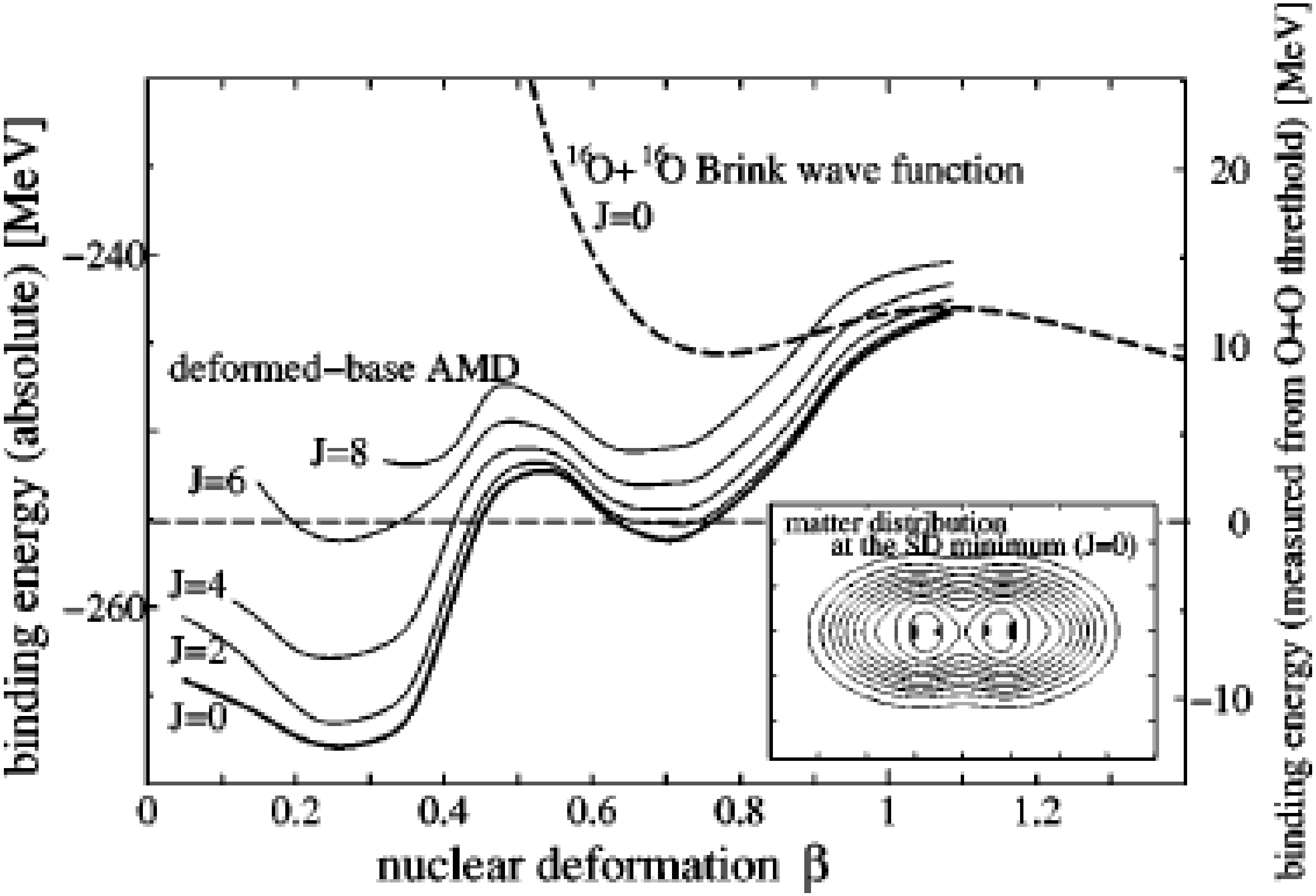




- **SD magic # N=Z=16**
- **HF(B) calculations predict the superdeformed minima.**
- **SD band starts about 9 MeV above the g.s.**
- **Density distribution shows two centre nature.**

R. R. Rodriguez-Guzma'n, et. al. Phys. Rev. C62 (2000), 054308.

AMD calculation for ^{32}S



M. Kimura and H. Horiuchi,
Phys. Rev. C69, 051304 (R) (2004)

The superdeformed band has a 4p-4h excited structure from the ground state.

Its main configuration has an SU_3 symmetry (24,0) and the following duality character:

$$4p - 4h \text{ excitation}$$
$$= (0,0,0)^4(1,0,0)^4(0,1,0)^4(0,0,1)^4(1,0,1)^4(0,1,1)^4(0,0,2)^4(0,0,3)^4$$
$$= n\mathcal{A}\{\underline{X_{(0,0,24)}(r_{\mathrm{O-O}})}\phi(^{16}\mathrm{O})\phi(^{16}\mathrm{O})\}\ \phi_G(r_G)$$

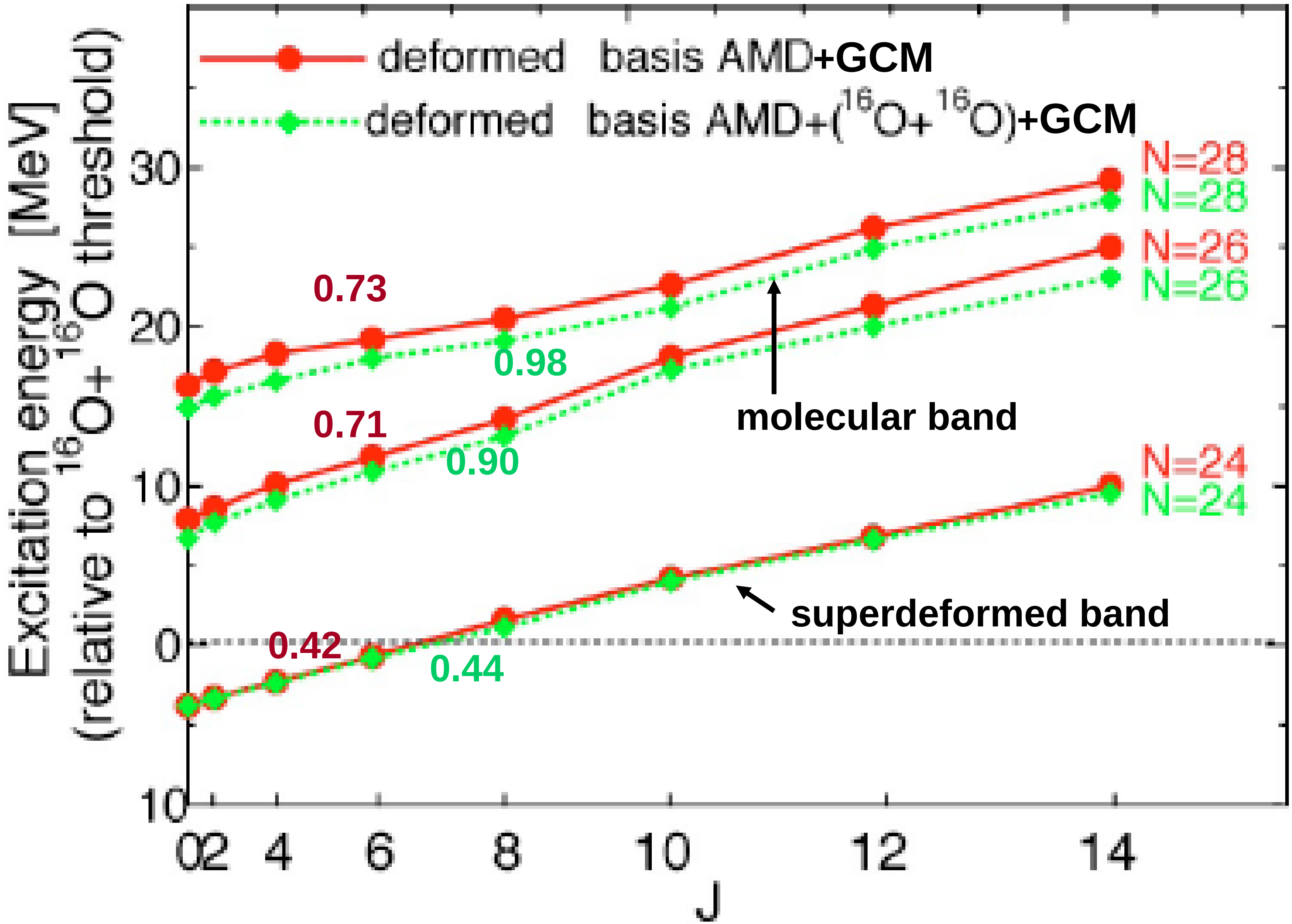
3-dimensional Cartesian H.O. w.f. $X_{(n_x,n_y,n_z)}(r)$

The excitation of the ^{16}O-^{16}O clustering degree of freedom imbedded in the superdeformed band states gives rise to the formation of ^{16}O-^{16}O molecular states.

Actually, the AMD+GCM calculation along the energy curve does give rise to the formation of ^{16}O-^{16}O molecular states.

^{32}S

Numbers are amount of $^{16}O + ^{16}O$ component in the wave function



5. The duality of the ground state appearing in nuclear reactions

<u>**Cluster transfer reactions**</u>

Alpha transfer reaction

Theoretical description is always based on the duality of the ground state wave function:

(6Li, d): 6Li = α + d,
(7Li, t): 7Li = α + t,
(16O, 12C): 16O = 12C + α ,
etc.

Related reaction processes :

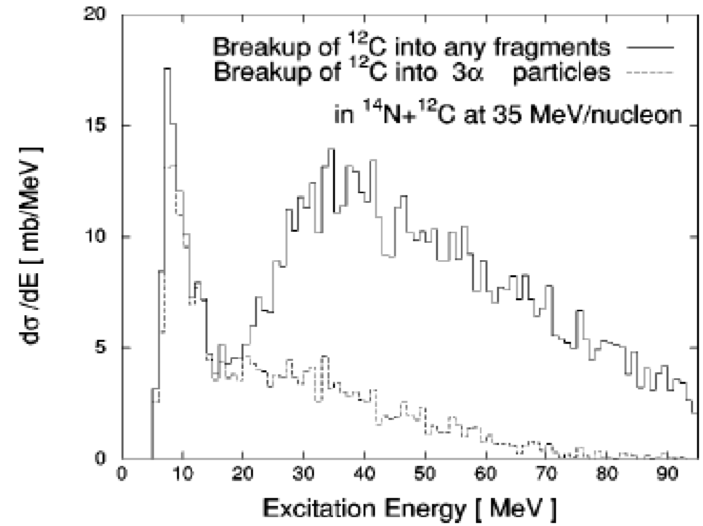
i. **breakup reaction into alpha + others,**
ii. **knockout reaction of alpha(s) such as quasi-elastic process,**
iii. **dynamical fragmentation reaction (non-statistical),**
(Takemoto et al. Phys. Rev. C54, 266 (1996))

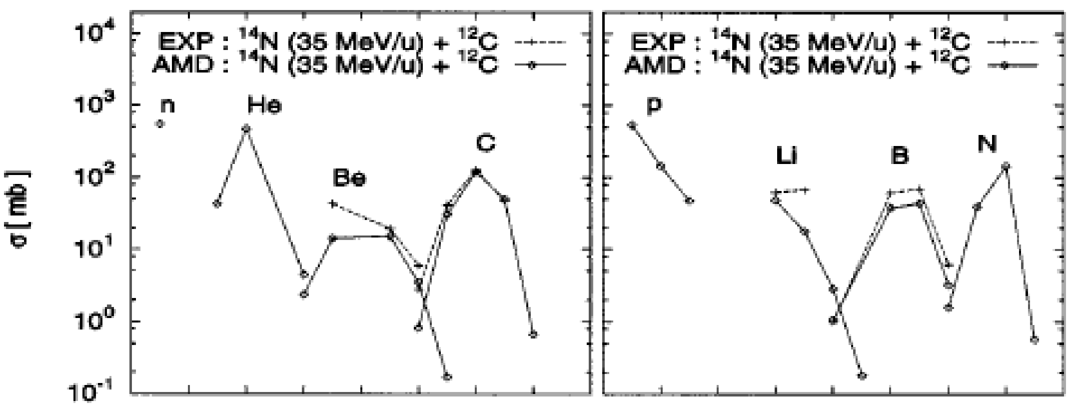
$^{14}N + ^{12}C$ collisions at 35 MeV/u

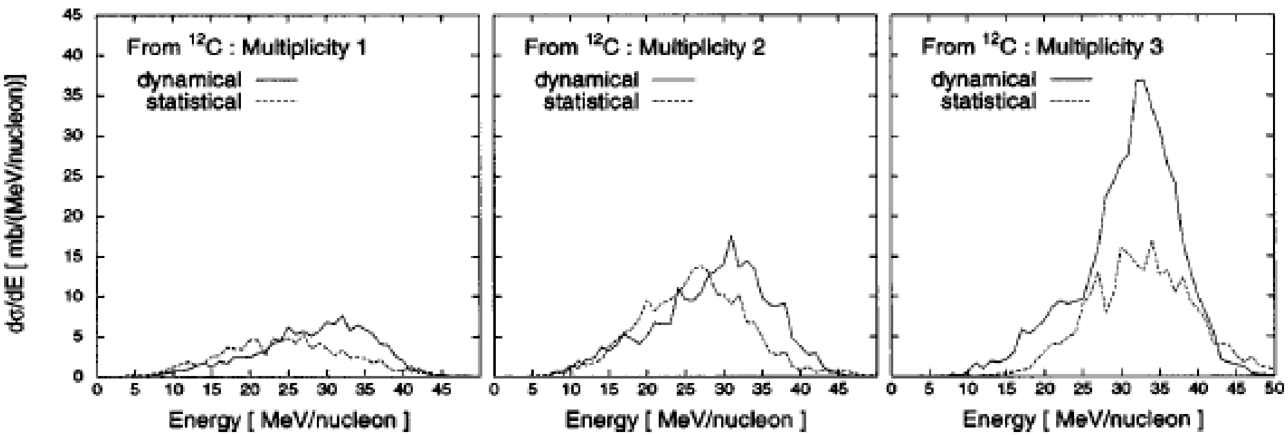
AMD calculation

H.Takemoto, H.Horiuchi, and A.Ono,
Phys.Rev.C54, 266(1996); C57, 811(1998)

3 alpha dynamical fragmentation via 3-alpha clustering states of ^{12}C



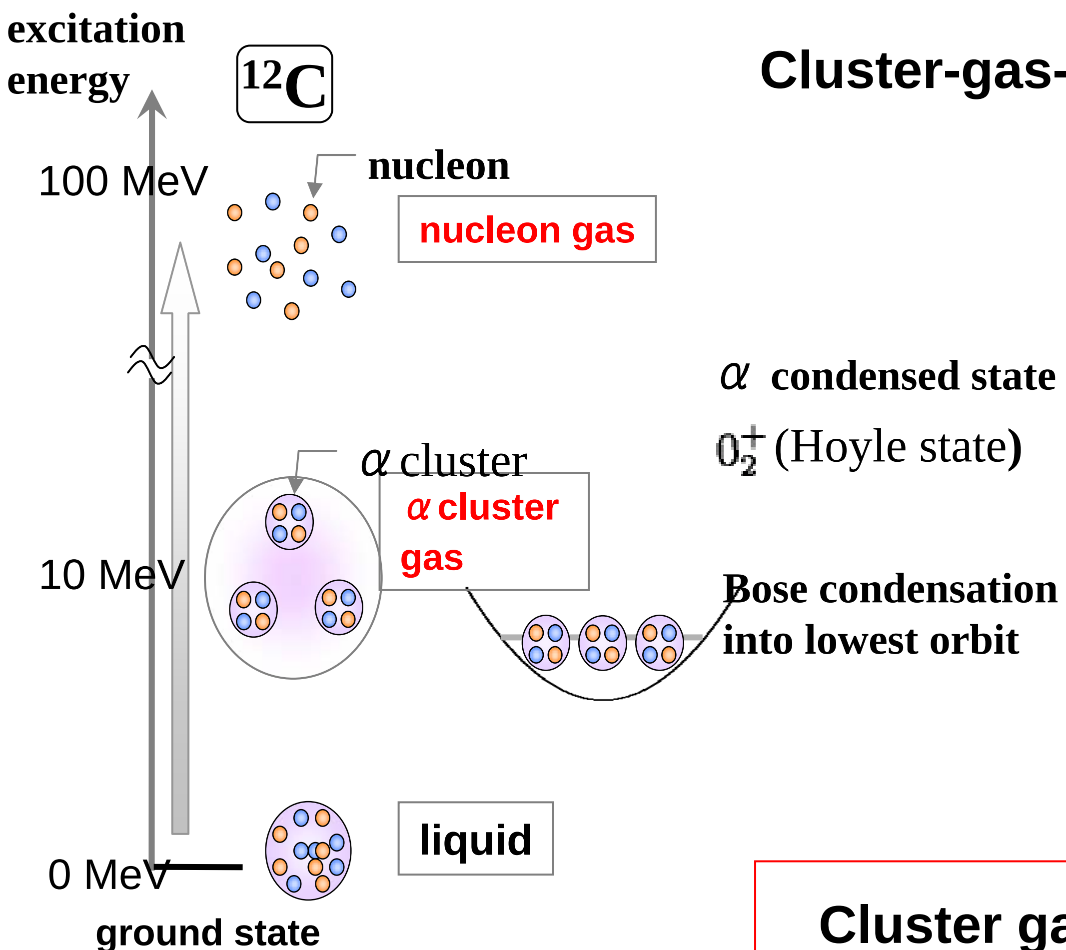




multiplicity 3 events of alpha are dominant in alpha fragments.

6. Cluster-gas state, liquid-gas phase transition, and the duality of compact nuclear states

Cluster-gas-like states



Cluster gas with zero T (temperature)

Nuclear states near zero temperature (T=0)

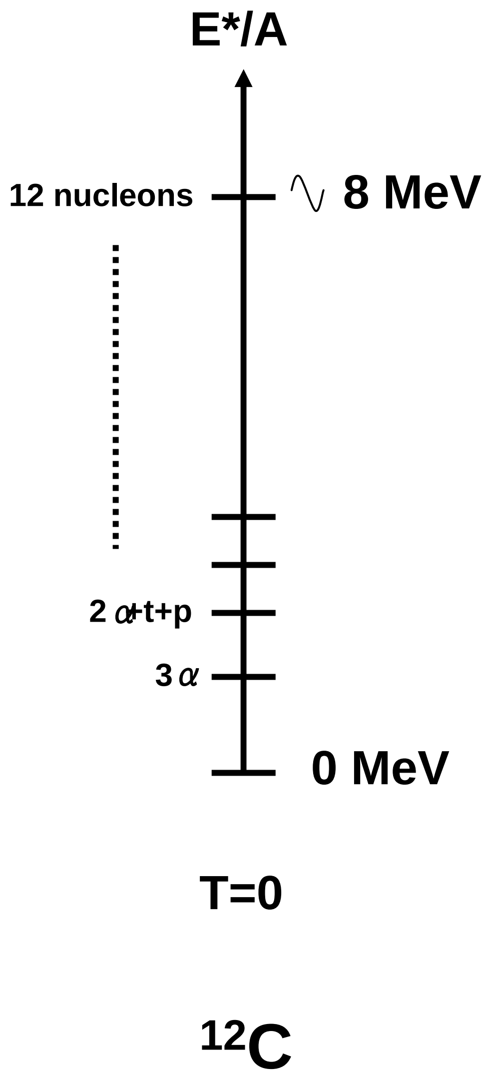
The lowest-energy spatially-localized cluster states, $C_1+C_2+......+C_n$, are states of almost zero temperature, which are located near the threshold of $C_1+C_2+......+C_n$.

The lowest-energy cluster states are not liquid-like, because the ''liquid-like compact cluster state'' is equivalent to mean-field-type state and so is not a real cluster state.
This is just due to the dual character of the compact cluster wave function.

When the temperature is raised from zero, two types of excitation occur:
one is the mean-field-type excitation of individual clusters with liquid nature,
and the other is the excitation of the inter-cluster relative motions.

These features can be studied by the AMD calculation of the caloric curve of finite nuclei.

Cluster dissociation thresholds



Caloric curves of a nucleus with N = Z = 18

AMD calculation (Furuta-Ono) PRC 74, 014612

Constant volume

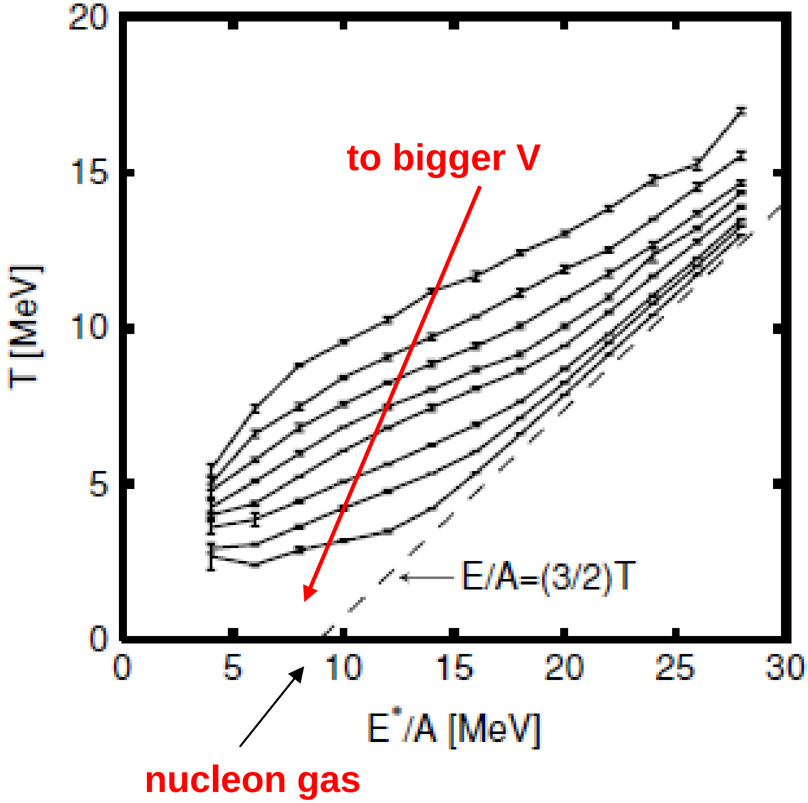


FIG. 3. The constant volume caloric curves for the $A = 36$ system obtained by AMD. The lines correspond to the container size $r_{\mathrm{wall}} =$ 5, 5.5, 6, 6.5, 7, 8, 9, and 11 fm from the top. Statistical uncertainty is shown by error bars. The line of $E/A = (E^* + E_{\mathrm{g.s.}})/A = (3/2)T$ is drawn for comparison.

Constant pressure

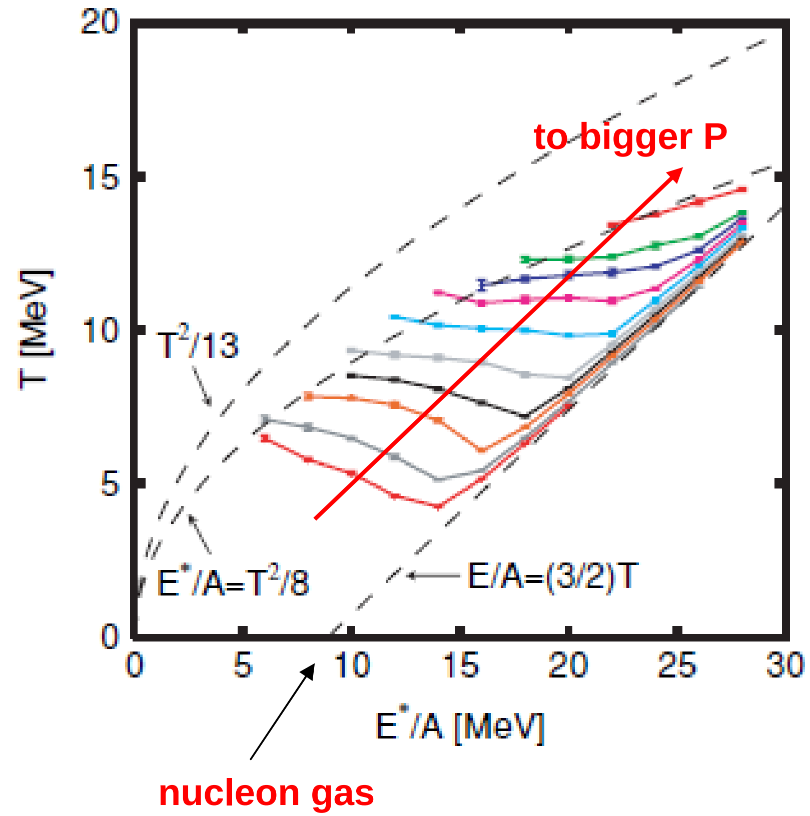


FIG. 5. (Color online) The constant pressure caloric curves for the $A = 36$ system obtained by AMD. The lines correspond to the pressure $P = 0.02$, 0.03, 0.05, 0.07, 0.10, 0.15, 0.20, 0.25, 0.30 and 0.40 MeV/fm^3 from the bottom. Statistical uncertainty is shown by error bars. The curves of $E^*/A = T^2/(8\text{ MeV})$ and $E^*/A = T^2/(13\text{ MeV})$, and the line of $E/A = (3/2)T$ are drawn for comparison.

Fragment mass distribution
on the caloric curve of a nucleus with N = Z = 18

AMD calculation (Furuta-Ono)   PRC 74, 014612

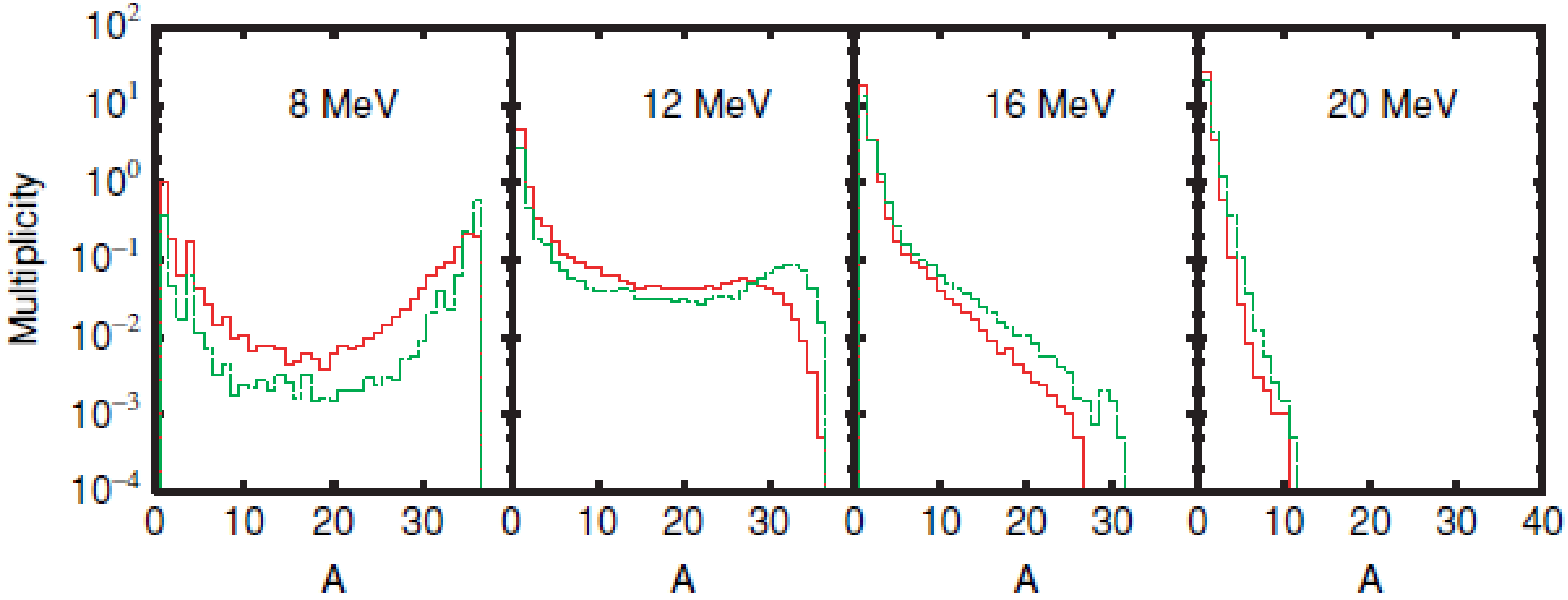


FIG. 6. (Color online) The fragment mass distributions along the $P = 0.05$ MeV/fm^3 line.
Full lines are the distributions obtained with $r_{\mathrm{clust}} = 2.5$ fm.
Dashed lines are the distributions obtained with $r_{\mathrm{clust}} = 3.0$ fm.

Constant-presuure caloric curve before reaching nucleon-gas phase comes down towards the zero-temperature line (abscissa) when the presuure becomes smaller.

(similarly, constant-volume caloric curve before reaching nucleon-gas phase comes down towards the zero-temperature line (abscissa) when the volume becomes larger.)

Therefore, the nuclear configuration along the caloric curve for low pressure has a close relation with the nuclear configuration along the zero temperature line.
The fragment-mass distribution along the caloric curve calculated with AMD looks consistent with the distribution of the dissociation thresholds into clusters.
It is to be noted that the lowest-energy cluster states on the zero-temperature line are not liquid-like as explained above.

7. Summary

(1) Although cluster states are very different from the ground state in structure, monopole transitions between them are strong in general having comparable magnitude with single-nucleon strength.

(2) Duality of mean-field-type structure and cluster structure possessed by the ground state explains the large *E0* strengths of cluster states to the ground state.

(3) Existence of cluster states is an inevitable consequence of the duality of the ground state.
This is supported by AMD studies of many nuclei.
There are cluster states which are formed due to the duality of excited mean-field-type states.

(4) The duality of the ground state appears in several types of nuclear reactions.

(5) The duality of compact nuclear states play important roles for the study of liquid-gas phase transition and cluster-gas states.