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# Neutron Halo in Deformed Nuclei

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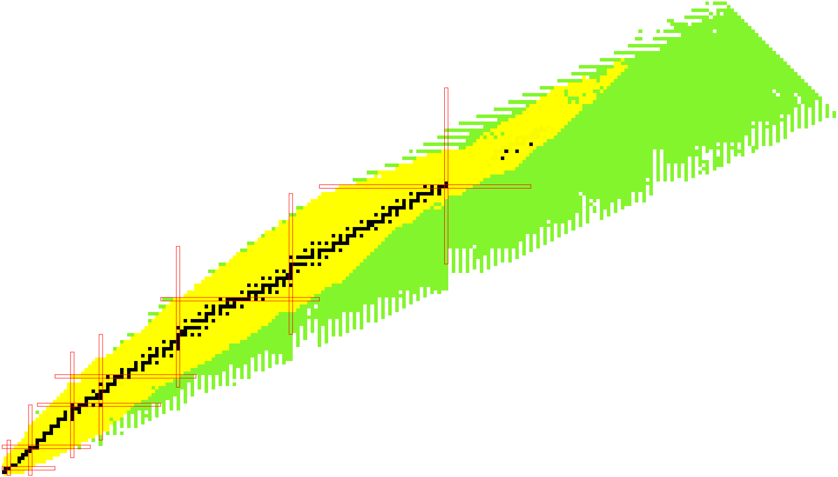
# Content

# Introduction

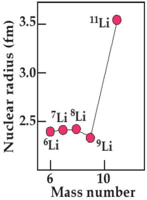


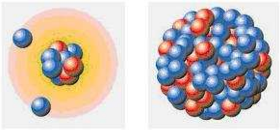
Provided by Jie Zhao (赵杰)

# Introduction

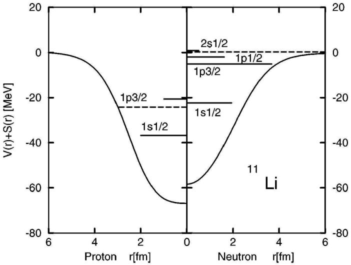
- Large spatial distribution in $^{11}$Li: HALO. Tanihata PRL1985

  $R \stackrel{?}{=} r_0 A^{1/3}$



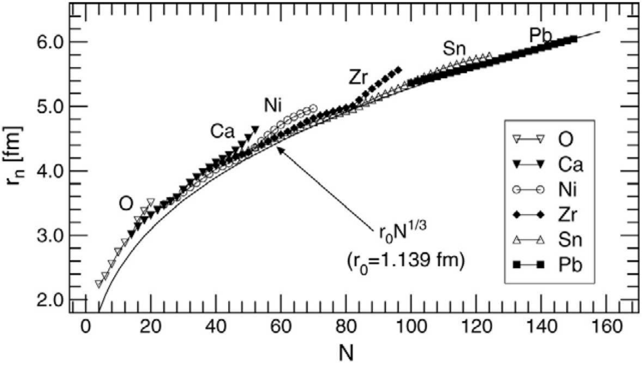
- Relativistic Hartree-Bogoliubov description




" In contrast to earlier investigations ... where the halo in $^{11}$Li could only be reproduced by an artificial shift of the $1p_{1/2}$ level close to the continuum limit, the halo is now reproduced in a self-consistent way, ... " Meng & Ring PRL1996

# Introduction

- Prediction of giant halo. Meng & Ring PRL1998
- Neutron halo and neutron skin. Meng *et al.* PPNP2006




- What about the shape of halo nuclei?

# Halo in deformed nuclei? Deformed halo?

- Halo in deformed nuclei? Deformed halo?
  - $^{11}$Be
    - The spin-parity of the ground state and first exciting state are $\frac{1}{2}^+$ and $\frac{1}{2}^-$.
      $\Rightarrow$ The parity-inversion phenomenon may caused by deformation.
      Li *et al.* PRC1996, Misu *et al.* NPA1997, Pei *et al.* NPA2006
  - $^{14}$Be
    - Halo structure is obtained with spherical RMF model. Ren *et al.* PLB1995
    - A kinematical complete measurement of the fragments suggests a large $(2s_{\frac{1}{2}})^2$ admixture and $(1d_{\frac{5}{2}})^2$ in $^{14}$Be.
      $\Rightarrow$ Deformation. Labiche *et al.* PRL2001
  - $^{31}$Ne
    - The major components of wave function might be $^{30}$Ne$(0_1^+) \otimes 2p_{3/2}(C^2S = 0.12)$, $^{30}$Ne$(2_1^+) \otimes 2p_{3/2}(C^2S = 0.27)$, $^{30}$Ne$(2_1^+) \otimes 2f_{7/2}(C^2S = 0.25)$.
      $\Rightarrow$ "...as such, suggests that it will be strongly deformed."
      Nakamura *et al.* PRL2009
  - Three body model:
    "Our results suggest that it is unlikely to find (two neutron) halo nuclei on the dripline of deformed nuclei." Nunes NPA2005

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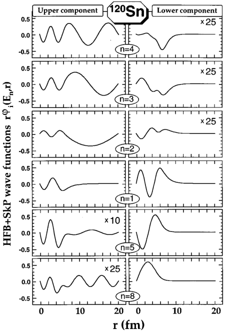
- Pairing effect and the contribution of continuum




HFB wave functions

Upper components

$$U(r) \sim \begin{cases} \cos(k_U^+ r + \delta_U), & \text{for } E > -\lambda, \\ \exp(-k_U^- r), & \text{for } E < -\lambda, \end{cases}$$

Lower components

$$V(r) \sim \begin{cases} \cos(k_V^+ r + \delta_V), & \text{for } E < \lambda, \\ \exp(-k_V^- r), & \text{for } E > \lambda. \end{cases}$$

Dobaczewski *et al.* PRC1996

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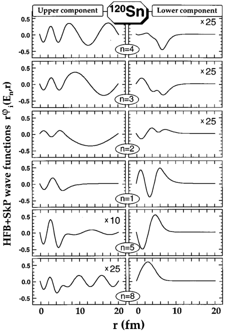
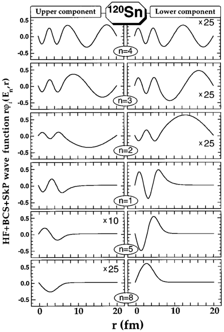



HFB wave functions




BCS wave functions

Dobaczewski *et al.* PRC1996

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A self consistent deformed Relativistic Hartree-Bogoliubov theory in continuum is established. S.G. Zhou, J. Meng, P. Ring, E.G. Zhao, Phys. Rev. C 82, 011301R (2010)

Present work: the halo phenomena in Mg isotopes is studied with the developed theory.

# Deformed relativistic hartree-bogoliubov theory

RHB Equation

$$\sum_{\sigma' p'} \int d^3\mathbf{r}' \begin{pmatrix} (h_D(\mathbf{r}\sigma p, \mathbf{r}\sigma' p') - \lambda) & \Delta(\mathbf{r}\sigma p, \mathbf{r}'\sigma' p') \\ -\Delta^*(\mathbf{r}\sigma p, \mathbf{r}'\sigma' p') & (-h_D(\mathbf{r}\sigma p, \mathbf{r}\sigma' p') + \lambda) \end{pmatrix} \begin{pmatrix} U_k(\mathbf{r}'\sigma' p') \\ V_k(\mathbf{r}'\sigma' p') \end{pmatrix} = E_k \begin{pmatrix} U_k(\mathbf{r}\sigma p) \\ V_k(\mathbf{r}\sigma p) \end{pmatrix}$$

Dirac hamiltonian

$$h_D = \alpha \cdot \mathbf{p} + V(r) + \beta(M + S(r))$$

Scalar and vector density read

$$\begin{aligned} S(r) &= g_\sigma \sigma(r) \\ V(r) &= g_\omega \omega^0(r) + g_\rho \tau_3 \rho^0(r) + e\frac{1-\tau_3}{2} A^0(r) \end{aligned}$$

Equations of corresponding meson and photon feilds

$$\begin{aligned} (-\Delta + m_\sigma^2)\sigma(r) &= -g_\sigma \rho_s(r) & \rho_s(r) &= \bar{\psi}\psi \\ (-\Delta + m_\omega^2)\omega^0(r) &= g_\omega \rho_v(r) & \rho_v(r) &= \psi^\dagger \psi \\ (-\Delta + m_\rho^2)\rho^0(r) &= g_\rho \rho_3(r) & \rho_3(r) &= \psi^\dagger \tau_3 \psi \\ -\Delta A^0(r) &= e\rho_p(r) & \rho_c(r) &= \psi^\dagger \frac{1-\tau_3}{2} \psi \end{aligned}$$

# Deformed relativistic hartree-bogoliubov theory

Coefficients of RHB equation are expanded with Dirac Woods-Saxon basis

$$\begin{pmatrix} U_k(\mathbf{r}\sigma p) \\ V_k(\mathbf{r}\sigma p) \end{pmatrix} = \begin{pmatrix} \sum_{i\kappa} u^{(m)}_{k,(i\kappa)} \varphi_{i\kappa m}(\mathbf{r}\sigma p) \\ \sum_{i\kappa} u^{(\bar{m})}_{k,(\widetilde{i\kappa})} \tilde{\varphi}_{i\kappa m}(\mathbf{r}\sigma p) \\ \sum_{i\kappa} v^{(m)}_{k,(i\kappa)} \varphi_{i\kappa m}(\mathbf{r}\sigma p) \\ \sum_{i\kappa} v^{(\bar{m})}_{k,(\widetilde{i\kappa})} \tilde{\varphi}_{i\kappa m}(\mathbf{r}\sigma p) \end{pmatrix}$$

Spherical Dirac spinor
$\{\varphi_{i\kappa m}(\mathbf{r}\sigma p);\ i = 0,\ 1,\ 2,\ \cdots, \kappa = \pm 1,\ \pm 2,\ \cdots,\ m = \pm\frac{1}{2},\ \pm\frac{3}{2},\ \cdots, \pm(|\kappa| - \frac{1}{2})\}$,

$$\varphi_{i\kappa m}(\mathbf{r}\sigma p) = \frac{1}{r} \begin{pmatrix} iG_{i\kappa}(r)\phi_{\kappa m}(\Omega\sigma) \\ -F_{i\kappa}(r)\phi_{-\kappa m}(\Omega\sigma) \end{pmatrix},$$

where

$$\begin{aligned} \phi_{\kappa m}(\Omega\sigma) &= \sum_{m_l m_s} \langle l m_l s m_s | jm\rangle Y_{l m_l}(\Omega)\chi_{m_s}(\sigma), \\ \phi_{-\kappa m}(\Omega\sigma) &= \sum_{m_l m_s} \langle \tilde{l} m_l s m_s | jm\rangle Y_{\tilde{l} m_l}(\Omega)\chi_{m_s}(\sigma), \end{aligned}$$

and $\Omega = (\theta, \phi)$,

$$\kappa = \pm(j + 1/2) \ \textit{for} \ j = l \mp 1/2,\ j = |\kappa| - \frac{1}{2},\ l = j + \mathrm{sign}(\kappa)/2,\ \tilde{l} = j - \mathrm{sign}(\kappa)/2$$

# Numerical details

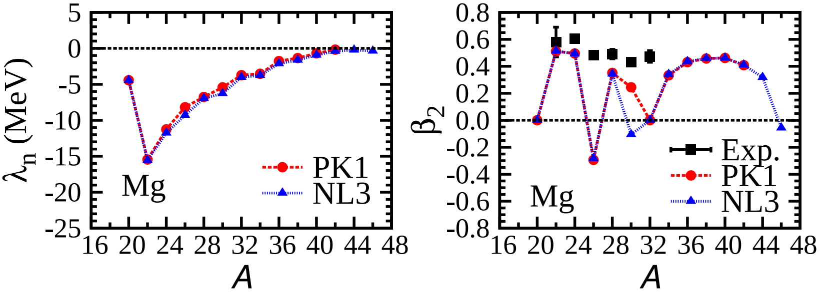
- Box size $R = 20$ fm and step size $\Delta r = 0.1$ fm are used for solving DWS basis.
- Energy cut off is chosen to be $E^{+}_{\mathrm{cut}} = 100$ MeV.
- Density dependent delta force is used for pairing, and smooth cut off is used.

$$s(E_k) = \frac{1}{2}\left(1 - \frac{E_k - E^{\mathrm{q.p.}}_{\mathrm{cut}}}{\sqrt{(E_k - E^{\mathrm{q.p.}}_{\mathrm{cut}})^2 + (\Gamma^{\mathrm{q.p.}}_{\mathrm{cut}})^2}}\right)$$

Parameters of pairing interaction is chosen by fitting the proton pairing energy of spherical $^{20}$Mg given by Gogny D1S.

| Model | Pairing force | Parameters | $E^{\mathrm{p}}_{\mathrm{pair}}$ (MeV) |
|---|---|---|---|
| SRHBHO | Gogny | D1S | $-9.2382$ |
| RCHB | Surface $\delta$ with sharp cutoff | $V_0 = 374$ MeV fm$^3$<br>$\rho_{\mathrm{sat}} = 0.152$ fm$^{-3}$<br>$E^{\mathrm{q.p.}}_{\mathrm{cut}} = 60$ MeV | $-9.2387$ |
| DRHBWS | Surface $\delta$ with smooth cutoff | $V_0 = 380$ MeV fm$^3$<br>$\rho_0 = 0.152$ fm$^{-3}$<br>$E^{\mathrm{q.p.}}_{\mathrm{cut}} = 60$ MeV<br>$\Gamma^{\mathrm{q.p.}}_{\mathrm{cut}} = 5.65$ MeV | $-9.2382$ |

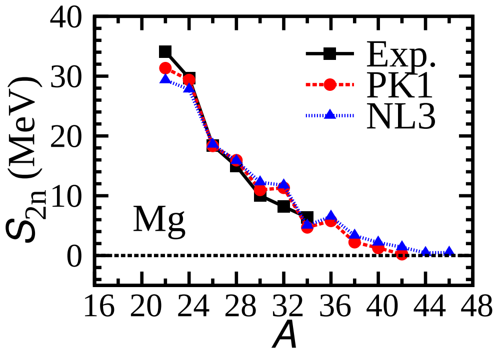
# Mg isotopes




- $^{42}$Mg is the drip line nucleus (PK1).
- $^{32}$Mg is spherical for both parameter sets.

Data taken from Raman *et al.* ADNDT2001

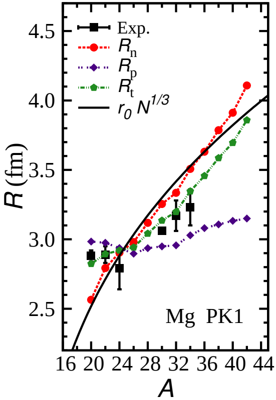
# Mg isotopes




- Reasonable two neutron separation energy obtained.
- Strong shell effect of $N = 20$ is observed with mean field theory.

Data taken from Audi *et al.* NPA2003

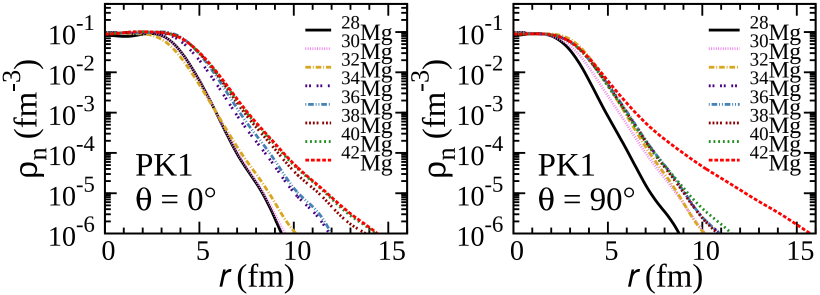
# Mg isotopes (radii)




- Slope of neutron radii changes suddenly at $^{32}$Mg $\Rightarrow$ Shell effect with mean field theory.
- Slope of neutron radii changes at $^{40}$Mg $\Rightarrow$ ?

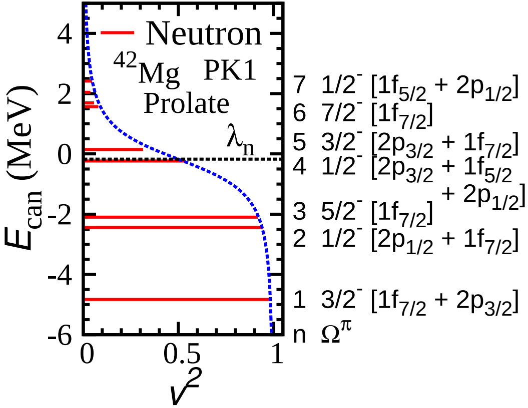
Data taken from Suzuki *et al.* NPA1997, NPA1998

# Mg isotopes (neutron density profiles)



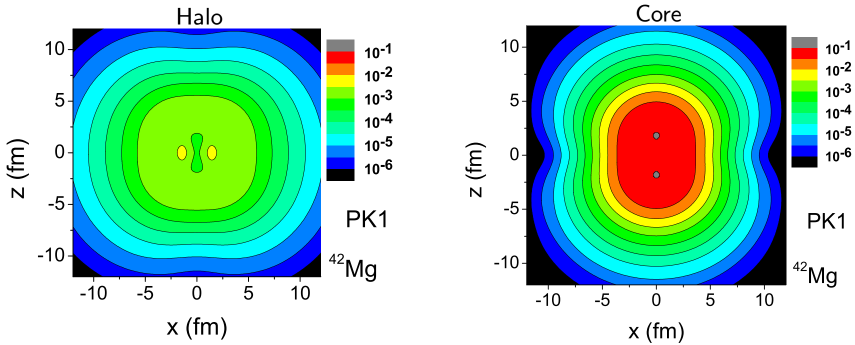

- In the direction of $z$ axis, neutron density profile of Mg isotopes changes smoothly.
- In the direction of $x$ axis, neutron density of $^{42}$Mg extend far away from the center . $\Rightarrow$ Oblate halo

# $^{42}$Mg: single particle levels




- Weakly bound and continuum orbitals → Halo
- Deeply bound orbitals → Core

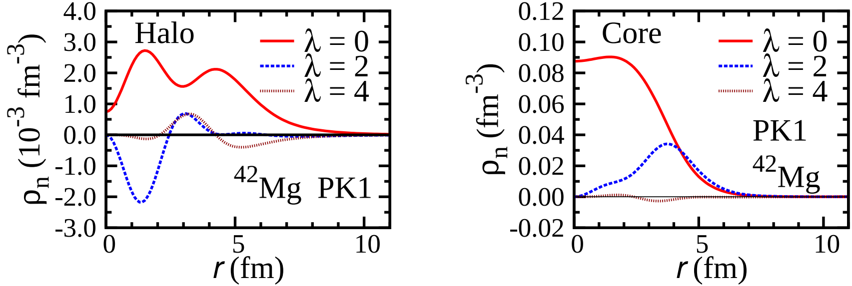
# $^{42}$Mg: density profile of halo and core




- Shape decoupling between core and halo.
  - Oblate halo
  - Prolate core

# $^{42}$Mg: density profile

$$\rho(\mathbf{r}) = \sum_{\lambda} \rho_{\lambda}(r) P_{\lambda}(\cos\theta), \qquad \lambda = 0, 2, 4 \ldots$$

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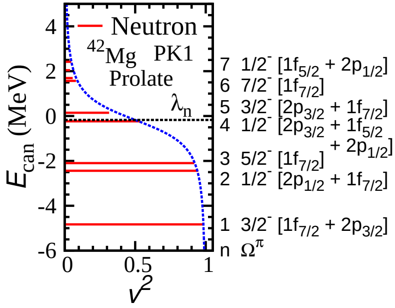
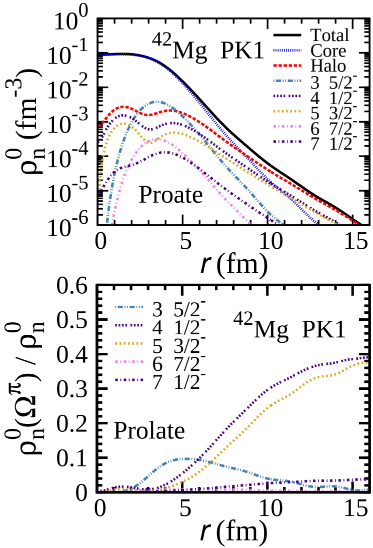
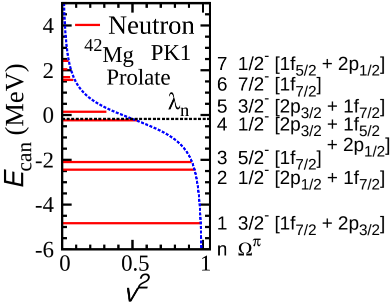
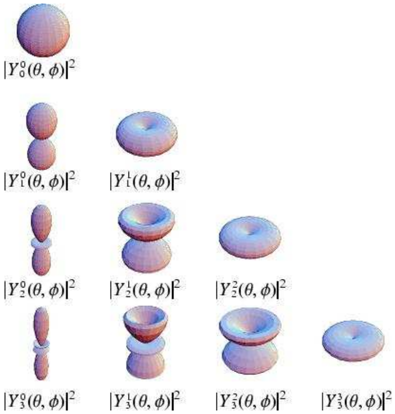



- Level 4, 5 give most contributions of the halo.

# $^{42}$Mg: shape of the halo




- Level 4
  - $2p_{3/2}$: gives 37% of the contribution, in which $\Lambda = 0$ component domains;
  - $1f_{5/2}$: gives 32% of the contribution, in which $\Lambda = 1$ component domains;
  - $2p_{1/2}$: gives 21% of the contribution, in which $\Lambda = 1$ component domains.
- Level 5
  - $2p_{3/2}$: gives 79% of the contribution, only has the component of $\Lambda = 1$.

# Summary

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- It is focused on halo phenomena in deformed nuclei (Mg isotopes).
- Halo may occur, depending on intrinsic properties of orbitals around the Fermi level.
- There might be shape decoupling between core and halo ($^{42}$Mg).

## Perspective

- How about the halo in odd-$A$ nuclei?
- What about the contribution of Fock term to halo phenomena?
  - The effect of $\pi$ meson,
  - The effect of tensor force,
  - ...

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*Thank you!*