

Global 3V data analysis: Past, present and future



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OUTLINE

Prologue: Why global analyses? (+ some past examples)

Main theme: 2012 global neutrino data analysis

Epilogue: Future challenges (a selection)

Based on work involving the Bari group:

G.L. Fogli, EL, A. Marrone, D. Montanino, A. Palazzo, A.M. Rotunno,

Prologue:

Why Global Analyses ?

- In general, global analyses of data from different experiments are necessary when some physical parameters are not (precisely) measured by any single experiment.
- In this case, the parameters may be at least constrained by joint, careful fits of various datasets. One can also find useful ways to properly show the allowed parameter space.
- Even when precise measurements become available, global analyses often remain useful to perform consistency tests of theoretical scenarios, where possible "tensions" may eventually emerge from the comparison of different datasets.
- Of course, such analyses have obvious limitations: they can never replace the experimental measurements! Nevertheless, they may provide some useful guidance about what can be expected.

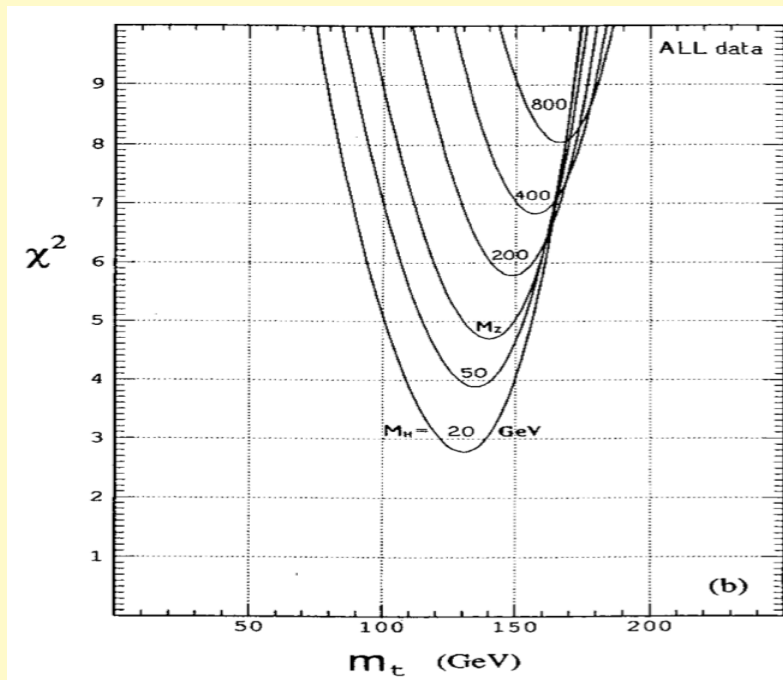
A few examples

(involving the Bari group)

Some results ~anticipated by global analyses of precision electroweak data:

... Before the 1995 top quark discovery:

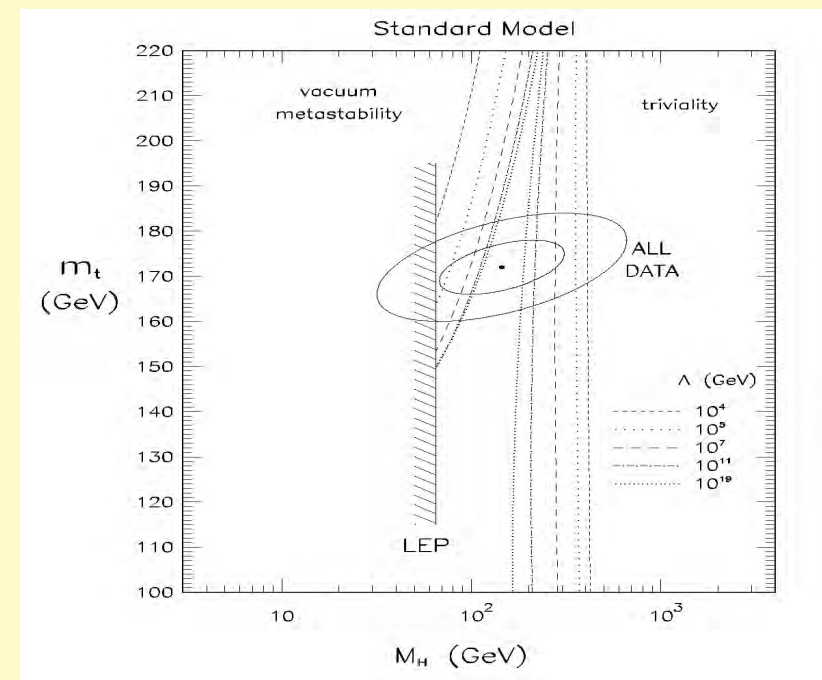
Top mass estimate: ~140 GeV
for Higgs mass around EW scale



J. Ellis, G.L. Fogli, EL (1993)

... Before the 2012 Higgs discovery:

Higgs mass estimate: ~150 GeV
within factor 2 uncertainty at 1σ



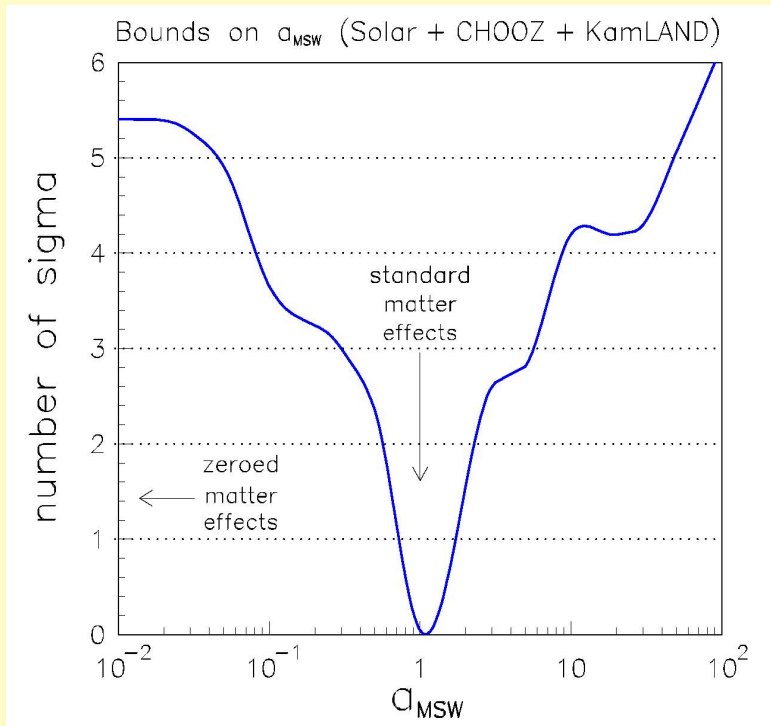
J. Ellis, G.L. Fogli, EL (1996)

Best fit estimates within ~20% from the direct measurement - not bad!

Some results anticipated by global neutrino oscillation analyses:

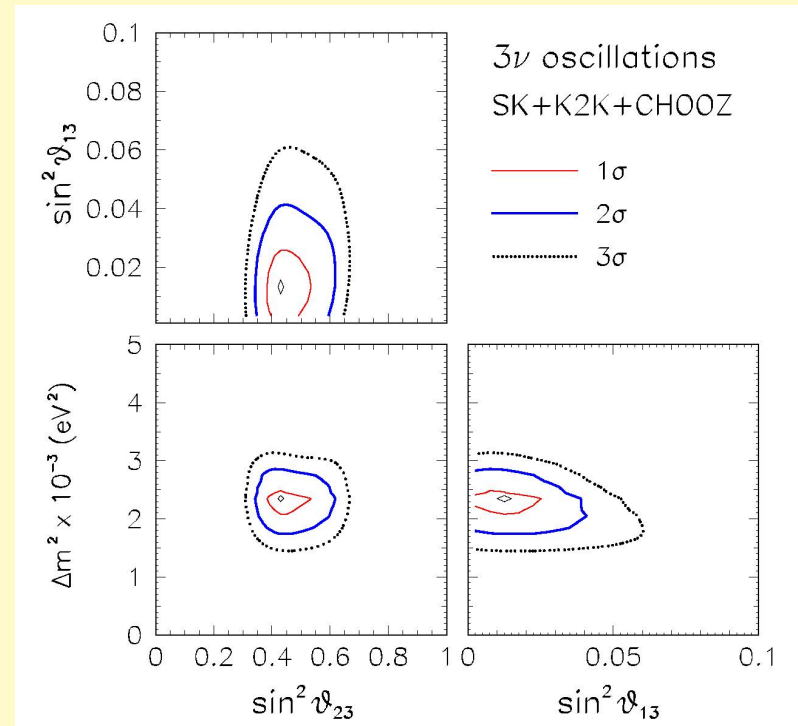
... Before Borexino:

Validation of MSW effect
(hypothetical scaling factor ~ 1)



... Before many ...:

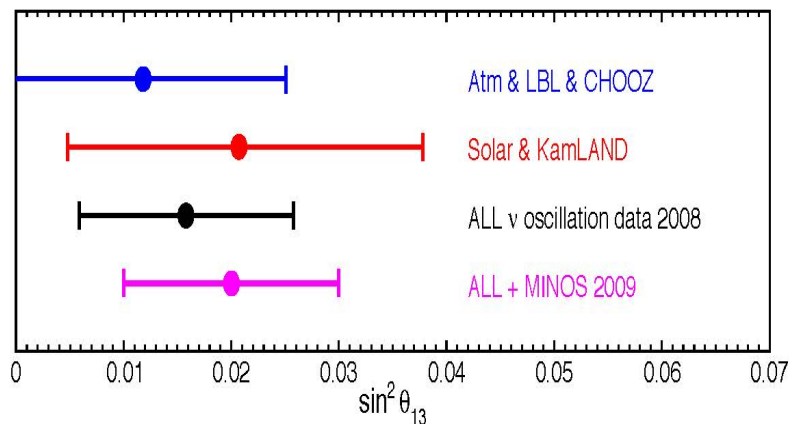
Possible hints of nonzero θ_{13}
and of nonmaximal θ_{23}



Fogli, EL, Marrone, Palazzo,
hep-ph/0506083 (2005)

... Before T2K 2011 & Reactors 2012:

$$\sin^2(2\theta_{13}) \sim 0.08 \pm 0.04$$



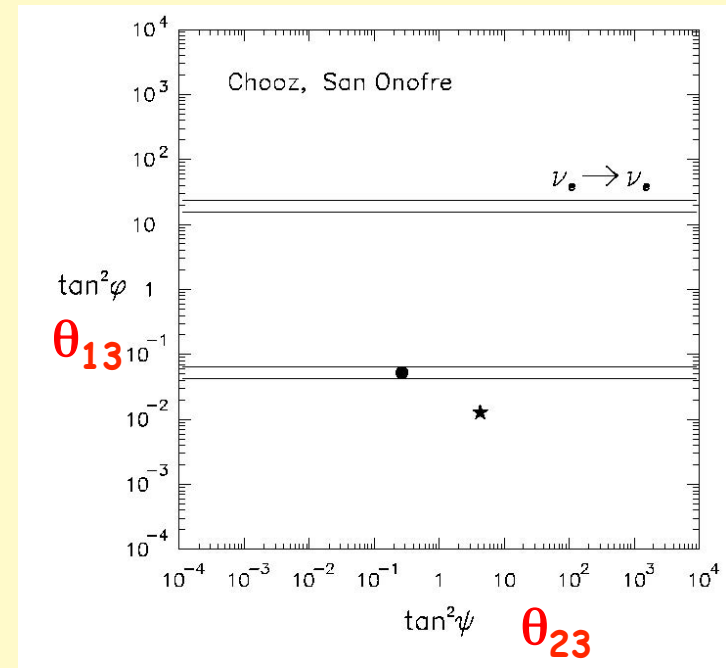
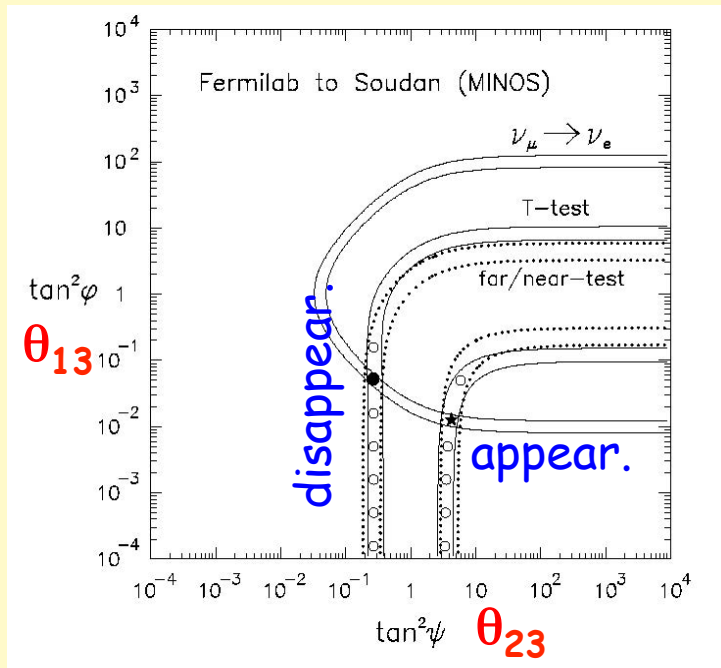
Wide skepticism until 2011/2012.

From this Symposium homepage:

"...the neutrino mixing angle θ_{13} is significantly larger than what was anticipated by most of us."

Fogli, EL, Marrone, Palazzo, Rotunno
arXiv:0806.2649, arXiv:0905.3549

Linking nonzero θ_{13} and nonmaximal θ_{23} (Fogli, EL, hep-ph/9604415)



For non-maximal mixing, LBL app. + disapp. data may allow two quasidegenerate solutions with anticorrelated θ_{13} and θ_{23}

Independent constraints on θ_{13} (sol.+KamLAND or SBL reactors) may then lift (at least in part) the θ_{23} octant degeneracy.

As we shall see, this hypothetical 1996 scenario might be emerging from 2012 data even though, at present, only at the level of hints!

This interplay is simply an effect of **3 ν** oscillation physics: in first approximation, **Δm^2** -driven vacuum oscillation probabilities are generalized as (**2 ν** \rightarrow **3 ν**):

$$P_{\alpha\beta} \simeq \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \longrightarrow P_{\alpha\beta} \simeq 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

$$P_{\alpha\alpha} \simeq 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \longrightarrow P_{\alpha\alpha} \simeq 1 - 4|U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right)$$

LBL appearance: $P_{\mu e} = \sin^2 \theta_{23} \sin^2(2\theta_{13}) \sin^2(\Delta m^2 L / 4E_\nu) + \text{corrections}$

NOT octant symmetric, anticorrelates θ_{23} and θ_{13} : the lower θ_{23} , the higher θ_{13}

SBL reactors: $P_{ee} = 1 - \sin^2(2\theta_{13}) \sin^2(\Delta m^2 L / 4E_\nu) + \text{corrections}$

So, they may distinguish "large" from "small" θ_{13}



3 ν combination of **LBL accelerator** and **SBL reactor** data may already provide some slight preference for one θ_{23} octant versus the other.

2012 Global data analysis

(1) Note about methodology:

We prefer to group **LBL accelerator** data with **solar+KamLAND** data, since the latter provide the “solar parameters” needed to calculate the full **3ν LBL probabilities in matter**. So, the sequence of constraints will be shown as:

$$(\text{LBL} + \text{Solar} + \text{KamLAND}) + (\text{SBL reactor}) + (\text{SK atm})$$

(2) Note about conventions:

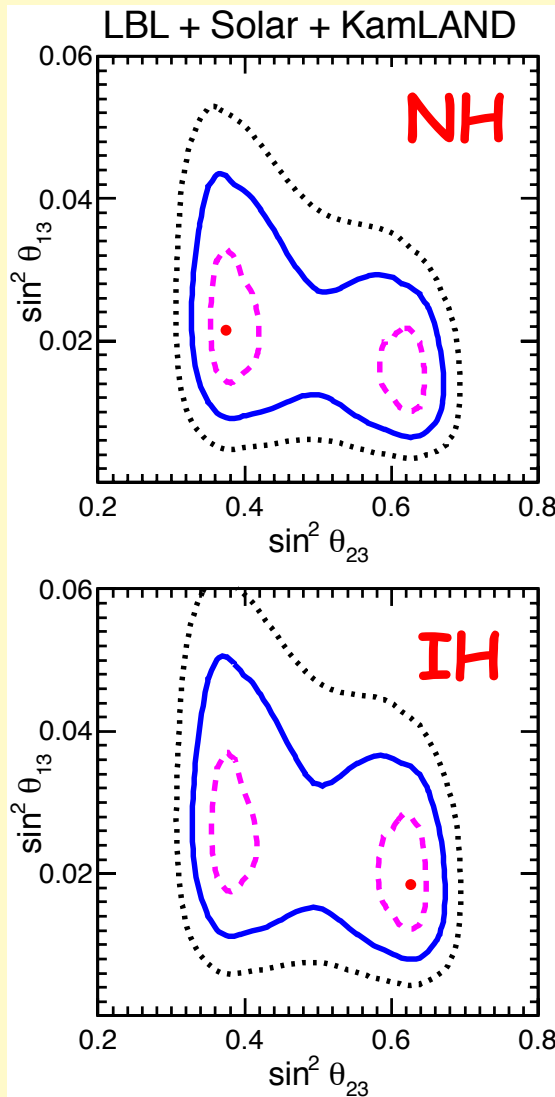
We show contours at **$N\sigma = \sqrt{\Delta\chi^2}$** [provide projected $N\sigma$ intervals for 1 dof]

We also use:
$$\Delta m^2 = \frac{1}{2} (\Delta m_{31}^2 + \Delta m_{32}^2)$$
 in both normal and inverted hierarchy (the sign just flips)

(3) Note about parameter space:

Since the squared mass differences are almost “fixed”, we prefer to show in more detail the **correlations among angles** (mixings and CP phase).

$(\sin^2\theta_{13}, \sin^2\theta_{23})$ from LBL app. + disapp. data plus solar + KamLAND data:



Latest LBL disappearance data from T2K and MINOS favor **nonmaximal** θ_{23}

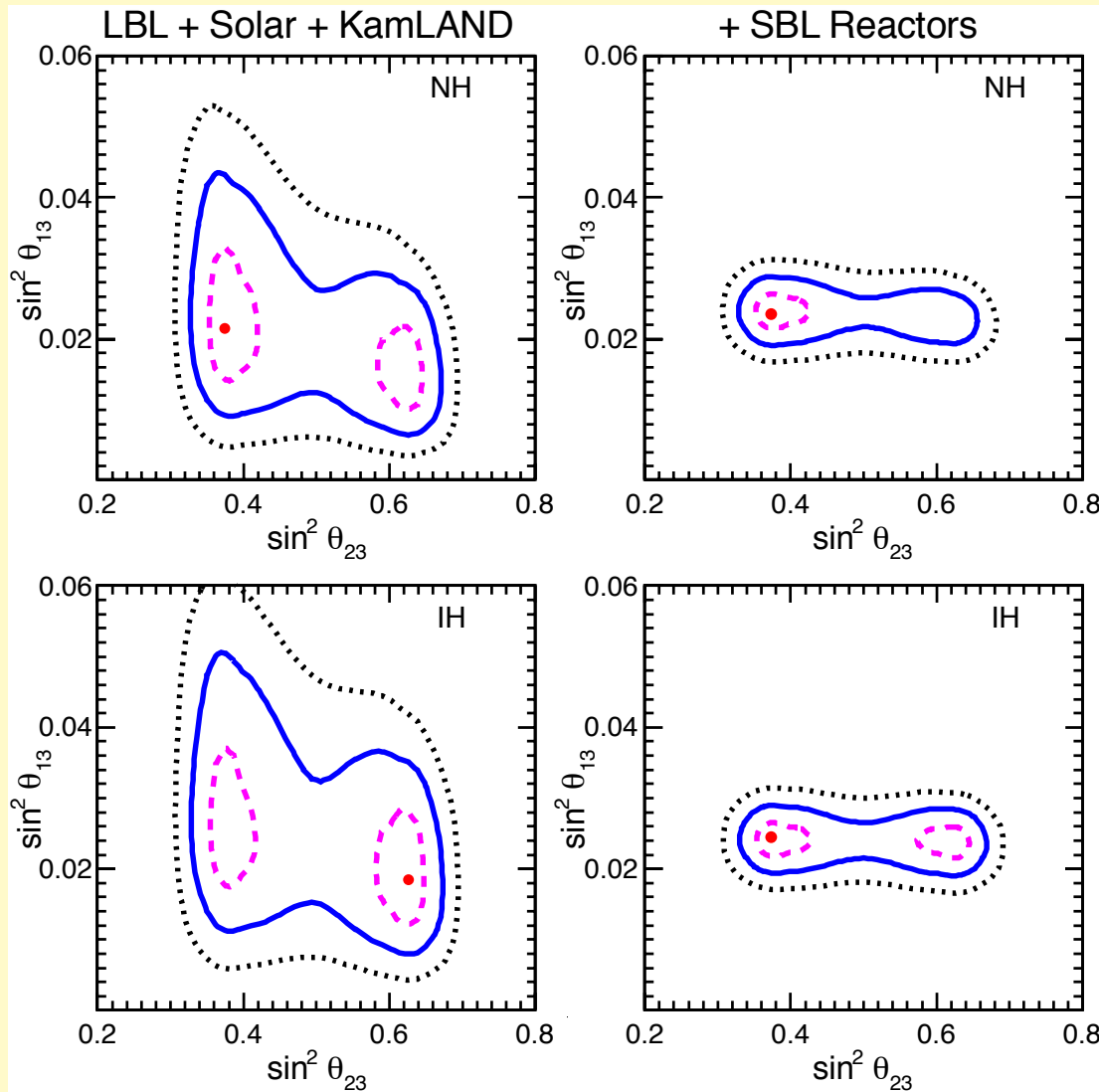
From LBL appearance+disappear. data, **two quasi-degenerate** θ_{23} solutions emerge, in **anticorrelation with** θ_{13} (one slightly above and the other slightly below $\sin^2\theta_{13} \sim 0.02$).

The two solutions merge above $\sim 1\sigma$.

[It would be nice to see these plots in the official T2K and MINOS data analyses!]

Solar+KamLAND data happen to prefer just $\sin^2\theta_{13} \sim 0.02$, and are unable to lift the octant degeneracy: **the depth of the two minima differ by only $\sim 0.3\sigma$.**

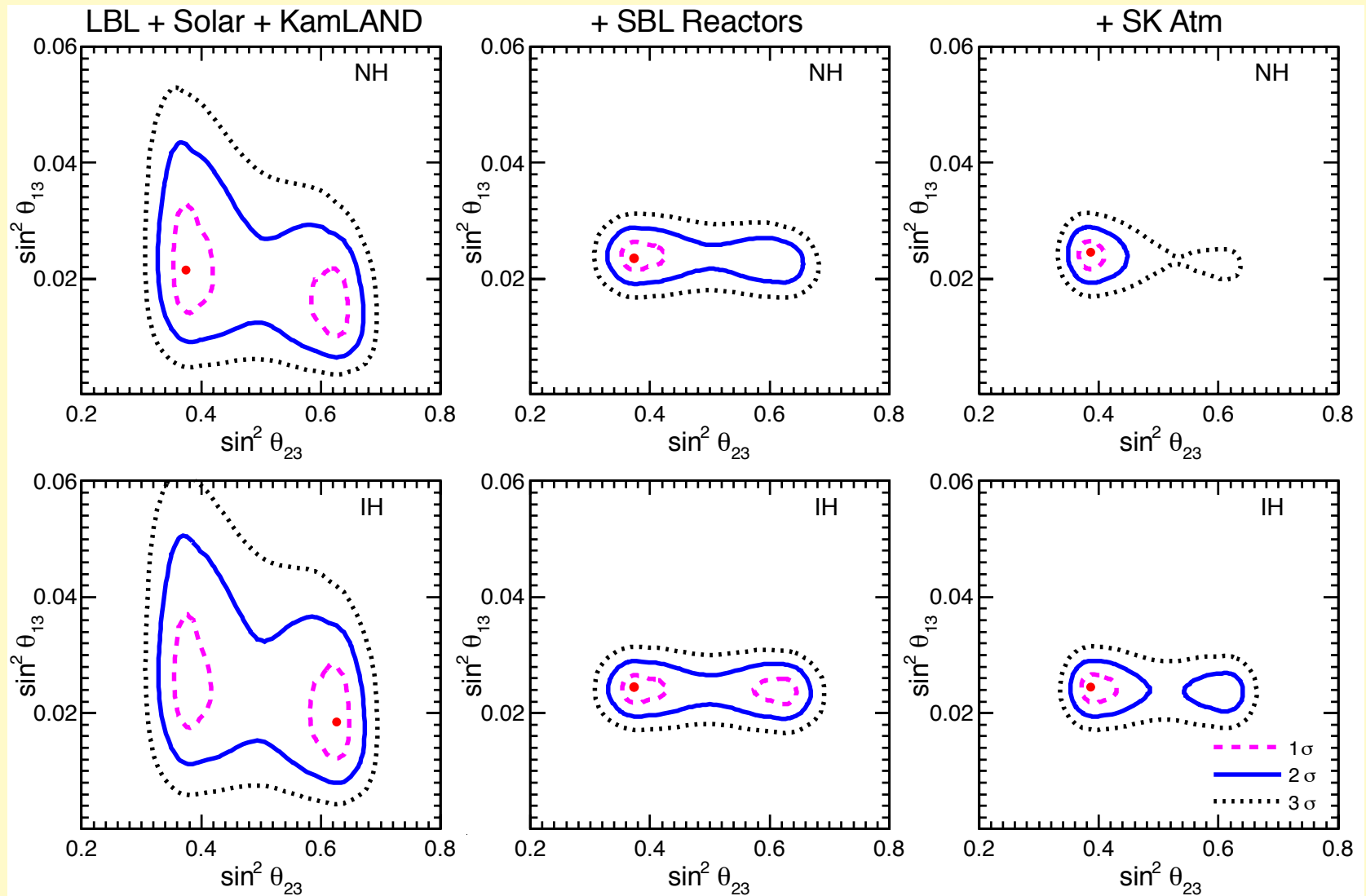
Adding 2012 SBL reactor constraints (Daya Bay, RENO, Double Chooz):



Overall preference emerges for the 1st octant solution at higher θ_{13} and lower θ_{23} , especially in NH.

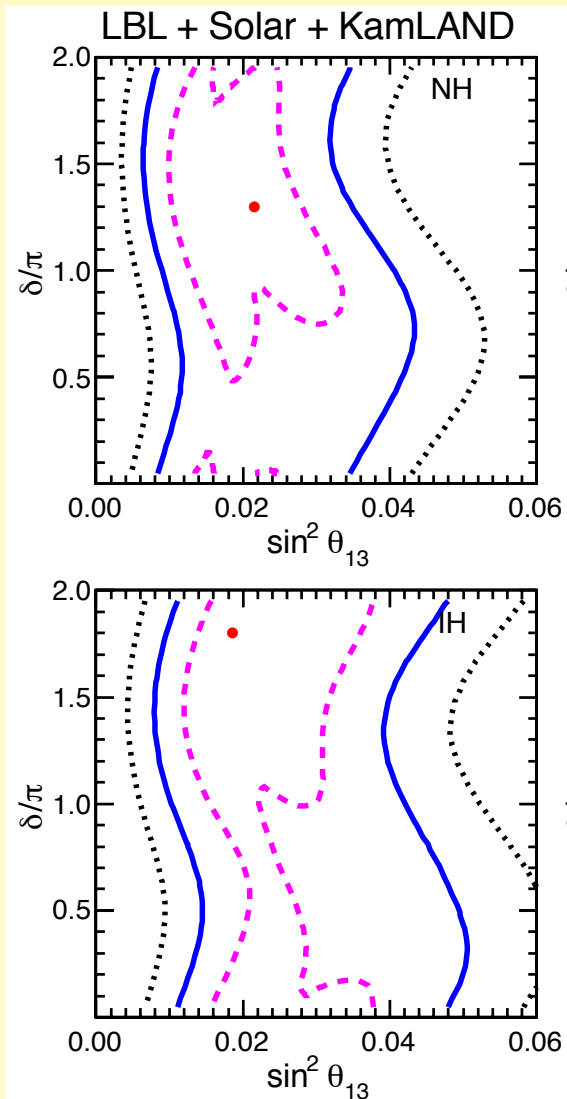
(Should future SBL reactor constraints prefer lower θ_{13} values, the preference might flip to the 2nd octant)

Adding 2012 SK atmospheric neutrino data:



Further hints for θ_{23} in 1st octant. But no significant hierarchy discrimination.

$(\sin^2 \theta_{13}, \delta)$ from LBL app. + disapp. data plus solar + KamLAND data:

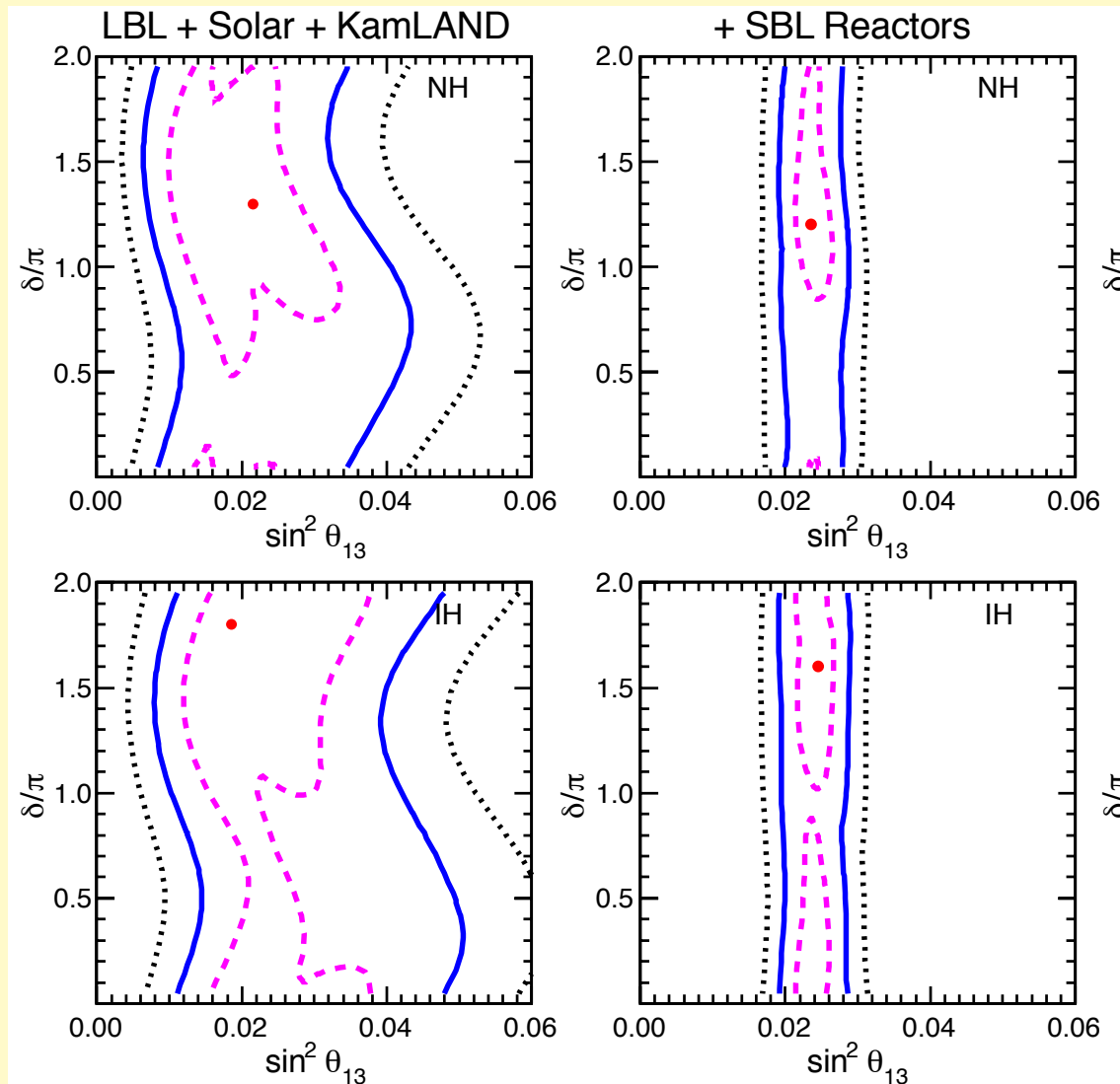


δ is basically unconstrained at $\sim 1\sigma$.

Fuzzy 1σ contours are a side effect of θ_{23} degeneracy: the two θ_{23} minima correspond to slightly different θ_{13} ranges and thus to two slightly overlapping "wavy bands" in the plot. Minima flip easily from one band to the other.

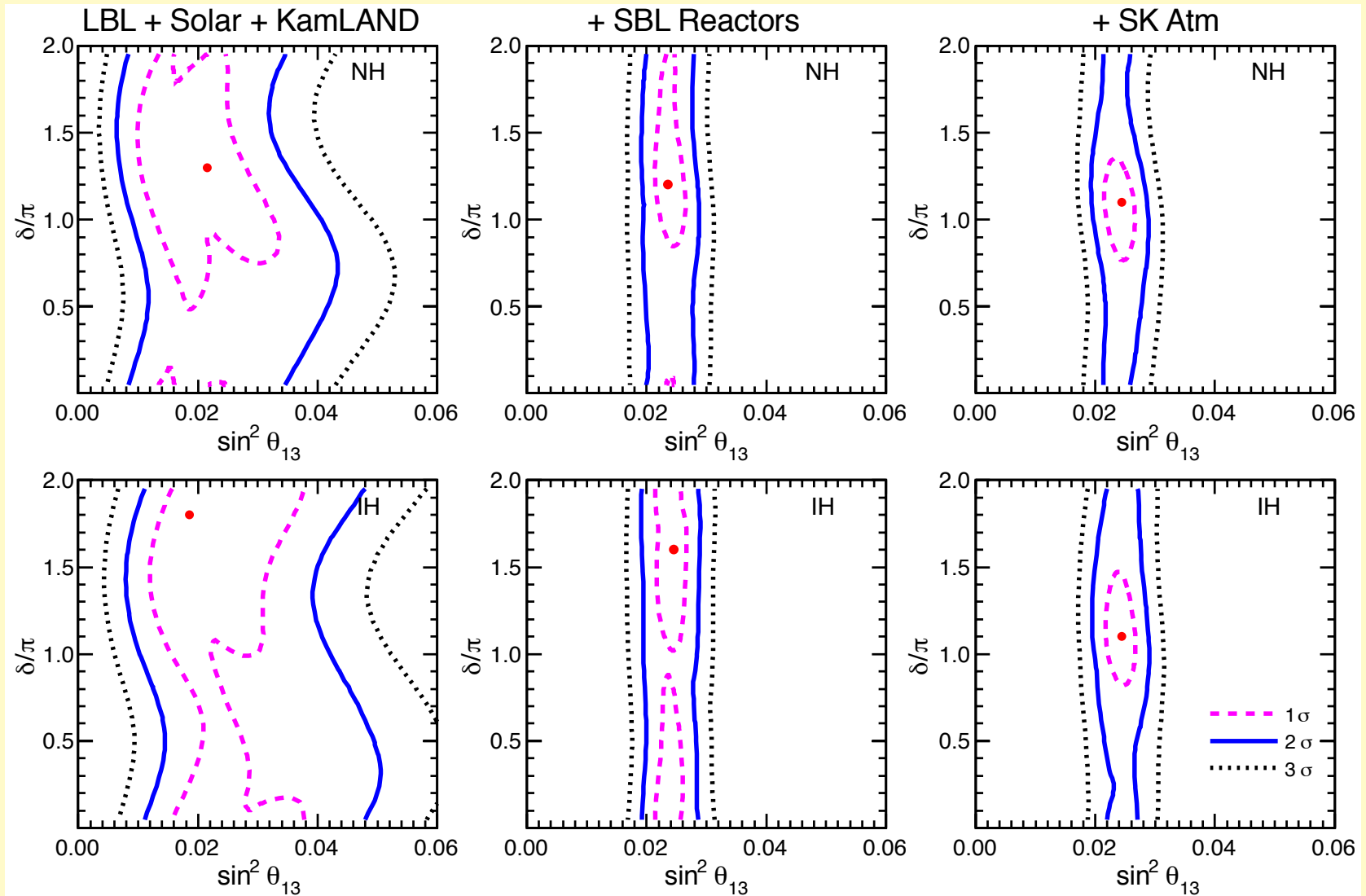
Fuzziness disappear at higher CL (degeneracy just enlarges bands).

Adding 2012 SBL reactor constraints (Daya Bay, RENO, Double Chooz):



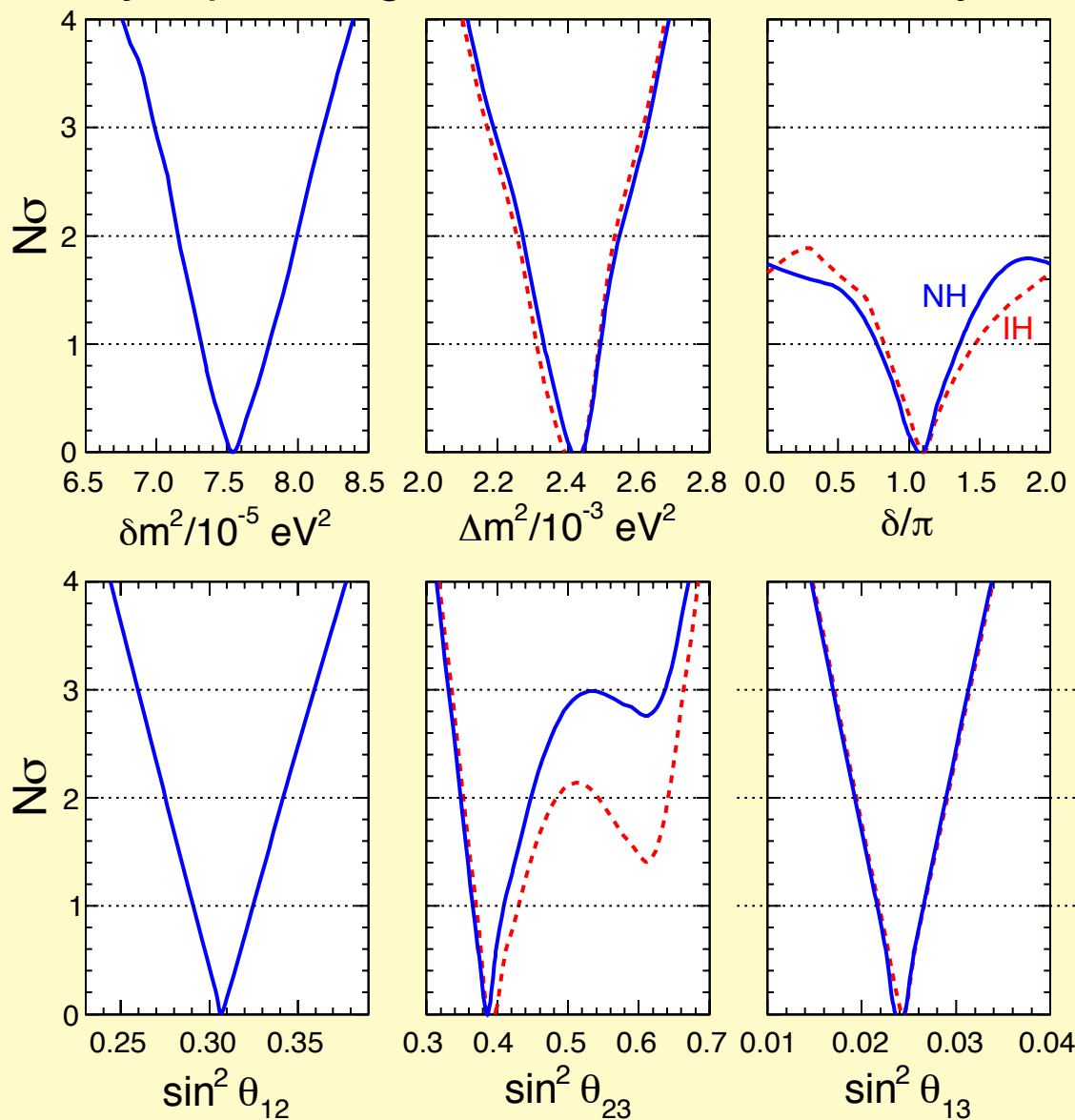
SBL reactor data
restrict θ_{13} and
reduce degeneracy
effects on the ns
contours.

Adding 2012 SK atmospheric neutrino data:



We find a preference for $\delta \sim \pi$ (helps fitting sub-GeV e-like excess in SK)

Synopsis of global 3ν oscillation analysis



Previous hints of $\theta_{13} > 0$ are now **measurements!** (and basically independent of old/new reactor fluxes)

Some hints of $\theta_{23} < \pi/4$ are emerging at $\sim 2\sigma$, worth exploring by means of atm. and LBL+reac. data

A possible hint of $\delta_{CP} \sim \pi$ emerging from **atm. data** [Is the PMNS matrix real?]

So far, **no hints** for
 NH \longleftrightarrow IH

Numerical 1σ , 2σ , 3σ ranges:

TABLE I: Results of the global 3ν oscillation analysis, in terms of best-fit values and allowed 1 , 2 and 3σ ranges for the 3ν mass-mixing parameters. We remind that Δm^2 is defined herein as $m_3^2 - (m_1^2 + m_2^2)/2$, with $+\Delta m^2$ for NH and $-\Delta m^2$ for IH.

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5} \text{ eV}^2$ (NH or IH)	7.54	7.32 – 7.80	7.15 – 8.00	6.99 – 8.18
$\sin^2 \theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25	2.75 – 3.42	2.59 – 3.59
$\Delta m^2/10^{-3} \text{ eV}^2$ (NH)	2.43	2.33 – 2.49	2.27 – 2.55	2.19 – 2.62
$\Delta m^2/10^{-3} \text{ eV}^2$ (IH)	2.42	2.31 – 2.49	2.26 – 2.53	2.17 – 2.61
$\sin^2 \theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66	1.93 – 2.90	1.69 – 3.13
$\sin^2 \theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67	1.94 – 2.91	1.71 – 3.15
$\sin^2 \theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10	3.48 – 4.48	3.31 – 6.37
$\sin^2 \theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31	$3.53 - 4.84 \oplus 5.43 - 6.41$	3.35 – 6.63
δ/π (NH)	1.08	0.77 – 1.36	—	—
δ/π (IH)	1.09	0.83 – 1.47	—	—

Fractional 1σ accuracy [defined as $1/6$ of $\pm 3\sigma$ range]

δm^2	Δm^2	$\sin^2 \theta_{12}$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{23}$
2.6%	3.0%	5.4%	10%	14%

Note: above ranges obtained for “old” reactor fluxes. For “new” fluxes, ranges are shifted (by $\sim 1/3 \sigma$) for two parameters only: $\Delta \sin^2 \theta_{12}/10^{-1} \simeq +0.05$ and $\Delta \sin^2 \theta_{13}/10^{-2} \simeq +0.08$

Hierarchy differences well below 1σ for various data combinations

There are some differences about octant preference and CP phase ranges in the two hierarchies, with respect to other recent global analyses:

Forero, Tortola & Valle arXiv:1205.4018v3

Gonzalez-Garcia, Maltoni, Salvado, Schwetz arXiv:1209.3023v2
SK official presentations at Neutrino 2012 and at NOW 2012

The differences seem to originate mainly in different approaches to atmospheric neutrino data. This dataset contains very rich physics, which is however smeared out over many decades in L and E; observables are also difficult to be reproduced and modeled (even in SK itself!).

I'll be happy to discuss in more detail these aspects with interested participants to this Symposium.

Epilogue: future challenges (a selection)

Hierarchy: no current hint! How can we get $\text{sign}(\pm\Delta m^2)$?
 Is the state with smallest ν_e component (ν_3) **lightest** or **heaviest**?

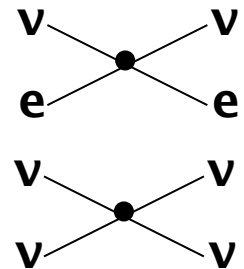
The hierarchy, namely, $\text{sign}(\pm\Delta m^2)$, can be probed (in principle), via interference of Δm^2 -driven oscillations with some other Q-driven oscillations, where Q is a quantity with known sign.

At present, the only known possibilities (barring new physics) are:

$Q = \delta m^2$ (e.g., high-precision oscillometry in vacuum)

$Q = \text{Electron density}$ (e.g., matter effects in Earth)

$Q = \text{Neutrino density}$ (SN ν - ν interaction effects)



Each of them is very challenging, for rather different reasons.

All of them are worth revisiting in more detail for “large” θ_{13} !

... A note: a 2001 analysis of CHOOZ spectral data observed that reactors could marginally feel the hierarchy

A. CHOOZ constraints

The general 3ν survival probability for electron antineutrinos at reactors (in vacuum) reads

$$\begin{aligned}
 P_{ee}^{\text{reac}} = & 1 - 4 \cos^4 \varphi \sin^2 \omega \cos^2 \omega \sin^2 \left(\frac{\delta k}{2} x \right) \\
 & - 4 \sin^2 \varphi \cos^2 \varphi \sin^2 \omega \sin^2 \left(\frac{k - \delta k/2}{2} x \right) \\
 & - 4 \sin^2 \varphi \cos^2 \varphi \cos^2 \omega \sin^2 \left(\frac{k + \delta k/2}{2} x \right), \quad (63)
 \end{aligned}$$

where x is the baseline. The above expression is invariant under the symmetry transformations $T_{\delta m^2} T_\omega$, $T_{\delta m^2} T_{m^2}$, and $T_{m^2} T_\omega$ [defined in Eqs. (36)–(38)], implying that the two spectra in Fig. 1 cannot be distinguished by reactor neutrino data (while they can be by solar ν data in the QEI regime, at least in principle).

In [16], Eq. (63) has been used in global 3ν oscillation fits by using the *total* CHOOZ rate [14]. However, since the low-energy part of the CHOOZ spectrum is more sensitive to relatively low values of δm^2 , we prefer to use the full CHOOZ data set (i.e., the binned spectra from the two reactors) rather than the total rate only. In particular, we have accurately reproduced the so-called “ χ^2 analysis A” of [14], by using two 7-bin positron spectra and one constrained normalization parameter, for a total of $14 + 1 - 1 = 14$ independent (but correlated) data. We obtain very good agreement with Fig. 9 in [14] for the two-flavor subcase (not shown).¹¹

Using our binned χ^2 analysis for CHOOZ, and setting $\tan^2 \varphi = 0.04$, $\tan^2 \omega = 0.5$, and $\delta m^2 = 0.6 \times 10^{-3} \text{ eV}^2$, we obtain $\chi^2/N_{\text{DF}} = 15.5/14$ for $m^2 = +1.5 \times 10^{-3} \text{ eV}^2$ and $\chi^2/N_{\text{DF}} = 13.4/14$ for $m^2 = -1.5 \times 10^{-3} \text{ eV}^2$. Due to the symmetry $T_{m^2} T_\omega$, the previous χ^2 values also apply for $\tan^2 \omega = 2$ by replacing $\pm m^2$ with $\mp m^2$. In any case, the choice of parameters adopted in Eqs. (60)–(62) gives $\chi^2/N_{\text{DF}} \simeq 1$, and thus passes the goodness-of-fit test.

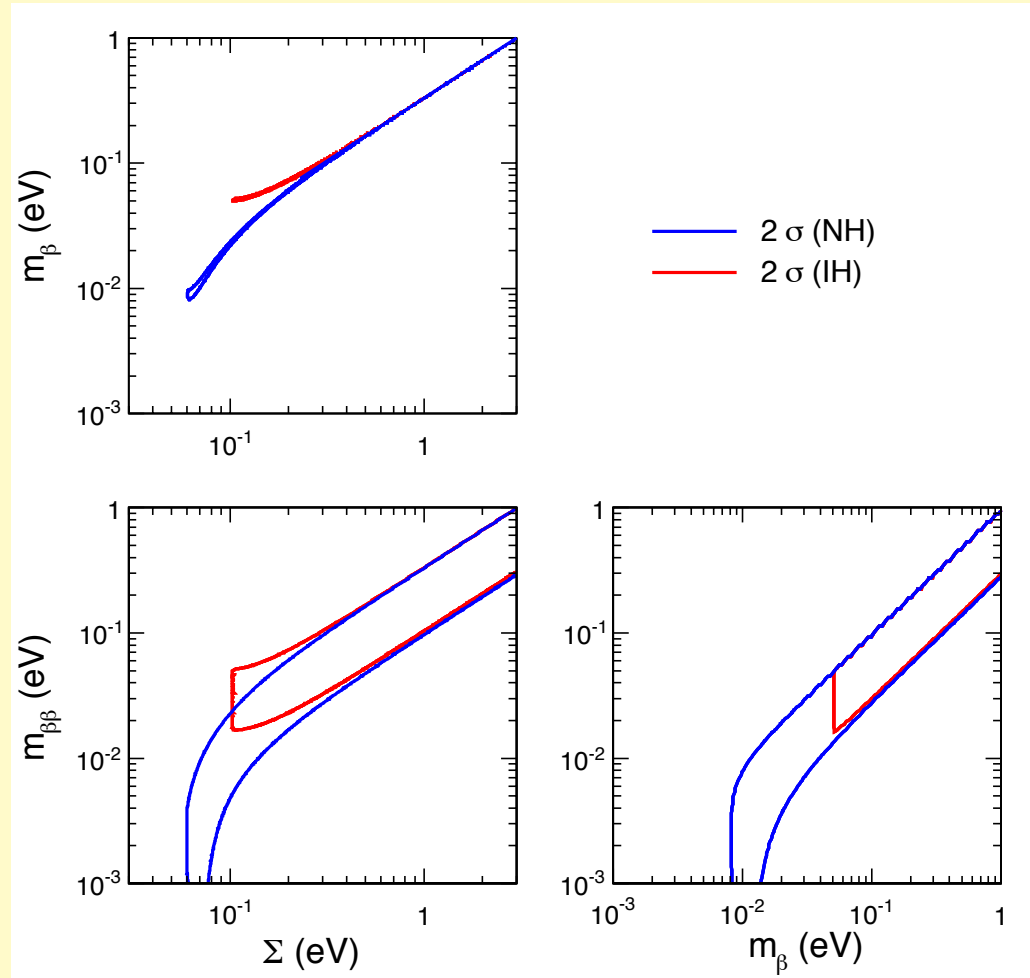
(Fogli, EL, Palazzo, hep-ph/0105080)

Another handle: 3V convergence of (osc, m_β , $m_{\beta\beta}$, Σ) data²³

Beta decay
Double beta decay
Cosmology

Experiments and quantification of expt+theo errors very challenging!

(Especially if upper limits will become measurements)



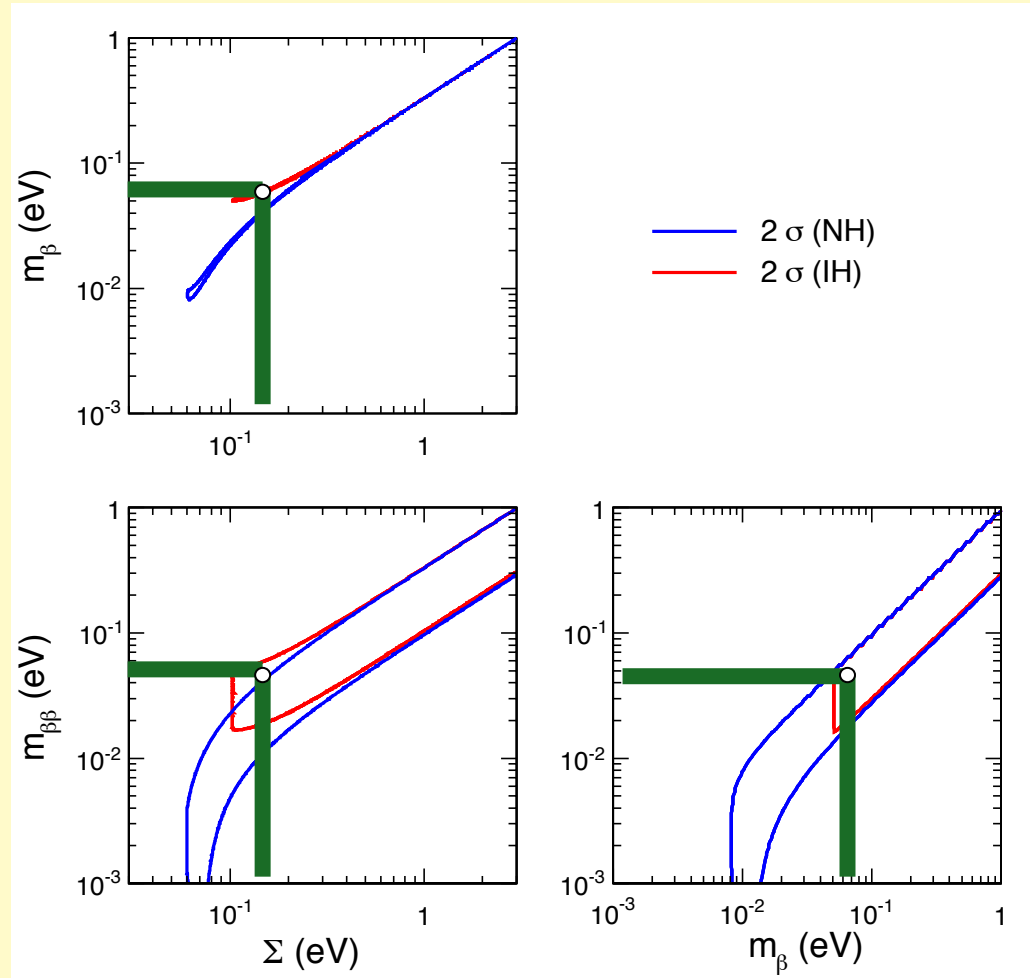
Dreaming about converging data with high accuracy...

If dreams come true, one might...

Determine the mass scale...

Identify the hierarchy ...

Probe the Majorana nature and phase(s)...



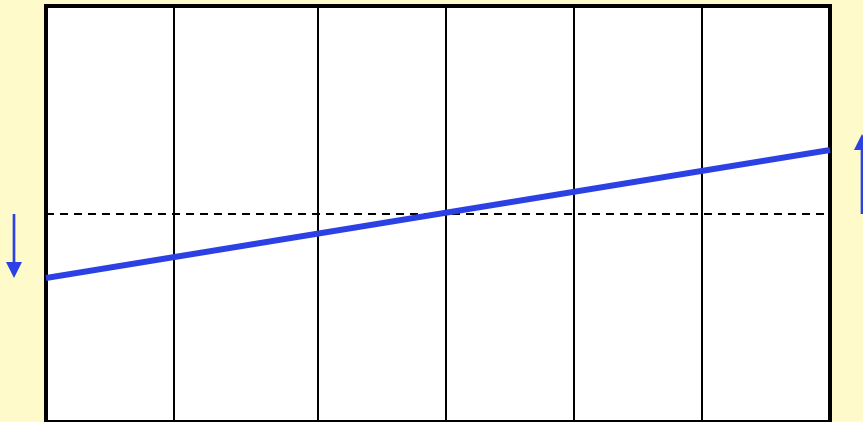
Finally, let me mention a subtle technical challenge for neutrino experiments with such high statistics to be dominated by systematics - a novel situation in neutrino physics!

(Discussed in more details at RCCN Tokyo Workshop in 2004)

The widely used "pull approach" to bin-by-bin correlated systematics implies that spectral shapes are modeled by a family of functions depending on N pulls. This may become a strong assumption.

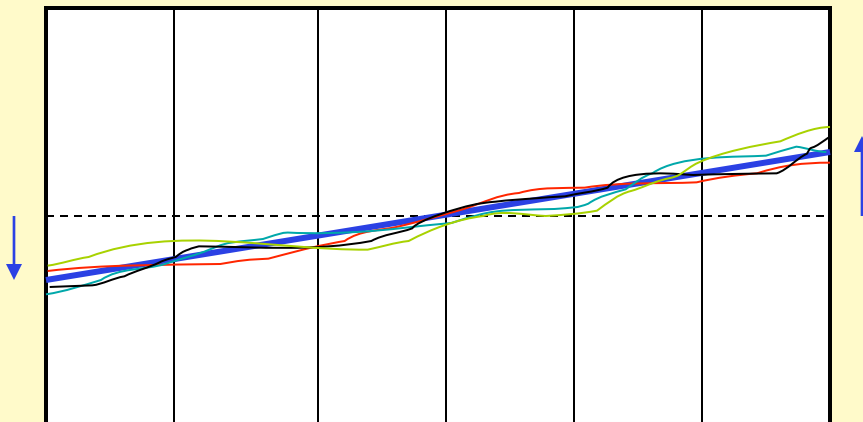
No matter how large is N (i.e., how "refined" is your systematic error budget): a generic measured shape will never fit your modeled family of shapes, if statistical errors are \sim zero!

A simple example:



bins in some parameter

Parametrization of shape errors through a single systematic pull, e.g., a linear “tilt”, linear in $\cos(\theta)$

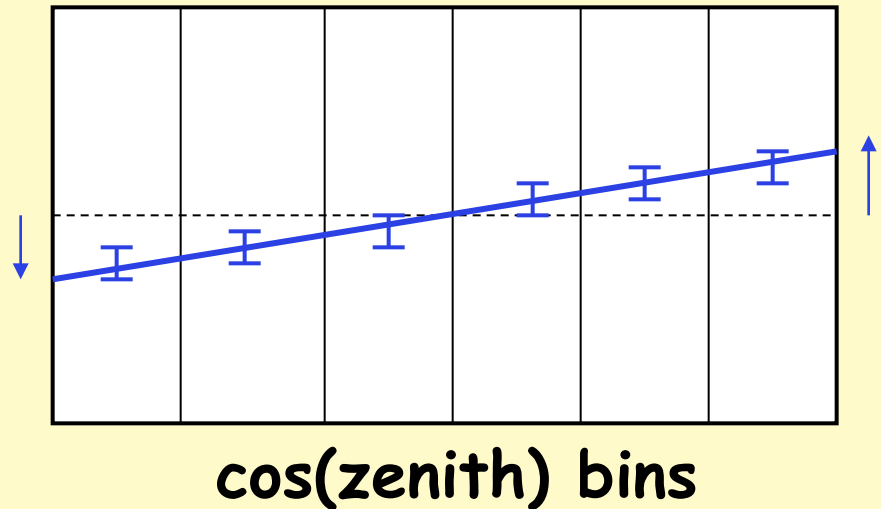
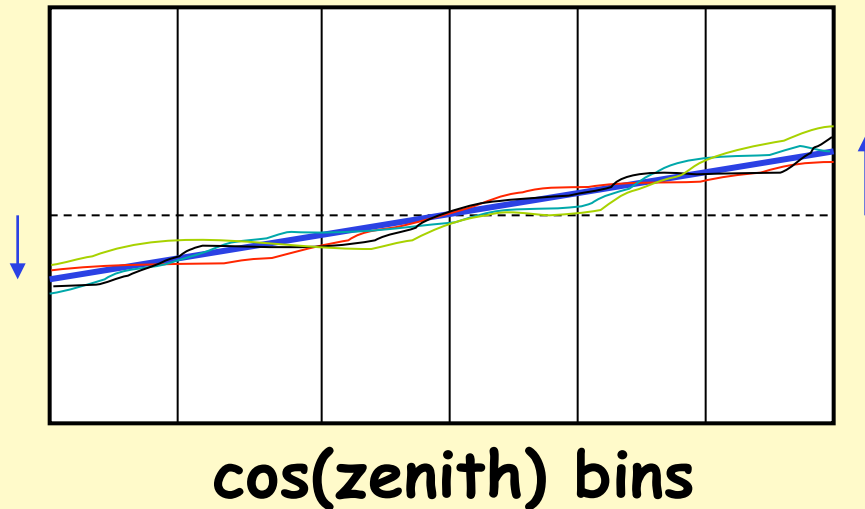


(e.g., zenith bins in atm ν)

In general, nature will provide a different measured shape, which can adapt to a linear tilt only if statistical errors give some tolerance! Fit impossible in the limit of zero statistical errors (even if further, nonlinear pulls are added).

Incomplete knowledge of error sources and shapes may be severe in some cases, e.g., in **atmospheric neutrino flux spectra**

This means that, on top of any dominant (correlated) systematic trend, one should allow small uncorrelated systematics, e.g.:



$$\chi_{\text{SK}}^2 = \min_{\{\xi_k\}} \left[\sum_{n=1}^{55} \left(\frac{\tilde{P}_n^{\text{theo}} - R_n^{\text{expt}}}{\sigma_n^{\text{stat}}} \right)^2 + \sum_{k,h=1}^{11} \xi_k [\rho^{-1}]_{hk} \xi_h \right]$$

$$\frac{1}{\sigma_n^2(\text{stat})} \rightarrow \frac{1}{\sigma_n^2(\text{stat}) + \sigma_n^2(\text{syst})}$$

Single-bin uncorrelated systematics will prevent χ^2 “explosion” and determine the ultimate sensitivity for vanishing stat. errors

At high statistics, need to evaluate uncorrelated systematics of each bin.

CONCLUSIONS

Global analyses involve reproducing and combining original observations in a well-defined theoretical framework, in order to explore features which might escape otherwise.

Reproductions cannot replace the originals (= real data)...
but can be fun anyway!



Thank you for your attention.