Remarks on hadronic molecules

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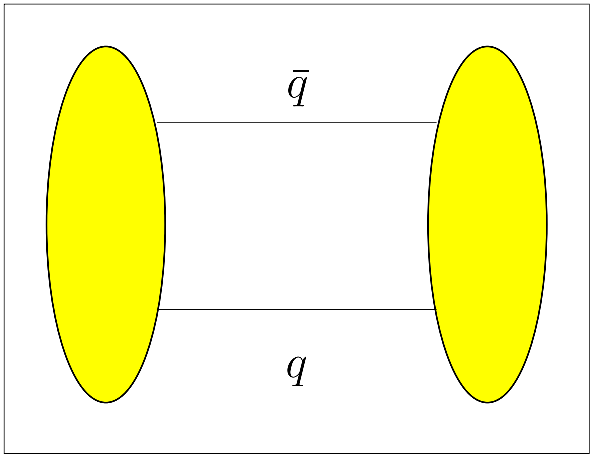
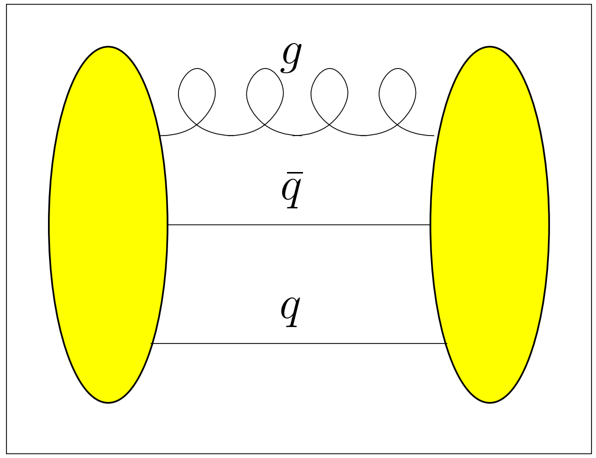
Exotics

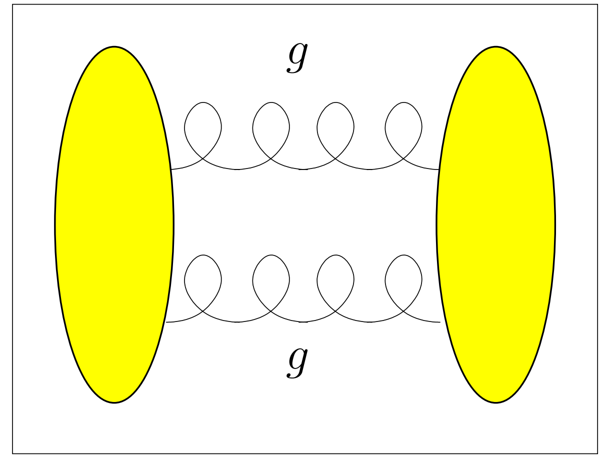
$\bar{q}q$



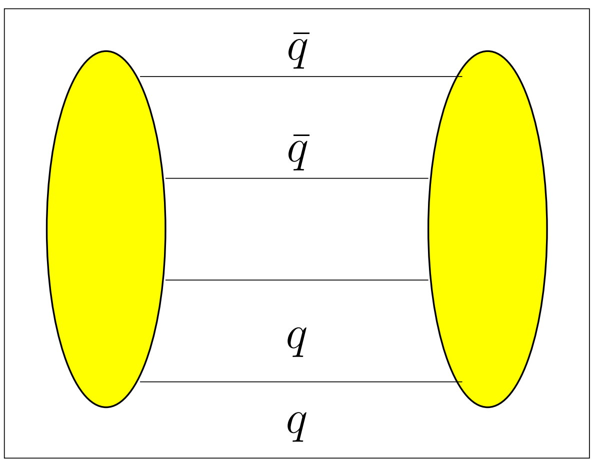
hybrid



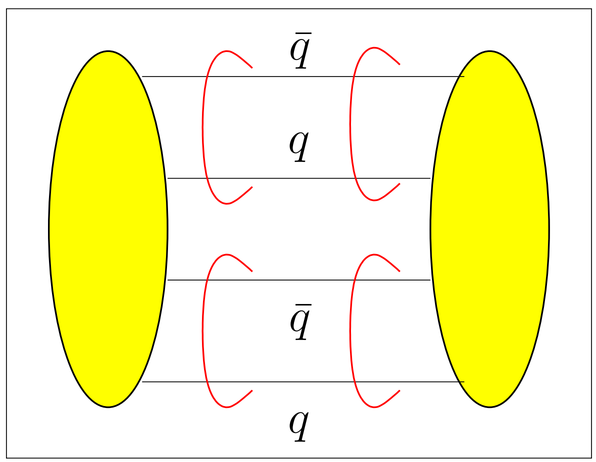
glueball



tetraquark



molecule



Only hadrons can go on-shell $\longrightarrow$ Unique analytic properties of molecular amplitude

non–rel. QM

Weinberg PR 130 (1963) 776

Expand in terms of non–interacting quark and meson states

$$|\Psi\rangle = \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi(\mathbf{p})|h_1 h_2\rangle \end{pmatrix},$$

here $|\psi_0\rangle$ = elementary state and $|h_1 h_2\rangle$ = two–hadron cont., then λ^2 equals probability to find the bare state in the physical state

$\rightarrow$ λ^2 is the quantity of interest!

After some algebra we get for the residue at the pole

$$\boxed{g_{\rm eff}^2 = 2(1-\lambda^2)\sqrt{\epsilon/m} \le 2\sqrt{\epsilon/m}}$$

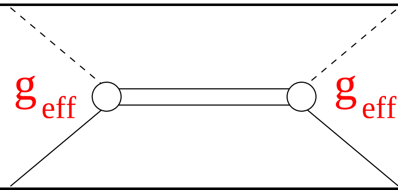
For bound state low E amplitude fixed in hh channel!

Picture not changed by far away threshold Baru et al. PLB586 (2004) 53

Equivalent to, e.g., Morgan NPA543 (1992) 63; Törnqvist PRD51 (1995) 5312

Remarks

- model independent only for s-waves
- corrections scale as $\sqrt{mE_B}R$, with R=range of forces
  $\longrightarrow$ model independent for near threshold states only
- one finds

$a =$ 

$+\mathcal{O}(R) \simeq \left(2\frac{1-\lambda^2}{2-\lambda^2}\right)\frac{-1}{\sqrt{mE_B}}$

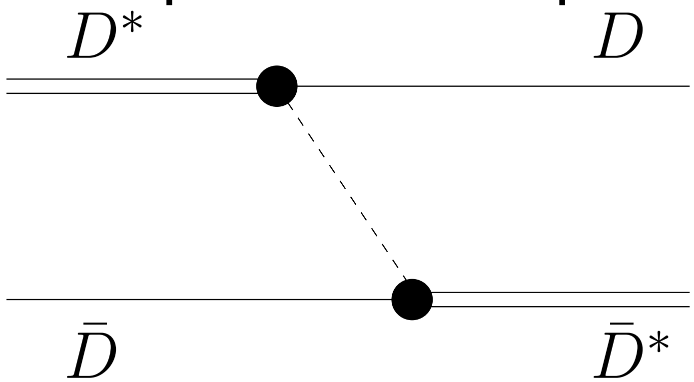
  $\longrightarrow$ for molecules: significant signature near threshold

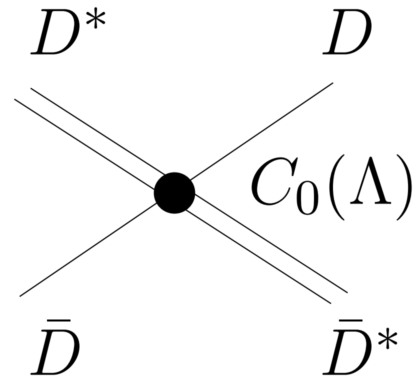
- applicable only, if there are no other small scales in the problem   C.H. et al., 'Interplay of quark and meson degrees of freedom' (2010,2011)
- Mass splittings from mother states:
  $D_{s1}(2460)/D_s(2317)$ as D^*K/DK molecules
  $M(D_{s1}(2460)) - M(D_s(2317)) = M(D^*) - M(D)$

X(3872): Dynamical model

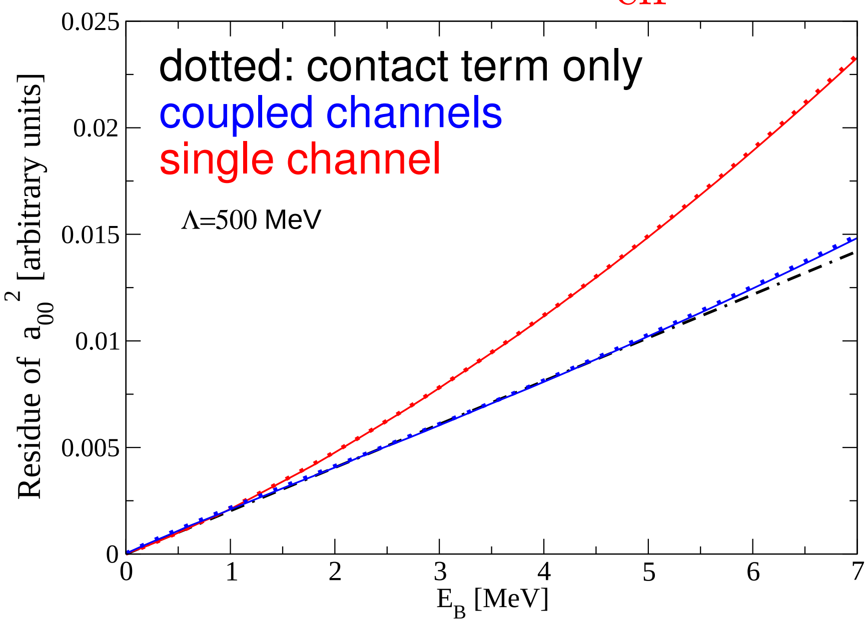
$\bar{D}D\pi$ Faddeev equations with potential

Baru et al. PRD 84 (2011) 074029

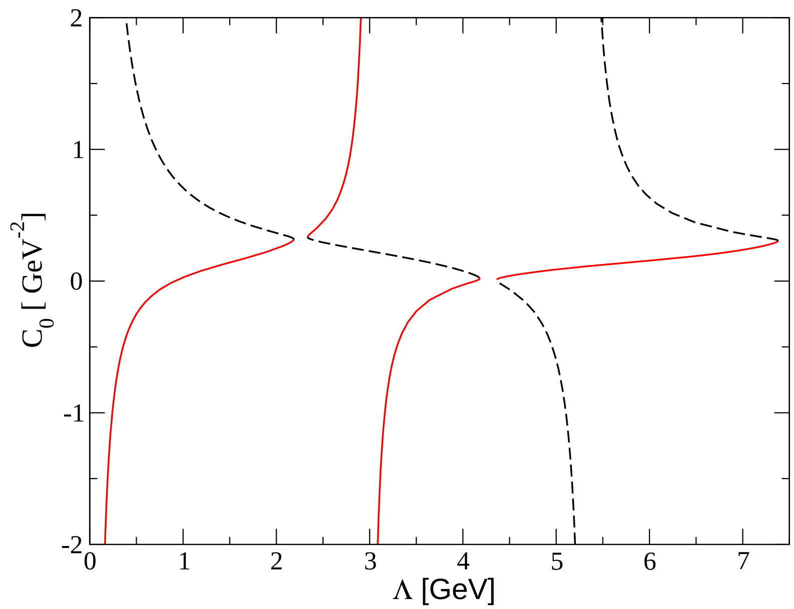




E_B dependence of $g_{\rm eff}^4$:



Role of pion exchange:



→ for $X(3872)$ pions perturbative

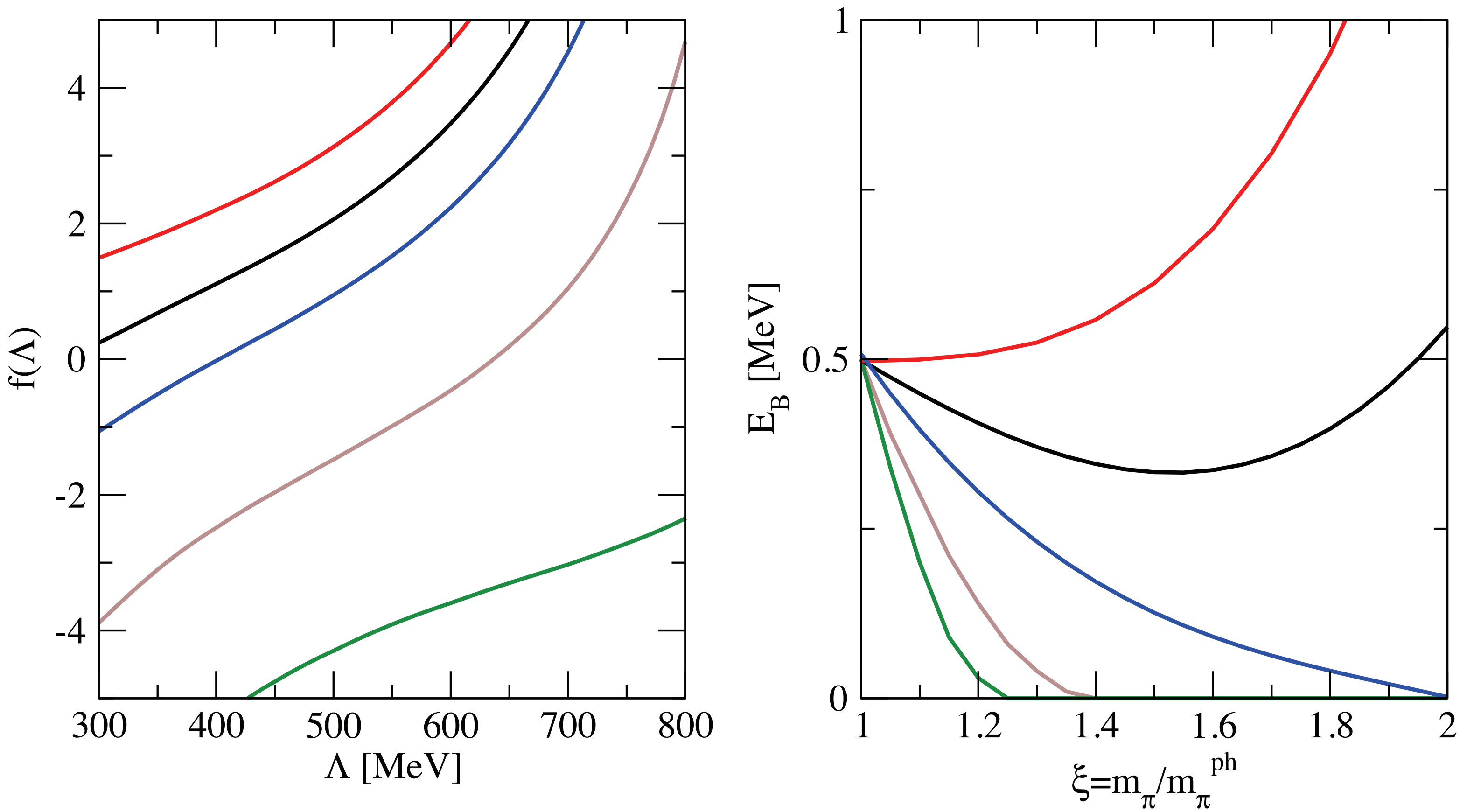
→ Explore SU(3) multiplets

Fleming et al. PRD76 (2007) 034006

Filin et al.; in preparation

Application: m_π dependence

$$C_0(\xi) = C_0(\Lambda)\left(1 + f(\Lambda)\frac{\delta m_\pi^2}{\mathrm{M}^2}\right) = C_0(\Lambda)\left(1 + f(\Lambda)\frac{(m_\pi^{\mathrm{ph}})^2}{\mathrm{M}^2}(\xi^2 - 1)\right)$$



Fix $f(\Lambda)$ via fixing $\partial E_B/\partial m_\pi = (0..-2)\times 10^{-2}$ Λ independent

Lattice will teach us m_π dependence of $\bar{D}^* D$ potential!

our results in conflict with Wang and Wang, arXiv:1304.0846v1

Thus ...

- For s–wave bound-states near a threshold:
  Amount of molecular admixture of the state is encoded in effective coupling constant to the continuum

- Spin symmetry allows to predict new states, since spin symmetry violation equal for molecules and mother states

  Exception: very near threshold states like Z_b's - symmetry violation enhanced due to strong final state interaction (c.f. NN scat. in ${}^1S_0$: $(a_{pn} - a_{nn})/a_{NN} \sim 25\%$)

  Cleven et al. , Eur.Phys.J. A47 (2011) 120

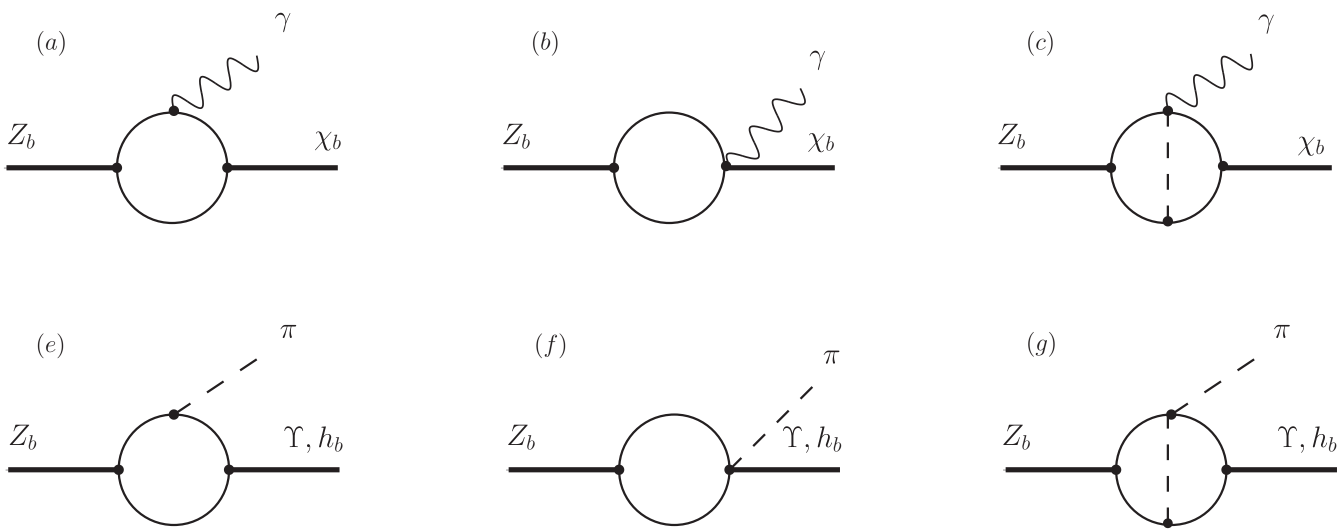
What to do in case of resonances (second sheet poles)?

$\rightarrow$ Relation between ϵ and $g_{\rm eff}$ lost, but still $g_{\rm eff}$ large

$\rightarrow$ for hadron-hadron states: all transitions through loops
$\longrightarrow$ relations between different reactions

Examples: Z_b and Z_b'

Cleven et al. PRD 87 (2013) 074006



if, e.g., (e) dominates (analogously for (a) dominance)

$$\frac{\Gamma[Z_b' \to h_b(mP)\pi]}{\Gamma[Z_b \to h_b(mP)\pi]} \simeq \frac{\mathrm{PS}_m'}{\mathrm{PS}_m} \frac{\Gamma(Z_b'^{+} \to B^{*+}\bar{B}^{*0})}{\Gamma(Z_b^{+} \to B^{+}\bar{B}^{*0} + B^{0}\bar{B}^{*+})},$$

with $\Gamma(Z_b'^{+} \to B^{*+}\bar{B}^{*0})/\Gamma(Z_b^{+} \to B^{+}\bar{B}^{0} + B^{0}\bar{B}^{*+}) \simeq 1.6$

other terms supp. for $\chi_b(m)$ & $h_b(m)$ final states $m = 2, 3$, then

even $\Gamma(Z_b^{(\prime)0} \to \chi_{bJ}(mP)\gamma)/\Gamma(Z_b^{(\prime)0} \to h_b(mP)\pi^0)$ predicted

$Z_c(3900)$

So far: given the Z_c, how can we test , if it is a D^*D–resonance?

Questions:

→ Why is Z_c seen in Y and not in B decays? see Q. Zhao's talk (NEXT!)

→ if it comes from non-perturbative D^*D interactions:

What is the dynamics that drives it above threshold?

Conventional potential scattering: no resonances in S-waves!

- ▷ Is the scattering potential energy dependent?

c.f. C.H., Pelaez, Rios, PRL 100(2008) 152001

- ▷ Can the inelastic channels move the pole?
- ▷ .... or is there a tetra-quark component?

I do not know ... → Round table

Summary

Amongst the various options for exotic states,

molecules near threshold are special

since the dynamics in controlled by the two-hadron cut

$\rightarrow$ for bound states: the coupling to the two-hadron state is fixed

$\rightarrow$ mass differences related to those of mother particles
e.g.: Z_b at $\bar{B}B^*$ threshold; Z_b' at $\bar{B}^*B^*$ threshold;

$\rightarrow$ Transitions to two-hadron channel large (c.f. tetra-quarks!)
e.g.: Z_b/Z_b' decay predominantly to $\bar{B}^{(*)}B^*$

$\rightarrow$ in general: seemingly unrelated reactions get related
(if higher order contributions are suppressed)

Molecular hypothesis can be tested experimentally!