QCD and relativistic corrections of spin singlet hadronic inclusive decays

Jin-Zhao Li (Peking University)

Based on the work:
H.-K. Guo, Y.-Q. Ma and K.-T. Chao Phys. Rev. D83, 114038 (2011)
J.-Z. Li, Y.-Q. Ma and K.-T. Chao hep-ph/1209.4011 (2012)

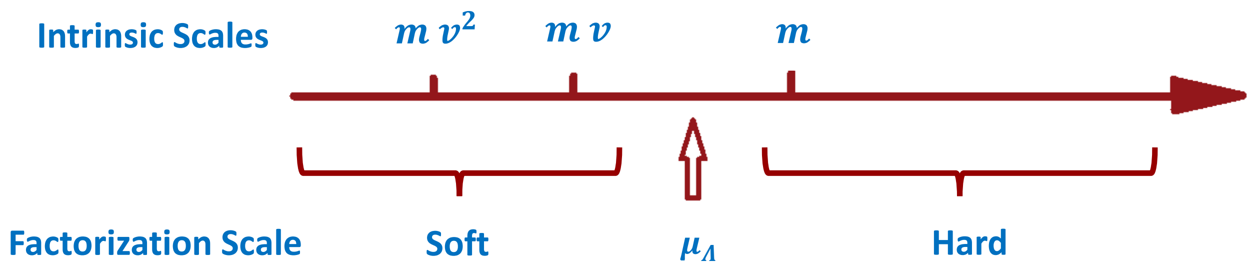
The 9th International Workshop on Heavy Quarkonium 2013
IHEP, P. R. China from 22 - 26 April, 2013

Outline

- Background
- Theoretical Setup
- Calculation
- Results
- Summary & Outlook

Background — NRQCD in Annihilation Decays

- Scales In Quarkonium Annihilation Decays



Quark mass: m

Relative Momentum: mv

Banding Energy: mv^2

$$\Lambda_{QCD} \sim mv^2 \ll mv \ll m$$

- Charm—anticharm system (Charmonium): $v_c^2 \sim 0.3$

  Bottom—antibottom system (Bottomonium): $v_b^2 \sim 0.1$

Relativistic effects of charmonium is more remarkable than bottomonium!

- NRQCD Factoriztion Formula：

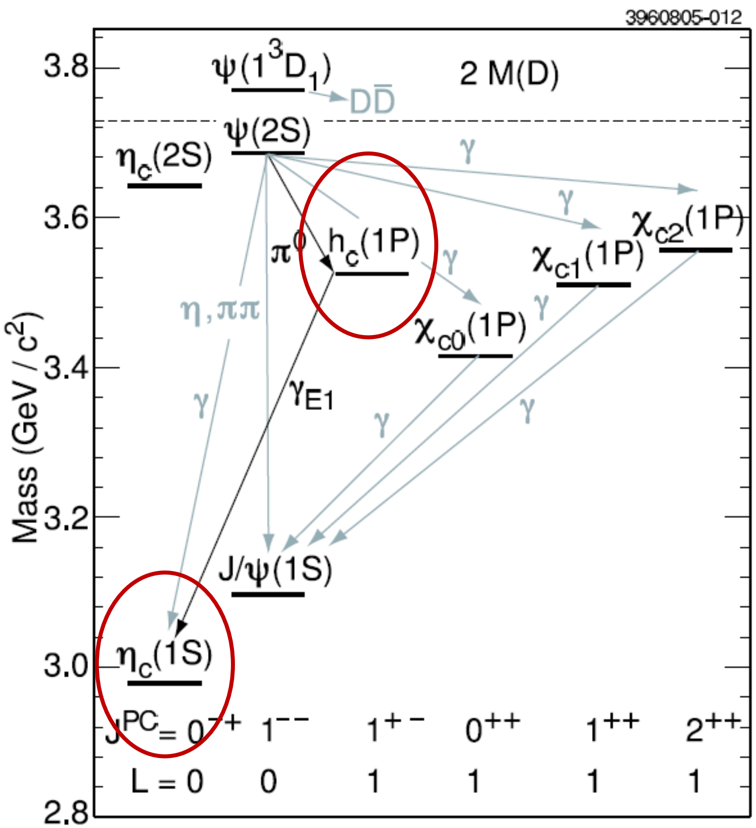
Bodwin, Braaten, and Lepage(1995,1997)

$$\Gamma(H[Q\bar{Q}] \to \text{light hadron}, \gamma\gamma) = \sum_n \frac{2\,\mathrm{Im}\, f_n^{\mathrm{LH,em}}(\mu_\Lambda)}{m^{d_n-4}} \langle H \,|\, O_n(\mu_\Lambda) \,|\, H \rangle_{\mathrm{LH,em}}$$

hard： SD Coefficients

soft: Matrix Elements

Background —η_c & h_c



- $\eta_c(h_c)$: S(P)-Wave Spin-singlet charmonium state.
- $M(\eta_c) = 2981.0 \pm 1.1$ MeV
  $M(h_c) = 3525.41 \pm 0.16$ MeV

PDG2012

- $\psi(2S) \longrightarrow \pi^0 \, h_c$, $h_c \longrightarrow \gamma\eta_c$

- Decay width— experiment data

$\Gamma^{LH}_{\exp}(\eta_c) = \Gamma^{total}_{\exp}(\eta_c) = 28.6 \ \pm 2.2$ MeV

$\Gamma^{\gamma\gamma}_{\exp}(\eta_c) = 7.2 \pm 0.7 \pm 2.0$ KeV

PDG2010

$\Gamma^{total}_{exp}(h_c) = 0.73^{+0.45}_{-0.28}$ MeV

$\mathrm{Br}_{exp}(h_c \longrightarrow \eta_c + \gamma) = 54.3 \pm 6.7 \pm 5.2\%$

BESIII, PRL. 104, 132002

Background — motivation

- Are QCD and relativistic corrections Important?

$$\Gamma(\eta_c \to \mathrm{LH}) = \frac{F({}^1S_0^{[1]})}{m^2}\left\langle \mathcal{O}({}^1S_0^{[1]})\right\rangle_{\eta_c} + \frac{G({}^1S_0^{[1]})}{m^2}\frac{\left\langle \mathcal{P}({}^1S_0^{[1]})\right\rangle_{\eta_c}}{m^2} + O(v^4).$$

Only relativistic correction: Bodwin and Petrelli(2002),...

SD Coefficients: $G^{(0)}({}^1S_0^{[1]})/F^{(0)}\left({}^1S_0^{[1]}\right) = -4/3$

Matrix Elements: $\left\langle \mathcal{P}\left({}^1S_0^{[1]}\right)\right\rangle_{\eta_c} \Big/ m^2 \left\langle \mathcal{O}\left({}^1S_0^{[1]}\right)\right\rangle_{\eta_c} \sim v_c^2 \sim 0.3$

The relativistic correction decreases the value by about 40%.

Only QCD correction: Petrelli, Cacciari, etc.(1998),...

SD Coefficients: $F^{(1)}({}^1S_0^{[1]})/F^{(0)}\left({}^1S_0^{[1]}\right) = 0.95 \quad (\mu_R = 2\,m)$

Matrix Elements: $\mathrm{ratio} = 1$

The QCD correction enhances the value by nearly 100% !

How about QCD and relativistic Corrections?

$$G^{(1)}({}^1S_0^{[1]})/G^{(0)}\left({}^1S_0^{[1]}\right) = ?$$

- P-wave: The argument is similar since ${}^1S_0^{[8]}$ dominates the decay width. (See below)

Theoretical Setup — Power Counting

- We adopt inhomogeneous power counting rule (Bodwin, Braaten, and Lepage(1995,1997)) instead of homogeneous one(Brambilla, Pineda, Soto and Vairo(2005)).

- Operators

Operator	Estimate	Description
α_s	v	effective quark-gluon coupling constant
ψ	$(Mv)^{3/2}$	heavy-quark (annihilation) field
χ	$(Mv)^{3/2}$	heavy-antiquark (creation) field
D_t	Mv^2	gauge-covariant time derivative
$\mathbf{D}$	Mv	gauge-covariant spatial derivative
$g\mathbf{E}$	M^2v^3	chromoelectric field
$g\mathbf{B}$	M^2v^4	chromomagnetic field
$g\phi$ (in Coulomb gauge)	Mv^2	scalar potential
$g\mathbf{A}$ (in Coulomb gauge)	Mv^3	vector potential

- Fock States

$$|H\rangle = O(1)\,|Q\bar{Q}\rangle + O(v)\,|Q\bar{Q}\,g^E\rangle + O\left(v^{3/2}\right)|Q\bar{Q}\,g^M\rangle + O(v^2).$$

- Only 2 terms for S-wave and 5 terms for P-wave in decay width are required up to $O(v^2)$ corrections.

Theoretical Setup — Decay Formula(I)

- S-wave

$$\Gamma(\eta_c \to \mathrm{LH}) = \frac{F({}^1S_0^{[1]})}{m^2}\left\langle \mathcal{O}({}^1S_0^{[1]})\right\rangle_{\eta_c} + \frac{G({}^1S_0^{[1]})}{m^4}\left\langle \mathcal{P}({}^1S_0^{[1]})\right\rangle_{\eta_c}.$$

$$\mathcal{O}\left({}^1S_0^{[1]}\right) = \psi^\dagger \chi\, \chi^\dagger \psi$$

$$\mathcal{P}\left({}^1S_0^{[1]}\right) = \frac{1}{2}\psi^\dagger \chi\, \chi^\dagger \left(-\frac{i\overleftrightarrow{\boldsymbol{D}}}{2}\right)^2 \psi + h.c.$$

$O(v^6)$ ← $\left\langle \mathcal{O}({}^1S_0^{[1]})\right\rangle_{\eta_c}$, $\left\langle \mathcal{O}_{em}({}^1S_0^{[1]})\right\rangle_{\eta_c}$

$O(v^8)$ ← $\left\langle \mathcal{P}({}^1S_0^{[1]})\right\rangle_{\eta_c}$, $\left\langle \mathcal{P}_{em}({}^1S_0^{[1]})\right\rangle_{\eta_c}$

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{F^{em}({}^1S_0^{[1]})}{m^2}\left\langle \mathcal{O}_{em}({}^1S_0^{[1]})\right\rangle_{\eta_c} + \frac{G^{em}({}^1S_0^{[1]})}{m^4}\left\langle \mathcal{P}_{em}({}^1S_0^{[1]})\right\rangle_{\eta_c}.$$

$$\mathcal{O}_{em}\left({}^1S_0^{[1]}\right) = \psi^\dagger \chi |0\rangle\langle 0| \chi^\dagger \psi$$

$$\mathcal{P}_{em}\left({}^1S_0^{[1]}\right) = \frac{1}{2}\psi^\dagger \chi |0\rangle\langle 0| \chi^\dagger \left(-\frac{i\overleftrightarrow{\boldsymbol{D}}}{2}\right)^2 \psi + h.c.$$

Theoretical Setup — Decay Formula(II)

- P-wave

$O(v^8)$

$$\Gamma(h_c \to \mathrm{LH}) = \frac{F({}^1P_1^{[1]})}{m^4}\left\langle \mathcal{O}({}^1P_1^{[1]})\right\rangle_{h_c} + \frac{F({}^1S_0^{[8]})}{m^2}\left\langle \mathcal{O}({}^1S_0^{[8]})\right\rangle_{h_c}$$
$$+ \frac{G({}^1P_1^{[1]})}{m^6}\left\langle \mathcal{P}({}^1P_1^{[1]})\right\rangle_{h_c} + \frac{G({}^1S_0^{[8]})}{m^4}\left\langle \mathcal{P}({}^1S_0^{[8]})\right\rangle_{h_c}$$
$$+ \frac{T({}^1S_0, {}^1P_1)}{m^5}\langle \mathcal{T}_{1-8}({}^1S_0, {}^1P_1)\rangle_{h_c}$$

$O(v^{10})$

$$\mathcal{O}\left({}^1P_1^{[1]}\right) = \psi^\dagger\left(-\frac{i\overleftrightarrow{\boldsymbol{D}}}{2}\right)\chi \cdot \chi^\dagger\left(-\frac{i\overleftrightarrow{\boldsymbol{D}}}{2}\right)\psi$$

$$\mathcal{O}\left({}^1S_0^{[8]}\right) = \psi^\dagger T^a \chi\, \chi^\dagger T^a \psi$$

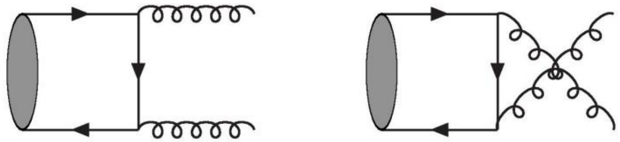
$$\mathcal{P}\left({}^1P_1^{[1]}\right) = \frac{1}{2}\psi^\dagger\left(-\frac{i\overleftrightarrow{\boldsymbol{D}}}{2}\right)\chi \cdot \chi^\dagger\left(-\frac{i\overleftrightarrow{\boldsymbol{D}}}{2}\right)^3\psi + h.c.$$

$$\mathcal{P}\left({}^1S_0^{[8]}\right) = \frac{1}{2}\psi^\dagger T^a \chi\, \chi^\dagger T^a\left(-\frac{i\overleftrightarrow{\boldsymbol{D}}}{2}\right)^2\psi + h.c.$$

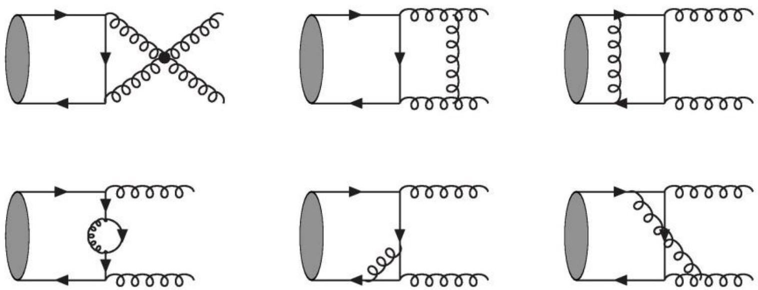
$$\mathcal{T}_{1-8}({}^1S_0, {}^1P_1) = \frac{1}{2}\psi^\dagger g\boldsymbol{E}\chi \cdot \chi^\dagger\overleftrightarrow{\boldsymbol{D}}\psi + h.c.$$

Calculation—Typical Feynman Diagrams

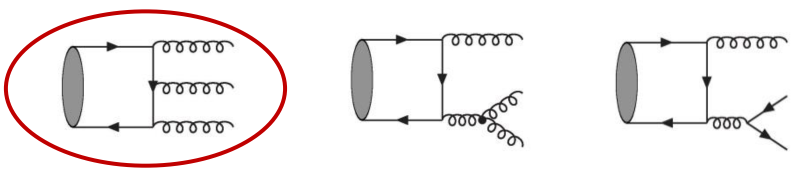
- LO



- Virtual



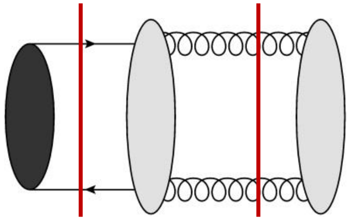
- Real



- Constrains on ${}^1P_1^{[1]}$

${}^1P_1^{[1]}$: C = −

$(gg)^{[1]}$: C = +



C—parity forbidden !

Calculation—Spin Singlet Projector

- Projected Amplitudes

C parity-Conserved spin singlet projector:

$$\Pi_0 = \frac{1}{2\sqrt{2}(E_{\mathbf{q}}+m_Q)}(\frac{\not P}{2}+\not q+m_Q)\frac{(\not P+2E_{\mathbf{q}})\gamma_5(-\not P+2E_{\mathbf{q}})}{8E_{\mathbf{q}}^2}(\frac{\not P}{2}-\not q-m_Q),$$

Color projector:

$$\mathcal{C}_1 = \frac{\mathbf{1}}{\sqrt{N_c}}, \qquad \mathcal{C}_8 = \sqrt{2}\mathbf{T}^c.$$

$$p_Q = \frac{1}{2}P+q, \qquad p_{\overline{Q}} = \frac{1}{2}P-q,$$

$$\mathcal{M}(q) = Tr[\mathcal{C}\,\Pi_0\mathcal{A}(q)].$$

- Expand in powers of relative momentum q

$$\mathcal{M}(q) = \mathcal{M}(0)+\frac{\partial\mathcal{M}(q)}{\partial q^\alpha}\Big|_{q=0}q^\alpha+\frac{1}{2!}\frac{\partial^2\mathcal{M}(q)}{\partial q^\alpha\partial q^\beta}\Big|_{q=0}q^\alpha q^\beta + \frac{1}{3!}\frac{\partial^3\mathcal{M}(q)}{\partial q^\alpha\partial q^\beta\partial q^\gamma}\Big|_{q=0}q^\alpha q^\beta q^\gamma+\cdots,$$

- Average the direction of q to get the replacement

$$\Pi_{\alpha\beta} \equiv -g_{\alpha\beta}+\frac{P_\alpha P_\beta}{P^2}$$

$$q_\alpha q_\beta \to \frac{\mathbf{q}^2}{D-1}\Pi_{\alpha\beta},$$

$$q_\alpha q_\beta q_\gamma q'_\lambda \to \frac{\mathbf{q}^2\mathbf{q}\cdot\mathbf{q}'}{D+1}(\Pi_{\alpha\beta}\Pi_{\gamma\lambda}+\Pi_{\alpha\gamma}\Pi_{\beta\lambda}+\Pi_{\alpha\lambda}\Pi_{\gamma\beta}),$$

Calculation—Phase Space Corrections

- Relative momentum appears not only in amplitudes

$$P^2 \equiv M^2 = 4m^2 + \mathbf{q}^2$$

$\mathbf{q}$ dependent term other than amplitudes:

flux factor: $1/2M$

phase space element: $\int \prod_i \frac{d\, k_i^{D-1}}{(2\pi)^{D-1}}\ \delta(P - \sum_i k_i)$

Polarization sum : $\Pi_{\alpha\beta} \equiv -g_{\alpha\beta} + \frac{P_\alpha P_\beta}{M^2}$

They all depend on $\mathbf{q}^2$

In our case, all the final state partons are massless

$$k_i^2 = 0 \implies x\, k_i^2 = 0.$$

- A math trick: absorb all the $\mathbf{q}^2$ dependence in to amplitudes.

$$P \longrightarrow P' \frac{E_q}{m} \qquad\qquad P'^2 = 4m^2$$

$$\implies$$

$$k_i \longrightarrow k_i' \frac{E_q}{m} \qquad\qquad k_i'^2 = 0$$

- The cost: more complicate in calculating amplitudes.

Calculation—Residue Infrared Divergence(I)

- ${}^1S_0^{[1]}$, ${}^1S_0^{[8]}$

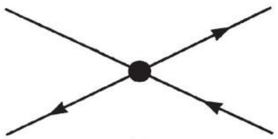
LO in $\boldsymbol{v}$:

$$\hat{\Gamma}^{(v^0)}(real) + \hat{\Gamma}^{(v^0)}(virtual) + \hat{\Gamma}^{(v^0)}(cttm) \Longrightarrow IR\ safe$$

$\boldsymbol{v}^{\mathbf{2}}$ corrections :

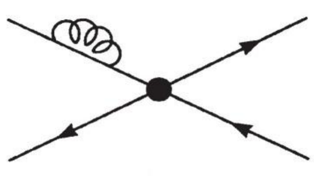
$$\hat{\Gamma}^{(v^2)}(real) + \hat{\Gamma}^{(v^2)}(virtual) + \hat{\Gamma}^{(v^2)}(cttm) \Longrightarrow residue\ IR\ deivergence$$

$$\left(\sum_{n=(v^0),(v^2)} [\hat{\Gamma}^{n}(real) + \hat{\Gamma}^{n}(virtual) + \hat{\Gamma}^{n}(cttm)]\right) \times \left(\quad \right)$$



Matching !

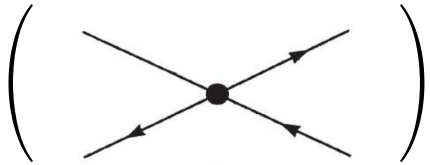
$$\left(\sum_{n=(v^0),(v^2)} [\hat{\Gamma}^{n}(born)]\right) \times \left(4 \quad \right)$$



Calculation—Residue Infrared Divergence(II)

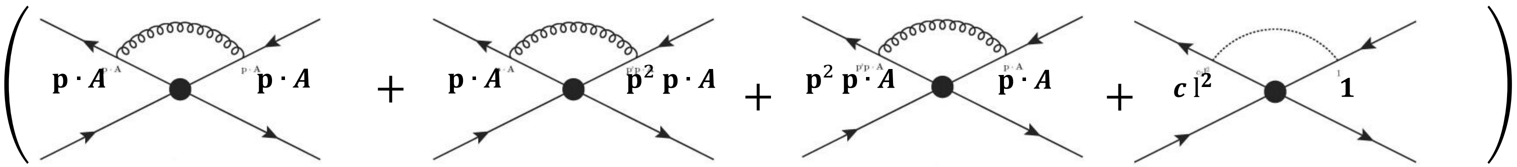
- $^{1}P_1^{[1]}$

$$\left(\sum_{n=(v^0),(v^2)}\left[\hat{\Gamma}^{n}_{^{1}P_1^{[1]}}\,(real)\right]\right)\times$$



Matching !

$$\left(\sum_{n=(v^0),(v^2)}\left[\hat{\Gamma}^{n}_{^{1}S_0^{[8]}}\,(born)\right]\right)\times$$



Calculation— $\mathcal{T}_{1-8}({}^1S_0, {}^1P_1)$

- The 5th term in h_c decay width

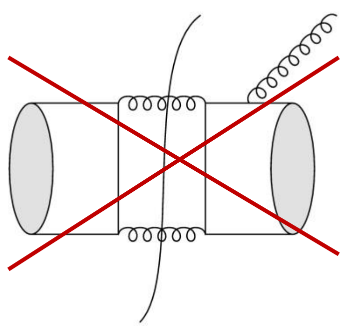
$$\frac{T({}^1S_0, {}^1P_1)}{m^5} \langle \mathcal{T}_{1-8}({}^1S_0, {}^1P_1) \rangle_{h_c}$$

$$\mathcal{T}_{1-8}({}^1S_0, {}^1P_1) = \frac{1}{2}\psi^\dagger g\boldsymbol{E}\chi \cdot \chi^\dagger \overleftrightarrow{\boldsymbol{D}}\psi + h.c.$$

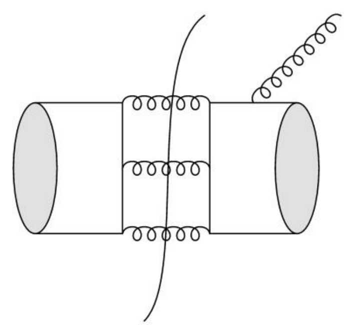
A dynamical gluon in initial state.

- Diagram:

LO



NLO



- C—parity forbidden at LO

  No Infrared entanglement with the other 4 terms

} We neglect this term in this work.

Results—Analytic Form

- SD coefficients:

$$G({}^1S_0^{[1]}) = -\frac{4}{3}\frac{4\pi\alpha_s^2}{9}\Big\{1-\frac{\alpha_s}{\pi}\frac{1}{144}[192\ln(\frac{\mu_\Lambda^2}{4m_Q^2})+72\beta_0\ln(\frac{4m_Q^2}{\mu_r^2})$$
$$+164n_f+237\pi^2-4964]\Big\},$$

$$G_{\gamma\gamma}({}^1S_0^{[1]}) = 2\pi\alpha^2 e_Q^4\Big\{-\frac{4}{3}+\frac{\alpha_s}{\pi}\frac{1}{27}\Big[48\ln\Big(\frac{\mu_\Lambda^2}{m_Q^2}\Big)-96\ln(2)-15\pi^2+196\Big]\Big\},$$

$$G({}^1S_0^{[8]}) = -\frac{4}{3}\frac{5\pi\alpha_s^2}{6}\Big\{1-\frac{\alpha_s}{\pi}\frac{1}{288}[168\ln(\frac{\mu_\Lambda^2}{4m_Q^2})+144\beta_0\ln(\frac{4m_Q^2}{\mu_r^2})$$
$$+328n_f+735\pi^2-12304]\Big\},$$

$$G({}^1P_1^{[1]}) = \frac{\alpha_s^3}{3645}\Big[1740\ln(\frac{\mu_\Lambda^2}{4m_Q^2})-555\pi^2+9236\Big],$$

- Answer the question above: $G^{(1)}/G^{(0)}$

	$F^{(1)}/F^{(0)}$	$G^{(1)}/G^{(0)}$
${}^1S_0^{[1]}$	95%	127%
${}^1S_0^{[8]}$	117%	121%
$\left({}^1S_0^{[1]}\right)_{\gamma\gamma}$	−29%	−11%

Results— Phenomenology of η_c (I)

- Determine the matrix elements

two undetermined parameters:

$$\left\langle \mathcal{O}\left({}^1S_0^{[1]}\right)\right\rangle_{\eta_c} \equiv \frac{N_c}{2\pi}\left|R_{\eta_c}(0)\right|^2, \qquad \langle \boldsymbol{v}^2\rangle_{\eta_c} \equiv \frac{\left\langle \mathcal{P}\left({}^1S_0^{[1]}\right)\right\rangle_{\eta_c}}{m^2\left\langle \mathcal{O}\left({}^1S_0^{[1]}\right)\right\rangle_{\eta_c}}$$

The constrains:

(i) Potential model with one undetermined parameter

$$V(r) = -\frac{\kappa}{r} + \sigma\, r, \qquad \sigma = 0.1682 \pm 0.0053\ \text{GeV}^2$$

(ii) Extract from experiment data

(A). Take $\Gamma_{\text{exp}}^{\gamma\gamma}(\eta_c) = 7.2 \pm 0.7 \pm 2.0$ KeV as input,

$$\left|\mathrm{R}_{\eta_c}^{\gamma\gamma}(0)\right|^2 = 0.881\,^{+0.382}_{-0.313}\ \text{GeV}^3$$
$$\langle \boldsymbol{v}^2\rangle_{\eta_c}^{\gamma\gamma} = 0.228\,^{+0.126}_{-0.100}$$
$$\Longrightarrow \Gamma^{LH}(\eta_c) = 31.4\,^{+29.3}_{-14.4}\ \text{MeV}$$

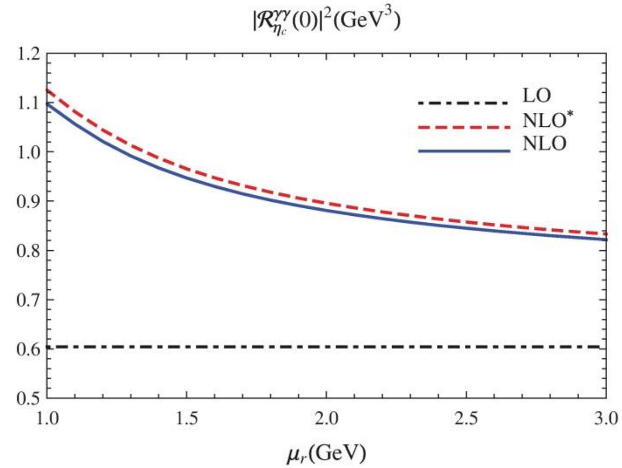
(B). Take $\Gamma_{\text{exp}}^{LH}(\eta_c) = 28.6 \pm 2.2$ MeV as input,

$$\left|\mathrm{R}_{\eta_c}^{LH}(0)\right|^2 = 0.814\,^{+0.332}_{-0.256}\ \text{GeV}^3$$
$$\langle \boldsymbol{v}^2\rangle_{\eta_c}^{\gamma\gamma} = 0.234\,^{+0.121}_{-0.099}$$
$$\Longrightarrow \Gamma^{\gamma\gamma}(\eta_c) = 6.61\,^{+2.77}_{-2.83}\ \text{KeV}$$

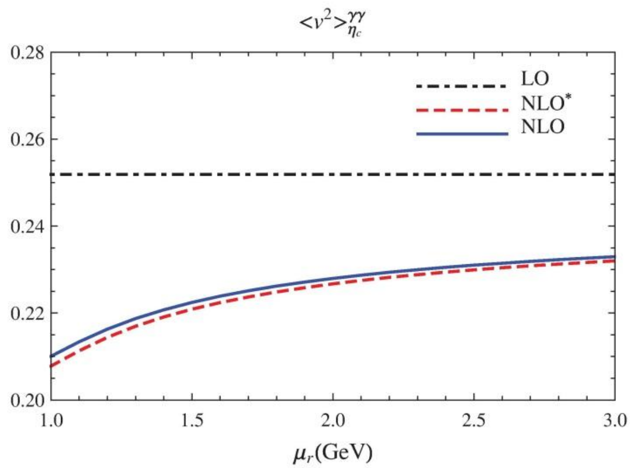
Two methods are both consistent with experiment.

Results— Phenomenology of η_c (II)

Method (A)

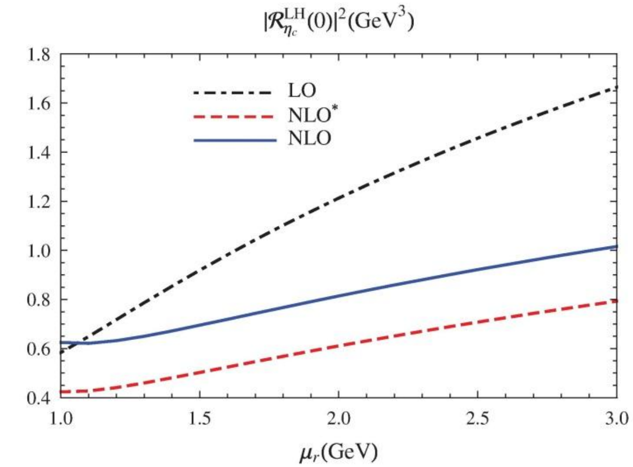


(a)μ_r dependence of $|\mathcal{R}_{\eta_c}^{\gamma\gamma}(0)|^2$

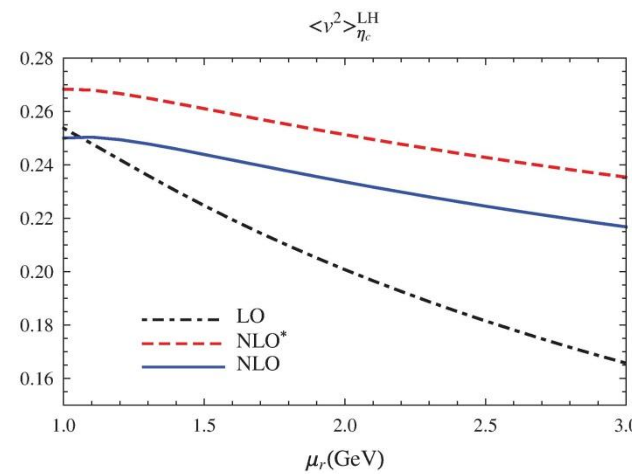


(b)μ_r dependence of $\langle \boldsymbol{v}^2 \rangle_{\eta_c}^{\gamma\gamma}$

Method (B)



(a)μ_r dependence of $|\mathcal{R}_{\eta_c}^{\mathrm{LH}}(0)|^2$



(b)μ_r dependence of $\langle \boldsymbol{v}^2 \rangle_{\eta_c}^{\mathrm{LH}}$

NLO*: Plots without $O(\alpha_s v^2)$ corrections.

Results— Phenomenology of h_c (I)

- Determine the matrix elements

$\left\langle \mathcal{O}\left({}^1P_1^{[1]}\right)\right\rangle_{h_c}$: potential model

$\left\langle \mathcal{P}\left({}^1P_1^{[1]}\right)\right\rangle_{h_c} \equiv m^2\langle v^2\rangle_{h_c} \left\langle \mathcal{O}\left({}^1P_1^{[1]}\right)\right\rangle_{h_c}$ Set: $\langle v^2\rangle_{h_c} \approx \langle v^2\rangle_{\eta_c} = 0.228$

$\left\langle \mathcal{O}\left({}^1S_0^{[8]}\right)\right\rangle_{h_c}$, $\left\langle \mathcal{P}\left({}^1S_0^{[8]}\right)\right\rangle_{h_c}$ } Operator Evolution Method

- Decay width in each channel

TABLE I: $\Gamma(h_c \to \mathrm{LH})$ expressed with the contributions of each LDME.

	$\langle\mathcal{O}({}^1S_0^{[8]})\rangle_{h_c}$	$\langle\mathcal{O}({}^1P_1^{[1]})\rangle_{h_c}$	$\langle\mathcal{P}({}^1S_0^{[8]})\rangle_{h_c}$	$\langle\mathcal{P}({}^1P_1^{[1]})\rangle_{h_c}$	Total
$\Gamma({}^{2S+1}L_J^{[c]} \to \mathrm{LH})(\mathrm{MeV})$	$0.83^{+0.37}_{-0.24}$	$-0.14^{+0.03}_{-0.05}$	$-0.21^{+0.09}_{-0.14}$	$0.07^{+0.04}_{-0.03}$	$0.55^{+0.27}_{-0.19}$

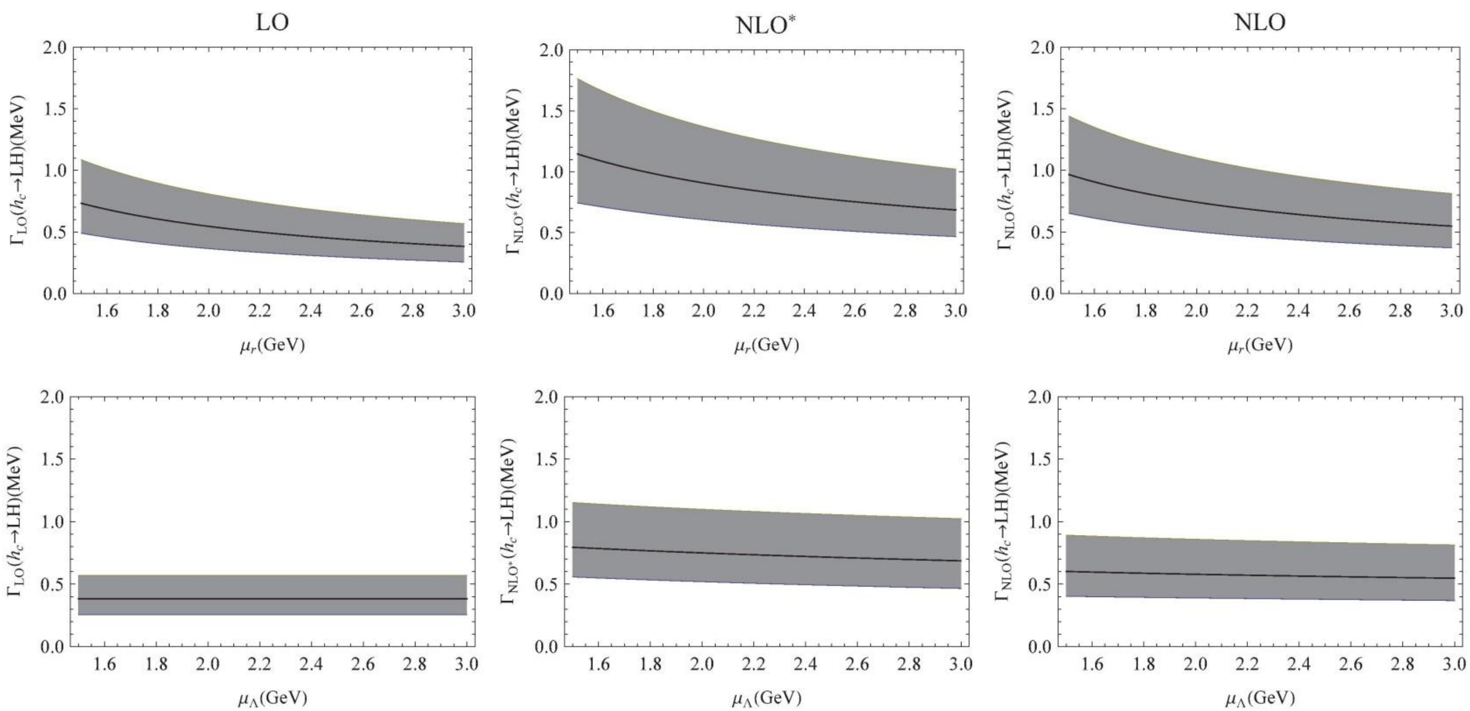
dominant — negative and also important — neglectable

TABLE II: $\Gamma(h_c \to \mathrm{LH})$ expressed with contributions at various orders of α_s and $\boldsymbol{v}$.

	$\alpha_s^0 v^0$	$\alpha_s^1 v^0$	$\alpha_s^0 v^2$	$\alpha_s^1 v^2$	Total
$\Gamma(h_c \to \mathrm{LH})(\mathrm{MeV})$	$0.40^{+0.18}_{-0.12}$	$0.29^{+0.17}_{-0.11}$	$-0.10^{+0.04}_{-0.07}$	$-0.04^{+0.03}_{-0.05}$	$0.55^{+0.27}_{-0.19}$

Relativistic corrections always pull down the decay width !

Results— Phenomenology of h_c (II)



μ_r and μ_Λ dependence of $\Gamma(h_c \to \mathrm{LH})$. The upper plots are for μ_r and lower ones for μ_Λ. From left to right the plots are shown for LO, NLO* and NLO respectively, where NLO* includes $O(\alpha_s)$ but excludes $O(\alpha_s v^2)$ corrections.

Results— Phenomenology of h_c (III)

- Total decay width

$$\Gamma^{total}(h_c) \approx \Gamma\left(h_c \longrightarrow LH\right) + \Gamma\left(h_c \longrightarrow \eta_c + \gamma\right)$$

- E1 transition

K. -T. Chao, Y. -B. Ding and D. -H. Qin, Phys. Lett. B **301**, 282 (1993)

$$\Gamma^{\rm th}\left(h_c \longrightarrow \eta_c + \gamma\right) = 385\ \mathrm{KeV}$$

It includes the relativistic effects, which is consistent with our calculations.

- Comparing with experiment data

$$\Gamma^{total}(h_c) = 0.93^{+0.27}_{-0.19}\ \mathrm{MeV}$$

$$\mathrm{Br}(h_c \longrightarrow \eta_c + \gamma) = 41^{+10}_{-9}\%$$

$$\Gamma^{total}_{(v^0)}(h_c) = 1.07^{+0.27}_{-0.19}\ \mathrm{MeV}$$

$$\mathrm{Br}_{(v^0)}(h_c \longrightarrow \eta_c + \gamma) = 36\ \%$$

better

worse

$$\Gamma^{total}_{exp}(h_c) = 0.73^{+0.45}_{-0.28}\ \mathrm{MeV}$$

$$\mathrm{Br}_{exp}(h_c \longrightarrow \eta_c + \gamma) = 54.3 \pm 6.7 \pm 5.2\%$$

Summary

- The first calculation of QCD and relativistic corrections of spin-singlet hadronic decays.

- The QCD corrections of hadronic decay is large and positive both for leading order in $\boldsymbol{v}$ and $\boldsymbol{v}^2$ corrections. While it is small and negative for electro-magnetic decay.

- For η_c, the theoretical results are consistent with the experiment data, both in two ways.

- For h_c, the relativistic corrections decrease the decay width and then fit the data better.

THANKS!