Quarkonium annihilation into photons

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Contents

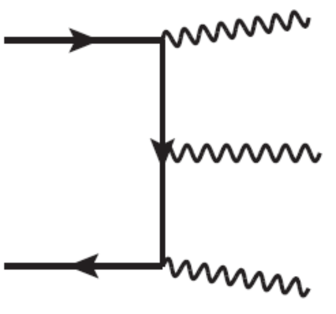
- **Investigation on** $J/\psi \rightarrow 3\gamma$
- **Investigation on** $\chi_{c0,2} \rightarrow 2\gamma$
- **Summary**

Investigation on J/ψ→ 3γ

Motivation

In cooperation with **Feng, Jia** (PRD87,051501)

- Analogous process: **ortho-positronium → 3γ** is well understood in non-relativistic QED (**NRQED)** framework



Excellent agreement between **Theory** (**Caswell, Lepage, Sapirstein, Adkins et al., from 1970 to 2000**) and **Experiment** (**Michigan group, and Tokyo group, 2003**)

Investigation on J/ψ→ 3γ

Motivation

- **Theoretical Prediction**: Ann. Phys. (N.Y.)295, 136 (2002)

$$\Gamma(\text{theory}) = 7.039979(11)\mu s^{-1}$$

- **Experiment data**: Phys. Lett. B 572, 117 (2003), Phys. Rev. Lett. 90, 203402 (2003)

$$\Gamma(\text{Tokyo}) = 7.0396(12\ \text{stat.})(11\ \text{syst.})\mu s^{-1}$$
$$\Gamma(\text{Michigan}) = 7.0404(10\ \text{stat.})(8\ \text{syst.})\mu s^{-1}$$

Theory and experiment are consistent with each other at relative high accuracy!

Investigation on J/ψ→ 3γ

Motivation

- However, this channel has suffered some long-standing problem in quarkonium physics

- **CLEO-c** Collaboration first measured this decay channel in 2008 (**PRL, 2008**)

$$\mathrm{Br}(J/\psi \to 3\gamma) = (1.2 \pm 0.3 \pm 0.2) \times 10^{-5}$$

- **BESIII** confirmed and improved the measurement (**PRD, 2013**)

$$\mathrm{Br}(J/\psi \to 3\gamma) = (11.3 \pm 1.8 \pm 2.0) \times 10^{-6}$$

Investigation on J/ψ→ 3γ

Motivation

- However, this channel has suffered some long-standing problem in quarkonium physics

- NRQCD Prediction:

$$\Gamma(J/\psi \to 3\gamma) = \frac{8(\pi^2-9)e_c^6\alpha^3}{9m_c^2}\left|\langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*\psi|J/\psi\rangle\right|^2 \left\{1 - 12.630\,\frac{\alpha_s}{\pi} + \left[\frac{132-19\pi^2}{12(\pi^2-9)} + \cdots\right]\langle v^2\rangle_{J/\psi} + \cdots\right\}$$

Investigation on J/ψ→ 3γ

Motivation

- NRQCD prediction:
  - LO prediction:

$$\mathrm{Br}(J/\psi \to 3\gamma) = 7.3 \times 10^{-5}$$

  - LO+NLO radiative correction:

$$\mathrm{Br}(J/\psi \to 3\gamma) < 0$$

  - LO+NLO relativistic correction:

$$\mathrm{Br}(J/\psi \to 3\gamma) < 0$$

Investigation on J/ψ→ 3γ

Motivation

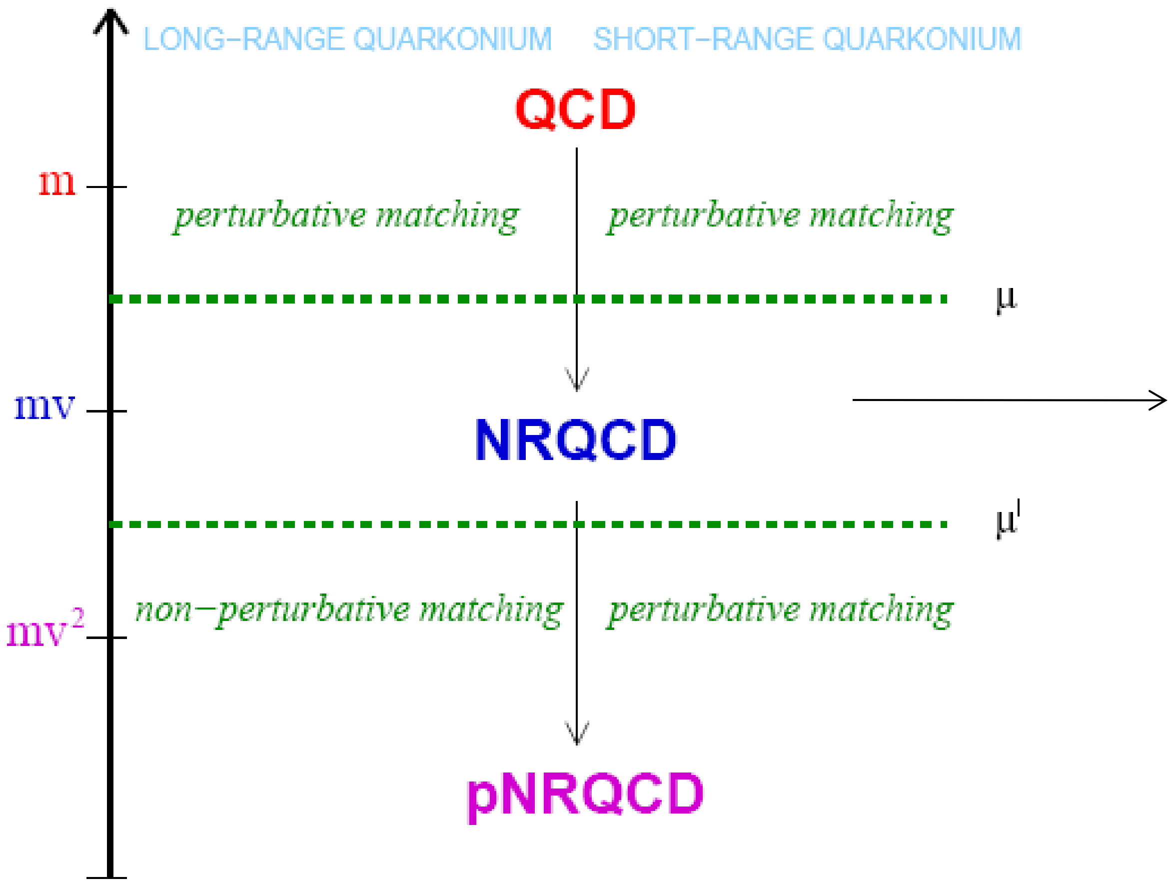
- To better understand the discrepancy between theory and experiment, and improve the theoretical prediction, we consider the $\mathcal{O}(\alpha_s v^2)$ correction to the decay rate.

$$\Gamma(J/\psi \to 3\gamma) = \frac{8(\pi^2-9)e_c^6\alpha^3}{9m_c^2}\left|\langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*\psi|J/\psi\rangle\right|^2 \left\{1 - 12.630\,\frac{\alpha_s}{\pi} + \left[\frac{132-19\pi^2}{12(\pi^2-9)} + \left(\frac{16}{9}\ln\frac{\mu_f^2}{m_c^2} + G\right)\frac{\alpha_s}{\pi}\right]\langle v^2\rangle_{J/\psi} + \cdots\right\}$$

（remains unkown！）

Investigation on J/ψ→ 3γ
Theoretical descriptions

- Non-relativistic QCD (**NRQCD**)



Bodwin, Braaten, Lepage PRD51,1125 (1995)

Investigation on J/ψ→ 3γ

Theoretical descriptions

- NRQCD factorization formula

$$\begin{aligned}\mathcal{A}(J/\psi \to 3\gamma) &= c_0 \langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*\psi|J/\psi\rangle \\ &+ c_1 \langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\psi|J/\psi\rangle + \cdots\end{aligned}$$

Investigation on J/ψ→ 3γ

Theoretical descriptions

- NRQCD factorization formula

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With the same short-distance coefficients!

$$\mathcal{A}(Q\bar{Q}(^3S_1) \to 3\gamma) = c_0 \langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^* \psi|Q\bar{Q}(^3S_1)\rangle + c_1 \langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^* (-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2 \psi|Q\bar{Q}(^3S_1)\rangle + \cdots$$

Investigation on J/ψ→ 3γ
Theoretical descriptions

- Method of Region: M. Beneke and V. A. Smirnov Nucl.Phys.B522:321-344,1998

$$\begin{aligned}\mathcal{A}(Q\bar{Q}(^3S_1) \to 3\gamma) &= c_0\langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*\psi|Q\bar{Q}(^3S_1)\rangle \\ &+c_1\langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\psi|Q\bar{Q}(^3S_1)\rangle + \cdots\end{aligned}$$

Investigation on J/ψ→ 3γ

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Dissect the amplitude (rate) into four distinct regions:
hard, soft, potential and ultrasoft

$$c_i = \mathcal{A} - \mathcal{A}\Big|_{\text{soft+potential+ultrasoft}}$$

Investigation on J/ψ→ 3γ

Theoretical descriptions

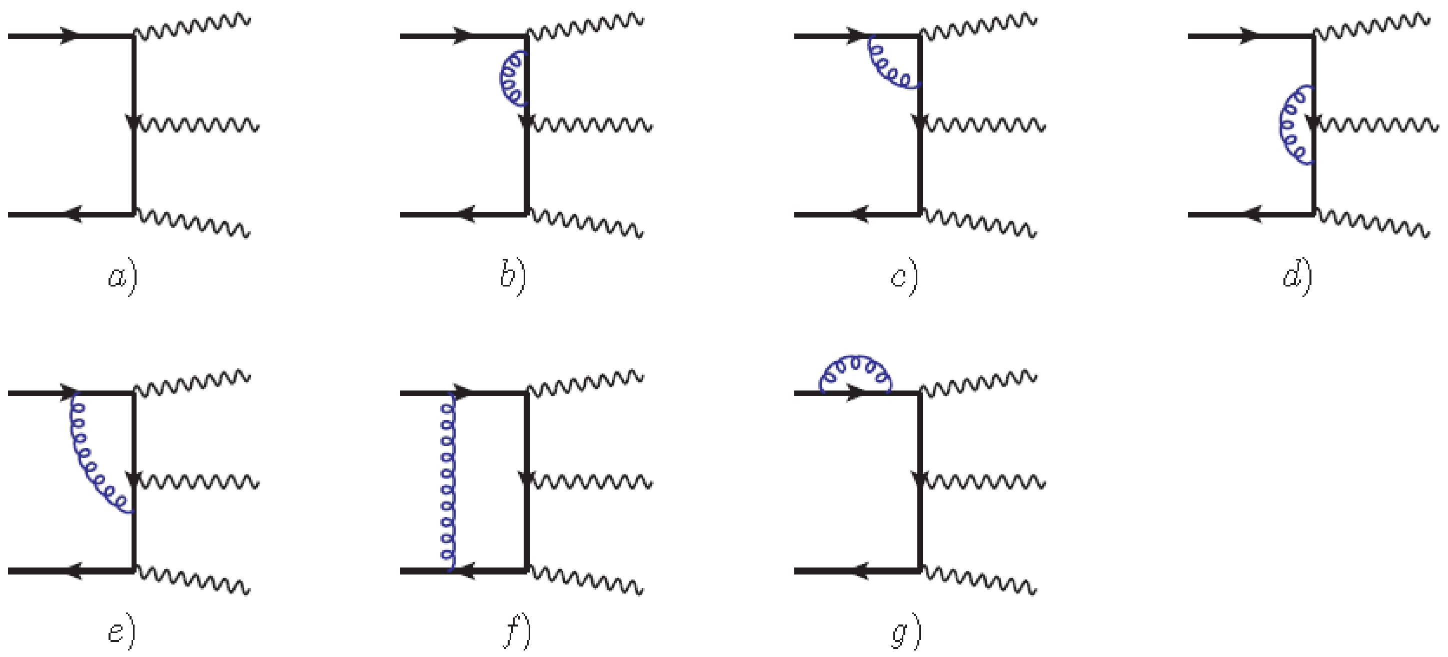
- Method of Region: M. Beneke and V. A. Smirnov Nucl.Phys.B522:321-344,1998

In our work, we expand the relative momentum q of the heavy quark pair before carrying out the integration. As a consequence, only the hard region involves and therefore we get the short-distance coefficients directly.

In fact, if we apply the same implementation in the NRQCD side, all the integrals are scaleless integration.

Investigation on J/ψ→ 3γ
Theoretical descriptions

- Typical QCD Feynman diagrams:



Investigation on J/ψ→ 3γ
Theoretical descriptions

- The missing piece is the unknown constant G in

$$\Gamma(J/\psi \to 3\gamma) = \frac{8(\pi^2-9)e_c^6\alpha^3}{9m_c^2}\left|\langle 0|\chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}^*\psi|J/\psi\rangle\right|^2 \left\{1 - 12.630\,\frac{\alpha_s}{\pi} + \left[\frac{132-19\pi^2}{12(\pi^2-9)} + \boxed{\left(\frac{16}{9}\ln\frac{\mu_f^2}{m_c^2} + G\right)\frac{\alpha_s}{\pi}}\right]\langle v^2\rangle_{J/\psi} + \cdots\right\}$$

It is extremely difficult to get analytic expression, we get it numerically

$$G = 68.913$$

huge and positive!

Investigation on J/ψ→ 3γ

Comparison with experiment

- Now the full NRQCD prediction is proportional to

$$\Gamma(J/\psi \to 3\gamma) = 1.35 \times 10^{-8} \times \left|\langle 0|\chi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}^* \psi|J/\psi\rangle\right|^2 \left[1 - 4.02\,\alpha_s(\mu) + \Big(-5.32 + 21.94\,\alpha_s(\mu)\Big)\langle v^2\rangle_{J/\psi}\right]$$

Select the following input parameters: (Bodwin et al., PRD,2008)

$$|\langle 0|\chi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}^* \psi|J/\psi\rangle|^2 = 0.446\ \mathrm{GeV}^3, \quad \langle v^2\rangle_{J/\psi} = 0.223.$$

$m_c = 1.4$ GeV and $\Lambda_{\mathrm{QCD}}^{(n_f=3)} = 390$ MeV

Investigation on J/ψ→ 3γ

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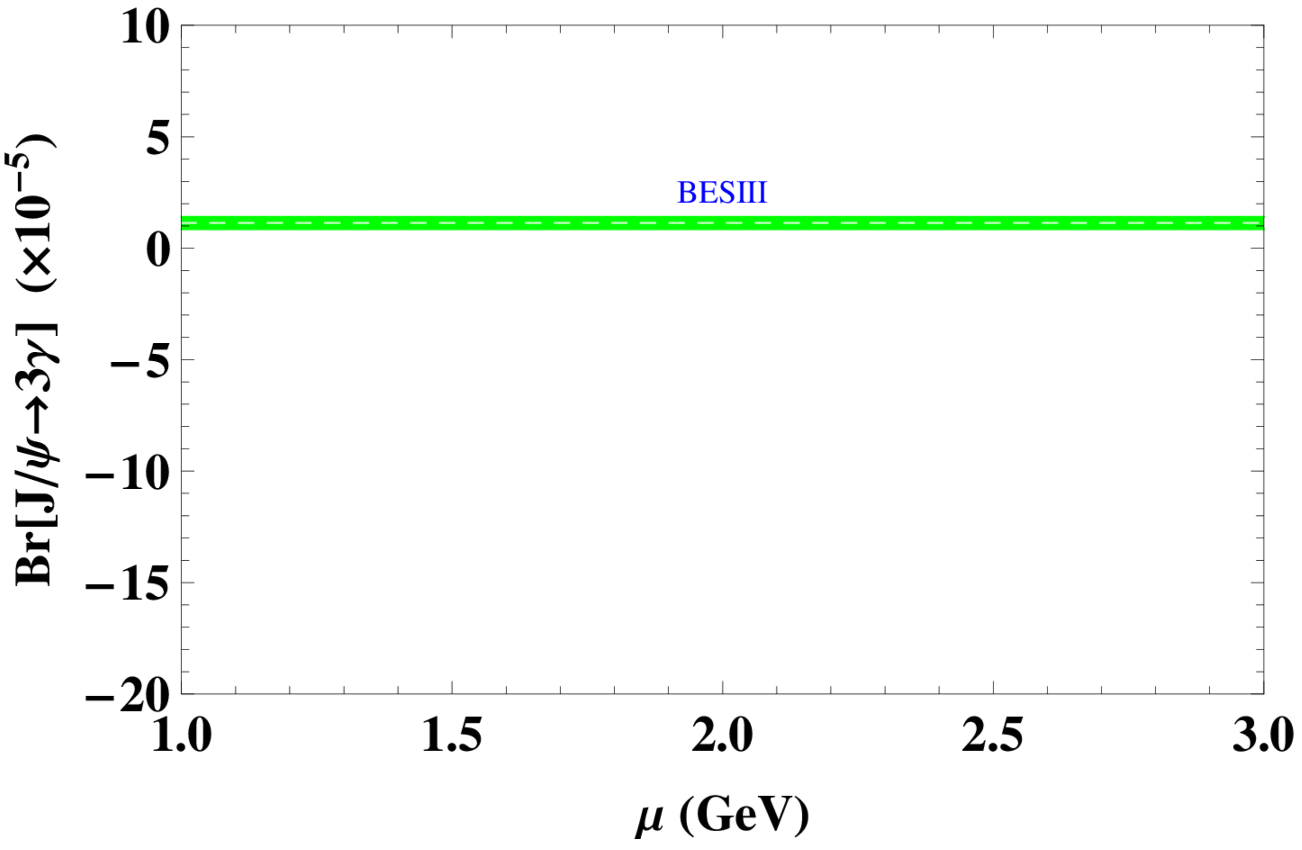
$m_c = 1.4$ GeV and $\Lambda_{\mathrm{QCD}}^{(n_f=3)} = 390$ MeV

Fitted from the process $\Gamma(J/\psi \to e^+e^-)$ accurate up to relative order v^2 .

Investigation on J/ψ→ 3γ

Comparison with experiment



Green band

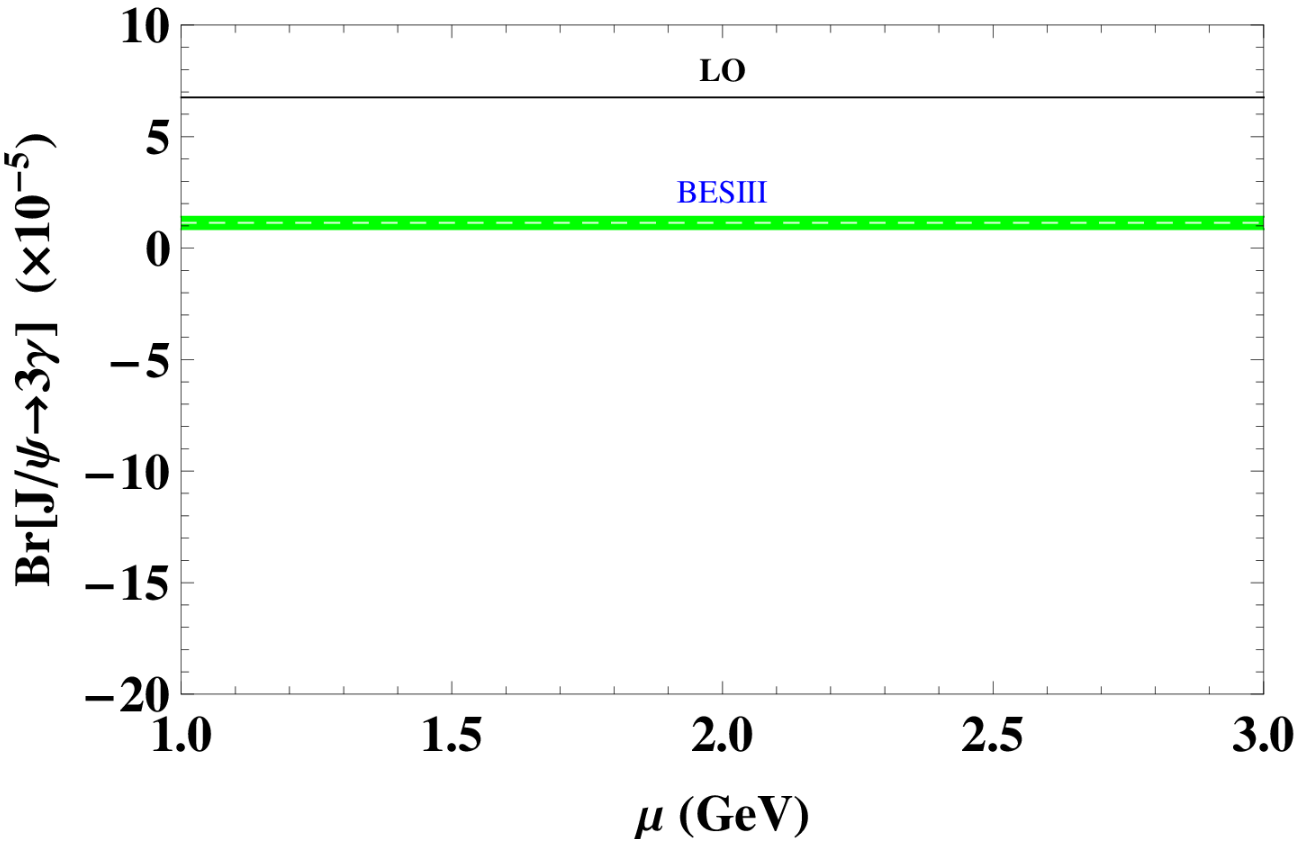
↓

BESIII data

The scale dependence stems from the coupling constant!

Investigation on J/ψ→ 3γ

Comparison with experiment

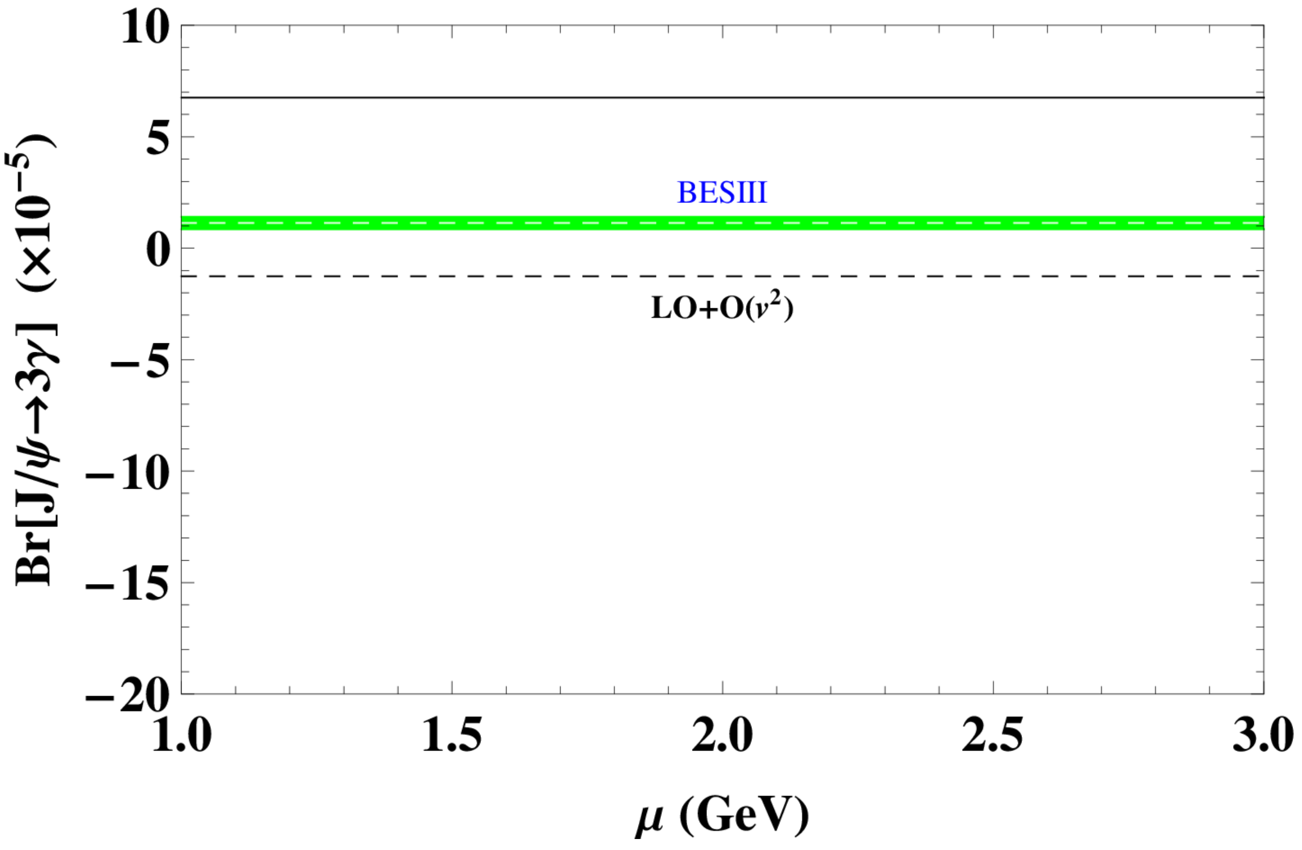


Green band → BESIII data

Solid line → LO prediction

Investigation on J/ψ→ 3γ

Comparison with experiment



Green band

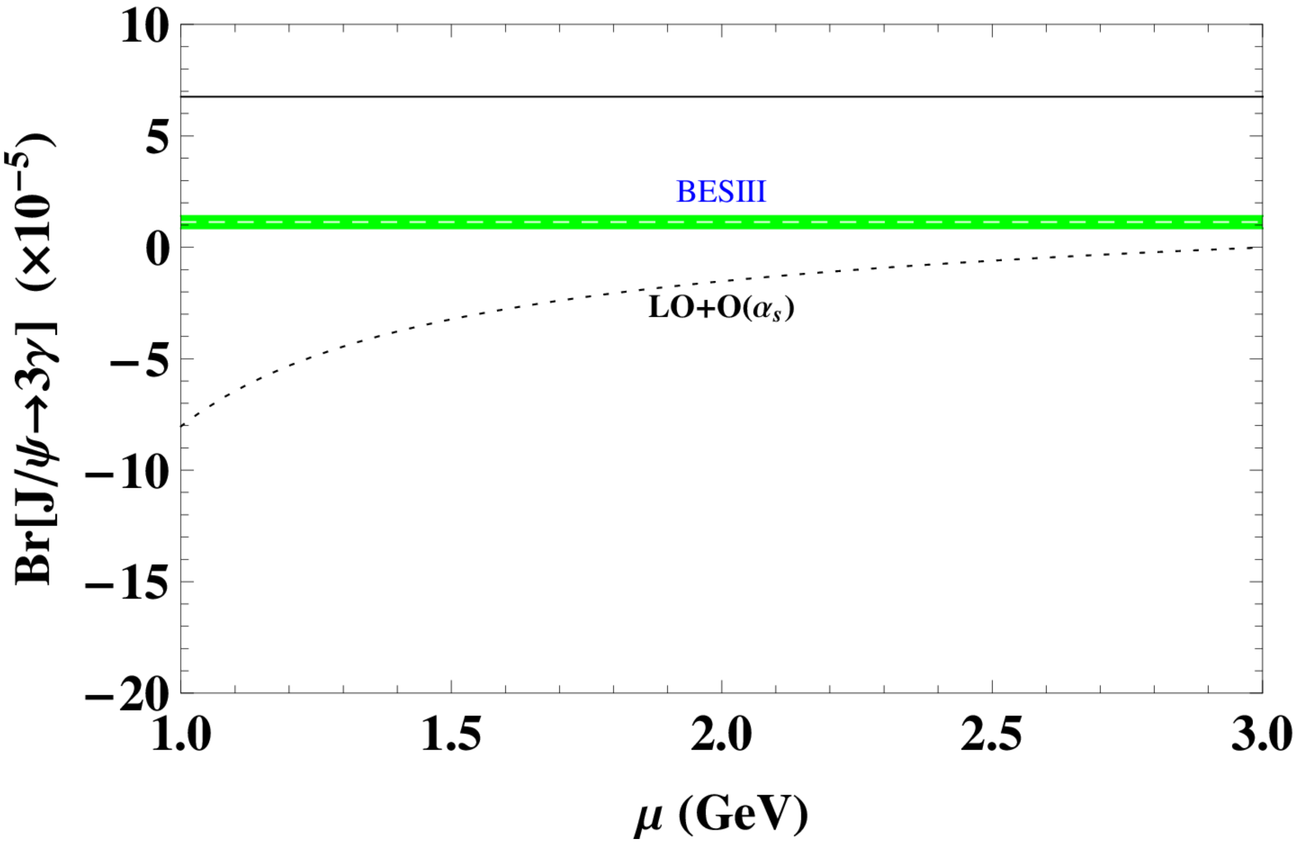
BESIII data

Dashed line

LO + O(v^2) prediction

Investigation on J/ψ→ 3γ

Comparison with experiment

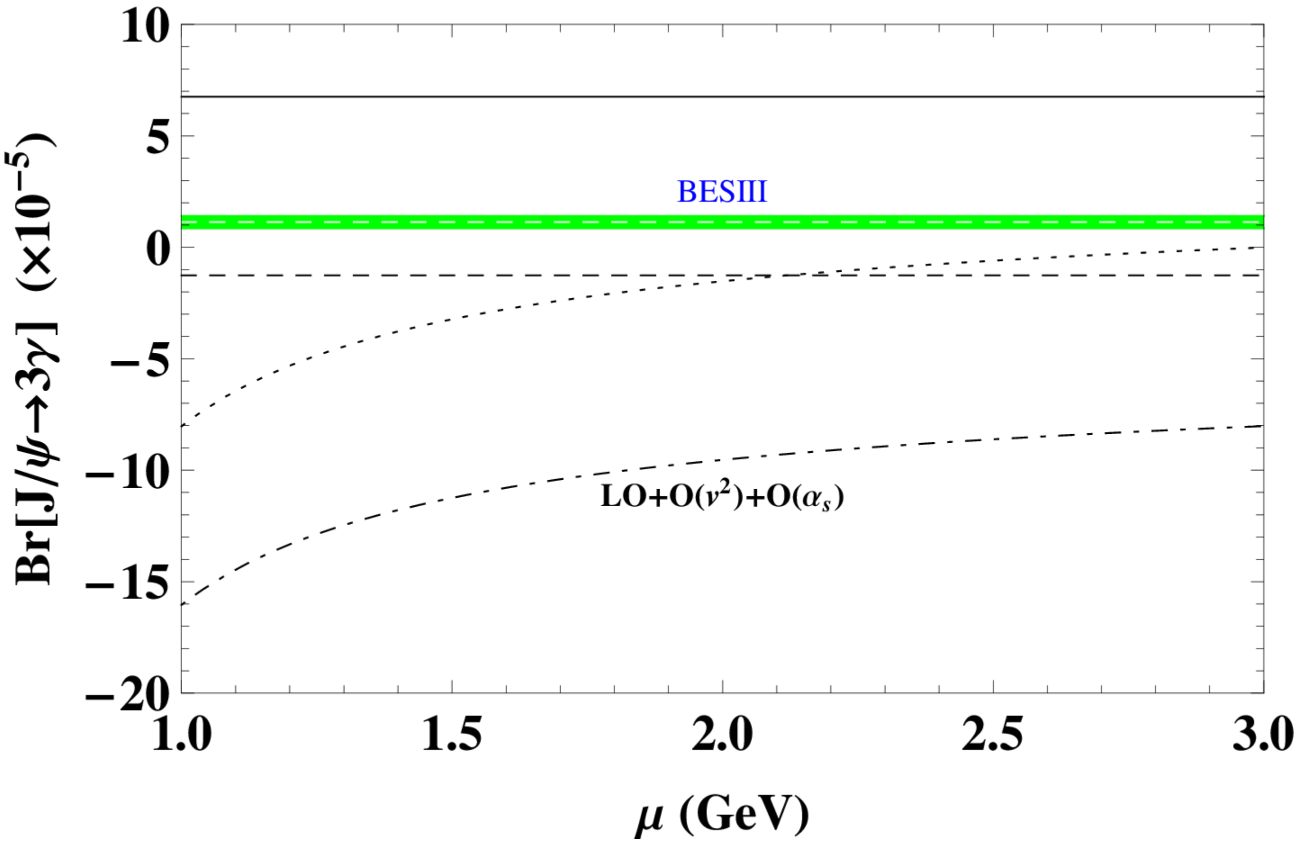


Green band → BESIII data

Dotted line → LO + O(α_s) prediction

Investigation on J/ψ→ 3γ

Comparison with experiment



Green band

↓

BESIII data

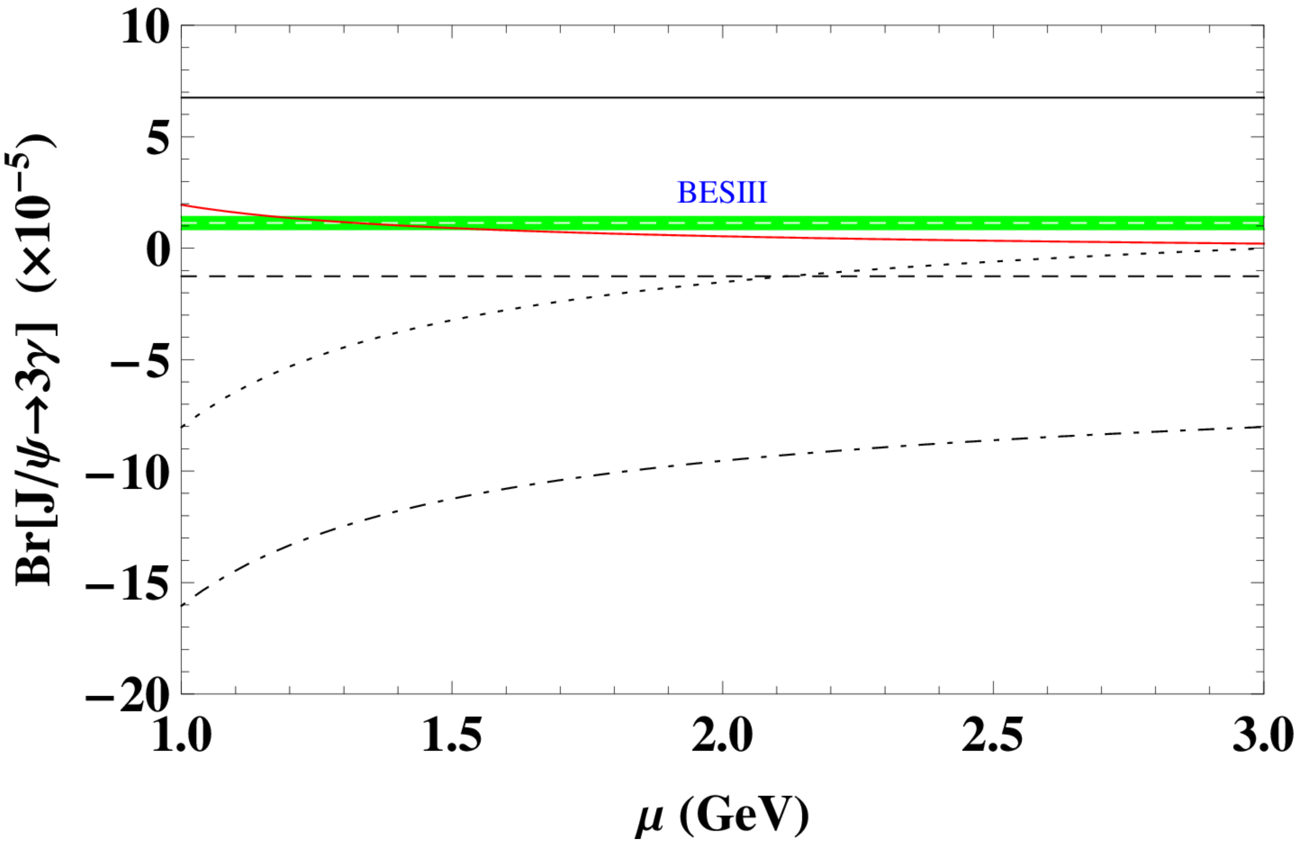
Dot-Dashed line

↓

LO + $O(v^2)$+$O(\alpha_s)$ prediction

Investigation on J/ψ→ 3γ

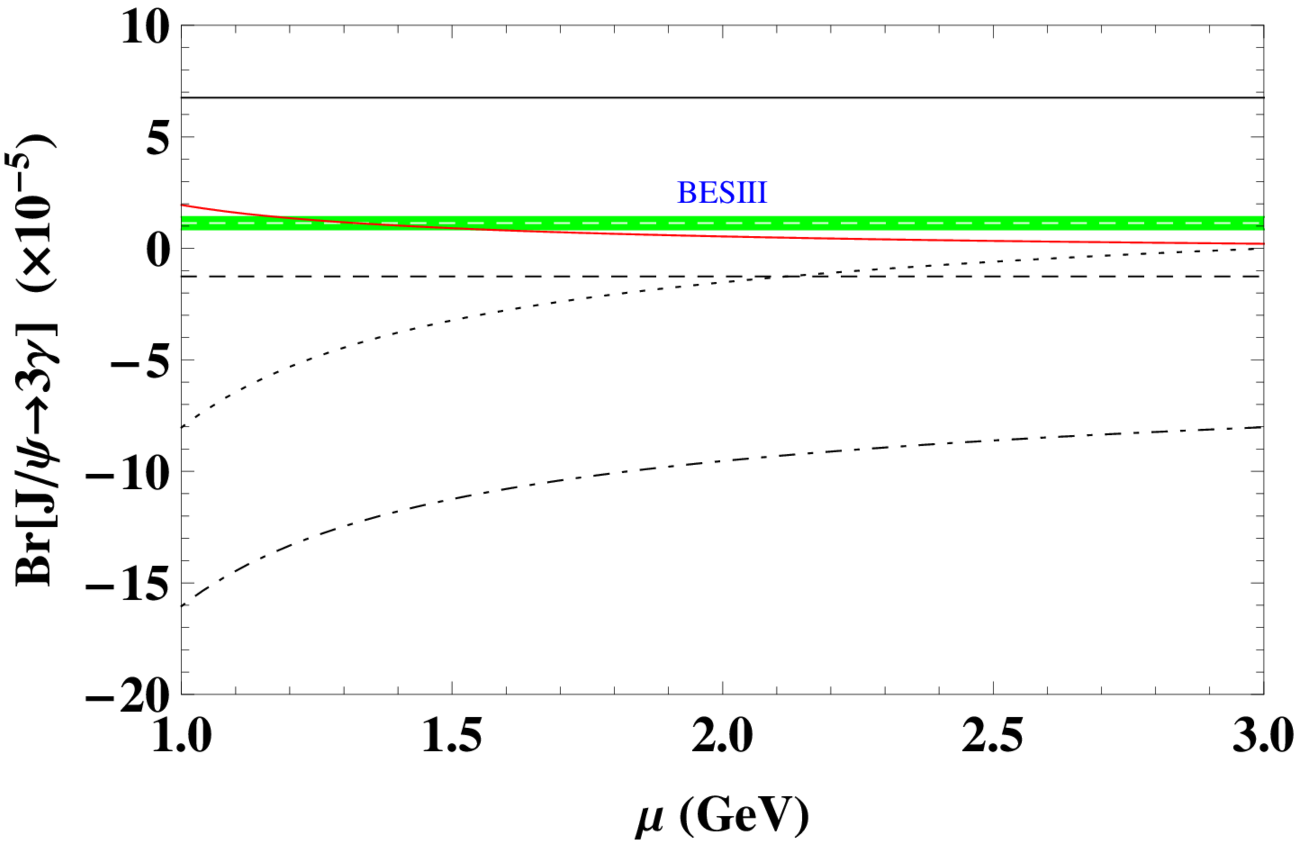
Comparison with experiment



Green band → BESIII data

Red line → Our prediction

Investigation on J/ψ→ 3γ
Comparison with experiment



For the first time, we can bring the NRQCD prediction in agreement with data !

Investigation on $\chi_{c0,2} \to 2\gamma$

Motivation

In cooperation with **Dong, Feng, Jia, Jugeau (to prepare)**

- Recently, BESIII give an updated measurements:

  (BESIII, PRD, 2012)

  - Decay width of $\chi_{c0} \to 2\gamma$

  $$\Gamma_{\gamma\gamma}(\chi_{c0}) = (2.33 \pm 0.20 \pm 0.13 \pm 0.17)\,\mathrm{keV}$$

  - Ratio of decay width of $\chi_{c2} \to 2\gamma$ to that of $\chi_{c0} \to 2\gamma$

  $$\mathcal{R} = \frac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})} = 0.271 \pm 0.029 \pm 0.013 \pm 0.027$$

  - Ratio of the two-photon widths for **helicity $\lambda = 0$ and helicity $\lambda = 2$** components in $\chi_{c2} \to 2\gamma$

  $$f_{0/2} = \frac{\Gamma^{\lambda=0}_{\gamma\gamma}(\chi_{c2})}{\Gamma^{\lambda=2}_{\gamma\gamma}(\chi_{c2})} = 0.00 \pm 0.02 \pm 0.02$$

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Motivation

♣ Review of previous theoretical work

- Numerous work based on phenomenological model
  - R. Barbieri, R. Gatto, R. Kogerler (PLB, 1976)
  - L. Bergstrom, G. Hulth, H. Snellman (Z. Phys. C, 1983)
  - Z. P. Li, F. E. Close, T. Barnes (PRD, 1991)
  - H. W. Huang, C. F. Qiao, K. T. Chao (PRD, 1996)
  - S. N. Gupta, J. M. Johnson, W. W. Repko (PRD, 1996)
  - G. L. Wang (PLB, 2007)
  - C. W. Hwang, R. S. Guo (PRD, 2010)
  - … … …
- Lattice QCD prediction for $\chi_{c0} \rightarrow 2\gamma$ by J. Dudek and R. Edwards (PRL 2006), obtains partial width of $(2.41 \pm 0.58 \pm 0.72 \pm 0.48)$ keV, based on the algorithm proposed by X. D. Ji and C. W. Jung (PRL 2001; PRD, 2001)

Investigation on $\chi_{c0,2} \to 2\gamma$

Motivation

- Review of previous theoretical work
  - NRQCD factorization approach
    - NLO radiative correction known long ago (1970s)
    - NLO relativistic correction known recently
      - ✓ J.P. Ma and Q. Wang (PLB, 2002)
      - ✓ N. Brambilla, E. Mereghetti and A. Vairo (JHEP, 2006)

Unfortunately, these two groups do not agree …

- To clarify the discrepancy, we restudy the relativistic correction in NRQCD. In addition, to compare with BESIII data, we study **helicity amplitudes** at v^2 .

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Theoretical descriptions

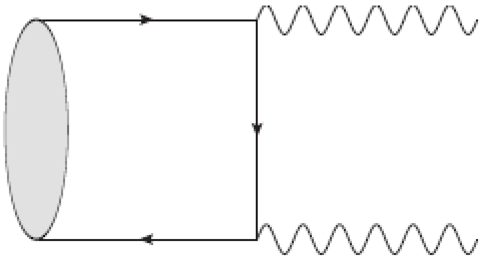
- Our strategy is different from the previous two groups

  - Extending the technique used by Braaten and Chen (PRD, 1997), we employ the fixed point (Fock-Schwinger) gauge, together with the Foldy-Wouthuysen-Tani transformation to descend from QCD to NRQCD

  - We also work at the helicity amplitude level, not only unpolarized decay rate

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Theoretical descriptions



- Fixed point gauge

$$z^\mu A_\mu(z) = 0 \qquad \Psi(z) = \Psi(0) + z_\mu D^\mu \Psi(0) + \cdots$$

- Obtain the amplitude in coordinate space

$$\mathcal{A}_{H\to\gamma\gamma} = -16 e_Q^2 e^2 \epsilon^*_{1\mu} \epsilon^*_{2\nu} \int d^4 z e^{i(k_1-k_2)\cdot z} \langle 0 | \overline{\Psi}(z) \gamma^\mu S(z,-z) \times \gamma^\nu \Psi(-z) + \overline{\Psi}(-z) \gamma^\nu S(-z,z) \gamma^\mu \Psi(z) | H \rangle$$

H with charge conjugation C=+1

- Foldy-Wouthuysen-Tani transformation

$$\Psi(z) = \exp[i \frac{\overrightarrow{\gamma} \cdot \overrightarrow{\mathbf{D}}}{2m}] \times \exp[-i \frac{g}{4m^2} \gamma^0 \overrightarrow{\gamma} \cdot \overrightarrow{\mathbf{E}}] \times \exp[-i \frac{1}{6m^3} (\overrightarrow{\gamma} \cdot \overrightarrow{\mathbf{D}})^3 + \frac{g}{8m^3} \overrightarrow{\gamma} \cdot D_0^{\rm ad} \overrightarrow{\mathbf{E}}] \times \begin{pmatrix} \psi(z) \\ \chi(z) \end{pmatrix}$$

Investigation on $\chi_{c0,2} \to 2\gamma$

Theoretical descriptions

- Using trick

$$e^A B e^{-A} = \sum_{n=0}^{\infty} \frac{1}{n!}[A,[A,\cdots[A,B]]]$$

- Gremm-Kapustin relation for χ_c

$$\langle 0|\chi^\dagger(-\frac{i}{2}\boldsymbol{\sigma}\cdot\overleftrightarrow{\mathbf{D}})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\psi|H\rangle = m_c E_B\langle 0|\chi^\dagger(-\frac{i}{2}\boldsymbol{\sigma}\cdot\overleftrightarrow{\mathbf{D}})\psi|H\rangle - im_c\langle 0|\chi^\dagger g\boldsymbol{\sigma}\cdot\mathbf{E}\psi|H\rangle$$

$$\langle 0|\chi^\dagger(-\frac{i}{2}\sigma^{(i}\overleftrightarrow{D}^{j)})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\psi|H\rangle = m_c E_B\langle 0|\chi^\dagger(-\frac{i}{2}\sigma^{(i}\overleftrightarrow{D}^{j)})\psi|H\rangle - im_c\langle 0|\chi^\dagger g\sigma^{(i}E^{j)}\psi|H\rangle$$

where the binding energy is $E_B = M_H - 2m_c$.
To derive above relations, we make use of equation of motion in NRQCD.

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Theoretical descriptions

- Finally, we obtain

$$\mathcal{A}^{[0]}_{1,1} = i\, e_Q^2 e^2 \left\{ \frac{1}{m_c^2}\langle 0|\chi^\dagger \sigma\cdot\mathbf{D}\psi|\chi_{c0}\rangle + \frac{1}{2m_c^3}\langle 0|\chi^\dagger g\mathbf{E}\cdot\sigma\psi|\chi_{c0}\rangle \right.$$
$$\left. + \frac{7}{6m_c^4}\langle 0|\chi^\dagger \sigma\cdot\mathbf{D}\mathbf{D}^2\psi|\chi_{c0}\rangle + \frac{\alpha_s}{\pi}\frac{1}{m_c^2}\frac{1}{2}\left(-\frac{28}{9}+\frac{\pi^2}{3}\right)\langle 0|\chi^\dagger \sigma\cdot\mathbf{D}\psi|\chi_{c0}\rangle \right\}$$

$$\mathcal{A}^{[2]}_{1,1} = i\, e_Q^2 e^2 \left\{ \frac{2\sqrt{6}}{15}\frac{1}{m_c^4}\langle 0|\chi^\dagger D^{(i}\sigma^{j)}\mathbf{D}^2\epsilon^*_{ij}\psi|\chi_{c2}\rangle \right.$$
$$\left. -\frac{\alpha_s}{\pi}\frac{1}{m_c^2}\frac{\sqrt{6}}{18}(8+3\pi^2-48\log 2)\langle 0|\chi^\dagger D^{(i}\sigma^{j)}\epsilon^*_{ij}\psi|\chi_{c2}\rangle \right\}$$

$$\mathcal{A}^{[2]}_{1,-1} = i\, e_Q^2 e^2 \left\{ \frac{2}{m_c^2}\langle 0|\chi^\dagger D^{(i}\sigma^{j)}\epsilon^*_{ij}\psi|\chi_{c2}\rangle + \frac{2}{m_c^4}\langle 0|\chi^\dagger D^{(i}\sigma^{j)}\mathbf{D}^2\epsilon^*_{ij}\psi|\chi_{c2}\rangle \right.$$
$$\left. -\frac{\alpha_s}{\pi}\frac{1}{m_c^2}\frac{16}{3}\langle 0|\chi^\dagger D^{(i}\sigma^{j)}\epsilon^*_{ij}\psi|\chi_{c2}\rangle \right\}$$

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Theoretical descriptions

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$$\left. -\frac{\alpha_s}{\pi}\frac{1}{m_c^2}\frac{16}{3}\langle 0|\chi^\dagger D^{(i}\sigma^{j)}\epsilon^*_{ij}\psi|\chi_{c2}\rangle \right\}$$

Our decay rates agree with results of N. Brambilla, E. Mereghetti and A. Vairo (JHEP, 2006)

Investigation on $\chi_{c0,2} \to 2\gamma$

Theoretical descriptions

- Finally, we obtain

$$\mathcal{A}^{[0]}_{1,1} = i\, e_Q^2 e^2 \left\{ \frac{1}{m_c^2}\langle 0|\chi^\dagger \sigma\cdot\mathbf{D}\psi|\chi_{c0}\rangle + \frac{1}{2m_c^3}\langle 0|\chi^\dagger g\mathbf{E}\cdot\sigma\psi|\chi_{c0}\rangle + \frac{7}{6m_c^4}\langle 0|\chi^\dagger\sigma\cdot\mathbf{D}\mathbf{D}^2\psi|\chi_{c0}\rangle + \frac{\alpha_s}{\pi}\frac{1}{m_c^2}\frac{1}{2}\left(-\frac{28}{9}+\frac{\pi^2}{3}\right)\langle 0|\chi^\dagger\sigma\cdot\mathbf{D}\psi|\chi_{c0}\rangle \right\}$$

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support

$$f_{0/2} = \frac{\Gamma^{\lambda=0}_{\gamma\gamma}(\chi_{c2})}{\Gamma^{\lambda=2}_{\gamma\gamma}(\chi_{c2})} = 0.00 \pm 0.02 \pm 0.02$$

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Phenomenology

- In the formulas, the emerging non-perturbative matrix elements are

$$\langle 0|\chi^\dagger \sigma\cdot\mathbf{D}\psi|\chi_{c0}\rangle, \quad \langle 0|\chi^\dagger \sigma\cdot\mathbf{D}\mathbf{D}^2\psi|\chi_{c0}\rangle \quad \langle 0|\chi^\dagger \sigma\cdot g\mathbf{E}\psi|\chi_{c0}\rangle,$$

$$\langle 0|\chi^\dagger \sigma^{(i}D^{j)}\epsilon^*_{ij}\psi|\chi_{c2}\rangle, \quad \langle 0|\chi^\dagger \sigma^{(i}D^{j)}\mathbf{D}^2\epsilon^*_{ij}\psi|\chi_{c2}\rangle, \quad \langle 0|\chi^\dagger \sigma^{(i}E^{j)}\epsilon^*_{ij}\psi|\chi_{c2}\rangle$$

Investigation on $\chi_{c0,2} \to 2\gamma$

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- In the formulas, the emerging non-perturbative matrix elements are

$$\langle 0|\chi^\dagger \sigma\cdot\mathbf{D}\psi|\chi_{c0}\rangle,\quad \langle 0|\chi^\dagger \sigma\cdot\mathbf{D}\mathbf{D}^2\psi|\chi_{c0}\rangle\quad \langle 0|\chi^\dagger \sigma\cdot g\mathbf{E}\psi|\chi_{c0}\rangle,$$

$$\langle 0|\chi^\dagger \sigma^{(i}D^{j)}\epsilon^*_{ij}\psi|\chi_{c2}\rangle,\quad \langle 0|\chi^\dagger \sigma^{(i}D^{j)}\mathbf{D}^2\epsilon^*_{ij}\psi|\chi_{c2}\rangle,\quad \langle 0|\chi^\dagger \sigma^{(i}E^{j)}\epsilon^*_{ij}\psi|\chi_{c2}\rangle$$

The two can be gotten rid of by using Gremm-Kapustin relation

$$\langle 0|\chi^\dagger(-\frac{i}{2}\boldsymbol{\sigma}\cdot\overleftrightarrow{\mathbf{D}})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\psi|H\rangle = m_c E_B\langle 0|\chi^\dagger(-\frac{i}{2}\boldsymbol{\sigma}\cdot\overleftrightarrow{\mathbf{D}})\psi|H\rangle - im_c\langle 0|\chi^\dagger g\boldsymbol{\sigma}\cdot\mathbf{E}\psi|H\rangle$$

$$\langle 0|\chi^\dagger(-\frac{i}{2}\sigma^{(i}\overleftrightarrow{D}^{j)})(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\psi|H\rangle = m_c E_B\langle 0|\chi^\dagger(-\frac{i}{2}\sigma^{(i}\overleftrightarrow{D}^{j)})\psi|H\rangle - im_c\langle 0|\chi^\dagger g\sigma^{(i}E^{j)}\psi|H\rangle$$

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$
Phenomenology

- To our desired order, there is the approximation

$$\frac{1}{\sqrt{3}}\langle 0|\chi^\dagger \sigma\cdot \mathbf{D}\mathbf{D}^2\psi|\chi_{c0}\rangle = \langle 0|\chi^\dagger \sigma^{(i}D^{j)}\mathbf{D}^2\epsilon^*_{ij}\psi|\chi_{c2}\rangle(1+\mathcal{O}(v^2))$$

- We will use the BESIII data to make constrains on the matrix elements

$$f_{0/2} = \frac{\Gamma^{\lambda=0}_{\gamma\gamma}(\chi_{c2})}{\Gamma^{\lambda=2}_{\gamma\gamma}(\chi_{c2})} = 0.00 \pm 0.02 \pm 0.02$$

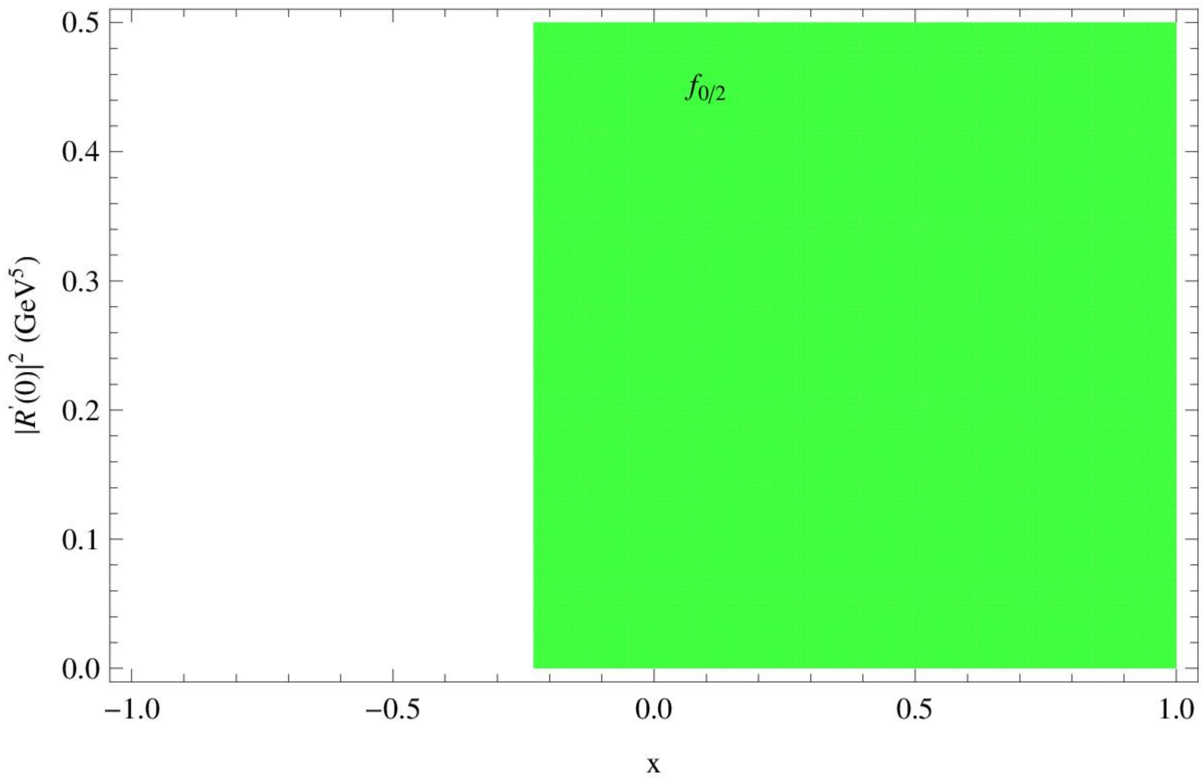
$$\Gamma_{\gamma\gamma}(\chi_{c0}) = (2.33 \pm 0.20 \pm 0.13 \pm 0.17)\,\mathrm{keV}$$

$$\mathcal{R} = \frac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})} = 0.271 \pm 0.029 \pm 0.013 \pm 0.027$$

Investigation on $\chi_{c0,2} \to 2\gamma$

Phenomenology

Constrain from $f_{0/2} = \frac{\Gamma^{\lambda=0}_{\gamma\gamma}(\chi_{c2})}{\Gamma^{\lambda=2}_{\gamma\gamma}(\chi_{c2})} = 0.00 \pm 0.02 \pm 0.02$



$$x \equiv \frac{\langle 0|\chi^\dagger \sigma \cdot \mathbf{D}\mathbf{D}^2 \psi|\chi_{c0}\rangle}{m_c^2 \langle 0|\chi^\dagger \sigma \cdot \mathbf{D}\psi|\chi_{c0}\rangle}$$

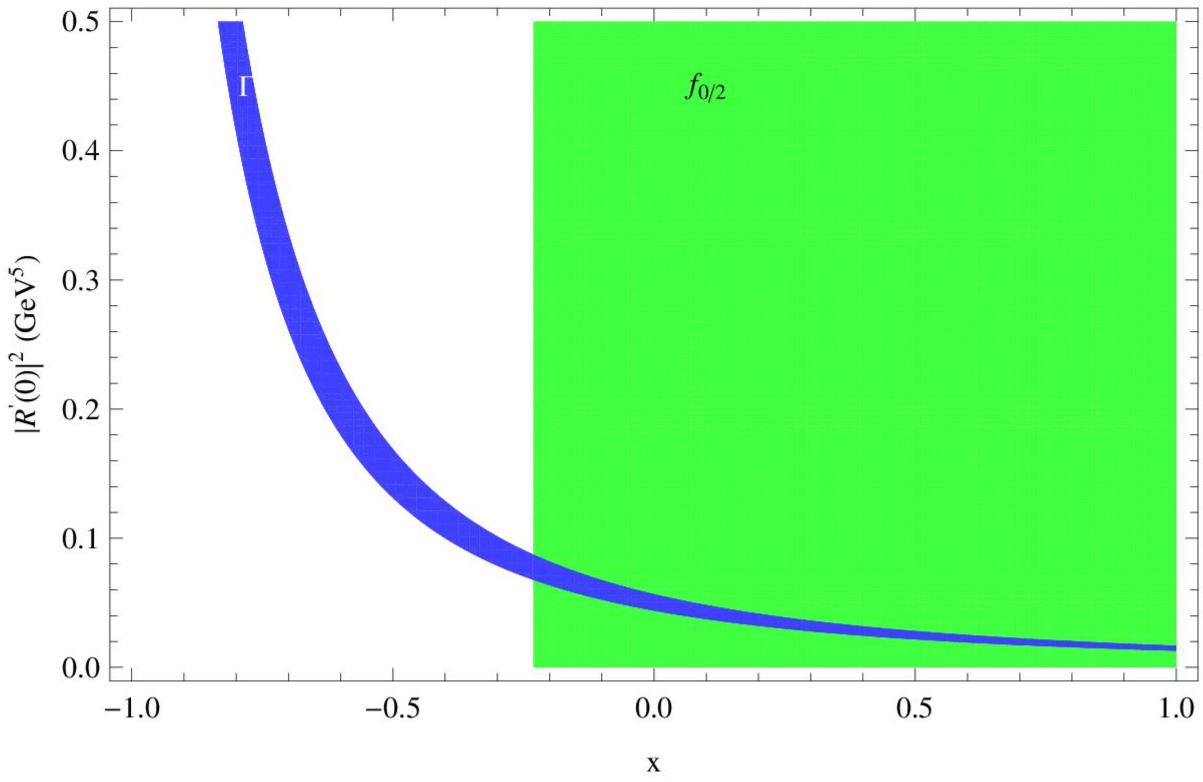
$$\frac{1}{\sqrt{3}}\langle 0|\chi^\dagger \sigma \cdot (-i\mathbf{D})\psi|\chi_{c0}\rangle$$

$$\equiv \sqrt{\frac{3N_c}{2\pi}} R'(0)$$

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Phenomenology

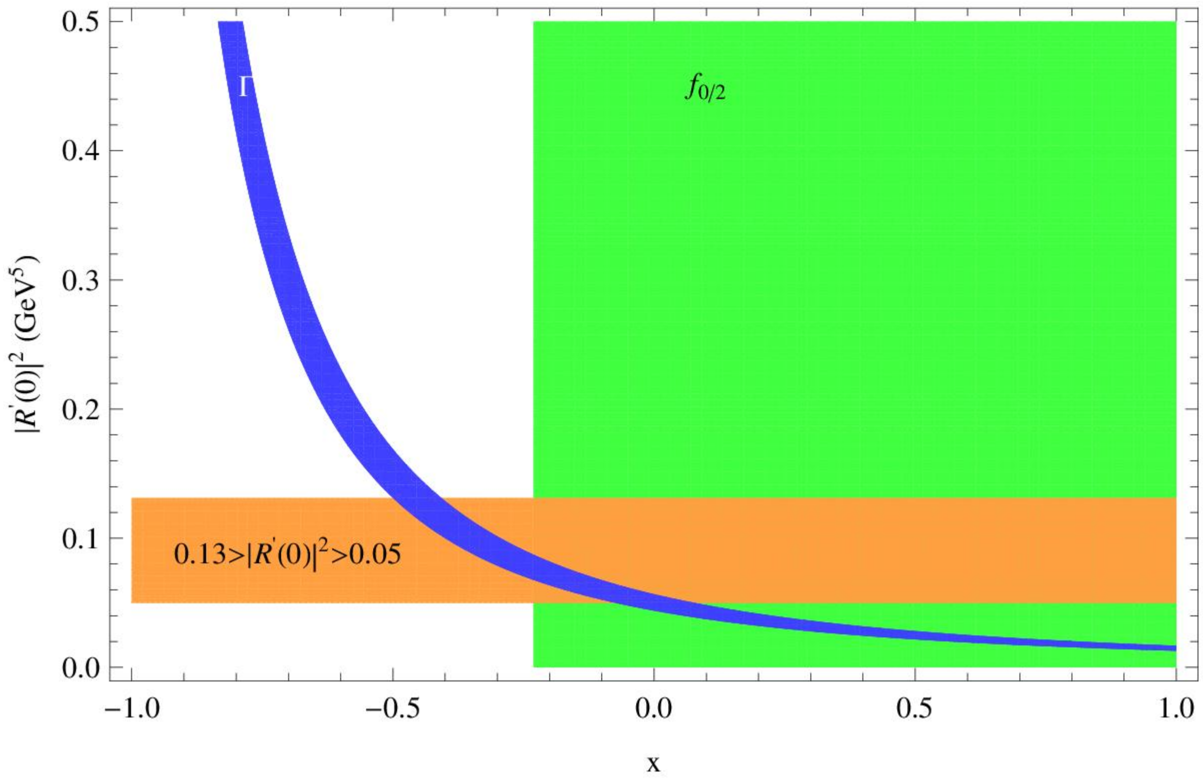
Constrain from $\Gamma_{\gamma\gamma}(\chi_{c0}) = (2.33 \pm 0.20 \pm 0.13 \pm 0.17)\,\text{keV}$



Investigation on $\chi_{c0,2} \to 2\gamma$

Phenomenology

If we require $0.13\,\mathrm{GeV}^5 > |R'(0)|^2 > 0.05\,\mathrm{GeV}^5$



For Buchmuller – Tye (BT) potential $|R'(0)|^2 = 0.075\,\mathrm{GeV}^5$

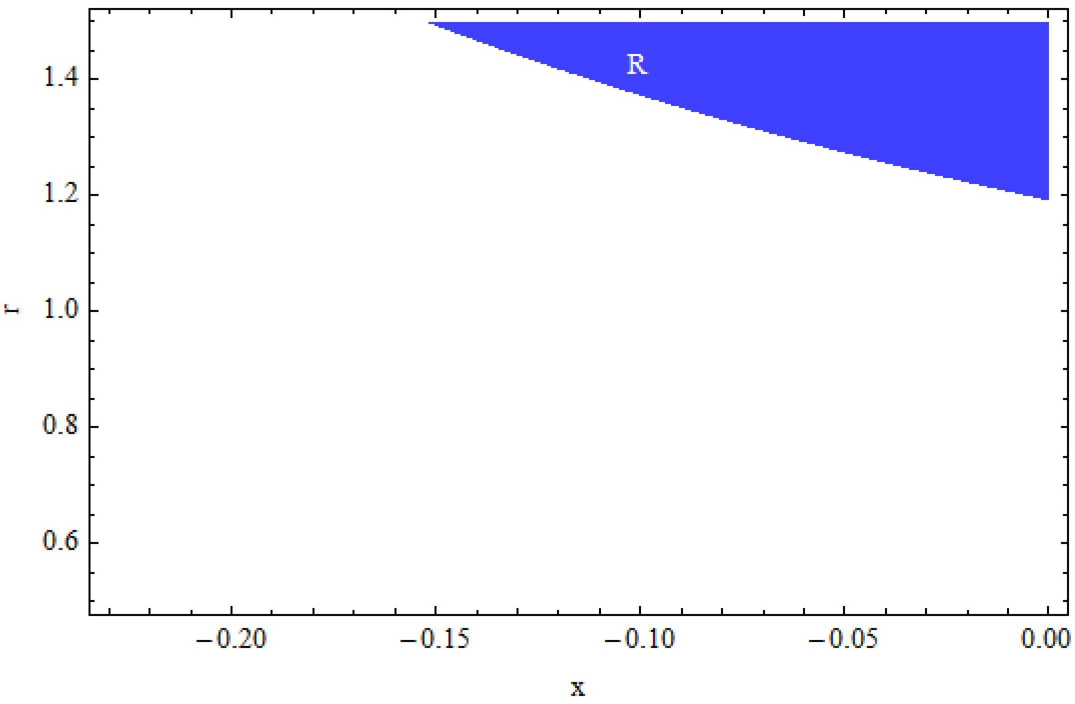
For Cornell potential $|R'(0)|^2 = 0.13\,\mathrm{GeV}^5$

There is still one experiment data, which we have not used! $\mathcal{R} = \dfrac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})}$

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Phenomenology

Constrain from $\mathcal{R} = \frac{\Gamma_{\gamma\gamma}(\chi_{c2})}{\Gamma_{\gamma\gamma}(\chi_{c0})} = 0.271 \pm 0.029 \pm 0.013 \pm 0.027$

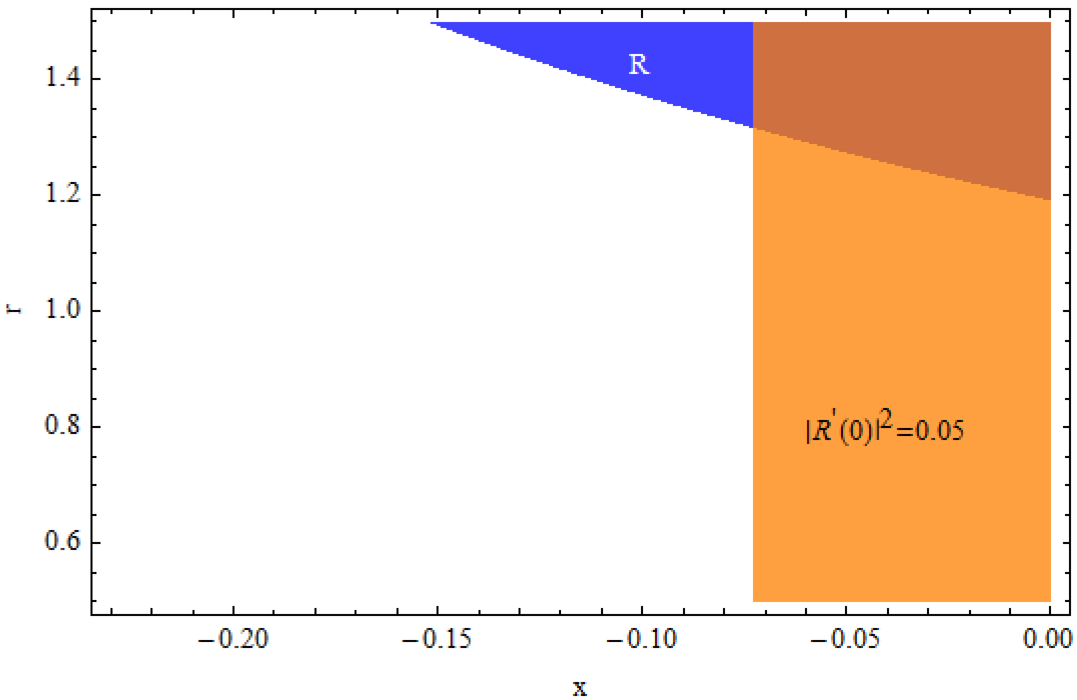


$$r \equiv \sqrt{3} \times \frac{\langle 0|\chi^\dagger \sigma^{(i} D^{j)} \psi|\chi_{c2}\rangle}{\langle 0|\chi^\dagger \sigma \cdot \mathbf{D} \psi|\chi_{c0}\rangle} = 1 + \mathcal{O}(v^2)$$

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Phenomenology

Constrain from $\Gamma_{\gamma\gamma}(\chi_{c0}) = (2.33 \pm 0.20 \pm 0.13 \pm 0.17)\,\text{keV}$



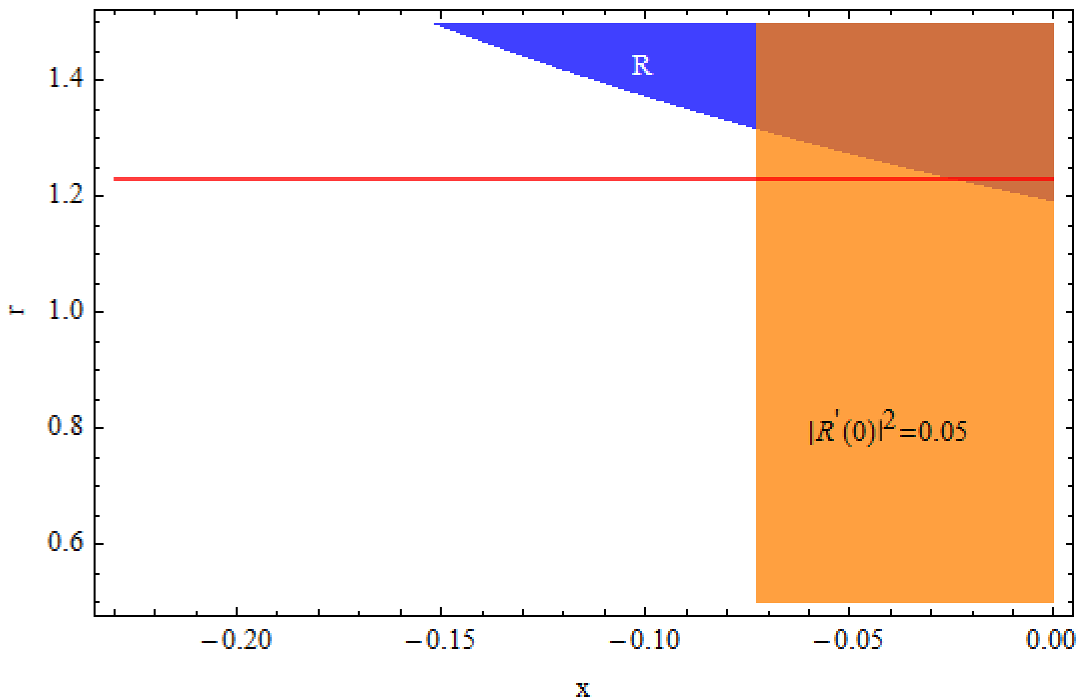
If we take

$$|R'(0)|^2 = 0.05\,\text{GeV}^5$$

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Phenomenology

Constrain from $\Gamma_{\gamma\gamma}(\chi_{c0}) = (2.33 \pm 0.20 \pm 0.13 \pm 0.17)\,\mathrm{keV}$



r may be a little large!

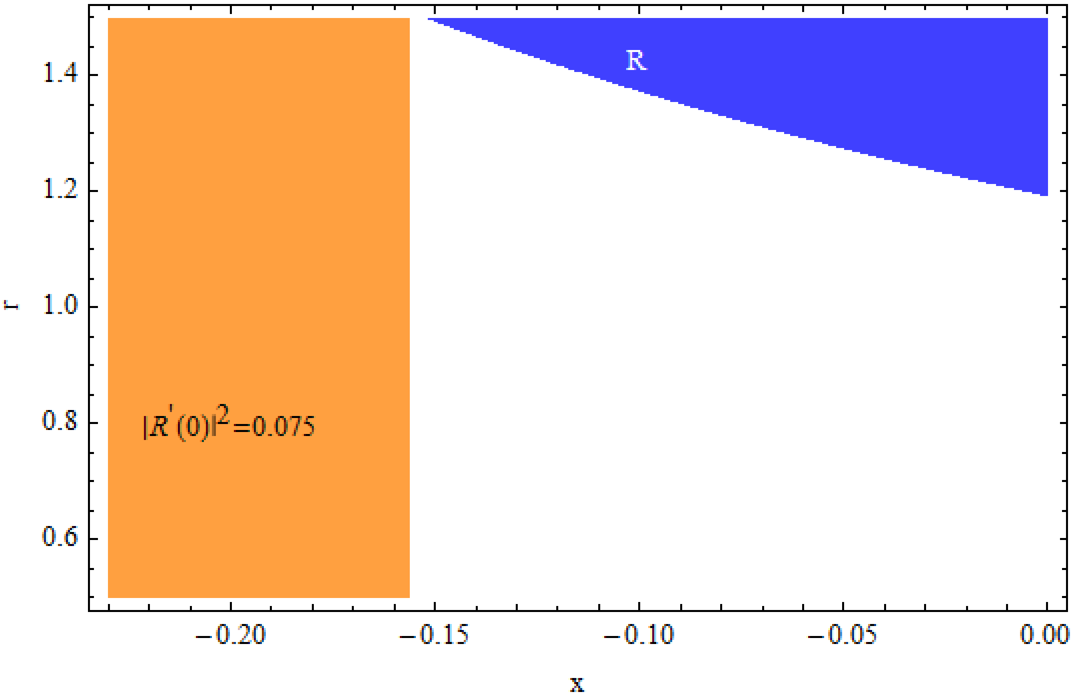
If we take

$$|R'(0)|^2 = 0.05\,\mathrm{GeV}^5$$

Investigation on $\chi_{c0,2} \to 2\gamma$

Phenomenology

If we take $|R'(0)|^2 > 0.075\,\mathrm{GeV}^5$

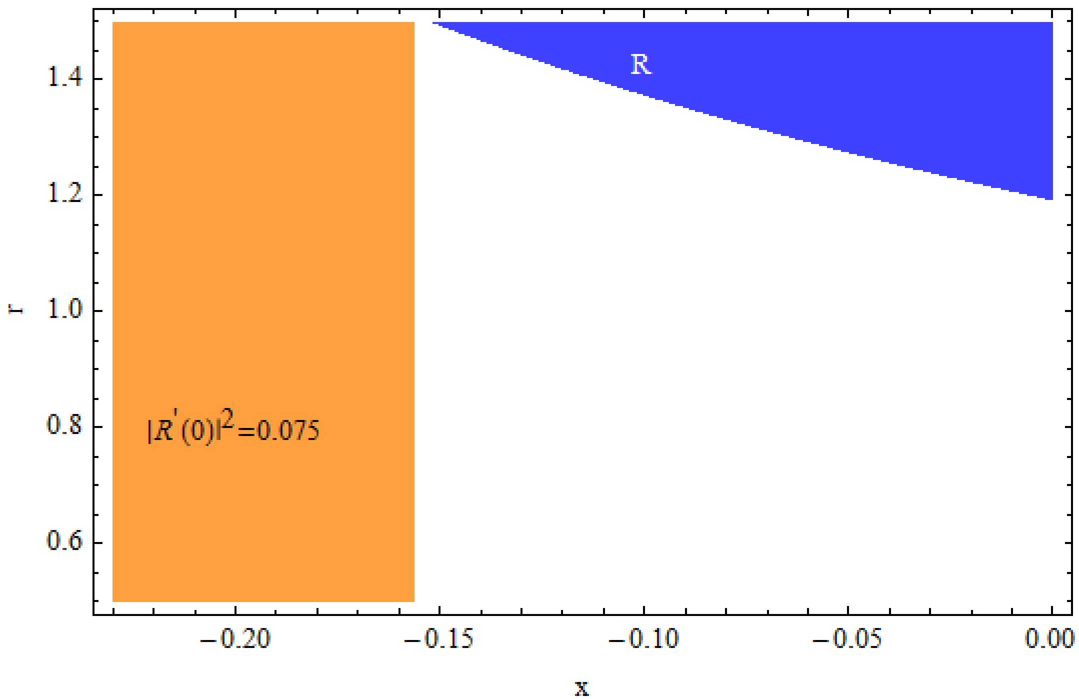


There is no parameter space!

Investigation on $\chi_{c0,2} \rightarrow 2\gamma$

Phenomenology

If we take $|R'(0)|^2 > 0.075\,\mathrm{GeV}^5$



There is no parameter space!

Can the NRQCD naturally explain the BESIII data at relative order v^2 ?

The further phenomenological analyses are still on going

Summary

- ✓ We study the $\mathcal{O}(\alpha_s v^2)$ correction to the decay rate of $J/\psi \to 3\gamma$. The correction is proved to be important and significantly improve the theoretical prediction, and even bring the agreement with experiment data.

- ✓ We study the relativistic & radiative correction to the helicity amplitudes of $\chi_{c0,2} \to 2\gamma$. We explore some analyses. The further phenomenological analyses are still on going.

Thank You!