The NLO Calculations of heavy quarkonium production at B factories

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2013.5.19, ACAT2013@IHEP.Beijing.China

Outline

- Introduction
- The frame of calculation
- Numerical result
- Summary

Collaborated with

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- Ying-Jia Gao
- Yan-Qing Ma
- Kai Wang

Based on PRL96(2006)092001 (hep-ph/0506076), PRL98(2007)092003 (hep-ph/0611086), PRD78(2008)054006 (0802.3655), PRL102(2009)162002 (arXiv:0812.5106), PRD81(2010)034015 (0911.2166)

Some detail can be found in Bin Gong and Feng Feng's talks.

Introduction

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\begin{aligned}
R &= \sum_n F_n < \mathcal{O}(n) > \\
F_n &= F_n^0 (1 + c_1\alpha_s + c_2\alpha_s^2 + ...) \\
< \mathcal{O}(n) > &\propto v^{d_n}
\end{aligned}
\tag{1}
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4. The short distance coefficients can be calculated perturbatively with the expansions by α_s.
5. The LDMES can be scaled by the relative velocity v between the quark and antiquark. v^2 is about $0.2 \sim 0.3$ for charmonium and about $0.08 \sim 0.1$ for bottomonium.

Inclusive J/ψ production at B factories

1. The cross section of $e^+e^- \to J/\psi c\bar{c}$ at $\sqrt{s} = 10.6$GeV was measured by Belle:

$$\sigma[e^+e^- \to J/\psi + c\bar{c} + X] = (0.74 \pm 0.08^{+0.09}_{-0.08}) \text{ pb} , \quad (2)$$

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3. $\sigma(e^+e^- \to J/\psi + X)$ was also measured by Belle, then Belle got:

$$R_{c\bar{c}} = \frac{\sigma(e^+e^- \to J/\psi + c\bar{c} + X)}{\sigma(e^+e^- \to J/\psi + X)} = 0.63 \pm 0.09^{+0.10}_{-0.09}. \tag{3}$$

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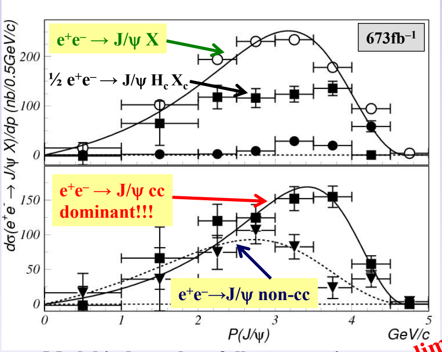
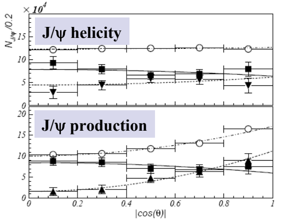
4. Which is larger than the theoretical prediction 0.1.

$e^+e^- \to J/\psi$ cc and non-cc cross sections





preliminary

Model independent full cross sections

$\sigma(e^+e^-\to J/\psi\ cc)$,pb	$0.74\pm0.08^{+0.09}_{-0.08}$
$\sigma(e^+e^-\to J/\psi$ non-cc), pb	$0.43\pm0.09\pm0.09$

No correction on for Nch requirement!
J/ψ from cascade decays included!

Perturbative QCD (no relativisitc corrections):
Kiselev et al. (1995)

$\sigma(e^+e^-\to J/\psi cc) \sim 0.05$pb

Perturbative QCD:
Berezhnoy-Likhoded (2003)

$$\frac{\sigma(e^+e^-\to J/\psi cc)}{\sigma(e^+e^-\to J/\psi gg)} \sim 0.1$$

Figure: **Belle's result of inclusive J/ψ production.**

Exclusive J/ψ production at B factories

1. The cross section of $e^+e^- \to J/\psi + \eta_c$ was measured by Belle and Babar:

$$\sigma[J/\psi + \eta_c] \times B^{\eta_c}[> 2] = 17.6 \sim 25.6 \text{ fb} \tag{4}$$

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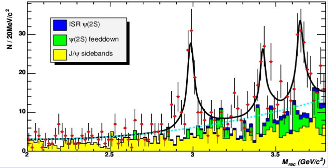
3. Similar discrepancy is appeared in $\sigma(e^+e^- \to J/\psi + \chi_{c0})$

Double $c\bar{c}$ production



124 fb^{-1}, preliminary, hep-ex in preparation.

	$J/\psi\ c\bar{c}$	η_c	χ_{c0}	$\eta_c(2S)$
Expt	$\sigma \times \mathcal{B}_{>2}$	17.6 $\pm$ 2.8 $\pm$ 2.1	10.3 $\pm$ 2.5 $\pm$ 1.8	16.4 $\pm$ 3.7 $\pm$ 3.0
	$\sigma \times \mathcal{B}_{>2}$	25.6 $\pm$ 2.8 $\pm$ 3.4	6.4 $\pm$ 1.7 $\pm$ 1.0	16.5 $\pm$ 3.0 $\pm$ 2.4
Th.	Braaten Lee PRD 67 054007(2003)	2.31 $\pm$ 1.09	2.28 $\pm$ 1.03	0.96 $\pm$ 0.45
	Liu, He, Chao hep-ph/0408141	5.5	6.9	3.7

Applicability of NRQCD : Bondar, Chernyak, hep-ph/0412335

Figure: **BaBar's result of double charmonium production.**

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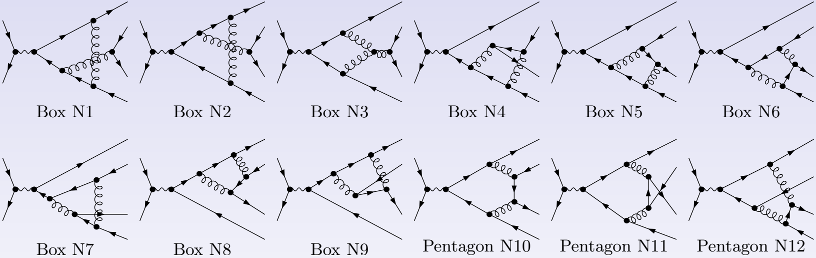
The frame of Calculation

Half of One-Loop box Feynman diagrams for double charm process



The frame of Calculation of inclusive process

Using the NRQCD factorization formalism, we can write down the scattering amplitude of $e^+e^- \to \gamma^* \to J/\psi + c\bar{c}$ as:

$$\begin{aligned} &\mathcal{A}(\gamma^* \to c\bar{c}({}^{2S_\psi+1}L_{J_\psi})(2p_1) + c(p_2) + \bar{c}(p_3)) \\ &\quad = \sqrt{C_{L_\psi}} \sum_{L_{\psi z} S_{\psi z}} \sum_{s_1 s_2} \sum_{jk} \\ &\qquad \times \langle s_1; s_2 | S_\psi S_{\psi z} \rangle \langle L_\psi L_{\psi z}; S_\psi S_{\psi z} | J_\psi J_{\psi z} \rangle \langle 3j; \bar{3}k | 1 \rangle \\ &\qquad \times \mathcal{A}(\gamma^* \to c_j(p_1) + \bar{c}_k(p_1) + c_l(p_2) + \bar{c}_i(p_3)) \end{aligned} \tag{5}$$

There are only three independent momentum p_1, p_2, p_3, but the loop integrate will be

$$\int \mathrm{d}^D q \frac{1}{N_0 N_1 N_2 N_3 N_4} \tag{6}$$

The frame of Calculation of exclusive process

The scattering amplitude of $e^+e^- \to \gamma^* \to J/\psi + \eta_c$ as:

$$
\begin{aligned}
&\mathcal{A}(\gamma^* \to c\bar{c}({}^{2S_\psi+1}L_{J_\psi})(2p_1) + c\bar{c}({}^{2S_{\eta_c}+1}L_{J_{\eta_c}})(2p_2)) \\
&\quad = \sqrt{C_{L_\psi} C_{L_{\eta_c}}} \sum_{L_{\psi z} S_{\psi z}} \sum_{L_{\eta_c z} S_{\eta_c z}} \sum_{s_1 s_2, s_3 s_4} \sum_{jk, il} \\
&\quad\quad \times \langle s_1; s_2 | S_\psi S_{\psi z} \rangle \langle L_\psi L_{\psi z}; S_\psi S_{\psi z} | J_\psi J_{\psi z} \rangle \langle 3j; \bar{3}k | 1 \rangle \\
&\quad\quad \times \langle s_3; s_4 | S_{\eta_c} S_{\eta_c z} \rangle \langle L_{\eta_c} L_{\eta_c z}; S_{\eta_c} S_{\eta_c z} | J_{\eta_c} J_{\eta_c z} \rangle \langle 3l; \bar{3}i | 1 \rangle \\
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\end{aligned}
\tag{7}
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There are only two independent momentum p_1, p_2, but the loop integrate will be

$$
\int \mathrm{d}^D q \frac{1}{N_0 N_1 N_2 N_3 N_4} \tag{8}
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&= \sqrt{C_{L_\psi} C_{L_{\chi_{c0}}}} \sum_{L_{\psi z} S_{\psi z}} \sum_{L_{\chi_{c0} z} S_{\chi_{c0} z}} \sum_{s_1 s_2, s_3 s_4} \sum_{jk,il} \\
&\quad \times \langle s_1; s_2 | S_\psi S_{\psi z} \rangle \langle L_\psi L_{\psi z}; S_\psi S_{\psi z} | J_\psi J_{\psi z} \rangle \langle 3j; \bar{3}k | 1 \rangle \\
&\quad \times \langle s_3; s_4 | S_{\chi_{c0}} S_{\chi_{c0} z} \rangle \langle L_{\chi_{c0}} L_{\chi_{c0} z}; S_{\chi_{c0}} S_{\chi_{c0} z} | J_{\chi_{c0}} J_{\chi_{c0} z} \rangle \langle 3l; \bar{3}i | 1 \rangle \\
&\quad \times \varepsilon^{*\alpha} \frac{\partial}{\partial q^\alpha} \mathcal{A}(\gamma^* \to Q_j(p_1)\bar{Q}_k(p_1) + Q_l(p_2+q)\bar{Q}_i(p_2-q)) \Big|_{q \to 0}
\end{aligned}$$

There are only two independent momentum p_1, p_2, but the loop integrate will be

$$\int \mathrm{d}^D q \frac{1}{N_0^2 N_1 N_2 N_3 N_4} \tag{9}$$

Key points of the calculation

Only two or three independent momentum $p_1, p_2, (p_3)$, but there are five points loop integrate

$$\int \mathrm{d}^D q \frac{1}{N_0 N_1 N_2 N_3 N_4}$$
$$\int \mathrm{d}^D q \frac{1}{N_0^2 N_1 N_2 N_3 N_4} \quad (10)$$

For the No. of independent momentum is less than 4, the Gram Determinant = 0.
The five point reduction, the Passarino-Veltman reduction do not work here.
The other part can be calculated in the same way of QCD.

Five point integrate of inclusive process

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$$\int \frac{\mathrm{d}^D q}{N_0 N_1 N_2 N_3 N_4} = \sum_{i=0}^{4} \frac{a_i}{C} \int \frac{N_i \mathrm{d}^D q}{N_0 N_1 N_2 N_3 N_4}. \tag{12}$$

   They become four point integrate and can be calculated directly.

Five point integrate of inclusive process

1. If we do not introduce the mass of gluon, $C = 0$ when the gluon connect with the both legs of J/ψ. Solve the equation

$$\begin{aligned} \sum_{i=1}^{4} a_i N_i &= N_0 \\ \sum_{i=1}^{4} a_i \frac{N_i}{N_0} &= 1, \end{aligned} \tag{13}$$

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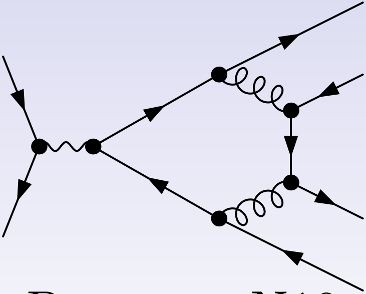
   They become four point integrate but N_0^2.

3. It is calculated in the same way with the exclusive process.

Five point integrate of exclusive process of double S wave

We need calculate the five-point function
$E_0[p_1, 2p_1, -p_2, -2p_2, m, 0, m, 0, m]$,



Pentagon N10

$$E_0^{fin}[p_1, 2p_1, -p_2, -2p_2, m, 0, m, 0, m]$$

$$= E_0 - \frac{2}{s} D_0[-p_1, -p_1 - p_2, p_1, 0, m, 0, m] - \frac{2}{s} D_0[p_1 \leftrightarrow p_2]$$

$$= \int \frac{\mathrm{d}^D q/(2\pi)^D \left(s/2 - 2(q^2 - m^2) - 4q \cdot p_1 + 4q \cdot p_2 - 8m^2\right) 2/s}{(q^2 - m^2)(q + p_1)^2((q + 2p_1)^2 - m^2)(q - p_2)^2((q - 2p_2)^2 - m^2)}$$

$$= \frac{-4}{s} D_0[p_1 + p_2, p_1 + 2p_2, -p_1, 0, 0, m, m] + \int \frac{\mathrm{d}^D q}{(2\pi)^D}$$

$$\frac{2/s(s/2 - 4q \cdot p_1 + 4q \cdot p_2 - 8m^2)}{(q^2 - m^2)(q + p_1)^2((q + 2p_1)^2 - m^2)(q - p_2)^2((q - 2p_2)^2 - m^2)}$$

$$= \text{First Term} + \int \frac{\mathrm{d}^D q}{(2\pi)^D} \int_0^1$$

$$\frac{\Pi_{i=1}^5 \mathrm{d}x_i \delta(\sum_{j=1}^5 x_j - 1) 4! (1 - 16m^2/s)(1 - X - Y)}{[(q + Xp_1 - Yp_2)^2 - m^2(1 - X - Y)^2 + XYs/4]^5}$$

1. where $X = x_1 + 2x_2, Y = x_3 + 2x_4$. The `First Term` is IR- and Coulomb-finite. It can be calculated in $D = 4$ space-time dimension and $v = 0$, it is

$$\frac{2\sqrt{4m^2 - s}\ \tan^{-1}\frac{\sqrt{s}}{\sqrt{4m^2-s}} - \sqrt{s}\ \ln\frac{-s}{m^2}}{-i\pi^2 m^2 s^{5/2}} \tag{15}$$

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2. The second term is IR- and Coulomb-finite too. Choose $\{x_1, x_2, x_3, x_4, x_5\} = \{1-a, ab(1-c), a(1-b), abcd, abc(1-d)\}$ and integrate a, b, d, c step by step in `Mathematica`,

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3. $$\frac{2(4m^2-s)^{3/2}\tan^{-1}\frac{\sqrt{s}}{\sqrt{4m^2-s}}+\sqrt{s}\left(i\pi(3m^2-s)+(s-4m^2)\ln\frac{-s}{m^2}\right)}{8im^4\pi^2(4m^2-s)s^{5/2}(16m^2-s)^{-1}} \tag{16}$$

and $\ln(-s/m^2) = \ln(-(s+i0)/m^2) = \ln(s/m^2) - i\pi$.

1. $D_0[-p_1, -p_1 - p_2, p_1, 0, m, 0, m]$ term in Eq. (15) is,

$$D_0[-p_1, -p_1 - p_2, p_1, 0, m, 0, m] = \frac{4}{s} C_0[-p_1, p_1, 0, m, m] + \frac{i}{(4\pi)^2} \frac{2i\pi - 2\ln 4}{m^2 s}. \tag{17}$$

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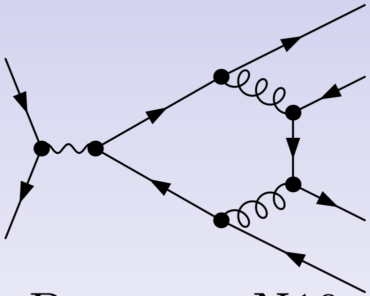
2. This term will appear in $\mathrm{Box\,N5, N8}$.

$$C_0[p_{1c}, -p_{1\bar{c}}, 0, m, m] = \frac{-i}{2m^2(4\pi)^2} \left(\frac{4\pi\mu^2}{m^2}\right)^{\epsilon} \Gamma(1+\epsilon) \left[\frac{1}{\epsilon} + \frac{\pi^2}{v} - 2\right] \quad (18)$$

where $v = |\overrightarrow{p_{1c}} - \overrightarrow{p_{1\bar{c}}}|/m$, defined in meson c.m. frame.

Five point integrate of exclusive process of S +P wave



Pentagon N10

$$\int \frac{\mathrm{d}^D q}{N_0^2 N_1 N_2 N_3 N_4} \tag{19}$$

p in this diagram is more complex, but it can be integrated analysis.

IBP ruduction in five point integrate of S +P wave

1. Solve the equation

$$\sum_{i=1}^{4} a_i \frac{N_i}{N_0} = 1, \tag{20}$$

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3. For there is only two independent momentum, we can reduce $\int \frac{N_i \mathrm{d}^D q}{N_0^3 N_1 N_2 N_3 N_4}$. again.

IBP ruduction in five point integrate of S +P wave

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$$\sum_{i=1}^{4} a_i \frac{N_i}{N_0} = 1, \qquad (20)$$

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4. Then we can get $\int \frac{\mathrm{d}^D q}{N_0^3 N_1^2 N_2}$, $\int \frac{\mathrm{d}^D q}{N_0^2 N_1^2 N_2^2}$....

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4. Then we can get $\int \frac{\mathrm{d}^D q}{N_0^3 N_1^2 N_2}, \int \frac{\mathrm{d}^D q}{N_0^2 N_1^2 N_2^2} \cdots$.
5. It can be calculated with IBP reduction. (Feng Feng's Talk)

Numerical Result

Numerical Result of $e^+e^- \to J/\psi + \eta_c$

1. Select $m_{J/\psi} = m_{\eta_c} = 2m$, $m = 1.5\ GeV$, $\Lambda^{(4)}_{\overline{\rm MS}} = 338\rm MeV$, then $\alpha_s(2m) = 0.259$, and the cross section at NLO is

$$\sigma(e^+ + e^- \to J/\psi + \eta_c) = 15.7\rm fb, \qquad (22)$$

which is larger than LO cross section 8.0 fb by a factor of 1.96 .

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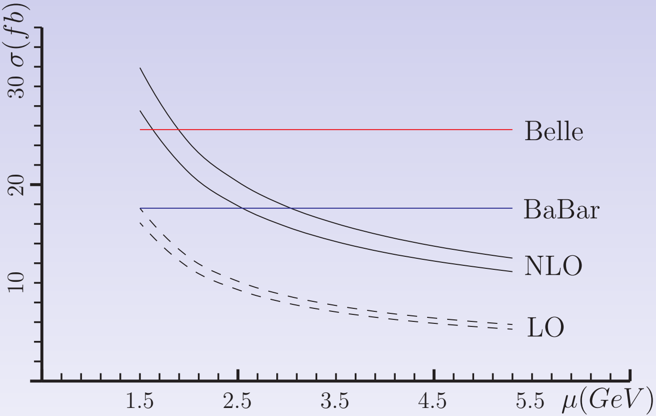
3. It is can be compared with the B factories data $17 \sim 25$ fb

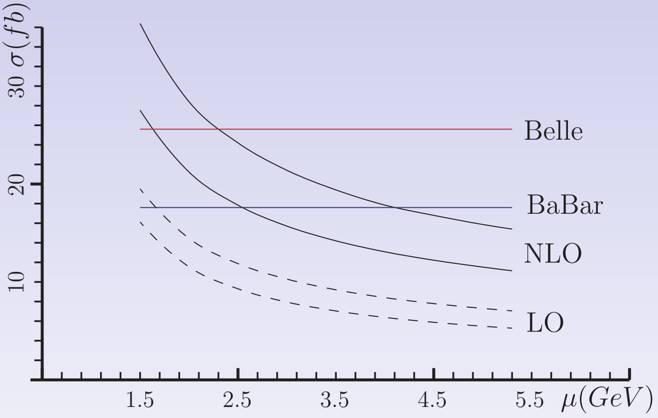


Cross sections as functions of the renormalization scale μ. Here $|R_S(0)|^2 = 0.978\text{GeV}^3$, $\Lambda = 0.338\text{GeV}$, $\sqrt{s} = 10.6\text{GeV}$; NLO results are represented by solid lines and LO one by dashed lines; the upper line is for $m = 1.4\text{GeV}$ and the corresponding lower line is for $m = 1.5\text{GeV}$.

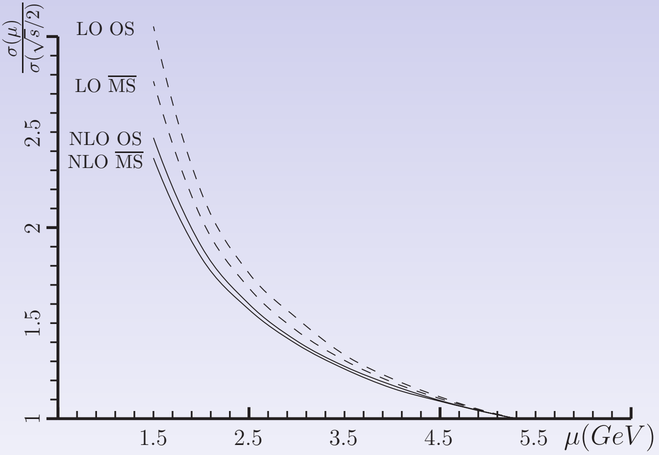
If we select $\overline{\mathrm{MS}}$ scheme for the charm mass, using $m_{OS} = 1.5\mathrm{GeV}$, the corresponding $\overline{\mathrm{MS}}$ mass is $\overline{m} = 1.30\mathrm{GeV}$ defined by $m_{\overline{\mathrm{MS}}}(\overline{m}) = \overline{m}$. And $m_{\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) = 1.16$ GeV. With the $\mu = 2m_{OS}$, we get the cross section at NLO of α_s

$$\sigma(e^+ + e^- \to J/\psi + \eta_c) = 21.4\ \mathrm{fb}, \tag{24}$$

which is a factor of 2.1 larger than the LO cross section 10.3 fb.



Cross sections as functions of the renormalization scale μ. Here $m_{OS} = 1.5\mathrm{GeV}$, and the corresponding $\overline{\mathrm{MS}}$ mass is $\overline{m} = 1.30\mathrm{GeV}$; the solid line is for $\overline{\mathrm{MS}}$ mass scheme and the dashed line is for on-shell mass scheme.



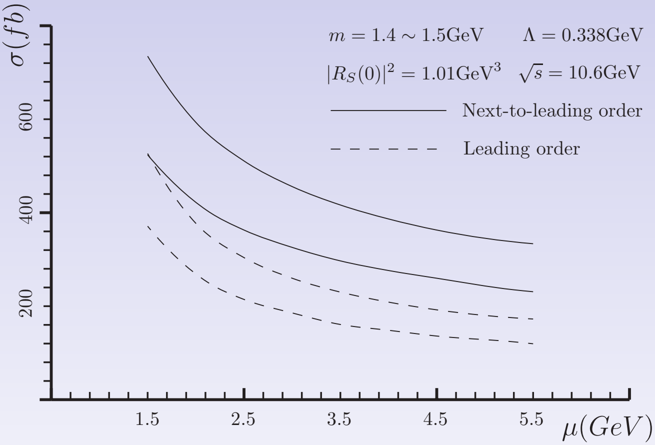
Cross sections rescaled by the corresponding value at $\mu = \sqrt{s}/2$ as functions of the renormalization scale μ. NLO results are represented by solid lines and LO one by dashed lines; the upper line is for on-shell mass scheme and the corresponding lower line is for $\overline{\text{MS}}$ mass scheme.

Numerical Result of $e^+e^- \to J/\psi + c\bar{c}$

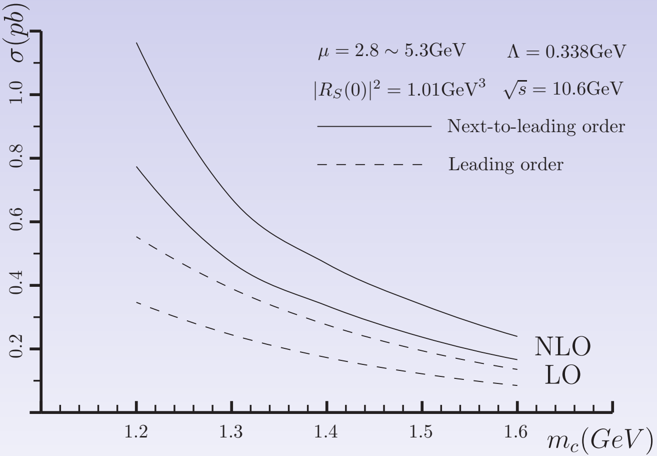
Using the experimental value $\Gamma(J/\psi \to e^+e^-) = 5.55 \pm 0.14 \pm 0.02$ KeV[*], we obtain $|R_S(0)|^2 = 1.01\text{GeV}^3$. Taking $\Lambda^{(4)}_{\overline{\text{MS}}} = 338\text{MeV}$, $m_{J/\psi} = m_{\eta_c} = 2m$ (in the nonrelativistic limit). If we set $m = 1.4\text{GeV}$ and $\mu = 2m$, the cross section at next-to-leading order of α_s is

$$\sigma(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.47 \text{ pb.} \qquad (25)$$

It is about a factor of 1.7 larger than leading order cross section 0.27 pb.



Cross sections as functions of the renormalization scale μ. The upper line is for $m = 1.4\mathrm{GeV}$ and the corresponding lower line is for $m = 1.5\mathrm{GeV}$.



Cross sections as functions of the charm quark mass m_c.
The upper line is for $\mu = 2.8\text{GeV}$ and the corresponding lower line is for $\mu = 5.3\text{GeV}$.

For the experiment date is the prompt $J/\psi + c\bar{c} + X$ cross section. Combine the feed down contributions, if we set $m = 1.5\mathrm{GeV}$ and $\mu = 2m$, then the prompt cross section of $e^+e^- \to J/\psi + c\bar{c} + X$ at next-to-leading order of α_s is

$$\sigma_{prompt}(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.50 \text{ pb}. \qquad (26)$$

It is 67% of the experiment date 0.74 pb in Eq. (2).
If we set $m = 1.4\mathrm{GeV}$ and $\mu = 2m$, ignore the other difference of other contributions, then the prompt cross section of $e^+e^- \to J/\psi + c\bar{c} + X$ at next-to-leading order of α_s is

$$\sigma_{prompt}(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.71 \text{ pb}. \qquad (27)$$

It is about 96% of the new Belle date 0.74 pb in Eq. (2).

Numerical result of $J/\psi gg$

	Belle Data	$\mu = 2.8$ GeV LO	$\mu = 2.8$ GeV NLO	$\mu = 5.3$ GeV LO	$\mu = 5.3$ GeV NLO
$\sigma(gg)$	0.43	0.57	0.67	0.36	0.53
$\sigma(c\bar{c})$	0.74	0.38	0.71	0.24	0.53
$R_{c\bar{c}}$	0.63	0.40	0.51	0.40	0.50

Table: Cross sections of prompt (feeddown included) $J/\psi gg$ and $J/\psi c\bar{c}$ production in e^+e^- annihilation at B factories in units of pb.

Summary

We calculated the NLO QCD corrections of J/ψ production at B factories, the NLO QCD corrections improved the cross sections can be compared with the B factories data.

Thanks!