Automatic one-loop calculations with OpenLoops

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In Collaboration with
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The NLO Frontier: Automation

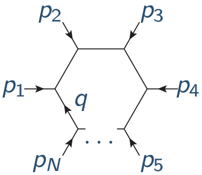
The list of completed ≥ 6 particle processes keeps growing ...

$pp \to WWb\bar{b}$	[Denner, Dittmaier, Kallweit, Pozzorini '11]
	[Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek '11]
$pp \to t\bar{t}b\bar{b}$	[Bredenstein, Denner, Dittmaier, Pozzorini '08, '09, '10]
	[Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09]
$pp \to t\bar{t}jj$	[Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '10]
$pp \to t\bar{t}t\bar{t}$	[Bevilacqua, Worek '12]
$pp \to WW + 2j$	[Melia, Melnikov, Rontsch, Zanderighi '10]
	[Greiner, Heinrich, Mastrolia, Ossola, Reiter, Tramontano '12]
$pp \to W + 3j$	[Ellis, Melnikov, Zanderighi '09]
$pp \to \gamma^*/Z/W + 3j$	[Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maître '09, '10]
$pp \to Z/W + 4j$	[Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maître '10, '11]
$pp \to W^{\pm} + 5j$	[Bern, Dixon, Febres Cordero, Höche, Ita, Kosower, Maître, Ozeren '13]
$pp \to 4j$	[Bern, Diana, Dixon, Febres Cordero, Höche, Ita, Kosower, Maître, Ozeren '11]
$pp \to b\bar{b}b\bar{b}$	[Greiner, Guffanti, Reiter, Reuter '11]
$pp \to W\gamma\gamma j$	[Campanario, Englert, Rauch, Zeppenfeld '11]
$pp \to WZjj$	[Campanario, Kerner, Ninh, Zeppenfeld '13]
$e^+e^- \to 7j$	[Becker, Goetz, Reuschle, Schwan, Weinzierl '11]

... but NLO automation is still a challenge.

- Focus on speed and usability.
- Arbitrary processes, decays, electroweak corrections, ...
- Progress in Monte Carlo generators: beyond parton level NLO.

From Loop Amplitudes to Scalar Integrals



$$= \int d^d q \frac{\mathcal{N}(q)}{D_0\, D_1\, \dots\, D_{N-1}}, \qquad D_i = \Big(q + \sum_{\ell=0}^{i} p_\ell\Big)^2 - m_i^2$$

Tensor integral reduction

Reduce amplitude to a linear combination of scalar basis integrals

On-shell methods

$$\int d^d q \left[\sum_{i_1} \frac{a_{i_1}}{D_{i_1}} + \sum_{i_1, i_2} \frac{b_{i_1 i_2}}{D_{i_1} D_{i_2}} + \sum_{i_1, i_2, i_3} \frac{c_{i_1 i_2 i_3}}{D_{i_1} D_{i_2} D_{i_3}} + \sum_{i_1, i_2, i_3, i_4} \frac{d_{i_1 i_2 i_3 i_4}}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} \right]$$

Tensor integral reduction combined with off-shell current recursion can compete with on-shell methods in gluon scattering with up to 10 gluons. [van Hameren '09]

Colour and Tensor Reduction

For each Feynman diagram
separate **colour factors** and tensor coefficients from tensor integrals.

$$\mathcal{A} = \mathcal{C} \cdot \sum_{r=0}^{R} \mathcal{N}_r^{\mu_1 \dots \mu_r} \cdot \int d^d q \, \frac{q_{\mu_1} \dots q_{\mu_r}}{D_0 \, D_1 \, \dots \, D_{N-1}}$$

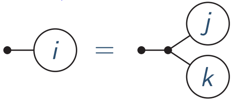
- Algebraic colour reduction and summation only once per process.
- Reduce tensor integrals to scalar basis integrals [Melrose; Passarino, Veltman; Denner, Dittmaier; Binoth et al.; Fleischer, Riemann; & many others].
  We use `Collier` [Denner, Dittmaier, Hofer]: cures numerical instabilities, e. g. by applying expansions in small Gram determinants.
- Alternatively use OPP reduction [Ossola, Papadopoulos, Pittau]: requires multiple evaluations of $\mathcal{N}_r^{\mu_1 \cdots \mu_r} q_{\mu_1} \dots q_{\mu_r}$ for complex q.

"Traditional" approach: construct $\mathcal{N}_r^{\mu_1 \cdots \mu_r}$ analytically in $d = 4 - 2\epsilon$.
Huge expressions & expensive algebraic simplifications limit applicability.

OpenLoops: **Recursive numerical construction of $\mathcal{N}_r^{\mu_1 \cdots \mu_r}$ in $d = 4$**

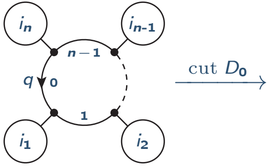
From Tree Recursion to Open Loops

Wave functions w^α of "sub-trees" are 4-tuples (for the spinor/Lorentz index) which are built by recursivly connecting lower sub-trees with vertices $X^\beta_{\gamma\delta}$ and propagators, starting from external legs.

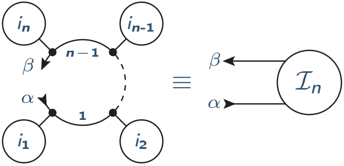


$$w^\beta(i) = \frac{X^\beta_{\gamma\delta}}{p_i^2 - m_i^2}\, w^\gamma(j)\, w^\delta(k)$$

A one-loop diagram is an ordered set of sub-trees $\mathcal{I}_n = \{i_1, \ldots, i_n\}$

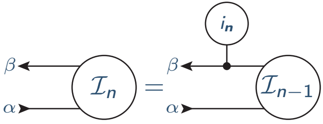


$\mathcal{N}^\beta_\alpha(\mathcal{I}_n; q) =$



Connect sub-trees along the loop to build the numerator $\mathcal{N} = \mathcal{N}^\alpha_\alpha$:

$$\mathcal{N}^\beta_\alpha(\mathcal{I}_n; q) = X^\beta_{\gamma\delta}\, \mathcal{N}^\gamma_\alpha(\mathcal{I}_{n-1}; q)\, w^\delta(i_n)$$



Open Loops Recursion

Separation of the loop momentum q

$$\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n; q) = \sum_{r=0}^{n} \mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)\, q^{\mu_1} \ldots q^{\mu_r}, \qquad X^{\beta}_{\gamma\delta} = Y^{\beta}_{\gamma\delta} + q^{\nu} Z^{\beta}_{\nu;\gamma\delta}$$

leads to the recursion formula for "Open loops" polynomials $\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}$:

$$\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n) = \left[Y^{\beta}_{\gamma\delta}\, \mathcal{N}^{\gamma}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1}) + Z^{\beta}_{\mu_1;\gamma\delta}\, \mathcal{N}^{\gamma}_{\mu_2\ldots\mu_r;\alpha}(\mathcal{I}_{n-1}) \right] w^{\delta}(i_n)$$

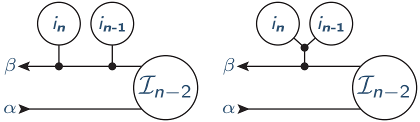
- Retains functional dependence on the loop momentum.
- $\mathcal{N}^{\alpha}_{\mu_1\ldots\mu_r;\alpha}$ are the coefficients of the tensor integrals.
- Also, once the polynomials are known, multiple evaluations of $\mathcal{N}(q) = \sum_{r=0}^{n} \mathcal{N}^{\alpha}_{\mu_1\ldots\mu_r;\alpha} q^{\mu_1} \ldots q^{\mu_r}$ are very fast. $\Rightarrow$ boosts OPP

Open loops can be interfaced with both tensor integrals and OPP in a straight forward way.

Recycling and Helicity Summation

Open loops recycling

Lower-point open-loops can be shared between diagrams if the cut it put appropriately.



Helicity summation

Perform **interference** with the Born amplitude $\mathcal{M}$, **colour and helicity sums** and the sum over the set of diagrams Δ with identical denominator structure on the level of open-loop coefficients.

$$\delta\mathcal{W}^{\Delta} = \sum_{\text{hel,col}} 2\,\mathrm{Re}\left[\mathcal{M}^*\left(\sum_{d'\in\Delta}\delta\mathcal{M}^{(d')}\right)\right]$$

$$\delta\mathcal{W}^{\Delta}_{\mu_1\dots\mu_R} = \sum_{\text{hel,col}} 2\times\left[\mathcal{M}^*\left(\sum_{d'\in\Delta}\mathcal{C}^{(d')}\mathcal{N}^{(d')}_{\mu_1\dots\mu_R}\right)\right]$$

Helicity sums with OPP as efficient as with tensor integrals

Implementation

User input: process definition file

- `FeynArts` [Hahn] generates Feynman diagrams.
- `Mathematica` organises recursion and recycling, reduces colour factors and generates `Fortran 90` code.
- Numerical routines for QCD corrections to Standard Model processes implemented in `Fortran 90`.
- Symmetrising $\mathcal{N}^{\beta}_{\mu_1 \dots \mu_r;\alpha}$ keeps the number of components manageable.
- Rational terms R_2 are calculated using the tree generator.
  [Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau '09, '10; Shao, Zhang, Chao '11]
- No user interaction required: process definition $\to$ compiled library.

Consistency checks

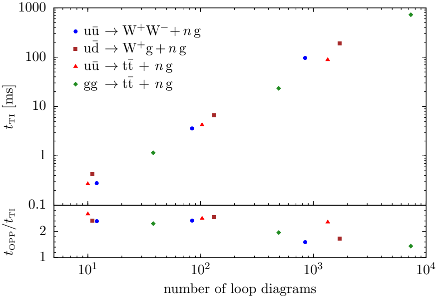
- UV/IR cancellations and Ward identities
- Tensor integrals / OPP reduction with different libraries
- "pseudo-tree": fix loop momentum and compare to tree generator

Speed and Flexibility

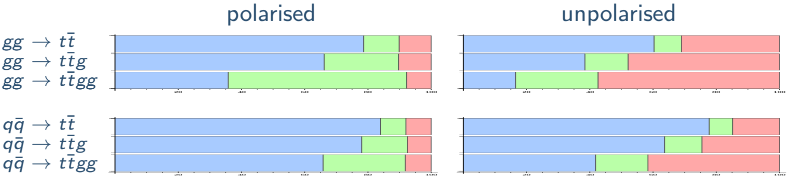
Time to generate code:
seconds to minutes

Compiled library size:
100 kB to a few MB

Runtime per phase space point:
< 1 s for a $2 \to 4$ process
(i7-750 single core, ifort 10.1)



Fractions of the runtime for scalar integrals, tensor reduction, coefficients



Full helicity sums cost only a factor ~ 2 for a $2 \to 4$ process.

Numerical Stability

The numerical precision can be estimated by a scaling test:

$m_i \to \xi m_i,\ p_i^\mu \to \xi p_i^\mu$ leads to $\delta\mathcal{W} \to \delta\mathcal{W}' = \xi^K \delta\mathcal{W}$

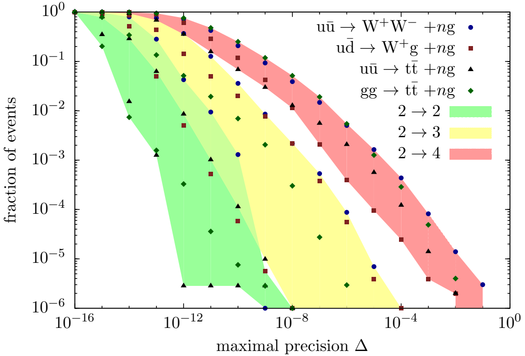
$\Rightarrow$ precision $\Delta = \left| \frac{\xi^{-K}\delta\mathcal{W}'}{\delta\mathcal{W}} - 1 \right|$, rsp. $d = -\log_{10}\Delta$ decimal digits.

12 processes, 10^6 phase space points each;

$\sqrt{s} = 1\,\mathrm{TeV}$,
$p_T > 50\,\mathrm{GeV}$, $\Delta R_{ij} > 0.5$;

using tensor integrals,
in double precision;

11-15 digits on average;
1 permille with <5 digits in the worst $2 \to 4$ case.



Automation of NLO Calculations

Combine OpenLoops with multi-purpose Monte Carlo programs

- `aMC@NLO, POWHEG, Sherpa`:
  IR subtraction, real emission, phase space integration,
  NLO matching with shower, jet merging, hadronisation.
- OpenLoops provides an easy to use API to directly access initialisation and matrix element routines.
- Seamless integration of tools desired.

Done: Sherpa+OpenLoops interface

- Use OpenLoops to generate and compile process libraries.
- Steered by standard Sherpa run cards.
- No hard-wiring or interface code generation required.
- Perform on-the-fly consistency and stability checks.

Process libraries for ATLAS and CMS

Libraries for a wide range of processes are available to ATLAS and CMS.

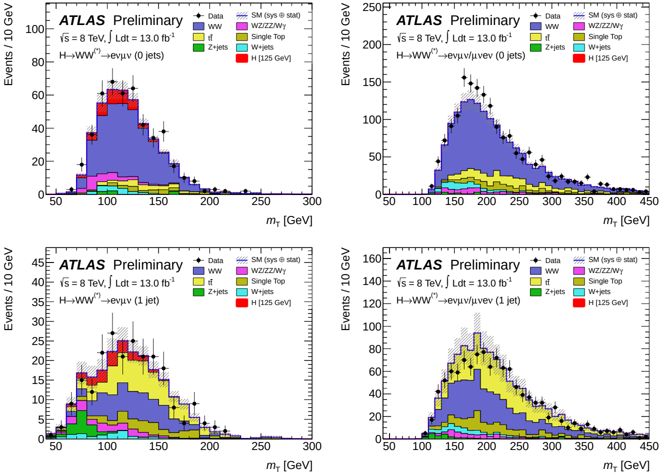
W/Z	γ	jets	HQ pairs	single-top	Higgs
$V+3j$	$\gamma+3j$	$3(4)j$	$t\bar{t}+1j$	$tb+1j$	$(H+2j)$
$VV+2j$	$\gamma\gamma+1(2)j$		$t\bar{t}V+0(1)j$	$t+1(2)j$	$VH+1j$
$gg \to VV+1j$	$V\gamma+2j$		$b\bar{b}V+0(1)j$	$tW+0(1)j$	$t\bar{t}H$
$VVV+0(1)j$					$qq \to Hqq+0(1)j$

(including lower jet multiplicities)

- Validated process-by-process.
- All contributing 1-loop diagrams, full colour.
- Off-shell leptonic W/Z decays (complex masses).
- First step towards a public OpenLoops release.

Irreducible background to $H \to WW^{*}$+0,1 jet

Signal: two opposite sign leptons + E_T^{miss}, binned in jet multiplicities.
Data driven analysis: normalise background (from MC simulation) to data in *control region* (right) and extrapolate to *signal region* (left).



$H \to WW^* \to e^-\bar{\nu}_e\mu^+\nu_\mu$ in exclusive 0-/1-jet bins

Previously available predictions for $pp \to e^-\bar{\nu}_e\mu^+\nu_\mu + 0/1\,\mathrm{jets}$

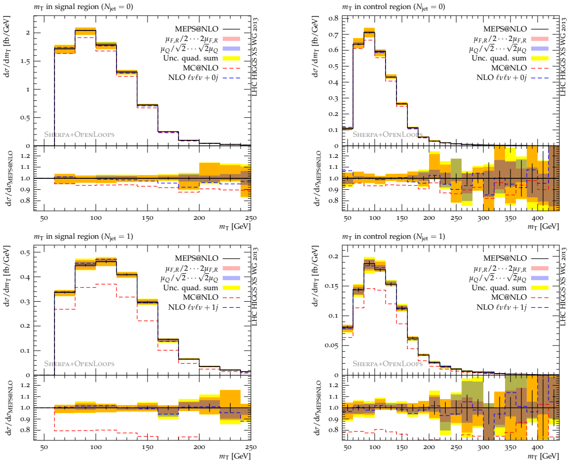
	NLO	gg induced	NLO+PS
0 jets	[Campbell, Ellis, Williams '11]	[Binoth et al. '05] [Campbell, Ellis, Williams '11]	[Melia et al. '11] [Frederix et al. '11]
1 jet	[Dittmaier, Kallweit, Uwer '07] [Campbell, Ellis, Zanderighi '07]	[Melia et al. '12] [Agrawal, Shivaji '12]	

Sherpa+OpenLoops (*preliminary*)

- $\ell\ell\nu\nu + 0/1\,\mathrm{jets}$ MEPS@NLO [Höche, Krauss, Schönherr, Siegert '12]: parton shower and jet merging, NLO+LL accuracy in 0- and 1-jet bins.
- More realistic error estimates, including p_T^{veto} logs.
- Compare to NLO (no resummation), and MC@NLO (LO in 1-jet bin).
- Include all spin correlation, off-shell, and interference effects.
- Studies for ATLAS and CMS experimental analysis.
- Gluon induced channels in progress.

Transverse WW mass distributions (CMS @ 8 TeV)

- %-level agreement between NLO/MC@NLO/MEPS@NLO in 0-jet bin.
- 20% discrepancies between MC@NLO and MEPS@NLO im 1-jet bin.
- Shape distorsions are small.



Cross sections in 0-jet and 1-jet bins (CMS @ 8 TeV)

…in the signal and control regions for NLO/MC@NLO/MEPS@NLO.

0-jets bin	NLO $\pm\Delta_{QCD}$	MC@NLO	MEPS@NLO $\pm\Delta_{QCD}$ $\pm\Delta_{res}$
σ_S [fb]	$159.34(18)\ ^{+1.8\%}_{-1.7\%}$	150.6(2)	$160.3(3)\ ^{+2.6\%}_{-3.8\%}\ ^{+1.4\%}_{-0.5\%}$
σ_C [fb]	$60.08(15)\ ^{+1.5\%}_{-1.4\%}$	56.60(11)	$60.31(22)\ ^{+3.6\%}_{-3.5\%}\ ^{+0.7\%}_{-0.2\%}$
σ_S/σ_C	2.65	2.66	2.66
1-jet bin	**NLO $\pm\Delta_{QCD}$**	**MC@NLO**	**MEPS@NLO $\pm\Delta_{QCD}$ $\pm\Delta_{res}$**
σ_S [fb]	$45.01(7)\ ^{+1.3\%}_{-2.6\%}$	34.75(9)	$44.88(23)\ ^{+3.0\%}_{-2.7\%}\ ^{+0.1\%}_{-0.3\%}$
σ_C [fb]	$22.09(5)\ ^{+1.2\%}_{-3.2\%}$	17.41(7)	$22.30(18)\ ^{+3.0\%}_{-2.7\%}\ ^{+0.5\%}_{-0.4\%}$
σ_S/σ_C	2.04	2.00	2.01

- Error estimation from QCD scales and resummation scale.
- Good agreement between NLO and MEPS@NLO, small scale uncertainties $\to$ Sudakuv logarithms turn out to be small.
- MC@NLO $\sim$ 20% smaller in 1-jet bin (only LO accuracy).

Summary

OpenLoops

- Diagrammatic, tree-like recursion for loop momentum polynomials to calculate one-loop amplitudes.
- Automatic, fast code generation, compact libraries.
- Fast and numerically stable evaluation of matrix elements.

Sherpa+OpenLoops

- Fully automated interface, NLO matching with parton shower and jet merging.
- Process libraries available to ATLAS and CMS.

MEPS@NLO predictions for $H \to WW^*$ background in 0/1-jet bins

- NLO accuracy and LL Sudakov resummation in individual jet bins.
- Small and more reliably estimated theoretical uncertainties.