NNLO mixed QCD-EW corrections to the Drell-Yan production of Z and W bosons

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Outline of the Talk

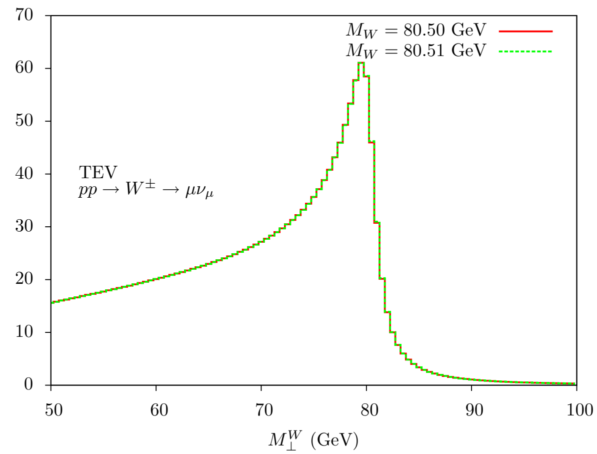
- Introduction and Motivation
  - Importance of an accurate calculation for the Drell-Yan process
  - Sensitivity and Measurement to M_W
  - The status of the radiative corrections
- Description of the Calculation
  - Narrow-width approximation
  - Using Cutkosky rule for the calculation of the real radiation
  - Laporta algorithm and differential equations
- Conclusions

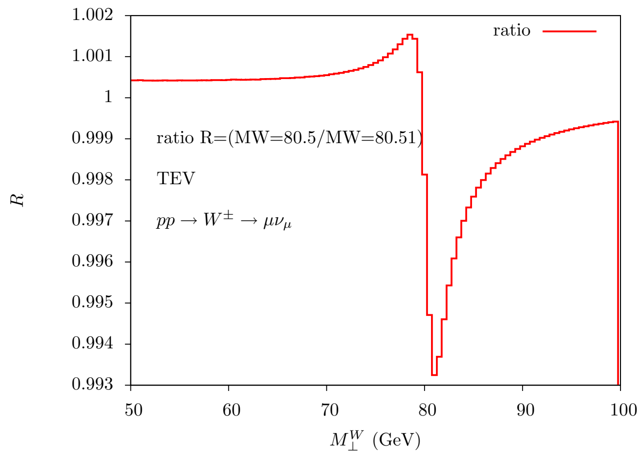
Introduction and Motivation

- Drell-Yan production of Z and W bosons, $pp(\bar{p}) \to Z \to l^+l^-$ and $pp(\bar{p}) \to W \to l\nu$
  - Check of the SM
  - Search new physics.
- DY process has big cross section and clean experimental signature
  - Calibration and monitoring machine
  - Detector performance Z and W production
  - Determine and monitor the hadronic and partonic luminosities at the LHC

Sensitivity and Measurement to M_W

- sensitivity to M_W





G.bozzi, J.Rojo, and A.Vicini '11

- With nominal M_W which differ by 10MeV
  - The total cross section (within cuts) is very weakly sensitive to a M_W variation
  - The ratio of the two distributions generated shows a deviation from unity at the level of few per mil, with non trivial shape.

Sensitivity and Measurement to M_W

- W Mass (transverse mass and p_T distributions) is suppose to be measured at Tevatron with $\Delta M_W \sim 15$ MeV and at LHC even more precisely ($\Delta M_W \sim 7$ MeV).
- At fixed (NLO)order, EW effects are tiny, but not negligible in the view of $\Delta M_W = 15 MeV$
- Mixed QCD-EW corrections important also for the stabilization of the scale dependence:
  - NLO EW (partonic cross section) is leading order in α_S for what concerns the hadronic observable.
  - The mixed corrections can reduce the scale variation

For this reasons require an accurate theoretical and reliable theoretical prediction.

The status of the QCD corrections

- A complete calculation of the NNLO corrections to vector (W,Z) total production rate

  Hamberg, van Neerven, Matsuura '91; van Neerven, Zijstra '92; Harlander and Kilgore '02

- Electroweak gauge boson rapidity distributions at NNLO in QCD

  Anatasiou, Dixon Melnikov and Petriello '04

- W-boson production cross section at the LHC at NNLO (including W decay products)

  Melnikov and Petriello '06

- The full exlcusive NNLO calculation, including the leptonic decay of the vector Boson ($V = Z/\gamma^*, W^+, or W^-$)

  Catani, Cieri, Ferrera, De Florian,Grazzini '09

- NLO matched with resummation NLL in p_T^W/M_W

  Bozzi, Catani, De Florian, Ferrera, Grazzini '09

The status of the eletroweak corrections

W production NLO

- Electroweak radiative corrections to resonant W boson production

  Wackeroth, Hollik '97; Baur et al. '99

- Eletroweak radiative corrections to $pp(\bar{p}) \to W \to l\nu$ beyond the pole appximation

  Zykunov et al. '01; Dittmaier, Krämer '02; Baur, Wackeroth '04; Arbuzov et al. '06; Carloni Calame et al. '06 (HORACE); Hollik, Kasprzik, Kniehl '08

- Photon induced processes of electroweak calculation for the charge current Drell-Yan

  Dittmaier, Krämer '05; Baur, Wackeroth '04; Carloni Calame et al. '06; Arbuzov et al. '07 ...

Z production NLO

- QED radiative corrections

  Baur et al.'98

- weak radiative corrections

  Zykunov et al.'07

- Electroweak radiative corrections

  Baur et al.'02 Carloni Calame et al.'07 (HORACE)

- Photon induced processes of production of a high transverse-momentum lepton pair

  Carloni Calame et al. '07 (HORACE)

Eletroweak and QCD mixed corrections

- Combined effect of the QCD resummation and QED radiative correction to W-boson observables at Tevatron

  Qing-Hong Cao and C.-P.Yuan '04

- Combination of electroweak and QCD corrections to single W production at the Fermilab Tevatron and the CERN LHC

  Balossini, Calame et al '10

- Two-loop form factors in theories with mass gap and Z-boson production

  Kotikov, Kuhn and Veretin.'07

- Two-Loop Virtual Corrections to Drell-Yan Production at order $\alpha^3\alpha_S$

  Kilgore, Sturm, '11

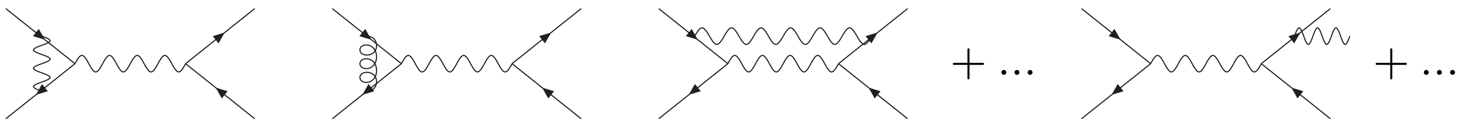
- Combination of electroweak and QCD corrections to resonant Z and W production

  Bonciani, Degrassi, Vicini, in preparation

Higher order corrections

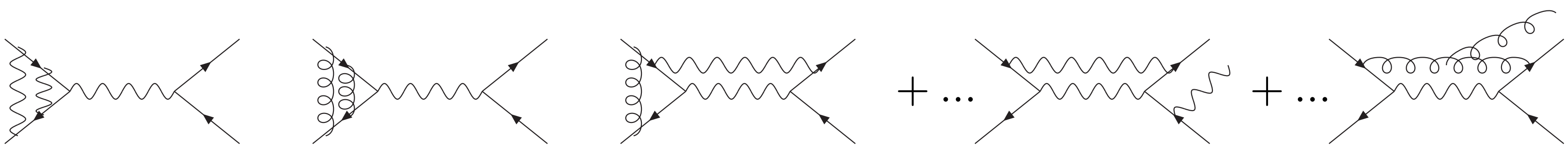
- Tree-level contributions

- NLO contribution



up to $1l$ box diagrams with massive propagators

- NNLO contribution



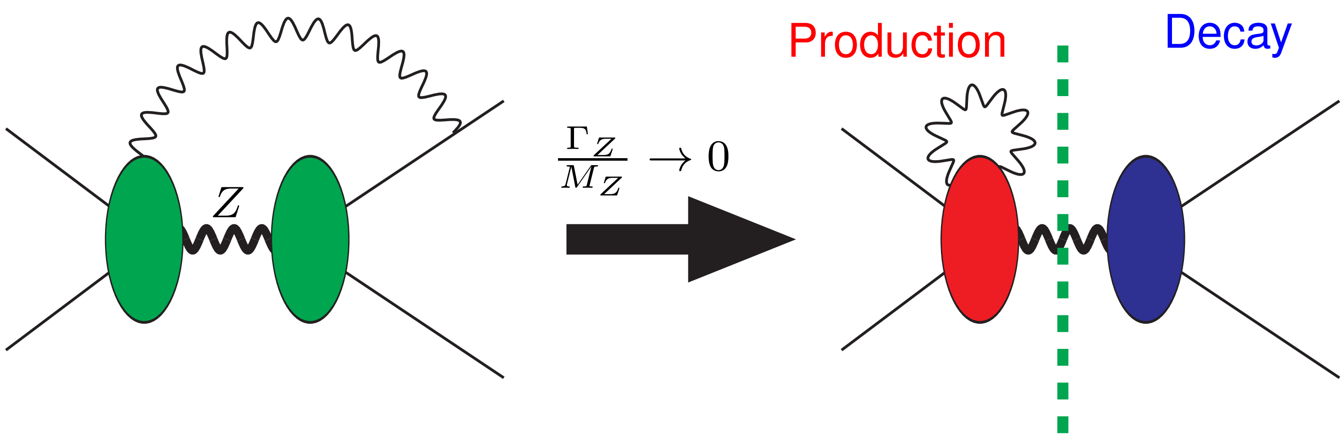
up to $2l$ boxes with masses, $1l$ boxes with masses, $1l$ pentagon + ...

First step: Narrow-width approximation

- Narrow-width approximation

$$\frac{\Gamma_Z}{M_Z}, \frac{\Gamma_W}{M_W} \ll 1$$

- Factorizable corrections: they do not mix production and decay stages

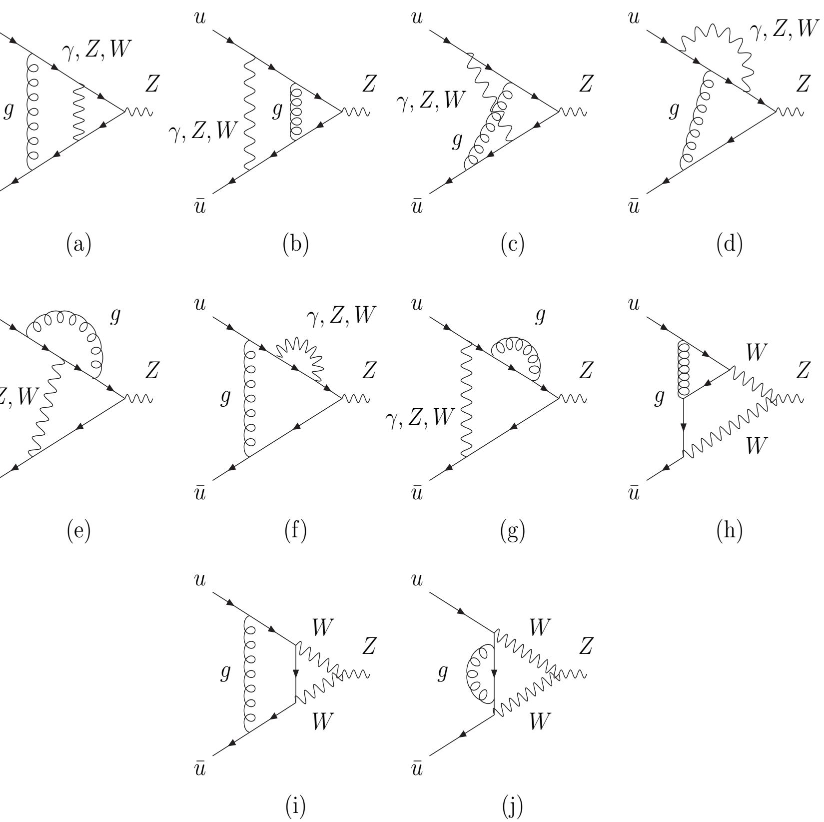


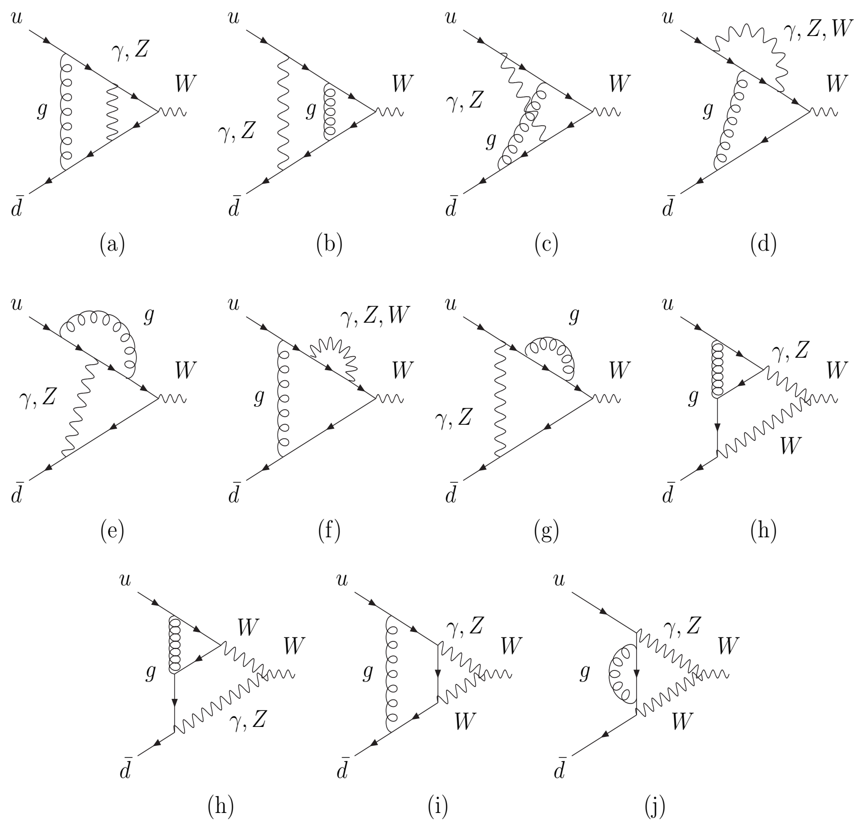
Non-factorized contribution are of $\sim \mathcal{O}(\Gamma_z/m_z)$

- Now we concentrate on the $\alpha\alpha_S$ corrections to the production process
  - virtual corrections: two-loop $2 \to 1$ processes
  - real correction: one-loop real-virtual $2 \to 2$ corrections and tree-level real $2 \to 3$ corrections

Feynman Diagrams for the vitual corrections

- Two-loop $\alpha\alpha_S$ virtual diagrams:





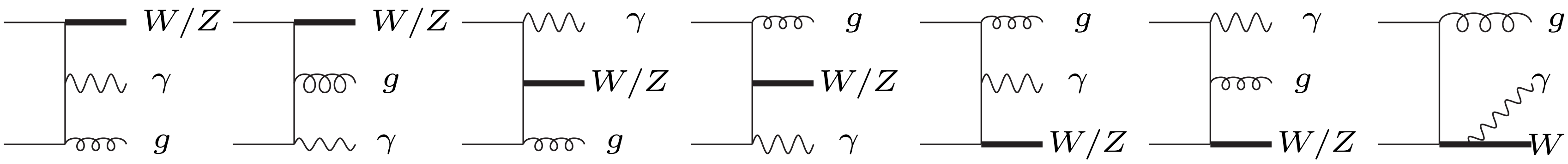
0 diagrams contribute to the Z production

44 diagrams contribute to the W production

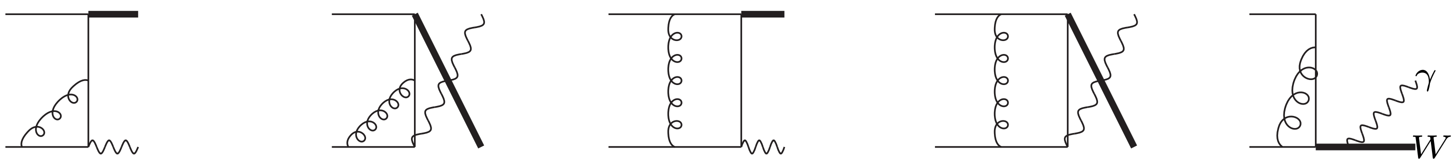
Bonciani, Degrassi, Vicini, in preparation

Feynman Diagrams for the real radiation

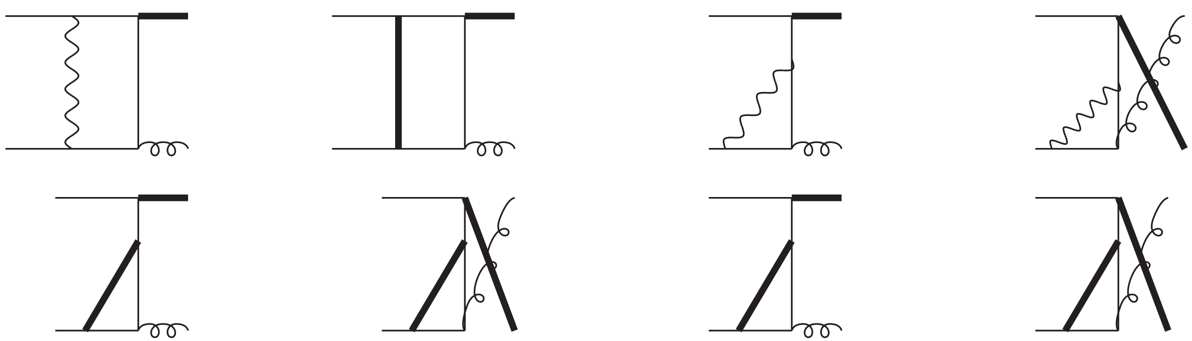
- Feynman diagrams for the double-real radiation at NNLO:



- Feynman diagrams for the single-real radiation at NNLO:
  - photon radiation in the final state



  - gluon radiation in the final state



Calculation of the Feynman diagrams

- Expression of the observable (modulus squared of the amplitude) in terms of (many) dimensionally regularized scalar integrals
- Use of the Cutkosky rules for the phase-space integrals (in order to use the same technique as for the virtual corrections)
- Generation of Integration-by-Parts Identities (IBPs) and solution of the corresponding algebraic system. Ouput: relations that link scalar integrals to Master Integrals (MIs)
- Calculation of the MIs: generation of the system of first-order linear differential equations
- Solution of the system in Laurant series of $(D-4)$. Coefficients of the series expressed in terms of Harmonic Polilogarithms (HPLs) or related functions

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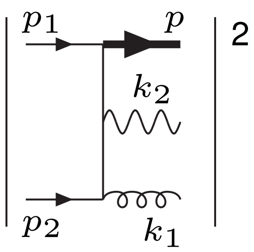
Cutkosky rules for the phase-space integrals

- Illustration of the method.
  - considere the following double-real contribution:

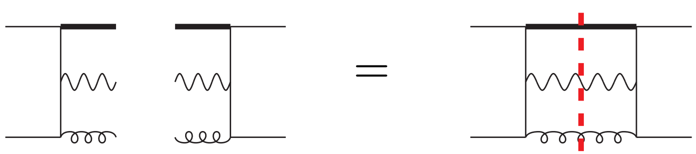


$$\sim \int \frac{d^d k_1 d^d k_2 \delta(p^2 - m^2)\delta(k_1^2)\delta(k_2^2)\,[\ldots]}{(p-p_1)^4\,(k_1-p_2)^4}$$

  - replace delta-functions in the above integral by the difference of two propagators with different causal prescription:

$$2\,i\,\pi\delta\left(p^2 - m^2\right) \to \frac{1}{p^2 - m^2 + i0} - \frac{1}{p^2 - m^2 - i0}$$

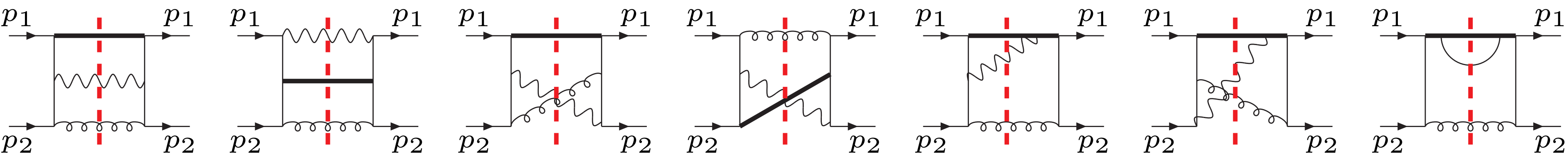
  - the r.h.s of last equation is equal to a forward scattering diagram:



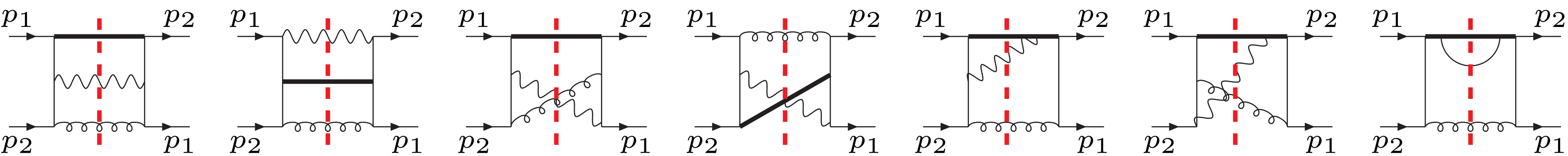
Anastasiou, Melnikov '02

Topologies for the real corrections

- 14 topologies with 3 cuts:
  - The "planar" diagrams with 3 cuts:

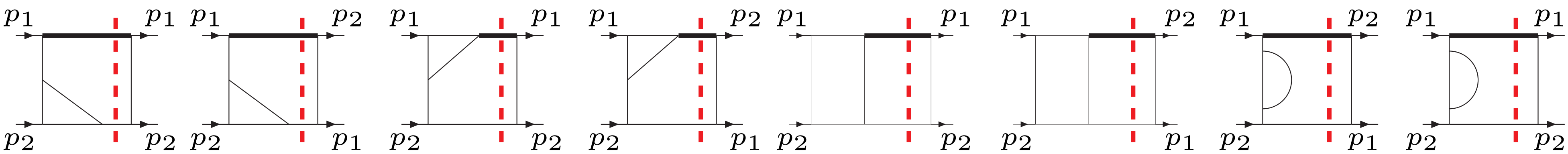


  - The "cross" diagrams with 3 cuts:

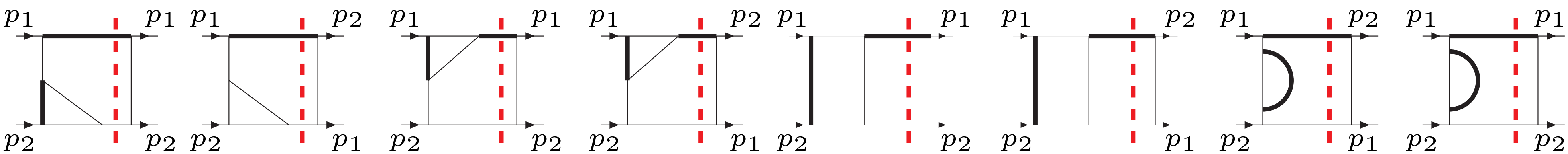


Topologies for the real corrections

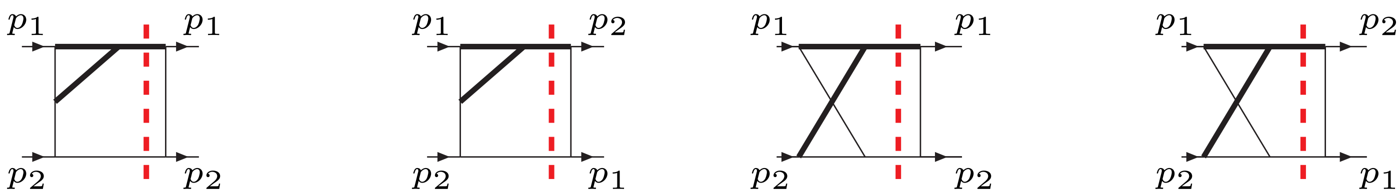
- 20 topologies with 2 cuts:
  - 2 cuts diagrams with one massive propagator:



  - 2 cuts diagrams with two massive propagators:



  - 2 cuts diagrams with three massive propagators:



the bold lines represent the massive W and Z bosons, the thin lines represent the massless gluon, photon and light quarks.

Reduction of the Feynman diagrams

- Expression of the observable (modulus squared of the amplitude) in terms of (many) dimensionally regularized scalar integrals
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Reduction of the Feynman diagrams

- Integration by parts identities (IBPs)

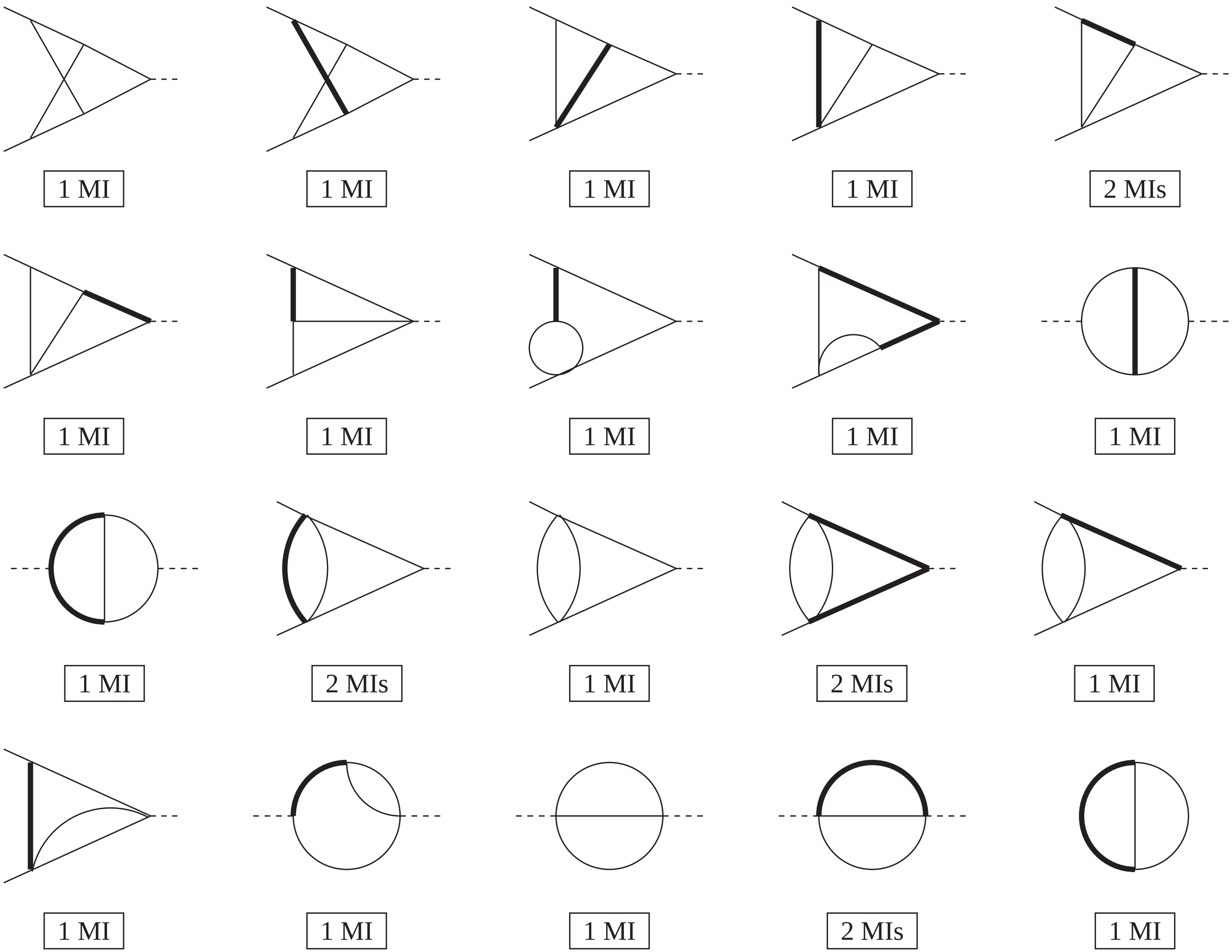
$$\int \frac{d^d k_1 d^d k_2}{(2\pi)^{(d-2)}} \frac{\partial}{\partial k^\mu_{1,2}} v^\mu \frac{S_1^n \, [\ldots] \, S_q^m}{D_1^a \, [\ldots] \, D_t^b} = 0$$

where $v^\mu = p_i^\mu, k_i^\mu, i = 1, 2.$

Tkachov, Chetyrkin '81

- Laporta Algorithm: For a certain power of the denominator and numerator the number of generated equations is bigger than the unknow amplitudes that come out because of the derivative of the IBPs.
- Based on this algorithm, some public programs are available
  - AIR – Maple package
    (C. Anastasiou and A. Lazopoulos, JHEP **0407** (2004) 046)
  - FIRE – Mathematica package (A. V. Smirnov, JHEP **0810** (2008) 107)
  - REDUZE – REDUZE2 C++/GiNaC packages
    (C. Studerus, Comput. Phys. Commun. **181** (2010) 1293;
    A. von Manteuffel and C. Studerus, arXiv:1201.4330)

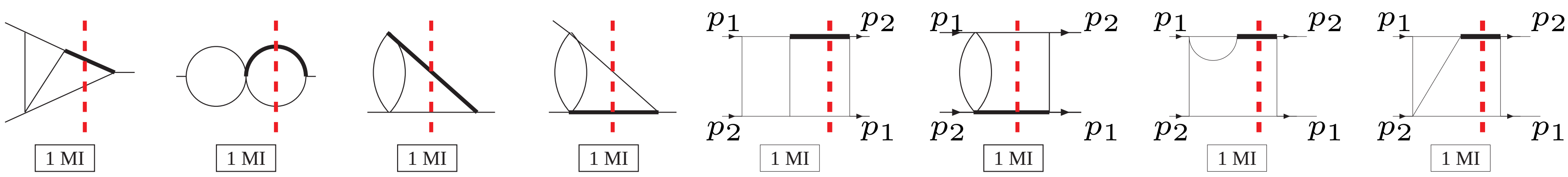
List of virtual Master Integrals



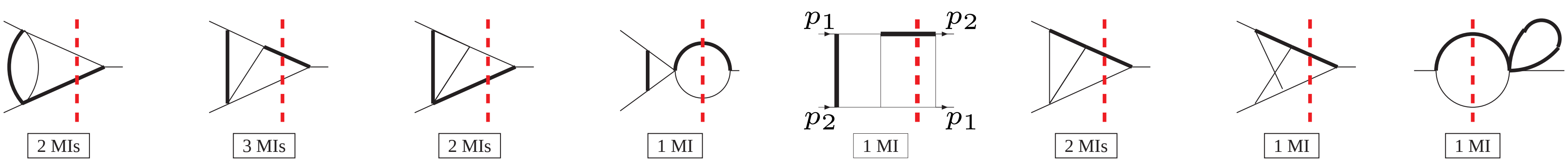
van Neerven '86; Gonsalves '86; Fleischer, Kotikov, Veretin '99; Davydychev, Kalmykov '03; Aglietti, Bonciani. '03-'04 ....

List of Master Integrals for the real radiation

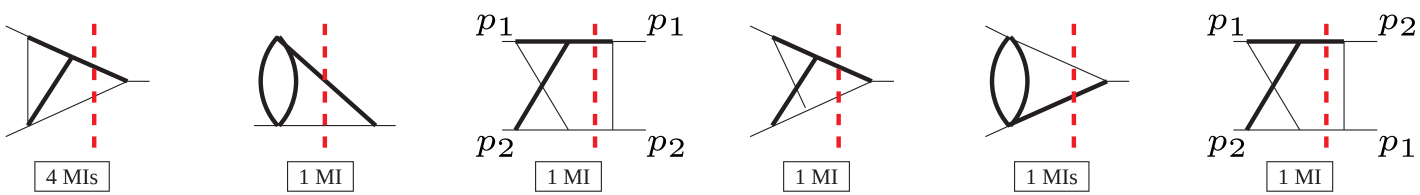
- 2 cuts MIs
  - 2 cuts Masters with one massive propagator:



  - 2 cuts Masters with two massive propagators:



  - 2 cuts Masters with three massive propagators:

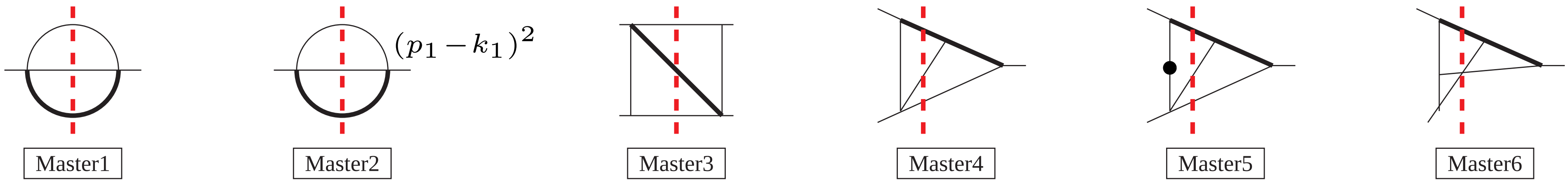


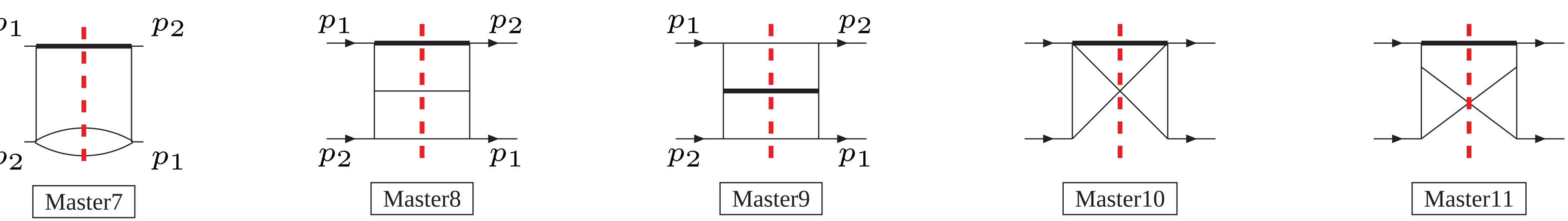
Some of them have been calculated by Anastasiou and Melnikov('02).

Bonciani and Pan, in preparation

List of Master Integrals for real radiation

- The masters with 3-cut





A dot on the propagator line means that the propagator is raised to power 2. Bold lines represent a massive propagator. Thin lines denote massless propagators.

Differential Equations for the MIs

- Expression of the observable (modulus squared of the amplitude) in terms of (many) dimensionally regularized scalar integrals
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Differential Equations for the MIs

or a given topology, when the system of identities is not reducible, we have a (hopefully) small umber of independent integrals called MI. In the case of three-point functions:

$$F_i(Q^2, p_1^2, p_2^2) = \int d^D k_1 d^D k_2 \frac{S_1^{n_1} \cdots S_q^{n_q}}{D_1^{m_1} \cdots D_t^{m_t}}$$

Jsing all the identity-relations (IBP's, LI, Sym) we can construct the following system of first-order near differential equations:

$$\frac{dF_i}{dQ^2} = \sum_j h_j(Q^2, m^2)\, F_j + \Omega_i$$

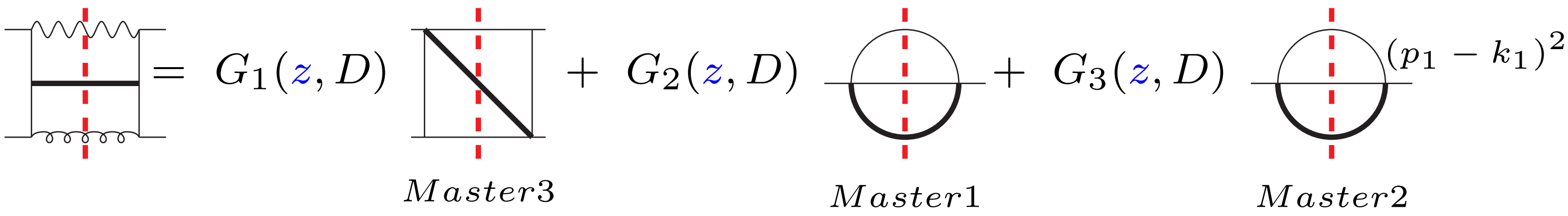
vhere $i, j = 1, ..., N_{MIs}$.

Ω_i | This term involves integrals of the class $I_{t-1,r,s}$ (sub-topologies) to be considered KNOWN

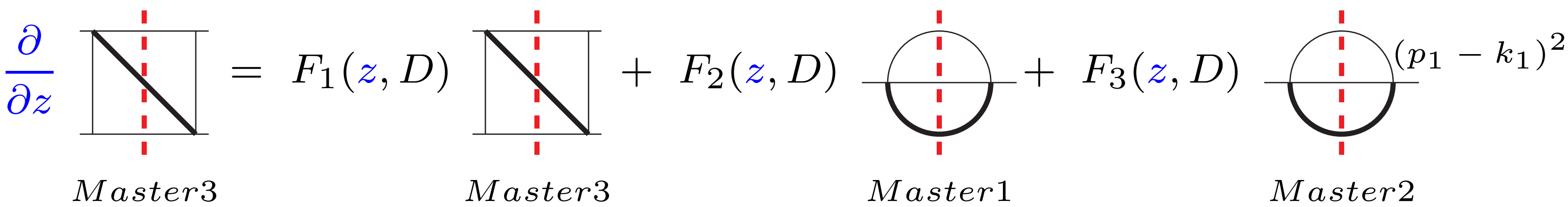
V. Kotikov, *Phys. Lett.* **B254** (1991) 158; **B259** (1991) 314; **B267** (1991) 123.
E. Remiddi, *Nuovo Cim.* **110A** (1997) 1435.

Example of the reduction

- Example of the IBPs for one reducible topology with 3 cuts



- Example of the DIFF for the MIs with 3 cuts

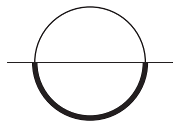
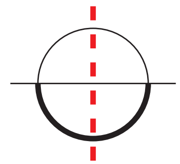


where $z = M^2/s$. The functions G_i, F_i are known directly from the reduction process.

- Calculation of the MIs
  - Master1 have to be integrated directly.
  - Master3 and Master2 can be calculated using the differential equations method. The constants of integration can be found imposing the initial condition (regularity at $z \to 1$).

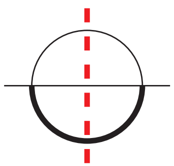
The solution of the base Master with three-cut

- The basic MIs with three cuts
  - Using the optical theorem, we can relate the cut diagrams to the imaginary part of the loop integral:

$2\ Im$  $=$ 

Master1

  - Extracting the imaginary part from the loop integral and using the relevant Kummer relation for the hypergeometric function, we find:



$$= N\Big[(z)^{-1+2\epsilon}(-\pi)\frac{\Gamma(1-\epsilon)^3}{\Gamma(\epsilon)^2\Gamma(3-3\epsilon)\Gamma(2-2\epsilon)}(1-z)^{3-4\epsilon}\,{}_2F_1(1-\epsilon,2-2\epsilon,\epsilon,z)$$

$$+(z)^{\epsilon}(-\pi)\frac{\Gamma(-1+\epsilon)\Gamma(1-\epsilon)\epsilon}{\Gamma(2-2\epsilon)\Gamma(1+\epsilon)\Gamma(\epsilon)}(1-z)^{3-4\epsilon}\,{}_2F_1(3-3\epsilon,2-2\epsilon,2-\epsilon,z)\Big]$$

l a nomalization factor.It's important to extract the $(1-z)^{3-4\epsilon}$ part. It will be explained by the ollowing example.

Harmonic Polylogarithms (HPLs)

- Weight $= 1$

$$H(0,x) = \ln x \quad H(-1,x) = \int_0^x \frac{dt}{1+t} = \ln(1+x) \quad H(1,x) = \int_0^x \frac{dt}{1-t} = -\ln(1-x)$$

- Weight > 1

  If $\vec{a} = \vec{0}$ we define $H(\vec{0},x) = \frac{1}{\omega!}\ln^{\omega} x$. If $\vec{a} \neq \vec{0}$:

$$H(\vec{a},x) = \int_0^x dt\, f(a_1,x)\, H(\vec{a}_{\omega-1},t) \qquad \frac{d}{dx} H(\vec{a},x) = f(a_1,x)\, H(\vec{a}_{\omega-1},x)$$

- The Algebra: $\omega_{\vec{a}} \times \omega_{\vec{b}} = \omega_{\vec{a}} \times \omega_{\vec{b}}$

$$H(\vec{a},x)\, H(\vec{b},x) = \sum_{\vec{c}=\vec{a}\uplus\vec{b}} H(\vec{c},x)$$

- Integration by Parts

$$H(m_1,...,m_q,x) = H(m_1,x)H(m_2,...,m_q,x) - ... + (-1)^{q+1} H(m_q,...,m_1,x)$$

- Connection with Nielsen's polylog and Spence functions:

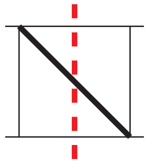
$$S_{n,p}(x) = H(\vec{0}_n,\vec{1}_p,x) \quad Li_n(x) = H(\vec{0}_{n-1},1,x)$$

A.B.Goncharov, *Math. Res. Lett.* **5** (1998), 497-516.

E. Remiddi and J. A. M. Vermaseren, *Int. J. Mod. Phys.* **A15** (2000) 725.

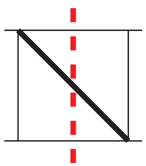
The solution of non-divergence factor Master3

- Using threshold condition to constrain the integral constants.
  - z = 1 is the threshold for the Z/W boson production, the integrals vanish in D dimension:



$$(z = 1) \quad = \quad 0$$

- The solution of the Master3.



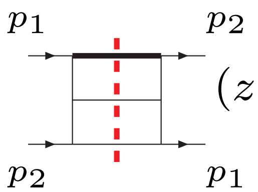
$$= N\frac{i\pi}{a^2}z(1-z)^{4\epsilon}\Bigg[\frac{1}{\epsilon^2}\Bigg(H(0,z)\Bigg) - \frac{1}{\epsilon}\Bigg(\frac{H(0,0,z)}{2} + H(1,0,z) + \zeta(2)\Bigg)$$

$$+\frac{H(0,z)}{2}\zeta(2) - \frac{H(0,0,0,z)}{4} - H(0,0,1,z) - \frac{H(0,1,1-z) * H(0,z)}{8} - \frac{H(0,1,1-z)}{2}$$

$$+\frac{H(1,0,0,z)}{2} + \frac{\zeta(3)}{4} + \mathcal{O}\left((D-4)\right)\Bigg]$$

Example of solving Master8

- The threshold condition
  - z = 1 is the threshold for the Z/W boson production, the integrals vanish in D dimensions:



$$(z=1) \quad = \quad 0$$

  - The behavior of the $(1-z)^{-1+a\epsilon}$.

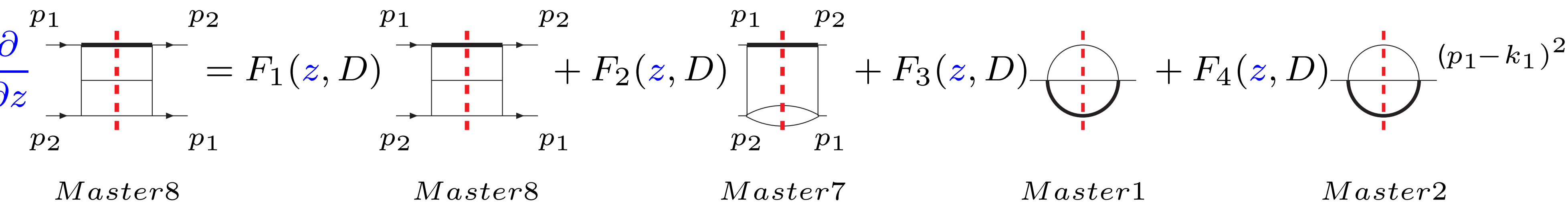
$$(1-z)^{-1+a\epsilon} = \frac{\delta(1-z)}{a\epsilon} + a\epsilon\left[\frac{1}{1-z}\right]_+ + \frac{(a\epsilon)^2}{2!}\left[\frac{(1-z)}{1-z}\right]_+$$

$$+\frac{(a\epsilon)^3}{3!}\left[\frac{(1-z)^2}{1-z}\right]_+ + \mathcal{O}\left((\epsilon^4)\right)$$

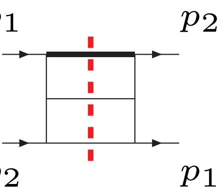
If this term expanded at $D=4$ dimension will give a soft divergence, It will be cancelled by the virtual singularity. Moreover when solving the differential equations and $z=1$, it should be extract before expanding in ϵ. Otherwise We cannot constrain the integral constant.

Example of solving Master8

- The differential equation of the Master8



- the solution of the Maser8



$$= z^3(1-z)^{-1-2\epsilon}\left[\frac{1}{\epsilon^3}\left(\frac{1}{2}-\frac{\delta(1-z)}{2}\right)+\frac{1}{\epsilon^2}\left(-H(0,z)-\frac{H(1,z)[1-z]_+}{2}\right)\right.$$

$$+\frac{1}{\epsilon}\left(\frac{3H(0,0,z)}{2}+\frac{H(0,1,z)}{2}+\frac{H(1,0,z)}{2}+\frac{H(1,1,z)[1-z]_+}{2}-\zeta(2)+\zeta(2)\delta(1-z)\right)$$

$$+\frac{15H(0,z)}{8}\zeta(2)-\frac{15H(0,0,0,z)}{8}-H(0,0,1,z)-\frac{9H(0,1,0,z)}{8}-\frac{H(0,1,1,z)}{2}$$

$$-\frac{H(0,1,1,z)}{2}[1-z]_- \frac{H(1,z)\zeta(2)}{8}-\frac{3H(1,0,0,z)}{8}-\frac{H(1,0,1,z)}{2}-\frac{5H(1,1,0,z)}{8}$$

$$\left.-\frac{H(1,1,1,z)[1-z]_+}{2}+\zeta(2)[1-z]_+\frac{13\zeta(2)}{8}-\frac{5\zeta(3)\delta(1-z)}{4}+\mathcal{O}\left((D-4)\right)\right]$$

Conclusions

- I summarized the importance of an accurate theoretical prediction for the Drell-Yan production of Z and W bosons at hadron colliders, and the status of the radiative corrections
- I reviewed the method for the analytic calculation of Feynman diagrams, based on the reduction to the Master Integrals via the "Laporta algorithm" and their calculation with the "differential equations method"
- I focused on the QCD-EW mixed radiative corrections. I summarized the calculation of the virtual corrections. I then focused on the calculation of the real radiation, needed to complete the NNLO corrections. I performed the reduction to the MIs using the method based on the Cutkowsky rules. I finished all the differential equation of Z boson production and am now working to the 3-cut solution of the corresponding differential equations of the W boson production.

Thank you for your attention