Compositeness of hadron resonances in chiral dynamics

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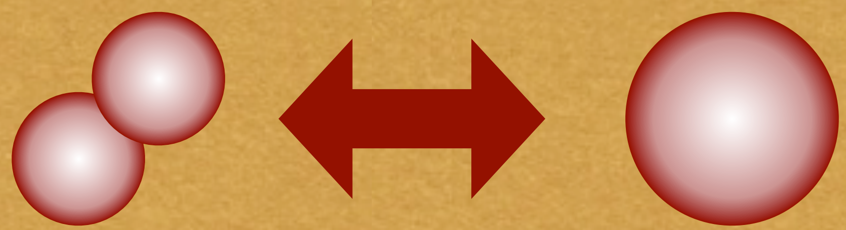
Contents

Structure of hadron resonances



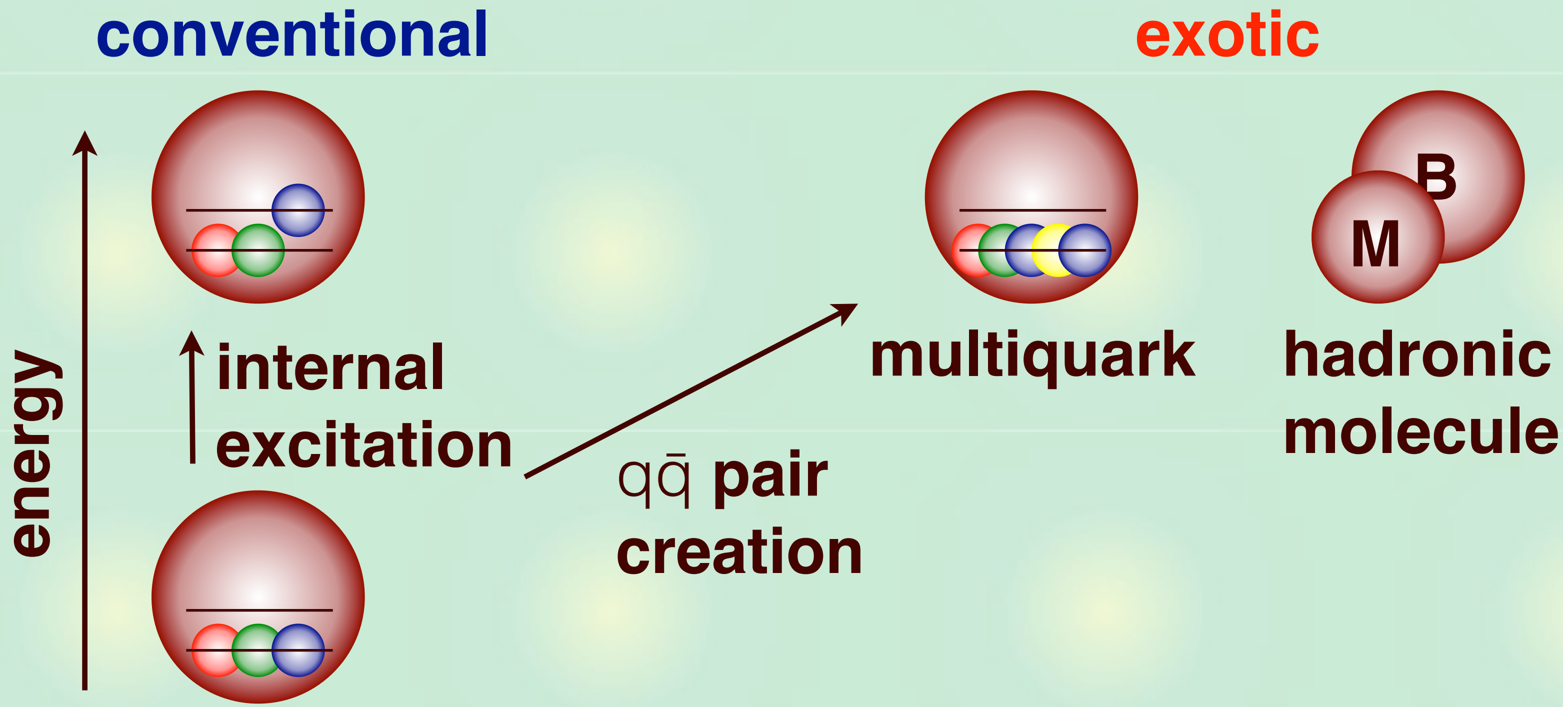
composite v.s. elementary

- **Field renormalization constant** Z
- **Negative effective range** r_e

T. Hyodo, Phy. Rev. Lett. 111, 132002 (2013)
T. Hyodo, arXiv:1310.1176 [hep-ph]

Exotic structure of hadrons

Various excitations of baryons



Physical state: superposition of 3q, 5q, MB, ...

$$|\,\Lambda(1405)\,\rangle = \underline{N_{3q}}|\,uds\,\rangle + \underline{N_{5q}}|\,uds\;q\bar{q}\,\rangle + \underline{N_{\bar{K}N}}|\,\bar{K}N\,\rangle + \cdots$$

Find out the dominant component among others.

Structure of resonances?

Excited states : finite width (unstable against strong decay)

- **stable (ground) states**
- **unstable states**

Most of hadrons are unstable!

p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1685)$		*
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	***
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	**
$N(1895)$	$1/2^-$	**
$N(1900)$	$3/2^+$	***
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	**
$N(2100)$	$1/2^+$	*
$N(2120)$	$3/2^-$	**
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	***
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	**
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	*
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	***
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$		*
$\Lambda(2020)$	$7/2^+$	*
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1480)$		*
$\Sigma(1560)$		**
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	**
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1690)$		**
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1770)$	$1/2^+$	*
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1840)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^-$	***
$\Sigma(2000)$	$1/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	**
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$		***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	***
$\Xi_c^{\prime +}$	$1/2^+$	***
$\Xi_c^{\prime 0}$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		*
$\Xi_c(2980)$		***
$\Xi_c(3055)$		**
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
Ξ_{cc}^+		*
Λ_b^0	$1/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
Ω_b^-	$1/2^+$	***

PDG12

State vector of resonance?

?

$$\boxed{|\,\Lambda(1405)\,\rangle} = N_{3q}|\,uds\,\rangle + N_{5q}|\,uds\ q\bar{q}\,\rangle + N_{\bar{K}N}|\,\bar{K}N\,\rangle + \cdots$$

We need a classification scheme applicable to resonances.

Compositeness of bound states

Compositeness approach for a bound state |B>

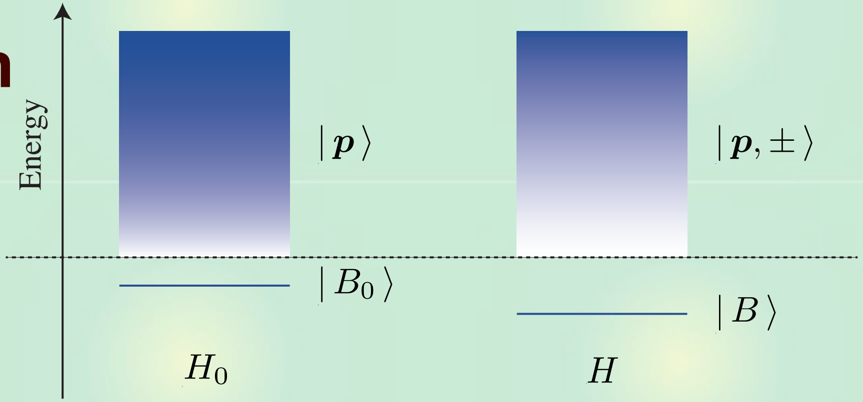
S. Weinberg, Phys. Rev. 137, B672 (1965); T. Hyodo, arXiv:1310.1176 [hep-ph]

$$H = H_0 + V \qquad H|B\rangle = -B|B\rangle, \quad \langle B|B\rangle = 1$$

Decompose H into free part + interaction part

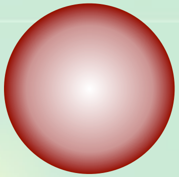
Complete set for free Hamiltonian : bare |B_0 > + continuum

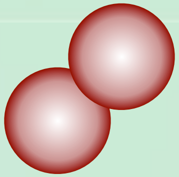
$$1 = |B_0\rangle\langle B_0| + \int d\boldsymbol{p}|\boldsymbol{p}\rangle\langle\boldsymbol{p}|$$



$$1 = \underbrace{\langle B|B_0\rangle\langle B_0|B\rangle}_{Z} + \int \underbrace{d\boldsymbol{p}\langle B|\boldsymbol{p}\rangle\langle\boldsymbol{p}|B\rangle}_{X}$$

Z : elementariness X : compositeness





Z, X : real and nonnegative --> probabilistic interpretation

$$\Rightarrow 0 \le Z \le 1, \quad 0 \le X \le 1$$

Weak binding limit

In general, Z depends on the choice of the potential V.

- Z : model-(scheme-)dependent quantity

$$1 - Z = \int d\boldsymbol{p} \frac{|\langle \boldsymbol{p} | V | B \rangle|^2}{(E_p + B)^2}$$

At the weak binding ($R \gg R_{\text{typ}}$), Z is related to observables.

S. Weinberg, Phys. Rev. 137, B672 (1965); T. Hyodo, arXiv:1310.1176 [hep-ph]

$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(R_{\text{typ}}), \quad r_e = \frac{-Z}{1-Z} R + \mathcal{O}(R_{\text{typ}}),$$

a **: scattering length,** r_e **: effective range**

$R = (2\mu B)^{-1/2}$ **: radius (binding energy)**

R_{typ} **: typical length scale of the interaction**

Criterion for the structure:

$$\begin{cases} a \sim R_{\text{typ}} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\text{typ}} & \text{(composite dominance)}. \end{cases}$$

Z ~ 1

Z ~ 0 **(deuteron)**

Interpretation of negative effective range

For $Z > 0$, effective range is always negative.

$$a = \frac{2(1-Z)}{2-Z}R + \mathcal{O}(R_{\mathrm{typ}}), \quad r_e = \frac{-Z}{1-Z}R + \mathcal{O}(R_{\mathrm{typ}}),$$

$$\begin{cases} a \sim R_{\mathrm{typ}} \ll -r_e & \text{(elementary dominance)}, \\ a \sim R \gg r_e \sim R_{\mathrm{typ}} & \text{(composite dominance)}. \end{cases}$$

Simple attractive potential: $r_e > 0$

--> only "composite dominance" is possible.

$r_e < 0$ **: energy- (momentum-)dependence of the potential**

D. Phillips, S. Beane, T.D. Cohen, Annals Phys. 264, 255 (1998)
E. Braaten, M. Kusunoki, D. Zhang, Annals Phys. 323, 1770 (2008)

<-- pole term/Feshbach projection of coupled-channel effect

Negative r_e --> Something other than $|p>$: CDD pole

Generalization to resonances

Compositeness approach of bound states

$$1 - Z = \int d\boldsymbol{p} \frac{|\langle \boldsymbol{p} | V | B \rangle|^2}{(E_p + B)^2}$$

Generalization to general resonances in chiral models

T. Hyodo, D. Jido, A. Hosaka, Phys. Rev. C85, 015201 (2012)
F. Aceti, E. Oset, Phys. Rev. D86, 014012 (2012)

$$1 - Z = -g^2 \left.\frac{dG(W)}{dW}\right|_{W \to M_B} \quad \to \quad 1 - Z = -g_{II}^2 \left.\frac{dG_{II}(W)}{dW}\right|_{W \to z_R}$$

- Z and X = 1 - Z are in general complex. Interpretation?

$$\langle R | R \rangle \to \infty, \quad \langle \tilde{R} | R \rangle = 1$$

$$1 = \underbrace{\langle \tilde{R} | B_0 \rangle \langle B_0 | R \rangle}_{\text{complex}} + \int d\boldsymbol{p} \langle \tilde{R} | \boldsymbol{p} \rangle \langle \boldsymbol{p} | R \rangle$$

$$\langle \tilde{R} | B_0 \rangle = \langle B_0 | R \rangle \neq \langle B_0 | R \rangle^*$$

Z of hadron resonances

Z can be calculated in chiral models

Table 1. Field renormalization constant Z of the hadron resonances evaluated on the resonance pole. The momentum cutoff $q_{\max}$ is chosen to be 1 GeV for the $\rho(770)$ and $K^*(892)$ mesons,[55, 59] 0.5 GeV for the $\Delta(1232)$ baryon, and 0.45 GeV for the $\Sigma(1385)$, $\Xi(1535)$, Ω baryons.[60]

Baryons	Z	$\lvert Z\rvert$	Mesons	Z	$\lvert Z\rvert$
$\Lambda(1405)$ higher pole[58]	$0.00+0.09i$	0.09	$f_0(500)$ or σ[58]	$\boxed{1.17-0.34i}$	$\boxed{1.22}$
$\Lambda(1405)$ lower pole[58]	$0.86-0.40i$	0.95	$f_0(980)$[58]	$0.25+0.10i$	0.27
$\Delta(1232)$[60]	$0.43+0.29i$	0.52	$a_0(980)$[58]	$0.68+0.18i$	0.70
$\Sigma(1385)$[60]	$0.74+0.19i$	0.77	$\rho(770)$[55]	$0.87+0.21i$	0.89
$\Xi(1535)$[60]	$\boxed{0.89+0.99i}$	$\boxed{1.33}$	$K^*(892)$[59]	$0.88+0.13i$	0.89
Ω[60]	0.74	0.74			
$\Lambda_c(2595)$[56]	$\boxed{1.00-0.61i}$	$\boxed{1.17}$			

F. Aceti, E. Oset, Phys. Rev. D86, 014012 (2012)
T. Sekihara, T. Hyodo, Phys. Rev. C87, 045202 (2013)
C.W. Xiao, F. Aceti, M. Bayar, Eur. Phys. J. A 49, 22 (2013)
F. Aceti, L. Dai, L. Geng, E. Oset, T, Zhang, arXiv:1301.2554 [hep-ph]

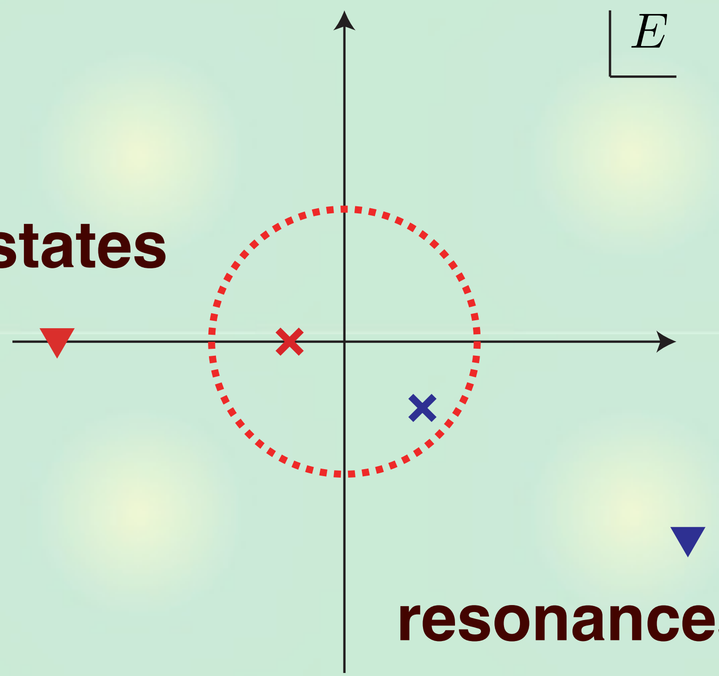
In some cases, Z and/or |Z| exceed unity. Interpretation?

Generalization to resonances

Compositeness approach at the weak binding:

- Model-independent (no potential, wavefunction, ...)
- Related to experimental observables
- Only for bound states with small binding

What about near-threshold resonances (~ small binding) ?



bound states

resonances : Z is complex

Poles of the amplitude

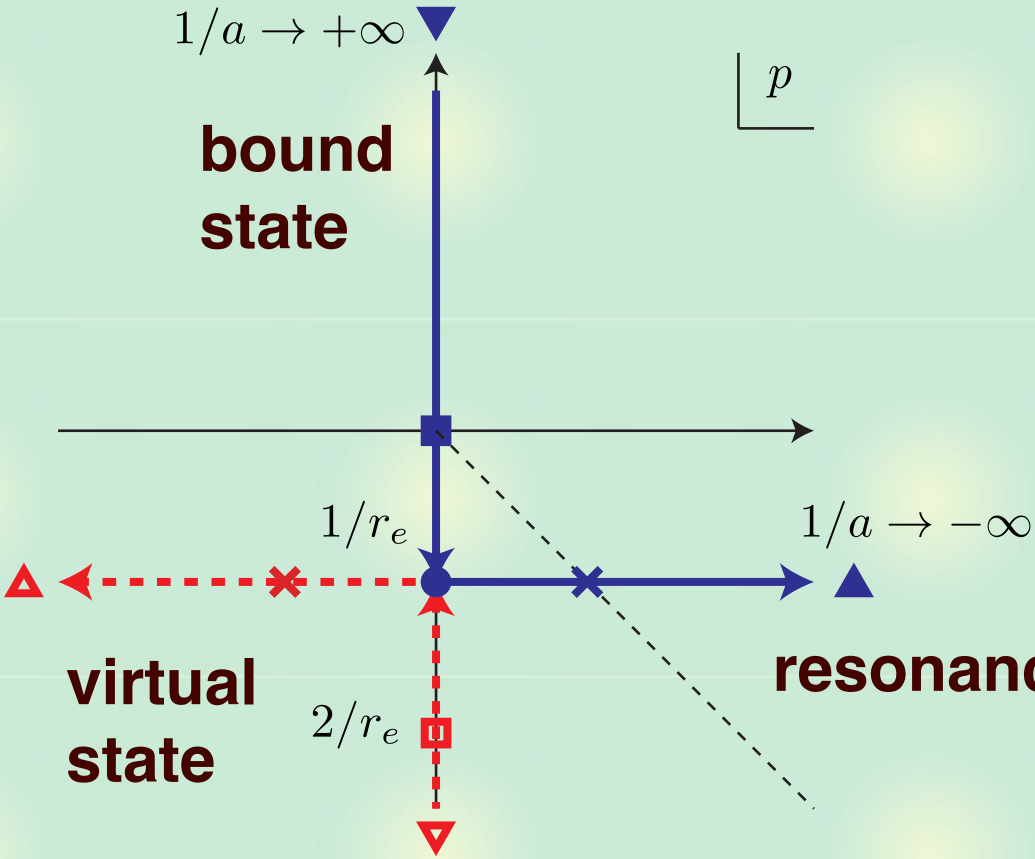
Near-threshold phenomena: effective range expansion

T. Hyodo, Phy. Rev. Lett. 111, 132002 (2013) with opposite sign of scattering length

$$f(p) = \left(-\frac{1}{a} - pi + \frac{r_e}{2} p^2 \right)^{-1}$$

$$p^{\pm} = \frac{i}{r_e} \pm \frac{1}{r_e} \sqrt{\frac{2r_e}{a} - 1}$$

Pole trajectories with a fixed $r_e < 0$



Resonance pole position <--> (a, r_e)

Example of resonance: $\Lambda_c(2595)$

Pole position of $\Lambda_c(2595)$ in $\pi\Sigma_c$ scattering

- central values in PDG

$$E = 0.67\ \mathrm{MeV}, \quad \Gamma = 2.59\ \mathrm{MeV}$$

- deduced threshold parameters

$$a = -\frac{p^+ + p^-}{ip^+p^-} = -10.5\ \mathrm{fm}, \quad r_e = \frac{2i}{p^+ + p^-} = -19.5\ \mathrm{fm}$$

- field renormalization constant: complex

$$Z = 1 - 0.608i$$

Large negative effective range

<-- substantial elementary contribution other than $\pi\Sigma_c$
(three-quark, other meson-baryon channel, or ...)

$\Lambda_c(2595)$ **is not likely a $\pi\Sigma_c$ molecule**

Summary

Composite/elementary nature of resonances

- Renormalization constant Z measures elementariness of a stable bound state.
- In general, Z of a resonance is complex.
- Negative effective range r_e : CDD pole
- Near-threshold resonance : pole position is related to r_e --> elementariness

T. Hyodo, Phy. Rev. Lett. 111, 132002 (2013)
T. Hyodo, arXiv:1310.1176 [hep-ph]