$\rho K\bar{K}$ system within the Framework of the Fixed Center Approximation to Faddeev equations

M. Bayar

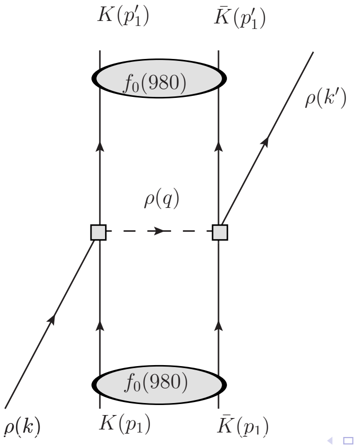
Physics Department
Kocaeli University

27 October, 2013
Collaborators: W. Liang, T. Uchino, C. W. Xiao
(work in progress)

Outline

- One of the important aims in the study of the strong interaction is to understand the nature and structure of hadronic resonances.
- The search for new resonances is goal both in theories and experiments
- Recently the rich spectrum of hadronic resonances is studied actively from various viewpoints.
- the $\rho K \bar{K}$ system with an aim to describe the $\rho(1700)$ resonance
- The Faddeev equations under the Fixed Center Approximation (FCA) is an effective tool to deal with multi-hadron interaction.

- A cluster of two bound particle ($K\bar{K}$ $(I = 0)$, $f_0(980)$)
- Third particle interacts with the cluster



Outline

1. Introduction
2. The $\rho K\bar{K}$ three-body scattering
   - Two body scattering
   - Three body scattering
3. Results
4. Conclusion

Two body cluster

$K\bar{K}$+coupled channel $\longrightarrow f_0(980)$

- J. A. Oller, E. Oset, Nuclear Physics A **620**,438 (1997)

$\rho K\,(\rho\bar{K})$ unitarized scattering amplitude

$\rho - (K\bar{K}) \longrightarrow$ one needs t for $\rho K\,(\rho\bar{K})$

• L. S. Geng, E. Oset, L. Roca and J. A. Oller, Phys. Rev. D **75**, 014017 (2007).
• L. Roca, E. Oset and J. Singh, Phys. Rev. D **72**, 014002 (2005).

The Bethe-Salpeter equation in coupled channels

$$T = [1 + V\hat{G}]^{-1}(-V)\vec{\epsilon}\cdot\vec{\epsilon}\,', \tag{1}$$

$\hat{G} = (1 + \frac{1}{3}\frac{q_l^2}{M_l^2})G$, The two meson loop function:

$$G_l(\sqrt{s}) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(P-q)^2 - M_l^2 + i\epsilon}\frac{1}{q^2 - m_l^2 + i\epsilon}, \tag{2}$$

the P is determined at the rest frame, $P = (\sqrt{s}, \vec{0})$

In the dimensional regularization scheme the loop function

$$\begin{aligned} G_l(\sqrt{s}) &= \frac{1}{16\pi^2}\Bigg\{a(\mu) + \ln\frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s}\ln\frac{m_l^2}{M_l^2} \\ &+ \frac{q_l}{\sqrt{s}}\Big[\ln(s - (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \\ &+ \ln(s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \\ &- \ln(-s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) \\ &- \ln(-s - (M_l^2 - m_l^2) + 2q_l\sqrt{s})\Big]\Bigg\}, \end{aligned} \quad (3)$$

q_l determined at the center of mass frame

$$q_l = \frac{\sqrt{[s - (M_l - m_l)^2][s - (M_l + m_l)^2)]}}{2\sqrt{s}}, \quad (4)$$

μ, a scale parameter in this scheme.$a(\mu)$,the subtraction constant

A finite width of the vector mesons in the loop function:

$$G_l(\sqrt{s}) = \frac{1}{C_l}\int_{(M_l-2\Gamma_l)^2}^{(M_l+2\Gamma_l)^2} ds_V G_l(\sqrt{s},\sqrt{s_V},m_l) \times \left(-\frac{1}{\pi}\right) \mathrm{Im}\left\{\frac{1}{s_V - M_l^2 + iM_l\Gamma_l}\right\}, \quad (5)$$

with the normalization for the lth component

$$C_l = \int_{(M_l-2\Gamma_l)^2}^{(M_l+2\Gamma_l)^2} ds_V \times \left(-\frac{1}{\pi}\right) \mathrm{Im}\left\{\frac{1}{s_V - M_l^2 + iM_l\Gamma_l}\right\}, \quad (6)$$

Outline

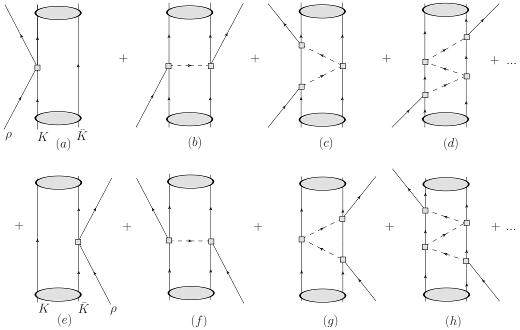


Figure: Diagrammatic representation of the fixed center approach.

- T1: all diagrams beginning with interaction in K meson.
- T2: all diagrams beginning with interaction in $\bar{K}$ meson.

The FCA to Faddeev equations for the three body interaction system

$$
\begin{aligned}
T_1 &= t_1 + t_1 G_0 T_2, \\
T_2 &= t_2 + t_2 G_0 T_1, \\
T &= T_1 + T_2,
\end{aligned} \tag{7}
$$

For the normalization

The S-matrix for the single scattering which corresponds to (a) or (e) in Fig. 1

$$
\begin{aligned}
S^{(1)} &= -i(2\pi)^4\delta^4(k_\rho + k_{f_0} - k_\rho^{'} - k_{f_0}^{'}) \\
&\times \frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_\rho}}\frac{1}{\sqrt{2\omega_\rho^{'}}}\frac{1}{\sqrt{2\omega_K}}\frac{1}{\sqrt{2\omega_K^{'}}}t, \qquad (8)
\end{aligned}
$$

S-matrix for the double scattering, (b) or (f) in Fig. 1

$$
\begin{aligned}
S^{(2)} &= -i(2\pi)^4\delta^4(k_\rho + k_{f_0} - k_\rho^{'} - k_{f_0}^{'}) \\
&\times \frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_\rho}}\frac{1}{\sqrt{2\omega_\rho^{'}}}\frac{1}{\sqrt{2\omega_K}}\frac{1}{\sqrt{2\omega_K^{'}}}\frac{1}{\sqrt{2\omega_K}}\frac{1}{\sqrt{2\omega_K^{'}}} \\
&\times \int \frac{d^3q}{(2\pi)^3}F_{f_0}(q)\frac{1}{q^{02} - \vec{q}^2 - m_\rho^2 + i\epsilon}tt, \qquad (9)
\end{aligned}
$$

The full three-body scattering is given by

$$\begin{aligned} S &= -i(2\pi)^4\delta^4(k_\rho + k_{f_0} - k_\rho^{'} - k_{f_0}^{'}) \\ &= \frac{1}{\mathcal{V}^2}\frac{1}{\sqrt{2\omega_\rho}}\frac{1}{\sqrt{2\omega_\rho^{'}}}\frac{1}{\sqrt{2\omega_{f_0}}}\frac{1}{\sqrt{2\omega_{f_0}^{'}}}T. \end{aligned} \tag{10}$$

Using the low energy reduction, $\sqrt{2\omega} \sim \sqrt{2m}$, we have

$$\tilde{t} = \frac{2m_{f_0}}{2m_K}t. \tag{11}$$

Finally

$$T(s) = 2\frac{\tilde{t}(s')}{1 - \tilde{t}(s')G_0(s)}, \tag{12}$$

$$s' = \frac{1}{2}(s + M_\rho^2 + 2m_K^2 - M_{f_0}^2). \tag{13}$$

The G_0 function

$$G_0(\sqrt{s}) = \frac{1}{2M_{f_0}} \int \frac{d^3q}{(2\pi)^3} F_{f_0}(q) \frac{1}{q^{02}(s) - \vec{q}^2 - m_\rho^2 + i\epsilon}, \tag{14}$$

The energy of the propagator q^0 is determined at the three-body rest frame

$$q^0(\sqrt{s}) = \frac{s + m_\rho^2 - M_{f_0}^2}{2\sqrt{s}}. \tag{15}$$

The form factor

$$F_{f_0}(q) = \frac{1}{\mathcal{N}} \int_{\substack{p<k_{\max} \\ |\vec{p}-\vec{q}|<k_{\max}}} d^3p \left(\frac{1}{2\omega_K(\vec{p})}\right)^2 \frac{1}{M_{f_0} - 2\omega_K(\vec{p})} \times \left(\frac{1}{2\omega_K(\vec{p}-\vec{q})}\right)^2 \frac{1}{M_{f_0} - 2\omega_K(\vec{p}-\vec{q})}, \quad (16)$$

where the normalization $\mathcal{N}$ is given by

$$\mathcal{N} = \int_{p<k_{\max}} d^3p \left[\left(\frac{1}{2\omega_K(\vec{p})}\right)^2 \frac{1}{M_{f_0} - 2\omega_K(\vec{p})}\right]^2 . \quad (17)$$

From J. A Oller, E. Oset(Nucl. Phys A, 620(1997),438-456.) we take $k_{\max} = \sqrt{\Lambda^2 - m_K^2}$ and $\Lambda = 1030$ MeV for getting the $f_0(980)$ from the $K\bar{K}$ cluster.

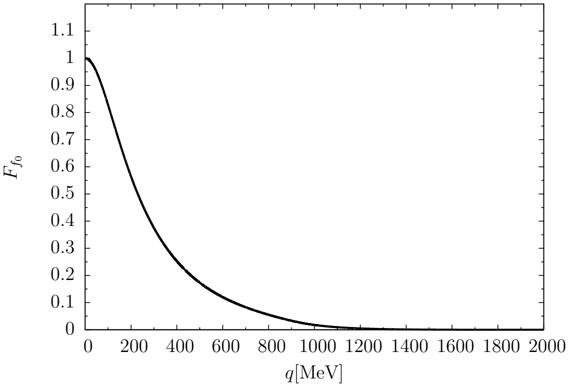


Figure: Form factor for the $K\bar{K}$ cluster.

$$\mu = 900 \text{ MeV}, \; a(\mu) = -1.85, \; f = 115 \text{ MeV}. \qquad (18)$$

L. S. Geng,E. Oset,L. Roca, J. A. Oller, Phys. Rev. D **75**, 014017 (2007)

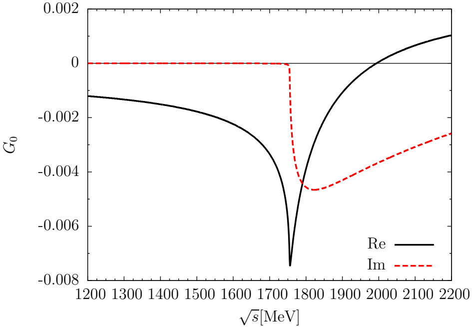


Figure: G_0 function.

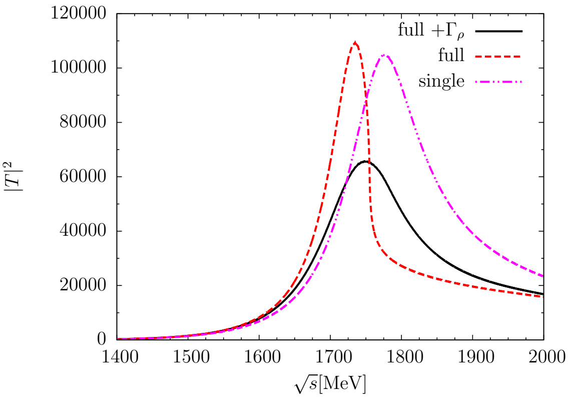


Figure: The $\rho K\bar{K}$ amplitude where “single", “full", “full + Γ_ρ" denote the amplitude of the single scattering, full scattering, full scattering with the ρ width effect respectively.

$$G_0(\sqrt{s}) = \frac{1}{2M_{f_0}} \int \frac{d^3q}{(2\pi)^3} F_{f_0}(q) \frac{1}{q^{02}(s) - \vec{q}^{\,2} - m_\rho^2 + i\epsilon}, \tag{19}$$

$$\frac{1}{q^{02} - \vec{q}^{\,2} - m_\rho^2 + i\epsilon} \rightarrow \frac{1}{q^{02} - \vec{q}^{\,2} - m_\rho^2 + i m_\rho \Gamma_\rho}, \tag{20}$$

$\Gamma_\rho \sim 150$ MeV

Table: The masses and widths of dynamically generated states. The amplitudes are given by the single scattering, the full scattering and the full scattering with the ρ width effect respectively.

	single	full	full + Γ_ρ	PDG
Mass(MeV)	1777.9	1734.8	1748.0	1720 ± 20
Width(MeV)	144.4	63.7	160.8	250 ± 100

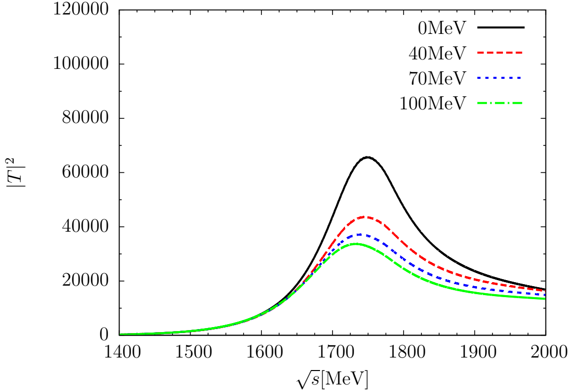


Figure: The $\rho K\bar{K}$ amplitude with the ρ and $f_0(980)$ width effect.

Table: The masses and widths of dynamically generated states with the ρ and $f_0(980)$ width effects.

	$\Gamma_{f_0} = 0$	$\Gamma_{f_0} = 40$	$\Gamma_{f_0} = 70$	$\Gamma_{f_0} = 100$ (MeV)
Mass(MeV)	1748.0	1743.6	1739.2	1734.8
Width(MeV)	160.8	216.4	227.2	224.6

$\Rightarrow$ The mass of the cluster M_{f_0} in Eqs. (16) and (17) is replaced by $M_{f_0} - i\Gamma_{f_0}/2$

- construct the $\rho K\bar{K}$ three-body amplitude by means of the fixed center approach
- a peak at the energy around 1748 MeV rather independent of the width of the $f_0(980)$
- the inclusion of the ρ and $f_0(980)$ width makes the peak wider
- the building block of the $\rho(1700)$ resonance are the ρ and $f_0(980)$

THANK YOU