Scattering lengths of Nambu-Goldstone bosons off D mesons and dynamically generated heavy-light mesons

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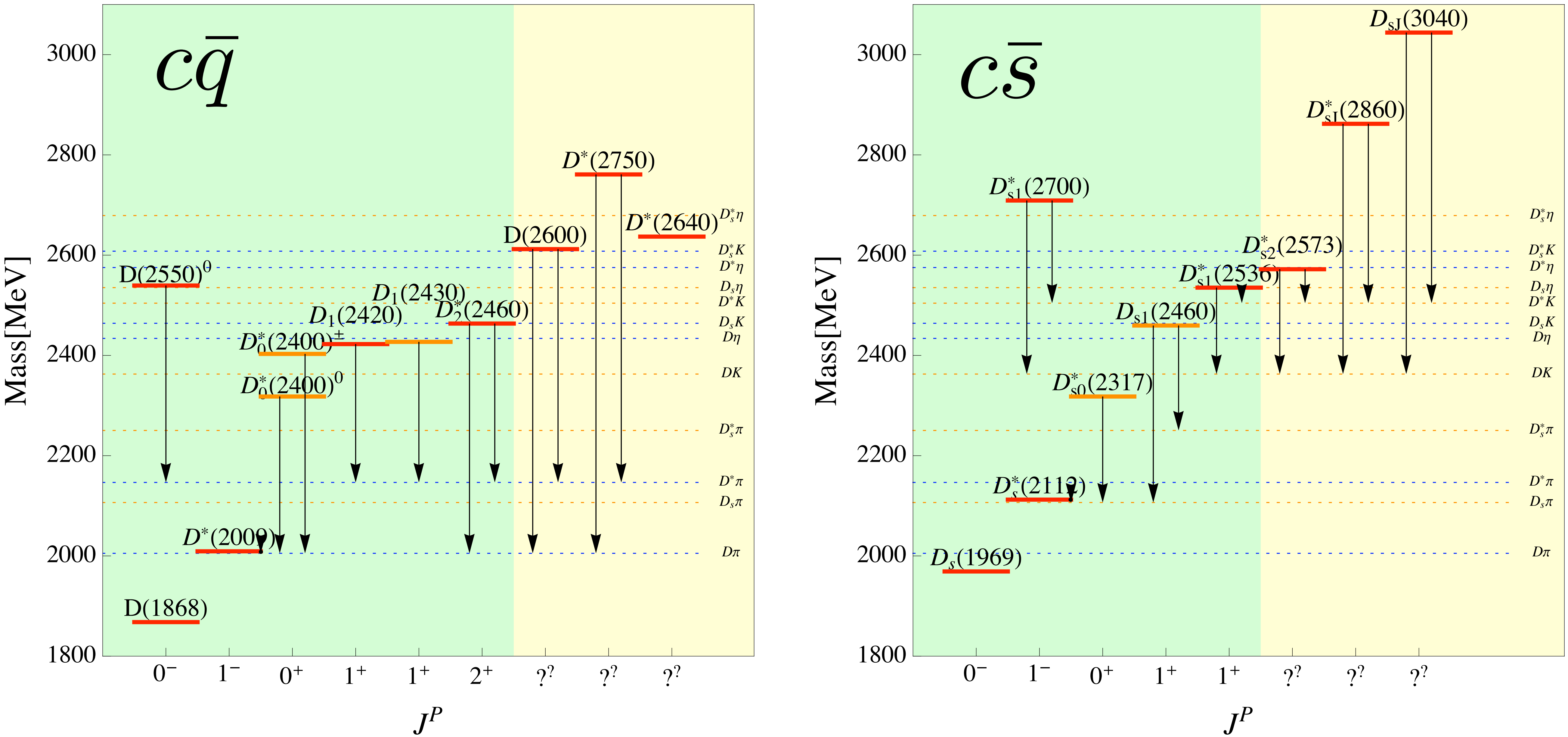
Excited $D(c\bar{q})$ and $D_s(c\bar{s})$ mesons



- $D_{s0}^*(2317)$ and $D_{s1}(2460)$ (BaBar, confirmed by CLEO)
  (PRL 90, 242001 (2003), PRD 68, 032002 (2003))
- $D_{s1}(2710)$ and $D_{sJ}(2860)$ (Belle, confirmed by BaBar)
  (PRL 100, 092001 (2008), PRL 80, 092001 (2009))
- $D_{sJ}(3040)$ (BaBar)
  (PRL 80, 092001 (2009))
- Various excited states: $D_0^*(2400)$, $D_1(2430)$, $D_1(2420)$, $D_2^*(2460)$, $D(2610)$, $D(2760)$, $D(2550)$, ...
  (Summarized in PRD 86, 010001 (2012))

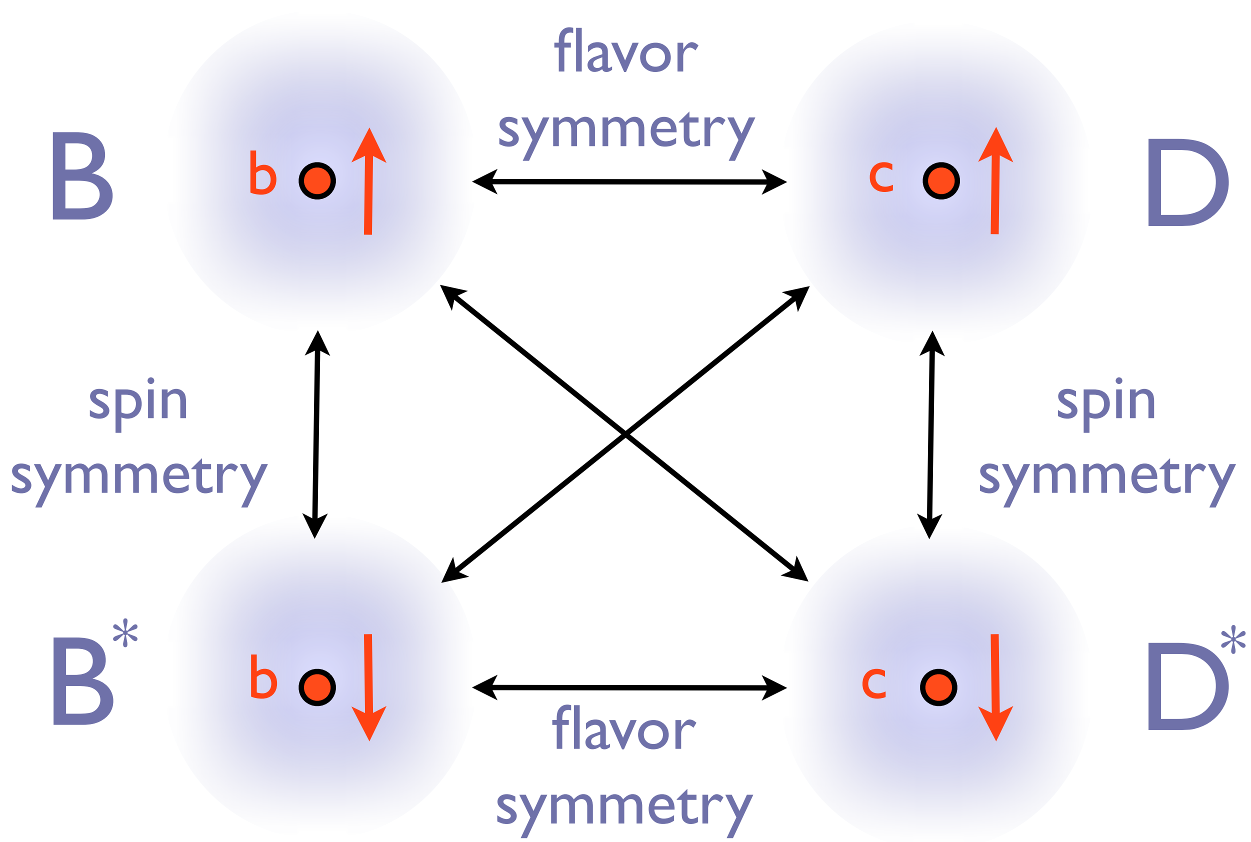
Symmetries in heavy-light mesons ($Q\bar{q}$)

- Two hadrons, each containing a single heavy quark:

$m_{Q_i},\ m_{Q_j} \gg \Lambda_{QCD}$

heavy quark $\mathbf{Q_i}$ acts as a static color source

- Consequence: $\mathbf{SU(2)_{HF}}$ **flavor and** $\mathbf{SU(2)}$ **spin symmetry**



Chiral effective Lagrangian I/II

Lowest order chiral Lagrangian $\mathcal{L}^{(1)} = \mathcal{L}_A^{(1)} + \mathcal{L}_B^{(1)}$**:**

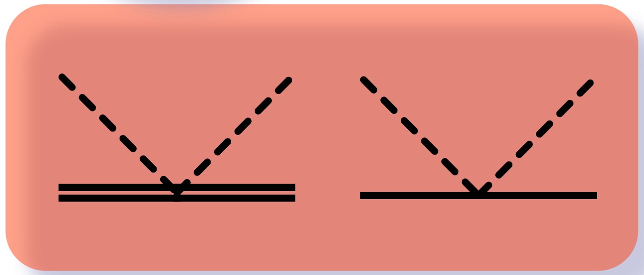
$$\mathcal{L}_A^{(1)} = \mathcal{D}_\mu P \mathcal{D}^\mu P^\dagger - m_P^2 P P^\dagger + \mathcal{D}_\mu P^{*\nu} \mathcal{D}^\mu P_\nu^{*\dagger} - m_{P^*}^2 P^{*\nu} P_\nu^{*\dagger}$$

$P = (D^0, D^+, D_s^+)$ and $P_\mu^* = (D^{*0}, D^{*+}, D_s^{*+})_\mu$

- **Covariant derivative:**

$$\mathcal{D}_\mu P_a = \partial_\mu P_a - \Gamma_\mu^{ba} P_b \qquad \mathcal{D}^\mu P_a^\dagger = \partial^\mu P_a^\dagger + \Gamma_{ab}^\mu P_b^\dagger$$

$\rightarrow$LO interaction with **Nambu-Goldstone bosons ϕ_i**



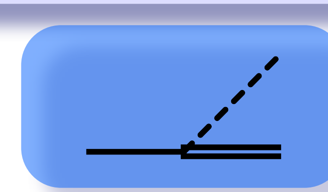
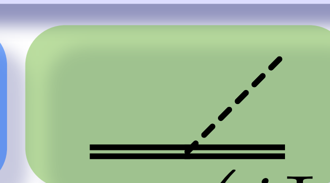
$$\Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger), \qquad u^2 = U = \exp\left(\frac{i\Phi}{f_0}\right)$$

$$\Phi = \Lambda_{\,i}\phi_i = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

Chiral effective Lagrangian II/II

Lowest order chiral Lagrangian $\mathcal{L}^{(1)} = \mathcal{L}_A^{(1)} + \mathcal{L}_B^{(1)}$:

$$\mathcal{L}_B^{(1)} = ig(P_\mu^* u^\mu P^\dagger - P u^\mu P_\mu^{*\dagger}) + \frac{g}{m_P}(P_\mu^* u_\alpha \partial_\beta P_\nu^{*\dagger} - \partial_\beta P_\mu^* u_\alpha P_\nu^{*\dagger})\epsilon^{\mu\nu\alpha\beta}$$

- **axial-vector current u_μ:**

$$u_\mu = i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger), \qquad u^2 = U = \exp\left(\frac{i\Phi}{f_0}\right)$$

- g fixed by

$$\left.\begin{aligned} \Gamma_{D^{*+}\to D^0\pi^+} &= 65 \pm 15\,\mathrm{keV} \\ BR_{D^{*+}\to D^0\pi^+} &= (67.7 \pm 0.5)\% \end{aligned}\right\} \Rightarrow g = 1177 \pm 137\,\mathrm{MeV}$$

fixed: $c_1 = -0.214$, $c_0 = 0.015$

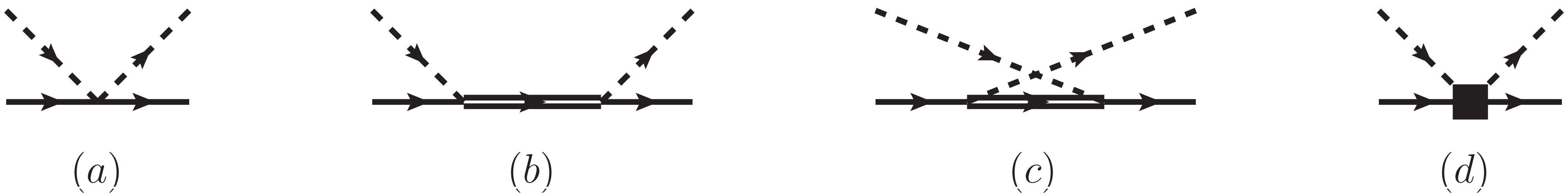
Next-to-leading order chiral Lagrangian:

$$\begin{aligned} \mathcal{L}^{(2)} &= -2\Big(c_0 PP^\dagger\langle\chi_+\rangle - c_1 P\chi_+ P^\dagger - c_2 PP^\dagger\langle u^\mu u_\mu\rangle \\ &\quad - c_3 P u^\mu u_\mu P^\dagger + c_4 \mathcal{D}_\mu P \mathcal{D}_\nu P^\dagger \langle\{u^\mu, u^\nu\}\rangle \\ &\quad + c_5 \mathcal{D}_\mu P\{u^\mu, u^\nu\}\mathcal{D}_\nu P^\dagger + c_6 \cancel{\mathcal{D}_\mu P[u^\mu, u^\nu]\mathcal{D}_\nu P^\dagger}\Big) \end{aligned}$$

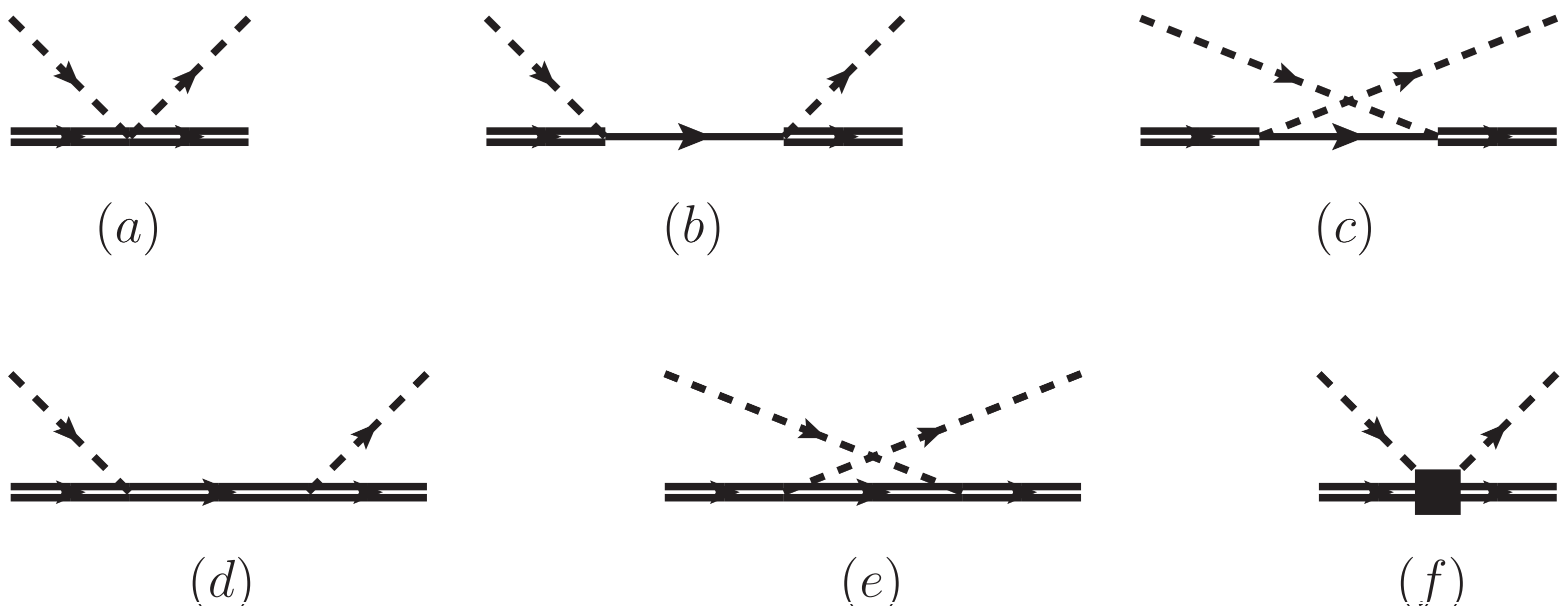
suppressed in 1/M

Potentials

- Contributions to the $D\phi \to D\phi$ potential up to NLO:



- Contributions to the $D^*\phi \to D^*\phi$ potential up to NLO:



Bethe-Salpeter equation and renormalization scheme motivated by heavy-quark symmetry

- The BS-equation symbolically

$$T = V + VGT$$

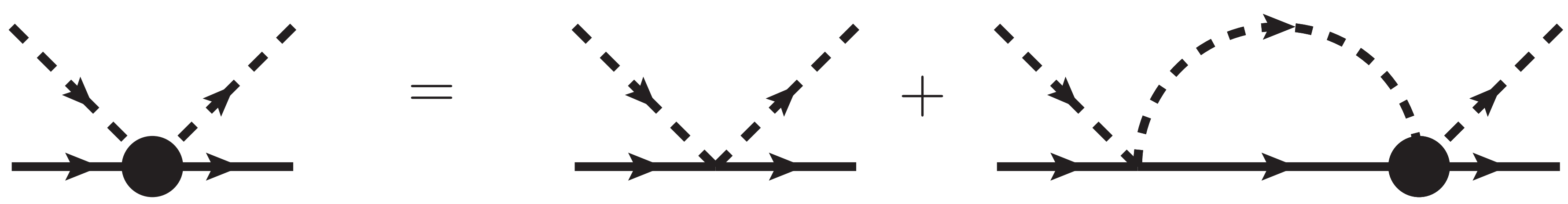


Figure: Diagrammatic representation of the BS equation

- loop function

$$\begin{aligned}
G(s, M^2, m^2) &\equiv i\int \frac{d^nq}{(2\pi)^n}\frac{1}{[(P-q)^2 - m^2 + i\epsilon][q^2 - M^2 + i\epsilon]} \\
&= \frac{1}{16\pi^2}\Bigg\{\frac{m^2 - M^2 + s}{2s}\log\left(\frac{m^2}{M^2}\right) \\
&\quad - \frac{q}{\sqrt{s}}\{\log[2q\sqrt{s} + m^2 - M^2 - s] + \log[2q\sqrt{s} - m^2 + M^2 - s] \\
&\quad - \log[2q\sqrt{s} + m^2 - M^2 + s] - \log[2q\sqrt{s} - m^2 + M^2 + s]\} \\
&\quad + \underline{\left(\log\left(\frac{M^2}{\mu^2}\right) - 2\right)}\Bigg\}
\end{aligned}$$

usually replaced by subtraction constant a
$\longrightarrow$ new parameter to be fixed

Bethe-Salpeter equation and renormalization scheme motivated by heavy-quark symmetry

- **The static limit $m_Q \to \infty$:** Loop function in HMChPT

$$G_{\text{HM}}(s, M^2, m^2) = \frac{1}{16\pi^2 \mathring{M}} \left\{ 2\sqrt{\Delta_{\text{HM}}^2 - m^2} \left(\operatorname{arccosh}\left(\frac{\Delta_{\text{HM}}}{m}\right) - \pi i \right) + \Delta_{\text{HM}} \left(\log\left(\frac{m^2}{\mu^2}\right) + a \right) \right\}$$

- Expand relativistic loop function:

$$\Delta_{\text{HM}} = \sqrt{s} - M$$

$$G(s, M^2, m^2) = \frac{1}{16\pi^2} \left(\log\left(\frac{\mathring{M}^2}{\mu^2}\right) - 2 \right) + \frac{1}{16\pi^2 \mathring{M}} \left\{ 2\sqrt{\Delta_{\text{HM}}^2 - m^2} \left(\operatorname{arccosh}\left(\frac{\Delta_{\text{HM}}}{m}\right) - \pi i \right) + \Delta_{\text{HM}} \log\left(\frac{m^2}{\mathring{M}^2}\right) \right\} + \ldots$$

- Renormalization prescription motivated by heavy-quark-symmetry:

$$G_{\text{HQS}}(s, M^2, m^2) \equiv G(s, M^2, m^2) - \underbrace{\frac{1}{16\pi^2} \left(\log\left(\frac{\mathring{M}^2}{\mu^2}\right) - 2 \right)}_{\text{Power-counting-breaking term}} + \frac{m_{\text{sub}}}{16\pi^2 \mathring{M}} \left(\log\left(\frac{\mathring{M}^2}{\mu^2}\right) + a \right)$$

$\Delta_{\text{HM}} = m \equiv m_{\text{sub}}$ fixed at threshold

Bethe-Salpeter equation and renormalization scheme motivated by heavy-quark symmetry

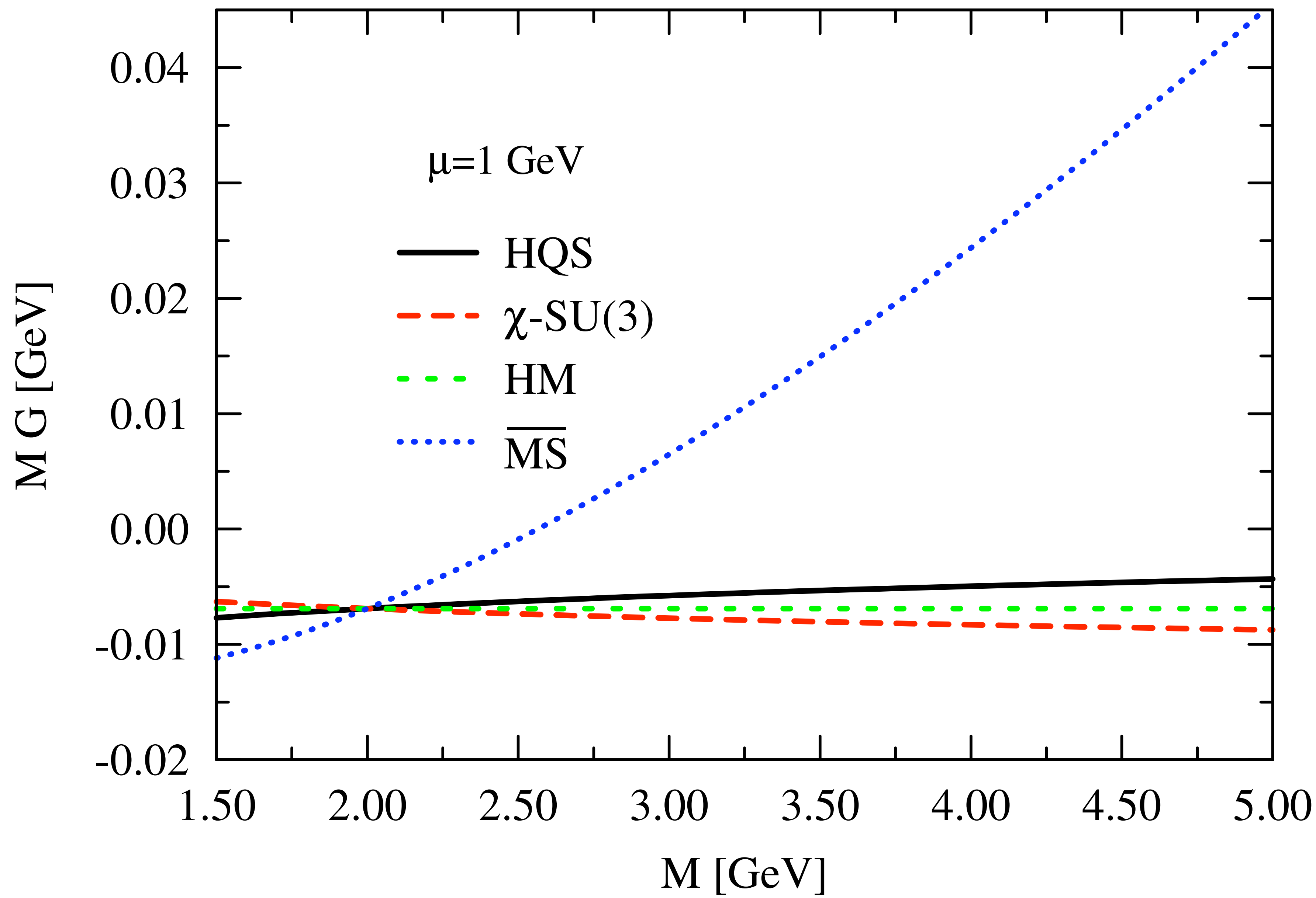
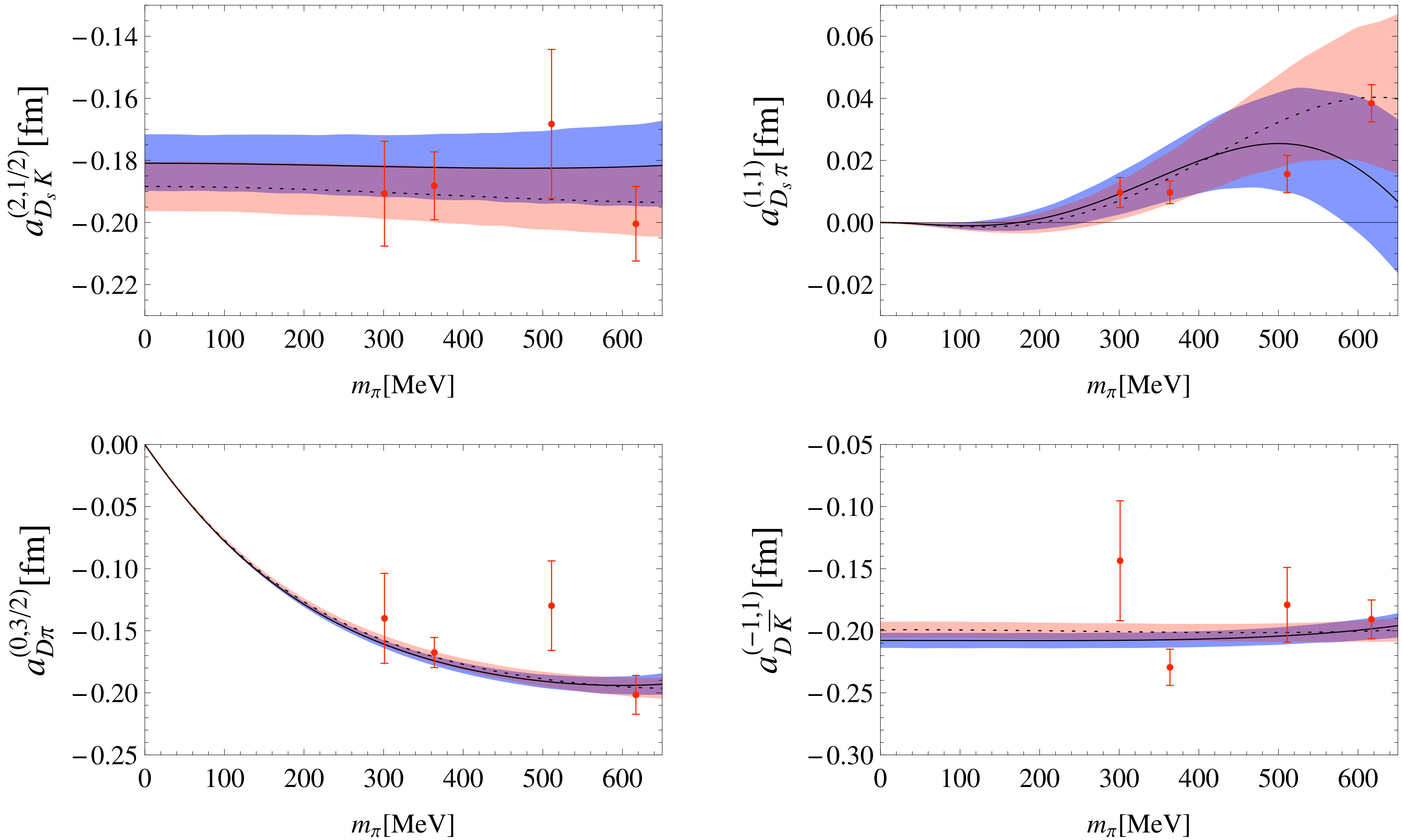
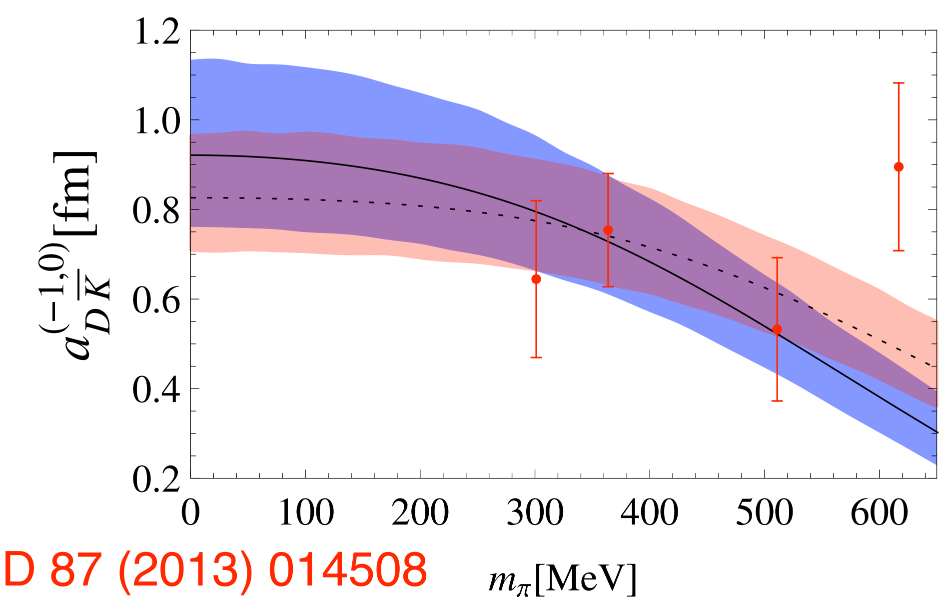


Figure: Dependence of loop functions (at threshold) on the heavy-light meson mass in different schemes with $\mu = 1$ GeV, $\mathring{M} = M$, $m = m_\pi = 0.138$ GeV, $\sqrt{s} = M + m$, and $m_{\text{sub}} = 0.3074$ GeV.

Fits to Lattice QCD



Blue bands and black solid lines: parameters fitted to the lowest 15 LQCD points



Red bands and black dashed lines: parameters fitted to all LQCD points

Red lattice points by [1] Liu et al., PRD 87 (2013) 014508

Fits to Lattice QCD within different approaches

Table: Low-energy constants and $\chi^2/\mathrm{d.o.f}$ from the best fits to LQCD[1] in perturbative ChPT.

	$c_2 - 2c_4$	$c_3 - 2c_5$	c_4	c_5	$\chi^2/\mathrm{d.o.f}$
Covariant ChPT	0.153(35)	$-0.126(71)$	0.760(186)	$-1.84(39)$	2.01
HM ChPT	0.012(6)	$0.167(17)$	-	-	3.10

Table: Low-energy constants, subtraction constants, and the $\chi^2/\mathrm{d.o.f}$ from the best fits to LQCD[1] in different approaches of unitarized ChPT

	a	$c_2 - 2c_4$	$c_3 - 2c_5$	c_4	c_5	$\chi^2/\mathrm{d.o.f}$
HQS UChPT	$-4.13(40)$	$-0.068(21)$	$-0.011(31)$	$0.052(83)$	$-0.96(30)$	1.23
χ-SU(3) UChPT	-	$-0.096(19)$	$-0.0037(340)$	$0.22(8)$	$-0.53(21)$	1.57
HM UChPT	2.52 (11)	4.86(30)	$-9.45(60)$	-	-	2.69

[1] Liu et al., PRD 87 (2013) 014508

Predicted pole positions

Table: Pole positions $\sqrt{s} = M - i\frac{\Gamma}{2}$ (in units of MeV) of charm mesons dynamically generated in the HQS UChPT.

(S,I)	$J^P = 0^+$	$J^P = 1^+$
(1,0)	2317 ± 10	2457 ± 17
(0,1/2)	$(2105 \pm 4) - i(103 \pm 7)$	$(2248 \pm 6) - i(106 \pm 13)$

identified with $D_{s0}^*(2317)$ and $D_{s1}(2460)$

Table: Dynamically generated 0^+ and 1^+ bottom states in $(S, I) = (1, 0)$ from different formulations of the UChPT (in units of MeV).

J^P	present work	NLO HMChPT[1]	LO UChPT[2]	LO χ-SU(3)[3]
0^+	5726 ± 28	5696 ± 36	5725 ± 39	5643
1^+	5778 ± 26	5742 ± 36	5778 ± 7	5690

[1] M. Cleven et al., EPJA 47, 19 (2011)
[2] F. K. Guo et al., PLB 641, 278 (2006)
[3] E. E. Kolomeitsev and M. F. M. Lutz, PLB 582, 39 (2004)

Summary and conclusions

- (U)ChPT describes the LQCD data better than its non-relativistic (heavy-meson) counterpart

- $D^*_{s0}(2317)$ can be dynamically generated without a priori assumption of its existence

  c.f. Liu et al., PRD 87 (2013) 014508
  and P. Wang and X. G. Wang, PRD 86, 014030 (2012)

- A heavy-quark-symmetry motivated renormalization prescription

  $\longrightarrow$ predictions for the $D^*_{s0}(2317)$ counterparts in the $J^P = 1^+$ sector and in the bottom sector