

# Recent developments on LQCD studies of nuclear force

H. Nemura<sup>1</sup>,

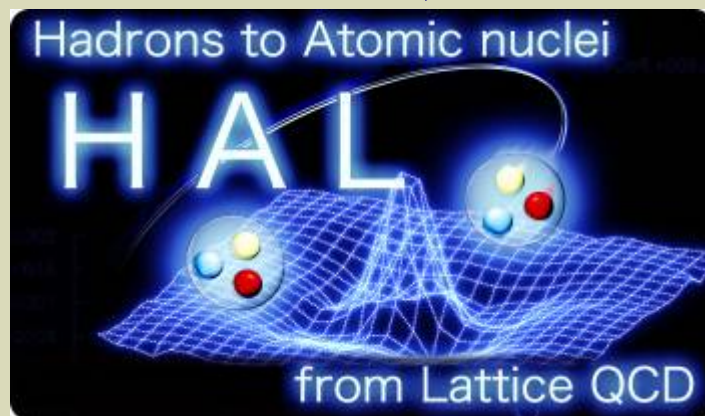
<sup>1</sup>*Center for Computational Science, University of Tsukuba, Japan*

# Recent developments on LQCD studies of nuclear force

H. Nemura<sup>1</sup>,

for HAL QCD Collaboration

S. Aoki<sup>2</sup>, B. Charron<sup>3</sup>, T. Doi<sup>4</sup>, F. Etminan<sup>1</sup>,  
T. Hatsuda<sup>4</sup>, Y. Ikeda<sup>4</sup>, T. Inoue<sup>5</sup>, N. Ishii<sup>1</sup>,  
K. Murano<sup>2</sup>, K. Sasaki<sup>1</sup>, and M. Yamada<sup>1</sup>,



<sup>1</sup>*Center for Computational Science, University of Tsukuba, Japan*

<sup>2</sup>*Yukawa Institute for Theoretical Physics, Kyoto University, Japan*

<sup>3</sup>*Department of Physics, University of Tokyo, Japan*

<sup>4</sup>*Theoretical Research Division, Nishina Center RIKEN, Japan*

<sup>5</sup>*College of Bioresource Science, Nihon University, Japan*

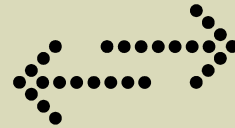
<sup>6</sup>*Strangeness Nuclear Physics, Nishina Center RIKEN, Japan*

# Plan of research

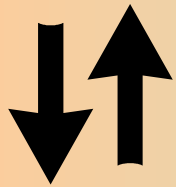
QCD



Baryon interaction



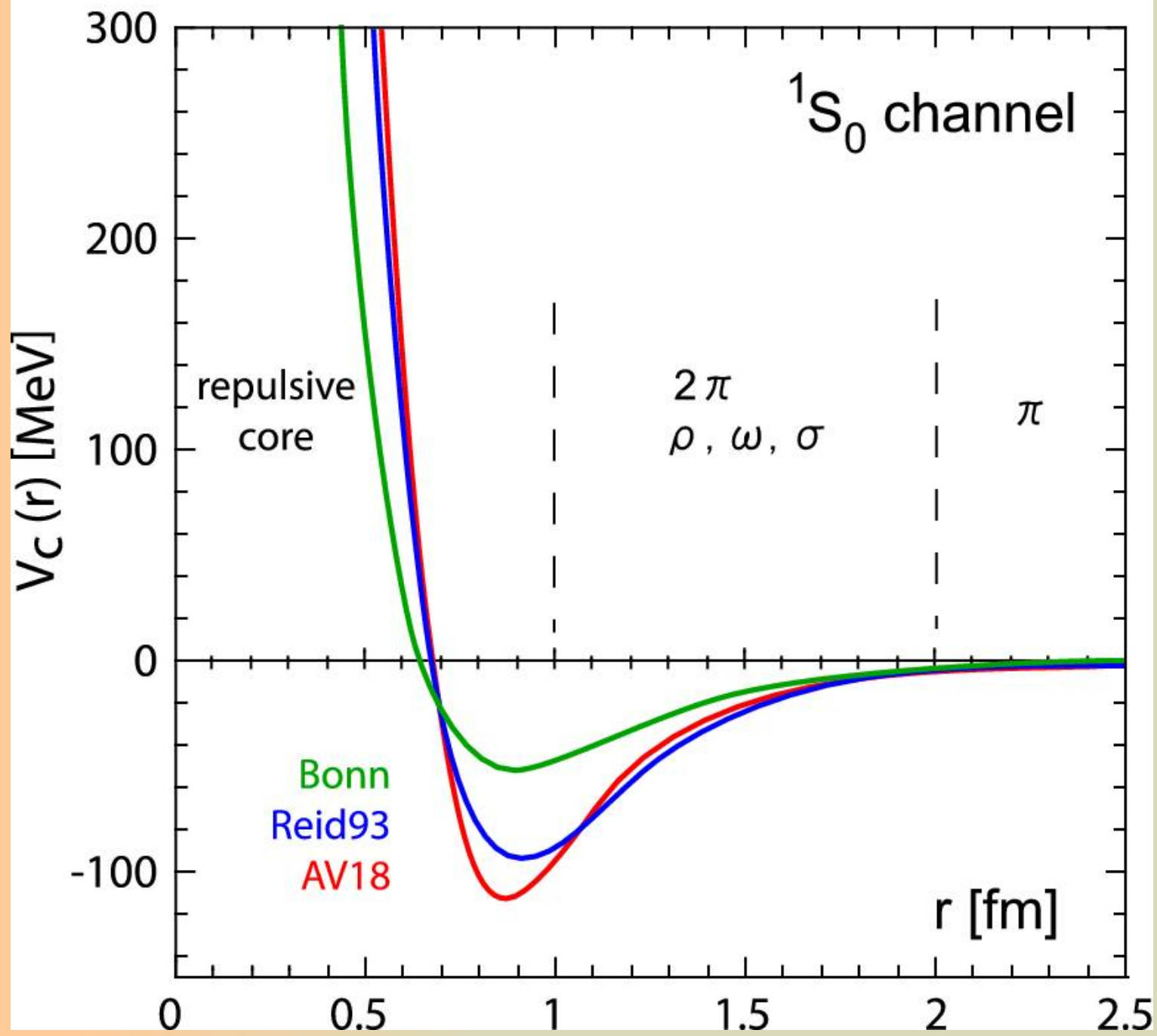
J-PARC  
hyperon-nucleon (YN)  
scattering



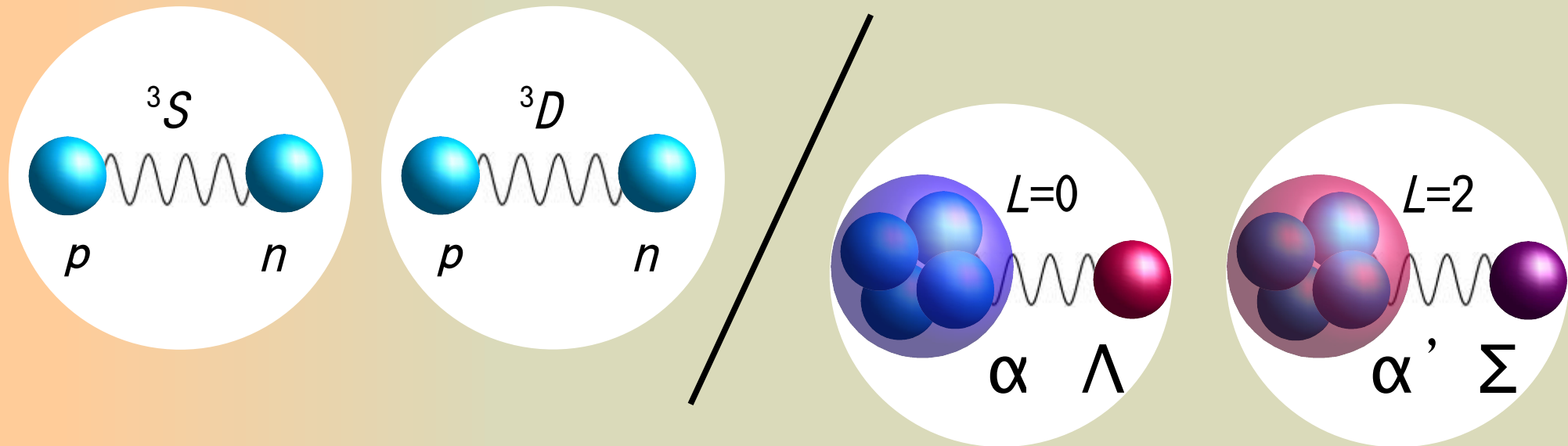
Structure and reaction of  
(hyper)nuclei

Equation of State (EoS)  
of nuclear matter

Neutron star and  
supernova



# Comparison between $d=p+n$ and $\text{core}+Y$



	$\langle T_S \rangle$ (MeV)	$\langle T_D \rangle$ (MeV)	$\langle V_{NN}(\text{central}) \rangle$ (MeV)	$\langle V_{NN}(\text{tensor}) \rangle$ (MeV)	$\langle V_{NN}(\text{LS}) \rangle$ (MeV)
AV8	8.57	11.31	-4.46	-16.64	-1.02
G3RS	10.84	5.64	-7.29	-11.46	0.00
	$\langle T_{Y-c} \rangle_\Lambda$	$\langle T_{Y-c} \rangle_\Sigma + \Delta \langle H_C \rangle$	$\langle V_{YN}(\text{のこり}) \rangle$	$2\langle V_{\Lambda N-\Sigma N}(\text{tensor}) \rangle$	
$^5_\Lambda\text{He}$	9.11	3.88+4.68	-0.86	-19.51	
$^4_\Lambda\text{H}^*$	5.30	2.43+2.02	0.01	-10.67	
$^4_\Lambda\text{H}$	7.12	2.94+2.16	-5.05	-9.22	

# Out line

- ⊗ Introduction
- ⊗ Formulation --- potential (central + tensor)
- ⊗ Numerical results:
  - ⊗  $M\Lambda$  force (  $V_C + V_T$  )
  - ⊗  $M\Sigma$  (I=3/2) force (  $V_C + V_T$  )
- ⊗ Recent improvement for  $V_C$  and  $V_T$
- ⊗ Stochastic variational calculation of  $4\text{He}$  with using a lattice potential
- ⊗ Summary and outlook

# Introduction:

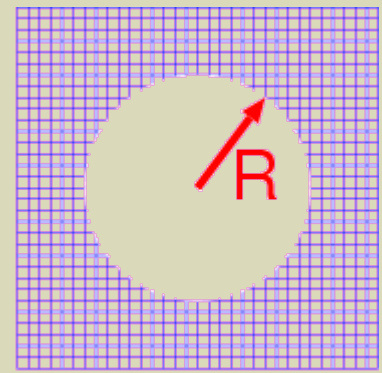
- ⊗ Study of hyperon-nucleon ( $YN$ ) and hyperon-hyperon ( $YY$ ) interactions is one of the important subjects in the nuclear physics.
  - ⊗ Structure of the neutron-star core,
    - ⊗ Hyperon mixing, softening of EOS, inevitable strong repulsive force,
  - ⊗ H-dibaryon problem,
    - ⊗ To be, or not to be,
- ⊗ The project at J-PARC:
  - ⊗ Explore the multistrange world,
- ⊗ However, the phenomenological description of  $YN$  and  $YY$  interactions has **large uncertainties**, which is in sharp contrast to the nice description of phenomenological  $NN$  potential.

# The purposes of this work

- ⊗  $NY$  forces from lattice QCD
- ⊗ Spin dependence
- ⊗ Potential (central + tensor)
- ⊗ Numerical calculation:
  - ⊗ Full lattice QCD by using  $N_F=2+1$  PACS-CS full QCD gauge configurations with the spatial lattice volume  $(2.86 \text{ fm})^3$



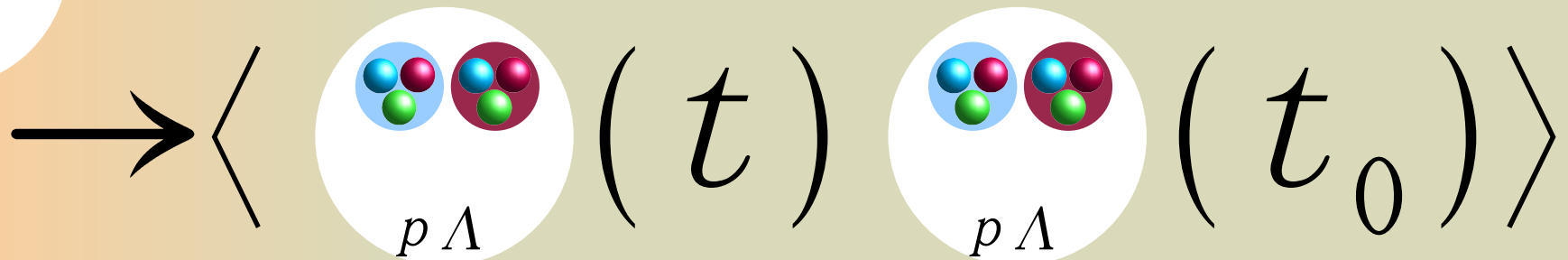
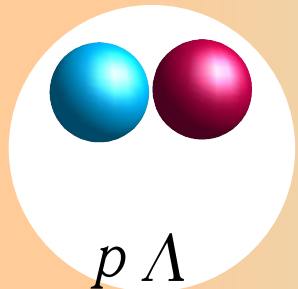
# Formulation



## Lattice QCD simulation

$$L = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$

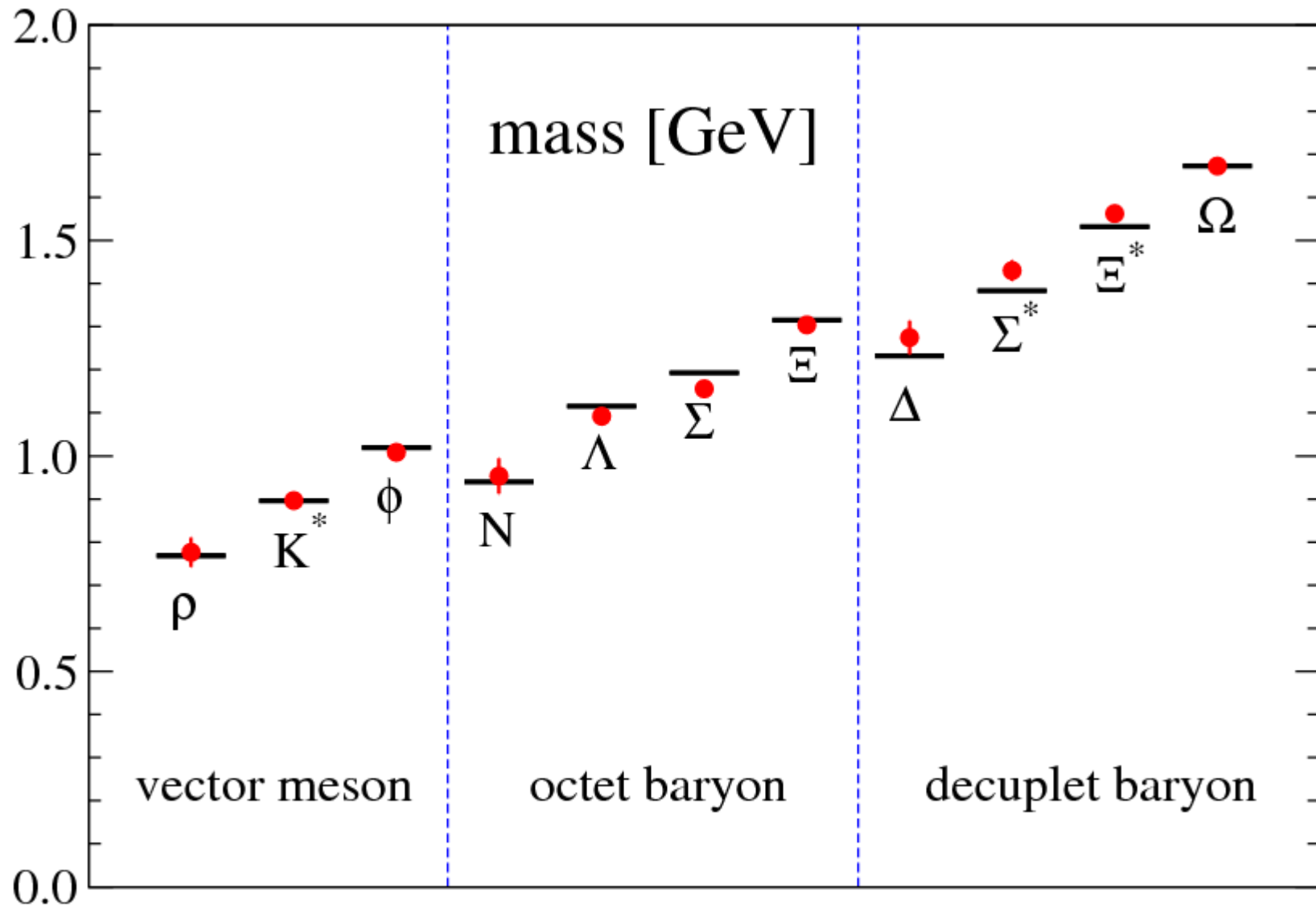
$$\begin{aligned} \langle O(\bar{q}, q, U) \rangle &= \int dU d\bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\ &= \int dU \det D(U) e^{-S_v(U)} O(D^{-1}(U)) \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N O(D^{-1}(U_i)) \end{aligned}$$



**2 + 1 flavor lattice QCD toward the physical point**

S. Aoki,<sup>1,2</sup> K.-I. Ishikawa,<sup>4</sup> N. Ishizuka,<sup>1,3</sup> T. Izubuchi,<sup>2,5</sup> D. Kadoh,<sup>3</sup> K. Kanaya,<sup>1</sup> Y. Kuramashi,<sup>1,3</sup> Y. Namekawa,<sup>3</sup>  
 M. Okawa,<sup>4</sup> Y. Taniguchi,<sup>1,3</sup> A. Ukawa,<sup>1,3</sup> N. Ukita,<sup>3</sup> and T. Yoshié<sup>1,3</sup>

(PACS-CS Collaboration)



PHYSICAL REVIEW D **81**, 111504(R) (2010)**Helium nuclei in quenched lattice QCD**T. Yamazaki,<sup>1</sup> Y. Kuramashi,<sup>1,2</sup> and A. Ukawa<sup>1</sup>

(PACS-CS Collaboration)

<sup>1</sup>*Center for Computational Sciences, University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan*<sup>2</sup>*Graduate School of Pure and Applied Sciences, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan*

(Received 7 December 2009; published 24 June 2010)

We present results for the binding energies for  ${}^4\text{He}$  and  ${}^3\text{He}$  nuclei calculated in quenched lattice QCD at the lattice spacing of  $a = 0.128$  fm with a heavy quark mass corresponding to  $m_\pi = 0.8$  GeV. Enormous computational cost for the nucleus correlation functions is reduced by avoiding redundancy of equivalent contractions stemming from permutation symmetry of protons or neutrons in the nucleus and various other symmetries. To distinguish a bound state from an attractive scattering state, we investigate the volume dependence of the energy difference between the nucleus and the free multinucleon states by changing the spatial extent of the lattice from 3.1 to 12.3 fm. A finite energy difference left in the infinite spatial volume limit leads to the conclusion that the measured ground states are bounded. It is also encouraging that the measured binding energies and the experimental ones show the same order of magnitude.

DOI: [10.1103/PhysRevD.81.111504](https://doi.org/10.1103/PhysRevD.81.111504)

PACS numbers: 12.38.Gc, 21.45.-v, 27.10.+h

The atomic nuclei have been historically treated as collections of protons and neutrons. The great success of the nuclear shell model since 1949 [1,2], explaining the nuclear magic numbers and detailed spectroscopy, has established that protons and neutrons are very good effective degrees of freedom at the nuclear energy scale of a few MeV. Nonetheless, 60 years later, we know for certain that protons and neutrons are made of quarks and gluons whose laws are governed by QCD. It is a great challenge to quantitatively understand the structure and property of

in quenched lattice QCD using a heavy quark mass at a single lattice spacing.

The binding energy  $\Delta E$  of the nucleus, consisting of  $N_N$  nucleons with the mass  $m_N$ , is very tiny compared with the mass  $M$  of the nucleus:  $\Delta E/M \sim O(10^{-3})$  with  $\Delta E = N_N m_N - M$ . This causes a complicated situation in that it is difficult to distinguish the physical binding energy from the energy shift due to the finite volume effect in the attractive scattering system [10]. One way to solve the problem is to investigate the volume dependence of the

PHYSICAL REVIEW D **81**, 111504(R) (2010)

# Helium nuclei in quenched lattice QCD

T. Yamazaki,<sup>1</sup> Y. Kuramashi,<sup>1,2</sup> and A. Ukawa<sup>1</sup>

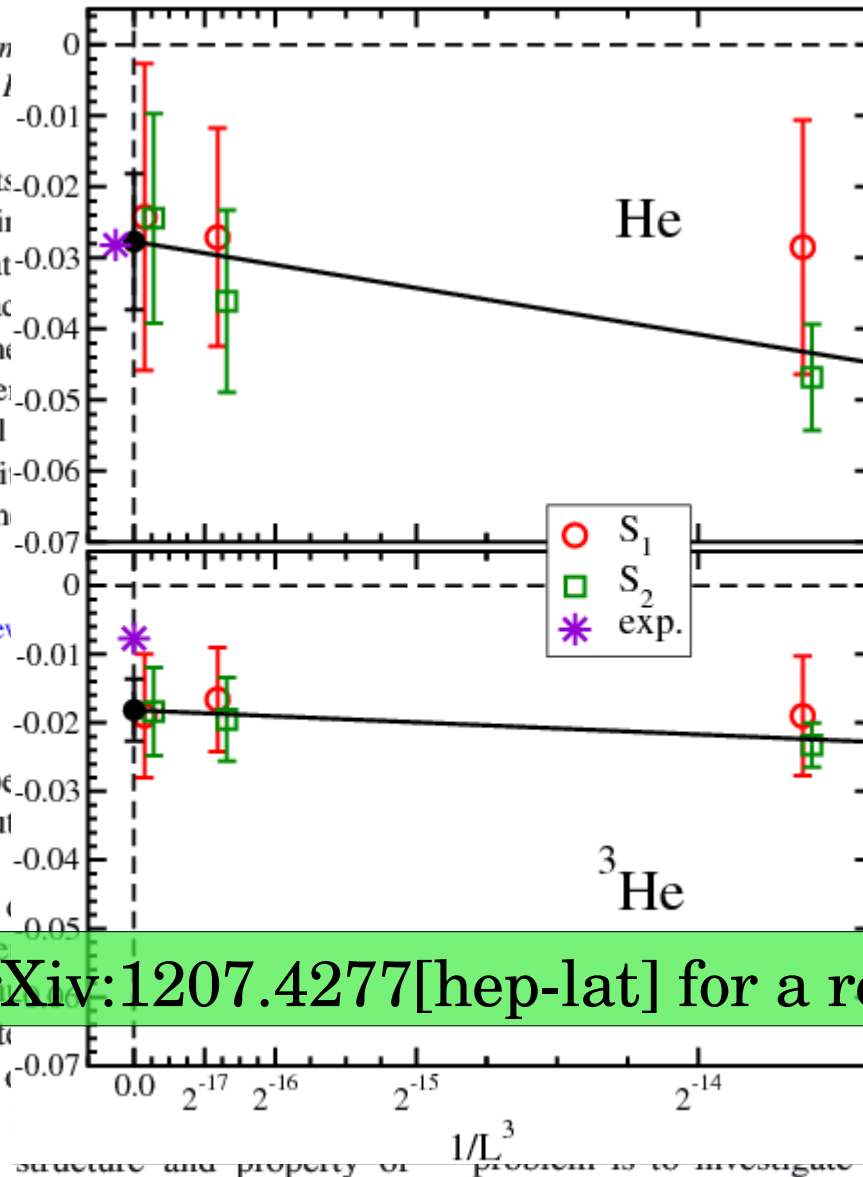
(PACS-CS Collaboration)

<sup>1</sup>Center for Computational Physics,  
<sup>2</sup>Graduate School of Physics,  
Kyushu University, Kasuga 816-8580, Japan

05-8577, Japan  
Yamaguchi University, Yamaguchi 753-8571, Japan

We present results at the lattice spacing  $a = 0.12$  fm. Enormous computational resources were used for equivalent contractions of various other symmetries. The volume dependence of the binding energy is investigated by changing the spatial volume. The spatial volume limit is encouraging that the magnitude of the finite volume effect is of the same order of magnitude as the physical binding energy.

DOI: [10.1103/PhysRevD.81.111504](https://doi.org/10.1103/PhysRevD.81.111504)

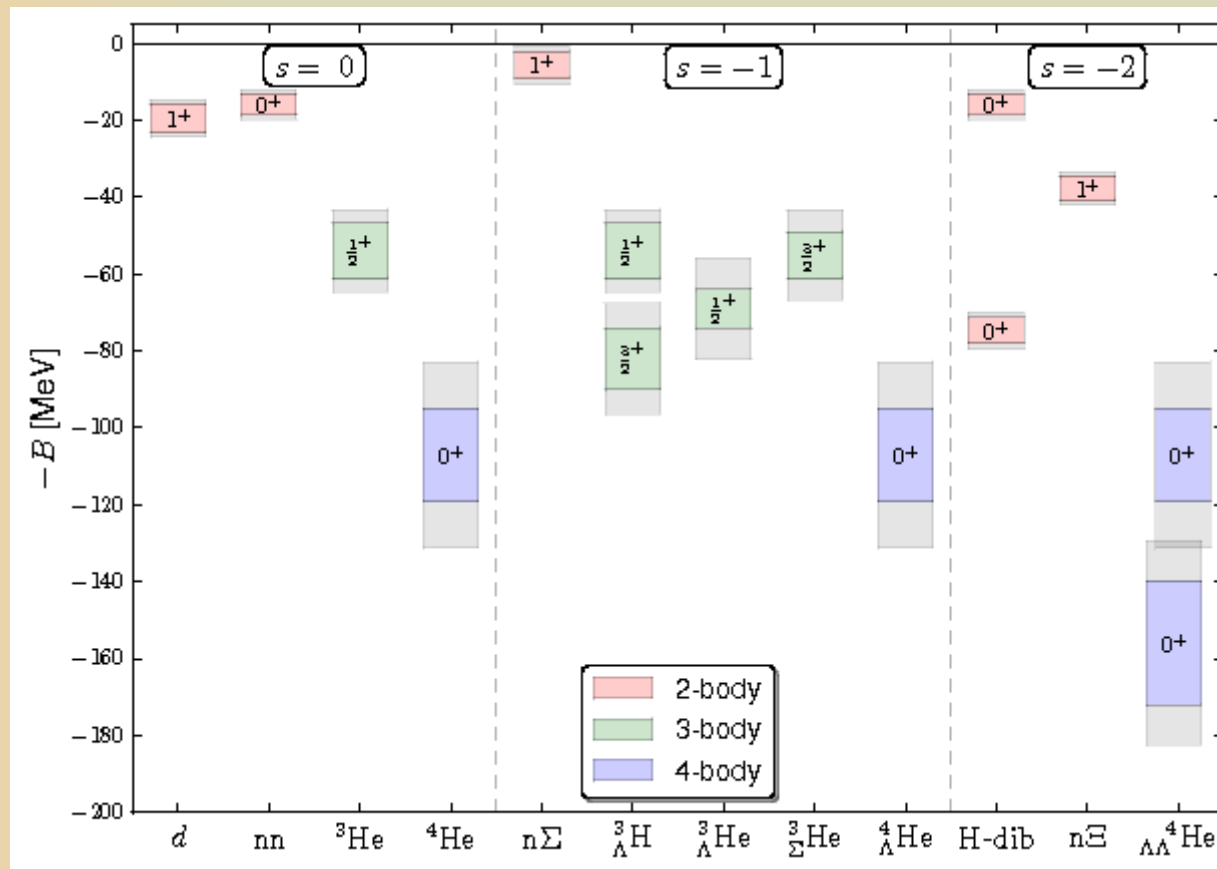


quenched lattice QCD to  $m_\pi = 0.8$  GeV. Avoiding redundancy in the nucleus and state, we investigate multinucleon states by leaving the infinite volume limit. It is also the same order of magnitude as the physical binding energy.

using a heavy quark mass at a lattice spacing of  $a = 0.12$  fm. The binding energy of the nucleus, consisting of  $N_N$  nucleons, is very tiny compared with the nucleon mass  $M_N$ .  $\Delta E/M \sim O(10^{-3})$  with  $\Delta E = E_{\text{bound}} - E_{\text{free}}$ . The finite volume effect in the binding energy is of the same order of magnitude as the physical binding energy [10]. One way to solve the problem is to investigate the volume dependence of the

See, e.g., arXiv:1207.4277[hep-lat] for a recent progress.

The atomic nuclei have been studied as collections of protons and neutrons. The nuclear shell model since the discovery of nuclear magic numbers and the establishment that protons and neutrons are fermions at the MeV scale. Nonetheless, 60 years later, the laws are governed by QCD. We aim to quantitatively understand the structure and property of



Light nuclei and hypernuclei from quantum chromodynamics in the limit of SU(3) flavor symmetry, PRD87, 034506 (2013)

# Formulation

i) basic procedure:

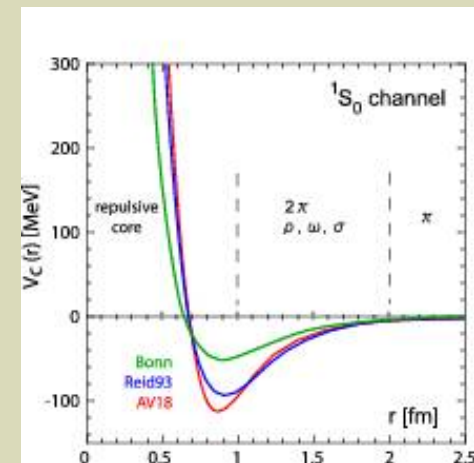
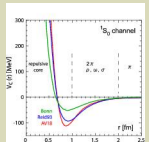
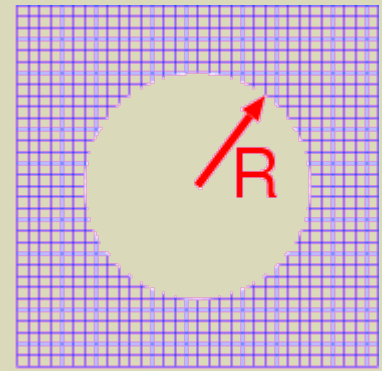
asymptotic region

→ phase shift

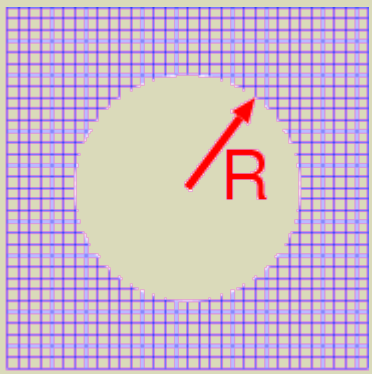
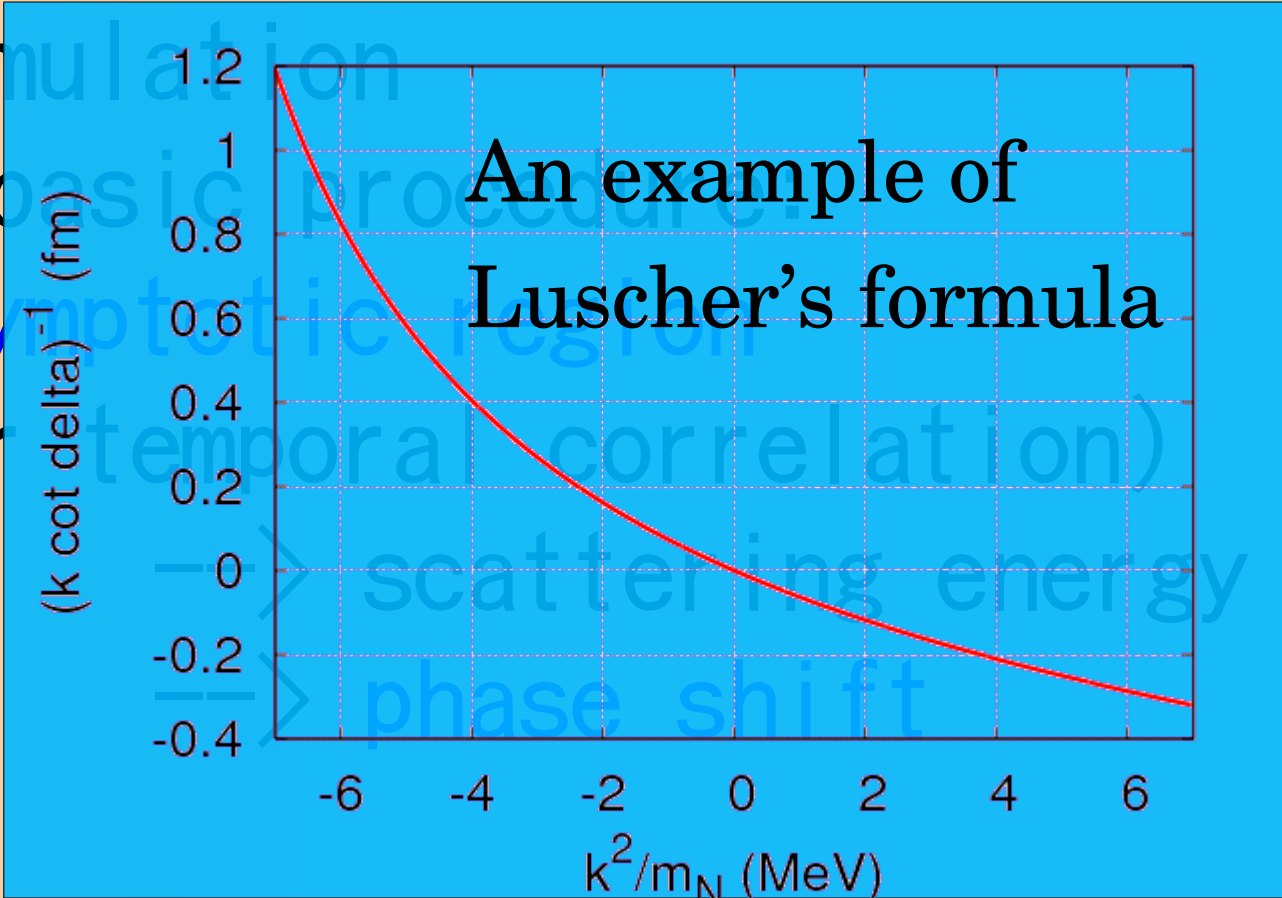
ii) advanced (HAL's) pro-

cedure: interacting region

→ potential



Formulation  
 i) basic  
 asymptotic region  
 (or scattering energy  
 phase shift)



$$E = \frac{k^2}{2\mu}$$

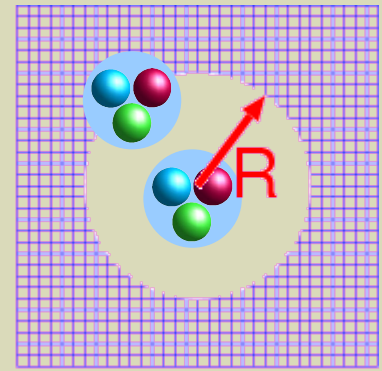
$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{(n^2 - q^2)^s} \quad \Re s > \frac{3}{2}$$

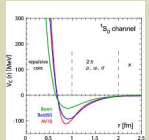
Luscher, NPB354, 531 (1991).  
 Aoki, et al., PRD71, 094504 (2005).

# Formulation

## Lattice QCD simulation



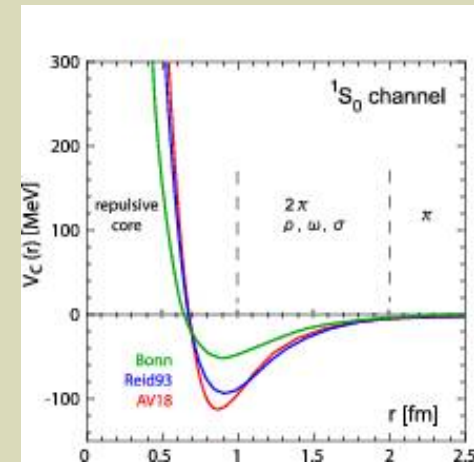
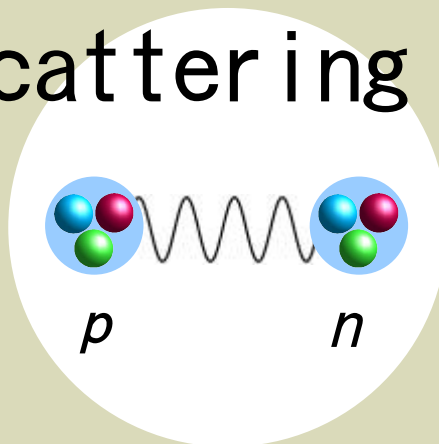
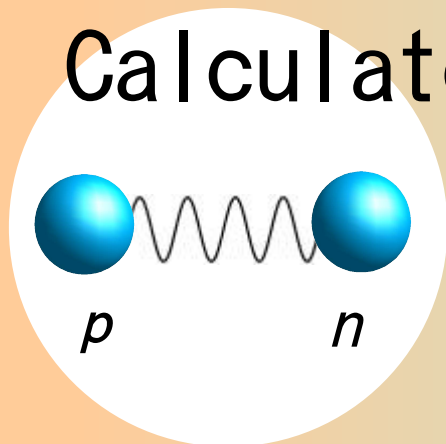
$$L = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$



$$\begin{aligned} \langle O(\bar{q}, q, U) \rangle &= \int dU d\bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\ &= \int dU \det D(U) e^{-S_v(U)} O(D^{-1}(U)) \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N O(D^{-1}(U_i)) \end{aligned}$$



Calculate the scattering state





# HAL formulation

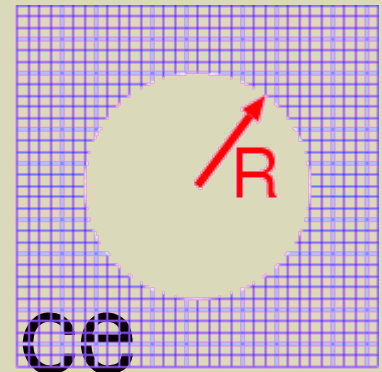
ii) advanced procedure:

make better use of the lattice

output ! (wave function)

interacting region

→ potential



Ishii, Aoki, Hatsuda,  
PRL99, 022001 (2007);  
ibid., arXiv:0805.2462[hep-ph].

NOTE:

- > Potential is not a direct experimental observable.
- > Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

# HAL formulation

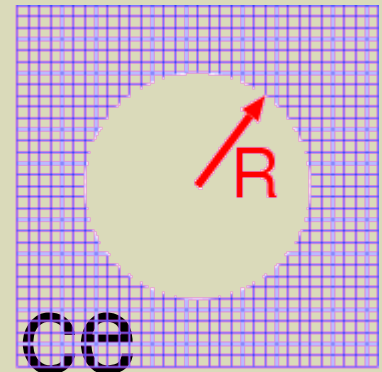
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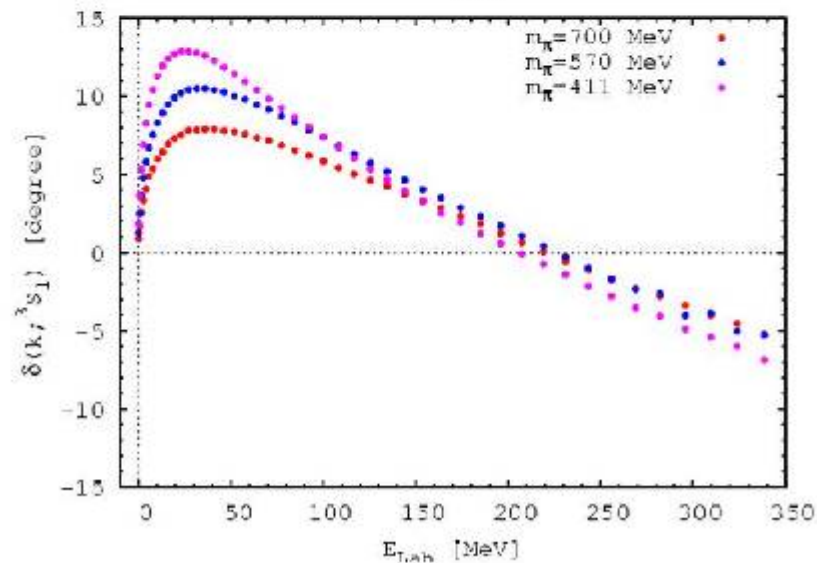
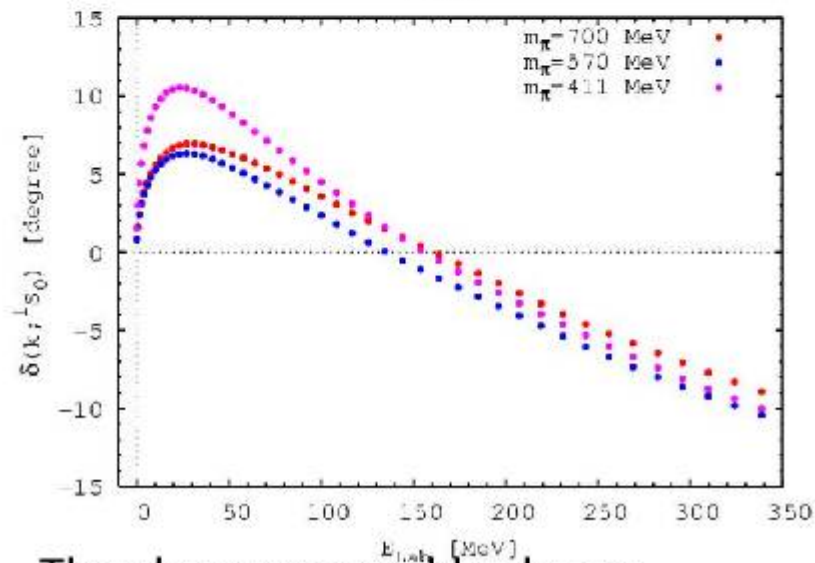
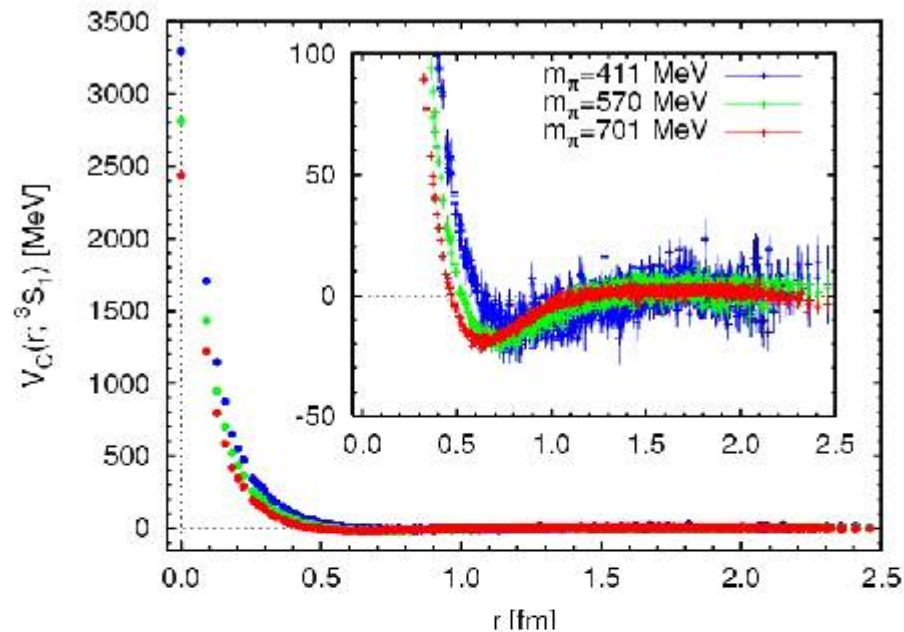
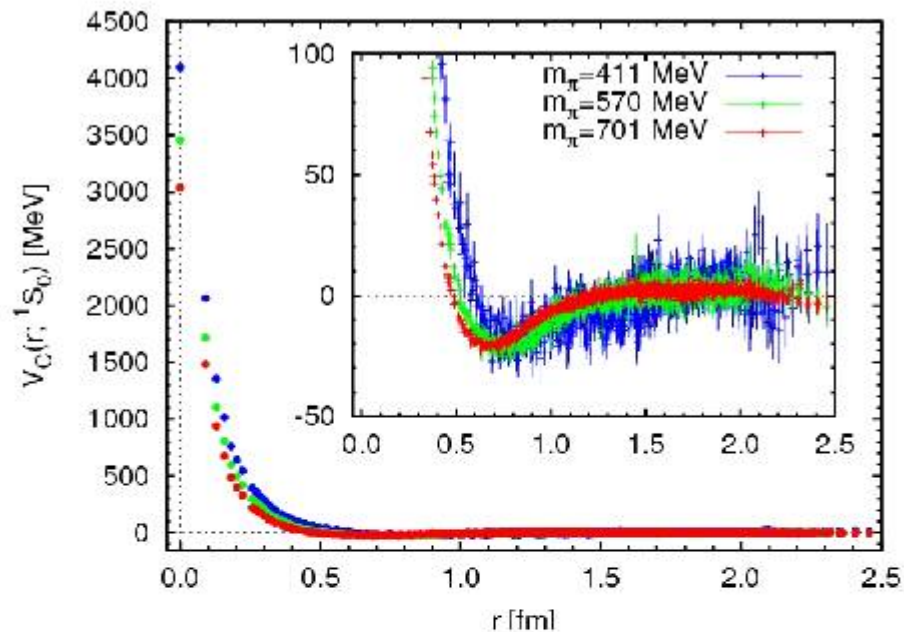
→ potential



Ishii, Aoki, Hatsuda,  
PRL99, 022001 (2007);  
ibid., arXiv:0805.2462[hep-ph].

- > Phase shift
- > Nuclear many-body problems

# NN (phase shift from potentials)



They have reasonable shapes.

# A recipe for $N\Lambda$ potential:

- The equal time BS wave function with angular momentum  $(J, M)$  on the lattice,

$$\Phi_{\alpha\beta}^{(JM)}(\vec{r}) = \sum_{\vec{x}} \langle 0 | p_{\alpha}(\vec{r} + \vec{x}) \Lambda_{\beta}(\vec{x}) | p\Lambda ; k, JM \rangle$$

$$p_{\alpha}(x) = \varepsilon_{abc} (u_a(x) C \gamma_5 d_b(x)) u_{c\alpha}(x),$$

$$\Lambda_{\alpha}(x) = \varepsilon_{abc} \left\{ (d_a C \gamma_5 s_b) u_{c\alpha} + (s_a C \gamma_5 u_b) d_{c\alpha} - 2(u_a C \gamma_5 d_b) s_{c\alpha} \right\}$$

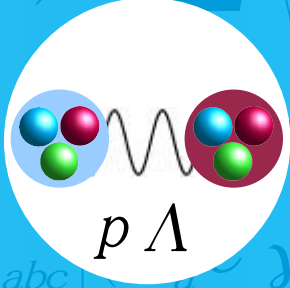
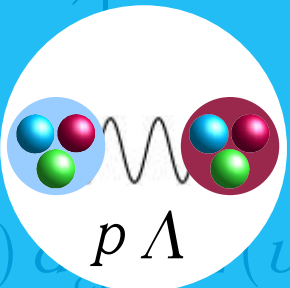
- The **4-point  $N\Lambda$  correlator** on the lattice,

$$\begin{aligned} F_{\alpha\beta}^{(JM)}(\vec{x}, \vec{y}, t - t_0) &= \langle 0 | p_{\alpha}(\vec{x}, t) \Lambda_{\beta}(\vec{y}, t) \overline{\Theta}_{p\Lambda}^{(JM)}(t_0) | 0 \rangle \\ &= \sum_n A_n^{(JM)} \langle 0 | p_{\alpha}(\vec{x}) \Lambda_{\beta}(\vec{y}) | E_n \rangle e^{-E_n(t - t_0)} \end{aligned}$$

$$\overline{\Theta}_{p\Lambda}^{(JM)}(t_0)$$

wall source at  $t = t_0$

# A recipe for $N\Lambda$ potential:

$$F_{\alpha\beta}^{(JM)}(\vec{r}, t - t_0) \rightarrow \left\langle \left( \text{p}\Lambda \right) (\vec{r}, t) \left( \text{p}\Lambda \right) (t_0) \right\rangle$$



• The **4-point  $N\Lambda$  correlator** on the lattice,

$$F_{\alpha\beta}^{(JM)}(\vec{x}, \vec{y}, t - t_0) = \left\langle 0 \left| p_\alpha(\vec{x}, t) \Lambda_\beta(\vec{y}, t) \overline{\Theta}_{p\Lambda}^{(JM)}(t_0) \right| 0 \right\rangle$$

$$= \sum_n A_n^{(JM)} \left\langle 0 \left| p_\alpha(\vec{x}) \Lambda_\beta(\vec{y}) \right| E_n \right\rangle e^{-E_n(t - t_0)}$$

wall source at  $t = t_0$

$$\overline{\Theta}_{p\Lambda}^{(JM)}(t_0)$$

# An improved recipe for lattice potential:

☉cf. Ishii (HAL QCD), PLB712 (2012) 437.

- ☉Take account of the temporal correlation as well as the spatial correlation of the NBS amplitude in terms of the R-correlator:

$$R(t, \vec{r}) = \frac{C_{YN}(t, \vec{r})}{C_Y(t)C_N(t)}$$

$$\begin{aligned} R(t + \Delta t, \vec{r}) &= e^{-\Delta t H} R(t, \vec{r}) \\ &= (1 - \Delta t H) R(t, \vec{r}) \end{aligned}$$

- ☉Time-dependent effective Schroedinger eq.:

$$-\frac{\partial}{\partial t} R(t, \vec{r}) = H R(t, \vec{r})$$

# An improved recipe for NY potential:

☉cf. Ishii (HAL QCD), PLB712 (2012) 437.

- ☉Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu} \nabla^2 R(t, \vec{r}) + \int d^3 r' U(\vec{r}, \vec{r}') R(t, \vec{r}') = -\frac{\partial}{\partial t} R(t, \vec{r})$$

$\rightarrow \frac{k^2}{2\mu} R(t, \vec{r})$

$$U(\vec{r}, \vec{r}') = V_{NY}(\vec{r}, \nabla) \delta(\vec{r} - \vec{r}')$$

- ☉A general expression of the potential:

$$\begin{aligned} V_{NY} = & V_0(r) + V_\sigma(r) (\vec{\sigma}_N \cdot \vec{\sigma}_Y) \\ & + V_T(r) S_{12} + V_{LS}(r) (\vec{L} \cdot \vec{S}_+) \\ & + V_{ALS}(r) (\vec{L} \cdot \vec{S}_-) + O(\nabla^2) \end{aligned}$$

# A recipe for $N\Lambda$ potential:

☉cf. Ishii (HAL QCD), PLB712 (2012) 437.

☉Effective central potential is obtained from the effective Schroedinger equation.

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + V(r)\right)R(t, \vec{r}) = -\frac{\partial}{\partial t}R(t, \vec{r})$$



$$V(r) = \frac{-\frac{\partial}{\partial t}R(t, \vec{r})}{R(t, \vec{r})} + \frac{\hbar^2}{2\mu} \frac{\nabla^2 R(t, \vec{r})}{R(t, \vec{r})}$$



# A recipe for NY potential: (contd.)

- For  $J = 1$ ,  $\phi$  comprises  $S$ -wave and  $D$ -wave,

$$|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle$$

where,

$$|\phi_S\rangle = \mathcal{P} |\phi\rangle = \left( \frac{1}{24} \right) \sum_{\mathcal{R} \in O} \mathcal{R} |\phi\rangle$$

$$|\phi_D\rangle = \mathcal{Q} |\phi\rangle = (1 - \mathcal{P}) |\phi\rangle$$

- Therefore, we have 2-component Schrödinger eq.

$S$ -wave:

$$\mathcal{P} (T + V_C + V_T S_{12}) |\phi\rangle = -\partial/\partial t \mathcal{P} |\phi\rangle$$

$D$ -wave:

$$\mathcal{Q} (T + V_C + V_T S_{12}) |\phi\rangle = -\partial/\partial t \mathcal{Q} |\phi\rangle$$

- Obtain the  $V_C(r)$  and the  $V_T(r)$  simultaneously.

# Numerical results

# Full QCD calculations by using $N_F=2+1$ PACS-CS gauge configurations:

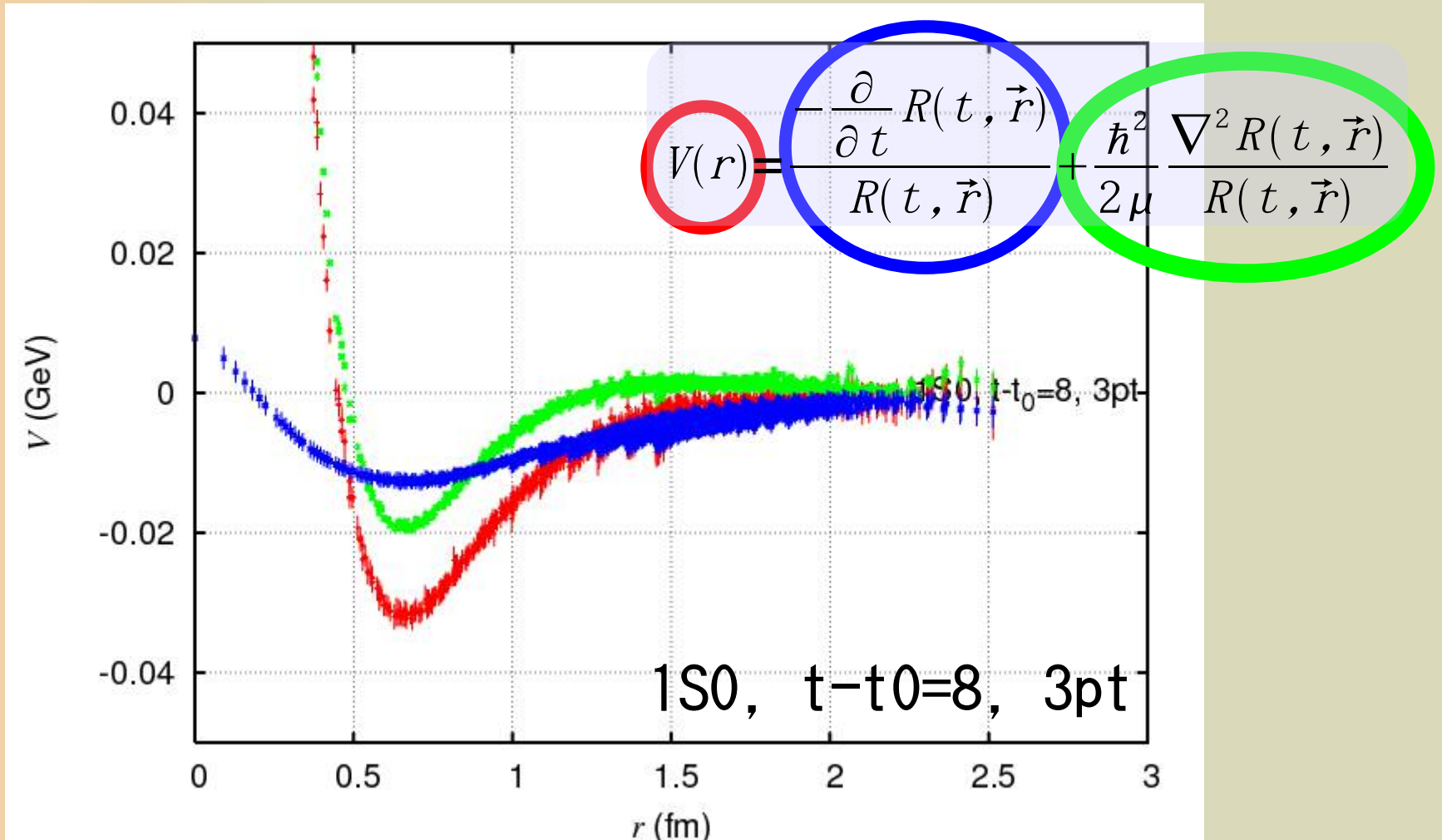
- ⊗ S. Aoki, et al., (PACS-CS Collaboration), PRD79, 034503 (2009), arXiv:0807.1661 [hep-lat].
- ⊗ Iwasaki gauge action at  $\beta=1.90$  on  $32^3 \times 64$  lattice
- ⊗  $O(a)$  improved Wilson quark action
- ⊗  $1/a = 2.17$  GeV ( $a = 0.0907$  fm)

$(\kappa_{ud})_{N_{\text{conf}}}$	$m_\pi$	$m_\rho$	$m_K$	$m_{K^*}$	$m_N$	$m_\Lambda$	$m_\Sigma$	$m_E$
<b>2+1 flavor QCD by PACS-CS with <math>\kappa_s = 0.13640</math> @ present calc (Dirichlet BC along T)</b>								
(0.13700)	700.0(4)	1108(3)	785.8(3)	1159(2)	1573(4)	1632(4)	1650(5)	1700(4)
(0.13754)	415(1)	903(5)	639.7(8)	1024(4)	1232(10)	1354(6)	1415(7)	1512(4)
Exp.	135	770	494	892	940	1116	1190	1320



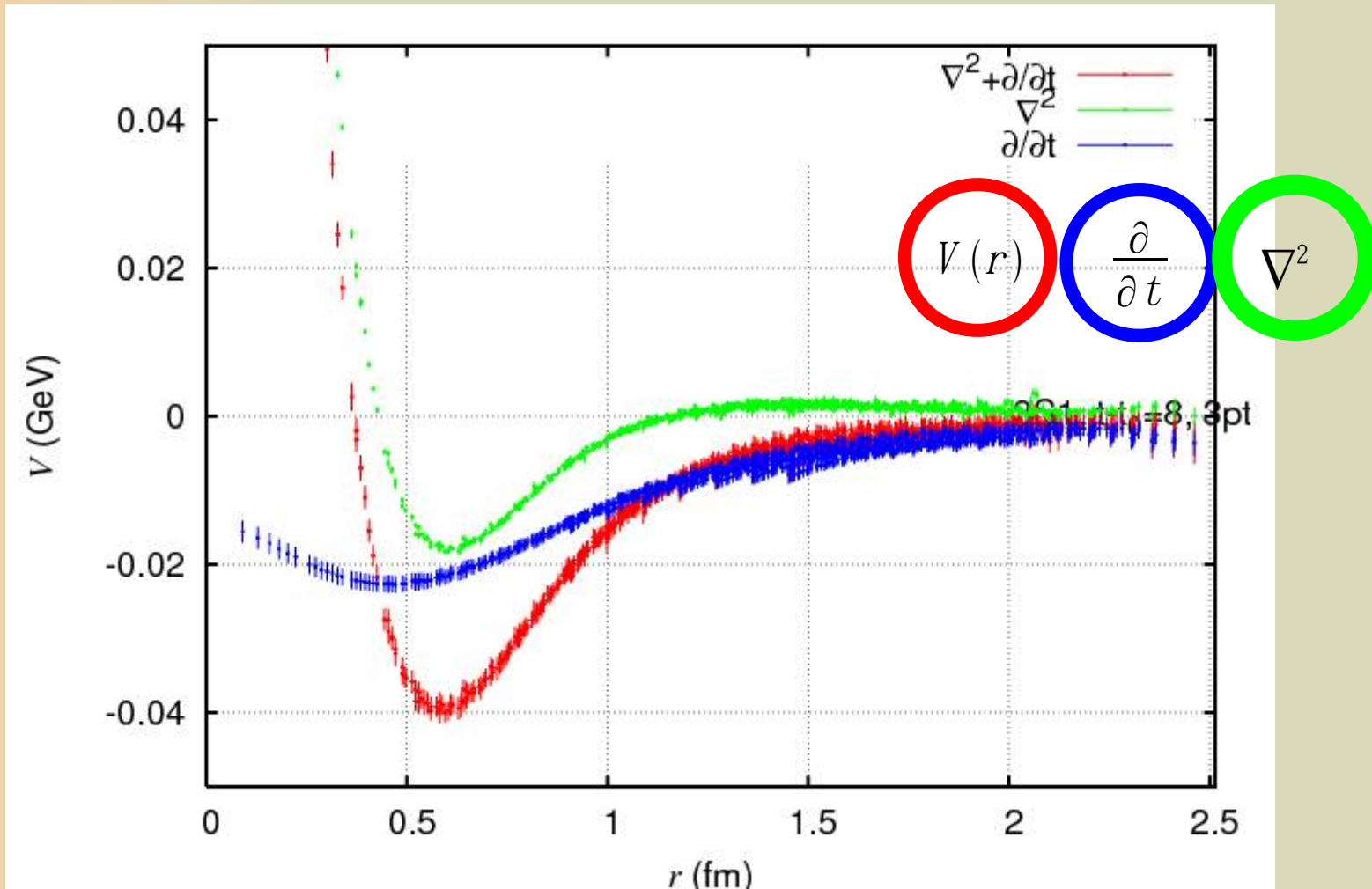
**$\Lambda N$  potential**

# $V_c(\Lambda N; 1S0)$



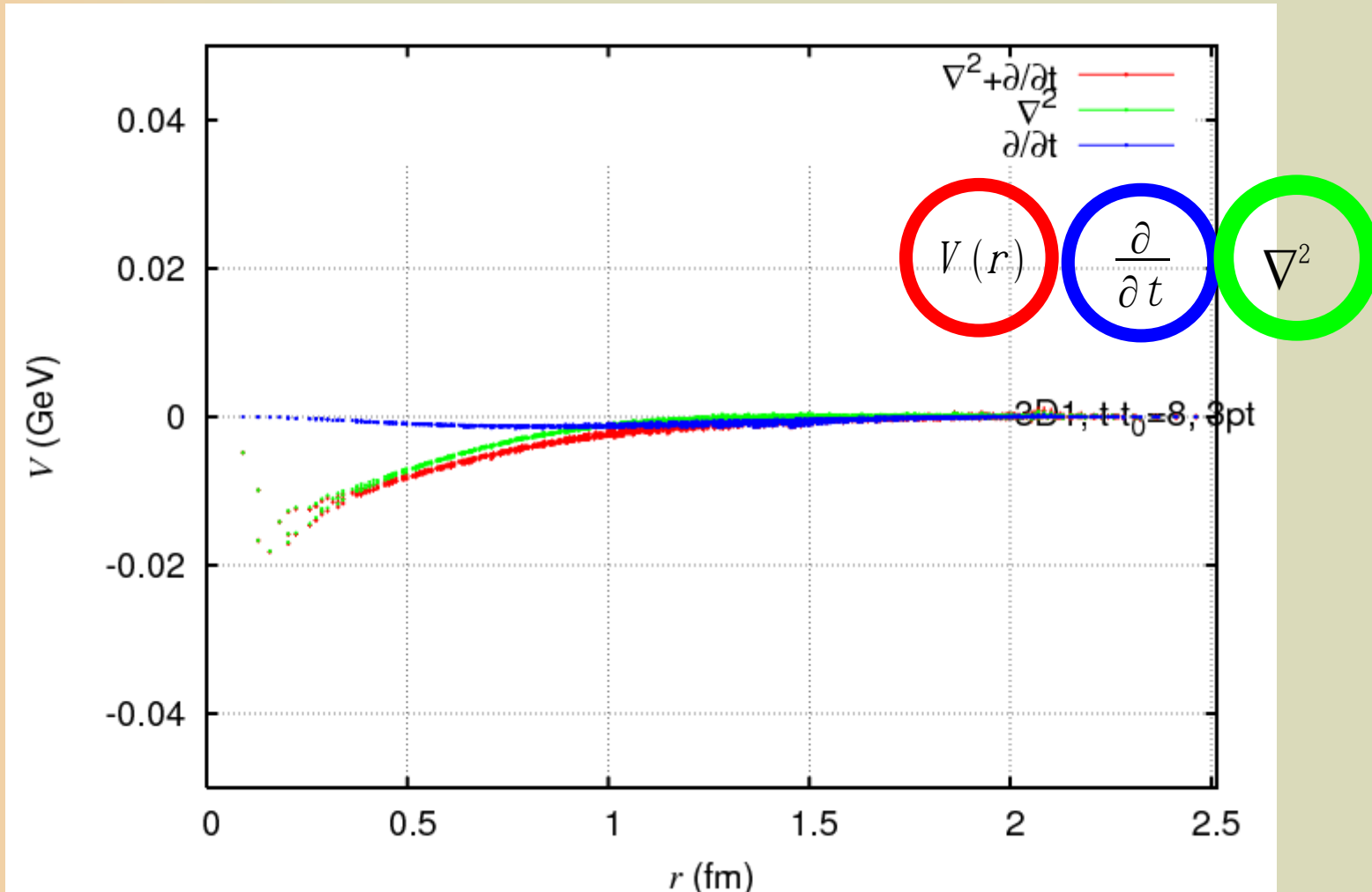
- $\{27\} + \{8s\}$
- Similar to NN (1S0)
- Sizable contribution from time-derivative part

# $V_C(\Lambda N; 3S1-3D1)$



- $\{10^*\} + \{8a\}$
- Sizable attractive contribution from time-derivative part

# $V_T(\Lambda N; 3S1-3D1)$

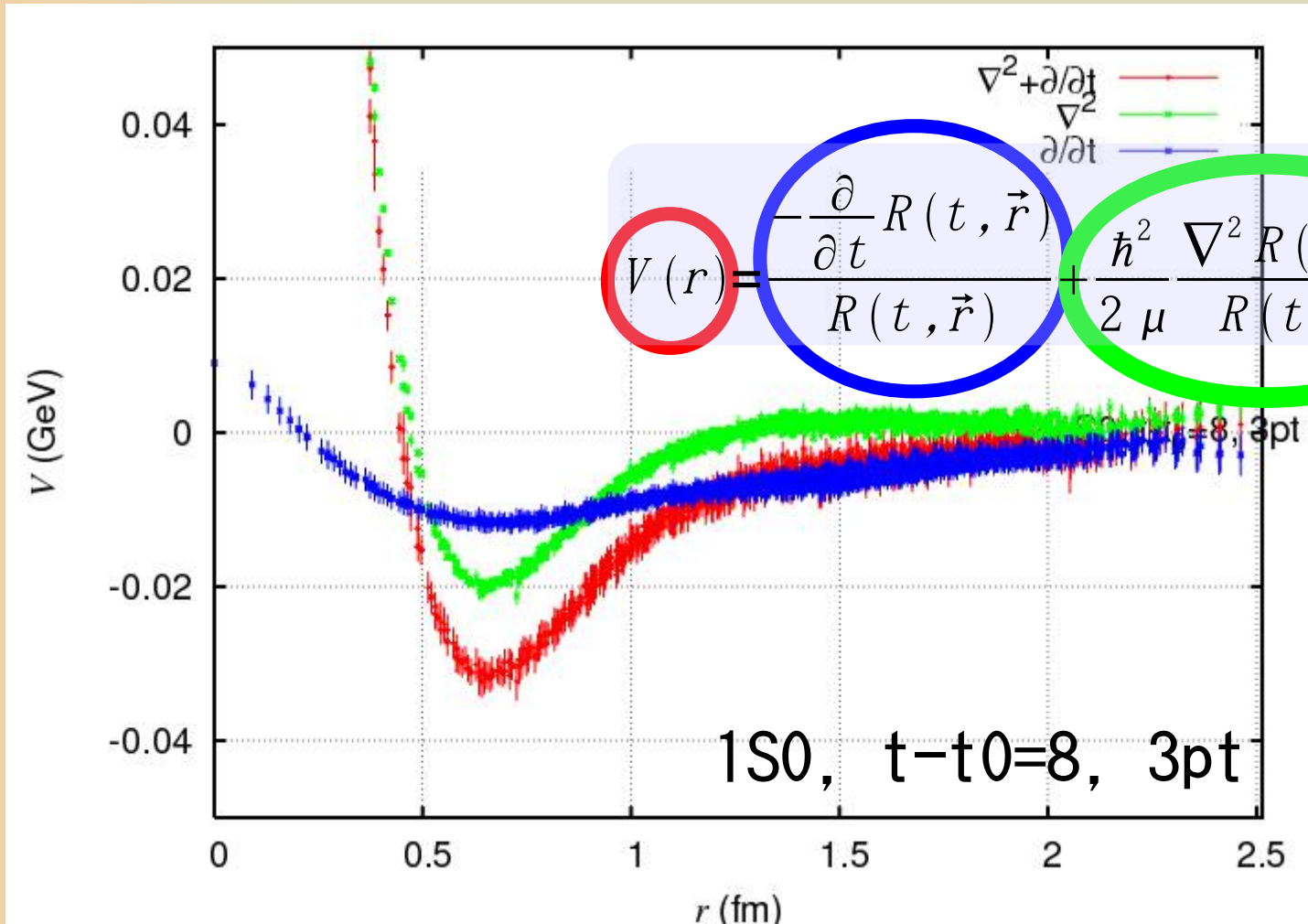


- Weaker tensor force than NN
- Small contribution from time-derivative part

**$\Sigma N(l=3/2)$  potential**

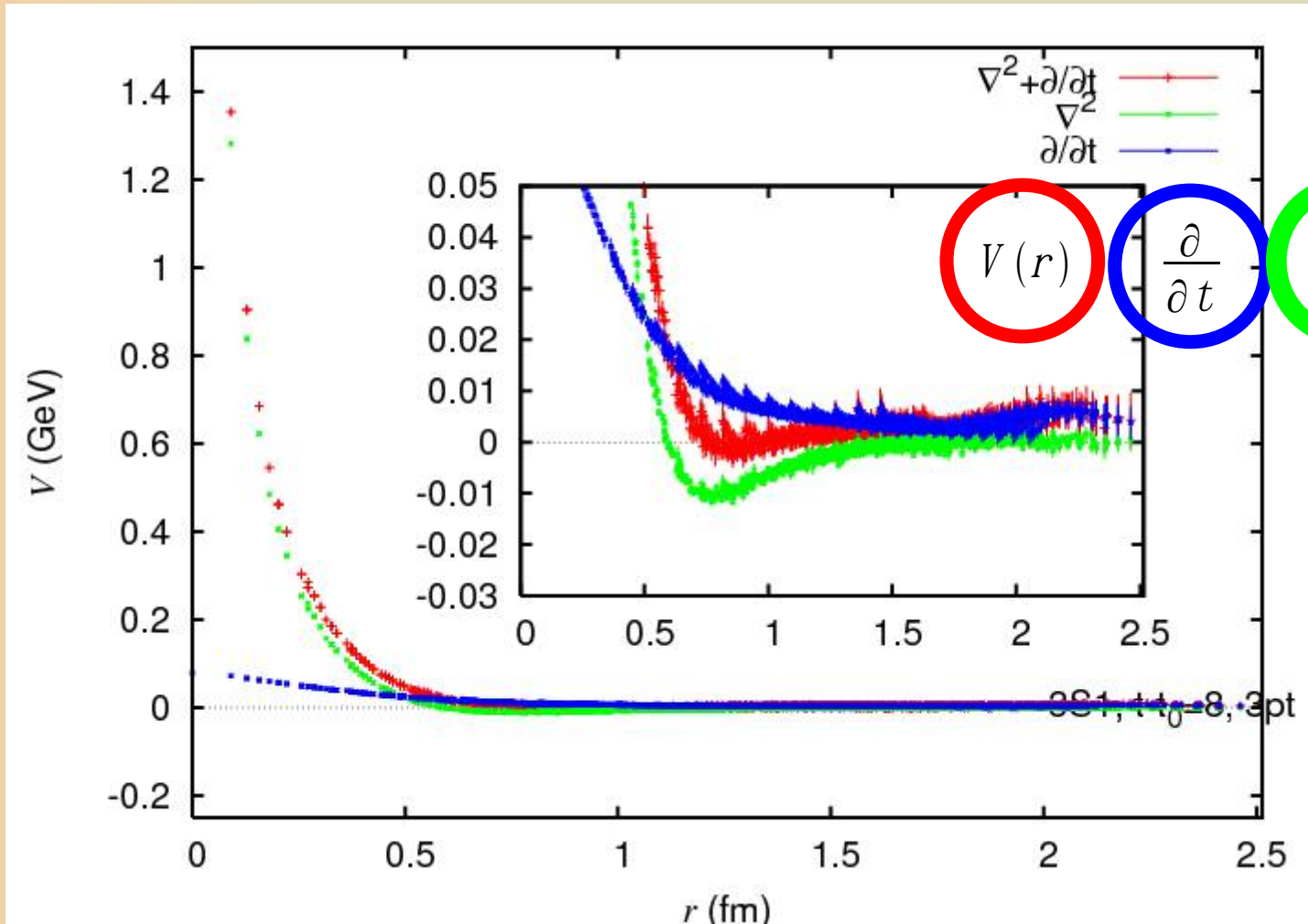


# $V_C(\Sigma N(I=3/2); 1S0)$



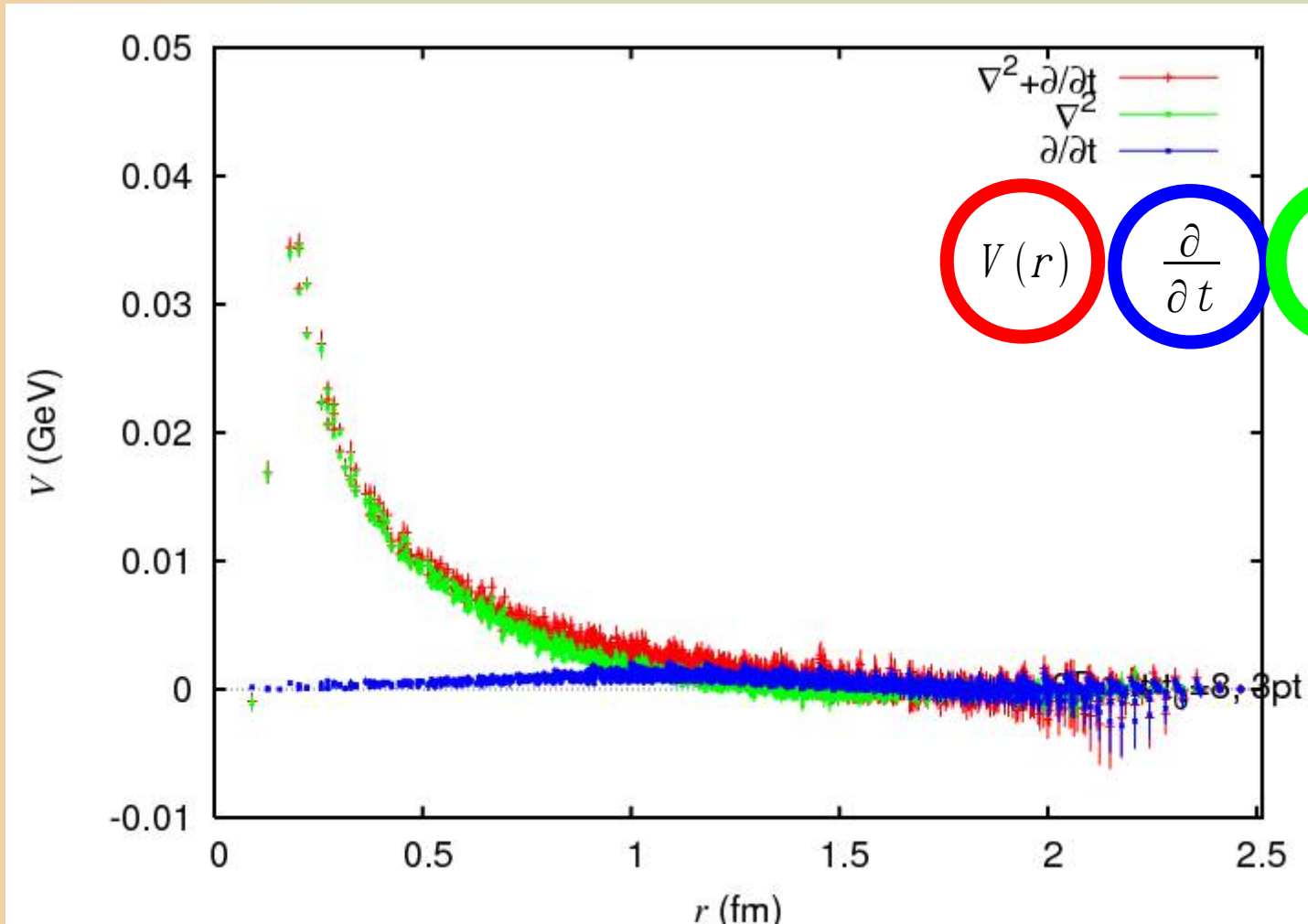
- {27}
- Similar to NN (1S0) (as well as Lambda-N (1S0))
- Sizable contribution from time-derivative part

# $V_C(\Sigma N(I=3/2); 3S1-3D1)$



- $\{10\}$
- Repulsive potential (consistent with quark model)
- sizable repulsive contribution from time-derivative part

# $V_T(\Sigma N(I=3/2); 3S1-3D1)$



- Weak tensor force
- Small contribution from time-derivative part

# Scattering phase shifts

Proton-Lambda scattering (preliminary)

**Parametrized  
potential**



**Phase shift**

# Summary:

- ⊗ The lattice QCD study for Lambda-Nucleon and Sigma-nucleon( $I=3/2$ ) interactions.
- ⊗  $p\Lambda$ :
  - ⊗ Central, tensor. For full QCD
  - ⊗ Time-derivative terms enhance the attractive force.
  - ⊗ Qualitatively similar to well-known nuclear forces.
    - ⊗ Repulsive at short distance.
    - ⊗ Attractive well at medium to long distance.
- ⊗  $N\Sigma(I=3/2)$ :
  - ⊗ Central, tensor. For full QCD
  - ⊗ The  $1S_0$  potential is similar to Lambda-N potential
  - ⊗ The  $3S_1$  potential is repulsive

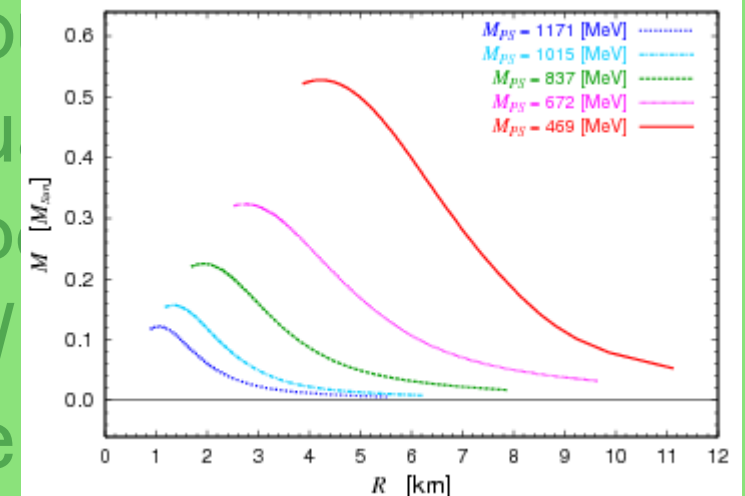
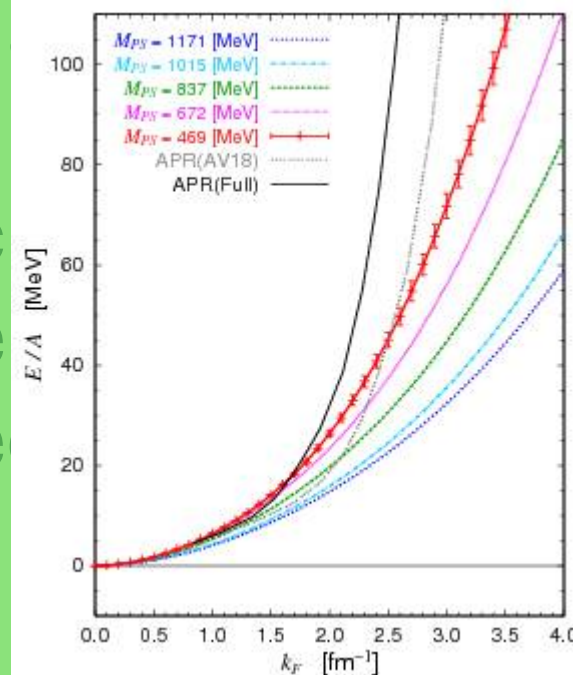
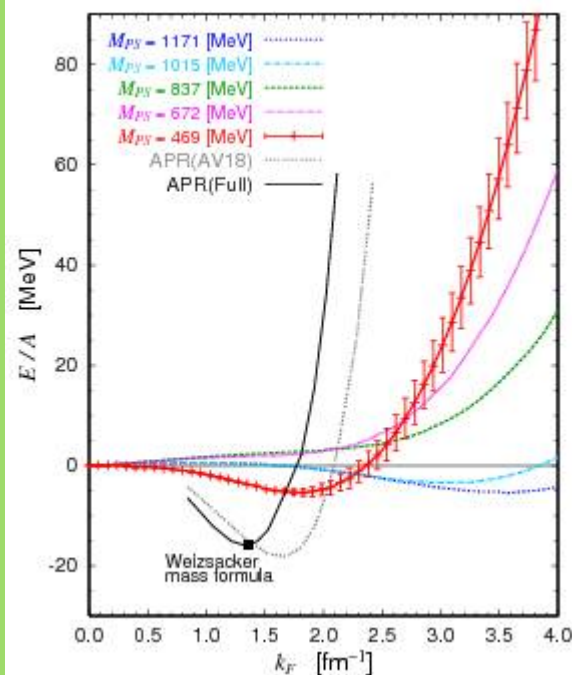
# Outlook:

- ⊗ Quark mass dependence.
- ⊗ Scattering lengths.
  - ⊗ spin-dependence.
  - ⊗ Comparison with the hypernuclear data.
- ⊗ Coupled-channel potential.
  
- ⊗ Application to nuclear physics (few-body systems)

# Stochastic variational calculation of $^4\text{He}$ with using a lattice potential

- ⊗ For NN potential, we use Inoue-san's SU(3) potential at the lightest quark mass ( $m_{ps} = 469 \text{ MeV}$ ), which has been reported to have a  $4N$  bound state (about  $5.1 \text{ MeV}$ ) within a tensor-included effective central potential; NPA881, 28-43 (2011).

# Stochastic variational calculation of $^4\text{He}$ with using a lattice potential



See also, PRL 111, 112503 (2013) for a recent work.



# Stochastic variational calculation of 4He with using a lattice potential

The wave function of  $A$ -body system is described by a linear combination of basis functions as

$$\Psi = \sum_{k=1}^K c_k \varphi_k, \quad \text{with} \quad \varphi_k = \mathcal{A}\{G(\mathbf{x}; A_k)[\theta_{(LL')_k}(\mathbf{x}; (uu')_k), \chi_{S_k}]_{JM\eta_{kIM_I}}\}, \quad (11)$$

where  $c_k$  is the linear variational parameter determined by the variational principle,  $\mathcal{A}$  is antisymmetrizer for identical particles.  $\chi_{S_k}$  ( $\eta_{kIM_I}$ ) is the spin (isospin) function of the system.  $G(\mathbf{x}; A_k)$  is the correlated Gaussian function which is given by

$$G(\mathbf{x}; A_k) = \exp \left\{ -\frac{1}{2} \sum_{i < j}^A \alpha_{kij} (\mathbf{r}_i - \mathbf{r}_j)^2 \right\} = \exp \left\{ -\frac{1}{2} \sum_{i,j=1}^{A-1} A_{kij} \mathbf{x}_i \cdot \mathbf{x}_j \right\}. \quad (12)$$

# Stochastic variational calculation of 4He with using a lattice potential

A set of relative coordinates  $\{\mathbf{x}_1, \dots, \mathbf{x}_{A-1}\}$  and the center-of-mass coordinate  $\mathbf{x}_A$  are given by a linear transformation of single particle coordinates  $\{\mathbf{r}_1, \dots, \mathbf{r}_A\}$  such as

$$\mathbf{x}_i = \sum_{j=1}^A U_{ij} \mathbf{r}_j, \quad (i = 1, \dots, A). \quad (13)$$

In order to obtain the accurate solution of the four-nucleon bound state with explicitly utilizing the the tensor potential, we consider nonzero orbital angular momentum states  $(L, S)J^\pi = (1, 1)0^+$  and  $(2, 2)0^+$  in addition to the  $(0, 0)0^+$  configuration. We employ the global vector representation[11] for these nonzero orbital angular momentum states. Therefore, the angular part of the basis function is given by

$$\theta_{(LL')_k}(\mathbf{x}; (uu')_k) = v_k^{L_k} v_k'^{L'_k} [Y_{L_k}(\hat{\mathbf{v}}_k) \times Y_{L'_k}(\hat{\mathbf{v}}'_k)]_{L_k}, \quad \begin{pmatrix} \mathbf{v} \\ \mathbf{v}' \end{pmatrix}_k = \sum_{i=1}^{A-1} \mathbf{x}_i \begin{pmatrix} u \\ u' \end{pmatrix}_{ki}. \quad (14)$$

The validity of the present choice of basis function is examined for several realistic  $NN$  potentials[11]. The  $A_{kij}$  and  $(u, u')_{ki}$  are the nonlinear variational parameters which are determined by the stochastic variational method[12].

# Inoue san's $SU(3)$ potential

## ★ Central potentials

- Two  $V_C$ 's for spin singlet and triplet

$$V_C(r) = V_1 \exp(\alpha_1 r^2) + V_2 \exp(\alpha_2 r^2) - V_3 (1 - \exp(\alpha_3 r^2))^2 (\exp(-\alpha_4 r)/r)^2$$

## ★ Tensor potential

$$V_T(r) = V_1 (1 - \exp(\alpha_1 r^2))^2 \left(1 + \frac{3}{\alpha_2 r} + \frac{3}{\alpha_2 r^2}\right) \frac{\exp(-\alpha_2 r)}{r} + V_2 (1 - \exp(\alpha_3 r^2))^2 \left(1 + \frac{3}{\alpha_4 r} + \frac{3}{\alpha_4 r^2}\right) \frac{\exp(-\alpha_4 r)}{r}$$

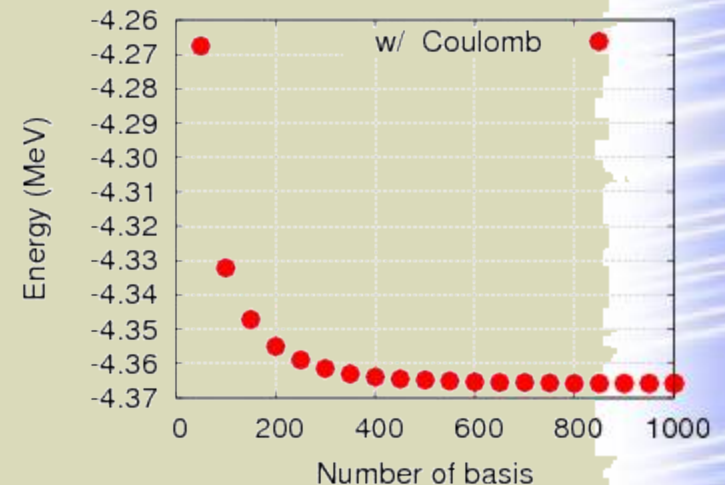
# Results of few-body calculation

## ★ Inputs:

- $m=1161.0$  MeV,
- $\hbar c = 197.3269602$  MeV fm
- $\hbar c/e^2 = 137.03599976$
- $V_{NN}$  is treated as a Serber-type potential.

## ★ Results:

- $B(4\text{He})=4.37$  MeV (w/ Coulomb)
  - Probabilities of (S, P, D) waves = (98.6%, 0.003%, 1.3%)
  - cf. roughly speaking (S,P,D)~(<90%, <0.1%, >10%) for a realistic NN force
- $B(4\text{He})=5.09$  MeV (w/o Coulomb)
  - Probabilities of (S, P, D) waves = (98.6%, 0.003%, 1.4%)



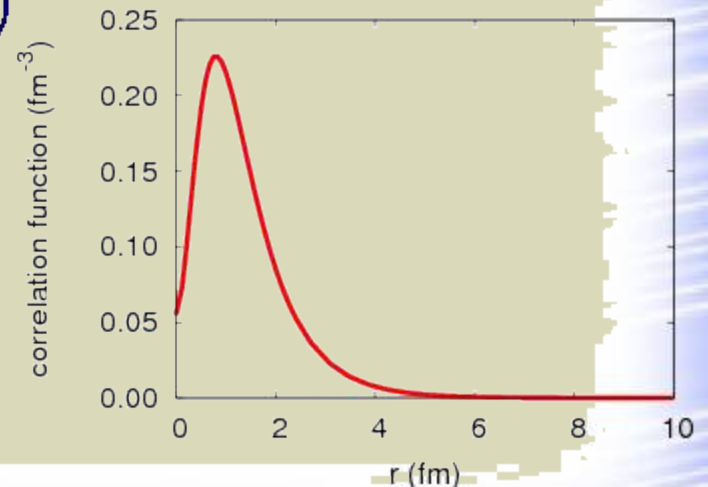
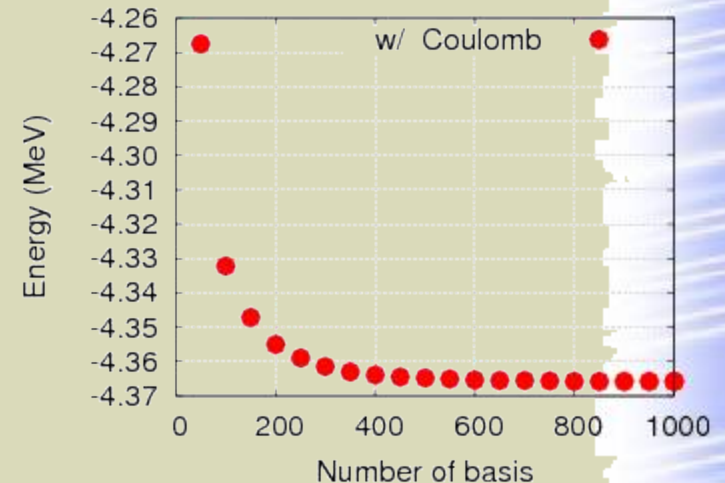
# Results of few-body calculation

## ★ Inputs:

- $m=1161.0$  MeV,
- $\hbar c = 197.3269602$  MeV fm
- $\hbar c/e^2 = 137.03599976$
- $V_{NN}$  is treated as a Serber-type potential.

## ★ Results:

- $B(4\text{He})=4.37$  MeV (w/ Coulomb)
  - Probabilities of (S, P, D) waves = (98.6%, 0.003%, 1.3%)
  - I also calculate the correlation function.



# Results when we cut off the tensor potential

## ★ Inputs:

- $m=1161.0$  MeV,
- $\hbar c = 197.3269602$  MeV fm
- $\hbar c/e^2 = 137.03599976$
- $V_{NN}$  is treated as a Serber-type potential **with just cutting off the tensor part.**

## ★ Results:

- $B(4\text{He})=1.61$  MeV (w/ Coulomb)
  - Probabilities of (S, P, D) waves = (100%, 0%, 0%)
  - cf. roughly speaking (S,P,D)~(<90%, <0.1%, >10%) for a realistic NN force
- $B(4\text{He})=2.25$  MeV (w/o Coulomb)
  - (Probability of each component is almost same as the case including Coulomb)

# Few-body calculations of $s$ -shell $\Lambda$ hypernuclei

	$A=3$	$A=4$	$A=5$	Tensor	$\Sigma$	
• Dalitz, <i>et al.</i>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	NPB47, 109 (1972).
• Shinmura, <i>et al.</i> ,	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	PTP71, 546 (1984).
• Gibson, <i>et al.</i> ,	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	PRC37, 679 (1988).
• Carlson,	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	AIP Conf. Proc. No. 224 (1991).
• Miyagawa, <i>et al.</i> ,	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	PRC51, 2905 (1995).
• Hiyama, <i>et al.</i> ,	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	PRC65, 011301 (2002).
• Sinha, <i>et al.</i> ,	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	PRC66, 024006 (2002).
• Nogga, <i>et al.</i> ,	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	PRL88, 172501 (2002).
• HN, <i>et al.</i> ,	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	PRL89, 142504 (2002).

:  $\Lambda NN$  3BF

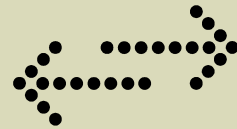


# Plan of research

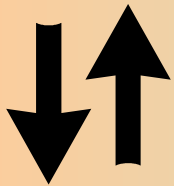
QCD



Baryon interaction



J-PARC  
hyperon-nucleon (YN)  
scattering



Structure and reaction of  
(hyper)nuclei

Equation of State (EoS)  
of nuclear matter

Neutron star and  
supernova