Correcting Dr. Weinberg's prescriptions

- Renormalization and power counting of chiral nuclear forces

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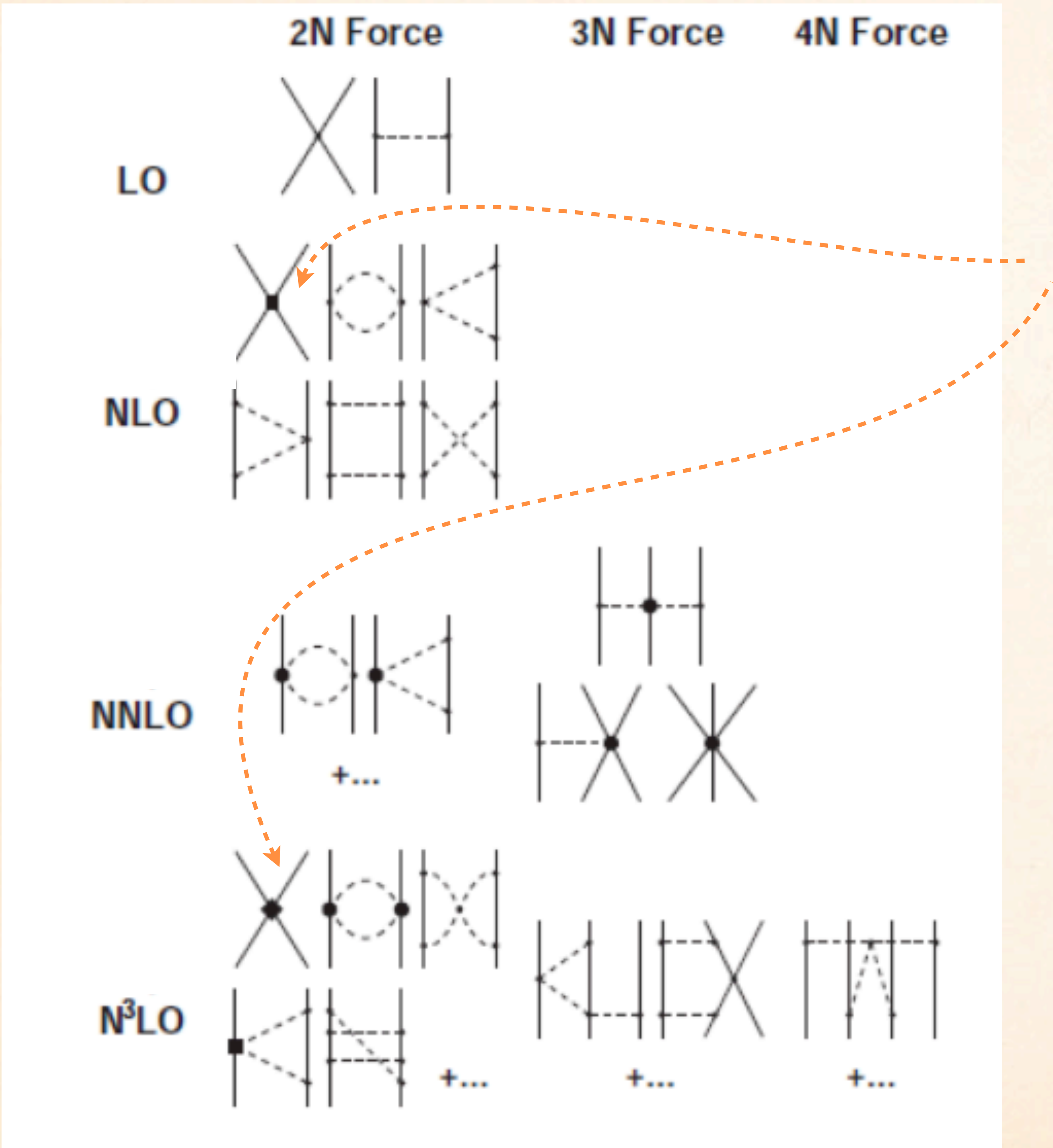
What are we really doing?



Modify power counting of NN contact interactions, so as

(1) to satisfy renormalization group invariance;

(2) to better understand how much of nuclear physics is decided by short-range interactions as opposed to chiral symmetry.

The big picture

Model-independent descriptions of low-energy nuclear physics, based on chiral effective field theory

1. Nuclear reactions

$$d + {}^3H \to {}^4He + n(14\ \text{MeV})$$
$$d + d \to {}^3He + n(2.5\ \text{MeV})$$
$$\ldots$$

2. Electro-weak probes

$$p + p \to d + e^+ + \nu_e$$

3. Nuclear structure

Binding energies of light nuclei

Effective field theory

EFT = Effective Lagrangian + Power Counting

. Low-energy Dofs
. Symmetries

organization principle:
a priori estimation of diagrams

Goal: expansion of amplitudes

$$\mathcal{M} = \sum_n \left(\frac{Q}{M_{hi}}\right)^n \mathcal{F}_n\left(\frac{Q}{M_{lo}}\right)$$

Q: generic external momenta,

$M_{hi} = \Lambda_{SB}, m_\rho, \cdots \sim 1\text{GeV}$

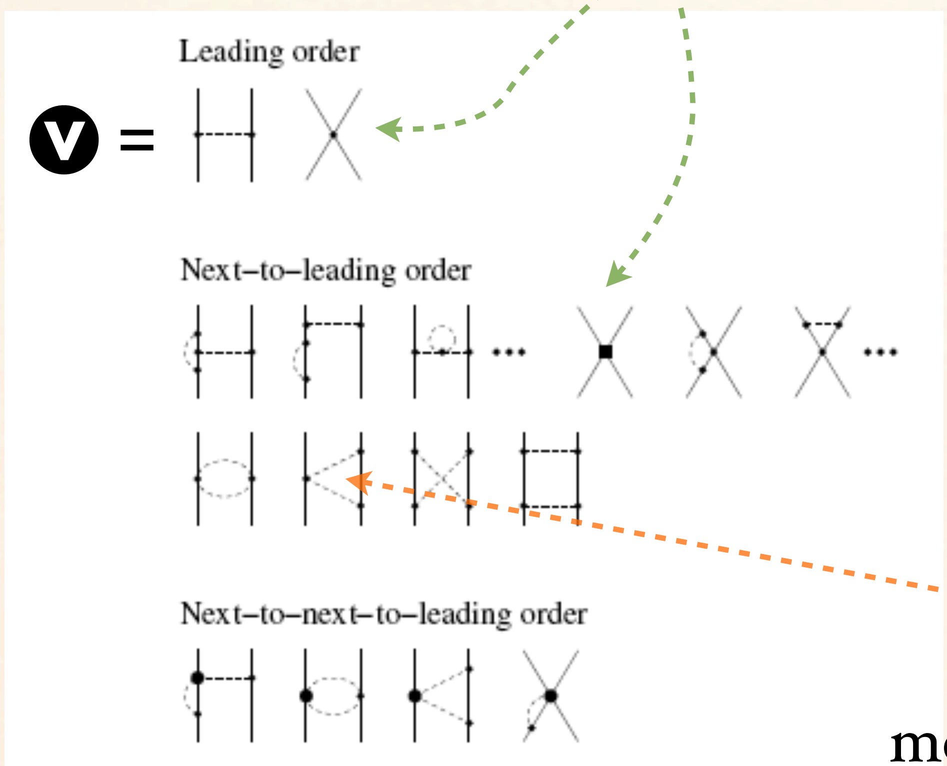
$M_{lo} = m_\pi, f_\pi \sim 100\text{MeV}$

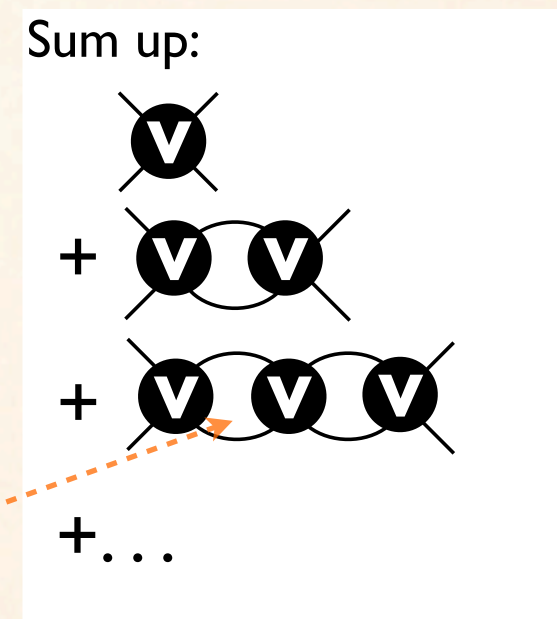
Main benefit: reliable estimates of theo. err.

Dr. W's prescriptions

1. Derivatives on couplings are always suppressed —— naive dimensional analysis.

2. Nonperturbative iterations don't affect power counting.





Λ

momentum cutoff

❖ Implemented by Epelbaum, et al (1999, 2003) and Entem et al (2002) with a small range of cutoffs

❖ However, cutoff dependence of resummed amplitudes was not addressed in W counting

Strength of OPE provides an infrared scale

$$V_{1\pi} = \frac{g_A^2}{4f_\pi^2}\frac{\vec{q}\cdot\vec{\sigma}_1\vec{q}\cdot\vec{\sigma}_2}{m_\pi^2+q^2} \quad \rightarrow \frac{1}{m_N}\frac{\lambda}{M_{NN}r^3}e^{-m_\pi r}$$

$M_{NN} = 100 \sim 300\text{MeV}$ varies for different partial waves

Naive dimensional analysis for estimating contact interactions is no longer reliable

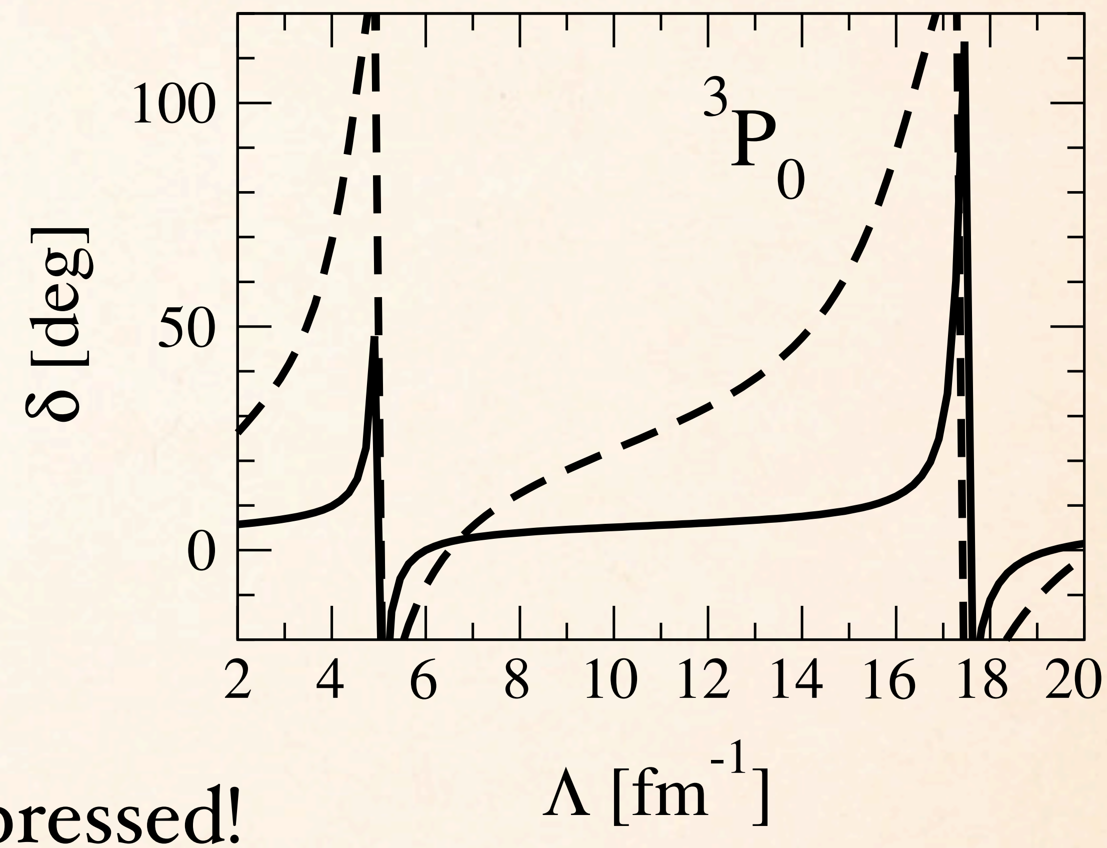
$$\frac{4\pi}{m_N}\left(\frac{\tilde{C}_0}{M_0}\delta^{(3)}(\vec{x}) + \frac{\tilde{C}_2}{M_2^2}\nabla^2\delta^{(3)}(\vec{x}) + \cdots\right)$$

Cutoff dependence of W counting

Nogga, Timmerman & van Kolck (2005)

E.g., 3P0

A singular attractive potential needs a counterterm — 4-nucleon operator with the same QM number as 3P0



→ A derivative coupling not suppressed!

Solid: Tlab = 10 MeV, dashed: 50 MeV

Very large cutoffs were used to illustrate the cutoff dependence, but we don't insist on using them in practical calculations once power counting is established.

Subleading orders in triplet channels

Renormalization of one insertion of two-pion exchange

BwL, van Kolck (2008)
BwL, Yang (2011, 2012)
Pavon Valderrama(2011, 2012)

for LO potential $\sim -1/r^3$,

$$\psi_k^{(0)}(r) \sim \left(\frac{\lambda}{r}\right)^{\frac{1}{4}} \left[u_0(r/\lambda) + k^2 r^2 \sqrt{\frac{r}{\lambda}} u_1(r/\lambda) + \mathcal{O}(k^4)\right]$$

$$\lambda = \frac{3 g_A^2 m_N}{8\pi f_\pi^2} \qquad u_{1,\,2}(x) \sim \mathcal{O}(1)$$

$$V_{2\pi} \sim \frac{1}{r^5} \; r \to 0$$

$$T^{(2)} = \langle \psi^{(0)} | V_{2\pi} | \psi^{(0)} \rangle$$

$$\sim \int_{\sim 1/\Lambda} dr r^2 |\psi^{(0)}(r)|^2 \frac{1}{r^5} \sim \alpha_0(\Lambda)\Lambda^{5/2} + \beta_0(\Lambda) k^2 + \mathcal{O}(k^4 \Lambda^{-5/2})$$

Cutoff independence has also been checked numerically

Original W counting for (attractive) triplet channels

e.g., $^3P_0$

LO	OPE
NLO	$C_2\, p^2$
NNLO	$C_4\, p^4$

Modified PC for (attractive) triplet channels

e.g., $^3P_0$

LO	OPE
NLO	$C_2\ p^2$
NNLO	$C_4\ p^4$

Modified PC for (attractive) triplet channels

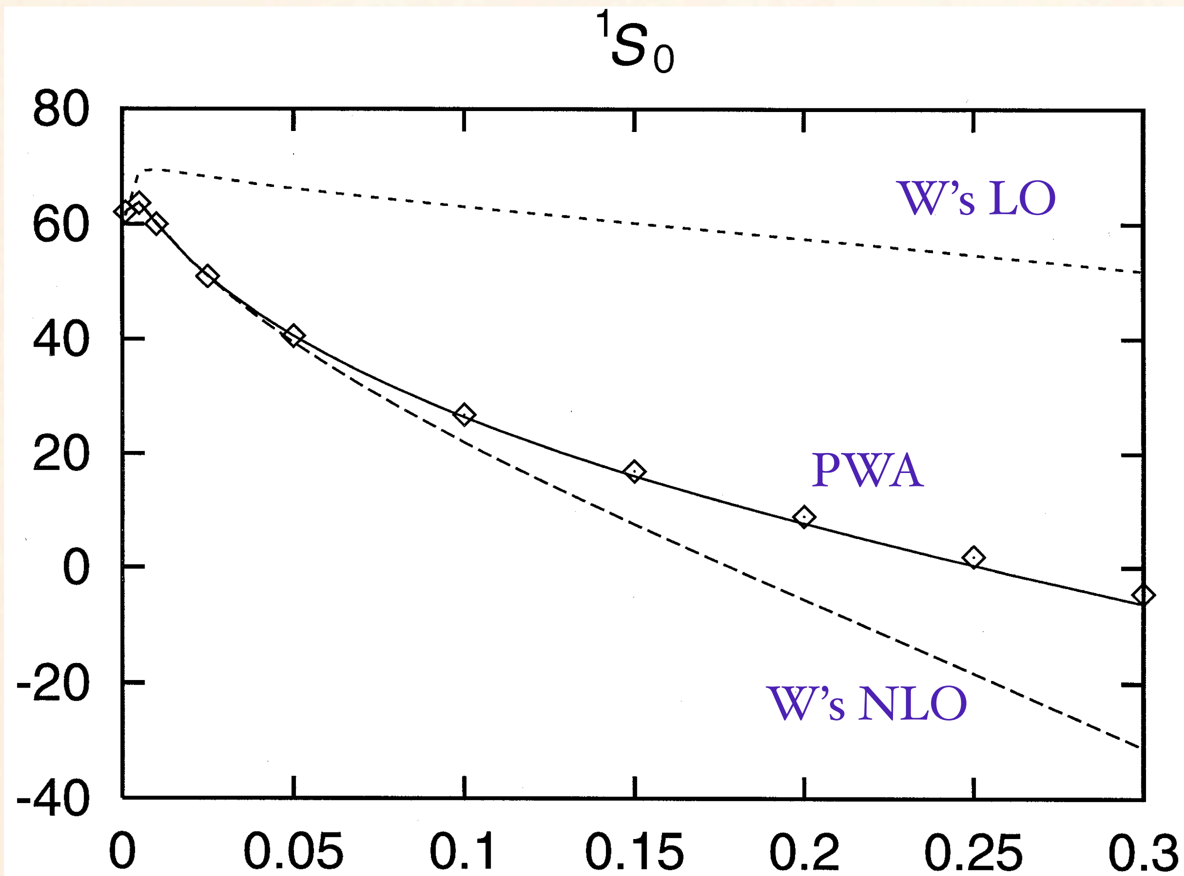
e.g., 3P0

LO	OPE C2 p^2
NLO	C4 p^4
NNLO	

Fine tuning in 1S0

1So phase shifts: comparison of EFT and PWA

Epelbaum et al (1999)



Tlab

LO pot. = OPE + Co

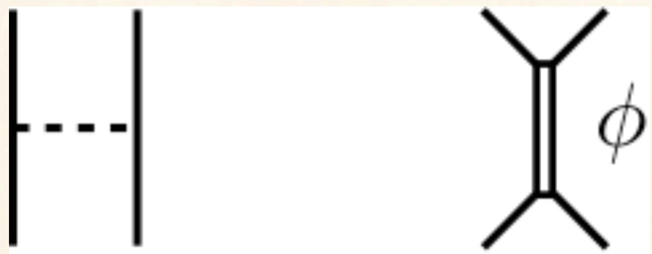
Large NLO correction suggests fine tuning of NLO counterterm --- C2 p^2

Promoting C2?

LO pot. = OPE + Co + C2 p^2 does not work for renormalization

A trick

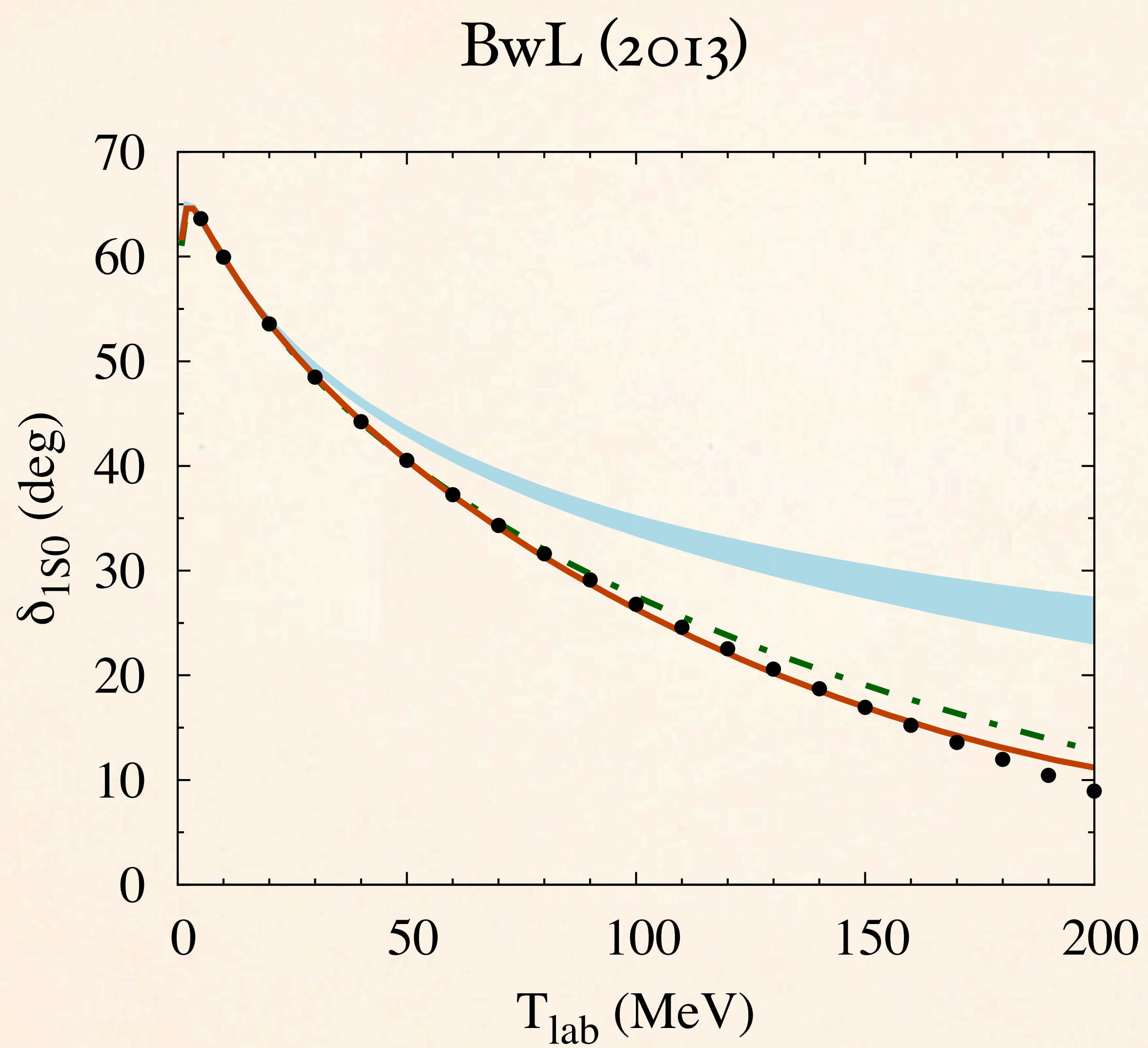
Introduce s-channel exchange by an auxiliary dibaryon field



Kaplan (1996), with modification by BwL (2013)

$$V^{(0)} = V_{Yukawa} + \frac{\sigma y^2}{E + \Delta} \longrightarrow T^{(0)}_{1S0} = T_{Yukawa} + \frac{\chi^2(k;k)}{\frac{E+\Delta}{\sigma y^2} - I_k}$$

Improved convergence



Blue band: LO

Green: NLO

Orange: NNLO

Summary

1. Weinberg's scheme for chiral nuclear forces needs modifications

2. Some of the NN contact operators need promotions
   - In attractive triplet channels, due to renormalization
   - In 1S0, due to fine tuning of underlying theory

3. Any other constraints?