The Seventh International Symposium on Chiral Symmetry in Hadrons and Nuclei

Tensor force and Deltas for the structure of light nuclei

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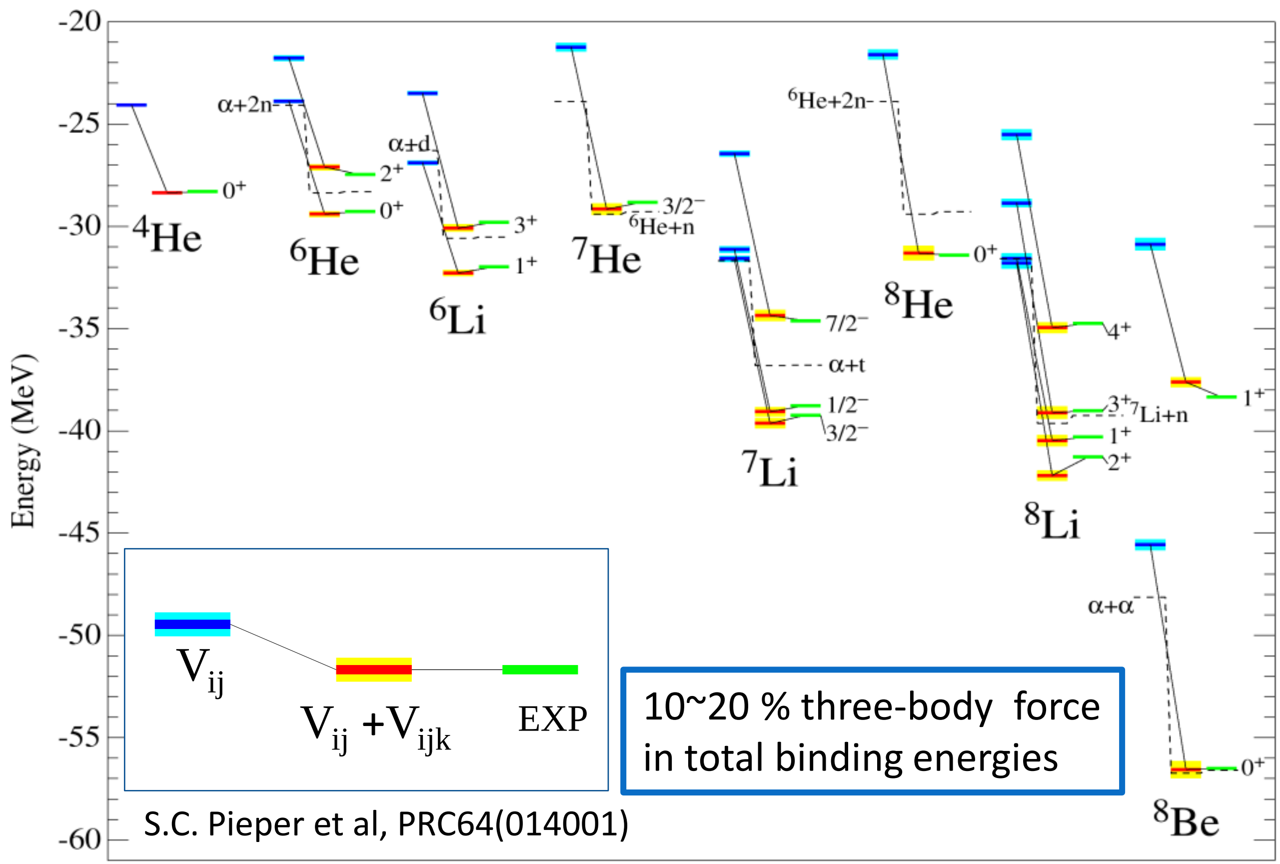
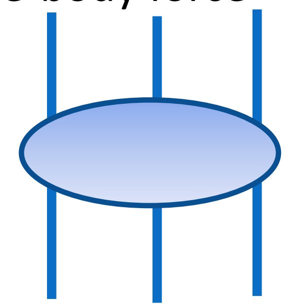
10/27~10/30 at BeiHang Univ, Beijing, China

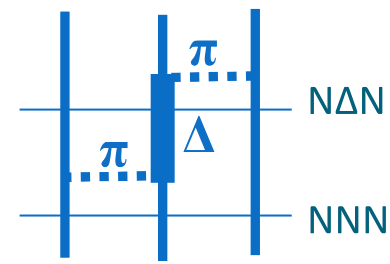
Three-body force and Δ(1232)



Phenomenological three-body force



Fujita-Miyazawa three-body force model



Delta can be generated by π exchange and play important roles in 3-body force.

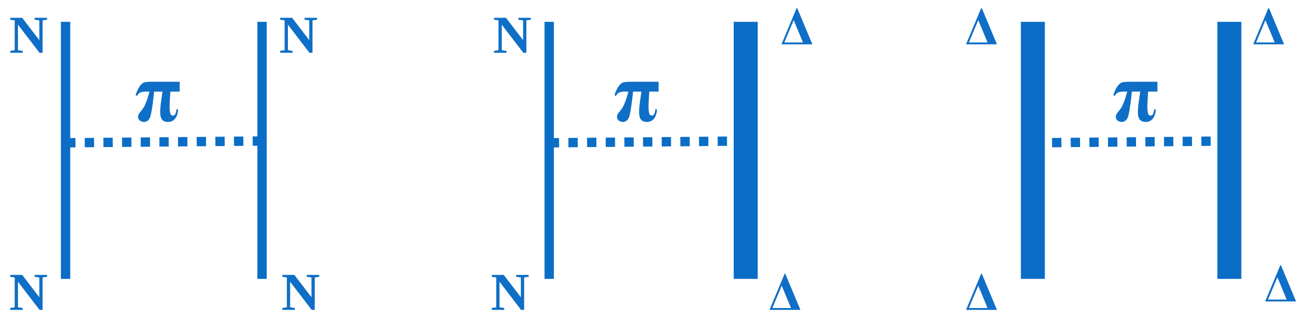
Delta（1232） Spin S=3/2 , Isospin T=3/2

We introduce Δ in two-body force as the origin of three-body force.

Treatment of the delta state

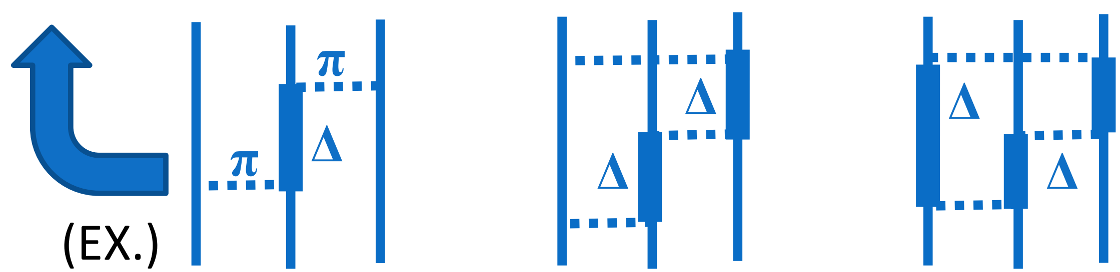
We add the delta degrees of freedom in two-body interaction.

(EX.)



NN channel NΔ channel ΔΔ channel

Many-body force processes can be treated by the two-body correlations with delta.



Two-body NN interaction with delta degrees of freedom → AV28 potential

We study the three-body system with delta degrees of freedom.

Effect of three-body force ? Tensor force from delta ?

AV28 potential

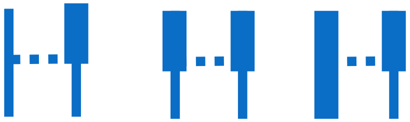
R.B. Wiringa et al, PRC 29, 1207(1984)   AV14 & AV28

AV14→bare NN interaction, 14 operators term
AV28→bare two-body interaction with Δ degree of freedom
NN,NΔ,ΔΔ channel , 28 operators term

Structure of Δ part potential

NΔ、ΔΔ without transition $V=V_S+V_I+V_\pi$ (Ex.)

N↔Δ transition $V=V_\pi$ (Ex.)



V_S+V_I The short- and intermediate range phenomenological part : same NN potential
V_π Pion exchange part : Yukawa & Tensor type function

Coupling constant $(f^2_{\pi N\Delta}/4\pi)=4(f^2_{\pi NN}/4\pi)$, $(f^2_{\pi\Delta\Delta}/4\pi)=0.04(f^2_{\pi NN}/4\pi)$

Brown and Weise, PRC(22)279 (1975)

Reproduce the deuteron properties and NN scattering phase-shifts

AV28 potential

R.B. Wiringa et al, PRC 29, 1207(1984) AV14 & AV28

AV14→bare NN interaction, 14 operators term

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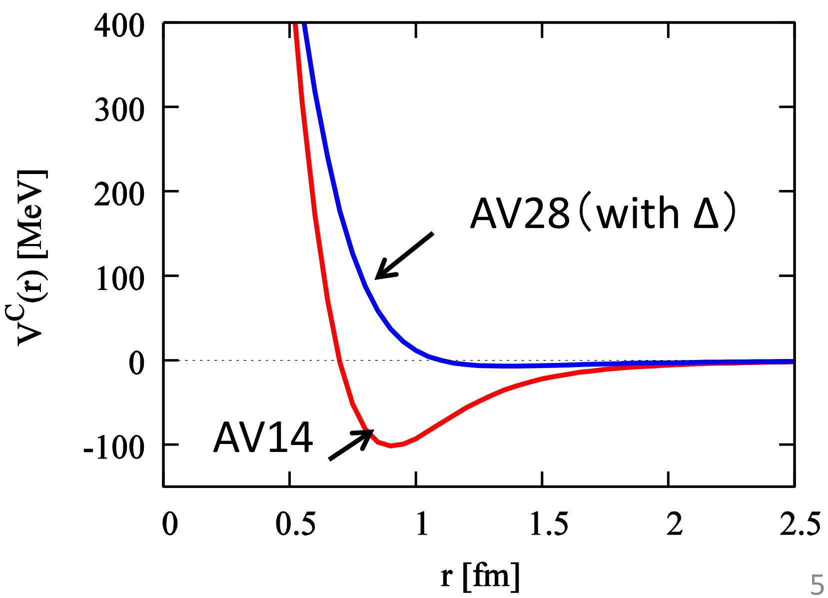
NN,NΔ,ΔΔ channel , 28 operators term

Structure of Δ part potential

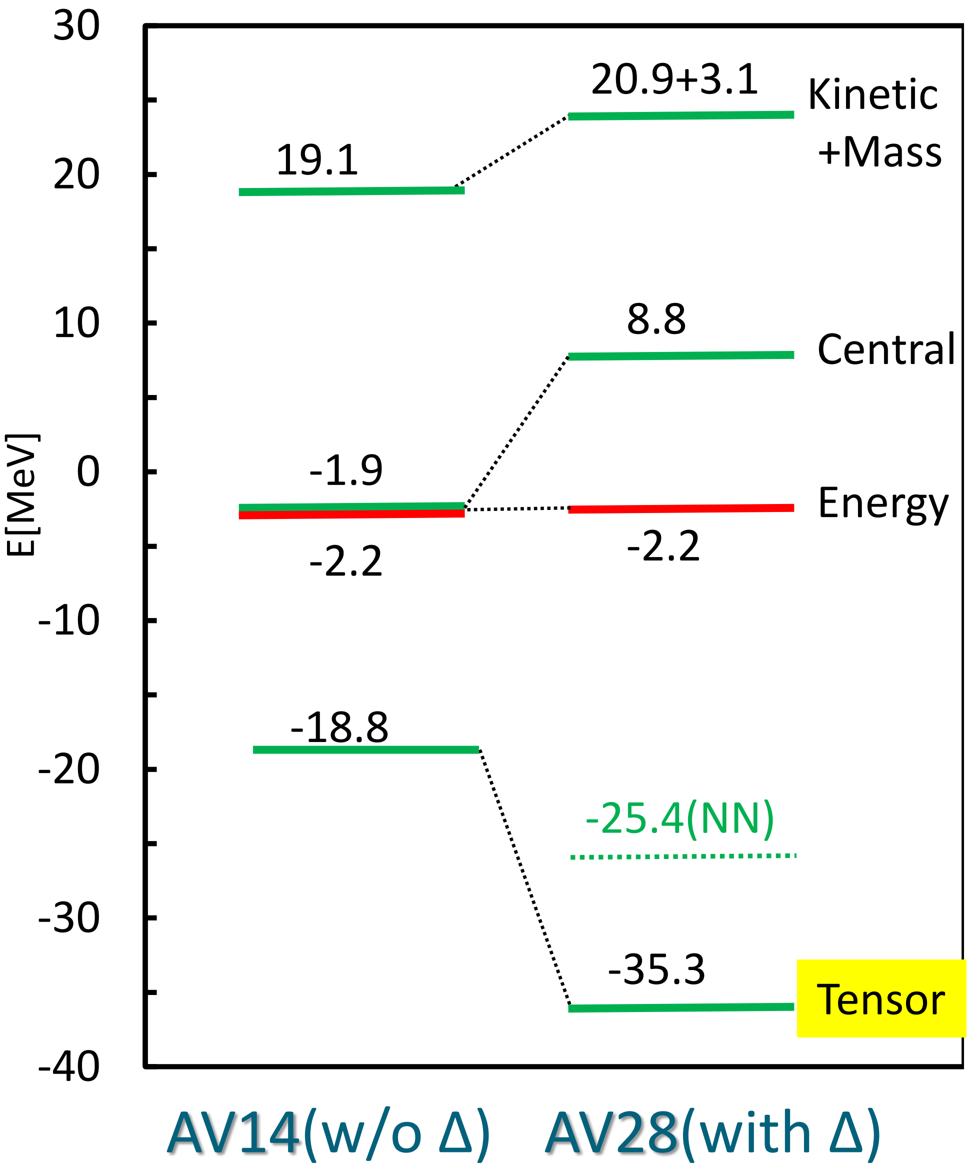
NΔ、ΔΔ without transition $V = V_S + V_I + V_\pi$

N↔Δ transition $V = V_\pi$

Difference of the central force (S=1,T=0 (^3E)) between AV 14 & AV28→



Effect of Δ in deuteron

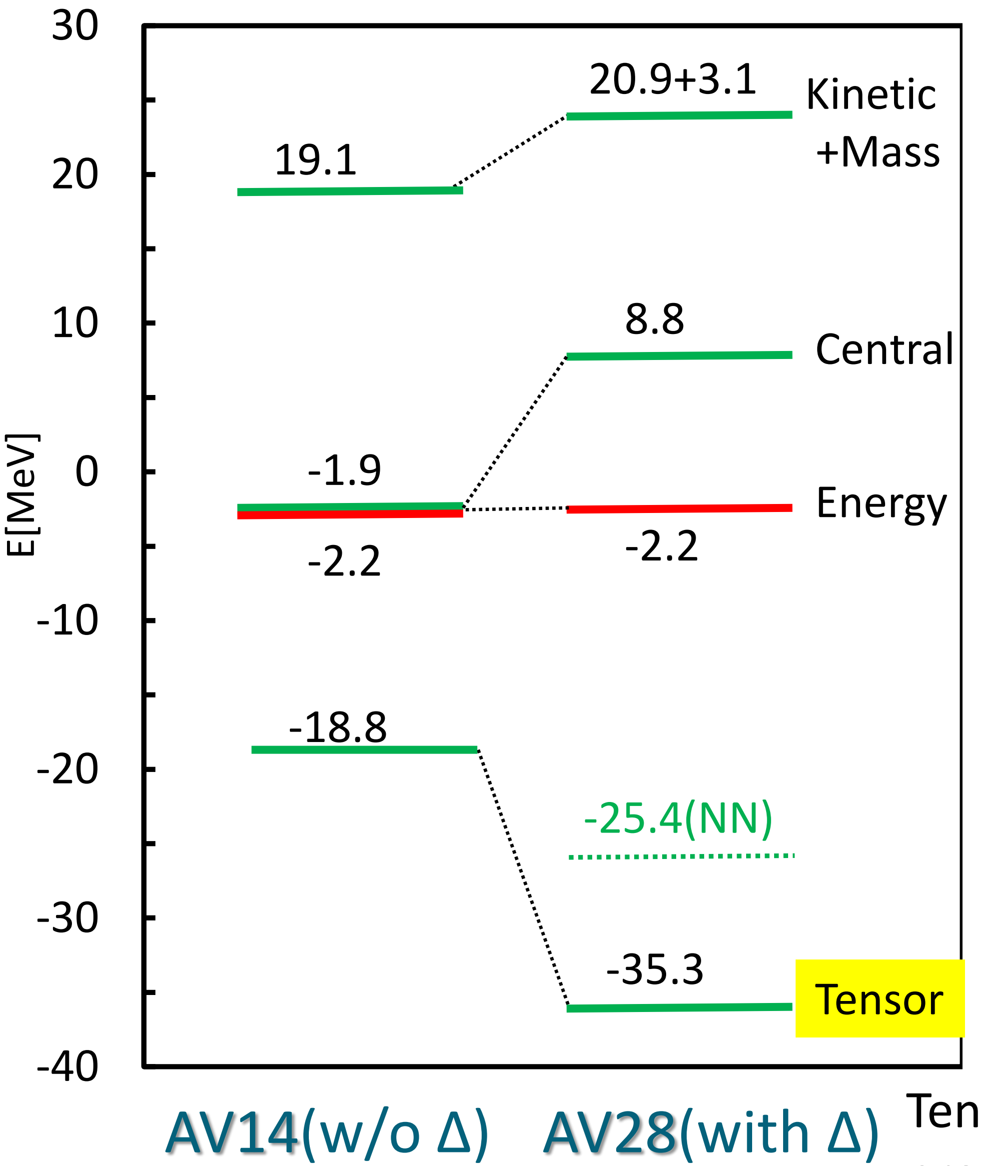


Deuteron 1^+	AV14	AV28
L・S	0.4	0.8
L^2	3.1	3.6
$(L\cdot S)^2$	- 4.0	- 4.1
P_{NN} $[^3S_1]$ %	93.9	93.3
P_{NN} $[^3D_1]$	6.1	6.2
$P_{\Delta\Delta}$ $[^3S_1]$		0.04
$P_{\Delta\Delta}$ $[^3D_1]$		0.02
$P_{\Delta\Delta}$ $[^7D_1]$		0.42
$P_{\Delta\Delta}$ $[^7G_1]$		0.04

Wave function

$$\Psi_{NN} = |^3S_1\rangle + |^3D_1\rangle$$
$$\Psi_{\Delta\Delta} = |^3S_1\rangle + |^3D_1\rangle + |^7D_1\rangle + |^7G_1\rangle$$

Effect of Δ in deuteron



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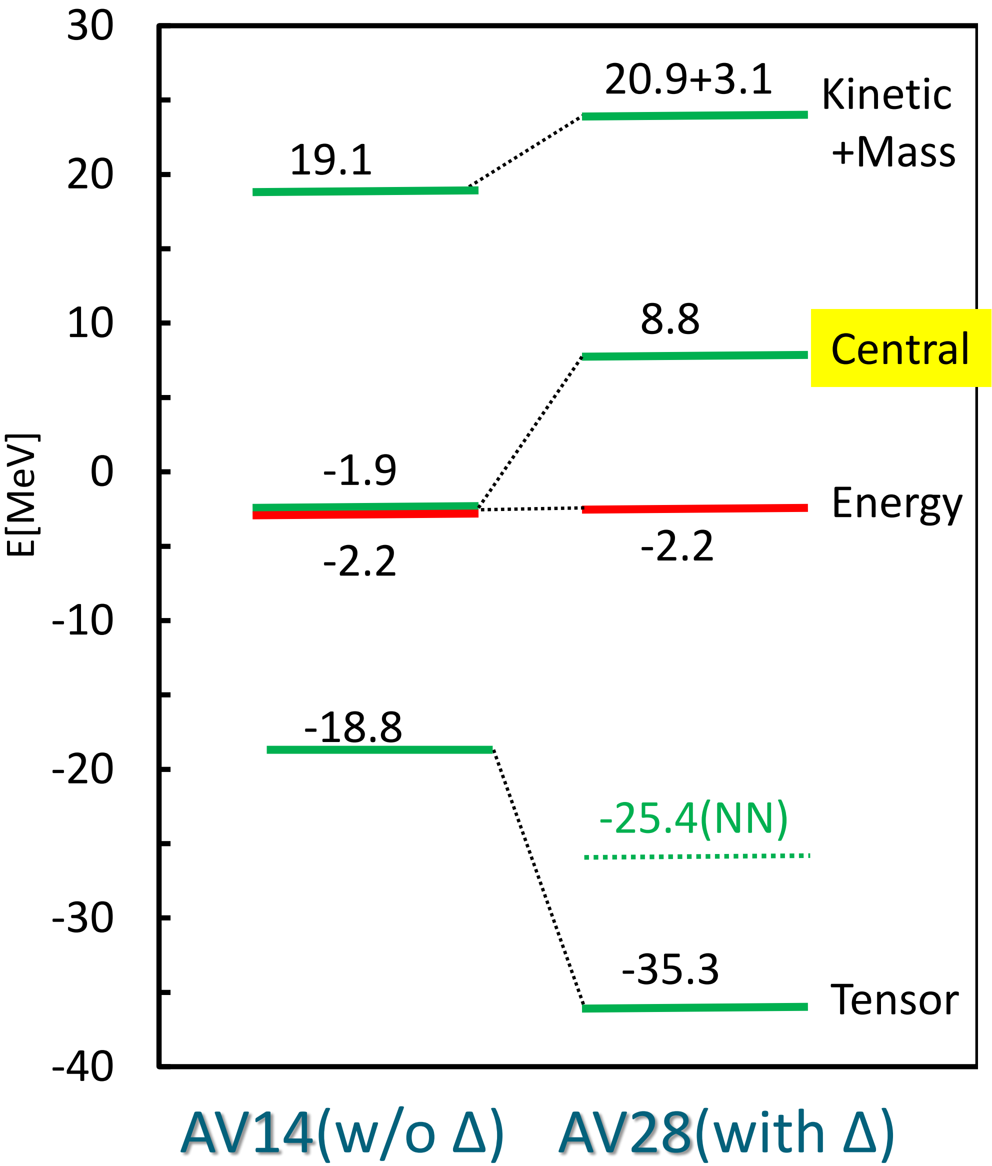
$$\Psi_{\Delta\Delta} = |^3S_1\rangle + |^3D_1\rangle + |^7D_1\rangle + |^7G_1\rangle$$

Tensor couplings between $^3S_1$(NN) and $^7D_1$(ΔΔ) provide larger attractions.

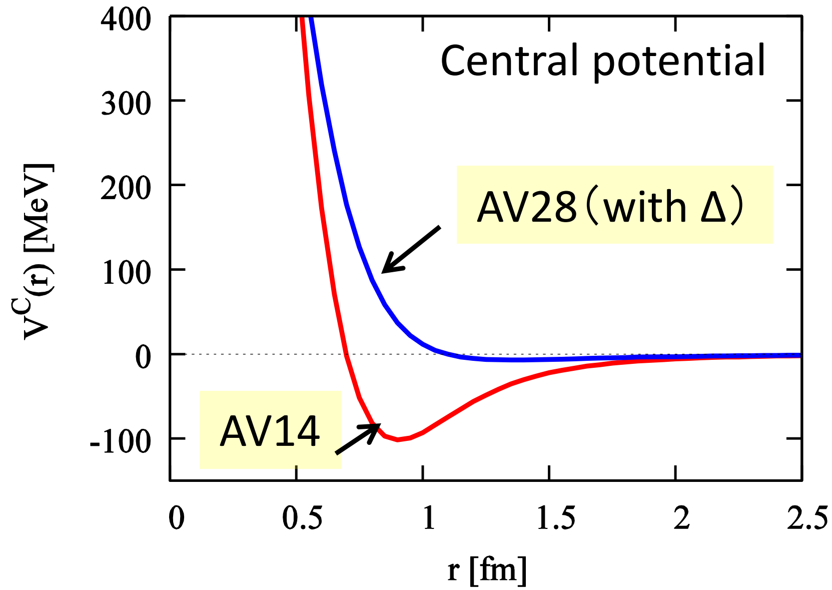
Effect of Δ in deuteron



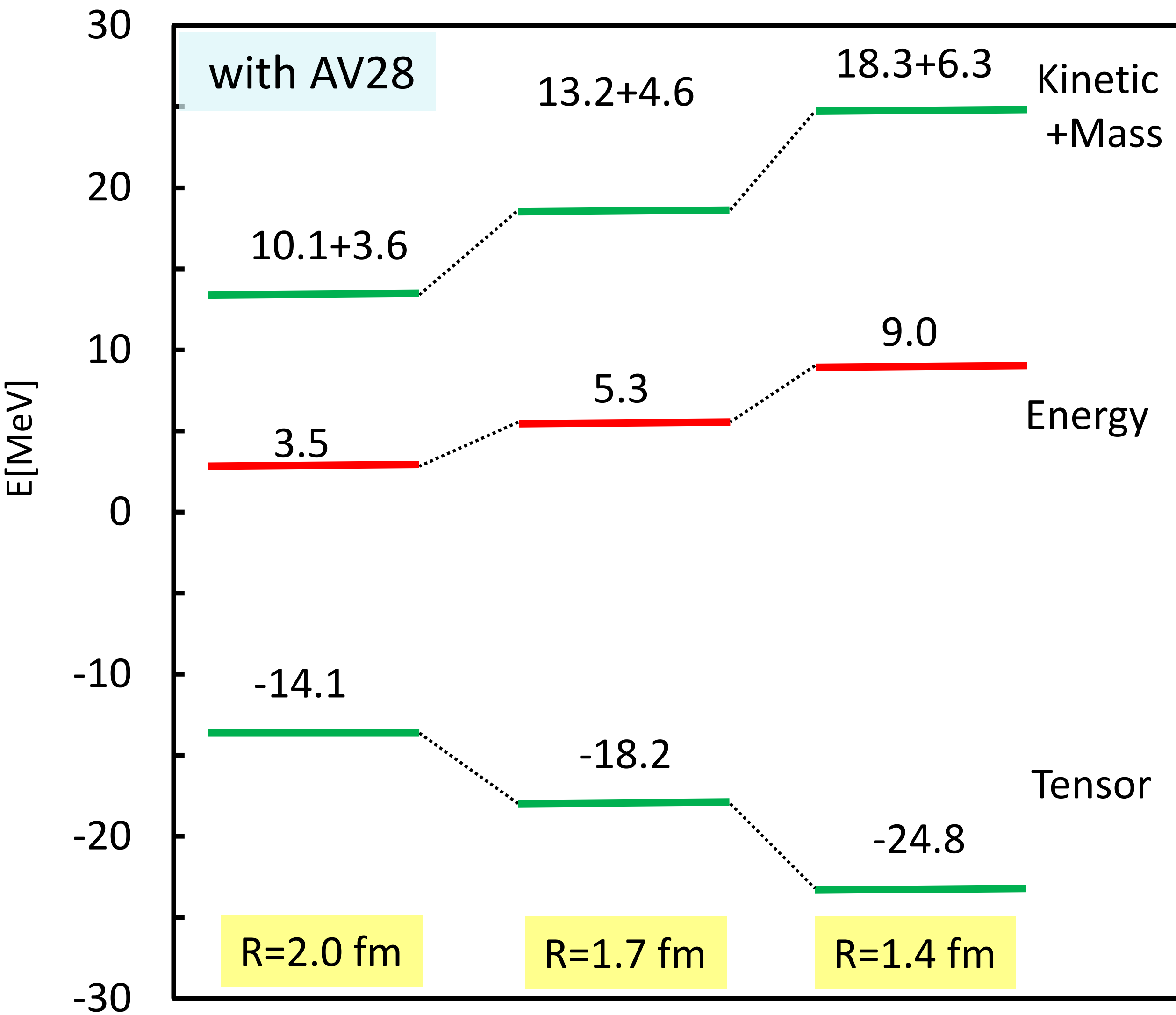
Central force components

AV14 = −1.9 MeV (attractive)

AV28 = 8.8 MeV (repulsive)



Effect of Δ in ^1E channel (T=1)



Wave function

$$\Psi_{NN} = |{}^1S_0\rangle$$
$$\Psi_{\Delta\Delta} = |{}^1S_0\rangle + |{}^5D_0\rangle$$
$$\Psi_{N\Delta} = |{}^5D_0\rangle$$

Hamiltonian
with radius constraint

$$\widetilde{H} = H + \lambda \hat{R}^2$$

$\langle S(NN)|V_T|D(N\Delta)\rangle$
$\langle S(NN)|V_T|D(\Delta\Delta)\rangle$

Previous work for triton including Δ

Two groups have studied ^3H including delta degree of freedom by Faddeev caluclation.

1.Hannover Group (A. Deltuva et al PRC68,024005,(2003))

NN＋NΔ model space with Bonn potential → (E_B= - 8.2 MeV)

ΔΔ states are not taken into account !!

2. Los Alamos Group (A.Picklesminer et al [1]~[5])

NN+NΔ+ΔΔ model space with AV28 potential → (E_B= - 7.3MeV, E_B(exp)= -8.4 MeV)

Model space restriction ? They truncated L=4 (G-wave) contributions in ΔΔ state.

Deuteron wave function

$$\Psi_{NN} = |^3S_1\rangle + |^3D_1\rangle$$

$$\Psi_{\Delta\Delta} = |^3S_1\rangle + |^3D_1\rangle + |^7D_1\rangle + |^7G_1\rangle$$

← 0.04 %

Transitions between D-wave(NN) and G-wave (ΔΔ) are not negligible !!

$$\langle\Psi_S|S_{12}|\Psi_G\rangle = 0$$

$$\langle\Psi_D|S_{12}|\Psi_G\rangle \neq 0$$

NO G-wave	E_B = −1.6 MeV, Tensor= −28 MeV
Full	E_B = −2.2 MeV, Tensor= −35 MeV

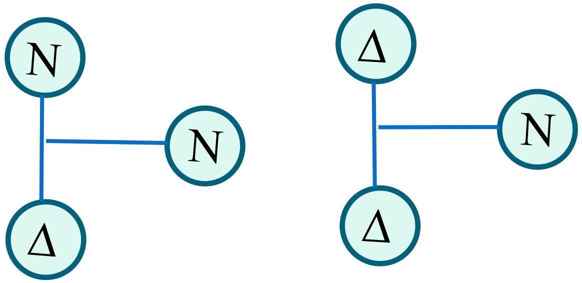
[1]PRC44,1359('91),[2]PRC45,547('92),[3]PRC45,2045('92),[4]PRC,2624('92),[5]PRL,68('92)

Triton including Δ degrees of freedom

^{3}H wave function $\Psi = \Psi_{NNN} + \Psi_{N\Delta N} + \Psi_{\Delta\Delta N}$

$$\Psi_{N\Delta N} = \Psi_{N\Delta_2 N} + \Psi_{N\Delta_3 N} + \Psi_{N\Delta_1 N}$$

$$\Psi_{\Delta\Delta N} = \Psi_{\Delta_1\Delta_2 N} + \Psi_{\Delta_2\Delta_3 N} + \Psi_{\Delta_3\Delta_1 N}$$



We take into account all nucleon can excite the delta state.

Valiational calculation with Stocathtic Variational Method(SVM)

Few Body System 42(2008)33 Y.Suzuki, et al

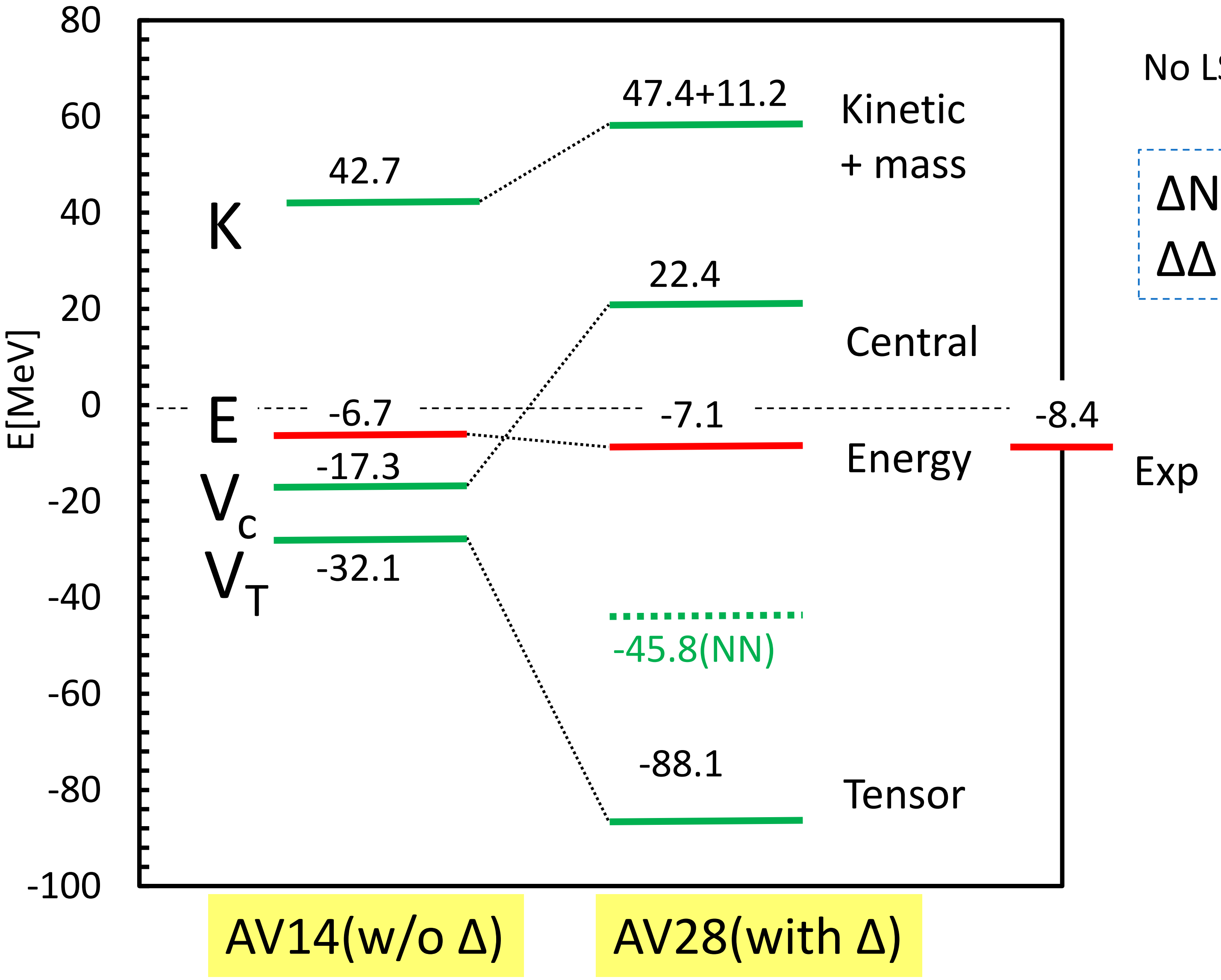
Most important configurations for tensor correlation (TOSM concept)

$$\begin{cases} \Psi_{NNN} = \Psi_S + \Psi_D \\ \Psi_{N\Delta N} = \Psi_S + \Psi_D \\ \Psi_{\Delta\Delta N} = \Psi_S + \Psi_D + \Psi_G \end{cases}$$

Myo, Toki, Ikeda PTP 121 (2009) 511
K.H., Myo, Toki, Ikeda PTP 127 (2011) 1109

Result ^3H with AV14 & AV28

^3H $J^\pi = 1/2^+$

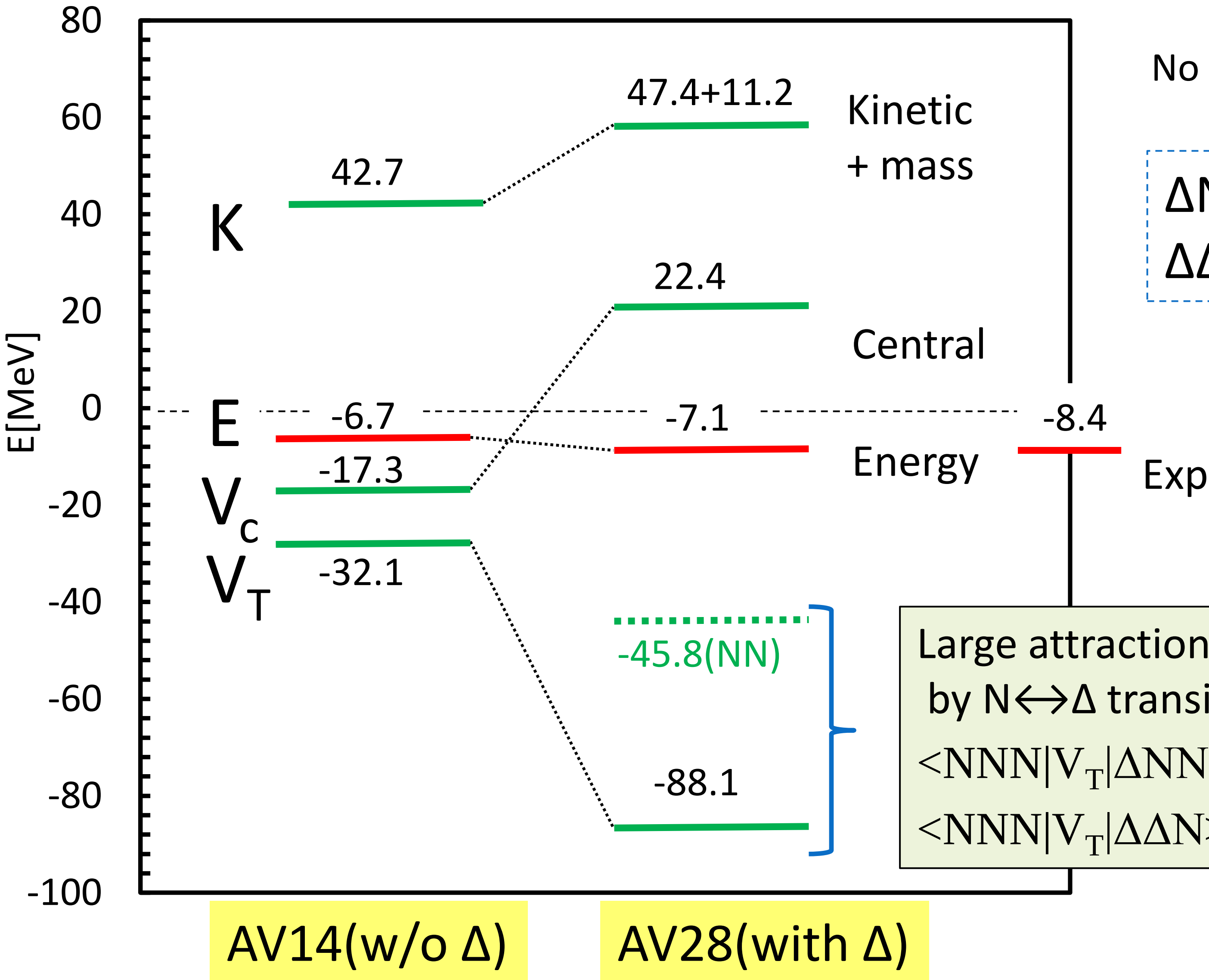


No LS, L^2,LS^2 forces

ΔNN 1.5 [%]
ΔΔN 1.1 [%]

Result ^{3}H with AV14 & AV28

^{3}H $J^\pi = 1/2^+$



No LS, L^2, LS^2 forces

ΔNN 1.5 [%]

ΔΔN 1.1 [%]

Large attractions are generated by N⟺Δ transition

$\langle NNN|V_T|\Delta NN\rangle \sim$ -22MeV

$\langle NNN|V_T|\Delta\Delta N\rangle \sim$ -22MeV

Summary & Outlook

1. As the origin of many-body force, we introduced the delta degrees of freedom in two-body interaction .

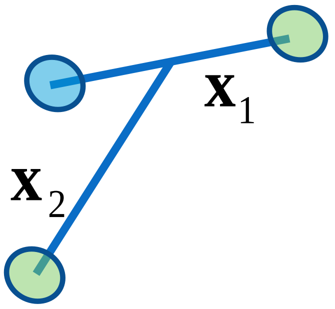
2. We used the two-body interaction AV28 including delta.

3. The transition between nucleon and delta provide large tensor correlations in deuteron and triton.

4. In the triton result, the tensor component extremely large and the changes of hamiltonian components are similar to the deuteron results.

5. We will develop the treatment of the delta state in the frameworks of TOFM・TOSM・EBHF.

Tensor Optimized Few-body Model for s-shell



Jacobi coordinate x_1,x_2

TOFM w.f. $$|\Psi\rangle = |\Psi\rangle_S + |\Psi\rangle_D$$

$$\Psi = \sum_i C_i \mathcal{A}[\psi_L^{space} \chi_S^{spin}]_J \chi_T^{isospin}$$

For ^{4}He Total J=0, S-wave (L=0,S=0), D-wave(L=2,S=2)

Basis function Correlated gaussian basis with the global vector

Few Body System 42(2008)33 Y.Suzuki, et al

$$\psi_L^{space} = \exp(-\frac{1}{2}\tilde{\mathbf{x}}A\mathbf{x})|\tilde{u}\mathbf{x}|^L Y_{LM}(\tilde{u}\mathbf{x})$$

Correlated gaussian $$\tilde{\mathbf{x}}A\mathbf{x} = \sum_{i,j}^{N-1} A_{ij}\mathbf{x}_i \cdot \mathbf{x}_j$$

global vector $$\tilde{u}\mathbf{x} = \sum_{i=1}^{N-1} u_i \mathbf{x}_i$$

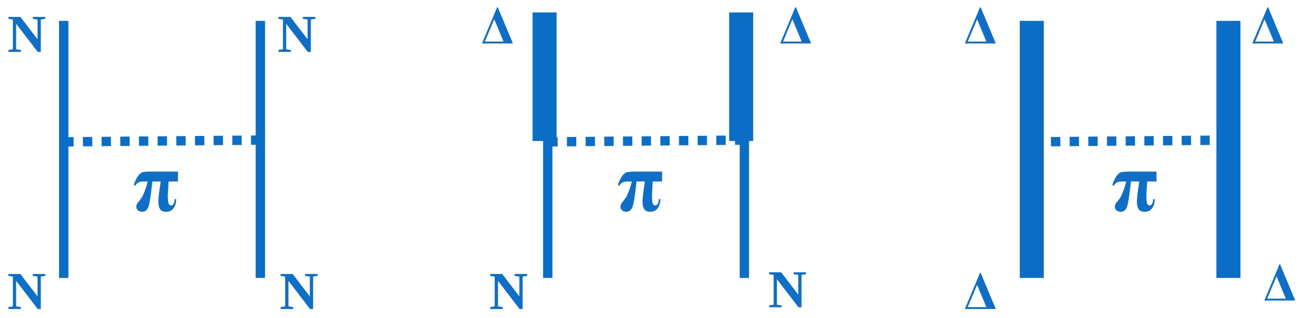
Variational calculation with the Stochastic Variational Method (SVM)

*All parameter of basis function are decided to obtain the energy minimum.
*The amplitude are decided by diagonalization.

Effect of Δ in deuteron

We would like to discuss the effect of delta in deuteron.

Deuteron $J^\pi = 1^+$, T=0 (No NΔ channel)



Wave function $$\Psi = \Psi_{NN} + \Psi_{\Delta\Delta}$$

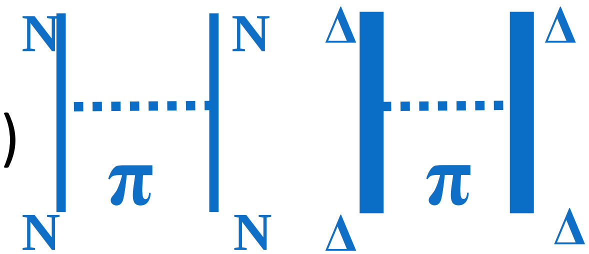
$$\Psi_{NN} = |^3S_1\rangle + |^3D_1\rangle$$
$$\Psi_{\Delta\Delta} = |^3S_1\rangle + |^3D_1\rangle + |^7D_1\rangle + |^7G_1\rangle$$

Hamiltonian $$H = (m_1 + m_2 - 2m_N) + \frac{\hbar^2 (m_1 + m_2)}{2m_1 m_2} \nabla_{12}^2 + V_{12}$$

Effect of Δ in two-body system

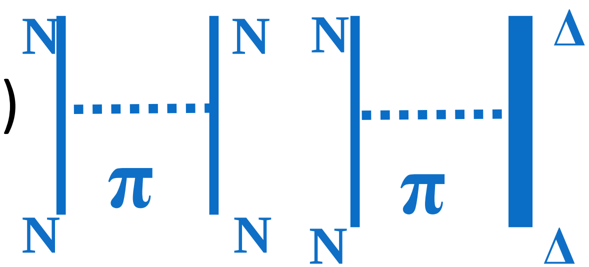
Effects of large tensor force by the transition between N and Δ

^{3}E channel (T=0)



$$\langle \Psi_{NN} | V_T | \Psi_{\Delta\Delta} \rangle \sim \underline{-10MeV}$$

^{1}E channel (T=1)



$$\langle \Psi_{NN} | V_T | \Psi_{N\Delta} \rangle \sim \underline{-8MeV}$$

NΔ states (T=1 channel) also generate large tensor correlations.

→ These transitions play important roles in three-body system.

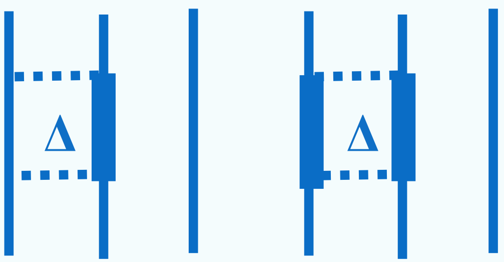
Effect of three-body force by Δ

We analyze the effect of three-body force by the Δ excitation.

We separate all the Δ excitation processes into 2-body & 3-body force contributions.

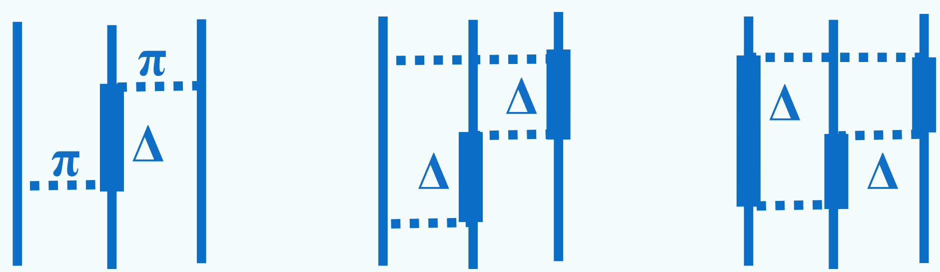
$$\langle \Psi | \sum_{i,j} V_{ij} | \Psi \rangle = \langle 2body \rangle + \langle 3body \rangle$$

2-body processes



The last one nucleon does not take part in the interaction.

3-body processes



All nucleon contributes to the Δ excitations.

$\Lambda N - \Sigma N$ coupling E. Hiyama et al, PRC65, 011301(R)

Feshbach projection operator method

Projection operator P and Q

Normalization $P + Q = 1$

Wave function with delta

$$\Psi = C\Psi_N + D\Psi_\Delta$$

$$P\Psi = C\Psi_N$$ Nucleon states

$$Q\Psi = D\Psi_\Delta$$ Delta states

Schroedinger equation $H\Psi = E\Psi$

$$PHP\Psi + PVQ\Psi = EP\Psi$$
$$QVP\Psi + QHQ\Psi = EQ\Psi$$

$\Rightarrow$

$$H^P C + GD = EC$$
$$H^Q D + G^\dagger C = ED$$

We eliminate the D using the second equation and obtain,

$$\left(H^P - G(H^Q - E)^{-1}G^\dagger - E\right) C = 0$$

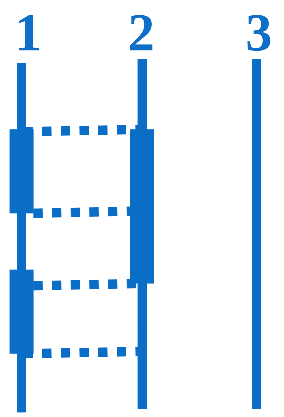
We can obtain the effective potential. $V_{eff} = V - G(H^Q - E)^{-1}G^\dagger$

Separation 2 and 3 body force with PQ method

Projection operator P and Q

$$P\Psi = C\Psi_N$$
$$Q\Psi = D\Psi_\Delta$$

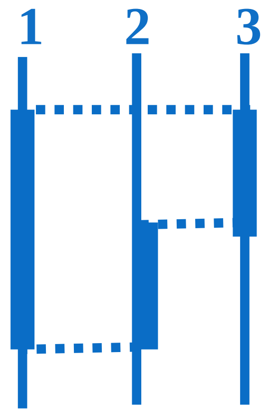
$$\left(H^P - G(H^Q - E)^{-1}G^\dagger - E\right)C = 0$$
$$G = \langle P|V|Q\rangle \qquad G^\dagger = \langle Q|V|P\rangle$$



Two-body forces are constructed by 1-2 pair interaction

$$\left(H^P - G_{12}(H^Q_{12} - E)^{-1}G^\dagger_{12} - E\right)C = 0$$

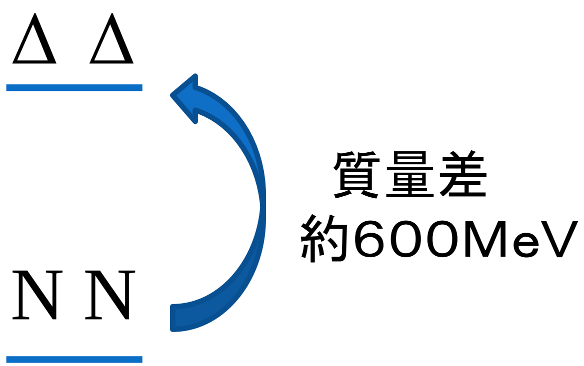
$$G_{12} = \langle P|V_{12}|Q\rangle \quad G^\dagger_{12} = \langle Q|V_{12}|P\rangle \qquad H^Q_{12} = \langle Q|H_{12}|Q\rangle$$



Three-body forces are constructed by 1-2 and other pairs interaction

$$\left(H^P - G_{12}(H^Q_{23} - E)^{-1}G^\dagger_{13} - E\right)C = 0$$

ダブルΔに効くテンソル力



$$\Psi_{NN} = |{}^3S_1\rangle + |{}^3D_1\rangle$$

$$\Psi_{\Delta\Delta} = |{}^3S_1\rangle + |{}^3D_1\rangle + |{}^7D_1\rangle + |{}^7G_1\rangle$$

NNの$^3S_1$からダブルΔの$^7D_1$に働く
テンソル力が大きい

テンソル力の行列要素から評価すると、

$$\left\langle {}^7D_{\Delta\Delta} \,|\, v_\pi S_{12} | {}^3S_{NN} \right\rangle \sim (f^2_{\pi N\Delta}/4\pi) M_S \sim 4.0$$

$$\left\langle {}^3D_{\Delta\Delta} \,|\, v_\pi S_{12} | {}^3S_{NN} \right\rangle \sim (f^2_{\pi N\Delta}/4\pi) M_S \sim 0.7$$

$$\left\langle {}^3D_{\Delta\Delta} \,|\, v_\pi S_{12} | {}^3S_{\Delta\Delta} \right\rangle \sim (f^2_{\pi\Delta\Delta}/4\pi) M_S \sim 0.1$$

結果 – ^{1}Odd チャンネル ---

J=1	Radius 1.70 fm
Energy [MeV]	25.6
Kinetic	23.25(NN=23.16)
Central	2.76(NN=2.79)
Tensor	-0.92
L・S	------
L^2	------
$(L\cdot S)^2$	------
P_{NN} $[^1P_1]$ %	99.93
$P_{\Delta\Delta}$ $[^1P_1]$	0.005
$P_{\Delta\Delta}$ $[^5P_1]$	0.032
$P_{\Delta\Delta}$ $[^5F_1]$	0.029

^{1}O チャンネル L=odd, S=even, T=0

$$\Psi_{NN} = |^1P_1\rangle$$

$$\Psi_{\Delta\Delta} = |^1P_1\rangle + |^5P_1\rangle + |^5F_1\rangle$$

ダブルΔとのテンソル力は、
ほとんど働かない。
3E（重陽子）で経験したように、
スピンは[3/2×3/2]=3の状態に
組むのがすき。2に組む状態は、
好まれない。

結果 – ^{3}Odd チャンネル ---

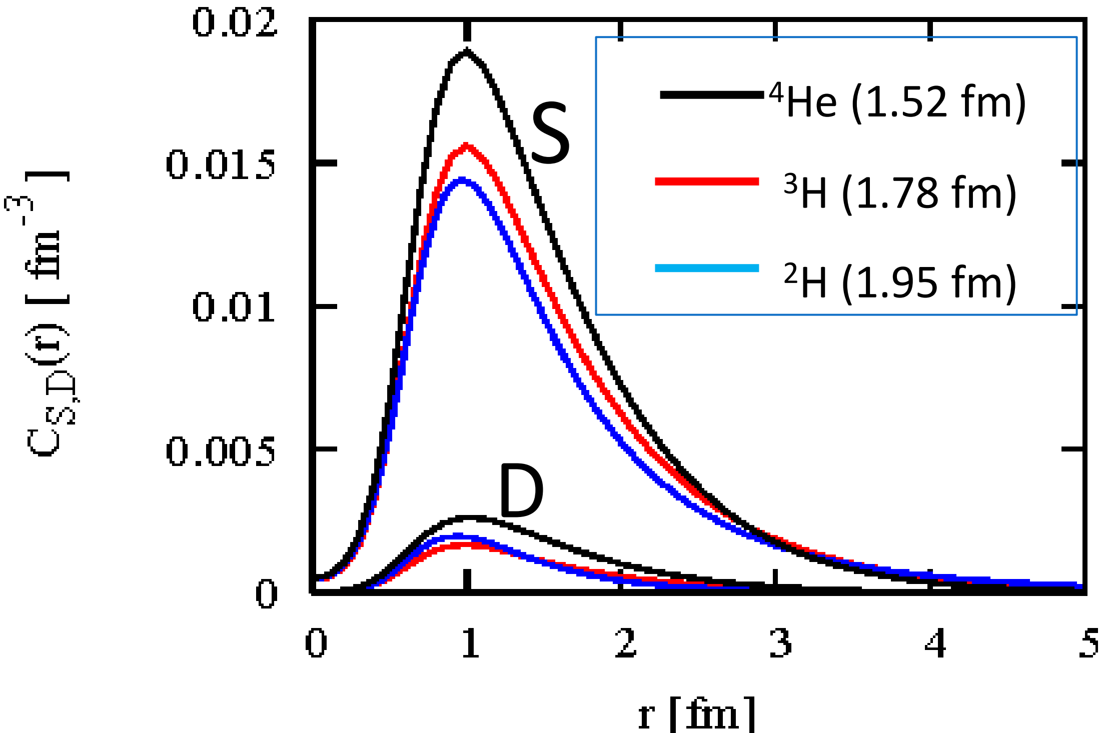
^{3}Oチャンネル　L=odd, S=odd, T=1

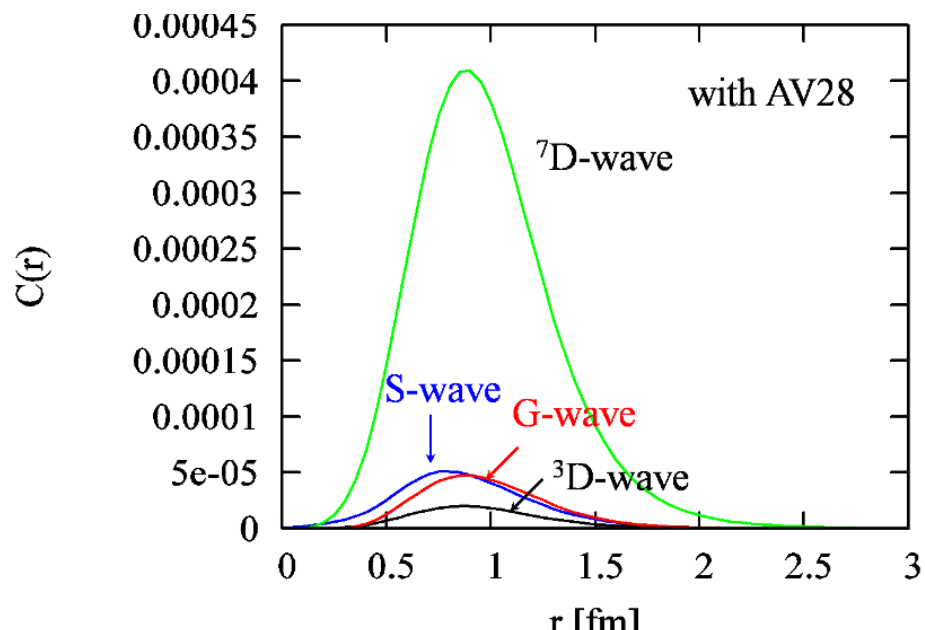
J=0	Radius 1.70 fm	NΔなし
Energy [MeV]	20.1	20.4
Kinetic	23.18(NN=22.99)	23.19 (NN=23.07)
Central	1.27(NN=1.31)	1.34 (NN=1.32)
Tensor	**-6.87**(NN=-5.41)	**-6.26**(NN=-5.31)
L・S	1.12	1.06
L^2	-0.07	-0.07
$(L・S)^2$	0.82	0.77
P_{NN} [$^3P_0$] %	99.85	99.94
$P_{\Delta\Delta}$ [$^3P_0$]	0.0004	0.0009
$P_{\Delta\Delta}$ [$^7F_0$]	0.06	0.05
$P_{N\Delta}$ [$^3P_0$]	0.08	------

$$\Psi_{NN} = |^3P_0\rangle$$
$$\Psi_{\Delta\Delta} = |^3P_0\rangle + |^7F_0\rangle$$
$$\Psi_{N\Delta} = |^3P_0\rangle$$

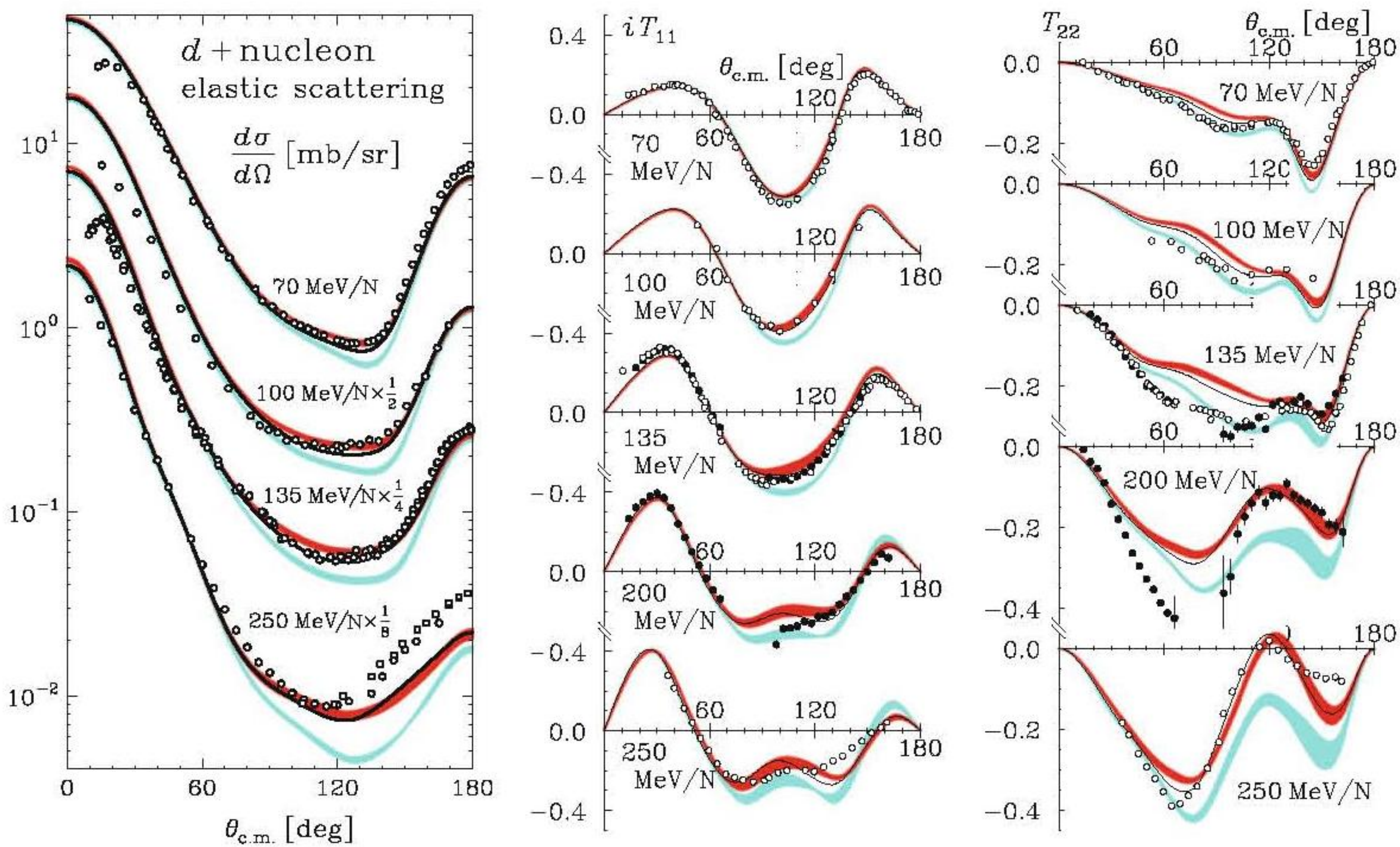
NNの$^3P_0$からダブルΔの$^7F_0$に働くテンソル力が大きい

J=1,2の状態には、テンソル力が効くようなチャネルはなかった


0.02
0.015
0.01
0.005
0
0
1
2
3
4
5
$C_{S,D}(r)$ [fm^{-3}]
r [fm]
S
D
^4He (1.52 fm)
^3H (1.78 fm)
^2H (1.95 fm)


0.00045
0.0004
0.00035
0.0003
0.00025
0.0002
0.00015
0.0001
5e-05
0
0
0.5
1
1.5
2
2.5
3
C(r)
r [fm]
with AV28
^{7}D-wave
S-wave
G-wave
^{3}D-wave

実験にみえる三体力の効果
青線・2体力　赤線・2体力＋3体力



K. Sekiguchi　Few-body System (2012)