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Collective Hamiltonian for chiral modes

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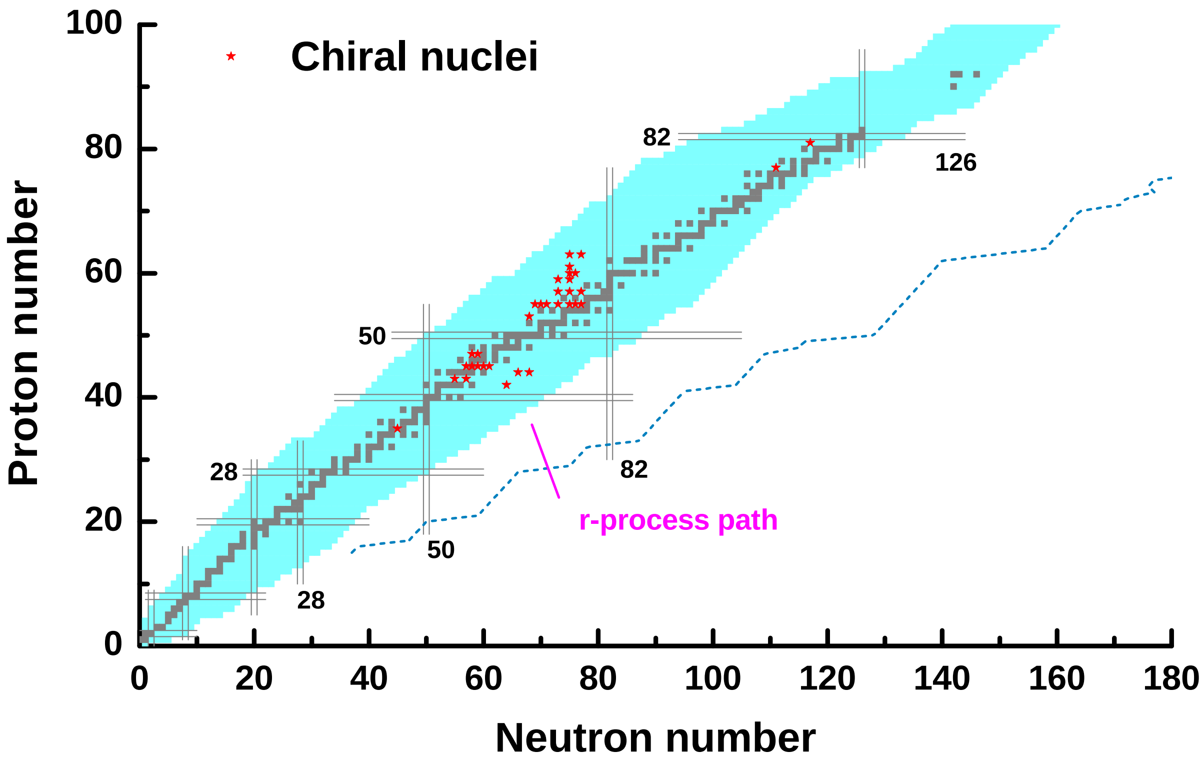
Outline

Introduction

- Chirality is a subject of general interest in molecular physics, elementary particle physics, and optical physics.
- In nuclear physics, the occurrence of chirality was originally suggested in 1997. Frauendorf and Meng1997NPA
- The chirality was firstly observed in the four $N = 75$ isotones: $^{130}\mathrm{Cs}$, $^{132}\mathrm{La}$, $^{134}\mathrm{Pr}$, and $^{136}\mathrm{Pm}$ in 2001. Starosta2001PRL
- So far, more than 30 candidate chiral nuclei have been reported experimentally in the $A \sim 80$, 100, 130, and 190 mass regions.



Introduction

- Chiral doublet bands were firstly predicted by tilted axis cranking (TAC) approach and particle rotor model (PRM). Frauendorf and Meng1997NPA
- Numerous efforts have been devoted to the development of both the TAC methods and PRM models.
- PRM: Peng2003PRC, Koike2004PRL, Zhang2007PRC, Droste2009EPJA, Qi2009PLB, Lawrie2010PLB
  - ✓ quantal model: the total angular momentum is a good quantum number; energies and transition probabilities are treated fully quantally.
  - × deformation parameters have to be assumed at the very beginning.
- TAC: Dimitrov2000PRL, Olbratowski2004PRL, Olbratowski2006PRC, Meng2013Front.Phys.
  - ✓ permits the calculation for the orientation of the density distribution relative to the angular momentum vector; easily extended to the multi-quasiparticle case.
  - × cannot give the quantum tunneling of chiral doublet bands.
- TAC+RPA: Mukhopadhyay2007PRL, Almehed2011PRC
  - ✓ go beyond the mean-field approximation.
  - × restricted in the description of the chiral vibration.

Motivation

- **It is imperative to search a unified method for studying both chiral rotation and vibration in the framework of TAC.**
- **In the present work, the collective Hamiltonian for chiral modes have been constructed.**

Theoretical framework

- **Determine the potential term:** starting from the tilted axis cranking model

$$\hat{h}' = \hat{h}_{\text{def}} - \vec{\omega}\cdot\hat{\vec{j}},$$
$$\vec{\omega} = (\omega\sin\theta\cos\varphi, \omega\sin\theta\sin\varphi, \omega\cos\theta). \quad (1)$$

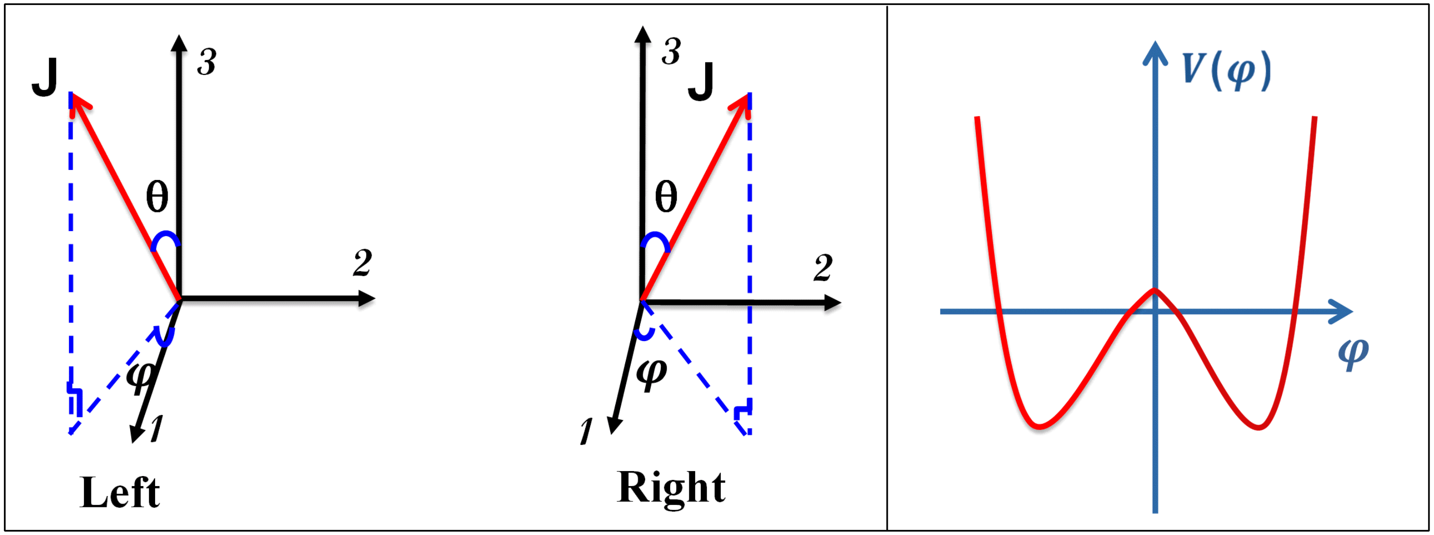
where $\hat{\vec{j}} = \hat{\vec{j}}_\pi + \hat{\vec{j}}_\nu$, and $\hat{h}_{\text{def}} = \hat{h}^\pi_{\text{def}} + \hat{h}^\nu_{\text{def}}$ is the single-j shell Hamiltonian with

$$\hat{h}^{\pi(\nu)}_{\text{def}} = \pm\frac{1}{2}C\Big\{(\hat{j}_3^2 - \frac{j(j+1)}{3})\cos\gamma + \frac{1}{2\sqrt{3}}(\hat{j}_+^2 + \hat{j}_-^2)\sin\gamma\Big\}. \quad (2)$$

Then minimize the total Routhian

$$E'(\theta,\varphi) = \langle h'\rangle - \frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k\omega_k^2, \quad \mathcal{J}_k = \mathcal{J}_0\sin^2(\gamma - \frac{2\pi}{3}k). \quad (3)$$

with respect to θ for given φ to obtain the collective potential $V(\varphi)$.



Theoretical framework

- **Determine the kinetic term:** the kinetic term is $T_{\rm kin} = \frac{1}{2}B\dot{\varphi}^2$ with mass parameter B, which is obtained from the cranking approximation as

$$B = 2\hbar^2 \sum_{l\neq 0} \frac{(E_l - E_0)\left|\frac{\partial\vec{\omega}}{\partial\varphi}\langle l|\hat{\vec{j}}|0\rangle\right|^2}{[(E_l - E_0)^2 - \hbar^2\Omega^2]^2}, \tag{4}$$

in which $|l\rangle$, $|0\rangle$ and E_l, E_0 are respective the eigen states of the cranking Hamiltonian and the corresponding eigenvalues. The Ω is chiral vibrational frequency and can be taken as $\Omega = 0$ for the case of chiral rotation.

- **Collective Hamiltonian:**
  - ✓ Classical form:

$$H_{\rm coll} = T_{\rm kin}(\varphi) + V(\varphi) = \frac{1}{2}B\dot{\varphi}^2 + V(\varphi), \tag{5}$$

where $V(\varphi)$ and B have been obtained from TAC calculation.

  - ✓ Quantal form:

$$\hat{H}_{\rm coll} = -\frac{\hbar^2}{2\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi}\frac{1}{\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi} + V(\varphi), \tag{6}$$

obtained according to the Pauli prescription:

$$\hat{H}_{\rm kin} = -\frac{\hbar^2}{2}\frac{1}{\sqrt{\det B}}\sum_{ij}\frac{\partial}{\partial q_i}\sqrt{\det B}(B^{-1})_{ij}\frac{\partial}{\partial q_j}. \tag{7}$$

Theoretical framework

$$\hat{H}_{\mathrm{coll}} = -\frac{\hbar^2}{2\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi}\frac{1}{\sqrt{B(\varphi)}}\frac{\partial}{\partial\varphi} + V(\varphi)$$

minimize

$$E' = \langle h' \rangle - \frac{1}{2}\sum_{k=1}^{3}\mathcal{J}_k\omega_k^2$$

TAC

$$\hat{h}' = \hat{h}_{\mathrm{def}} - \vec{\omega}\cdot\hat{\vec{j}}$$

$$B = 2\hbar^2\sum_{l\neq 0}\frac{(E_l - E_0)\left|\frac{\partial\vec{\omega}}{\partial\varphi}\langle l|\hat{\vec{j}}|0\rangle\right|^2}{\left[(E_l - E_0)^2 - \hbar^2\Omega^2\right]^2}$$

V(φ)

B(φ)

Collective Hamiltonian

E

ψ

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Numerical details

- Angular moments of valence nucleons: $j_\pi = j_\nu = 11/2\ \hbar$;
- Single-j shell Hamiltonian coefficients: $C_\pi = 0.25$ MeV, $C_\nu = -0.25$ MeV;
- Triaxial deformation: $\gamma = -30^\circ$;
- Moment of inertia: $\mathcal{J}_0 = 40\ \hbar^2/\mathrm{MeV}$;
- Potential energy surface mesh points (θ_i, φ_j) is represented as

$$\theta_i = (i-1)\times 1^\circ, (i=1,...,91),$$
$$\varphi_j = (j-91)\times 1^\circ, (j=1,...,181).$$

Total Routhian

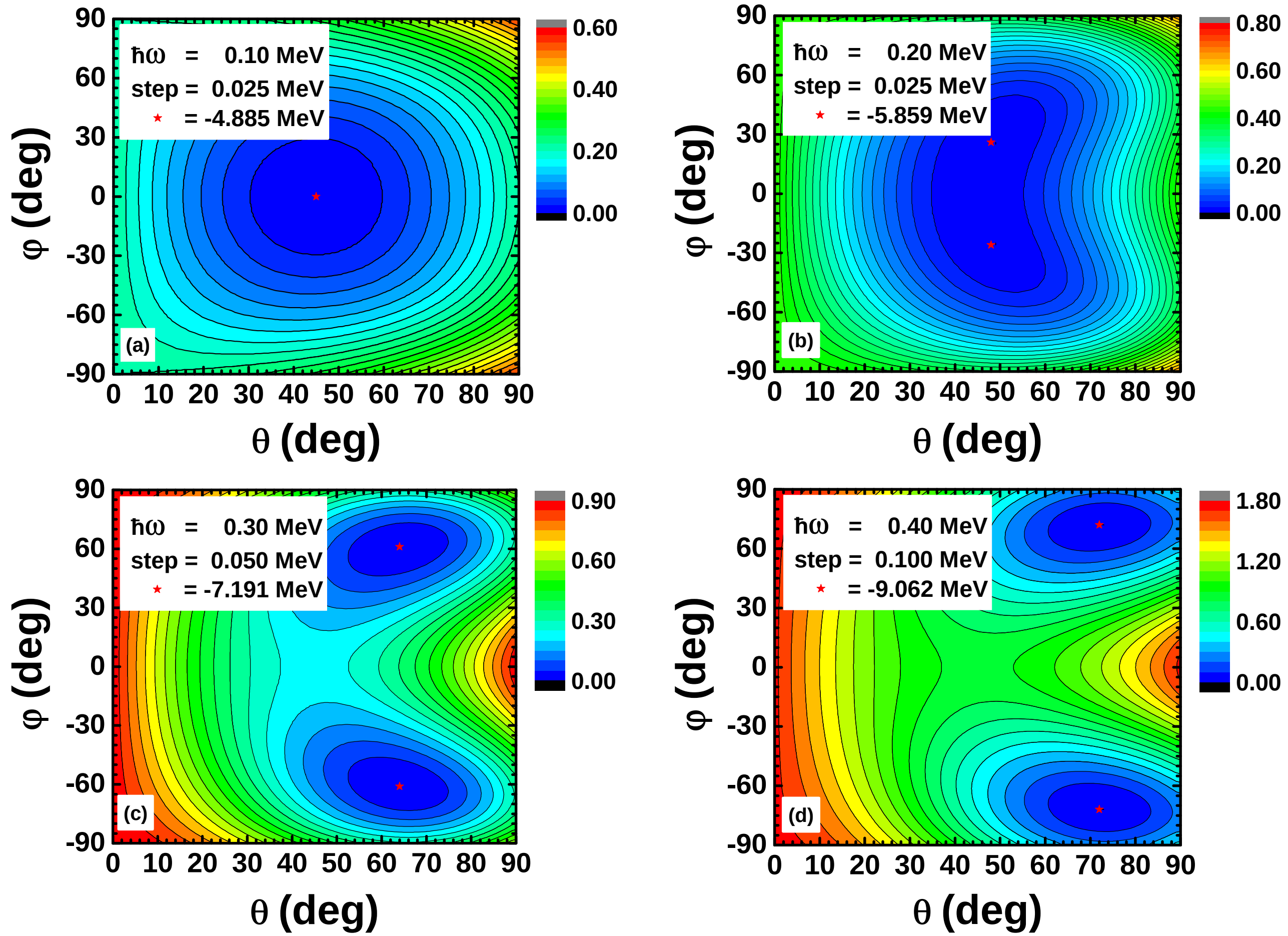

Figure: Total Routhian surface calculations for the $h_{11/2}$ proton particle and the $h_{11/2}$ neutron hole coupled to a triaxial rotor with $\gamma = -30^\circ$ at the frequencies $\hbar\omega = 0.1$, 0.2, 0.3, 0.4 MeV.

Remarks

a. Potential energy surfaces are symmetrical with the $\varphi = 0^\circ$.

b. The minimal points change from $\varphi = 0^\circ$ to $\varphi \neq 0^\circ$.

Collective potential

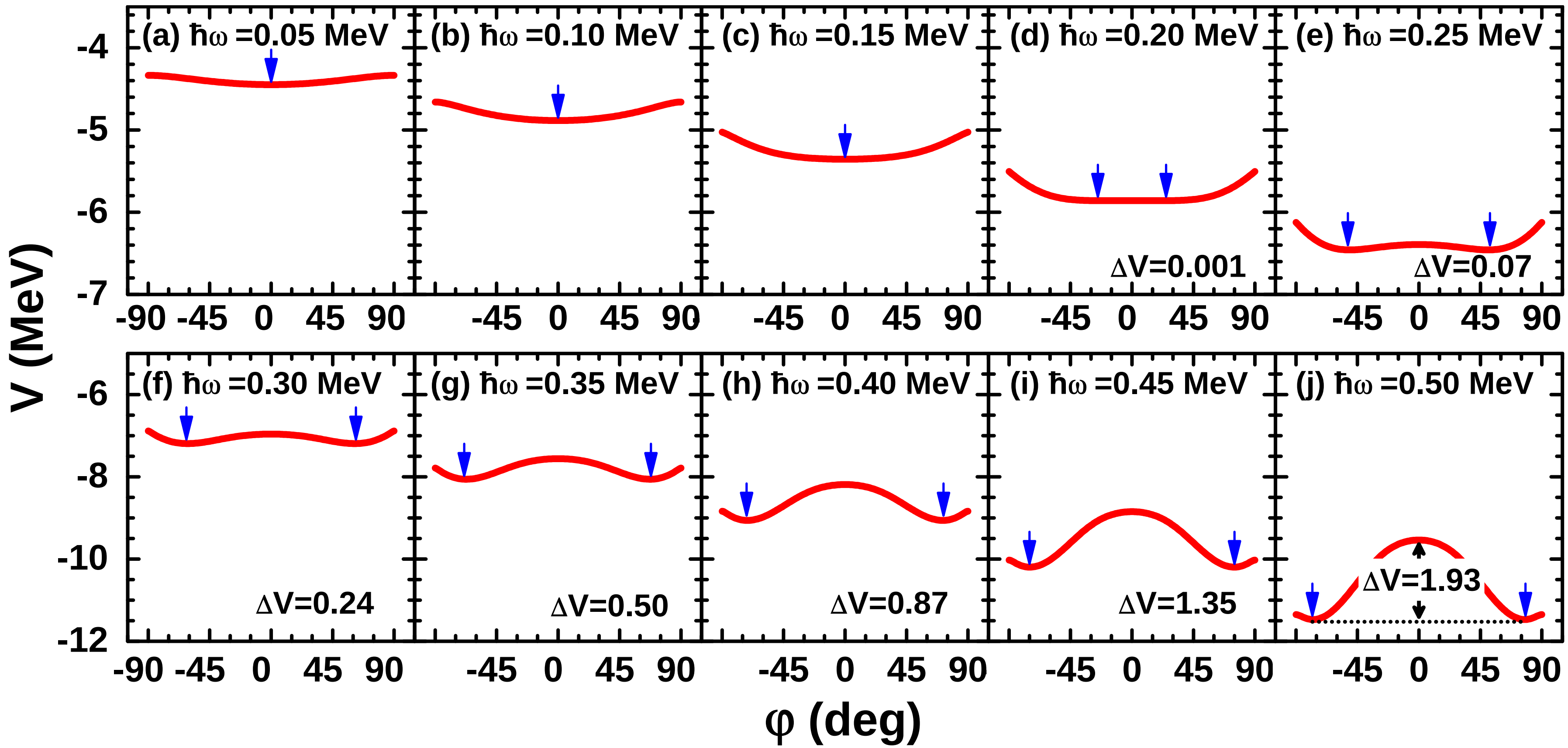

Figure: The potential energy $V(\varphi)$ as a function of φ extracted from the total Routhian surface calculations. The arrow labels the position of the potential minimum $V_{\min}$.

Remarks

a. The shape of potential change from harmonic oscillator type to double well type.

b. The potential barrier increases from $1\ \mathrm{keV}$ at $\hbar\omega = 0.2\ \mathrm{MeV}$ to about $2\ \mathrm{MeV}$ at $\hbar\omega = 0.50\ \mathrm{MeV}$.

Mass parameter

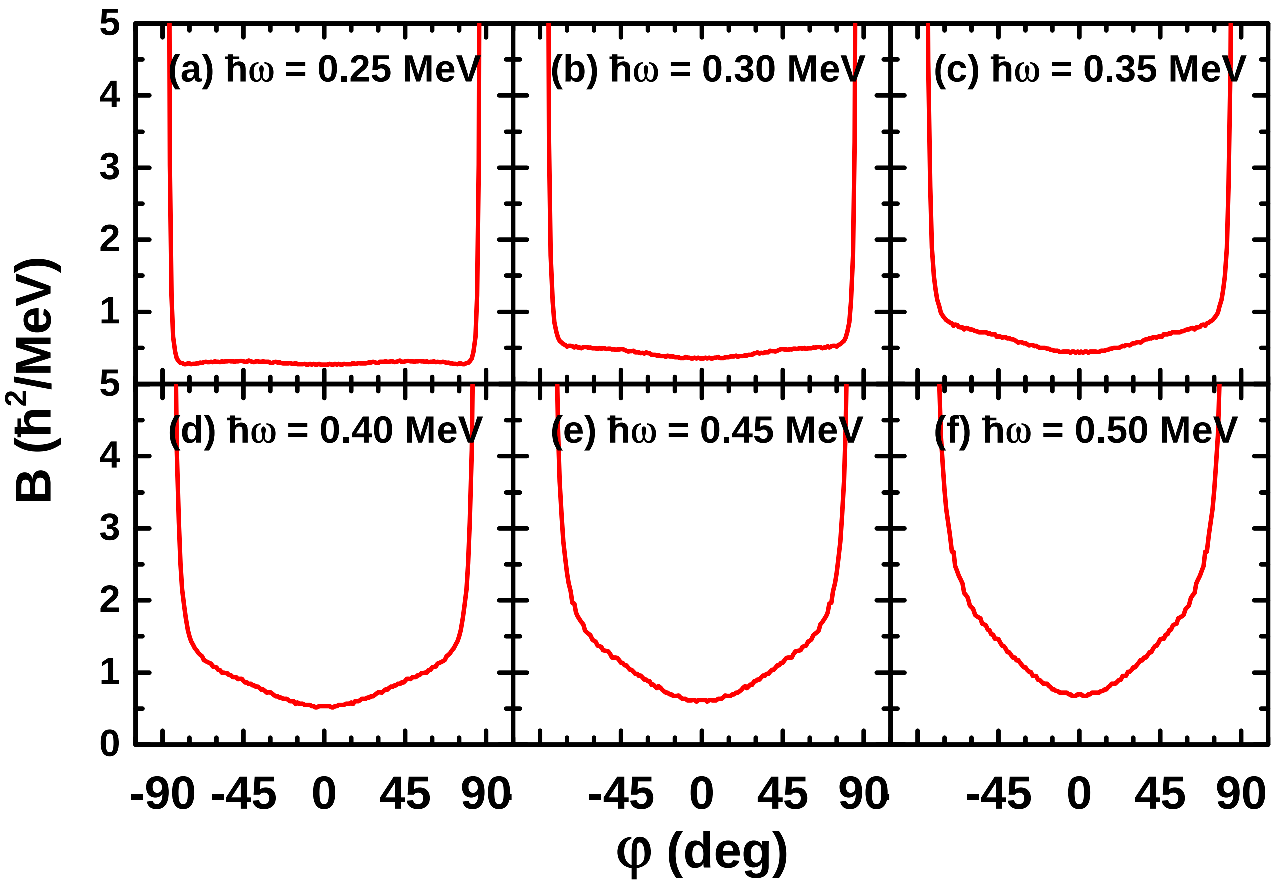

Figure: The mass parameter $B(\varphi)$ as a function of φ for the chiral rotation cases obtained based on TAC.

Remarks

a. $B(\varphi)$ is symmetric with respect to $\varphi = 0^\circ$ and increase dramatically when φ approach to $\pm 90^\circ$.

b. In the interior part, $B(\varphi)$ is increased remarkably with $|\varphi|$ for $\hbar\omega \geq 0.35$ MeV, while its dependence on φ is weak for $\hbar\omega = 0.25$ and 0.35 MeV.

Energy levels

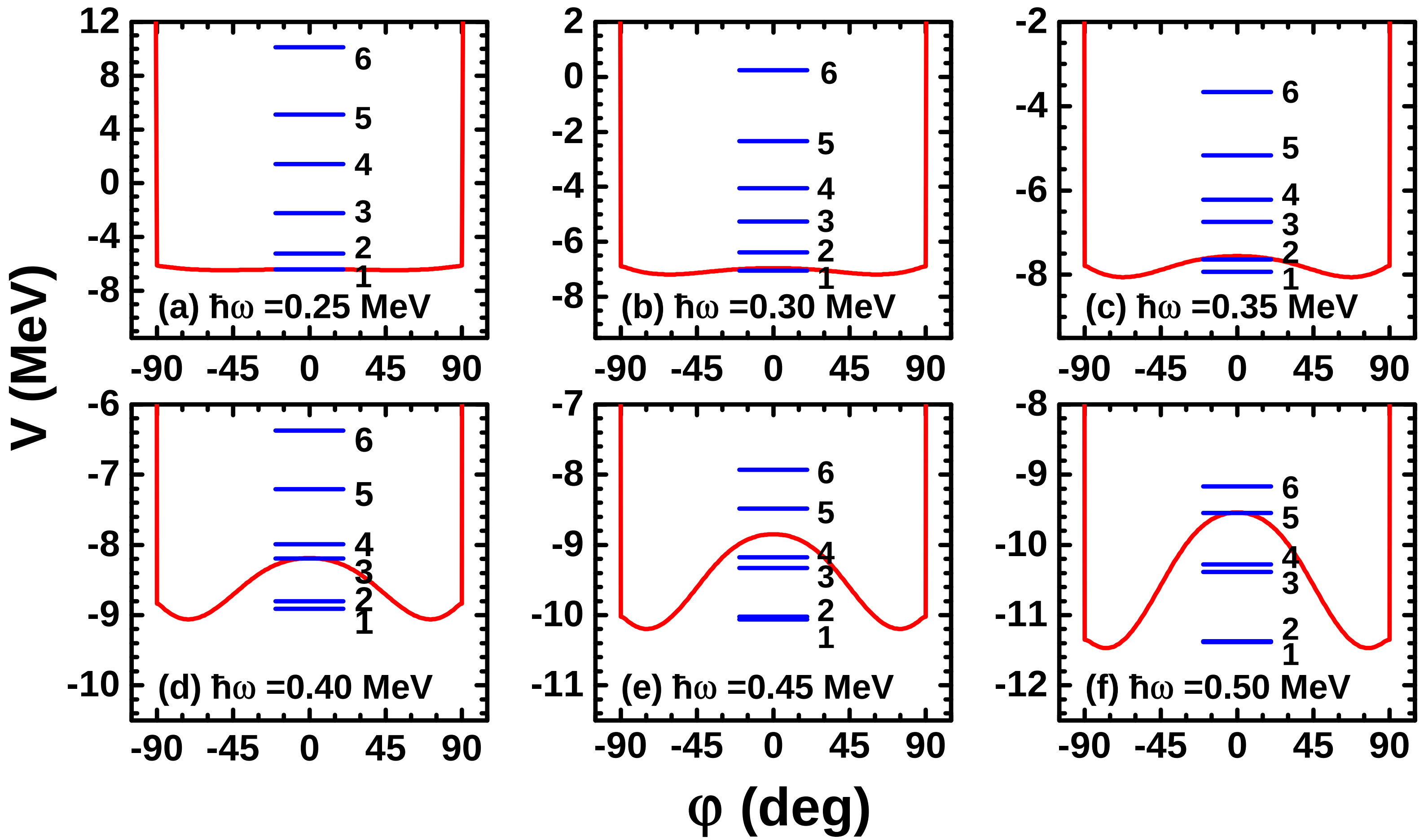

Figure: The six lowest energy levels, labeled as 1-6, obtained from the collective Hamiltonian.

Remarks

a. The three pairs of energy levels become close to each other.

b. Are the levels 2-4 corresponding to the **excited chiral doublet bands**? Droste2009EPJA, Chen2010PRC, Hamamoto2013PRC

Wave function

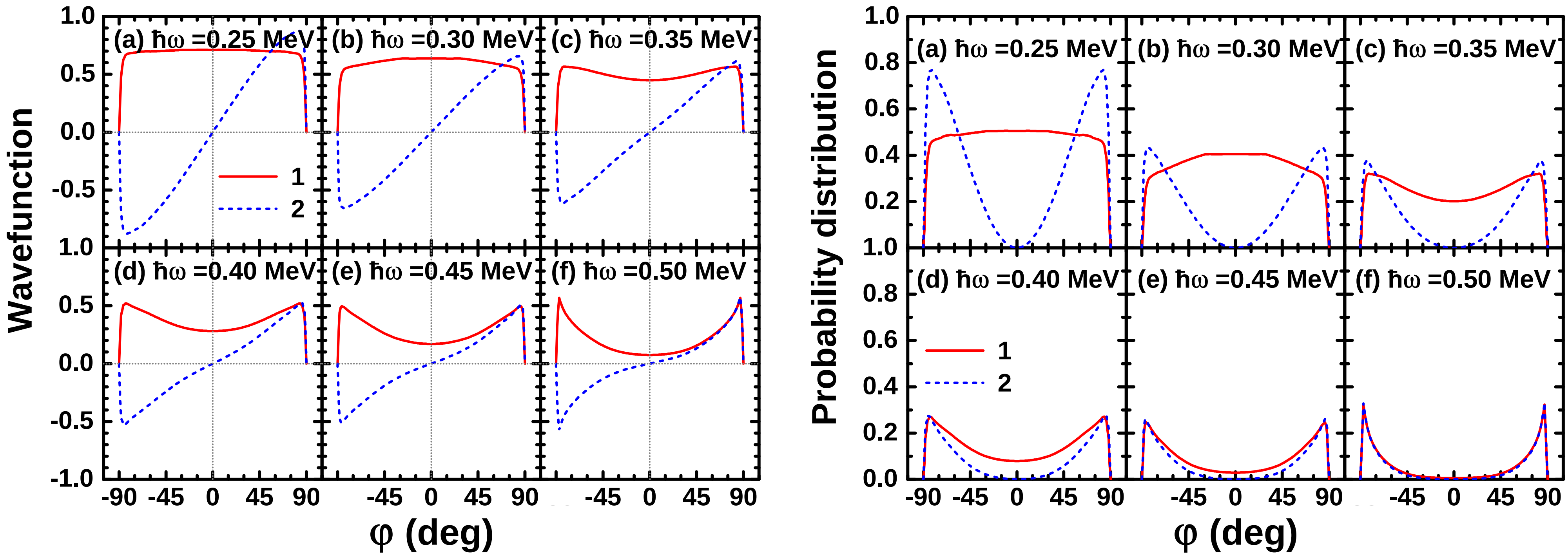

Figure: Wave functions $\psi(\varphi)$ and probability distributions $|\psi(\varphi)|^2$ for the lowest two levels 1 and 2 obtained from collective Hamiltonian.

Remarks

a. The wave functions are symmetric for level 1 and antisymmetric for level 2 with respect to $\varphi \to -\varphi$.

b. The wave functions of level 1 tends to show similar pattern with level 2.

Comparison with exact solutions

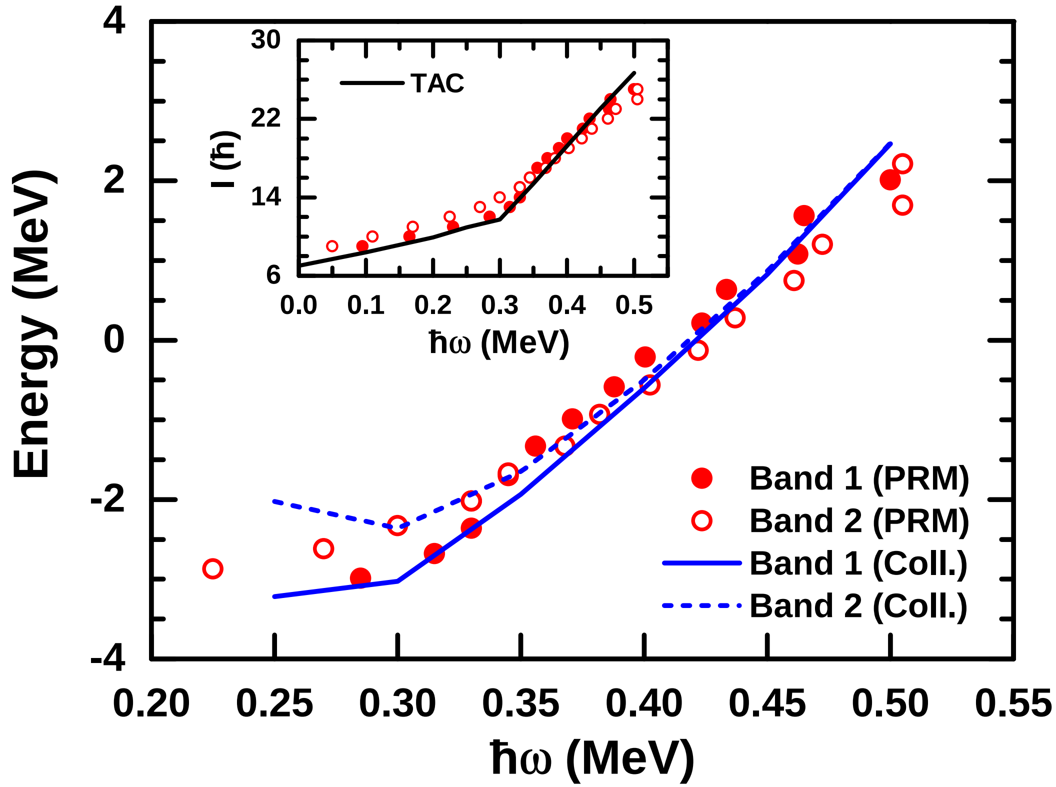

Figure: The energy spectra of the doublet bands obtained from the collective Hamiltonian in comparison with the exact solutions by the PRM.

Remarks

a. Apart from the agreement of collective Hamiltonian and PRM results for the yrast band, the partner band of PRM can also be reasonably reproduced by the collective Hamiltonian.

b. The second chiral vibration character obtained by PRM is not taken into account by the present collective Hamiltonian investigation.

Summary and perspective

Summary:

- A collective model which is able to describe the chiral rotation and vibration is proposed and applied to a system of one $h_{11/2}$ proton particle and one $h_{11/2}$ neutron hole coupled to a triaxial rigid rotor.
- Based on the tilted axis cranking approach, both the potential energy and mass parameter as functions of φ are obtained and included in the collective Hamiltonian.
- It is found that for chiral rotation, the partner states become more degenerate with the increase of the cranking frequency, and the wave function of levels 1 and 2 tend to show similar pattern.

Perspective:

- The collective Hamiltonian is expected to describe the chiral doublet bands in more realistic nuclei, such as tilted axis cranking relativistic mean-field theory, to obtain a more microscopic collective potential.
- The fluctuation of the potential energy with θ will be taken into account.
- The adiabatic self-consistent collective coordinate (**ASCC**) method will be introduced to calculate the mass parameter. Marumori1980PTP, Matsuo2000PTP, Hinohara2010PRC

Thank you!