Lowest-lying octet baryon masses in covariant baryon chiral perturbation theory

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OUTLINE

- Introduction
- Theoretical Framework
- Numerical Details
- Results and Discussion
- Summary

Origin of masses

☐ Current quark masses --- Explained

- Standard Model $\rightarrow$ Higgs Mechanism
- LHC @ CERN $\rightarrow$ Higgs particle

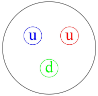
ATLAS Collaboration, PLB716(2012)1

CMS Collaboration, PLB716(2012)30

Nobel Prize 2013

☐ Light hadron masses --- Complicated

QCD



Proton

$M_u \sim 3$ MeV

$M_d \sim 6$ MeV

$M_p = 938$ MeV

(Strong force)

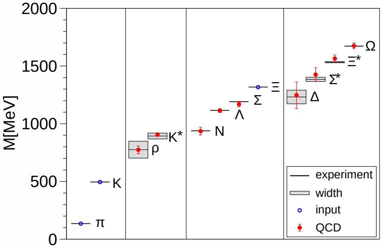
$$M_p \;(938\mathrm{MeV}) \gg m_u + m_u + m_d \;(12\mathrm{MeV})$$

- Current quark masses (1-3%)
- Non-perturbative strong interaction (>95%)
  - Lattice QCD
  - Chiral Perturbation Theory
  - Other Models

Octet baryon masses in LQCD

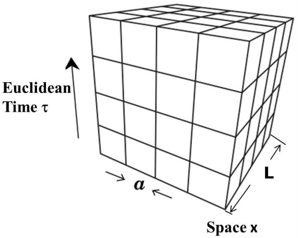
◻ $N_f = 2+1$ lattice calculation

- BMW, *S. Dürr et al., Science 322 (2008) 1224*
- PACS-CS, *S. Aoki et al., PRD 79 (2009) 034503*
- LHPC, *A. Walker-Loud et al., PRD 79 (2009) 054502*
- HSC, *H.-W. Lin et al., PRD 79 (2009) 034502*
- UKQCD, *W. Bietenholz et al., PRD 84 (2011) 054509*
- NPLQCD, *S. Beane et al., PRD 84 (2011) 014507*



☞Test the consistency --- **crucial**

- Lattice simulations:
  - **different fermion/gauge actions**
  - **different quark masses**
  - **different lattice volumes ($V = L^3$)**
  - **different lattice spacings (a)**
- In continuum:
  should lead to the same theory --- QCD



LQCD supplemented BChPT

- Cost of LQCD

$$\textbf{Cost} \propto \left(\frac{L}{a}\right)^4 \frac{1}{a} \frac{1}{m_{u/d}a}.$$

- Limitation of LQCD

Input of LQCD	Simulation	Physical World
Light quark masses $m_{u/d}$	~ 10 MeV	$3-5$ MeV
Lattice box size L	$2-5$ fm	Infinite space time
Lattice spacing a	$a \sim 0.1$fm	Continuum

In order to obtain the physical values

$$m_{u/d}^{\text{Lat.}} \xrightarrow{\textbf{Chiral extrapolation}} m_{u/d}^{\text{Phys.}}$$

$$L \xrightarrow{\textbf{Finite-volume corrections}} \infty$$

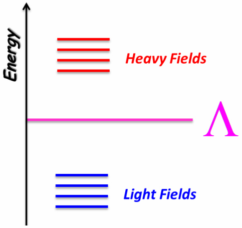
$$a \xrightarrow{\textbf{Continuum extrapolation}} 0$$

Baryon Chiral Perturbation Theory (BChPT) is a powerful tool to perform **the multi-extrapolation** for LQCD simulations.

Baryon Chiral Perturbation Theory

☐ Effective Field Theory of low-energy QCD

- Degrees of freedom
  ✓ Pseudoscalar mesons, ✓ Baryons (Octet and Decuplet)
- Chiral symmetry $SU(3)_L \times SU(3)_R$
- Explicit and spontaneous symmetry breaking



☐ Solving the Power Counting Breaking problem

Non-Relativistic	Relativistic	
Heavy-Baryon ChPT *E.E. Jenkins et al., PLB(1991)*	Infrared BChPT *T. Becher et al., EPJC(1999)*	Extended-on-mass-shell (EOMS) *J. Gegelia et al., PRD(1999), T. Fuchs et al., PRD(2003)*
Baryon as static source	$H = I + R$	PCB terms subtracted
Strict power-counting	$\int_0^1 \cdots = \int_0^\infty \cdots - \int_1^\infty \cdots$	Redefinition of the LECs
breaks analyticity	breaks analyticity	satisfies analyticity
converges slowly	converges slowly	converges relatively fast

Octet baryon masses in BChPT

Up to NNLO

- HBChPT
  - ☞ **failed** to describe the lattice data *PACS-CS,PRD(2009), LHPC,PRD(2009)*
- EOMS-BChPT
  - ☞ **Improved description** of the PACS-CS and LHPC data *J. Martin-Camalich et al., PRD(2010)*
  - ☞ Finite-volume effects in LQCD simulations are very important *L.S. Geng et al., PRD(2011)*
- Finite-range regularization + HBChPT
  - ☞ nice description of the PACS-CS and LHPC data *R.D. Young et al., PRD(2010)*

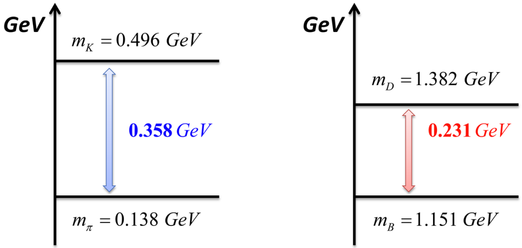
Up to N^3LO --- **Few** calculations

Many low-energy constants (LECs) need to be fixed

- Partial summation BChPT
  - ☞ nice description of the BMW, PACS, LHPC and UKQCD data *A. Smeke et al., PRD(2012), M.F.M. Lutz et al., PRD(2013)*
- Infrared BChPT
  - ☞ nice description of UKQCD data *P.C. Bruns et al.,PRD(2013)*

Decuplet resonances in BChPT

Baryon Spectrum in SU(3)-BChPT



Perturbative parameters

$$\frac{m_K}{\Lambda_{\rm ChPT}} > \frac{\delta}{\Lambda_{\rm ChPT}}$$

Effects of virtual decuplet baryons should be studied

In this work

Calculate the octet baryon masses in EOMS BChPT up to N^{3}LO without and with virtual decuplet baryons

- Take into account **finite volume corrections (FVCs)** self-consistently
- Perform **a simultaneous fit** of all the $N_f = 2 + 1$ lattice results
  - Determine the values of LECs
  - Test the consistency of LQCD
  - Predict the sigma terms of octet baryons
- Study **virtual decuplet effects** on the octet baryon masses
  - Effects on the chiral extrapolation
  - Effects on the finite-volume corrections
- Include **discretization effects** up to $\mathcal{O}(a^2)$ and perform the **continuum extrapolation** of LQCD

Theoretical Framework

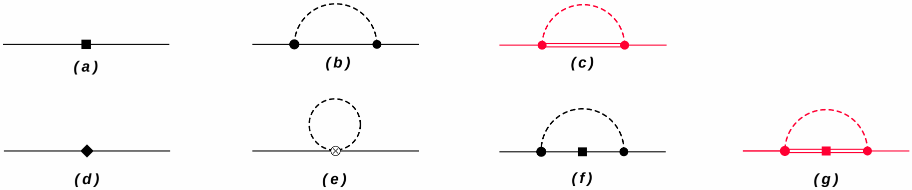
Effective Lagrangians up to N^{3}LO

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\phi^{(2)} + \mathcal{L}_\phi^{(4)} + \mathcal{L}_{\phi B}^{(1)} + \mathcal{L}_{\phi B}^{(2)} + \mathcal{L}_{\phi B}^{(3)} + \mathcal{L}_{\phi B}^{(4)} + \mathcal{L}_{T}^{(1)} + \mathcal{L}_{T}^{(2)} + \mathcal{L}_{\phi BT}^{(1)}$$

$$\begin{aligned}
= \; & \frac{F_\phi{}^2}{4}\langle D_\mu U (D_\mu U)^\dagger\rangle + \frac{F_\phi{}^2}{4}\langle \chi U^\dagger + U\chi^\dagger\rangle + \sum_{i=4}^{8} L_i \hat{\mathcal{O}}_\phi^{(4)} \\
& + \langle \bar{B}(i\not{D} - M_0)B\rangle + \frac{D/F}{2}\langle \bar{B}\gamma^\mu\gamma_5[u_\mu, B]_\pm\rangle + b_0\langle\chi_+\rangle\langle B\bar{B}\rangle + b_{D/F}\langle\bar{B}[\chi_+, B]_\pm\rangle \\
& + \sum_{j=1}^{8} b_j \hat{\mathcal{O}}_{\phi B}^{(2)} + \sum_{k=1}^{7} d_k \hat{\mathcal{O}}_{\phi B}^{(4)} \\
& + \bar{T}_\mu^{abc}\left(i\gamma^{\mu\nu\alpha}D_\alpha - m_D\gamma^{\mu\nu}\right)T_\mu^{abc} + \frac{t_0}{2}\bar{T}_\mu^{abc} g^{\mu\nu} T_\nu^{abc}\langle\chi_+\rangle + \frac{t_D}{2}\bar{T}_\mu^{abc} g^{\mu\nu}(\chi_+, T_\nu)^{abc} \\
& + \frac{i\mathcal{C}}{m_D F_\phi}\varepsilon^{abc}(\partial_\alpha \bar{T}_\mu^{ade})\gamma^{\alpha\mu\nu} B_c^e \partial_\nu \phi_b^d + \text{H.c.}.
\end{aligned}$$

- The meson Lagrangians. *J. Gasser et al.,NPB(1985)*
  - LECs from $\mathcal{L}_\phi^{(2)}$, $\mathcal{L}_\phi^{(4)}$: $F_\phi,\ L_i,\ i\in(4,5,6,7,8)$
- The meson-baryon Lagrangians. *B. Borasoy et al.,A.P.(1996), J. A. Oller et al.,JHEP(2006)*
  - LECs from $\mathcal{L}_{\phi B}^{(1)}$: m_0, D, F
  - LECs from $\mathcal{L}_{\phi B}^{(2)}$: $b_0, b_D, b_F, b_j,\ j\in(1,\cdots,8)$
  - LECs from $\mathcal{L}_{\phi B}^{(4)}$: $d_k,\ k\in(1,\cdots,7)$
  - LECs from $\mathcal{L}_{\phi T}$: $m_D,\ t_0,\ t_D,\ \mathcal{C}$

Feynman diagrams up to N^{3}LO



Fields: *Solid lines --- Octet baryons,* *Double lines --- Decuplet baryons,* *Dashed lines --- Pseudoscalar mesons*

Vertex: *Boxes ---* $\mathcal{L}_{\phi B}^{(2)}$, $\mathcal{L}_{\phi T}^{(2)}$; *Diamonds ---* $\mathcal{L}_{\phi B}^{(4)}$; *Solid dot ---* $\mathcal{L}_{\phi B}^{(1)}$, $\mathcal{L}_{\phi BT}^{(1)}$; *circle-cross ---* $\mathcal{L}_{\phi B}^{(2)}$

① Calculate **the baryon self-energy** in covariant BChPT

② Subtract **PCB terms** with EOMS scheme

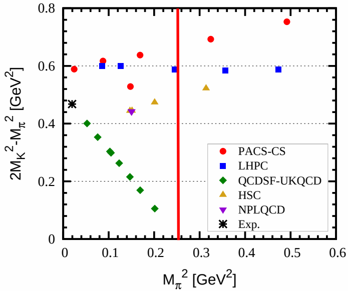
③ Include **FVCs** self-consistently

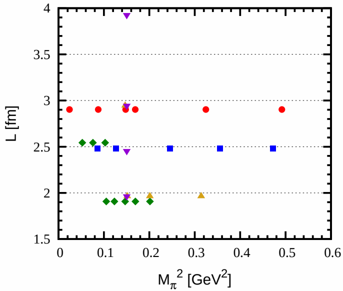
Octet baryon masses in finite volume

$$
\begin{aligned}
m_B(M_\phi) &= m_0 + m_B^{(2)}(M_\phi) + m_B^{(3)}(M_\phi) + m_B^{(4)}(M_\phi) + m_B^{(D)}(M_\phi) \\
&= m_0 + \sum_{\phi=\pi,K} \xi^{(a)}_{B,\phi} M_\phi^2 + \sum_{\phi_1,\phi_2=\pi,K,\eta} \xi^{(d)}_{B,\phi_1,\phi_2} M_{\phi_1}^2 M_{\phi_2}^2 \\
&\quad + \frac{1}{(4\pi F_\phi)^2} \sum_{\phi=\pi,K,\eta} \xi^{(b)}_{B,\phi} \left[H^{(b)}_{\rm loop} - H^{(b)}_{\rm pcb} - \Delta H^{(b)}_{\rm FVC} \right] \\
&\quad + \frac{1}{(4\pi F_\phi)^2} \sum_{\phi=\pi,K,\eta} \xi^{(e)}_{B,\phi} \left[H^{(e)}_{\rm loop} - \Delta H^{(e)}_{\rm FVC} \right] \\
&\quad + \frac{1}{(4\pi F_\phi)^2} \sum_{\phi=\pi,K,\eta} \xi^{(f)}_{B,\phi} \left[H^{(f)}_{\rm loop} - H^{(f)}_{\rm pcb} - \Delta H^{(f)}_{\rm FVC} \right] \\
&\quad + \frac{1}{(4\pi F_\phi)^2} \sum_{\phi=\pi,\ K,\ \eta} \xi^{(c)}_{BD,\phi} \left[H^{(c)}_{\rm loop} - H^{(c)}_{\rm pcb} - \Delta H^{(c)}_{\rm FVC} \right] \\
&\quad + \frac{1}{(4\pi F_\phi)^2} \sum_{\phi=\pi,\ K,\ \eta} \xi^{(g)}_{BD,\phi} \cdot \left[H^{(g)}_{\rm loop} - H^{(g)}_{\rm pcb} - \Delta H^{(g)}_{\rm FVC} \right].
\end{aligned}
$$

Numerical Details

- Fitting data: **LQCD results (11-sets)** + **Exp. values**
  - PACS-CS, LHPC, QCDSF-UKQCD, HSC, NPLQCD
    - Lattice data with $M_\pi < 500$ MeV
      - ☞ reduce the higher order contributions of chiral expansions
    - Lattice data with $M_\phi L > 4$
      - ☞ minimize finite-volume effects of LQCD
  - **Fitting points:** $44(\mathrm{LQCD}) + 4(\mathrm{Exp.}) = 48$





Results and Discussion

STRATEGY ONE --- Octet only EOMS BChPT

☞ Assuming: **virtual decuplet effects can be absorbed by LECs.**

- **Fitting methods**

	Fitting formula	Free parameters	
NLO	$m_0 + m_B^{(2)}$	m_0, b_0, b_D, b_F	**4**
NNLO	$m_0 + m_B^{(2)} + m_B^{(3)}$	m_0, b_0, b_D, b_F	**4**
N^{3}LO	$m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)}$	$m_0, b_0, b_D, b_F, b_i, d_j$	**19**

- **Other parameters**
  - $L^r_{4,5,6,7,8}$, *J. Bijnens et al., NPB(2012), with $\mu = 1$ GeV*
  - $F_0 = 0.0871$ GeV, *G. Amoros et al., NPB(2001)*
  - $D = 0.80$, $F = 0.46$

Best fitting results

	NLO	NNLO	N^{3}LO
m_0 [MeV]	$900(6)$	$767(6)$	**$880(22)$**
b_0 [GeV^{-1}]	$-0.273(6)$	$-0.886(5)$	$-0.609(19)$
b_D [GeV^{-1}]	$0.0506(17)$	$0.0482(17)$	$0.225(34)$
b_F [GeV^{-1}]	$-0.179(1)$	$-0.514(1)$	$-0.404(27)$
b_1 [GeV^{-1}]	--	--	$0.550(44)$
b_2 [GeV^{-1}]	--	--	$-0.706(99)$
b_3 [GeV^{-1}]	--	--	$-0.674(115)$
b_4 [GeV^{-1}]	--	--	$-0.843(81)$
b_5 [GeV^{-2}]	--	--	$-0.555(144)$
b_6 [GeV^{-2}]	--	--	$0.160(95)$
b_7 [GeV^{-2}]	--	--	$1.98(18)$
b_8 [GeV^{-2}]	--	--	$0.473(65)$
d_1 [GeV^{-3}]	--	--	$0.0340(143)$
d_2 [GeV^{-3}]	--	--	$0.296(53)$
d_3 [GeV^{-3}]	--	--	$0.0431(304)$
d_4 [GeV^{-3}]	--	--	$0.234(67)$
d_5 [GeV^{-3}]	--	--	$-0.328(60)$
d_7 [GeV^{-3}]	--	--	$-0.0358(269)$
d_8 [GeV^{-3}]	--	--	$-0.107(32)$
χ^2/d.o.f.	11.8	8.6	1.0

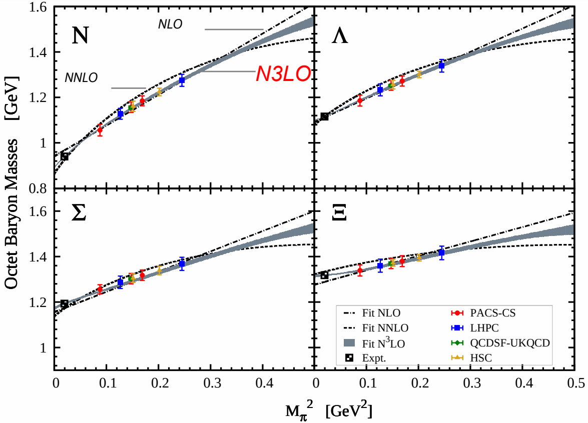
Table: Values of the LECs.

- EOMS-BChPT shows a clear improvement order by order
- Different lattice QCD calculations are consistent with each other
- Values of LECs from EOMS-N^{3}LO look very natural
- $m_0 = 880$ MeV consistent with the SU(2)-BChPT results.

  M. Procura et al.,PRD(2003,2006)

  L. Alvarez-Ruso et al., PRD(2013)

Chiral extrapolation



- **NLO fitting** linear and can not describe the experimental value
- **NNLO fitting** more curved and can not well describe lattice data
- **N^{3}LO fitting** can give a good description of LQCD and Exp. data, confirm the linear dependence of the lattice data on M_π^2

Pion- and strangeness-octet baryon sigma terms

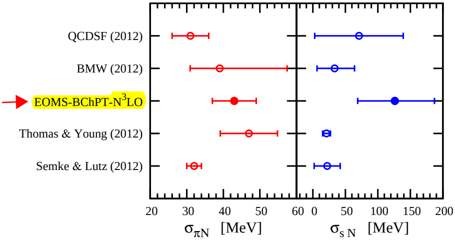
Nucleon-sigma term

- Related to chiral quark condensate $\langle \bar{q}q \rangle$
- Important to understand the composition of the nucleon mass and its strangeness content
- Useful for direct **dark matter** searches

$$\sigma_{\pi B} = m_l \langle B(p)|\bar{u}u + \bar{d}d|B(p)\rangle$$
$$\sigma_{sB} = m_s \langle B(p)|\bar{s}s|B(p)\rangle.$$

	$\sigma_{\pi B}$ [MeV]	σ_{sB} [MeV]
N	43(1)(6)	126(24)(54)
Λ	19(1)(7)	269(23)(66)
Σ	18(2)(6)	296(21)(50)
Ξ	4(2)(3)	397(22)(56)

Comparison with the latest LQCD and BChPT

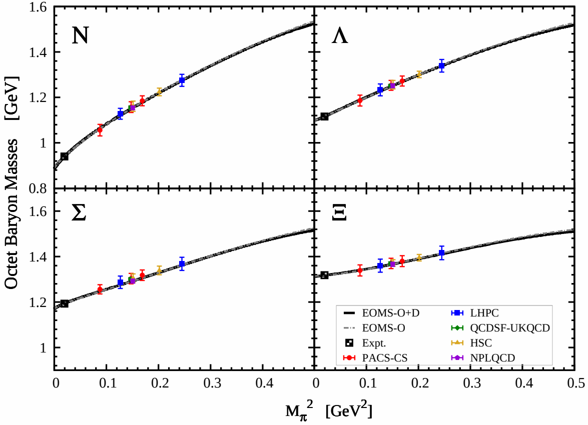


- $\sigma_{\pi N}$ is consistent with others. *almost the same as the empirical value* 45 ± 8 *MeV.* *J. Gasser et al., PLB(1991)*
- σ_{sN} is larger than others. *Strong correlation between the LECs ?* *Lattice QCD scale setting problem ?* *P.E. Shanahan, A.W. Thomas, R.D. Young, PRD (2013)*

STRATEGY TWO --- Octet + Decuplet EOMS BChPT

- **Virtual decuplet baryons** are explicitly included in BChPT
- Fit the **same** lattice data as previous (N^3LO)
- There is **no new unknown LECs**
  - Octet-decuplet mass splitting: $\delta = 0.231$ GeV
  - Meson-octet-decuplet coupling constant: $\mathcal{C} = 0.85$ *J. M. Alarcon et al., 1209.2870*
  - Fixed from the experimental decuplet masses *J. Martin-Camalich et al., PRD(2010)*
    - $m_D = m_0 + \delta = m_0 + 0.231$ GeV
    - $t_0 = (m_0 + 0.231 - 1.215)/0.507$ GeV^{-1}
    - $t_D = -0.326$ GeV^{-1}

Virtual decuplet effects on the chiral extrapolation

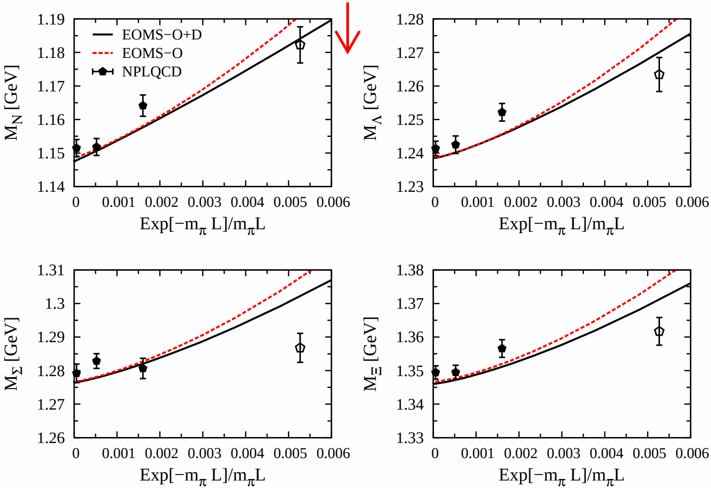


Fit the 11 LQCD data sets **with and without decuplet**

- **Decuplet effects on the chiral extrapolation are small**
- **Previous assumption is confirmed**: virtual decuplet contributions can be absorbed by 19 LECs of octet only version (loop diagrams)

Virtual decuplet effects on finite-volume corrections

- Use the previous best fit results to describe the NPLQCD lattice data
- Virtual decuplet contributions can give a better description of the FVCs at small volume region



Up to now...

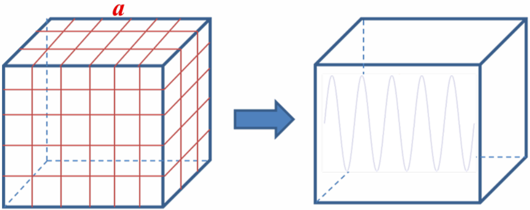
Multi-extrapolation of LQCD

$m_{u/d}^{\text{Lat.}}$ —Chiral extrapolation→ $m_{u/d}^{\text{Phys.}}$ ✓

L —Finite−volume corrections→ ∞ ✓

a —Continuum extrapolation→ 0

↓



Discretization effects of LQCD

- In the above two fitting strategies and other BChPT studies
  - Discretizsation effects of LQCD are **assumed small and neglected**
- In principle, **continuum extrapolation** should be first performed
  - BChPT describes the continuum QCD and is not valid for $a \neq 0$
- Several LQCD collaborations employed **different lattice spacings**

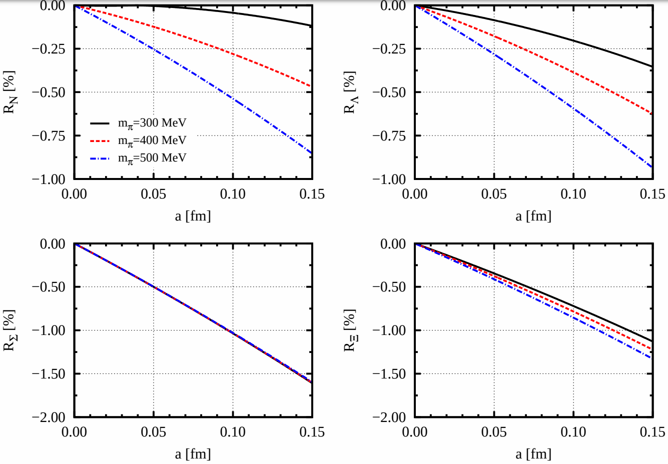
Continuum extrapolation and discretization effects of LQCD

- ☑ Formulate the discretization effects $m_B^{(a)}$ **up to $\mathcal{O}(a^2)$** for the Wilson action
- ☑ **Self-consistently including FVCs and discretization effects** in the octet baryon masses

$$m_B = m_0 + m_B^{(2)} + m_B^{(3)} + m_B^{(4)} + \boldsymbol{m_B^{(a)}}.$$

  - 19 LECs + **4 new** LECs (related to lattice spacing)
- ☑ Study the LQCD results obtained with $\mathcal{O}(a)$-improved Wilson actions
  - **10 sets**: PACS-CS, QCDSF-UKQCD, HSC, NPLQCD

Details will be reported soon: *XLR, L.S. Geng and J. Meng, in preparation*



$$R_B = \frac{m_B^{(a)}}{m_B}$$

- Pion mass m_π fixed:

$$a \Uparrow \quad \sim \quad m_B^{(a)} \Uparrow$$

- Lattice spacing a fixed:

$$m_\pi \Uparrow \quad \sim \quad m_B^{(a)} \Uparrow$$

- Discretization effects on baryon masses **do not exceed 2% for $a = 0.15$ fm**
- **Consistent** with early studies *S. Durr et al., Phys. Rev. D79, (2009) 014501.*
- Up to $\mathcal{O}(a^2)$, discretization effects are **small** and can be safely **ignored**

Summary

- **We have studied the lowest-lying octet baryon masses with the EOMS BChPT up to N^{3}LO without and with virtual decuplet baryons**

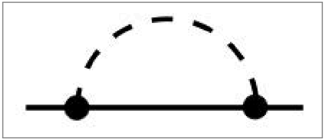
- **Finite-volume and disretizaion effects on the lattice data are taken into account self-consistently**

- **Through simultaneously fitting "all" the current LQCD data:**
  - Covariant BChPT shows a clear improvement order by order
  - LQCD results are consistent with each other, though their setups are quite different
  - Pion- and strangeness-nucleon sigma terms are $\sigma_{\pi N} = 43(1)(6)$ MeV and $\sigma_{sN} = 126(24)(54)$ MeV
  - Virtual decuplet effects on the baryon masses cannot be distinguished from those of the virtual octet baryons and the tree level diagrams
  - Up to $\mathcal{O}(a^2)$, the discretization effects on the LQCD baryon masses are shown to be small and can be safely ignored

Thank you!

STEP 1 --- Self-energy calculation

- Take loop diagram-(b) for example



- ϕBB vertices

$$\mathcal{L}^{(1)}_{\phi B} = \frac{D/F}{2}\langle \bar{B}\gamma^{\mu}\gamma_5[u_\mu, B]_{\pm}\rangle.$$

- Loop function

$$H_B^{(b)} = i\int \frac{d^4k}{(2\pi)^4}\frac{\not{k}(\not{k}-\not{p}+m_0)\not{k}}{(k^2-M_\phi^2+i\epsilon)((p-k)^2-m_0^2+i\epsilon)},$$

- After feynman-parameter integral

$$H_B^{(b)} = -\frac{m_0}{2}\int_0^1 dx\left[\left(m_0^2x^3+3(x+1)\mathcal{M}_B^{(b)\,2}\right)\left(\gamma_\varepsilon+\ln\frac{\mathcal{M}_B^{(b)\,2}}{\mu^2}\right)-2(x+1)\mathcal{M}_B^{(b)\,2}\right]$$

with $\gamma_\varepsilon = \frac{2}{\varepsilon}+\Gamma'(1)+\ln(4\pi)$, $\varepsilon = 4-d$.

Dimension renormalization scheme ($\overline{\mathrm{MS}}$) to remove the divergent terms γ_ε.

STEP 2 --- PCB terms and EOMS scheme

☐ Power-couning-breaking terms

- Loop results (expand with M_ϕ)

$$H_B^{(b)}(M_\phi) = \boldsymbol{a m_0^3 + b m_0 M_\phi^2} + c M_\phi^3 + \cdots \quad (\mathcal{O}(p^3))$$

- However, if the systematic power counting exists

$$H_B^{(b)}(M_\phi) = c M_\phi^3 + \cdots \quad (\mathcal{O}(p^3))$$

☐ EOMS renormalization scheme

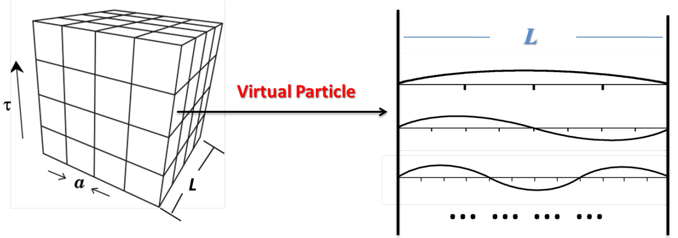
- Drops the PCB terms $\boldsymbol{a = 0,\ b = 0}$
- Equivalently, **redefinition the corresponding LECs**

Loop-b's contribution to octet baryon masses in infinite space-time

$$m_B^{(3)-b} = \frac{1}{(4\pi F_0)^2} \sum_{\phi=\pi,\ K,\ \eta} \xi_{B,\phi}^{(b)} \times \left\{ H_B^{(b)}(M_\phi) - 2m_0 \left[m_0^2 + 2M_\phi^2 + (m_0^2 + M_\phi^2) \ln \frac{\mu^2}{m_0^2} \right] \right\}$$

STEP 3 --- Finite-Volume Corrections

☐ Physical picture of FVCs



- Momentum of virtual particle discretized

$$k_i = \frac{2\pi}{L} \cdot n_i, \quad (i = 0, 1, 2, 3) \quad \Longrightarrow \quad \int_{-\infty}^{\infty} dk \sim \sum_{n=-\infty}^{\infty} \frac{2\pi}{L} \cdot n$$

- Definition of FVCs:

$$\Delta H^{(b)}_{\rm FVC} = \int \frac{dk_0}{2\pi} \cdot \left(\frac{1}{L^3} \sum_{\vec{k}} \Box - \int \frac{d\vec{k}}{(2\pi)^3} \Box \right) \qquad \text{with} \;\; \mathrm{L}_{\rm time} \sim 5\mathrm{L}_{\rm space}.$$

Power-counting in mesonic and baryonic sector

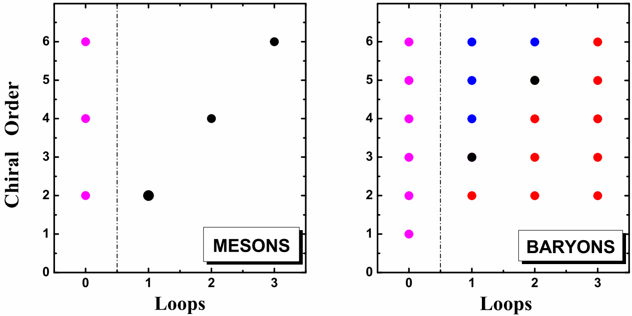
Mesonic sector

- ChPT has gained great achievements
- Calculation up to $\mathcal{O}(p^6)$ is standard

Baryonic sector --- Baryon ChPT

- A systematic power-counting lost
- Because $m_B \neq 0$ in the chiral limit

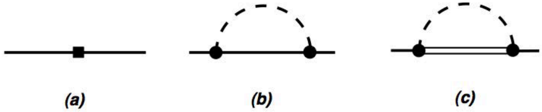
Chiral Order $$4L - 2N_M - N_B + \sum_k k v_k.$$



NPB(1988)

Power-Counting Breaking Problem

- Take the nucleon mass up to $\mathcal{O}(p^3)$ for example



$$m_N = m_0 + bM_\pi^2 + \text{loops}.$$

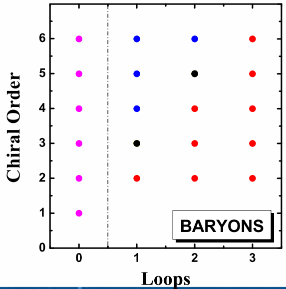
(b or c) Chiral order: 1+1+4-1-2=3.

If the systematic power counting exists:

$$\text{loop} = cM_\pi^3 + \cdots .$$

However the truth:

$$\text{loop} = \alpha m_0^3 + \beta m_0 M_\pi^2 + cM_\pi^3 + \cdots .$$



Relatively large Strangeness Sigma terms?

☐ Lattice-scale setting

- PACS-CS data with mass independent scale-setting:
  assume that the lattice scale, at constant bare coupling, is independent of the bare quark mass.

$$\sigma_{sN} = 59 \pm 7 \; (MeV)$$

- PACS-CS data with mass dependent scale-setting:
  The scale for the PACS-CS lattice data was set assuming that the dimensionful Sommer scale r_0 is independent of quark mass.

$$\sigma_{sN} = 21 \pm 6 \; (MeV)$$

P.E. Shanahan, A.W. Thomas and R.D. Young, PRD 87, 074503 (2013)

☐ Whether other LQCD data will show the same trend?

Continuum effective action

$$S_{\text{eff}} = S_0 + aS_1 + a^2 S_2 + \cdots$$
$$= \int d^4x (\mathcal{L}^{(4)} + a\mathcal{L}^{(5)} + a^2\mathcal{L}^{(6)} + \cdots)$$

K. Symanzik, NPB 226(1983)187, 226(1983)205

- QCD Lagrangian $\mathcal{L}^{(4)} = \bar{\psi}(i\not{D} - m_q)\psi$
  - ✓ m_q: quark mass; ψ: quark field
- At $\mathcal{O}(a)$: Pauli term $\mathcal{L}^{(5)} = c_{\text{SW}}\bar{\psi}\sigma^{\mu\nu}G_{\mu\nu}\omega_q\psi$
  - ✓ c_{SW}: Sheikholeslami-Wohler coefficient *Nucl.Phys.B259(1985)572*
  - ✓ ω_q**=1** for Wilson fermion, ω_q**=0** for Ginsparge-Wilson fermion
- At $\mathcal{O}(a^2)$: five types of operators
  - ✓ Break/conserve **Chiral symmetry**
  - ✓ Break/conserve **O(4) rotation symmetry**

$$\mathcal{L}^{(6)} = \sum_{i=1}^{18} c_i \mathcal{O}_i$$

Chiral Lagrangian and Feynman Diagrams

Chiral Effective Lagrangians in SU(3) sector

$$\mathcal{L}_a^{\text{eff}} = \mathcal{L}_a^{(1)} + \mathcal{L}_a^{(2)}$$

$$\mathcal{L}_a^{(1)} = \mathcal{L}^{\mathcal{O}(a)} + \mathcal{L}^{\mathcal{O}(am_q)},$$

$$\mathcal{L}_a^{(2)} = \mathcal{L}_1^{\mathcal{O}(a^2)} + \mathcal{L}_2^{\mathcal{O}(a^2)} + \mathcal{L}_3^{\mathcal{O}(a^2)} + \mathcal{L}_4^{\mathcal{O}(a^2)} + \mathcal{L}_5^{\mathcal{O}(a^2)}$$

Low Energy Constants (LECs): 3 + 11 + 4 + 0 + 7 + 7 + 4 = 36

Feynman Diagrams

(a) (b) (c) (d)