Masses of hadrons

Scattering and the *XYZ* states

Radiative transitions/decays of charmonium

The story of f_{D_s}

Anomalous magnetic moment of muon

Summary and outlooks

Recent results from Lattice QCD

with an emphasis on spectroscopy containing charmed quark

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A few words in advance

- It would be impossible for me to cover ALL recent lattice results
- Nor would it be possible to cover even MOST of the results
- For details, please look into relevant papers being cited
- A good place: Lattice 2013 homepage

Basic procedure of a lattice QCD calculation

QCD action in terms of basic fields ($\bar{\psi}$, ψ, U_μ):

$$S_{LQCD}[\bar{\psi}, \psi, U_\mu(x)] = S_g[U_\mu(x)] + \bar{\psi}_x \mathcal{M}[U_\mu(x)]_{xy}\psi_y \ , \qquad (1)$$

where $\mathcal{M}[U_\mu(x)]$ is called the fermion matrix, which differentiate different lattice fermions (staggered, Wilson, twisted mass, etc.). Any physical quantity (observable) $\mathcal{O}[\bar{\psi}, \psi, U_\mu]$ is given by

$$\begin{cases} \langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int DU_\mu D\bar{\psi} D\psi \mathcal{O}[\bar{\psi}, \psi, U_\mu] e^{-S_{LQCD}[\bar{\psi},\psi,U_\mu(x)]} \\ \mathcal{Z} = \int DU_\mu D\bar{\psi} D\psi e^{-S_{LQCD}} = \int DU_\mu e^{-S_g[U_\mu]} \det \mathcal{M}[U_\mu] \end{cases} \qquad (2)$$

This looks like a ensemble average with probability density:

$$\mathcal{P}[U_\mu] = \frac{1}{\mathcal{Z}} e^{-S_g[U_\mu]} \det \mathcal{M}[U_\mu] \qquad (3)$$

Omitting the $\det \mathcal{M}$ gives the so-called quenched approximation

Outline

1 Masses of hadrons

2 Scattering and the XYZ states

3 Radiative transitions/decays of charmonium

4 The story of f_{D_s}

5 Anomalous magnetic moment of muon

6 Summary and outlooks

1. Masses of hadrons

Mass computations on a lattice

1. Within a definite symmetry, use the basic fields to construct a class of interpolating operators $\{\mathcal{O}_\alpha(t) : \alpha = 1, \cdots N\}$
2. Form a $N \times N$ correlation matrix:

$$\mathcal{C}_{\alpha\beta}(t) = \langle\Omega|\mathcal{O}_\alpha(t)\mathcal{O}^\dagger_\beta(0)|\Omega\rangle \ . \tag{4}$$

   and calculate $\mathcal{C}_{\alpha\beta}(t)$ using numerical simulations
3. On the other hand, it is known that

$$\mathcal{C}_{\alpha\beta}(t) = \sum_n \frac{e^{-E_n t}}{2E_n} Z^{(n)}_\alpha Z^{(n)*}_\beta \tag{5}$$

4. Solve the Generalized Eigenvalue Problem/Variational Method:

$$\mathcal{C}_{\alpha\beta}(t)v^{(n)}_\beta = \lambda^{(n)}(t)\mathcal{C}_{\alpha\beta}(t_0)v^{(n)}_\beta \tag{6}$$

5. $\lambda^{(n)}(t)$'s give the eigenvalues of Hamiltonian, namely E_n's, while the eigenvectors yields the overlap:

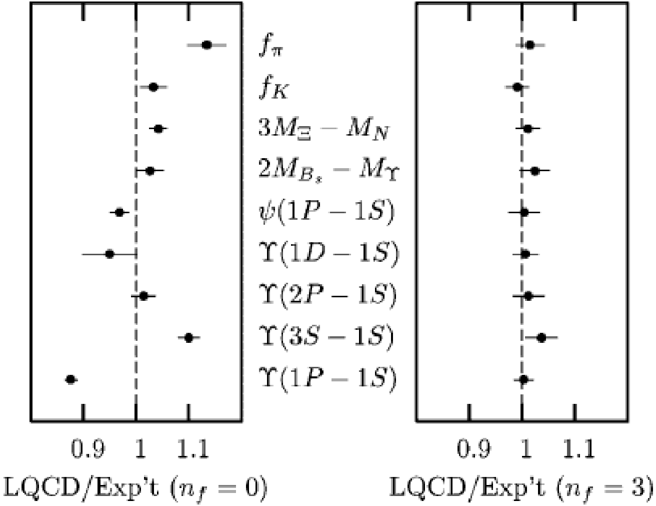
$$\lambda^{(n)}(t) \sim e^{-E_n(t-t_0)}\left[1 + O\left(e^{-\Delta E(t-t_0)}\right)\right] \tag{7}$$

Not done yet!

- Note that E_n's are NOT the physical mass values of the hadron!
  - They are eigenvalues of the QCD Hamiltonian
  - Most hadrons are unstable in QCD, i.e. they are resonances
  - Need to study hadron-hadron scattering processes
  - Need two-hadron operators for $\mathcal{O}_\alpha(t)$
  - Take for example the ρ meson case. One needs both single-hadron type operator $\bar{\psi}\gamma_i\psi$ and the two-pion operators in the $I = J = 1$ channel. (More on this issue later)
  - ☞ $E_n \simeq M$ only for "narrow resonances"
  - In principle, single and multi-hadron operators do mix within QCD.
- Need all sorts of extrapolations/interpolations (chiral, continuum, infinite volume, etc.)
- Scale setting and parameter tuning

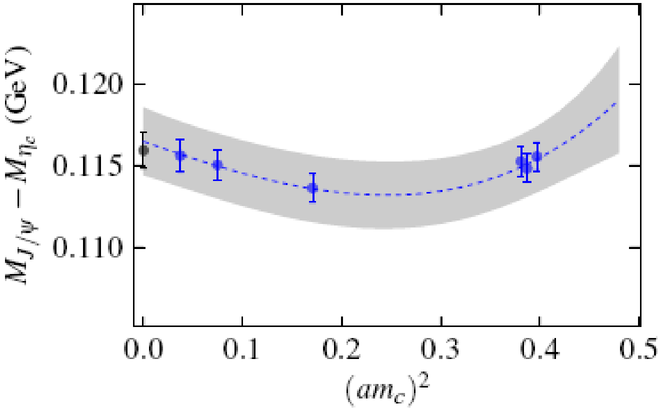
gold-plated variables: quench vs. unquench

- from HPQCD & UKQCD & MILC & Fermilab Lattice, PRL92 022001 (2004)



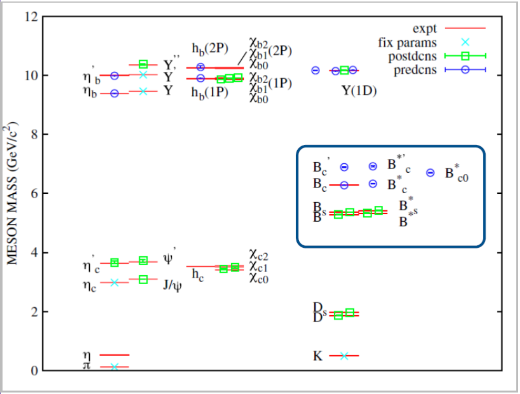
Hyperfine splitting

- ☞ from HPQCD, PRD86 094501 (2012); 2+1 flavor staggered



Spectrum for heavy-heavy and heavy-light mesons

☞ see HPQCD, PRD86 094510 (2012)



- Dynamical (HISQ) $N_f = 2+1+1$ confs
- NRQCD b quark action with $O(\alpha_s)$ corrected coefficients

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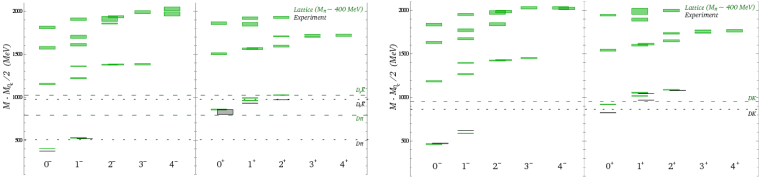
Excited D and D_s mesons

figures from Had Spec Collab, JHEP 05 021 (2013)



- PRD80 054506 (2009), PRL103 262001, PRD82 034508, PRD85 014507 (2012); 2+1 flavor aniso. clover imp. Wilson
- large class of single-hadron "subduced ops" with up to 2 covariant derivatives:

$$\mathcal{O}(t) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \bar{\psi}(t,\mathbf{x}) \left[\Gamma \times \overleftrightarrow{D} \times \overleftrightarrow{D} \cdots \right] \psi(t,\mathbf{x}) \qquad (8)$$

- Subduce ops into irreps of reduced symm. group
- Use the eigenvectors (the Z's) to do spin identification
- they used a smearing technique called "distillations"

Hybrids

figures from Had Spec Collab, JHEP 05 021 (2013)

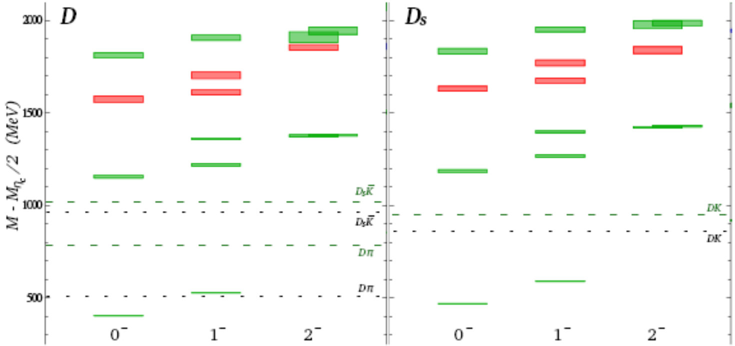
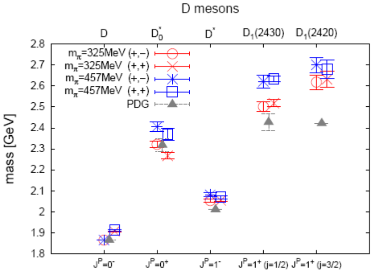


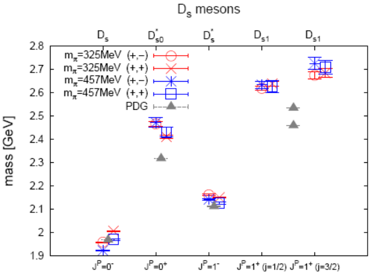
Figure 9. The negative-parity D (left panel) and D_s (right panel) meson spectra showing only channels where we identify hybrid candidates. The red boxes are identified as states belonging to the lightest hybrid supermultiplet as discussed in the text and other notation is as in Figs. 7 and 8.

- the red states have a large overlap with the gluonic operators

Results from ETMC

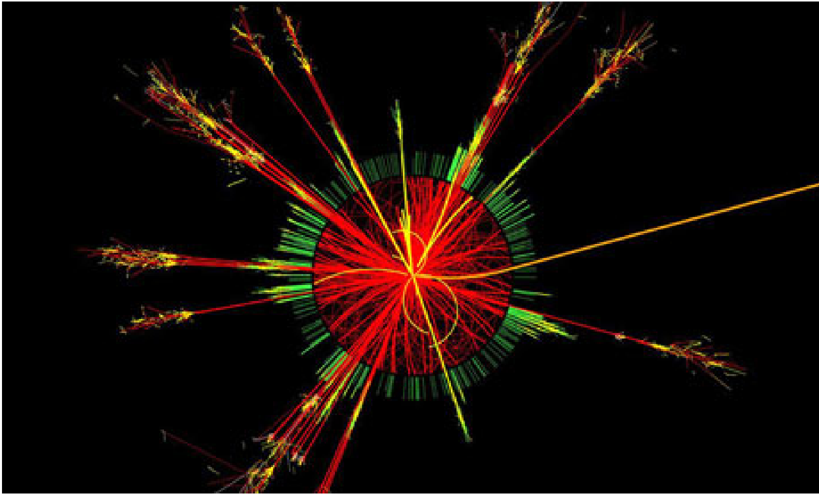
figures from ETMC, arXiv1304.7974





- 2+1+1 flavor twisted mass

2. Scattering of hadrons and the XYZ states



Scattering of hadrons

Lüscher's formula, see e.g. NPB354, 531 (1991)

- Hadronic resonances ⇒ hadron scattering
- Eigenvalues of Hamiltonian ⇔ scattering matrix elements
- Consider a system in a finite box of size L (cubic, rectangular,...), the idea of Lüscher's formula looks like the following:

$$E(L) \quad \Leftrightarrow \text{Lüscher formula} \Leftrightarrow \quad S(E) \quad \overset{elastic}{\twoheadrightarrow\twoheadrightarrow} \quad \delta(E)$$

$E(L)$ ⇑ Correlators ⇑ MC simulations

$S(E)$ ⇓ X-sections ⇓ Experiments

Lüscher's formula

- In the simplest case (single channel, s-wave scattering, neglecting higher l contributions...):

$$\tan \delta_0(E) = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1, q^2)} , \qquad (9)$$

where q is related to E via

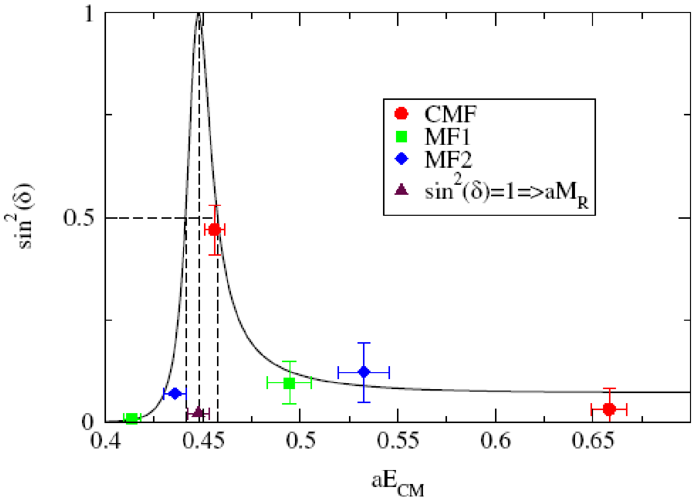
$$E = \sqrt{m_1^2 + \bar{k}^2} + \sqrt{m_2^2 + \bar{k}^2} , \quad q \equiv \frac{\bar{k} L}{2\pi} . \qquad (10)$$

- For small momentum, we will use

$$k \cot \delta(E) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \cdots \qquad (11)$$

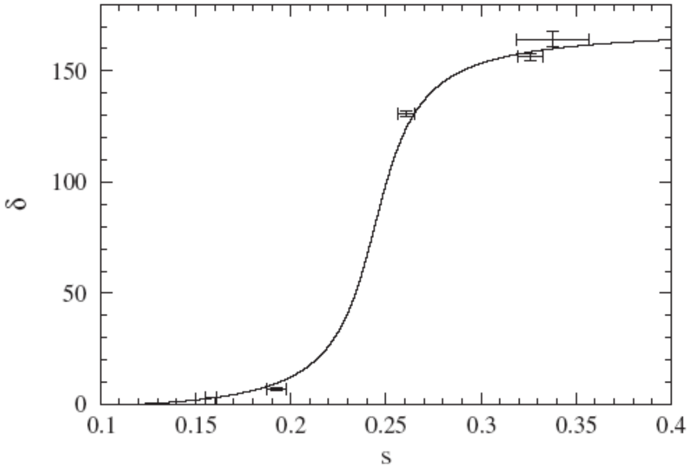
with a_0 the scattering length and r_0 the effective range

The ρ meson



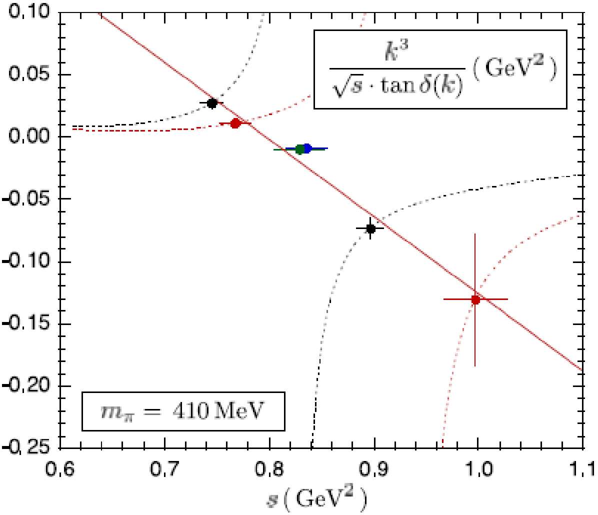
Feng, Jansen, Renner (ETMC, $N_f = 2$, $m_\pi = 480, 420, 330, 290$MeV), PRD83 094505 (2011)

The ρ meson



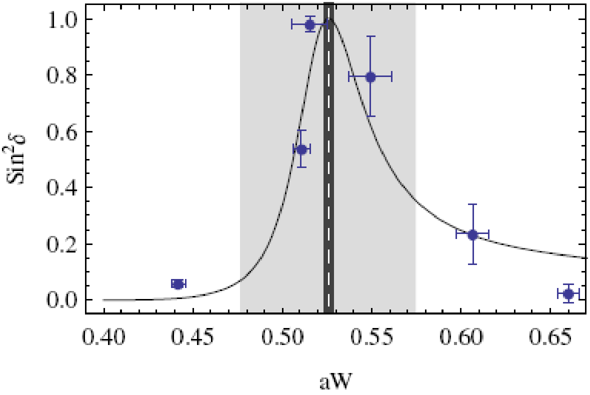
Lang, Mohler, Prelovsek, Vidmar ($N_f = 2$ imp Wilson, $m_\pi = 266$MeV), PRD84 054503 (2011)

The ρ meson



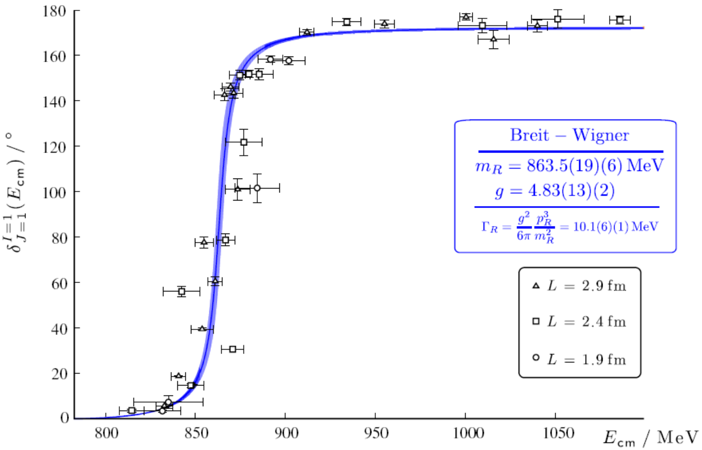
Aoki et al (PACS-CS, $N_f = 2+1$ imp Wilson, $m_\pi = 410, 300$MeV), PRD84 094505 (2011)

The ρ meson



Pelissier, Alexandru ($N_f = 2$ Wilson, $m_\pi = 300$MeV), PRD87 014503 (2013)

The ρ meson



Dudek et al (Had Spec Collab, $N_f = 2+1$ aniso. clover, $m_\pi = 400$MeV), PRD87 034505 (2013)

The newly discovered XYZ states
the $X(3872)$, $Z(4430)$ and $Z(3900)$

- Four (valance) quarks: $q'\bar{q}Q\bar{Q}$, with two heavy quarks and two light ones
- Charged, if $q' \neq q$
- Shows up in both charm and bottom systems
- Close to threshold of two existing mesons (charmed or beautiful) $q'\bar{Q}$ and $Q\bar{q}$
  - $Z_b(10610)$ close to threshold of B and B^*
  - $Z_b(10650)$ clsoe to threshold of B^* and B^*
  - $Z^{\pm}(4430)$ close to threshold of D^* and D_1
  - $Z_c(3900)$ close to threshold of D and D^*

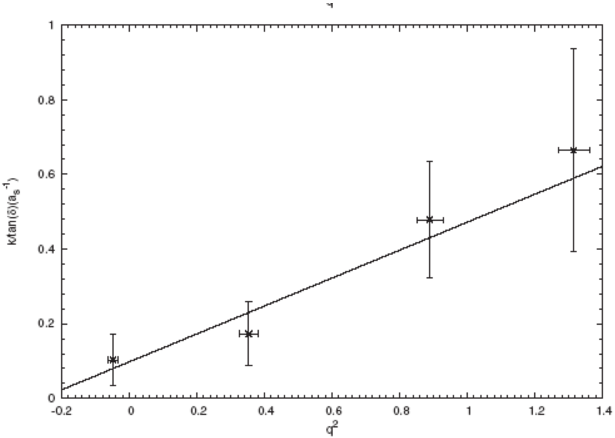
¿¿ What is the nature of these states??

☞ One has to study scattering of the relevant charmed mesons close to their threshold

  - Effective field theories
  - lattice study $\rightarrowtail$ charmed meson scattering

The $Z^{\pm}(4430)$

$$k \cot \delta(E) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \cdots \quad (12)$$

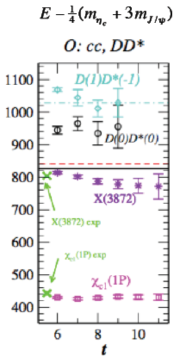


CLQCD (quenched), PRD80 034503 (2009)

The $X(3872)$

from S. Prelovsek's talk at Lattice 2013, see also arXiv:1307.5172

DD* scattering phase shift in s-wave (I=0)



- δ from levels 2,3 using Luscher's f.: $p \cdot \cot\delta(p) = \dfrac{2\, Z_{00}(1;q^2)}{\sqrt{\pi} L}$;
- effective range approx. $p\cot\delta(p) = \dfrac{1}{a_0} + \dfrac{1}{2} r_0 p^2$
  $a_0 = -1.7 \pm 0.4$ fm
  $r_0 = 0.5 \pm 0.1$ fm
- large negative a_0 agrees with one shallow BS according to Levinson's t. [Sasaki & Yamazaki 2006]
- L→∞ bound st. X $S \propto [\cot\delta - i]^{-1} = \infty, \quad \cot\delta(p_{BS}) = i$
  $p^2_{BS} = -0.020(13)\ \mathrm{GeV}^2$
  $m_X^{lat, L\to\infty} = E_D(p_{BS}) + E_{D^*}(p_{BS})$

Similar phenomenon observed for:

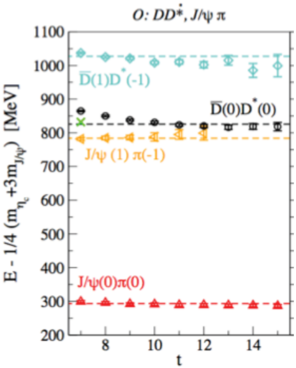
- pn bound st.: NPLQCD:1301.5790, PACS-CS PRD84 (2011) 054506]
- DK bound st.: [talk my D. Mohler]

$X(3872)$	$m_X - \frac{1}{4}(m_{\eta_c} + 3m_{J/\psi})$	$m_X - (m_{D^0} + m_{D^{0*}})$
$\mathrm{lat}^{L\to\infty}$	815 ± 7 MeV	-11 ± 7 MeV
exp	804 ± 1 MeV	-0.14 ± 0.22 MeV

The $Z(3900)$

from S. Prelovsek's talk at Lattice 2013, see also arXiv:1308.2097

Lattice search for $Z_c^+(3900)$ in $J^{PC}=1^{+-}$, I=1 channel



- only scattering states found
- small energy shifts → small interaction
- **we find no candidate for $Z_c^+(3900)$ in 1^{+-} channel**
- Possible reasons:

✧ perhaps $J^{PC} \neq 1^{+-}$ (exp unknown)

✧ perhaps our interpolators (all of scat. type) are not diverse enough : calls for further simulations

✧ does the state really exist ?

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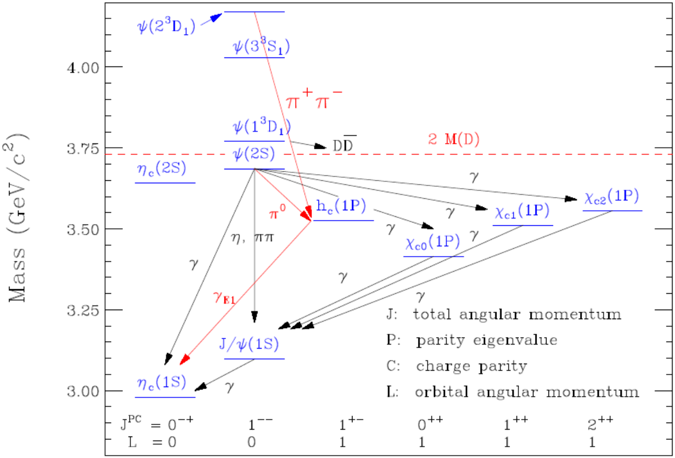


Radiative transitions/decays of charmonium

- To lowest order in QED, amplitude for $J/\psi \to \gamma H$ is given by

$$M_{r,r_\gamma,r_H} = \epsilon^*_\mu(\vec{q}, r_\gamma)\langle H(\vec{p}_f, r_H)|j^\mu(0)|J/\psi(\vec{p}_i, r)\rangle, \qquad (13)$$

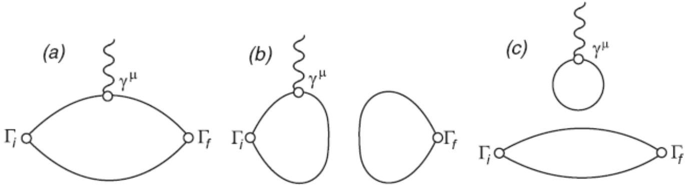
  - $\vec{p}_i$: initial momentum of J/ψ
  - $\vec{p}_f$: final momentum of hadron H
  - $\vec{q} = \vec{p}_i - \vec{p}_f$: momentum of the real photon
  - r/r_γ: polarization of initial J/ψ/photon
  - r_H: polarization of final hadron (if needed)
  - $\epsilon(\vec{q}, r_\gamma)$: polarization vector of photon
  - $j^\mu(0)$: electromagnetic current operator

☞ Matrix element $\langle H(\vec{p}_f, r_H)|j^\mu(0)|J/\psi(\vec{p}_i, r)\rangle$ is non-perturbative in nature

Three-point function

$$\Gamma^{(3)}_{i,\mu,j}(\vec{p}_f, \vec{q}; t_f, t) = \frac{1}{T} \sum_{\vec{y},\tau=0}^{T-1} e^{-i\vec{q}\cdot\vec{y}} \langle \Phi^{(i)}(\vec{p}_f, t_f+\tau) J_\mu(\vec{y}, t+\tau) O_{V,j}(\vec{0}, \tau) \rangle \quad (14)$$

- $J_\mu = \bar{c}\gamma_\mu c$: vector current for the charm quark
- $O_{V,j} = \bar{c}\gamma_j c$: interpolating operator for J/ψ
- $\Phi^{(i)}$ interpolating operator for final hadron H



Multipole decompostions

$$
\begin{aligned}
\langle H(\vec{p}_f, r_H)|j^\mu(0)|J/\psi(\vec{p}_i, r)\rangle &= \alpha_1^\mu E_1(Q^2) + \alpha_2^\mu M_2(Q^2) \\
&+\alpha_3^\mu E_3(Q^2) + \alpha_4^\mu C_1(Q^2) + \alpha_5^\mu C_2(Q^2) \qquad (15)
\end{aligned}
$$

where the first is for scalar while the second is for tensor glueball.

- α's are known functions of p_f and p_i;
- $Q^2 = -(p_i - p_f)^2$ is the photon four-momentum squared

☞ Physical photon point: $Q^2 = 0$

- $E_i(Q^2)$, $M_2(Q^2)$, $C_i(Q^2)$'s form factors
- For scalar: only $E_1(0)$ enters the physical decay rate
- For pseudoscalar: only $V(0)$ enters the physical decay rate
- For tensor: only $E_1(0)$, $M_2(0)$ and $E_3(0)$ enter the physical decay rate

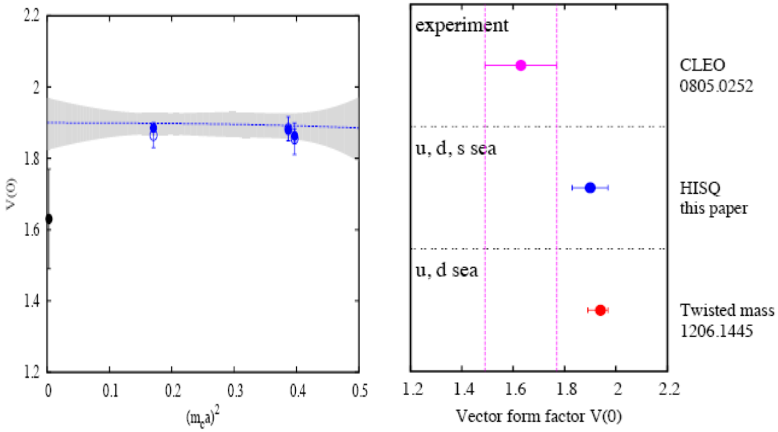
Existing lattice calculations

- J. Dudek et al, PRD73 074507 (2006) (quenched)
- CLQCD PRD84 034503 (2011) ($N_f = 2$ twisted mass)
- D. Becirevic and F. Sanfilippo, JHEP 01 028 (2013) ($N_f = 2$ twisted mass)
- HPQCD PRD86 094501 (2012) (2+1 flavor HISQ)

Example: $J/\psi \to \gamma\eta_c$

see HPQCD, PRD86 094501 (2012), Becirevic& Sanfilippo JHEP 01 028 (2013)

$$\langle \eta_c(p')|\bar{c}\gamma^\mu c|J/\psi(p)\rangle = \frac{2V(q^2)}{M_{J/\psi}+M_{\eta_c}}\epsilon^{\mu\alpha\beta\gamma}p'_\alpha p_\beta \epsilon(p)_\gamma \tag{16}$$



One can also calculate $J/\psi \to \gamma G$

quenched, see CLQCD, PRL110 021601 (2013); PRL111 091601 (2013)

- scalar glueball

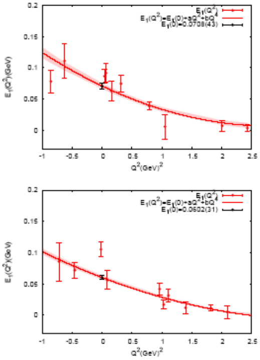


FIG. 2: The extracted form factors $E_1(Q^2)$ in the physical units. The upper panel is for $\beta = 2.4$ and the lower one for $\beta = 2.8$. The curves with error bands show the polynomial fit with $E_1(Q^2) = E_1(0) + aQ^2 + bQ^4$, as the black dot is the interpolated value $E_1(0)$ at $Q^2 = 0$.

- tensor glueball

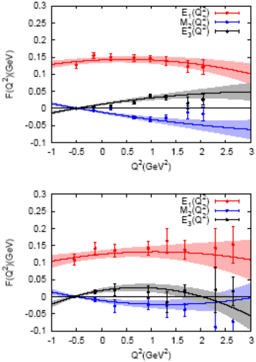


FIG. 2: The extracted form factors $E_1(Q^2)$ $M_2(Q^2)$ a
$E_3(Q^2)$ in the physical units. The upper panel is for $\beta =$
and the lower one for $\beta = 2.8$. The curves with error bar
show the polynomial fit with $F_i(Q^2) = F_i(0) + a_iQ^2 + b_iQ$

4. The story of f_{D_s}

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The decay constant f_{D_s}

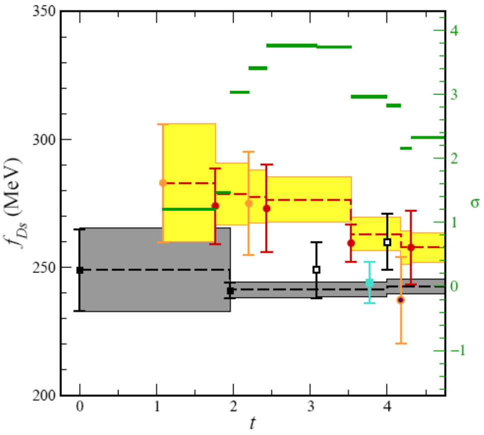
For a pseudoscalar meson, the decay constant is defined via the matrix elements

$$\begin{cases} \langle\Omega|\bar{s}\gamma_\mu\gamma_5 c|D_s(p)\rangle = if_{D_s}p_\mu \ , \\ (m_c+m_s)\langle\Omega|\bar{s}\gamma_5 c|D_s(p)\rangle = -m_{D_s}^2 f_{D_s} \end{cases} \tag{17}$$

☞ A very well-measured quantity in lattice QCD

History of f_{D_s}

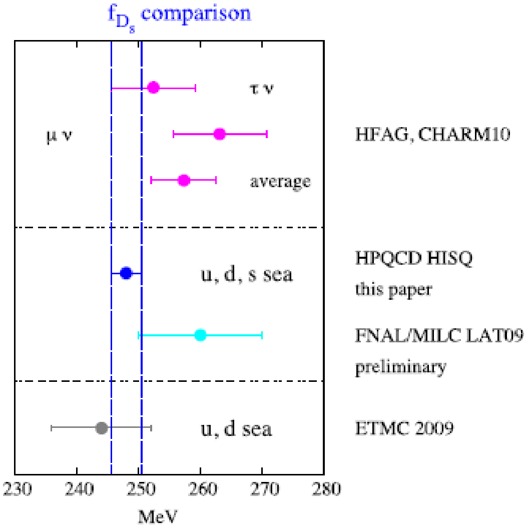
☞ see A. Kronfeld, "The f_{D_s} puzzel", Proceedings of PIC2009



- $t = 0 \mapsto$ June 2005
- 2007.06, lattice QCD's error dropped significantly, pushing up the tension to about 3.8σ
- 2009.01, CLEO's new measurement, -0.8σ
- reinterpretation of Barbar experiments by HFAG: -0.67σ
- ...

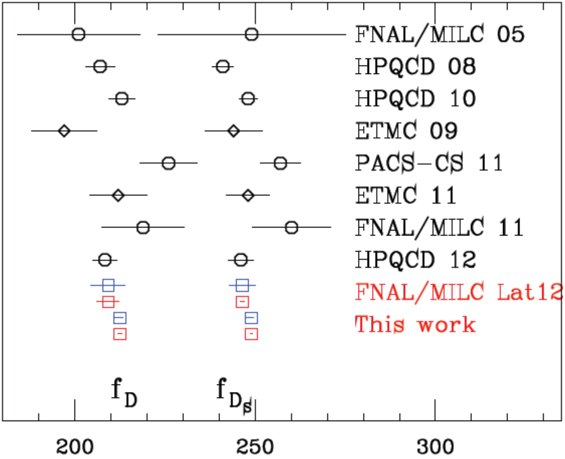
Update from HPQCD

see HPQCD, PRD82 114504 (2010)



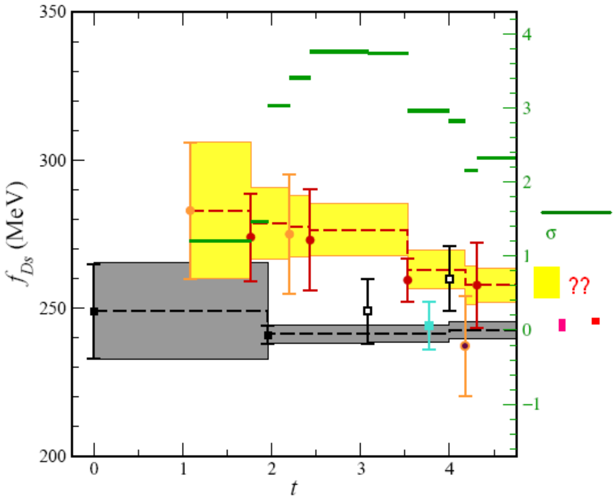
☞ $f_{D_s} = 248.0(2.5)$MeV; only 1.6σ lower than experiment

This years new result
see C. Bernard's talk at Lattice 2013



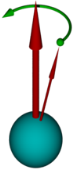
☞ $f_{D_s} = 248.9(0.2)(^{+0.5}_{-1.6})\text{MeV}$

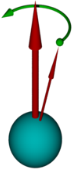
Waiting for more accurate experiments



☞ It would be nice to have more accurate (say, 1%) result for f_{D_s} experimentally (BESIII??)

5. Anomalous magnetic moment of muon



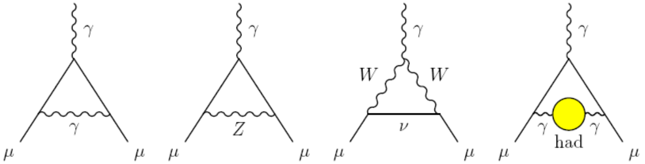


Why anomalous magnetic moment of muon?

- Definition: $a_\mu \equiv (g_\mu - 2)/2$
- One of the most accurately measured quantities: $a_\mu^{\rm exp} = 11659208.9(5.4)(3.3) \times 10^{-10}$
- The same quantity, predicted by Standard Model (SM): $a_\mu^{\rm SM} = 11659180.2(0.2)(4.2)(2.6) \times 10^{-10}$
- Measured value differs from SM prediction: $\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = 287(63)(49) \times 10^{-11}$
- This is roughly a 3.6σ effect. Possible room new physics (SUSY, dark photons,...)??

☞ But, before we jump into such conclusions, we should first check $a_\mu^{\rm SM}$ more carefully!

Why anomalous magnetic moment of muon?



- The so-called $a_\mu^{\rm SM}$ is a sum of three types of contributions:

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm Had} \tag{18}$$

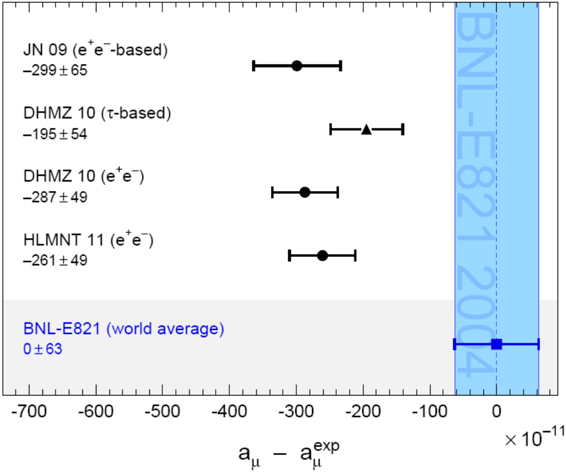
☞ $a_\mu^{\rm QED} \gg a_\mu^{\rm Had} \gg a_\mu^{\rm EW}$

☞ Errors dominated by $a_\mu^{\rm Had}$ (factor of 10 or more)

$$a_\mu^{\rm Had} = a_\mu^{\rm Had}(HVP) + a_\mu^{\rm Had}(HLbL) + \cdots \ , \tag{19}$$

- Leading contribution: Hadronic Vacuum Polarisation $a_\mu^{\rm Had}(HVP)$, which can be measured experimentally
- Next-order correction: Light-by-light scattering $a_\mu^{\rm Had}(HLbL)$ which can not be measured easily (modelling)

A summary figure from PDG



- Is it possible to compute $a_\mu^{\rm Had}$ from the lattice?

Lattice attempts for HVP: a partial list

- T. Blum, PRL 91 052001 (2003) (quenched)
- QCDSF: M. Göckeler et al., NPB 688 135 (2004) (quenched)
- Aubin and Blum: PRD75 114502 (2007) (2+1 imp. staggered)
- X. Feng et al, PRL107 081802 (2011) (Twisted mass $N_f = 2$)
- UKQCD: PRD85 074504 (2012) (2+1 DWF)
- Mainz: JHEP1203 055 (2012) ($N_f = 2$ O(a) imp. Wilson)
- New results
  - ETMC: arXiv1308.4327 (2+1+1 twisted mass), see also talk by Hotzel at Lattice 2013
  - BMW group: see E. Gregory talk at Lattice 2013
  - Mainz: arXiv1306.2532, see also A. Francis talk at Lattice 2013

Basic Ingredients for a lattice calculation

Focus on the HVP first:

$$a_\mu^{HVP} = \alpha^2 \int_0^\infty \frac{dQ^2}{Q^2} w(Q^2/m_\mu^2)\hat{\Pi}(Q^2) \tag{20}$$

where $w(Q^2/m_\mu^2)$ is a known function; $\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$ which is defined via:

$$\begin{cases} \Pi_{\mu\nu}(Q) \equiv (Q_\mu Q_\nu - Q^2\delta_{\mu\nu})\Pi(Q^2) \\ \Pi_{\mu\nu}(Q) = \int d^4x e^{iQ\cdot x}\langle\Omega|T[J_\mu(x)J_\nu(0)]|\Omega\rangle \end{cases} \tag{21}$$

☞ $\langle\Omega|T[J_\mu(x)J_\nu(0)]|\Omega\rangle$ can be computed using standard methods in lattice QCD

Challenges: I

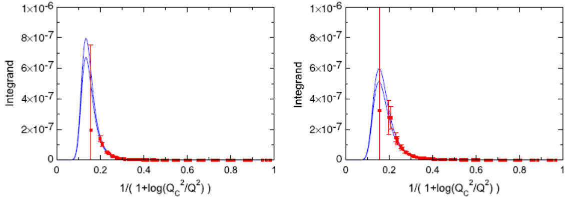
- First we need $\Pi(0)$ to form $\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$
- Conventional treatment relies on extrapolations from $Q^2 > 0$ to $Q^2 = 0$
- This introduces systematic errors
- It would be nice to compute it directly on the lattice (see G.M. de Divitiis et al, arXiv1208.5914)

Challenges: II

- a_μ^{HVP} is dominated by contributions from low-Q^2

$$a_\mu^{HVP} \sim \int_0^{Q_c^2} dQ^2 F(Q^2) \,, \qquad (22)$$

where $F(Q^2)$ is peaked around m_μ^2, indicating its non-perturbative nature. Change variable to $t = 1/(1+\log(Q_c^2/Q^2))$, and the integrand looks like



(a) $\beta = 2.25$ $am_u = 0.004$ $Q_C^2 = 11\ \mathrm{GeV}^2$

(b) $\beta = 1.75$ $am_u = 0.0042$ $Q_C^2 = 4\ \mathrm{GeV}^2$

- Red data points are from Boyle et al, PRD85 074504 (2012)

Challenges: III

Recent results from Lattice QCD

Chuan Liu

Masses of hadrons

Scattering and the XYZ states

Radiative transitions/decays of charmonium

The story of f_{D_s}

Anomalous magnetic moment of muon

Summary and outlooks

- Not so many low-Q^2 values on a lattice
- Periodic boundary conditions yields discrete Q^2
- Use twisted boundary conditions, but transverse property violated! But it could be cured (see C. Aubin et al, arXiv1307.4701).

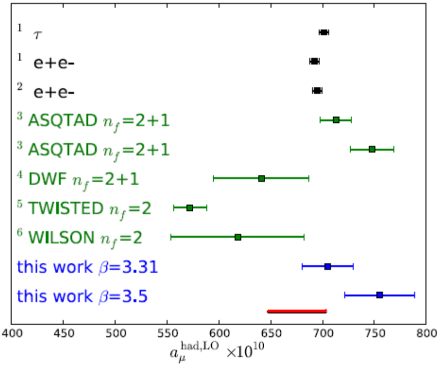
Challenges: IV

- Need a parametrization (functional form) of $\hat{\Pi}(Q^2)$ to integrate
- M. Goltermann et al suggested that Padè should be used (arXiv1205.3695).
- One could try the new analytically continuation method, but the fluctuation seems larger, (see X.Feng et al, arXiv1305.5878)

Challenges: V

- need extremely well-controlled errors, both statistically and systematically
- statistically: tricks like All-Mode-Averaging (AMA, see arXiv1208.4349)
- systematically:
  - chiral extrapolations, if any
  - finite volume effects,
  - finite lattice spacing errors
  - other possible sources of errors

Summary of the lattice calculations



Partly taken from Gregory's talk at Lattice 2013

[1]M. Davier, *et al.*(2011)

[2]K. Hagiwara, *et al.*(2011)

[3]C. Aubin and T. Blum (2007)

[4]P. Boyle,*et al.*(2012)

[5]X. Feng, *et al.* (2011)

[6]M. Della Morte *et al.* (2012)

lowest data from ETMC 2+1+1 flavor calculation: arXiv:1308.4327

- more results are coming in the future...

Summary and outlook

1. As a theoretical tool from first principles, Lattice QCD has become an important player in relevant physics
   - ☞ more and more unquenched results are available
2. For the hadron masses and decays
   - both post-dictions and pre-dictions
   - helps to clarify nature of newly discovered states
   - precision tests: some quantities can be obtained with very good precision in lattice QCD:
     - Some agree with experiment perfectly: $M_{J/\psi} - M_{\eta_c}, \ldots$
     - Some awaits further experiments: $\Gamma_{J/\psi \to \gamma\eta_c}$, f_{D_s}
3. For the muon $g-2$,
   - It is feasible to compute the leading hadronic contributions from QCD first principles
   - More efforts are needed to bring down the error bars

Thank you for your attention!