Introduction to Standard Model

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Three Lectures

- Lecture 1: What is the Standard Model (SM)?
- Lecture 2: SM parameters and tests
- Lecture 3: Neutrinos and the physics beyond SM

- Ref:
  - M. Peskin and D. Schroeder, An introduction to QFT
  - Particle Data Group, reviews on SM
  - R. Mohapatra, Unification and Supersymmetry

Lecture 1: What Is the SM?

- SM particles
- SM Fields:
- Constructing the Lagrangian, mass dimension counting
- QCD
- SU(2)xU(1) theory.
- Mass generation: spontaneous breaking of gauge symmetry
- CP violation
- The final chapter of SM: Higgs discovery!

A short summary of the SM

- It is a beautiful theory! (lecture 1)
  - It is based on quantum field theory, the result of marrying quantum mechanics with special relativity.
  - It is based on the simple gauge symmetry principle.
  - It unifies electromagnetism and weak interaction.
- It is a theory that can explain almost everything that happens in the world! (lecture 2)
  - It contains 24 (26) parameters.
  - Precision tests of strong interactions and electroweak.

A short summary of the SM

- It is a theory that is fined tuned, as many parameters appear "un-natural." (Lecture 2)
- It is an incomplete theory because it lacks explanation for many curious facts→ a more fundamental theory? (Lecture 3)

A century of discoveries of particles

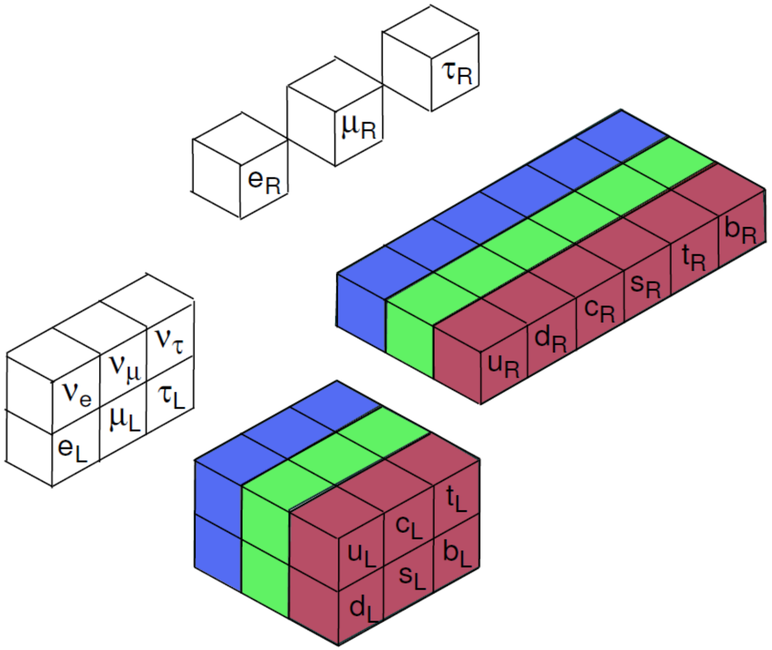
- 1897: Electron discovered by J.J. Thomson[4]
- 1919: Proton discovered by Ernest Rutherford[8]
- 1932: Neutron discovered by James Chadwick[9] (predicted by Rutherford in 1920[10])
- 1932: Antielectron (or positron) the first antiparticle, discovered by Carl D. Anderson[11]
- 1937: Muon (or mu lepton) discovered by Seth Neddermeyer et al.
- 1956: Electron neutrino detected by Frederick Reines and Clyde Cowan
- 1962: Muon neutrino (or mu neutrino) shown to be distinct from the electron neutrino by Lederman et al.

A century of discoveries of particles

- 1969: Partons and thus the discovery of the up quark, down quark, and strange quark in deep-inelastic scattering.
- 1974: J/ψ meson (charm quark) discovered by groups headed by Burton Richter and Samuel Ting.
- 1975: Tau discovered by a group headed by Martin Perl
- 1977: Upsilon meson discovered at Fermilab, demonstrating the existence of the bottom quark
- 1979: Gluon observed indirectly in three jet events at DESY
- 1983: W and Z bosons discovered by Carlo Rubbia, Simon van der Meer, and the CERN UA1 collaboration
- 1995: Top quark discovered at Fermilab
- 2000: Tau neutrino first observed directly at Fermilab
- 2012: Higgs bosons at LHC

Particle content

- Leptons (18)
  3 generations x 3 types $(e_L,\ e_R, \nu_L)$ x 2 charges
- Quarks (72)
  3 generations x 3 colors
  x 4 types (u_L, d_L, u_R, d_R)
  x 2 charges
- There are a total of 90 fermions
- There is no evidence for 4th generation
- There is no evidence for existence or non-existence ν_R



Gauge particles

- Generating interactions , starting from Global Symmetries
- Photons, from SU(2)xU(1) symmetry, 2 d.o.f
- Gluons, from SU(3) symmetry, 2x8 = 16 d.o.f
- W and Z bosons, from SU(2)xU(1), 3x3 = 9 d.o.f
- A total of 27 d.o.f

- Higgs boson, for generating mass, 1 d.o.f

- SM has a total of 90+27+1= 118 d.o.f

- Spins: 0, ½, 1

Dirac Fields: fermions

- All fermions can be generated from Dirac field

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

Define the projection operator ($\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$)

$$P_{L,R} = \left(\frac{1}{2}\right)(1 \mp \gamma_5)\psi$$

we have $\psi_{L,R} = P_{L,R}\psi$

These are left and right-handed chiral fields which form Weyl spinors.

8 real d.o.f's and 4 of them are eliminated by equation of motion (Dirac equation): L, R particles and antiparticles.

Boson Fields

- Gauge particles are presented by vector potentials

$$A^{\mu}$$

in which one of the d.o.f's is gauge d.o.f and has no dynamics, one represents static interactions, the remaining two are dynamical d.o.f's : transverse polarizations

- Scalar Higgs potential (SU(2) complex doublet)

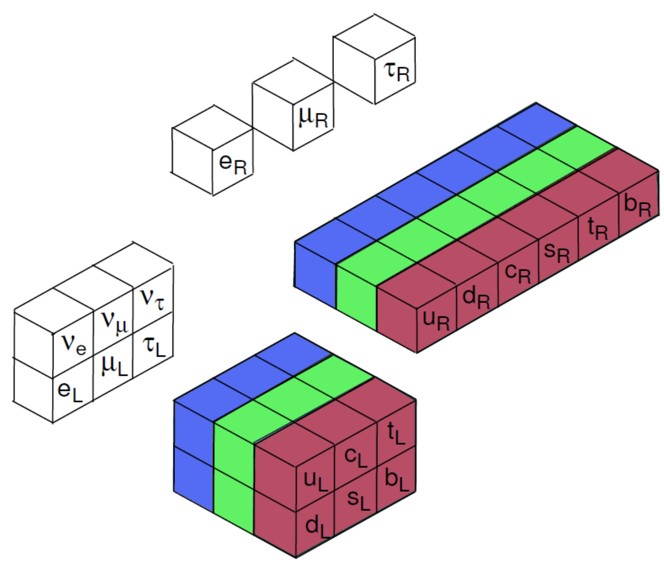
$$\varphi=\begin{pmatrix}\phi^{+}\\ \phi^{0}\end{pmatrix}$$

has 4 d.o.f.

3 of them become the longitudinal components of W and Z, and the last one is the neutral higgs particle.

Symmetries of SM: Origin of interactions!

- SU(3) color symmetry, T^a, gauged, QCD.
- SU(2)L weak isospin symmetry, I_L, gauged,
- U(1)Y weak hypercharge symmetry, Y, gauged
- When all particles are massless, we have SU(6)LxSU(6)R chiral symmetry, breaks down to SU(3)LxSU(3)R when top, bottom and charm quarks are heavy, down to SU(2)LxSU(2)R when the strange is considered heavy.
- U(1) Baryon number symmetry.
- U(1) Lepton number symmetry.
- U(1) Axial symmetry
- C, P, T
- Lorentz symmetry



SM Lagrangian L_{SM}

- SM lagrangian density is made of local terms with fields and parameters.
- Mass dimension MD:

  Using the natural unit ($\hbar = c = 1$):

  Energy, momentum and mass have dimension of mass;

  Time and space coordinates have dimension of $(\text{mass})^{-1}$
- Lagrangian density has MD-4, $\int Ld^4x$ has no MD.
- Fermion field ψ has mass-dimension (MD) 3/2, vector and scalar fields, A^{μ} and ϕ, have (MD) 1.
- The renormalizable lagrangian density, L_{SM}, is made of operators of DM 4 or less.

Renormalizability allows consistent calculations with finite number of parameters in field theory.

QCD lagrangian: the simple example

- Quarks and gluons ($8 = 3^2 - 1$)
- All quarks have color charges. Gluons do too.
- All flavors of quarks have the same strong interaction dynamics (flavor blind).
- For kinetic energy terms and gauged minimal coupling terms, there is no dimension-ful parameter.
- For example, QCD lagrangian is made of

$$L = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}\big((i\partial^{\mu} - gA^{\mu})\gamma_{\mu}\big)\psi$$

where $F_a^{\mu\nu} = \partial^{\mu} A_a^{\nu} - \partial^{\nu} A_a^{\mu} - g f_{abv} A_b^{\mu} A_c^{\nu}$ where the only parameter is the dimensionless coupling g.

- QCD is the fundamental theory of strong interactions.

Mass terms

- For fermions, dimension-3 operator $\bar{\psi}\psi$ has a coefficient as mass parameter

  $$m\,\bar{\psi}\psi$$ (violate SU(2)L gauge symmetry)

- Fermion masses are protected by chiral symmetry. The quantum correction to the mass can only be logarithmic, proportional to the bare mass.

  $$\delta m = m\,(1 + \alpha\,\ln\left(\frac{\Lambda}{\mu}\right))$$

- Bosonic mass term,

  $$m^2\phi^2, \quad \mathrm{m}^2\mathrm{A}^\mu\mathrm{A}_\mu$$ (violate gauge symmetry)

  there is quadratic quantum correction,

  $$\delta m^2 = m^2 + \alpha\Lambda^2$$

- There can be no masses for quarks, leptons and gauge fields.

EW gauge theory: SU(2)$_L$x U(1)$_Y$

- Consider SU(2)xU(1) symmetry. The first generation of fermions is assigned in SU(2) quantum numbers,

  Doublets: $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}; \quad Q = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}, \quad \mathrm{i} = 1, 2, 3 \text{ color}$

  Singlet: e_R, u_R^i, d_R^i

  As for the U(1) hypercharge, the assignment is

  $$Q = Y + I_3$$

- After gauging the symmetry, there are 4 gauge potentials, $W_a^\mu (a = 1,2,3)$ and B^μ . And the gauge-field part of the lagrangian is standard,

$$L = -\frac{1}{4} W^{\mu\nu} W_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$

Mass generation

- Fermion and vector particle mass terms violate gauge symmetry.
- Weak interactions are short-ranged, and immediate gauge particles must be massive.
- To get masses for them, one needs to use **spontaneous breaking** of SU(2)xU(1) symmetries, which generates a mass scale.
- However, one must keep the electromagnetic interaction long range, photon massless.
- Therefore, the **symmetry breaking pattern** is

$$SU(2)_L \times U(1)_Y \to U(1)_{EM}$$

Spontaneous symmetry breaking (SSB)

- Consider a continuous symmetry group G of the lagrangian, with generators T^a (Lie algebra)
- Normally the vacuum (ground state) of the system, $|0\rangle$, is unique, and invariant under the symmetry transformation, $T^a|0\rangle = 0$.
- However, if for some reason, the vacuum is no longer symmetric under a certain subset of $\{T^a\}$, such as $\{X^i\}$,

$$X^i|0\rangle \neq 0$$

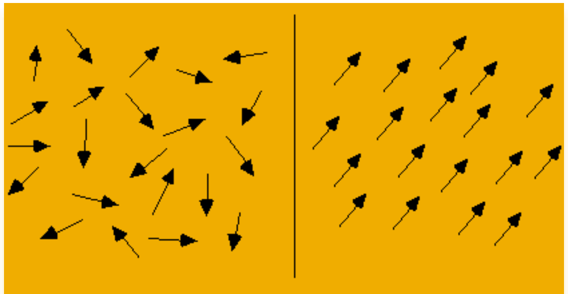
Then we say the symmetry is spontaneously broken. The remaining generators $\{H^i\} = \{T^a\} - \{X^i\}$ still annihilates the vacuum and forms a subgroup H.

We say that group G is spontaneously broken into subgroup H.

Example of Magnetization

- Consider a piece of paramagnetic material. The underlying lagrangian is symmetric under SO(3) rotation. The grand state of the system has no magnetic moment and hence is symmetric under SO(3) rotation as well.
- As temp lowers, the paramagnet gets spontaneously magnetized. There is now a special direction for magnetization.
- The ground state of the system is no longer symmetric under SO(3) rotation. However, it is still symmetric when rotating around the magnetization direction, SO(2). So we say the system has a SS breaking from

$$SO(3) \rightarrow SO(2)$$



Nambu-Goldstone Theorem

- SSB can happen not only for continuous symmetry groups, but also for discrete symmetries. (e.g. the double well potential in quantum mechanics)
- However, when the symmetry is continuous, there is Nambu-Goldstone theorem:

For every generator X_i that does not annihilate the vacuum, there exists an associated massless scalar particle, called Nambu-Goldstone boson or Goldstone boson.

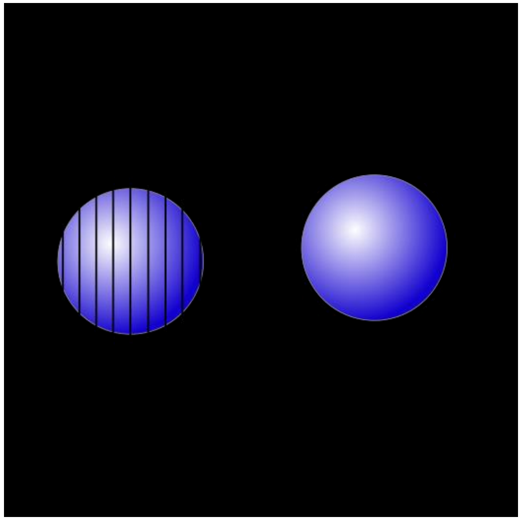
Consider a magnetic system with symmetry group SO(3), after spontaneous magnetization in z-direction, the symmetry becomes SO(2), there is two NG bosons associated with rotation in x and y directions. There are called spin waves.

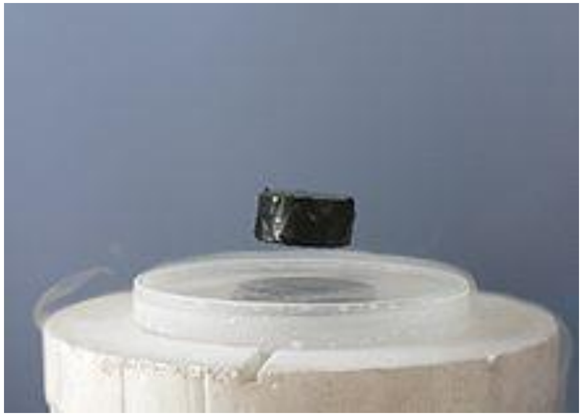
SSB of Gauge Symmetry: Higgs mechanism

- If group G is a gauge symmetry, then there is a vector boson A_a^μ associated with every generator T^a
- Before SSB, all the gauge bosons are massless
- After SSB, the NG bosons associated with broken generators X^i will appear as the third polarization of the corresponding vector boson, and thus the latter becomes a massive particle, conducting a short range force.
- This is the so-called Higgs Mechanism. Higgs particles are the massive particles left over in the scalar multiplet after others in the same multiplet become part of the massive gauge particles.

Superconductivity

- 1911, Onnes discovered superconductivity, the resistence drops to zero at low-temp for certain metal (Nobel Prize 1913)
- 1933, Meissner discovered the meissner effect, magnetic field cannot penetrate into a superconductor





Massive photon

- In a conductor, an electric field is not allowed because the electrons are free to move
- In a superconductor, neither electric nor magnetic field is allowed to exist
- In this environment, a massless photon cannot propagate. The interaction between two charges do not follow Columb's law, rather

$$e^{-mr}/r$$

We say photon now has a finite mass m!

Electroweak Interactions

- The coupling with quarks and leptons generates usual electroweak interactions.

- The couplings among Higgs fields generate SSB.
- The coupling between Higgs particles and Dirac particles generate mass terms.
- The coupling between Higgs particles and W and B bosons generates their masses.

Quarks and lepton weak interactions

- Define the new gauge field:

$$A = B\cos\theta_W + W^3\sin\theta_W$$
$$Z = -B\sin\theta_W + W^3\cos\theta_W$$
$$W^{\pm} = (W^1 \mp iW^2)/\sqrt{2}$$

$$-\frac{g}{2\sqrt{2}}\sum_i \overline{\Psi}_i\,\gamma^\mu\,(1-\gamma^5)(T^+\,W_\mu^+ + T^-\,W_\mu^-)\,\Psi_i$$
$$-e\sum_i q_i\,\overline{\psi}_i\,\gamma^\mu\,\psi_i\,A_\mu$$
$$-\frac{g}{2\cos\theta_W}\sum_i \overline{\psi}_i\,\gamma^\mu(g_V^i - g_A^i\gamma^5)\,\psi_i\,Z_\mu\ .$$

where $e = g\sin\theta_W$; $g_V = t_{3L}(i) - 2q_i\sin^2\theta_w$; $g_A^i = t_{3L}\,(i)$

Relation with Fermi Coupling, $\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} = \frac{g^2}{8M_W^2}$

Electroweak symmetry breaking and Higgs mechanism

- Introduce SU(2)L doublet

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

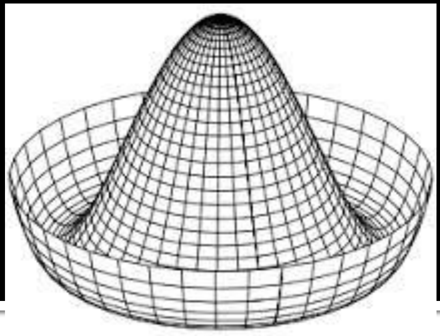
$$\mathcal{L}_{\mathrm{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

where $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_\mu$ and

$$\boxed{V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda|\,(\phi^\dagger \phi)^2}$$

- SSB happens when μ^2 is less than zero.

SSB



- Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$
  Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{\text{em}}$ invariance

V=246 GeV

Higgs coupling with the fermions

- Higgs coupling with fermions generate mass as well as the Yukawa interactions which are proportional to the quark mass,

$$\mathscr{L}_F = \sum_i \overline{\psi}_i \left(i \not\partial - m_i - \frac{g m_i H}{2M_W} \right) \psi_i$$

- Fermion mixing: mass eigenstates of quarks

$$L = h_{ij}\ \bar{Q}_{Li} \phi d_{Ri} + \tilde{h}_{ij} \bar{Q}_{Li}\, \tilde{\phi} u_{Ri} + h.c.$$

Where h_{ij} and $\tilde{h}_{ij}$ are general complex matrix. After SSB diagonalization, (using 18-3 degrees of freedom)

$$h_{ij} = D_L m_D D_R^{+} \qquad \tilde{h}_{ij} = U_L m_U U_R^{+}$$

Where m_D and m_U are diagonal and positive.

Charged particle coupling

- All fields are non-mass eigenstates before diagonalization.
- After diagonalization, one needs to use the mass eigenstates.
- All kinematic terms are easy to change to mass eigenstates, so are the electromagnetic and neutral weak coupling.
- However, charged weak interaction is

$$-\frac{e}{\sqrt{2}\sin\theta_W} W_\mu^- \bar{d}_{Li}\gamma^\mu u_{Li} + h.c.$$

- After changing to mass eigenstates, one has

$$-\frac{e}{\sqrt{2}\sin\theta_W} W_\mu^- \bar{d}_{Li}\gamma^\mu D_L^+ U_L u_{Li} + h.c.$$

where U = $D_L^+ U_L$ is the unitary Cabbibo-Kobayashi-Moskawa CKM matrix.

CP violation in SM

- The CP violation in the SM comes only from the CKM matrix.
- The unitary matrix U has in general 9 elements.
- However, one can wrote the mass eigenstates by constant phase factors.
- There are a total of 6-1 phase factors that one rotate.
- Thus there are 9-5 parameters in the CKM matrix, 3 rotational angle and 1 so-called Dirac phase δ.

$$V_{\mathrm{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Masses of W and Z boson

- The scalar particle has the kinetic energy term which must be gauged.

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^{\mu}\phi)^{\dagger}(\mathcal{D}_{\mu}\phi) - V(\phi^{\dagger}\phi),$$

where $\mathcal{D}_{\mu} = \partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_{\mu}$ and

$$\boxed{V(\phi^{\dagger}\phi) = \mu^2(\phi^{\dagger}\phi) + |\lambda|\,(\phi^{\dagger}\phi)^2}$$

- After SSB, this gauged term generates interactions between Higgs particle and W and Z bosons, as well as W and Z mass.
- W-mass,

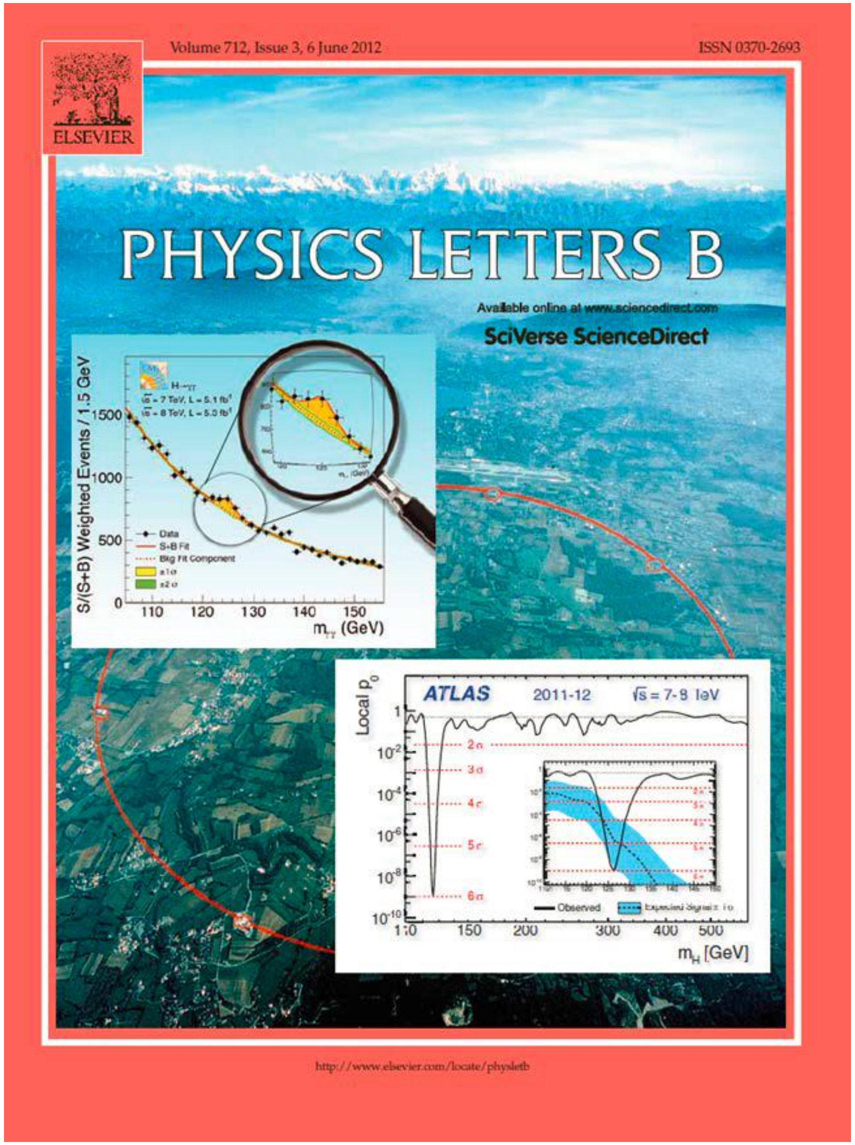
$$M_w = \frac{1}{2}gv = \frac{ev}{2\sin\theta_W}$$

$$M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}v = M_W/\cos\theta_W$$

Higgs Physics

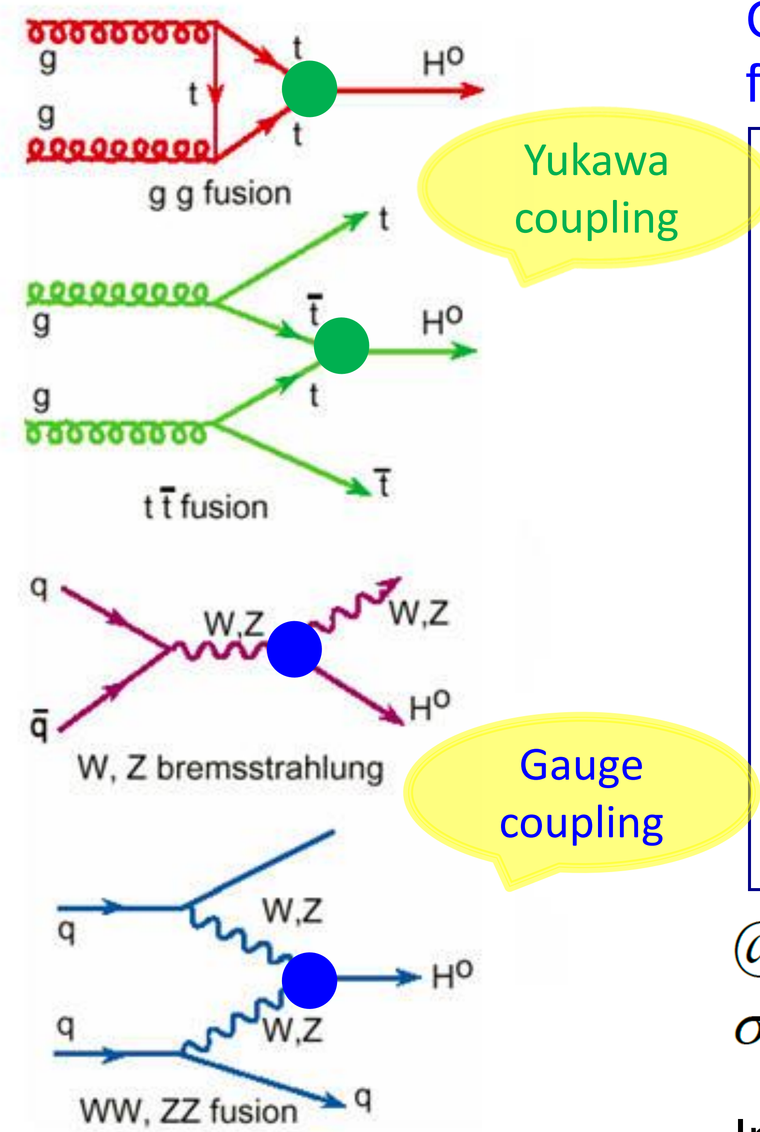
- In the process, Higgs particle interacts with W and Z boson through kinetic term of the scalar fields
- Higgs particle interacts with fermions through coupling proportional to fermion masses.
- Higgs self-interactions.

Observation of a new Particle (2012.7.4) !

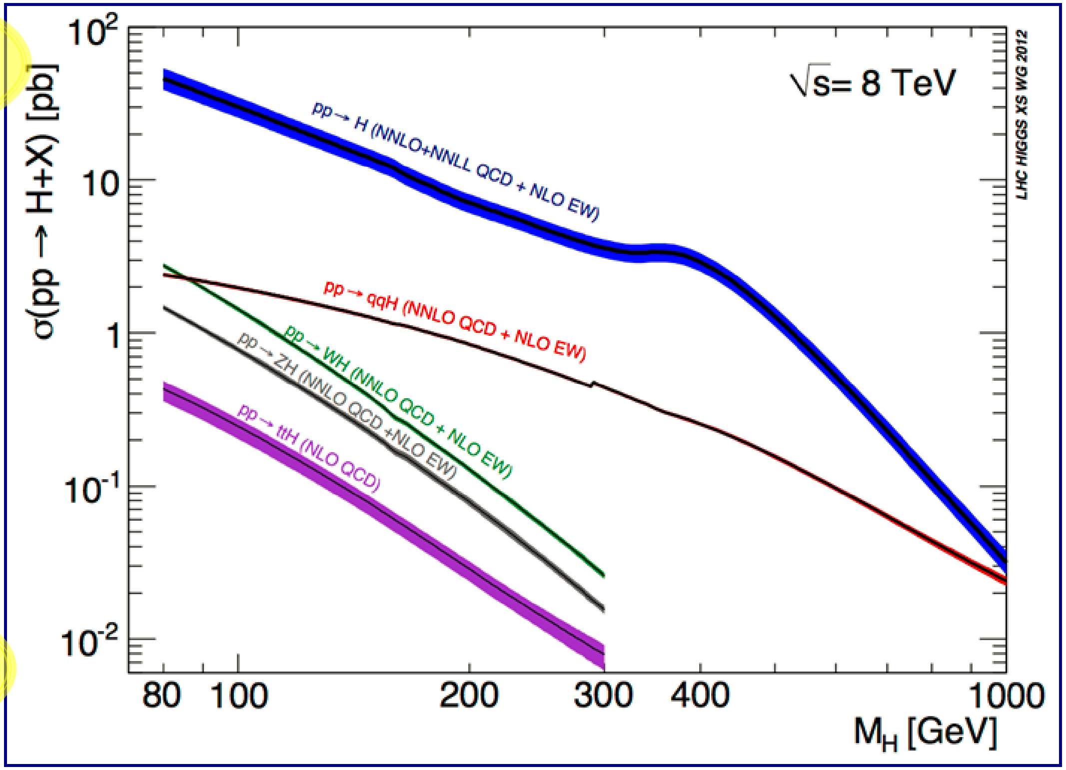


Phys. Lett. B 716 (2012) 1-29 (ATLAS) **Phys. Lett. B 716 (2012) 30-61 (CMS)**

Higgs Boson Production at LHC



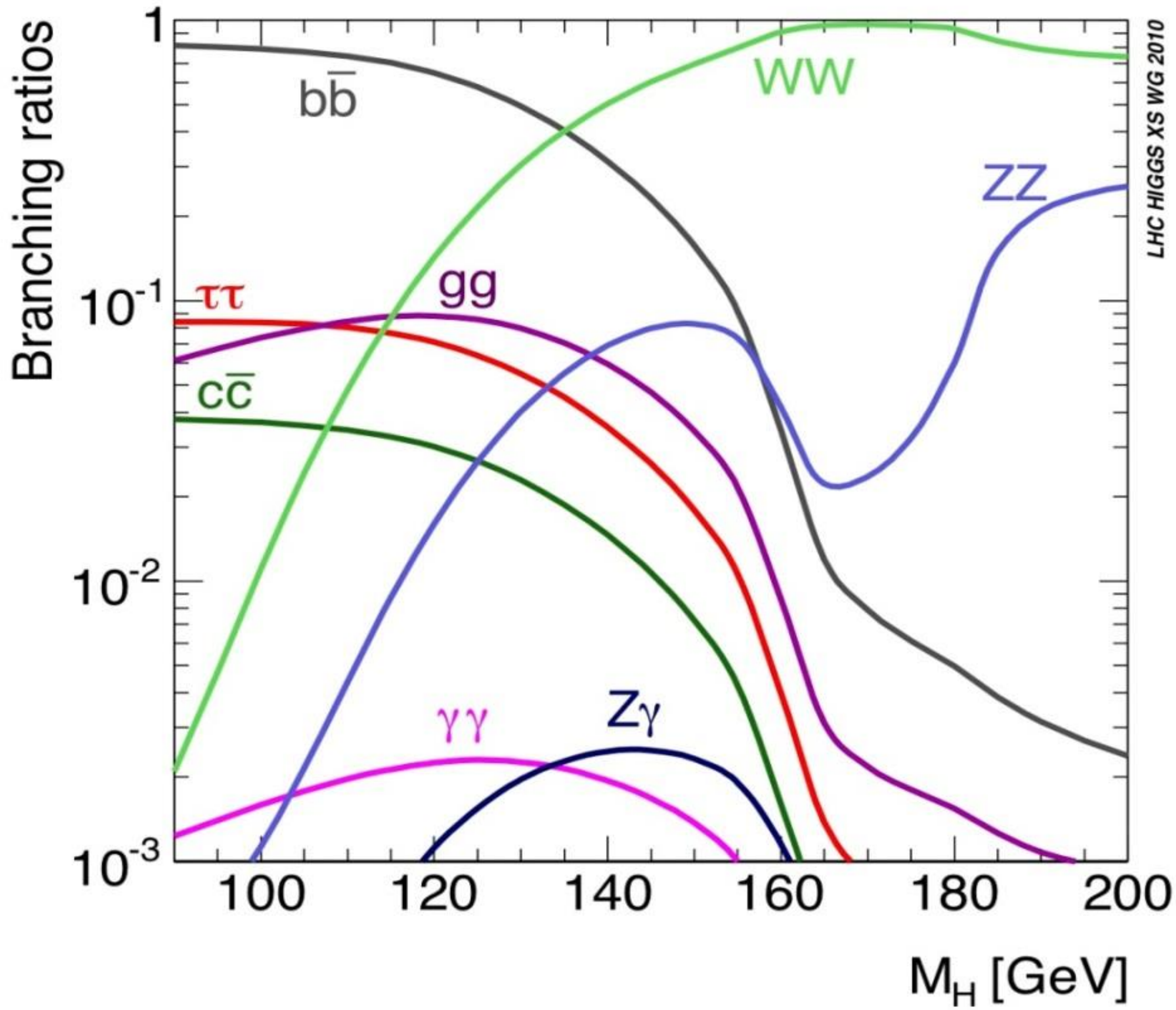
Gluon-gluon fusion gg→H and vector-boson fusion qq→qqH are dominant



$$@125\ \text{GeV:}\quad \sigma_{ggH}=19.5\ \text{pb},\ \sigma_{VBF}=1.6\ \text{pb},$$
$$\sigma_{WH}=0.70\ \text{pb},\ \sigma_{ZH}=0.39\ \text{pb},\ \sigma_{ttH}=0.13\ \text{pb}$$

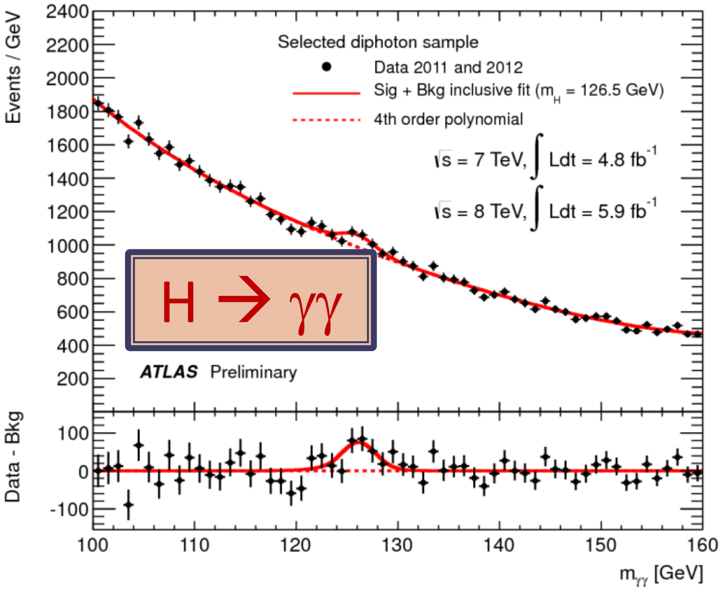
Inelastic pp cross section at 7 TeV is ~ 60 mb

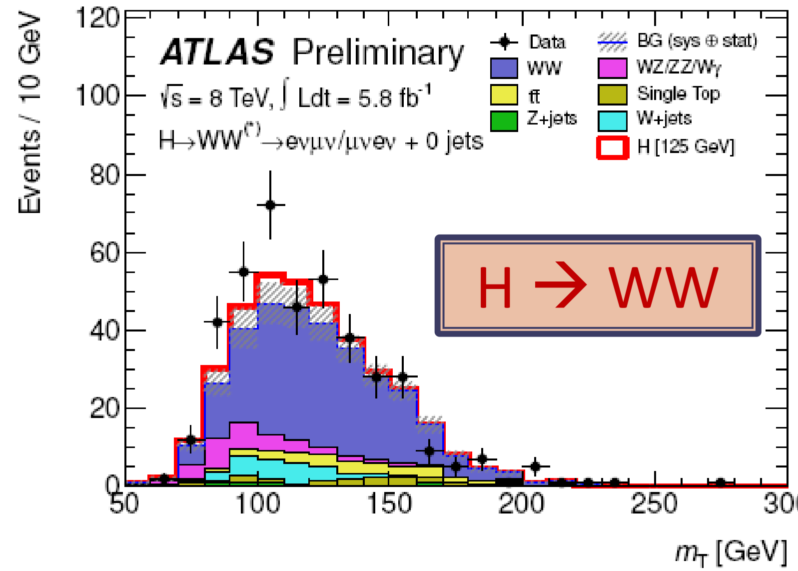
Higgs Decay

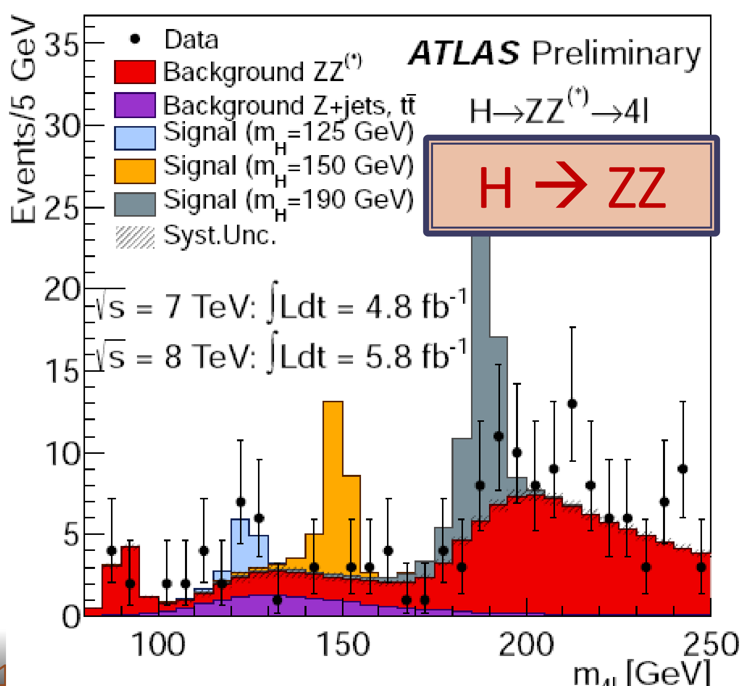


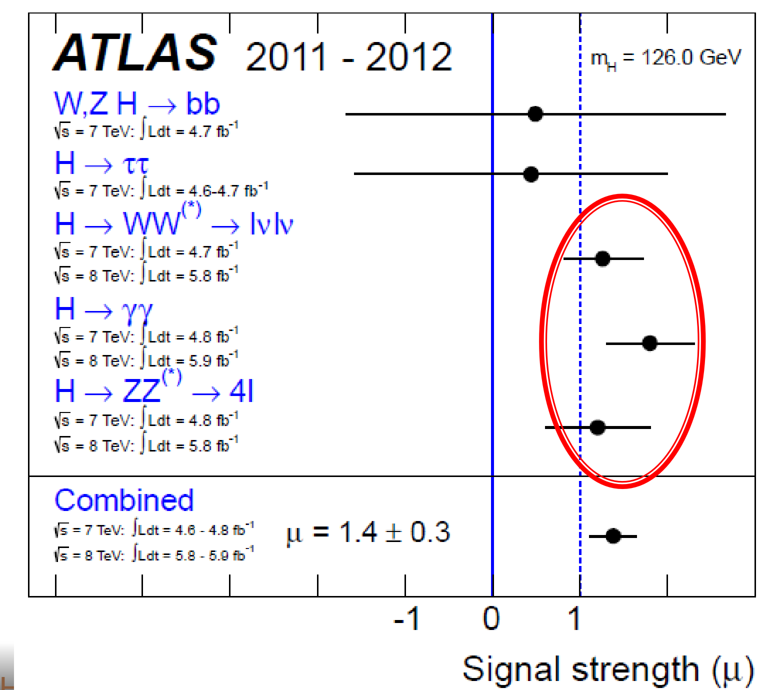
Branching ratios
1
10^{-1}
10^{-2}
10^{-3}
$b\bar{b}$
WW
ZZ
$\tau\tau$
gg
$c\bar{c}$
$\gamma\gamma$
$Z\gamma$
100
120
140
160
180
200
M_H [GeV]
LHC HIGGS XS WG 2010

Observation of a new Particle (July 4, 2012)



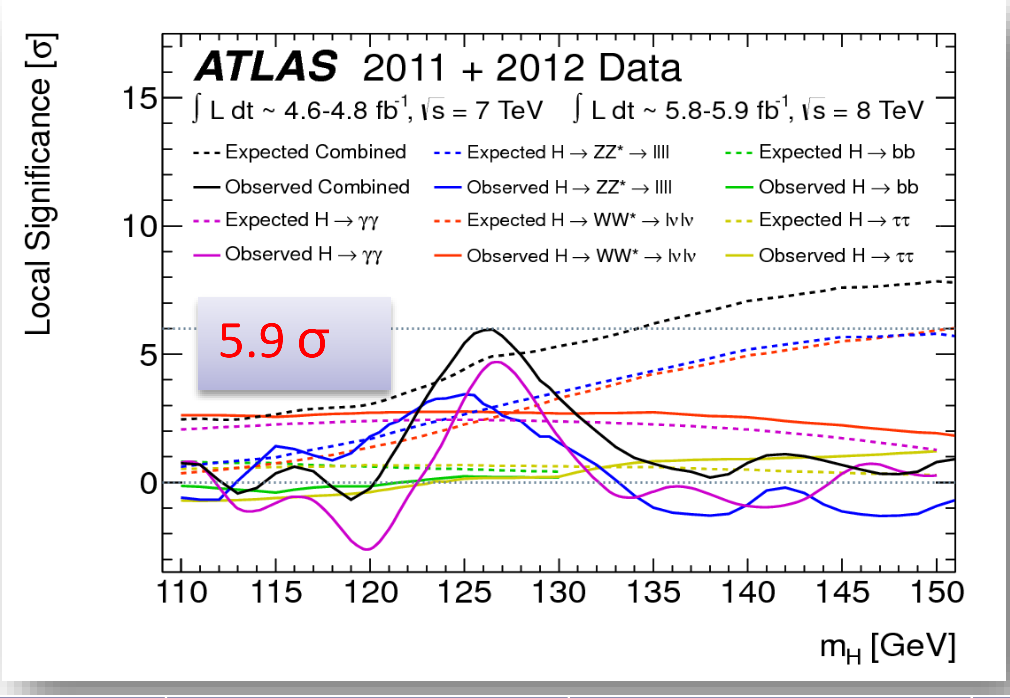






ATLAS Combined Results

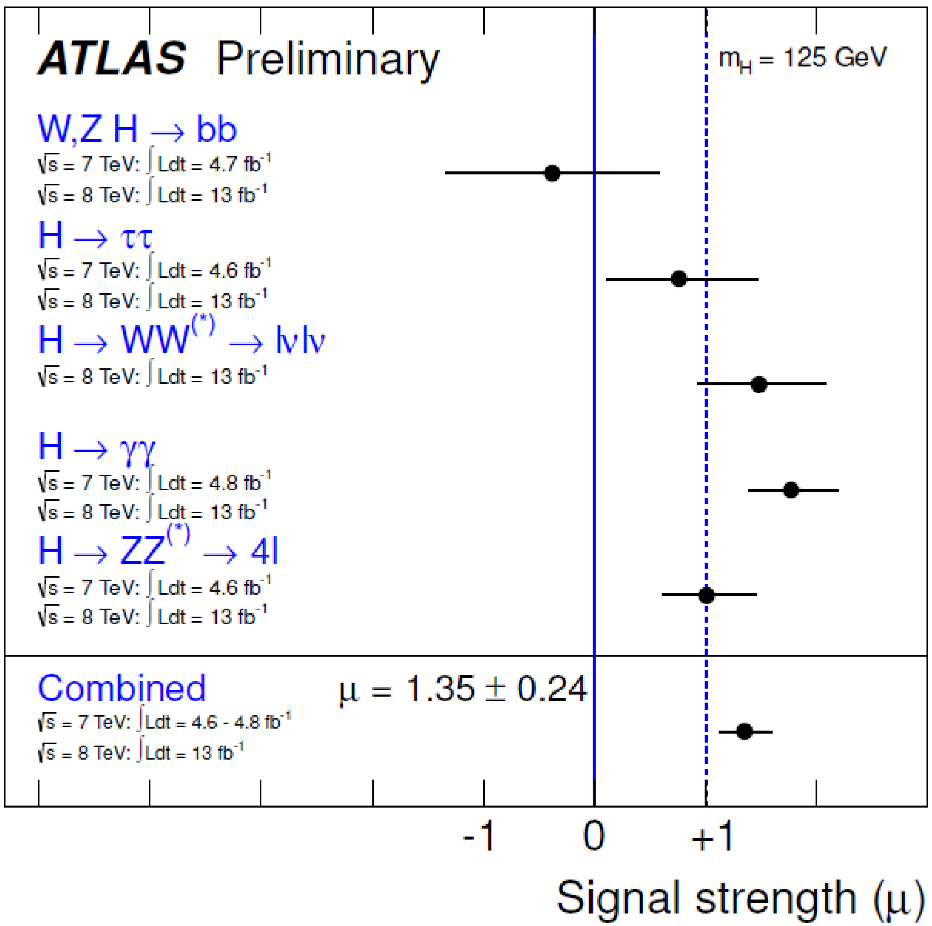
➔ **Discovery of a particle with a local significance of 5.9σ.**

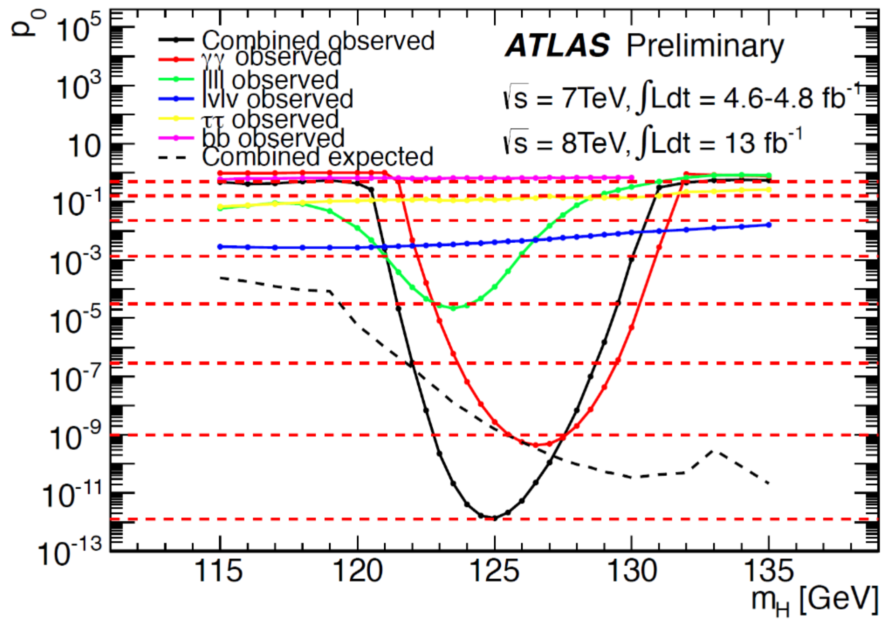


Channel	Fitted m_H	Observed	Expected
H→γγ	126.5 GeV	4.5σ	2.5σ
H→ZZ*→4l	125.0 GeV	3.6σ	2.7σ
H→WW*→lνlν	125.0 GeV	2.8σ	2.3σ
Combined	**126.0 GeV**	**5.9σ**	**4.9σ**

Update of Higgs Signal Strength

- ➔The observed significance is ~ 7.0σ (expected 5.9σ)
- ➔ The signal strength: μ = 1.35 ±0.24





Higgs Boson Decay	μ (m_H=125 GeV)
$VH \rightarrow Vbb$	-0.4 ± 1.0
$H \rightarrow \tau\tau$	0.8 ± 0.7
$H \rightarrow WW^{(*)}$	1.5 ± 0.6
$H \rightarrow \gamma\gamma$	1.8 ± 0.4
$H \rightarrow ZZ^{(*)}$	1.0 ± 0.4
Combined	1.35 ± 0.24

Is it the SM Higgs ?

❑ Verify the new observed particle

✓ Spin-0 particle

❖ Spin-1: excluded by $H \rightarrow \gamma\gamma$

❖ Spin-2: look at angular correlations

Spin of particle	$\gamma\gamma$	ZZ^*	$\tau\tau$	bb
Spin 0	☺	☺	☺	☺
Spin 1	☹	☺	☺	☺
Spin 2	☺	☺	☹	☺
Seen?	Yes	Yes	Not yet	Not yet

✓ CP-nature

❖ SM Higgs CP-even, extended Higgs sectors has CP-odd or mixed states

❖ Look at angular correlations

✓ Couplings

❖ Gauge / Yukawa couplings → $g_{VVH}, g_{ffH} \propto m$

❖ Unitarity in $W_L W_L$ scattering → $g_{WWH} \propto m_W$

❖ Higgs self-couplings, determine shape of Higgs potential via trilinear and quartic couplings $V = \mu^2|\Phi|^2 + \lambda|\Phi|^4 + \text{constant}$

Creators of Electroweak theory

- Glashow (1963), Salam(1967),Weinberg(1967),

- Nobel Prize in 1979