# Light Quark Mass Dependence of the $X(3872)$ in XEFT

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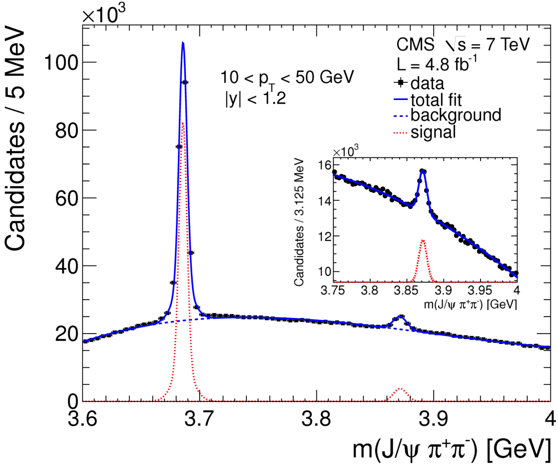
Huangshan, Nov. 20-22, 2013

# Outline

- First observation by the Belle Collaboration [Choi et al., 2003]
- Determination $J^{PC} = 1^{++}$ by LHCb [Aaij et al., 2013]




[Chatrchyan et al., 2013]

# Introduction and Motivation

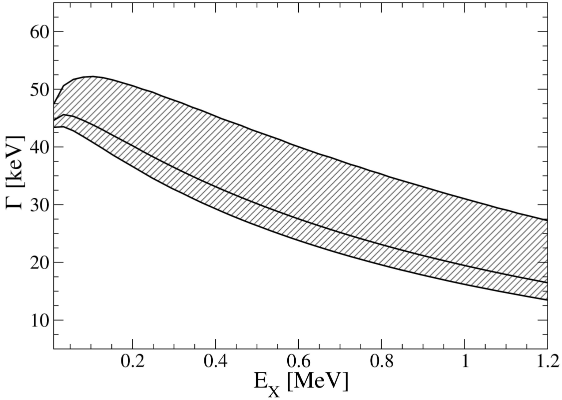
- Interpretations: tetraquark, charmonium, hadronic molecule
- Mass of the $X(3872)$ close to $D^0 D^{*0}$ threshold

**Particle Content of the $X(3872)$**

$$X = \frac{1}{\sqrt{2}}\left(\bar{D}^0 D^{*0} + D^0 \bar{D}^{*0}\right)$$

- Recent observation of a candidate for the $X$ on the lattice [Prelovsek and Leskovec, 2013]
- Performed on rather small lattices for large quark masses
- Previous work:
  - Unitarized heavy meson ChpT: no sensitivity to contact interactions [Wang and Wang, 2013]
  - Non-relativistic Faddeev-type three-body equations: contact interactions essential [Baru et al., 2013]

- Universal properties due to small binding energy
  $E_X = m_{D^*} + m_D - M_X = (0.17 \pm 0.26)$ MeV
- Corrections calculable in XEFT [Fleming et al., 2007]




Decay rate for $X \to D^0 \bar{D}^0 \pi^0$ as a function of $E_X$

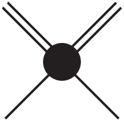
# Basics of XEFT

- Similar to KSW theory for $NN$ scattering [Kaplan et al., 1998]
  - Includes pions perturbatively
  - Unnaturally large NNLO coefficients [Fleming et al., 2000]
- Nearness of $D^0 D^{*0}$ hyperfine splitting and pion mass induces small mass scale $\mu^2 = \Delta^2 - m_\pi^2$
- Mass scale $\mu$, $D^{(*)0}$ and pion momenta and binding momentum of same order $Q \ll m_\pi, m_D, m_{D^*}$
- Pions and $D^{(*)0}$ mesons treated non-relativistically
- Integrated out charged $D^{(*)\pm}$ mesons
  - Effective field theory: $1/a$ suppression [Braaten and Kusunoki, 2004]
  - Charmonium- hadronic molecule hybrid: charged states small contribution [Takizawa and Takeuchi, 2013]
- Takes finite width of the $D^{*0}$ into account

# XEFT Lagrangian

$$
\begin{aligned}
\mathcal{L} =& \boldsymbol{D}^\dagger \left( i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_{D^*}} \right) \boldsymbol{D} + D^\dagger \left( i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_D} \right) D \\
&+ \bar{\boldsymbol{D}}^\dagger \left( i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_{D^*}} \right) \bar{\boldsymbol{D}} + \bar{D}^\dagger \left( i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_D} \right) \bar{D} + \pi^\dagger \left( i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_\pi} + \delta \right) \pi \\
&+ \frac{g}{\sqrt{2}f} \frac{1}{\sqrt{2m_\pi}} \left( D \boldsymbol{D}^\dagger \cdot \overrightarrow{\nabla} \pi + \bar{D}^\dagger \bar{\boldsymbol{D}} \cdot \overrightarrow{\nabla} \pi^\dagger \right) + \text{h.c.} \\
&- \frac{C_0}{2} \left( \bar{\boldsymbol{D}} D + \boldsymbol{D} \bar{D} \right)^\dagger \cdot \left( \bar{\boldsymbol{D}} D + \boldsymbol{D} \bar{D} \right) \\
&+ \frac{C_2}{16} \left( \bar{\boldsymbol{D}} D + \boldsymbol{D} \bar{D} \right)^\dagger \cdot \left( \bar{\boldsymbol{D}} \overleftrightarrow{\nabla}^2 D + \boldsymbol{D} \overleftrightarrow{\nabla}^2 \bar{D} \right) + \text{h.c.} \\
&- \frac{D_2 \mu^2}{2} \left( \bar{\boldsymbol{D}} D + \boldsymbol{D} \bar{D} \right)^\dagger \cdot \left( \bar{\boldsymbol{D}} D + \boldsymbol{D} \bar{D} \right) + \ldots,
\end{aligned}
$$

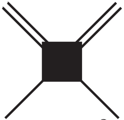
# Power Counting in XEFT



$-iC_0$

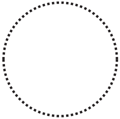
$\sim Q^{-1}$

$\sim Q^{-2}$

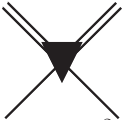


$-iC_2 p^2$

$\sim Q^{0}$



$\sim Q^{5}$



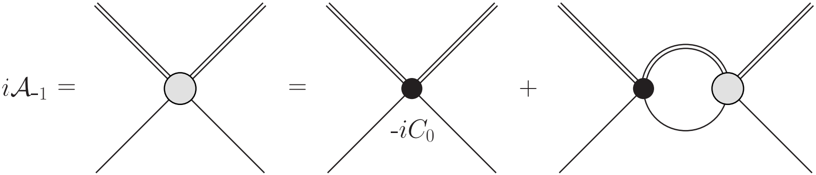
$-iD_2\mu^2$

$\sim Q^{0}$

$-i\frac{g}{\sqrt{2}f}\frac{1}{\sqrt{2m_\pi}}\left(\boldsymbol{\varepsilon}\cdot\mathbf{p}_\pi\right)$

$\sim Q^{1}$

# LO Scattering Amplitude



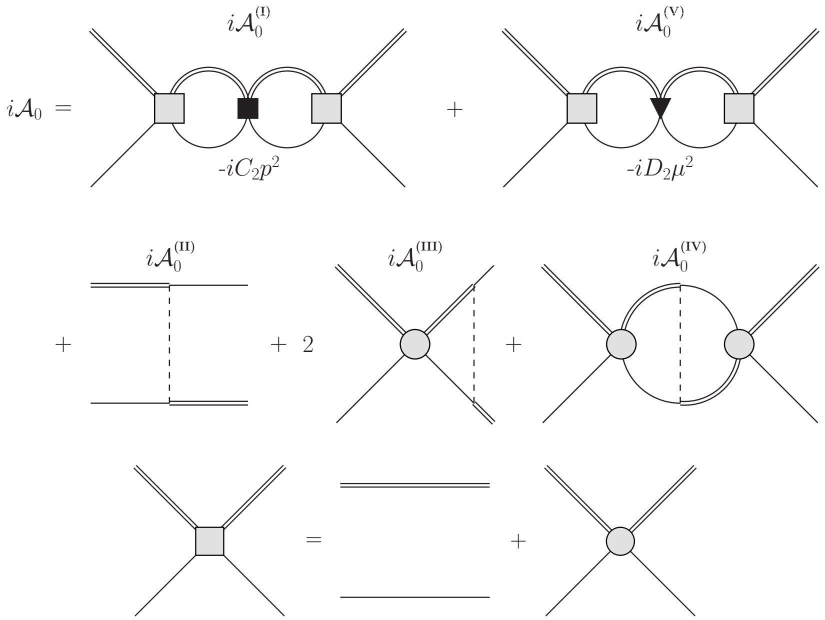

$\bar{D}^0 D^{*0}$ scattering amplitude to LO

$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \sqrt{-2M_{DD^*}E - i\epsilon}}$$

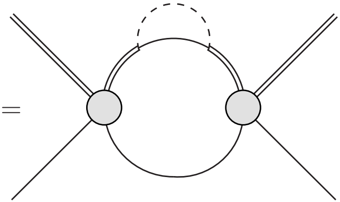
$$\gamma \equiv \frac{2\pi}{M_{DD^*} C_0(\Lambda)} + \Lambda$$

$$\text{Pole at} - E = \frac{\gamma^2}{2M_{DD^*}}$$

# NLO Contributions to the Scattering Amplitude



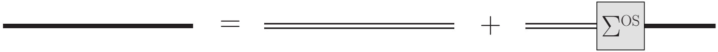
# Infrared Divergences in XEFT




$$i\mathcal{A}_0^{(\mathrm{VI})} = \frac{ig^2}{6f^2}\frac{1}{p}\left(i\Lambda - \mu\right)\frac{\mu^2}{2}\left(\frac{M_{DD^*}}{2\pi}\right)^2 \mathcal{A}_{-1}^2$$

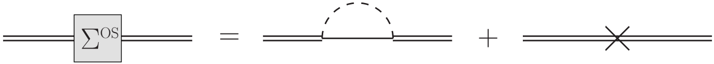
- Infrared divergent
- Renormalization scale dependent
- Pion bubbles give contribution to the $D^*$ self energy

# Resummation for the $D^{*0}$ Propagator

$iG =$ 

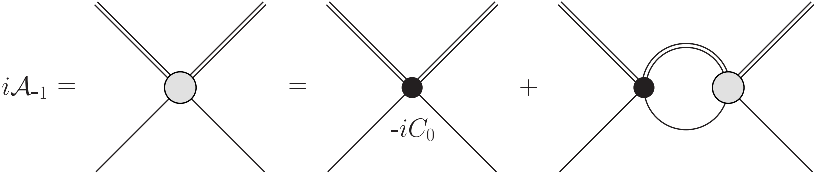

Full $D^{*0}$ propagator

$$iG = \frac{i}{p_0 - p^2/2m_{D^*} + \Sigma^{\rm OS} + i\epsilon}$$

$i\Sigma^{\rm OS} =$ 


$$\Sigma^{\rm OS} = \begin{cases} \frac{g^2}{24\pi f^2} i\mu^3, & m_\pi < \Delta \\ 0, & m_\pi \geq \Delta \end{cases}$$

$$\mu = \sqrt{\Delta^2 - m_\pi^2} \xrightarrow{m_\pi > \Delta} i\,|\mu|$$



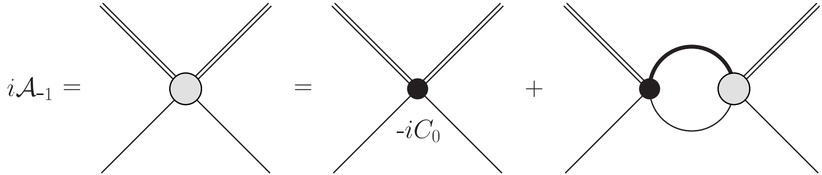

$\bar{D}^0 D^{*0}$ scattering amplitude to LO

$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \sqrt{-2M_{DD^*}E - i\epsilon}}$$

$$\gamma \equiv \frac{2\pi}{M_{DD^*} C_0(\Lambda)} + \Lambda$$

$$\text{Pole at} - E = \frac{\gamma^2}{2M_{DD^*}}$$

# LO Scattering Amplitude



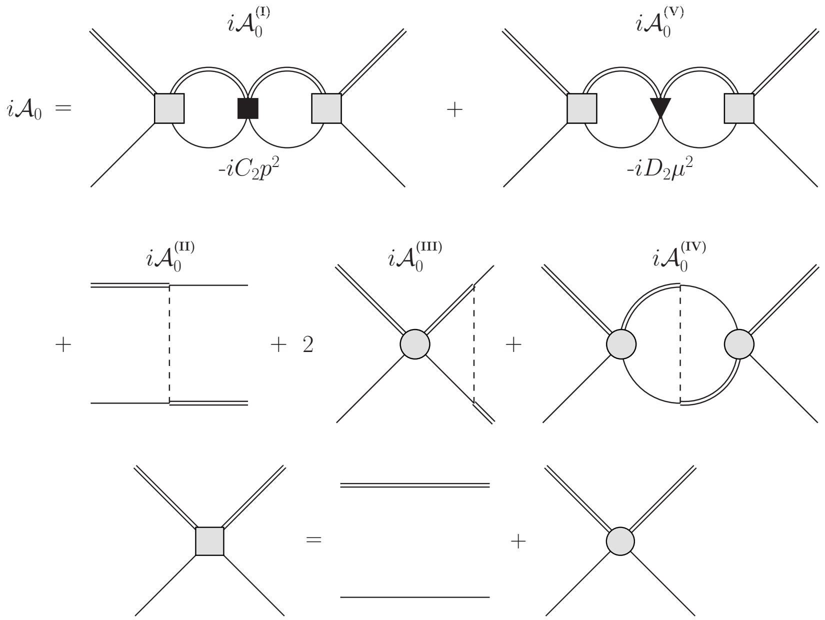

$\bar{D}^0 D^{*0}$ scattering amplitude to LO

$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \sqrt{-2M_{DD^*}E - 2M_{DD^*}\Sigma^{os} - i\epsilon}}$$

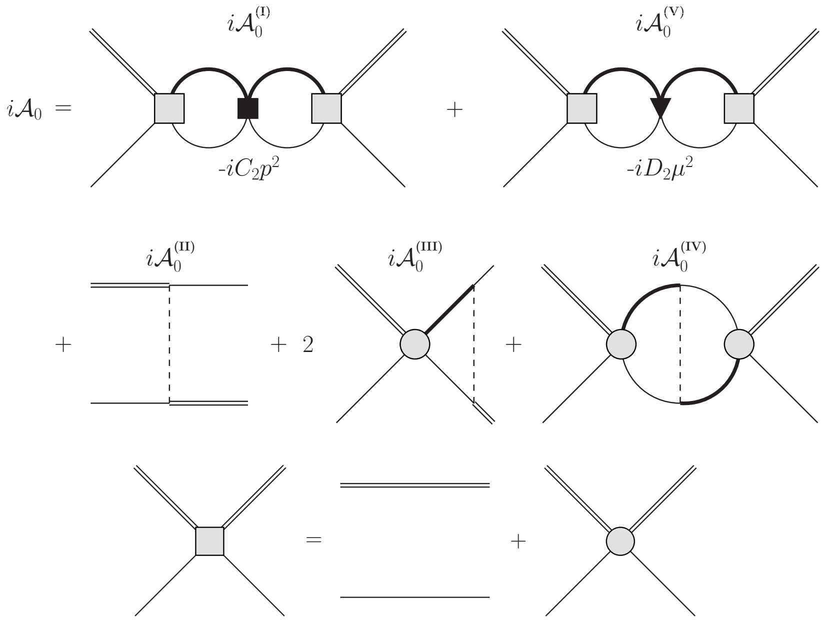
$$\gamma \equiv \frac{2\pi}{M_{DD^*}C_0(\Lambda)} + \Lambda$$

$$\text{Pole at} - E = \frac{\gamma^2}{2M_{DD^*}} + \Sigma^{os}$$

# NLO Contributions to the Scattering Amplitude



# NLO Contributions to the Scattering Amplitude



$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \eta}$$

$$i\mathcal{A}_0^{(\mathrm{I})} = \frac{-iC_2}{C_0^2} \left( p^2 + 2M_{DD^*}\Sigma^{\mathrm{os}} \frac{-\eta + \Lambda}{-\gamma + \Lambda} \right) \mathcal{A}_{-1}^2$$

$$i\mathcal{A}_0^{(\mathrm{II})} = \frac{ig^2}{6f^2} \left( 1 + \frac{\mu^2}{4p^2} \log \left( 1 - \frac{4p^2}{\mu^2} \right) \right)$$

$$i\mathcal{A}_0^{(\mathrm{III})} = \frac{ig^2}{3f^2} \left( (-\eta + \Lambda) + \frac{i\mu^2}{2p} \log \left( 1 + \frac{2p}{i\eta + \mu - p} \right) \right) \frac{M_{DD^*}}{2\pi} \mathcal{A}_{-1}$$

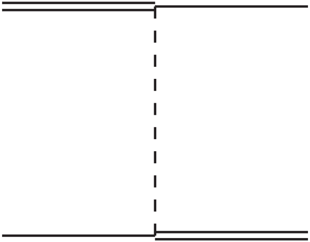
$$i\mathcal{A}_0^{(\mathrm{IV})} = \frac{ig^2}{6f^2} \left( (-\eta + \Lambda)^2 + \mu^2 \left( \log \left( \frac{\Lambda}{2\eta - i\mu} \right) + 1 + R \right) \right) \left( \frac{M_{DD^*}}{2\pi} \right)^2 \mathcal{A}_{-1}^2$$

$$i\mathcal{A}_0^{(\mathrm{V})} = \frac{-iD_2\mu^2}{C_0^2} \mathcal{A}_{-1}^2$$

$$\eta \equiv \sqrt{-p^2 - 2M_{DD^*}\Sigma^{\mathrm{os}} - i\epsilon}$$

$$R \equiv \frac{1}{2} \left( -\gamma_E + \log\left(\tfrac{\pi}{4}\right) + \tfrac{2}{3} \right)$$

# One-Pion Exchange



$$i\hat{\mathcal{A}}_0^{(\mathrm{II})}{}_{ij} = \frac{ig^2}{2f^2}\frac{(\varepsilon_i \cdot \mathbf{p}_\pi)(\varepsilon_j \cdot \mathbf{p}_\pi)}{\mathbf{p}_\pi^2 - \mu^2}$$

$$\xrightarrow{\text{S-wave}} \delta_{ij} \cdot \frac{ig^2}{6f^2}\left(1 + \frac{\mu^2}{4p_\pi^2}\log\left(1 - \frac{4p_\pi^2}{\mu^2}\right)\right) \equiv \delta_{ij} \cdot i\mathcal{A}_0^{(\mathrm{II})}$$

- Seperate amplitudes $\hat{\mathcal{A}}_{ij} = \delta_{ij} \cdot \mathcal{A}$

- Relation between scattering amplitude and S-matrix

$$S - 1 = e^{2i\delta_s} - 1 = i\frac{pM_{DD^*}}{\pi}\mathcal{A}$$

- Apply effective range expansion

$$p\cot\delta_s = ip + \frac{2\pi}{M_{DD^*}\mathcal{A}} = -\frac{1}{a_s} + \frac{1}{2}r_s p^2 + \ldots$$

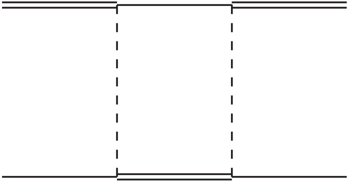
- OPE in coordinate space oscillatory [Suzuki, 2005]

$$\frac{ig^2}{2f^2}\frac{(\varepsilon_i\cdot\mathbf{p}_\pi)(\varepsilon_j\cdot\mathbf{p}_\pi)}{\mathbf{p}_\pi^2-\mu^2}$$

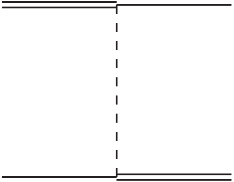
$$\xrightarrow{\text{F.T.}}\frac{ig^2}{8\pi f^2}\left(\varepsilon_i\cdot\varepsilon_j - 3(\varepsilon_i\cdot\hat{\mathbf{r}})(\varepsilon_j\cdot\hat{\mathbf{r}})\right)\frac{\cos(\mu r)+\mu r\sin(\mu r)}{r^3} + \ldots$$

- Effective range expansion only valid up to order $p^0$

# Suppression of the Two-Pion Exchange

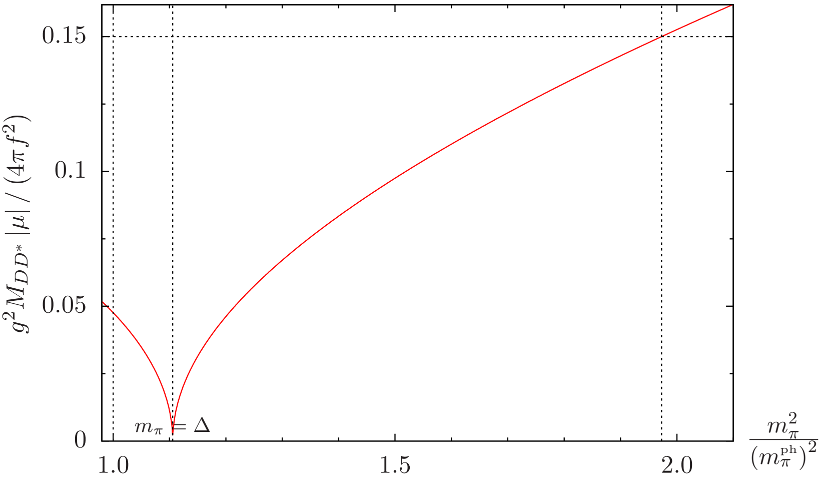


$$\sim \frac{g^2 M_{DD^*}\mu}{4\pi f^2} \times$$

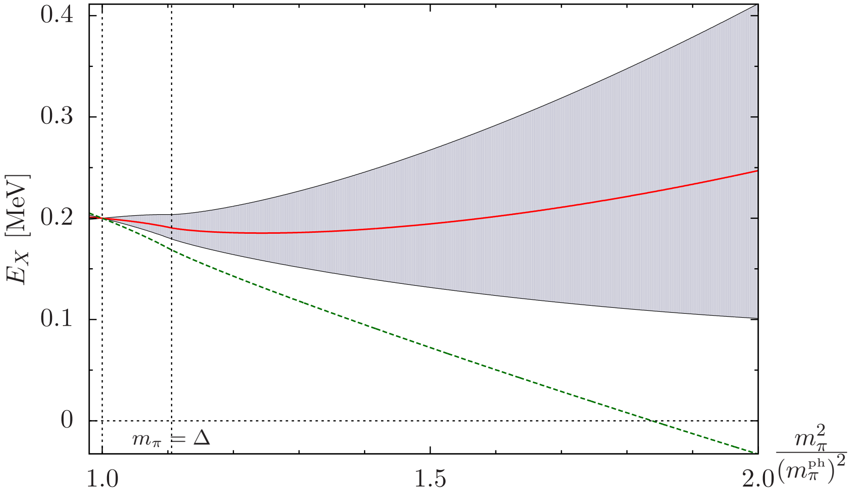


- Expansion factor in KSW for $NN$ scattering $\left(\frac{g_A^2 M_N m_\pi}{8\pi f^2}\right)^{\text{ph}} \sim 0.5$ [Kaplan et al., 1998]
- Expansion factor in XEFT for $DD^*$ scattering $\left(\frac{g^2 M_{DD^*}\mu}{4\pi f^2}\right)^{\text{ph}} \sim 0.05$ [Fleming et al., 2007]
- Quark mass dependent $\rightarrow$ estimate range of validity

# Expansion factor



# Results for the Binding Energy

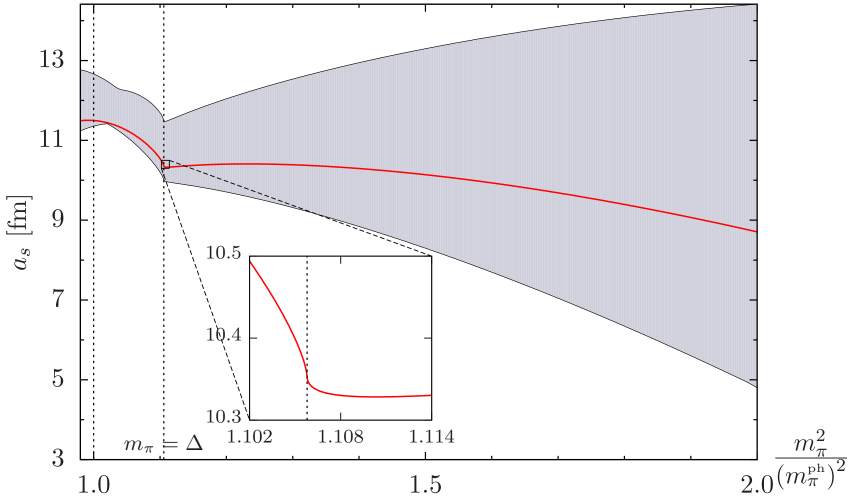



Red: LO contact interaction and OPE only

Bounds: Natural ranges for NLO coefficients

Green: Unnaturally large NLO coefficient

# Results for the Scattering Length




Red: LO contact interaction and OPE only

Bounds: Natural ranges for NLO coefficients

# Conclusion and Outlook

## Conclusion

- XEFT applicable to calculate chiral extrapolations analytically
- Quark mass dependent contact interaction essential for renormalization
- $X(3872)$ should be observable on the lattice
- High sensitivity of scattering length (cusp effect)
- Qualitative agreement with results from non-relativistic Faddeev-type three-body equations [Baru et al., 2013]
- Discrepancy with results from unitarized heavy meson ChpT [Wang and Wang, 2013]

## Outlook

- Extension to NNLO; Inclusion of charged $D$-mesons
- Relativistic pion fields for extrapolation to chiral limit
- Calculation of finite volume effects

# Renormalization of $C_2$ and $D_2$

$$C_2 = \frac{M_{DD^*}}{2\pi}\frac{r_0}{2}\left(C_0\right)^2 \equiv c_2\left(C_0\right)^2$$

$$D_2 = \frac{6f^2}{g^2}\left(\frac{2\pi}{M_{DD^*}}\right)^2\left(d_2 + \log\left(\frac{\Lambda}{\mu^{\mathrm{ph}}}\right) - R\right)\left(C_0\right)^2$$