

Light Quark Mass Dependence of the $X(3872)$ in XEFT

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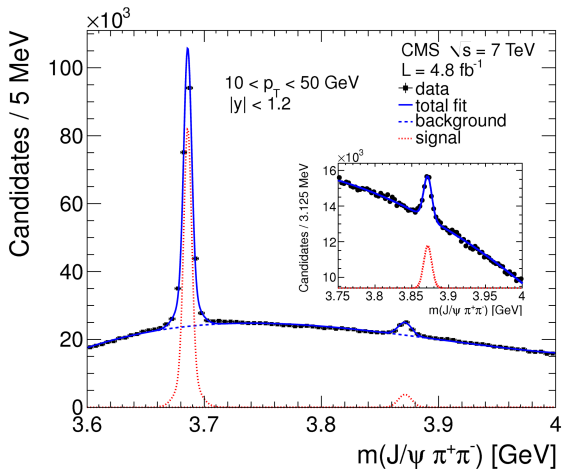
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- 1 Introduction and Motivation
- 2 XEFT and the $\bar{D}^0 D^{*0}$ Scattering Amplitude
- 3 Binding Energy and Scattering Length
- 4 Conclusion and Outlook

Introduction and Motivation

- First observation by the Belle Collaboration [Choi et al., 2003]
- Determination $J^{PC} = 1^{++}$ by LHCb [Aaij et al., 2013]



[Chatrchyan et al., 2013]

Introduction and Motivation

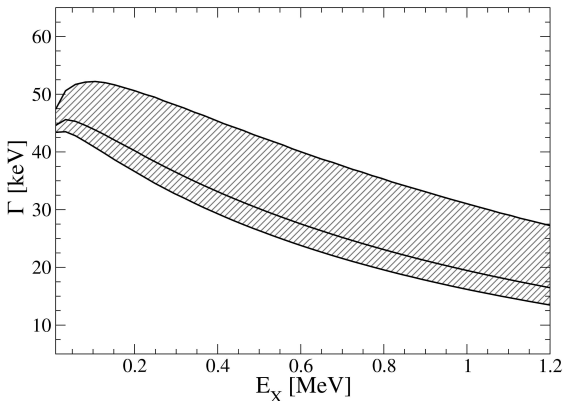
- Interpretations: tetraquark, charmonium, **hadronic molecule**
- Mass of the $X(3872)$ close to $D^0 D^{*0}$ threshold

Particle Content of the $X(3872)$

$$X = \frac{1}{\sqrt{2}} (\bar{D}^0 D^{*0} + D^0 \bar{D}^{*0})$$

- Recent observation of a candidate for the X on the lattice
[Prelovsek and Leskovec, 2013]
- Performed on rather small lattices for large quark masses
- Previous work:
 - Unitarized heavy meson ChPT: no sensitivity to contact interactions [Wang and Wang, 2013]
 - Non-relativistic Faddeev-type three-body equations: contact interactions essential [Baru et al., 2013]

- Universal properties due to small binding energy
 $E_X = m_{D^*} + m_D - M_X = (0.17 \pm 0.26) \text{ MeV}$
- Corrections calculable in XEFT [Fleming et al., 2007]



Decay rate for $X \rightarrow D^0 \bar{D}^0 \pi^0$ as a function of E_X

- Similar to KSW theory for NN scattering [Kaplan et al., 1998]
 - Includes pions perturbatively
 - Unnaturally large NNLO coefficients [Fleming et al., 2000]
- Nearness of $D^0 D^{*0}$ hyperfine splitting and pion mass induces small mass scale $\mu^2 = \Delta^2 - m_\pi^2$
- Mass scale μ , $D^{(*)0}$ and pion momenta and binding momentum of same order $Q \ll m_\pi, m_D, m_{D^*}$
- Pions and $D^{(*)0}$ mesons treated non-relativistically
- Integrated out charged $D^{(*)\pm}$ mesons
 - Effective field theory: $1/a$ suppression [Braaten and Kusunoki, 2004]
 - Charmonium- hadronic molecule hybrid: charged states small contribution [Takizawa and Takeuchi, 2013]
- Takes finite width of the D^{*0} into account

$$\begin{aligned}
 \mathcal{L} = & \mathbf{D}^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_{D^*}} \right) \mathbf{D} + \mathbf{D}^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_D} \right) \mathbf{D} \\
 & + \bar{\mathbf{D}}^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_{D^*}} \right) \bar{\mathbf{D}} + \bar{\mathbf{D}}^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_D} \right) \bar{\mathbf{D}} + \pi^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_\pi} + \delta \right) \pi \\
 & + \frac{g}{\sqrt{2}f} \frac{1}{\sqrt{2}m_\pi} \left(\mathbf{D}\mathbf{D}^\dagger \cdot \vec{\nabla}\pi + \bar{\mathbf{D}}^\dagger\bar{\mathbf{D}} \cdot \vec{\nabla}\pi^\dagger \right) + \text{h.c.} \\
 & - \frac{C_0}{2} \left(\bar{\mathbf{D}}\mathbf{D} + \mathbf{D}\bar{\mathbf{D}} \right)^\dagger \cdot \left(\bar{\mathbf{D}}\mathbf{D} + \mathbf{D}\bar{\mathbf{D}} \right) \\
 & + \frac{C_2}{16} \left(\bar{\mathbf{D}}\mathbf{D} + \mathbf{D}\bar{\mathbf{D}} \right)^\dagger \cdot \left(\bar{\mathbf{D}}\overleftrightarrow{\nabla}^2\mathbf{D} + \mathbf{D}\overleftrightarrow{\nabla}^2\bar{\mathbf{D}} \right) + \text{h.c.} \\
 & - \frac{D_2\mu^2}{2} \left(\bar{\mathbf{D}}\mathbf{D} + \mathbf{D}\bar{\mathbf{D}} \right)^\dagger \cdot \left(\bar{\mathbf{D}}\mathbf{D} + \mathbf{D}\bar{\mathbf{D}} \right) + \dots,
 \end{aligned}$$

Power Counting in XEFT



$$\sim Q^{-1}$$



$$\sim Q^{-2}$$



$$\sim Q^0$$



$$\sim Q^5$$



$$\sim Q^0$$

$\sim Q^1$

$$-i \frac{g}{\sqrt{2}f} \frac{1}{\sqrt{2}m_\pi} (\boldsymbol{\varepsilon} \cdot \mathbf{p}_\pi)$$

$$\sim Q^1$$

LO Scattering Amplitude

$$i\mathcal{A}_{-1} = \text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3}$$

$\bar{D}^0 D^{*0}$ scattering amplitude to LO

$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \sqrt{-2M_{DD^*} E - i\epsilon}}$$

$$\gamma \equiv \frac{2\pi}{M_{DD^*} C_0(\Lambda)} + \Lambda$$

$$\text{Pole at } -E = \frac{\gamma^2}{2M_{DD^*}}$$

NLO Contributions to the Scattering Amplitude

$$i\mathcal{A}_0 =$$

$i\mathcal{A}_0^{(i)}$
 $-iC_2 p^2$

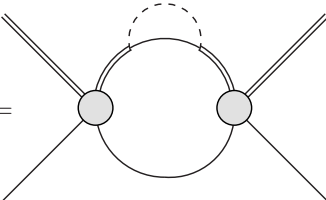
$i\mathcal{A}_0^{(v)}$
 $-iD_2 \mu^2$

$i\mathcal{A}_0^{(iii)}$

$i\mathcal{A}_0^{(iii)}$

$i\mathcal{A}_0^{(iv)}$

$=$

$$i\mathcal{A}_0^{(VI)} =$$


The diagram shows a central bubble structure. On the left and right sides, there are two external lines each, meeting at two vertices (represented by grey circles). The bubble is formed by two internal lines connecting these vertices. A dashed line forms a loop on top of the bubble, representing a pion loop. A solid line forms a loop on the bottom of the bubble, representing a D^* self-energy insertion.

$$i\mathcal{A}_0^{(VI)} = \frac{ig^2}{6f^2} \frac{1}{p} (i\Lambda - \mu) \frac{\mu^2}{2} \left(\frac{M_{DD^*}}{2\pi} \right)^2 \mathcal{A}_{-1}^2$$

- Infrared divergent
- Renormalization scale dependent
- Pion bubbles give contribution to the D^* self energy

Resummation for the D^{*0} Propagator

$$iG = \text{---} = \text{====} + \text{====} \boxed{\Sigma^{\text{OS}}} \text{---}$$

Full D^{*0} propagator

$$iG = \frac{i}{p_0 - p^2/2m_{D^*} + \Sigma^{\text{OS}} + i\epsilon}$$

$$i\Sigma^{\text{OS}} = \text{====} \boxed{\Sigma^{\text{OS}}} \text{====} = \text{====} \text{---} \text{---} \text{====} + \text{====} \times \text{====}$$

$$\Sigma^{\text{OS}} = \begin{cases} \frac{g^2}{24\pi f^2} i\mu^3, & m_\pi < \Delta \\ 0, & m_\pi \geq \Delta \end{cases}$$

$$\mu = \sqrt{\Delta^2 - m_\pi^2} \xrightarrow{m_\pi > \Delta} i|\mu|$$

LO Scattering Amplitude

$$i\mathcal{A}_{-1} = \text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3}$$

$\bar{D}^0 D^{*0}$ scattering amplitude to LO

$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \sqrt{-2M_{DD^*} E - i\epsilon}}$$

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LO Scattering Amplitude

$$i\mathcal{A}_{-1} = \text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3}$$

$\bar{D}^0 D^{*0}$ scattering amplitude to LO

$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \sqrt{-2M_{DD^*}E - 2M_{DD^*}\Sigma^{os}} - i\epsilon}$$

$$\gamma \equiv \frac{2\pi}{M_{DD^*} C_0(\Lambda)} + \Lambda$$

$$\text{Pole at } -E = \frac{\gamma^2}{2M_{DD^*}} + \Sigma^{os}$$

NLO Contributions to the Scattering Amplitude

$$\begin{aligned}
 i\mathcal{A}_0 = & \quad i\mathcal{A}_0^{(i)} \quad + \quad i\mathcal{A}_0^{(v)} \\
 & \quad + \quad i\mathcal{A}_0^{(ii)} \quad + \quad 2 \quad i\mathcal{A}_0^{(iii)} \quad + \quad i\mathcal{A}_0^{(iv)} \\
 & \quad = \quad \text{[Diagram 1]} \quad + \quad \text{[Diagram 2]}
 \end{aligned}$$

The diagrams are:

- $i\mathcal{A}_0^{(i)}$: Two loops with a black square vertex in the middle. Label: $-iC_2p^2$.
- $i\mathcal{A}_0^{(v)}$: Two loops with a black triangle vertex in the middle. Label: $-iD_2\mu^2$.
- $i\mathcal{A}_0^{(ii)}$: Two parallel horizontal lines connected by a vertical dashed line.
- $i\mathcal{A}_0^{(iii)}$: A central grey circle with four external lines. A vertical dashed line connects the top and bottom lines.
- $i\mathcal{A}_0^{(iv)}$: A central grey circle with two loops on top and bottom. A vertical dashed line connects the two loops.
- Diagram 1: A central grey square with four external lines.
- Diagram 2: A central grey circle with four external lines.

NLO Contributions to the Scattering Amplitude

$$\begin{aligned}
 i\mathcal{A}_0 = & \quad i\mathcal{A}_0^{(i)} \quad + \quad i\mathcal{A}_0^{(v)} \\
 & \quad + \quad i\mathcal{A}_0^{(ii)} \quad + \quad 2 \quad i\mathcal{A}_0^{(iii)} \quad + \quad i\mathcal{A}_0^{(iv)} \\
 & \quad = \quad \text{[Diagram 1]} \quad + \quad \text{[Diagram 2]}
 \end{aligned}$$

The diagrams are:

- $i\mathcal{A}_0^{(i)}$: A box diagram with two internal loops. The top-left loop is shaded black, and the bottom-right loop is shaded grey. The propagator between the loops is labeled $-iC_2p^2$.
- $i\mathcal{A}_0^{(v)}$: A box diagram with two internal loops. The top-left loop is shaded black, and the bottom-right loop is shaded grey. The propagator between the loops is labeled $-iD_2\mu^2$.
- $i\mathcal{A}_0^{(ii)}$: A diagram with two external lines on the left and two on the right. A vertical dashed line connects the two lines on the left to the two lines on the right.
- $i\mathcal{A}_0^{(iii)}$: A diagram with two external lines on the left and two on the right. A vertex (grey circle) is connected to the two lines on the left. A diagonal line connects this vertex to the top-right line. A vertical dashed line connects the top-right line to the bottom-right line.
- $i\mathcal{A}_0^{(iv)}$: A diagram with two external lines on the left and two on the right. A vertex (grey circle) is connected to the two lines on the left. A curved line connects this vertex to the top-right line. A vertical dashed line connects the top-right line to the bottom-right line.
- Diagram 1 (bottom row): A box diagram with two external lines on the left and two on the right. A vertex (grey square) is connected to the two lines on the left and the two lines on the right.
- Diagram 2 (bottom row): A diagram with two external lines on the left and two on the right. A vertex (grey circle) is connected to the two lines on the left and the two lines on the right.

NLO Scattering Amplitudes

$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \eta}$$

$$i\mathcal{A}_0^{(I)} = \frac{-iC_2}{C_0^2} \left(p^2 + 2M_{DD^*} \Sigma^{\text{os}} \frac{-\eta + \Lambda}{-\gamma + \Lambda} \right) \mathcal{A}_{-1}^2$$

$$i\mathcal{A}_0^{(II)} = \frac{ig^2}{6f^2} \left(1 + \frac{\mu^2}{4p^2} \log \left(1 - \frac{4p^2}{\mu^2} \right) \right)$$

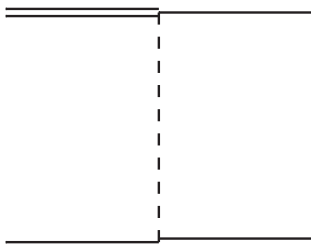
$$i\mathcal{A}_0^{(III)} = \frac{ig^2}{3f^2} \left((-\eta + \Lambda) + \frac{i\mu^2}{2p} \log \left(1 + \frac{2p}{i\eta + \mu - p} \right) \right) \frac{M_{DD^*}}{2\pi} \mathcal{A}_{-1}$$

$$i\mathcal{A}_0^{(IV)} = \frac{ig^2}{6f^2} \left((-\eta + \Lambda)^2 + \mu^2 \left(\log \left(\frac{\Lambda}{2\eta - i\mu} \right) + 1 + R \right) \right) \left(\frac{M_{DD^*}}{2\pi} \right)^2 \mathcal{A}_{-1}^2$$

$$i\mathcal{A}_0^{(V)} = \frac{-iD_2\mu^2}{C_0^2} \mathcal{A}_{-1}^2$$

$$\eta \equiv \sqrt{-p^2 - 2M_{DD^*} \Sigma^{\text{os}} - i\epsilon}$$

$$R \equiv \frac{1}{2} \left(-\gamma_E + \log \left(\frac{\pi}{4} \right) + \frac{2}{3} \right)$$



$$i\hat{\mathcal{A}}_0^{(II)}{}_{ij} = \frac{ig^2}{2f^2} \frac{(\boldsymbol{\varepsilon}_i \cdot \mathbf{p}_\pi)(\boldsymbol{\varepsilon}_j \cdot \mathbf{p}_\pi)}{\mathbf{p}_\pi^2 - \mu^2}$$

$$\xrightarrow{\text{S-wave}} \delta_{ij} \cdot \frac{ig^2}{6f^2} \left(1 + \frac{\mu^2}{4p_\pi^2} \log \left(1 - \frac{4p_\pi^2}{\mu^2} \right) \right) \equiv \delta_{ij} \cdot i\mathcal{A}_0^{(II)}$$

- Separate amplitudes $\hat{\mathcal{A}}_{ij} = \delta_{ij} \cdot \mathcal{A}$

Effective Range Expansion

- Relation between scattering amplitude and S-matrix

$$S - 1 = e^{2i\delta_s} - 1 = i \frac{p M_{DD^*}}{\pi} \mathcal{A}$$

- Apply effective range expansion

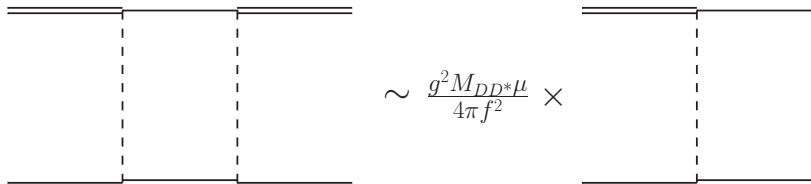
$$p \cot \delta_s = ip + \frac{2\pi}{M_{DD^*} \mathcal{A}} = -\frac{1}{a_s} + \frac{1}{2} r_s p^2 + \dots$$

- OPE in coordinate space oscillatory [Suzuki, 2005]

$$\frac{ig^2}{2f^2} \frac{(\boldsymbol{\varepsilon}_i \cdot \mathbf{p}_\pi)(\boldsymbol{\varepsilon}_j \cdot \mathbf{p}_\pi)}{\mathbf{p}_\pi^2 - \mu^2}$$
$$\xrightarrow{\text{F.T.}} \frac{ig^2}{8\pi f^2} (\boldsymbol{\varepsilon}_i \cdot \boldsymbol{\varepsilon}_j - 3(\boldsymbol{\varepsilon}_i \cdot \hat{\mathbf{r}})(\boldsymbol{\varepsilon}_j \cdot \hat{\mathbf{r}})) \frac{\cos(\mu r) + \mu r \sin(\mu r)}{r^3} + \dots$$

- Effective range expansion only valid up to order p^0

Suppression of the Two-Pion Exchange

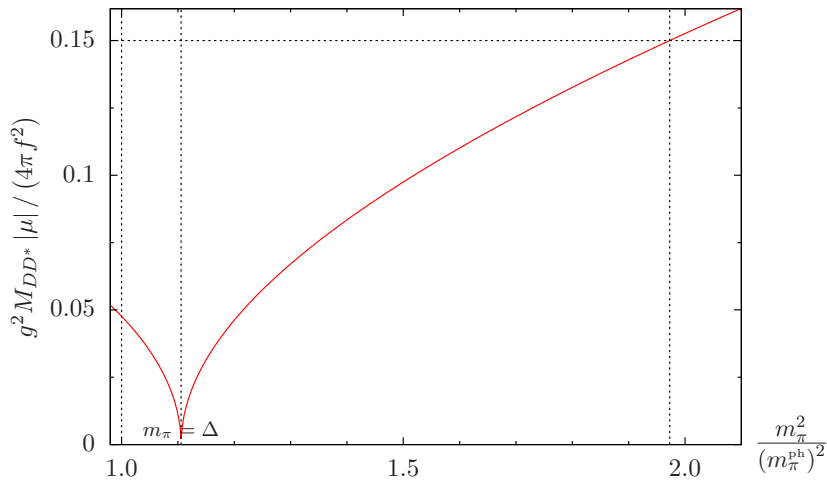


The diagram shows two Feynman diagrams representing scattering processes. On the left, a diagram for NN scattering shows two horizontal lines (nucleons) with two vertical dashed lines (pions) connecting them. On the right, a diagram for DD* scattering shows two horizontal lines (deuteron and deuteron*) with two vertical dashed lines (pions) connecting them. The two diagrams are separated by a tilde symbol and a fraction $\frac{g^2 M_{DD^*} \mu}{4\pi f^2}$, followed by a multiplication sign \times .

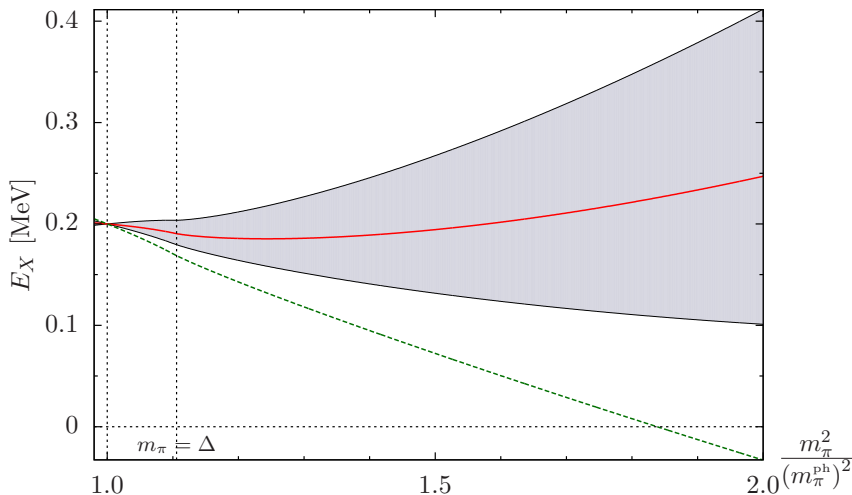
$$\sim \frac{g^2 M_{DD^*} \mu}{4\pi f^2} \times$$

- Expansion factor in KSW for NN scattering
 $\left(\frac{g_A^2 M_N m_\pi}{8\pi f^2}\right)^{\text{ph}} \sim 0.5$ [Kaplan et al., 1998]
- Expansion factor in XEFT for DD^* scattering
 $\left(\frac{g^2 M_{DD^*} \mu}{4\pi f^2}\right)^{\text{ph}} \sim 0.05$ [Fleming et al., 2007]
- Quark mass dependent \rightarrow estimate range of validity

Expansion factor



Results for the Binding Energy

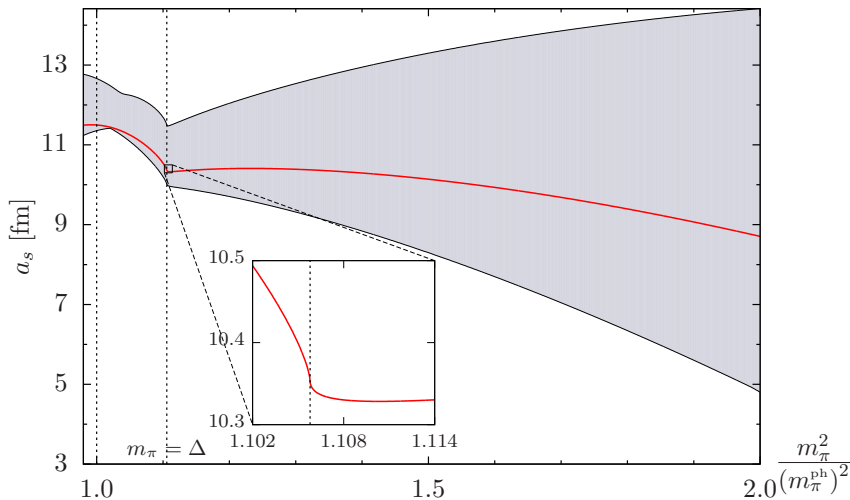


Red: LO contact interaction and OPE only

Bounds: Natural ranges for NLO coefficients

Green: Unnaturally large NLO coefficient

Results for the Scattering Length



Red: LO contact interaction and OPE only

Bounds: Natural ranges for NLO coefficients

Conclusion

- XEFT applicable to calculate chiral extrapolations analytically
- Quark mass dependent contact interaction essential for renormalization
- $X(3872)$ should be observable on the lattice
- High sensitivity of scattering length (cusp effect)
- Qualitative agreement with results from non-relativistic Faddeev-type three-body equations [Baru et al., 2013]
- Discrepancy with results from unitarized heavy meson ChpT [Wang and Wang, 2013]

Outlook

- Extension to NNLO; Inclusion of charged D -mesons
- Relativistic pion fields for extrapolation to chiral limit
- Calculation of finite volume effects

$$C_2 = \frac{M_{DD^*}}{2\pi} \frac{r_0}{2} (C_0)^2 \equiv c_2 (C_0)^2$$

$$D_2 = \frac{6f^2}{g^2} \left(\frac{2\pi}{M_{DD^*}} \right)^2 \left(d_2 + \log \left(\frac{\Lambda}{\mu^{\text{ph}}} \right) - R \right) (C_0)^2$$