中国科学院大学
University of Chinese Academy of Sciences

Hidden Charm Tetraquark States & Molecular States in QCD Sum Rules

Cong-Feng Qiao

University of Chinese Academy of Sciences

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Outline

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- New Exotic States.
- Theoretical Interpretations.

Part II: Tetraquark States

- QCD Sum Rules.
- 1^+ Charged Hidden Charm Tetraquark State.

  C.-F. Qiao & L. Tang, arXiv:1307.6654.
- 1^- & 2^+ Charged Hidden Charm Tetraquark States.

  C.-F. Qiao & L. Tang, arXiv:1308.3439.
- Summary of Tetraquark State.

Part III: Molecular States

- Hidden Charm & Strange Molecular States.

  C.-F. Qiao & L. Tang, arXiv:1309.7596.
- Discussions and Predictions.

Part I

Exotic States

New Exotic States

List of New Exotic States

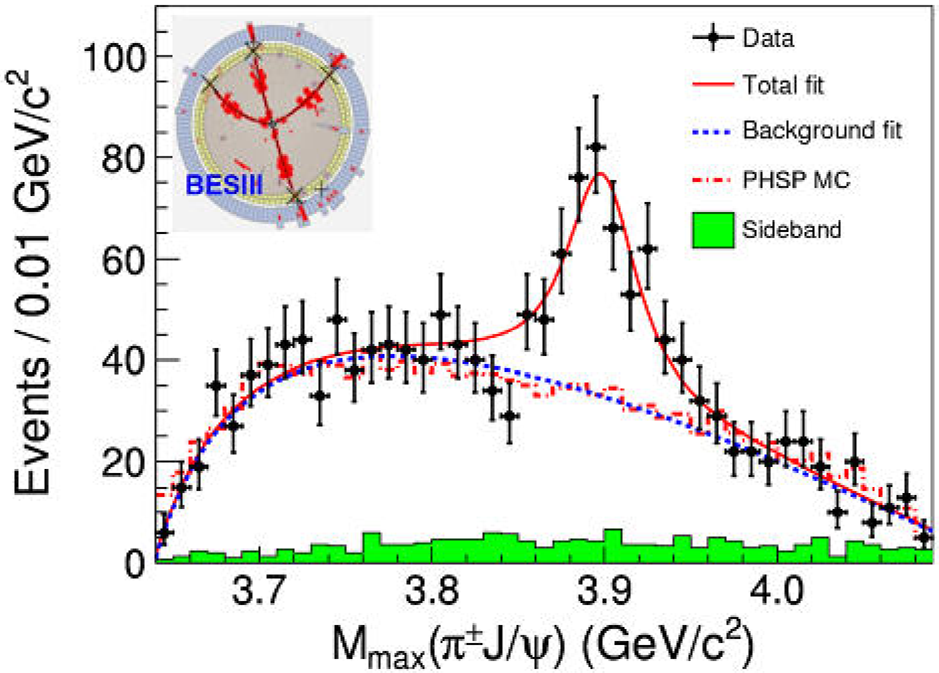
State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)	Experiment
$X(3823)$	3823.1	< 24	$?^{?-}$	$B \to K + (\chi_{c1}\gamma)$	Belle [1]
$\boldsymbol{X(3872)}$	3871.68	< 1.2	1^{++}	$B \to K + (J/\psi\,\pi^+\pi^-)$	Belle [2, 3], *BABAR* [4]
				$p\bar{p} \to (J/\psi\,\pi^+\pi^-) + \ldots$	CDF [5, 6, 7], DØ [8]
				$B \to K + (J/\psi\,\pi^+\pi^-\pi^0)$	Belle [9], *BABAR* [10]
				$B \to K + (D^0\bar{D}^0\pi^0)$	Belle [11, 12], *BABAR* [13]
				$B \to K + (J/\psi\,\gamma)$	Belle [14], *BABAR* [15, 16]
				$B \to K + (\psi(2S)\,\gamma)$	*BABAR* [16], Belle [14]
				$pp \to (J/\psi\,\pi^+\pi^-) + \ldots$	LHCb [17]
$Z_c^+(3885)$	3883.9	24.8	1^+	$Y(4260) \to \pi^- + (D\bar{D}^*)^+$	BESIII [18]
$\boldsymbol{Z_c^+(3900)}$	3898	51	$1^{?-}$	$Y(4260) \to \pi^- + (J/\psi\,\pi^+)$	BESIII [19], Belle [20]
					CLEO-c [21]
$\boldsymbol{X(3915)}$	3917.5	20	0^{++}	$B \to K + (J/\psi\,\omega)$	Belle [22], *BABAR* [23]
				$e^+e^- \to e^+e^- + (J/\psi\,\omega)$	Belle [24], *BABAR* [10, 25]
$\boldsymbol{\chi_{c2}(2P)}$	3927.2	24	2^{++}	$e^+e^- \to e^+e^- + (D\bar{D})$	Belle [26], *BABAR* [27]
$X(3940)$	3942	37	$?^{?+}$	$e^+e^- \to J/\psi + (D^*\bar{D})$	Belle [28]
				$e^+e^- \to J/\psi + (\ldots)$	Belle [29]
$\boldsymbol{G(3900)}$	3943	52	1^{--}	$e^+e^- \to \gamma + (D\bar{D})$	*BABAR* [30], Belle [31]
				$e^+e^- \to \pi^- + (J/\psi\,\pi^+)$	Xiao *et al.* [21]
$Y(4008)$	4008	226	1^{--}	$e^+e^- \to \gamma + (J/\psi\,\pi^+\pi^-)$	Belle [32]
$Z_c^+(4020)$	4022.9	7.9	?	$e^+e^- \to (h_c\pi^+)\pi^-$	BESIII [33]
$Z_c^+(4025)$	4026.3	24.8	?	$e^+e^- \to (D^*\bar{D}^*)^\pm\pi^\mp$	BESIII [34]

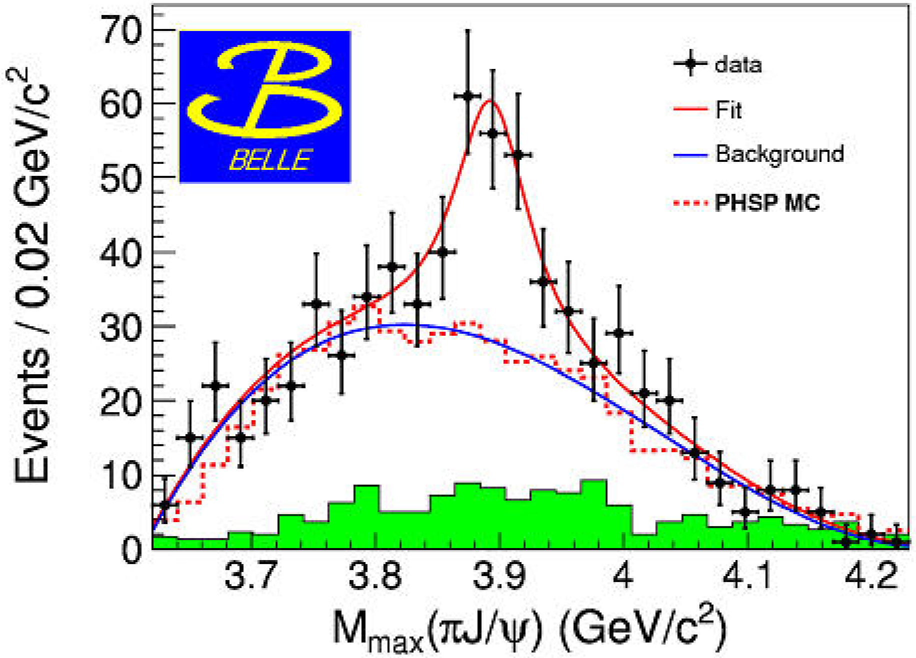
Observed by BESIII, this year!

New Exotic States

State	M (MeV)	Γ (MeV)	J^{PC}	Process (decay mode)	Experiment
$Z_1^+(4050)$	4051	82	?	$B \to K + (\chi_{c1}(1P)\,\pi^+)$	Belle [35], *BABAR* [36]
$\boldsymbol{Y(4140)}$	4144.5	15	$?^{?+}$	$B \to K + (J/\psi\,\phi)$	CDF [37, 38], CMS [39] (>5)
$X(4160)$	4156	139	$?^{?+}$	$e^+e^- \to J/\psi + (D^*\bar{D}^*)$	Belle [28]
$Z_2^+(4250)$	4248	177	?	$B \to K + (\chi_{c1}(1P)\,\pi^+)$	Belle [35], *BABAR* [36]
$\boldsymbol{Y(4260)}$	4263	95	1^{--}	$e^+e^- \to \gamma + (J/\psi\,\pi^+\pi^-)$	*BABAR* [40, 41], CLEO [42]
					Belle [32]
				$e^+e^- \to (J/\psi\,\pi^+\pi^-)$	CLEO [43] (11)
				$e^+e^- \to (J/\psi\,\pi^0\pi^0)$	CLEO [43]
$Y(4274)$	4274.4	32	$?^{?+}$	$B \to K + (J/\psi\,\phi)$	CDF [38]
$X(4350)$	4350.6	13.3	$0/2^{++}$	$e^+e^- \to e^+e^-\,(J/\psi\,\phi)$	Belle [44]
$\boldsymbol{Y(4360)}$	4361	74	1^{--}	$e^+e^- \to \gamma + (\psi(2S)\,\pi^+\pi^-)$	*BABAR* [45], Belle [46]
$Z^+(4430)$	4443	107	?	$B \to K + (\psi(2S)\,\pi^+)$	Belle [47, 48], *BABAR* [49]
$X(4630)$	4634	92	1^{--}	$e^+e^- \to \gamma\,(\Lambda_c^+\Lambda_c^-)$	Belle [50]
$Y(4660)$	4664	48	1^{--}	$e^+e^- \to \gamma + (\psi(2S)\,\pi^+\pi^-)$	Belle [46]
$Y_b(10888)$	10888.4	30.7	1^{--}	$e^+e^- \to (\Upsilon(nS)\,\pi^+\pi^-)$	Belle [51, 52]
$Z_b^+(10610)$	10607.2	18.4	1^{+-}	$\Upsilon(5S) \to \pi^- + (\Upsilon(nS)\pi^+)$	Belle [53, 54]
				$n = 1, 2, 3;$	
				$\Upsilon(5S) \to \pi^- + (h_b(nP)\pi^+)$	Belle [53, 54]
				$n = 1, 2;$	
$Z_b^+(10650)$	10652.2	11.5	1^{+-}	$\Upsilon(5S) \to \pi^- + (\Upsilon(nS)\pi^+)$	Belle [53, 54]
				$n = 1, 2, 3;$	
				$\Upsilon(5S) \to \pi^- + (h_b(nP)\pi^+)$	Belle [53, 54]
				$n = 1, 2,$	

- $Z_c^+(3900)$ observed by BESIII and Belle.





$M=3899.0\pm3.6\pm4.9\ MeV$,

$\Gamma=46\pm10\pm20\ MeV$,

BESIII, arXiv:1303.5949.

$M=3894.5\pm6.6\pm4.5\ MeV$,

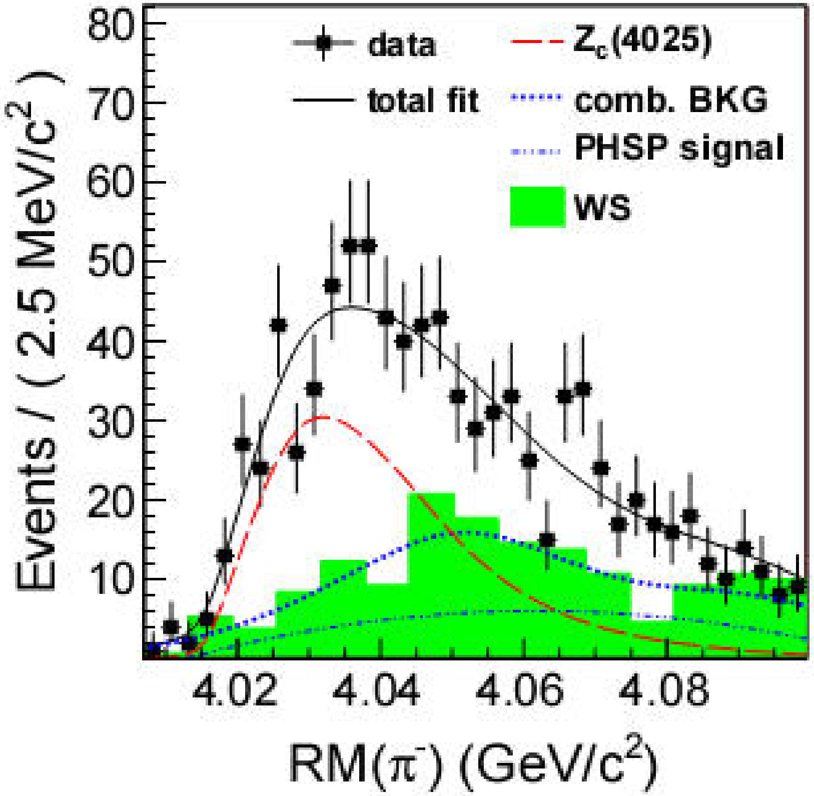
$\Gamma=63\pm24\pm26\ MeV$,

Belle, arXiv:1304.0121.

- Confirmed with CLEOc data. CLEOc, arXiv:1304.3036.

New Charged Charmonium-like Structures

- $Z_c^+(4025)$ observed by BESIII.

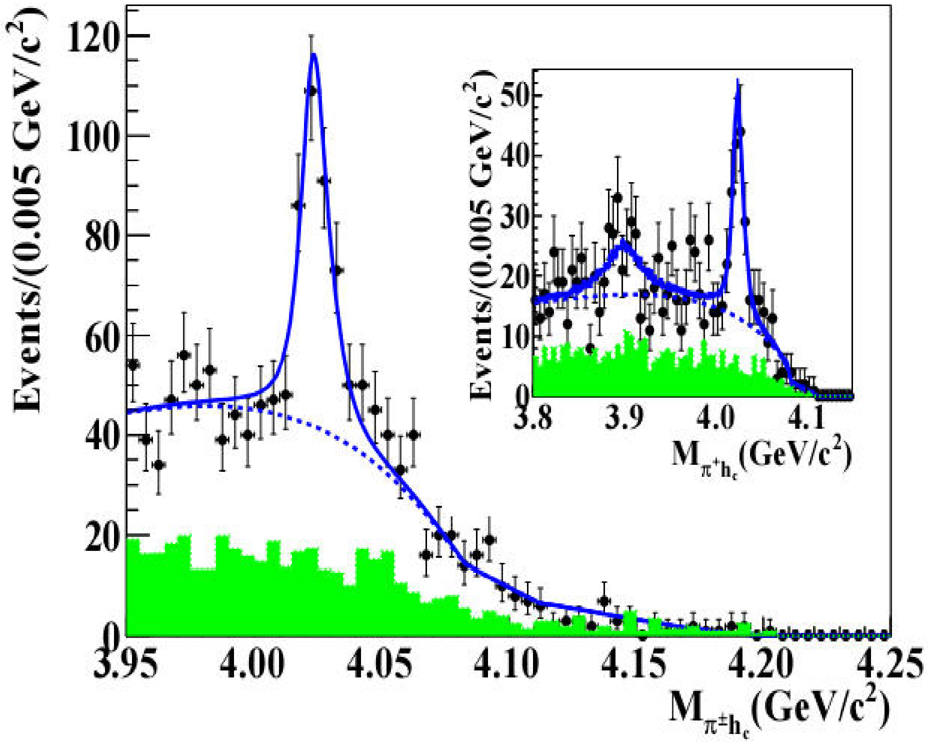


$M=4026.3 \pm 2.6\ MeV$,

$\Gamma=24.8 \pm 5.6\ MeV$, $e^+e^- \to (D^*\bar{D}^*)^\pm \pi^\mp$

arXiv:1308.2760, BESIII Colaboration.

- $Z_c^+(4020)$ observed by BESIII.



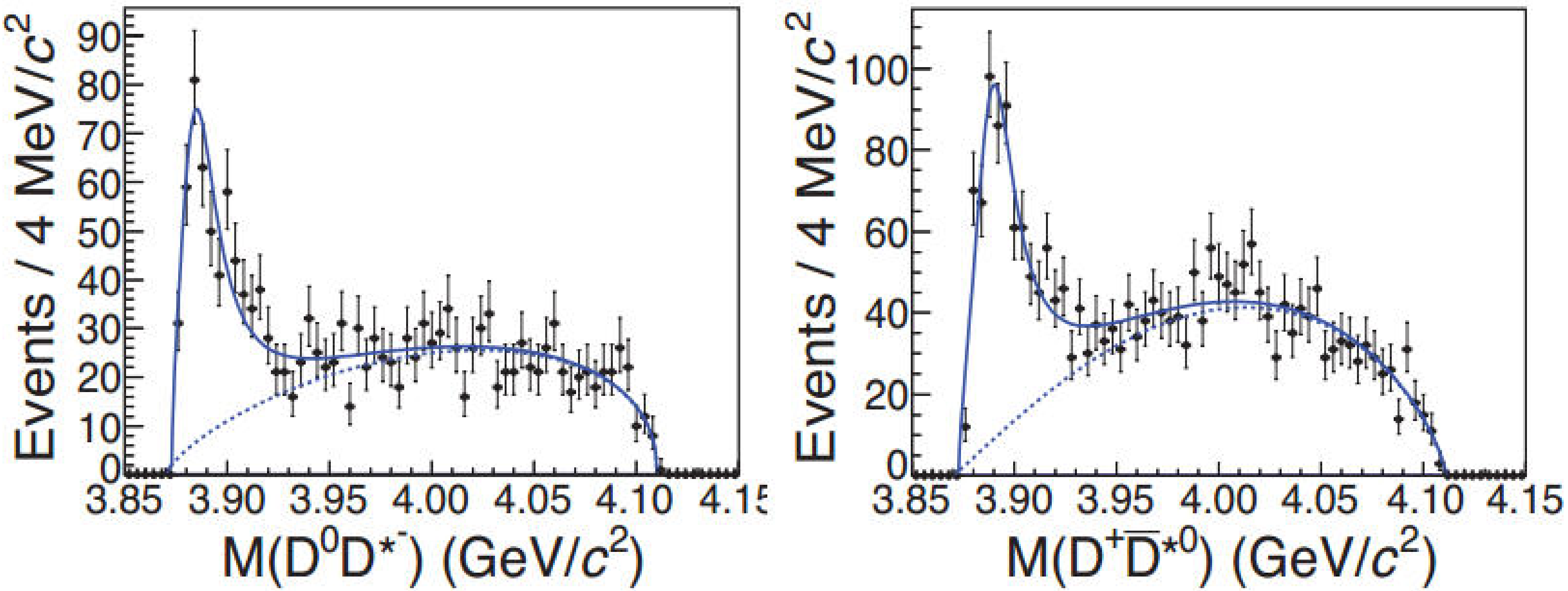
$M=4022.9 \pm 0.8 \pm 2.7\ MeV$,

$\Gamma=7.9 \pm 2.7 \pm 2.3$, $e^+e^- \to \pi^+\pi^- h_c$

arXiv:1309.1896, BESIII Colaboration.

New Charged Charmonium-like Structures

- $Z_c^+(3885)$ observed by BESIII .



$$M = 3883.9 \pm 1.5 \pm 4.2\ MeV,$$

$$\Gamma = 24.8 \pm 3.3 \pm 11.0\ MeV,$$

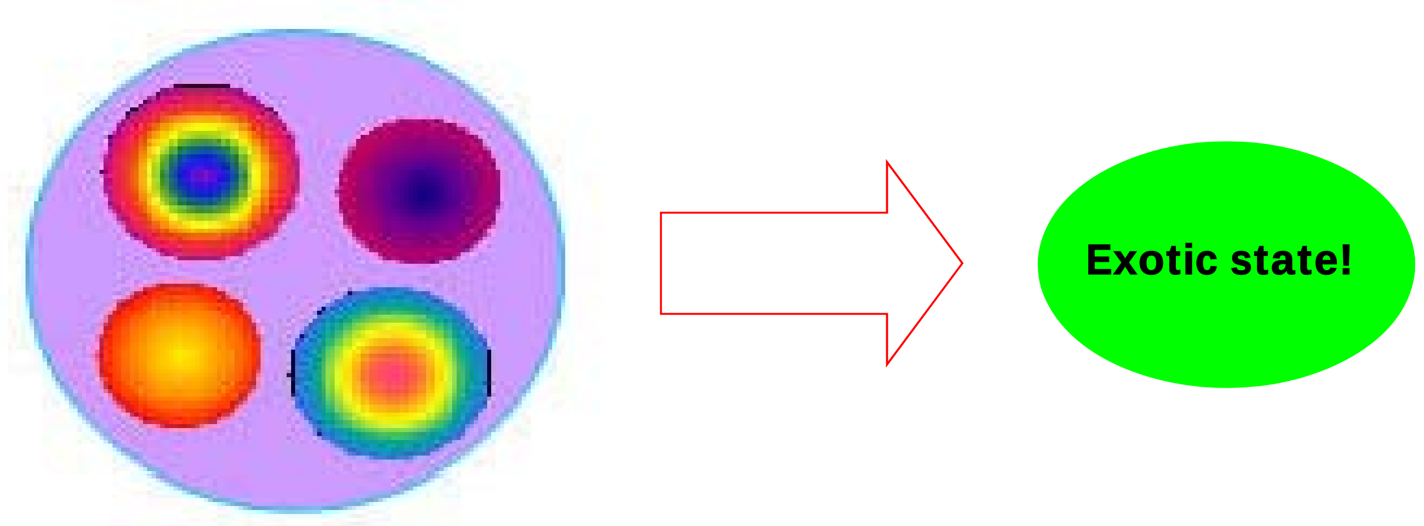
$$e^+e^- \to \pi^\pm (D\bar{D}^*)^\mp$$

arXiv:1310.1163,
BESIII Colaboration.

Theoretical Interpretations

- Decays to charmonium ⇒ Contain a $c\bar{c}$ pair
- With electric charge ⇒ More than 2 quarks!

  At least four quarks!



Theoretical Interpretations

- **Explanations on $Z_c^+(3900)$**

  - **D*D molecular state:**
    - F-K Guo, et al., 1303.6608. → Heavy quark flavor symmetry
    - C.-Y. Cui, et al., 1304.1850. → QCDSR

  - **Tetraquark state:**
    - L. Maiani, et al., PRD87, 111102 (R) (2013). → Dynamic analysis
    - J.M. Dias, et al., 1304.6433. → Decay
    - C.-F. Qiao and L. Tang, 1307.6654 → Mass

  - **Hadron-charmonium:**
    - M.B. Voloshin, 1304.0380. → Be argued

  - **Initial Single Pion Emission mechanism (ISPE):**
    - D.Y. Chen, X. Liu, PRD84 (2011) 034032. → Predicted

Theoretical Interpretations

- **Explanations on $Z_c^+(4025)$**

  - **D*D* molecular state:**

    F.-K. Guo, et al., 1303.6608.
    Jun He, et al., arXiv:1308.2999.
    C.-Y. Cui, et al., arXiv:1308.3625.
    Wei Chen, et al., arXiv:1308.5060.
    K.P. Khemchandani, et al., arXiv:1310.0862.

    Heavy quark flavor symmetry
    Effective Lagrangian
    QCDSR-P-wave
    QCDSR
    Projected from 2^+ current

  - **Tetraquark state:**

    C.-F. Qiao & L Tang, arXiv:1308.3439.
    Z.-G. Wang, arXiv:1311.1046.

    Disfavor 1^-, Favor 2^+
    Disfavor 1^-

Theoretical Interpretations

- **Explanations on $Z_c^+(4020)$:**

It does not seem to be clear if the states $Z_c^+(4020)$ and $Z_c^+(4025)$ are the same or not.

Z.-G. Wang, arXiv:1311.1046. → Disfavor 1^-

- **Explanations on $Z_c^+(3885)$:**

It does not seem to be clear if the states $Z_c^+(3885)$ and $Z_c^+(3900)$ are the same or not.

Part II

Tetraquark States

QCD Sum Rules

- **Review of hidden charm tetraquark states with QCD Sum Rules.**

  - **$I^G(J^{PC})=0^+(1^{++})$ hidden charm tetraquark state was studied to interprete X(3872).**

    R.D. Matheus, S. Narison, M. Nielsen, J.M. Richard, PRD 75, 014005 (2007).

  - **In a systematic way, the vector and axial-vector hidden charm tetraquark states was investigated by W. Chen & S.L. Zhu. They found 1^{--} hidden charm tetraquark was consistent with Y(4660) .**

    W. Chen, and S.L. Zhu, PRD 83, 034010 (2011).

  - **1^{--} and 0^{++} hidden charm tetraquark states have been studied.**

    R.M. Albuquerque, et. al. PLB 715, 129 (2012).

QCD Sum Rules

✡ **A 2^- hidden charm tetraquark state was investigated to explore the X(3872).**

C.Y. Cui, et al., CTP, 57,1033 (2012).

- **Note, these works only refer to the neutral tetraquark states.**

We estimate the charged hddien charm tetraquark states via QCD Sum Rules.

C.-F. Qiao & L. Tang,
arXiv:1307.6654 & 1308.3534.

1^+ Charged Hidden Charm Tetraquark State

- The interpolating current of the 1^+ charged hidden charm tetraquark state is used as:

$I^G(J^P) = 1^+(1^+)$

$$j_\mu(x) = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}\left[\left(u_a^T(x)C\gamma_5 c_b(x)\right)\left(\bar{d}_d\gamma_\mu C\bar{c}_e^T\right) - \left(u_a^T(x)C\gamma_\mu c_b(x)\right)\left(\bar{d}_d\gamma_5 C\bar{c}_e^T\right)\right]$$

- Insert this current into the two-point correlation function:

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle 0|T\left\{j_\mu(x)j_\nu^\dagger(0)\right\}|0\rangle$$

This correlation function has the following Lorentz covariance form:

$$\Pi_{\mu\nu}(q) = -(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})\Pi_1(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi_0(q^2)\ .$$

where the spin-1 part corresponds to 1^+ charged hidden charm tetraquark state.

1^+ Charged Hidden Charm Tetraquark State

- On the phenomenological side, the correlation function is expressed as:

$$\Pi_1(q^2) = \frac{\lambda_{1^+}^{c\,2}}{m_{1^+}^{c\,2} - q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\rho^h(s)}{s - q^2} ,$$

- In the Operator Product Expansion (OPE) side, it is described as:

$$\Pi_1^{OPE}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho^{OPE}(s)}{s - q^2} ,$$

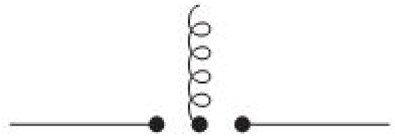
where

$$\rho^{OPE}(s) = \rho^{pert}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle g_s^2 G^2\rangle}(s) + \rho^{\langle g_s\bar{q}\sigma\cdot Gq\rangle}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle g_s^3 G^3\rangle}(s) + \rho^{\langle g_s\bar{q}\sigma\cdot Gq\rangle\langle\bar{q}q\rangle}(s) + \cdots ,$$

1^+ Charged Hidden Charm Tetraquark State

- **The full propagator of light quark is used as:**

$$S_{ij}^{q}(x) = \frac{i\delta_{ij}\hat{x}}{2\pi^2 x^4} - \frac{m_q\delta_{ij}}{4\pi^2 x^2} - \frac{ig_s t_{ij}^a G_{\kappa\lambda}^a}{32\pi^2 x^2}(\sigma^{\kappa\lambda}\hat{x} + \hat{x}\sigma^{\kappa\lambda}) + \frac{i\delta_{ij}\hat{x}}{48} m_q\langle\bar{q}q\rangle - \frac{\delta_{ij}\langle\bar{q}q\rangle}{12}$$
$$- \frac{\delta_{ij}\langle g_s\bar{q}\sigma G q\rangle x^2}{192} - \frac{t_{ij}^a\sigma^{\kappa'\lambda'}}{192}\langle g_s\bar{q}\sigma\cdot G' q\rangle + \cdots ,$$



R.M. Albuquerque, 1306.4671.

- **The full propagator of heavy quark is writen as:**

$$S_{ij}^{Q}(p) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\left\{ \frac{i}{\hat{p} - m_Q}\delta_{ij} - \frac{i}{4} g_s (t^c)_{ij} G_{\kappa\lambda}^c \frac{1}{(p^2 - m_Q^2)^2} \right.$$
$$\times\ [\sigma^{\kappa\lambda}(\hat{p} + m_Q) + (\hat{p} + m_Q)\sigma^{\kappa\lambda}] + \frac{i}{12} g_s^2 \delta_{ij} G_{\alpha\beta}^a G_{\alpha\beta}^a m_Q \frac{p^2 + m_Q\hat{p}}{(p^2 - m_Q^2)^4}$$
$$\left. + \frac{i\delta_{ij}}{48}\left[\frac{(\hat{p} + m_Q)[\hat{p}(p^2 - 3m_Q^2) + 2m_Q(2p^2 - m_Q^2)](\hat{p} + m_Q)}{(p^2 - m_Q^2)^6}\right]\langle g_s^3 G^3\rangle + \cdots \right\} ,$$

1^+ Charged Hidden Charm Tetraquark State

- **Matching these two sides (quark-hadron dualtiy), and performing the Borel transformation, we obtain the mass function:**

$$m^c_{1^+}(s_0, M_B^2) = \sqrt{-\frac{R_1(s_0, M_B^2)}{R_0(s_0, M_B^2)}} \, ,$$

where

$$\begin{aligned} R_0(s_0, M_B^2) &= \int_{4m_c^2}^{s_0} ds \; \rho^{OPE}(s) e^{-s/M_B^2} + \Pi_1^{\langle g_s \bar{q}\sigma \cdot G q\rangle \langle \bar{q} q\rangle}(M_B^2) \, , \\ R_1(s_0, M_B^2) &= \frac{\partial}{\partial M_B^{-2}} R_0(s_0, M_B^2) \, . \end{aligned}$$

M.A. Shifman, et.al., NPB147,385 (1979).

1^+ Charged Hidden Charm Tetraquark State

- Generally, two criteria are employed to determine the threshold parameter s_0 and the Borel parameter M_B.

  (A). The OPE convergence. Determine the lower limit constraint of M_B^2.

  (B). Pole Contribution dominant. PC> 50% is required here. Determine the upper limit constraint of M_B^2.

  These criteria give a reliable window of M_B^2 with a definite s_0.

- Additionally, s_0 is chosen to ensure the minimum variation of m^c_{1+} with M_B^2.

- In the end, with these input parameters, we can obtain the mass of 1^+ charged hidden charm tetraquark state.

1^+ Charged Hidden Charm Tetraquark State

- **OPE convergence of 1^+ charged hidden charm tetraquark state:**

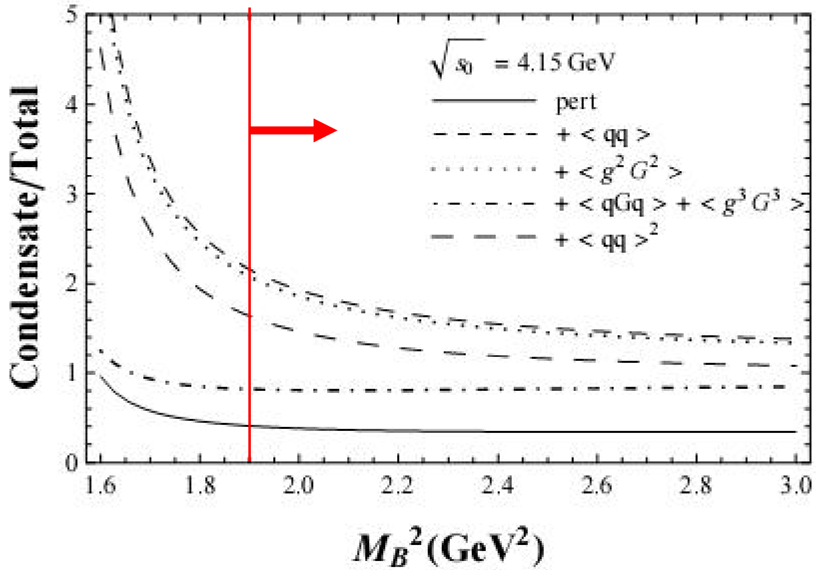


Figure 1: The OPE convergence in the M_B^2 range from 1.6 to 3.0 GeV^2 for $\sqrt{s_0} = 4.15$ GeV. The black line denotes the perturbative contribution, and each subsequent line denotes the addition of one extra condensate, *i.e.*, $+\langle\bar{q}q\rangle$ (short-dashed line), $+\langle g_s^2G^2\rangle$ (dotted line), $+\langle g_s\bar{q}\sigma\cdot Gq\rangle$ + $\langle g_s^3G^3\rangle$ (dotted-dashed line),$+\langle\bar{q}q\rangle^2$ (long-dashed line).

M_B^2 >1.9 GeV^2

1^+ Charged Hidden Charm Tetraquark State

- **Pole contribution of 1^+ charged hidden charm tetraquark state:**

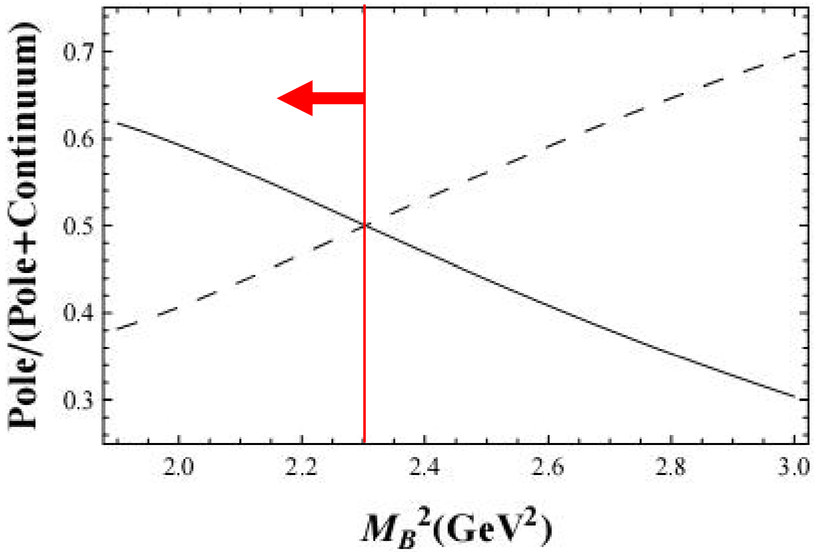


Figure 2: The relative pole contribution for $\sqrt{s_0} = 4.15$ GeV. The black line represents the relative contribution, whereas the dotted line corresponds to the continuum contribution.

$M_B^2 < 2.3\ \mathrm{GeV}^2$

1^+ Charged Hidden Charm Tetraquark State

- **Mass curves of 1^+ charged hidden charm tetraquark state:**

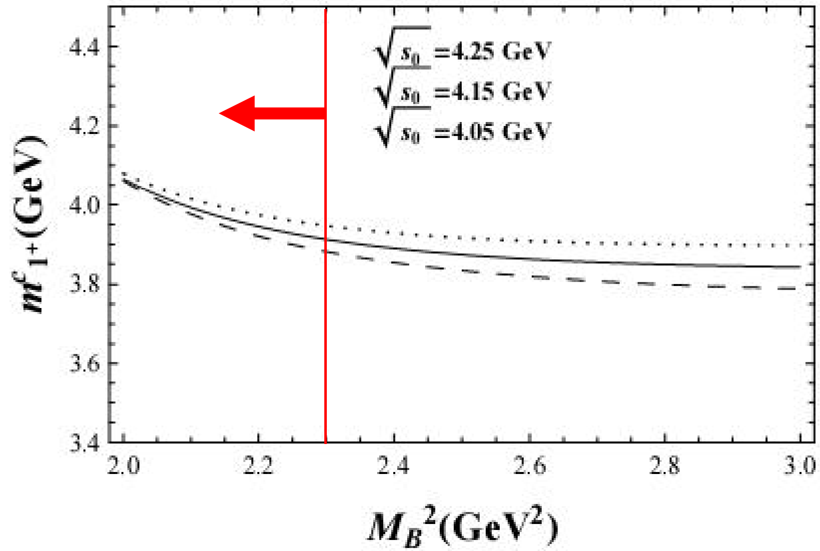


Figure 3: Dependence of $m^c_{1^+}$ on Borel parameter M_B^2, where the continuum thresholds $\sqrt{s_0}$ are taken as 4.05, 4.15, 4.25 GeV, from down to up, respectively.

$$m^c_{1^+} = (3912^{+106}_{-103})\,\text{MeV}$$

1^- & 2^+ Charged Hidden Charm Tetraquark States

Possible quantum numbers of $Z_c^+(4025)$ are 1^+, 1^-, 2^+ and so on.

We concentrate on 1^- and 2^+, since our former study favors $Z_c^+(3900)$ as a 1^+ tetraquark state.

C.-F. Qiao & L. Tang, arXiv:1308.3534.

- The interpolating currents of 1^- & 2^+ tetraquark states are used as:

$$j_\mu^{1^-}(x) = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}\left[\left(u_a^T(x)C\gamma_5 c_b(x)\right)\left(\bar{d}_d\gamma_\mu\gamma_5 C\bar{c}_e^T\right) - \left(u_a^T(x)C\gamma_\mu\gamma_5 c_b(x)\right)\left(\bar{d}_d\gamma_5 C\bar{c}_e^T\right)\right],$$

$$j_{\mu\nu}^{2^+}(x) = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}\left[\left(u_a^T(x)C\gamma_\mu c_b(x)\right)\left(\bar{d}_d\gamma_\nu C\bar{c}_e^T\right) - \left(u_a^T(x)C\gamma_\nu c_b(x)\right)\left(\bar{d}_d\gamma_\mu C\bar{c}_e^T\right)\right],$$

- The mass of 1^- state has been estimated in previous literatures.

$$m_{1^-} = 4.6 \sim 4.7 GeV$$

W. Chen & S-L Zhu, PRD 83, 034010 (2011).

$$m_{1^-} = 4818 MeV$$

R.M. Albuquerque, et al., PLB 715, 129 (2012).

- We reestimate the mass of 1^- with the aforementioned criteria.

1^- Charged Hidden Charm Tetraquark States

- **OPE convergence of 1^- charged hidden charm tetraquark state:**

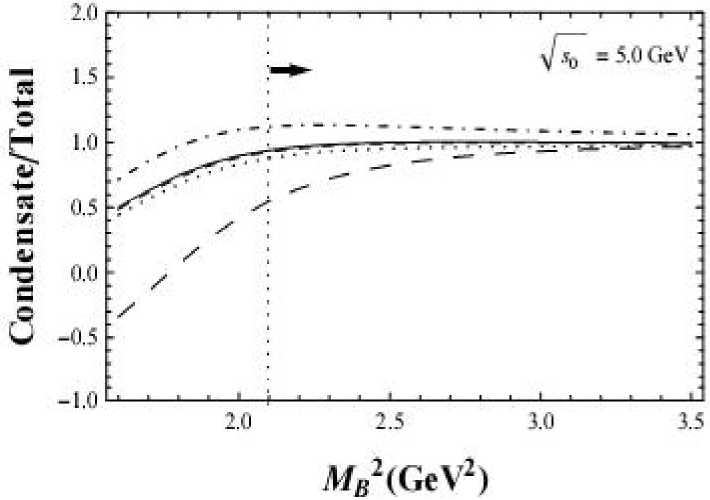


Figure 1: The OPE convergence in the region $1.6 \leq M_B^2 \leq 3.5$ GeV^2 for $J^P = 1^-$ hidden charm tetraquark state with $\sqrt{s_0} = 5.0$ GeV. The black line denotes the perturbative contribution, and each subsequent line denotes the addition of one extra condensate, *i.e.*, $+\langle\bar{q}q\rangle$ (short-dashed line), $+\langle g_s^2G^2\rangle$ (dotted line), $+\langle g_s\bar{q}\sigma\cdot Gq\rangle + \langle g_s^3G^3\rangle$ (dotted-dashed line), $+\langle\bar{q}q\rangle^2$ (long-dashed line).

M_B^2 >2.1 GeV^2

1^- Charged Hidden Charm Tetraquark States

- **Pole contribution of 1^- charged hidden charm tetraquark state:**

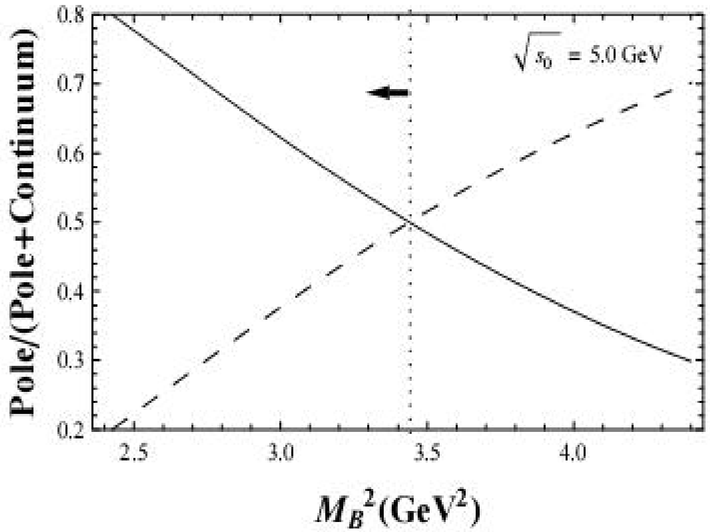


Figure 2: The relative pole contribution for $J^P = 1^-$ hidden charm tetraquark state with $\sqrt{s_0} = 5.0\,\mathrm{GeV}$. The black line represents the relative contribution, whereas the dashed line corresponds to the continuum contribution.

$M_B^2 < 3.5\ \mathrm{GeV}^2$

1^- Charged Hidden Charm Tetraquark States

- **Mass curves of 1^- state**

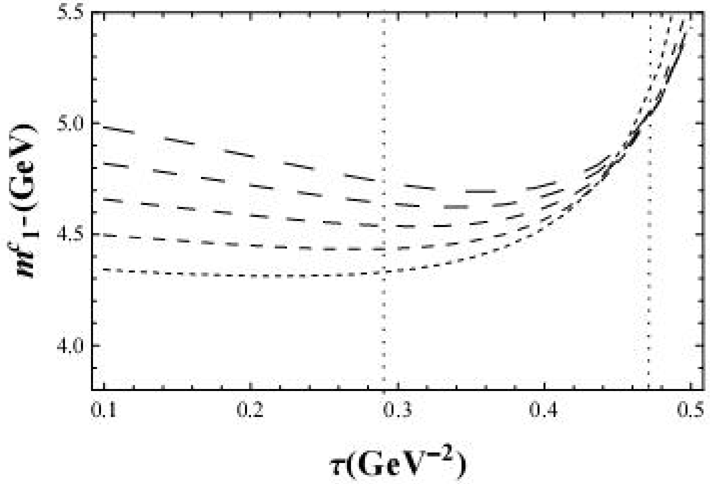


Figure 3: Dependence of $m^c_{1^-}$ on the parameter τ for $J^P = 1^-$ hidden charm tetraquark state, where $\tau = 1/M_B^2$, and the continuum thresholds $\sqrt{s_0}$ are taken as 4.6, 4.8, 5.0, 5.2 and 5.4 GeV, from down to up, respectively. We deliberately put two vertical lines denoting the chosen Borel window.

where, $2.1\ GeV^2 < M_B^2 < 3.5\ GeV^2$, and we find the optimal threshold parameter is $s_0^{1/2} = 5.0\ GeV$.

- **Mass of 1^- state is:**

$$m_{1^{-(-)}} = 4.54 \pm 0.15 GeV$$

2^+ Charged Hidden Charm Tetraquark State

- **OPE Convergence of 2^+ state**

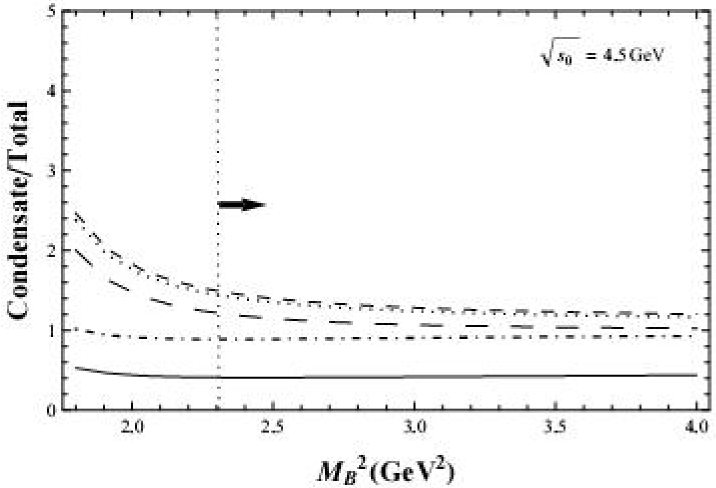


Figure 4: The OPE convergence in the region $1.6 \le M_B^2 \le 4.0$ GeV^2 for 2^+ hidden charm tetraquark state with $\sqrt{s_0} = 4.5$ GeV. The black line denotes the perturbative contribution, and each subsequent line denotes the addition of one extra condensate, *i.e.*, $+\langle\bar{q}q\rangle$ (short-dashed line), $+\langle g_s^2 G^2\rangle$ (dotted line), $+\langle g_s\bar{q}\sigma\cdot Gq\rangle + \langle g_s^3 G^3\rangle$ (dotted-dashed line), $+\langle\bar{q}q\rangle^2$ (long-dashed line).

$M_B^2 > 2.3$ GeV^2

2^+ Charged Hidden Charm Tetraquark State

- **Pole Contribution of 2^+ state**

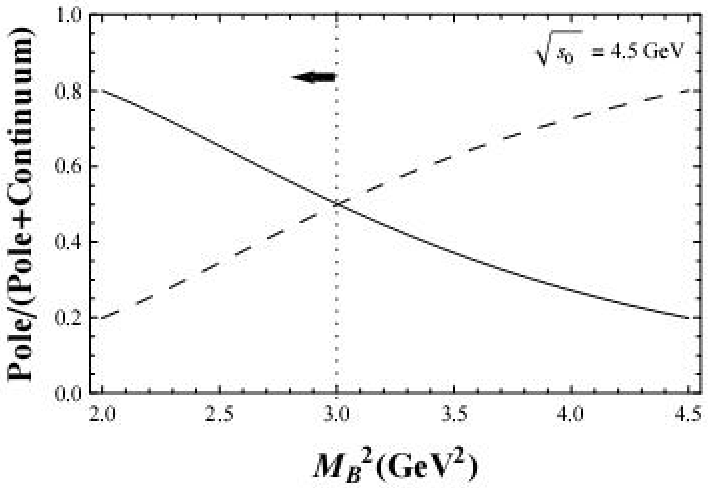


Figure 5: The relative pole contribution of $J^P = 2^+$ hidden charm tetraquark state with $\sqrt{s_0} = 4.5$ GeV. The black line represents the relative contribution, whereas the dashed line corresponds to the continuum contribution.

$M_B^2 < 3.0$ GeV^2

2^+ Charged Hidden Charm Tetraquark State

- **Mass curves of 2^+ state**

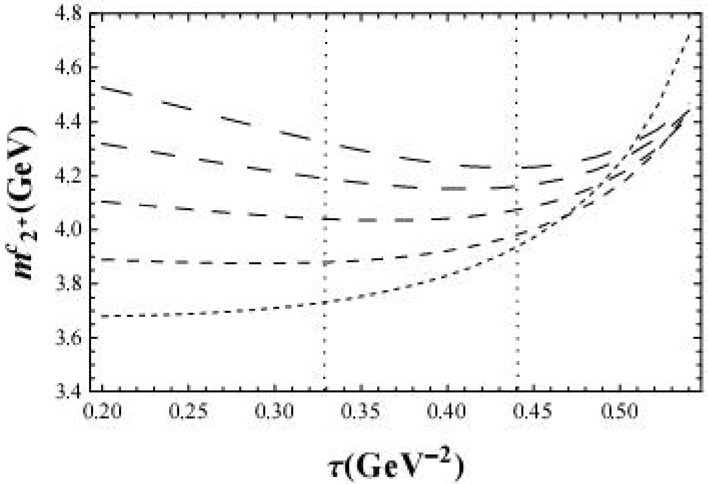


Figure 6: Dependence of $m^c_{2^+}$ on the parameter τ, where $\tau = 1/M_B^2$, and the continuum threshold parameter $\sqrt{s_0}$ are taken as 3.9, 4.2, 4.5, 4.8, and 5.1 GeV, from down to up, respectively. We deliberately put two vertical lines denoting the chosen Borel window.

- **Mass of 2^+ state is:**

$$m_{2^+} = 4.04 \pm 0.19 GeV$$

Summary of Tetraquark States

- The mass of 1^+ charged hidden charm tetraquark state gives support to the tetraquark picture of $Z_c^+(3900)$.

  Extending to the b-quark sector, we have: $m^b_{1^+} = (10561^{+345}_{-113})\,\mathrm{MeV}$,
  which supports the tetraquark picture of $Z_b^+(10610)$.

- The mass of 1^- charged hidden charm tetraquark state is much higher than $Z_c^+(4025)$ or $Z_c^+(4020)$, so the possible quantum numbers 1^- of $Z_c^+(4025)/Z_c^+(4020)$ is ruled out.

  Maybe it corresponds the charged partner of Y(4360) or Y(4660)!

  Extending to the b-quark sector, we have: $m^b_{1^-} = (10.97 \pm 0.25)\,\mathrm{GeV}$.

- We suggest that $Z_c^+(4025)$ is a 2^+ charged hidden charm tetraquark state.

  Extending to the b-quark sector, we have: $m^b_{2^+} = (10.35 \pm 0.25)\,\mathrm{GeV}$.

Part III

Molecular States

Hidden Charm and Hidden Strange Molecular States

Motivation

DD^* & D^*D^* molecular states were used to interpret $Z_c^+(3900)$ and $Z_c^+(4025)$, so what about $D_sD_s^*$ and $D_s^*D_s^*$ in QCD Sum Rules?

C-Y Cui, et al., arXiv:1304.1850;
Wei Chen, et al., arXiv:1308.5060.

- **The interpolating currents of $D_sD_s^*$ & $D_s^*D_s^*$ with $J^{pc}=1^{+-}$ are constructed as:**

$$j_\mu^{D_s\bar{D}_s^*} = \frac{i}{\sqrt{2}}\left[(\bar{s}_a\gamma_5 c_a)(\bar{c}_b\gamma_\mu s_b) + (\bar{s}_a\gamma_\mu c_a)(\bar{c}_b\gamma_5 s_b)\right] ,$$

$$j_\mu^{D_s^*\bar{D}_s^*} = \frac{i}{\sqrt{2}}\left[(\bar{s}_a\gamma^\alpha c_a)(\bar{c}_b\sigma_{\alpha\mu}\gamma_5 s_b) - (\bar{s}_a\sigma_{\alpha\mu}\gamma_5 c_a)(\bar{c}_b\gamma^\alpha s_b)\right] ,$$

Hidden Charm and Hidden Strange Molecular States

- **OPE convergences:**

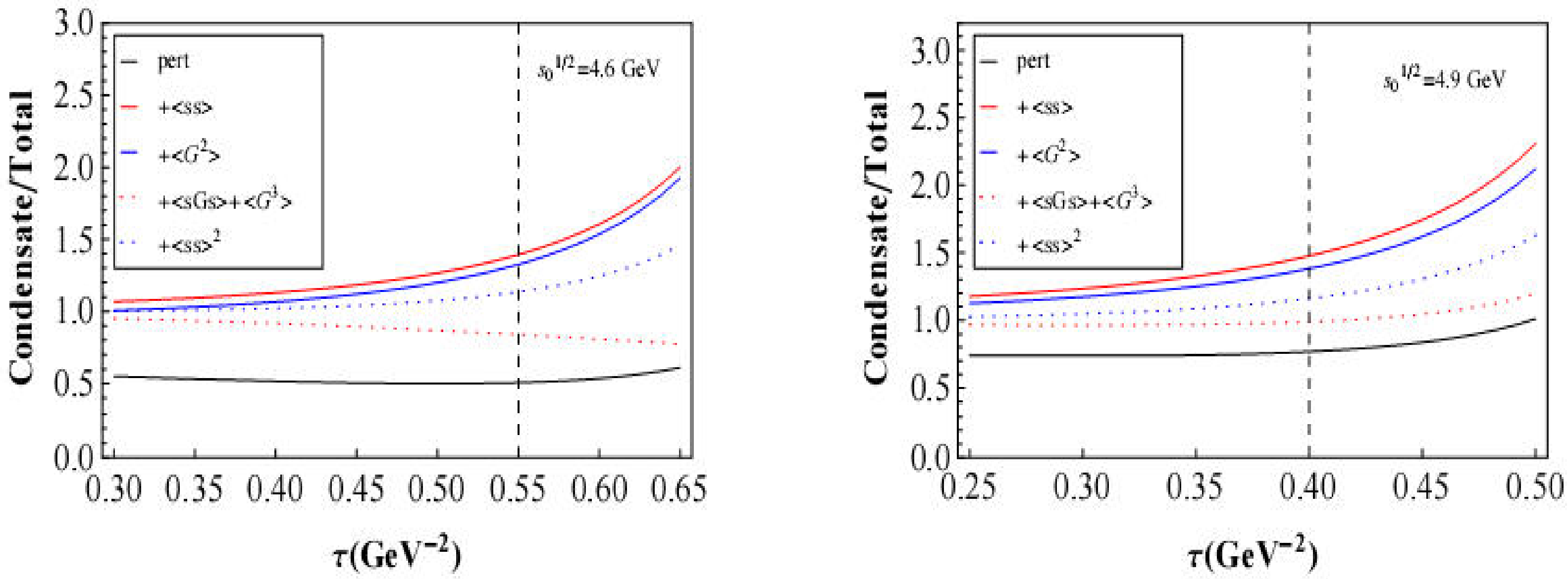
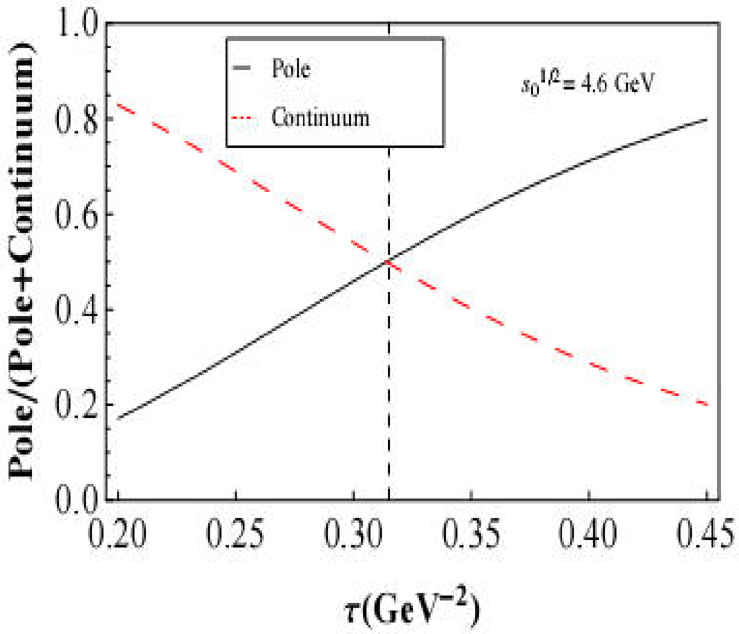


Figure 1: (**Left**) OPE convergence in the region $0.30 \leq \tau \leq 0.65$ GeV^{-2} for the $D_s\bar{D}_s^*$ molecular state with $\sqrt{s_0} = 4.6$ GeV, where $\tau = 1/M_B^2$. (**Right**) OPE convergence in the region $0.25 \leq \tau \leq 0.50$ **GeV**$^{-2}$ for $D_s^*\bar{D}_s^*$ molecular state with $\sqrt{s_0} = 4.9$ GeV. The black line denotes the perturbative contribution, and each subsequent line denotes the addition of one extra condensate, *i.e.*, $+\langle\bar{s}s\rangle$ (red line), $+\langle g_s^2G^2\rangle$ (blue line), $+\langle g_s\bar{s}\sigma\cdot Gs\rangle + \langle g_s^3G^3\rangle$ (red dotted line), $+\langle\bar{s}s\rangle^2$ (blue dotted line). The vertical lines respectively denote the upper limit constraints of the τ for $D_s\bar{D}_s^*$ and $D_s^*\bar{D}_s^*$.

- **Pole contributions:**



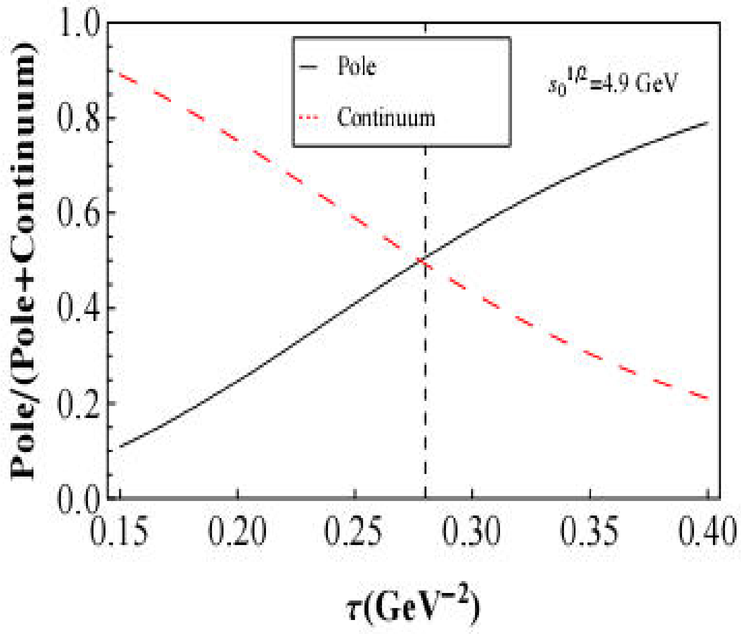


Figure 2: (**Left**) The relative pole contribution of the $D_s\bar{D}_s^*$ state with $\sqrt{s_0} = 4.6$ GeV. (**Right**) the relative pole contribution of the $D_s^*\bar{D}_s^*$ state with $\sqrt{s_0} = 4.9$ **GeV**. The black line represents the relative contribution, whereas the red dashed line corresponds to the continuum contribution. The vertical lines respectively denote the lower limit constraints of the τ for $D_s\bar{D}_s^*$ and $D_s^*\bar{D}_s^*$.

Hidden Charm and Hidden Strange Molecular States

- **Mass curves:**

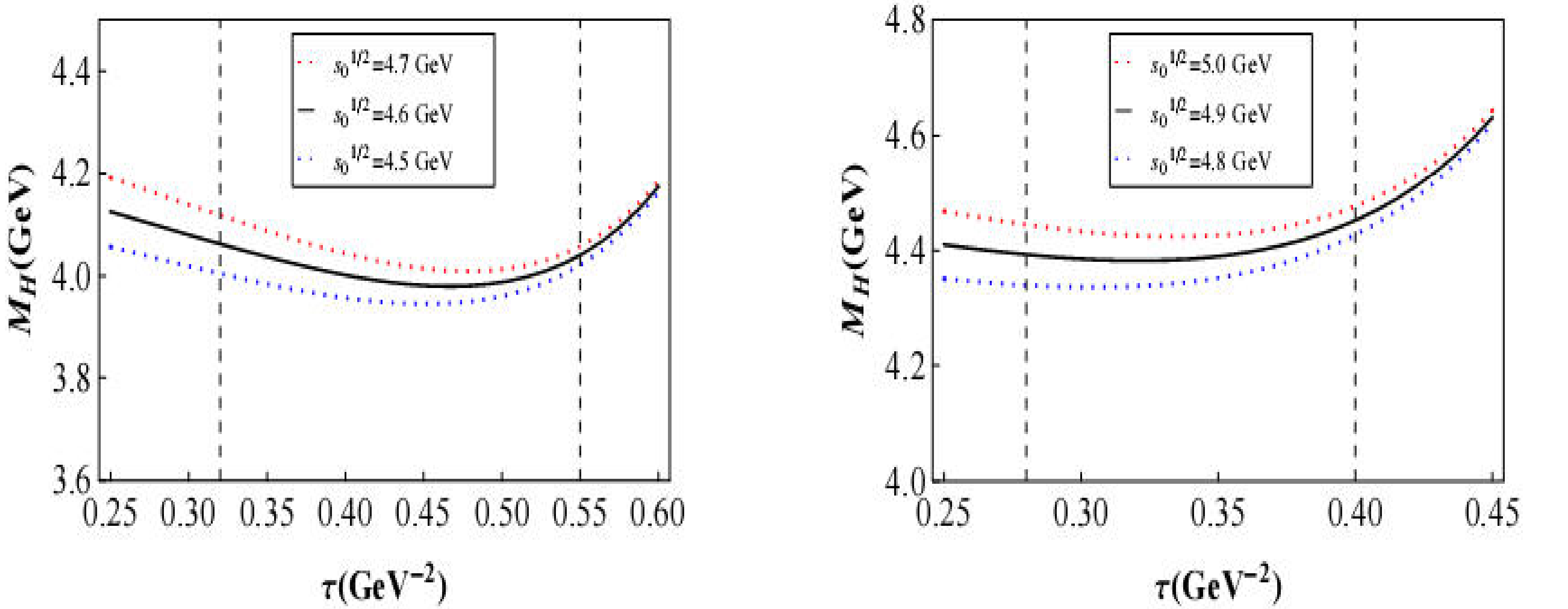


Figure 3: (**Left**) The mass of the $D_s\bar{D}_s^*$ state as a function of the sum rule parameter τ, for different values of $\sqrt{s_0}$. (**Right**) the mass of the $D_s^*\bar{D}_s^*$ state as a function of the sum rule paramter τ, for different values of $\sqrt{s_0}$. The two vertical lines indicate the upper and lower limits of a valid Borel window.

Hidden Charm and Hidden Strange Molecular States

- **Results:**

$$M_H^{D_s\bar{D}_s^*} = (3.98 \pm 0.15)\,\mathrm{GeV}$$

$$M_H^{D_s^*\bar{D}_s^*} = (4.38 \pm 0.16)\,\mathbf{GeV}$$

Hidden Charm and Strange Molecular States

- **Discussion**

  - **The central value of the $D_sD_s^*$ <** $E_{th}[D_s\bar{D}_s^*]$ **(4.08 GeV).**

  - **The center value of the $D_s^*D_s^*$ >** $E_{th}[D_s^*\bar{D}_s^*]$ **(4.22 GeV).**

Hidden Charm and Hidden Strange Molecular States

- **Predictions**

Possible decay modes for D_sD_s* bound state:

$$e^+e^- \to (\eta_c + \omega) + \eta,$$

$$e^+e^- \to (J/\psi + \eta) + \eta$$

$$e^+e^- \to (J/\psi + f_0(500)) + \eta$$

Possible decay modes for D_s*D_s* resonance:

$$e^+e^- \to (\eta_c + \phi) + \eta,$$

$$e^+e^- \to (J/\psi + \eta') + \eta$$

$$e^+e^- \to (J/\psi + f_0(980)) + \eta$$

$$e^+e^- \to (h_c + \eta) + \eta$$

$$e^+e^- \to (h_c + f_0(500)) + \eta$$

$$e^+e^- \to (D_s^* + \bar{D}_s^*) + \eta$$

Thank you!