Some new hadrons from QCD sum rules

Jian-Rong Zhang (张建荣)

National University of Defense Technology
（国防科技大学）

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Outline

1. Introduction

2. Some new hadrons from QCDSR

3. Summary

1. Introduction

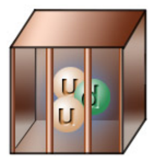
QCD: the correct theory of strong interaction

1) asymptotic freedom (Gross, Wilczek, Politzer)

at high energy: up to 0.1% accuracy

perturbative calculations

2) color confinement



no ~~free quark and gluon~~

Color confinement is related to the dynamics of QCD at low energy

↕

hadron Physics

Study for the hadron Physics

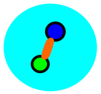
1) **precision test of the standard model (SM) and probing new physics beyond the SM**

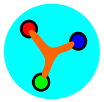
2) **understanding of confinement & the dynamic behavior of QCD at low energy**

Quark model and QCD

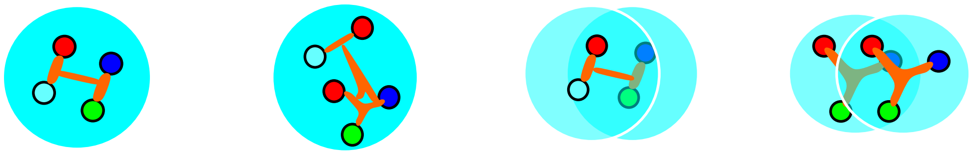
1) **nonpertubative QCD at large distances, hard to get hadronic spectroscopy by solving QCD**

2) **quark model succeeds in classifying hadron spectroscopy**

meson 

baryon 

exotic states: glueball, hybrid, multiquark states, molecular states, baryonium



B-factories achievement: hadron spectroscopy- Charmonium states

X(3872), X(3940),Y(3930), Z(3930), Y(4260), Y(4360), Y(4660), Z⁺(4430)……

State	Mass (MeV)	Production mode	Experiment
$X(3872)$	$3872.0 \pm 0.6 \pm 0.5$	$B \to KX$	Belle[65]
	3873.4 ± 1.4	$B \to KX$	BABAR[68]
	$3871.3 \pm 0.7 \pm 0.4$	$p\bar{p} \to X$	CDF[67]
	$3871.8 \pm 3.1 \pm 3.0$	$p\bar{p} \to X$	D0[66]
$X(3940)$	$3943 \pm 6 \pm 6$	$e^+e^- \to J/\psi X$	Belle[72]
$Y(3930)$	$3943 \pm 11 \pm 13$	$B \to KY$	Belle[69]
	$3914.6^{+3.8}_{-3.4} \pm 2$	$B \to KY$	BABAR[70]
$Z(3930)$	$3931 \pm 4 \pm 2$	$\gamma\gamma \to Z$	Belle[71]
$Y(4260)$	$4259 \pm 8 \pm 4$	$e^+e^- \to \gamma_{ISR}Y$	BABAR[75]
	$4284^{+17}_{-16} \pm 4$	$e^+e^- \to \gamma_{ISR}Y$	CLEO[76]
$Y(4360)$	4324 ± 24	$e^+e^- \to \gamma_{ISR}Y$	BABAR[78]
	$4361 \pm 9 \pm 9$	$e^+e^- \to \gamma_{ISR}Y$	Belle[79]
$Y(4660)$	$4664 \pm 11 \pm 5$	$e^+e^- \to \gamma_{ISR}Y$	Belle[79]
$Z^+(4430)$	$4433 \pm 4 \pm 2$	$B \to KZ$	Belle[73]
$Z_1^+(4050)$	$4051 \pm 14^{+20}_{-41}$	$\bar{B}^0 \to K^-Z$	Belle[74]
$Z_2^+(4250)$	$4248^{+44+180}_{-29-35}$	$\bar{B}^0 \to K^-Z$	Belle[74]
$Y(4140)$	$4143.0 \pm 2.9 \pm 1.2$	$B^+ \to K^+Y$	CDF[81]
$X(4350)$	$4350.6^{+4.6}_{-5.1} \pm 0.7$	$\gamma\gamma \to X$	Belle[82]

→ **candidates for exotic states**

Dynamics of QCD at low energy:

The strong interaction dynamics of hadronic systems is governed by nonperturbative QCD effects completely. It is quite difficult to calculate the hadron spectrum from QCD first principles.

➢ **understanding limited!**

Some nonperturbative QCD approaches:

- **Quark model**
- **Effective field theory**
- **Lattice**
- **QCD sum rules**
- **Ads/QCD**
- **NRQCD**
- **……**

Each of them has advantages and disadvantages

QCD Sum Rules (QCDSR):

Shifman, Vainshtein, Zakharov, NPB**147**, 385 (1979)

basic point：current j(x) for hadron

correlation function: 2-point or 3-point correlator

$$\Pi(q^2) = i\int d^4x e^{iq.x}\langle 0|T\left[j(x)j^\dagger(0)\right]|0\rangle,$$

three steps：

1) phenomenological descriptions of correlators

$$\Pi^{\text{phen}}(q^2) = \int ds \frac{\rho(s)}{s - q^2 + i\epsilon} + \text{subtractions}$$

$$\rho(s) = \frac{1}{\pi}\text{Im}[\Pi(s)]$$

2) QCD descriptions of correlators (Wilson's Operator Product Expansion)

$$\Pi^{\text{OPE}}(q^2) = \sum_n C_n(Q^2)\hat{O}_n,$$

short-distance——perturbative calculations

long-distance effects——vacuum condensates

3) matching the two sides $\Pi^{\text{phen}}(Q^2) \leftrightarrow \Pi^{\text{OPE}}(Q^2),$

Equating the two sides + quark hadron duality + making a Borel transform

Borel transform

$$\mathcal{B}_{M^2}[(q^2)^n] = 0; \qquad \mathcal{B}_{M^2}[\frac{1}{(m^2-q^2)^n}] = \frac{e^{-m^2/M^2}}{(n-1)!(M^2)^{n-1}}, \qquad n > 0.$$

- → **enhance the role of the lowest resonance;**
- → **suppress contributions of higher states and continuum states;**
- → **improve the convergence of the nonperturbative series and eliminate subtractions**

Sum rule:

$$\lambda_H^2 e^{-M_H^2/M^2} = \int_{s_{min}}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2}.$$

Eliminating the coupling constant:

$$M_H^2 = \int_{s_{min}}^{s_0} ds \rho^{\text{OPE}} s e^{-s/M^2} \Big/ \int_{s_{min}}^{s_0} ds \rho^{\text{OPE}} e^{-s/M^2}.$$

input values

Choice of work windows

QCDSR: advantages & disadvantage

1) **Firmly based on the basic theory of QCD;**

2) **Approximation in the OPE of the correlator, and there is a very complicated and largely unknown structure of the hadronic dispersion integral in the phenomenological side: accuracy limited;**

3) **The uncertainty could be well estimated.**

Widely used: Cited> 4100 times

Shifman, Vainshtein, Zakharov……	● **mesons**
Ioffe, Chung……	● **baryons**
S. L. Zhu, PRL **91,** 232002 (2003)……	● **multiquark states**
	●……

Molecular states：

M. B. Voloshin and L. B. Okun, JETP Lett. **23**, 333 (1976)

A. D. Rujula, H. Georgi, and S. L. Glashow, PRL **38**, 317 (1977)

N. A. Tornqvist, Z. Phys. C **61**, 525 (1994)

Many new hadrons have been explained as molecular states!

Y(3930) →	$D^*\bar{D}^*$	X. Liu, Z. G. Luo, Y. R. Liu & S. L. Zhu; Y. C. Yang & J. L. Ping
Y(4140) →	$D_s^*\bar{D}_s^*$	X. Liu & S. L. Zhu; N. Mahajan; T. Branz *et al.;* G. J. Ding...
Y(4260) →	$\chi_c\rho^0$	X. Liu, X. Q. Zeng & X. Q. Li
	$\omega\chi_{c1}$	C. Z. Yuan, P. Wang & X. H. Mo
	$\bar{D}D_1(2420)+D\bar{D}_1(2420)$	Q. Wang, C. Hanhart & Q. Zhao
Z(4430) →	$D^*\bar{D}_1$	C. Meng & K. T. Chao; X. Liu, Y. R. Liu, W. Z. Deng & S. L. Zhu
X(4350) →	$D_s^*D_s^{0*}$	Zhang&Huang; Y. L. Ma
Y(4274) →	$D_sD_{s0}(2317)$	X. Liu, Z. G. Luo & S. L. Zhu
......		......

2. Some new hadrons from QCDSR

Zc(3900)

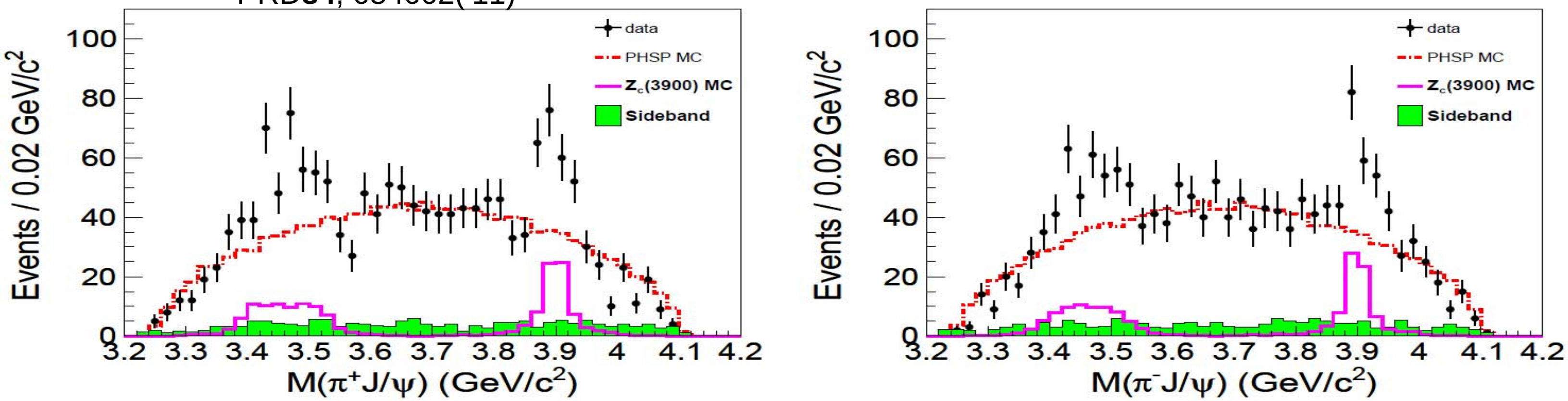
BESIII , PRL **110**, 252001 (2013)

Predicted in

PRD**84**, 034032('11)

PRD**84**, 054002('11)

$$(3899.0 \pm 3.6 \pm 4.9)\ \mathrm{MeV}$$



Confirmed by Belle, PRL **110**, 252002 (2013)

CLEO, hep-ex/1304.3036

Y(4260) → $\bar{D}D_1(2420) + D\bar{D}_1(2420)$

Zc(3900) → $\bar{D}D^*$ Q. Wang, C. Hanhart, Q. Zhao, PRL **111**, 132003 (2013)

Zc(3900) → tetraquark or $\bar{D}D^*$ L. Maiani *et al.*, arXiv:1303.6857

arXiv:1303.6857,1303.6608,1303.6842,1303.6857,1304.1850,1304.0345,1304.0380,1304.1301,1304.2882,1304.4458,1304.5845,1304.6433,1304.7467,1305.1997 ……

$3.88 \pm 0.10\,(\mathrm{GeV})$ Predicted in QCDSR: Zhang&Huang, PRD **80**, 056004 (2009)

4-body problem

Meson	configuration	J^P	Meson	configuration	J^P
D	$(c\bar{q})$	0^-	B	$(b\bar{q})$	0^-
D^*	$(c\bar{q})^*$	1^-	B^*	$(b\bar{q})^*$	1^-
D_0^*	$(c\bar{q})_0^*$	0^+	B_0^*	$(b\bar{q})_0^*$	0^+
D_1	$(c\bar{q})_1$	1^+	B_1	$(b\bar{q})_1$	1^+
D_s	$(c\bar{s})$	0^-	B_s	$(b\bar{s})$	0^-
D_s^*	$(c\bar{s})^*$	1^-	B_s^*	$(b\bar{s})^*$	1^-
D_{s0}^*	$(c\bar{s})_0^*$	0^+	B_{s0}^*	$(b\bar{s})_0^*$	0^+
D_{s1}	$(c\bar{s})_1$	1^+	B_{s1}	$(b\bar{s})_1$	1^+

Symbol	Current	J^P
D_0	$\bar{q}c$	0^+
D	$i\bar{q}\gamma_5 c$	0^-
D^*	$\bar{q}\gamma_\mu c$	1^-
D_1	$\bar{q}\gamma_\mu\gamma_5 c$	1^+

Zc(3900) (3899.0±3.6±4.9) MeV

Zhang & Huang, PRD **80**, 056004 (2009)

Y(3930)

Z(4430)

TABLE I. The mass spectra of molecular states with same heavy quarks.

Hadron	Configuration	Mass (GeV)	Hadron	Configuration	Mass (GeV)
$D\bar{D}$	$(c\bar{q})(\bar{c}q)$	3.76 ± 0.10	$B\bar{B}$	$(b\bar{q})(\bar{b}q)$	10.58 ± 0.10
$D^*\bar{D}$	$(c\bar{q})^*(\bar{c}q)$	3.88 ± 0.10	$B^*\bar{B}$	$(b\bar{q})^*(\bar{b}q)$	10.62 ± 0.10
$D^*\bar{D}^*$	$(c\bar{q})^*(\bar{c}q)^*$	3.91 ± 0.11	$B^*\bar{B}^*$	$(b\bar{q})^*(\bar{b}q)^*$	10.67 ± 0.10
$D_0^*\bar{D}_0^*$	$(c\bar{q})_0^*(\bar{c}q)_0^*$	4.56 ± 0.11	$B_0^*\bar{B}_0^*$	$(b\bar{q})_0^*(\bar{b}q)_0^*$	11.28 ± 0.08
$D_1\bar{D}_0^*$	$(c\bar{q})_1(\bar{c}q)_0^*$	4.62 ± 0.11	$B_1\bar{B}_0^*$	$(b\bar{q})_1(\bar{b}q)_0^*$	11.32 ± 0.09
$D_1\bar{D}_1$	$(c\bar{q})_1(\bar{c}q)_1$	4.66 ± 0.13	$B_1\bar{B}_1$	$(b\bar{q})_1(\bar{b}q)_1$	11.33 ± 0.12
$D\bar{D}_0^*$	$(c\bar{q})(\bar{c}q)_0^*$	4.21 ± 0.07	$B\bar{B}_0^*$	$(b\bar{q})(\bar{b}q)_0^*$	11.03 ± 0.09
$D_1\bar{D}$	$(c\bar{q})_1(\bar{c}q)$	4.34 ± 0.07	$B_1\bar{B}$	$(b\bar{q})_1(\bar{b}q)$	11.04 ± 0.09
$D^*\bar{D}_0^*$	$(c\bar{q})^*(\bar{c}q)_0^*$	4.26 ± 0.07	$B^*\bar{B}_0^*$	$(b\bar{q})^*(\bar{b}q)_0^*$	11.02 ± 0.09
$D^*\bar{D}_1$	$(c\bar{q})^*(\bar{c}q)_1$	4.44 ± 0.09	$B^*\bar{B}_1$	$(b\bar{q})^*(\bar{b}q)_1$	11.03 ± 0.09

TABLE II. The mass spectra of molecular states with differently heavy quarks.

Hadron	Configuration	Mass (GeV)	Hadron	Configuration	Mass (GeV)
$B\bar{D}$	$(b\bar{q})(\bar{c}q)$	7.12 ± 0.09	$B^*\bar{D}_0^*$	$(b\bar{q})^*(\bar{c}q)_0^*$	7.67 ± 0.06
$B^*\bar{D}$	$(b\bar{q})^*(\bar{c}q)$	7.28 ± 0.09	$B^*\bar{D}_1$	$(b\bar{q})^*(\bar{c}q)_1$	7.74 ± 0.07
$B^*\bar{D}^*$	$(b\bar{q})^*(\bar{c}q)^*$	7.29 ± 0.10	$D^*\bar{B}$	$(c\bar{q})^*(\bar{b}q)$	7.21 ± 0.09
$B_0^*\bar{D}_0^*$	$(b\bar{q})_0^*(\bar{c}q)_0^*$	8.04 ± 0.08	$D_1\bar{B}_0^*$	$(c\bar{q})_1(\bar{b}q)_0^*$	8.04 ± 0.10
$B_1\bar{D}_0^*$	$(b\bar{q})_1(\bar{c}q)_0^*$	8.06 ± 0.13	$D\bar{B}_0^*$	$(c\bar{q})(\bar{b}q)_0^*$	7.70 ± 0.06
$B_1\bar{D}_1$	$(b\bar{q})_1(\bar{c}q)_1$	8.07 ± 0.11	$D_1\bar{B}$	$(c\bar{q})_1(\bar{b}q)$	7.74 ± 0.07
$B\bar{D}_0^*$	$(b\bar{q})(\bar{c}q)_0^*$	7.68 ± 0.06	$D^*\bar{B}_0^*$	$(c\bar{q})^*(\bar{b}q)_0^*$	7.76 ± 0.06
$B_1\bar{D}$	$(b\bar{q})_1(\bar{c}q)$	7.77 ± 0.06	$D^*\bar{B}_1$	$(c\bar{q})^*(\bar{b}q)_1$	7.76 ± 0.07

Zhang, PRD **87**, 116004 (2013)

Motivation: Improved QCDSR study of Zc(3900) as a molecular state

1) It may be more reliable to test the OPE convergence by including higher dimension condensate contributions than six and considering the work windows minutely.

2) Even higher condensate contributions may not radically influence the character of OPE convergence in some case, one still could attempt to improve the theoretical result because some higher condensates are helpful to stabilize the Borel curves.

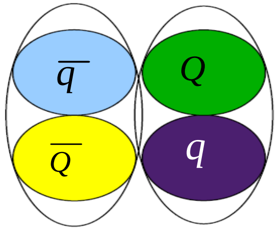
3) Particularly for the newly observed *Zc*(3900) states, they can not be simple charmonium mesons since they are electric charged. It may be a new hint for the existence of exotic hadrons and *Zc*(3900) are some ideal candidates for them.

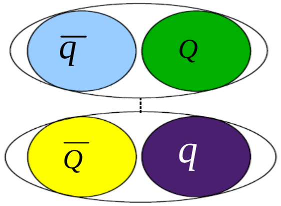
Current:

Current	J^P
$q_a^T C\gamma_5 c_b$	0^+
$q_a^T C c_b$	0^-
$q_a^T C\gamma_5\gamma_\mu c_b$	1^-
$q_a^T C\gamma_\mu c_b$	1^+

Symbol	Current	J^P
D_0	$\bar{q}c$	0^+
D	$i\bar{q}\gamma_5 c$	0^-
D^*	$\bar{q}\gamma_\mu c$	1^-
D_1	$\bar{q}\gamma_\mu\gamma_5 c$	1^+

Discussions:





diquark-antidiquark *Vs* meson-meson

Fiertz rearrangements

e.g.

$$\frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}[(q_a^T C\gamma_5 c_b)(\bar{q}_d\gamma_\mu C\bar{c}_e^T) + (q_a^T C\gamma_\mu c_b)(\bar{q}_d\gamma_5 C\bar{c}_e^T)]$$

$$= \frac{i}{\sqrt{2}}\left(-\frac{1}{8}\right)\Big[2(\bar{c}\lambda^\alpha\gamma_5 q)(\bar{q}\lambda^\alpha\gamma_\mu c) - 2(\bar{c}\lambda^\alpha\gamma_\mu q)(\bar{q}\lambda^\alpha\gamma_5 c) - 2i(\bar{c}\lambda^\alpha\sigma_{\mu\nu} q)\times(\bar{q}\lambda^\alpha\gamma_\nu\gamma_5 c)$$

$$+ 2i(\bar{c}\lambda^\alpha\gamma_\nu\gamma_5 q)(\bar{q}\lambda^\alpha\sigma_{\mu\nu} c)\Big] + \frac{i}{\sqrt{2}}\left(\frac{1}{6}\right)\Big[2(\bar{c}\gamma_5 q)(\bar{q}\gamma_\mu c) - 2(\bar{c}\gamma_\mu q)(\bar{q}\gamma_5 c)$$

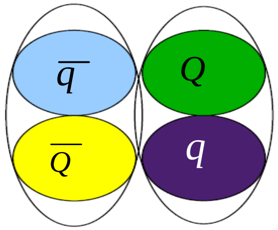
$$- 2i(\bar{c}\sigma_{\mu\nu} q)(\bar{q}\gamma_\nu\gamma_5 c) + 2i(\bar{c}\gamma_\nu\gamma_5 q)(\bar{q}\sigma_{\mu\nu} c)\Big].$$

M. Nielsen, et al. Phys. Rep. **497**, 41 (2010)

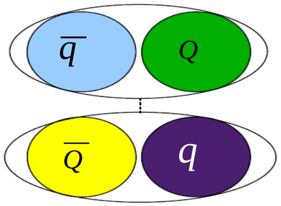
Current:

$$j^{\mu}_{\bar{D}D^*} = (\bar{Q}_a i\gamma_5 q_a)(\bar{q}_b\gamma^{\mu}Q_b)$$

Discussions:



antidiquark - diquark



molecule

diquark-antidiquark *Vs* meson-meson

Fiertz rearrangements

two-point correlator:

$$\Pi^{\mu\nu}(q^2) = i\int d^4x e^{iq.x}\langle 0|T[j^{\mu}_{\bar{D}D^*}(x)j^{\nu+}_{\bar{D}D^*}(0)]|0\rangle$$

Phenomenological side:

$$\Pi^{(1)}(q^2) = \frac{[\lambda^{(1)}]^2}{M^2_{\bar{D}D^*}-q^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds\frac{\mathrm{Im}\Pi^{(1)\mathrm{phen}}(s)}{s-q^2} + \mathrm{subtractions},$$

$$\langle 0|j^{\mu}_{\bar{D}D^*}|\bar{D}D^*\rangle = \lambda^{(1)}\epsilon^{\mu}$$

OPE side:

$$\Pi^{(1)}(q^2) = \int_{4m_Q^2}^{\infty} ds \frac{\rho^{\mathrm{OPE}}(s)}{s-q^2} + \Pi_1^{\mathrm{cond}}(q^2)$$

$$\rho^{\mathrm{OPE}}(s) = \frac{1}{\pi}\mathrm{Im}\Pi^{(1)}(s)$$

Equating the two sides + quark hadron duality + making a Borel transform

$$[\lambda^{(1)}]^2 e^{-M_{\bar{D}D^*}^2/M^2} = \int_{4m_Q^2}^{s_0} ds \rho^{\mathrm{OPE}} e^{-s/M^2} + \hat{B}\Pi_1^{\mathrm{cond}}$$

Mass sum rule:

$$M_{\bar{D}D^*}^2 = \left\{ \int_{4m_Q^2}^{s_0} ds \rho^{\mathrm{OPE}} s e^{-s/M^2} + \frac{d\hat{B}\Pi_1^{\mathrm{cond}}}{d(-\frac{1}{M^2})} \right\} / \left\{ \int_{4m_Q^2}^{s_0} ds \rho^{\mathrm{OPE}} e^{-s/M^2} + \hat{B}\Pi_1^{\mathrm{cond}} \right\}$$

OPE calculations

One calculates the heavy-quark part of the correlation function in the momentum space, and calculates the light-quark part in the coordinate space, which is then Fourier-transformed to the momentum space in D dimension. The resulting light-quark part is combined with the heavy-quark part before it is dimensionally regularized at D=4.

$$S_{ab}(x) = \frac{i\delta_{ab}}{2\pi^2 x^4}\not{x} - \frac{m_q\delta_{ab}}{4\pi^2 x^2} - \frac{i}{32\pi^2 x^2} t^A_{ab} g G^A_{\mu\nu}(\not{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\not{x}) - \frac{\delta_{ab}}{12}\langle\bar{q}q\rangle + \frac{i\delta_{ab}}{48} m_q \langle\bar{q}q\rangle\not{x}$$
$$- \frac{x^2\delta_{ab}}{3\cdot 2^6}\langle g\bar{q}\sigma\cdot Gq\rangle + \frac{ix^2\delta_{ab}}{2^7\cdot 3^2} m_q\langle g\bar{q}\sigma\cdot Gq\rangle\not{x} - \frac{x^4\delta_{ab}}{2^{10}\cdot 3^3}\langle\bar{q}q\rangle\langle g^2G^2\rangle.$$

$$S_{free}(p) = \frac{i}{\not{p} - m_Q},$$

$$S_G(p) = -\frac{i}{4} g t^A G^A_{\kappa\lambda}(0)\frac{1}{(p^2-m_Q^2)^2}[\sigma_{\kappa\lambda}(\not{p}+m_Q) + (\not{p}+m_Q)\sigma_{\kappa\lambda}],$$

$$S_{G^2}(p) = -\frac{i}{4} g^2 t^A t^B G^A_{\kappa\lambda}(0) G^B_{\mu\nu}(0)\frac{\not{p}+m_Q}{(p^2-m_Q^2)^5}(f_{\alpha\beta\mu\nu} + f_{\alpha\mu\beta\nu} + f_{\alpha\mu\nu\beta})(\not{p}+m_Q),$$

$$S_{G^3}(p) = \frac{i}{48} g^3 f^{ABC} G^A_{\gamma\delta} G^B_{\delta\varepsilon} G^C_{\varepsilon\gamma}\frac{1}{(p^2-m_Q^2)^6}(\not{p}+m_Q)[\not{p}(p^2-3m_Q^2) + 2m_Q(2p^2-m_Q^2)]$$
$$\times(\not{p}+m_Q).$$

spectral densities

$$\rho^{\rm pert}(s) = \frac{3}{2^{12}\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^3}(1-\alpha-\beta)(1+\alpha+\beta)r(m_Q,s)^4$$

$$\rho^{\langle\bar{q}q\rangle}(s) = -\frac{3\langle\bar{q}q\rangle}{2^7\pi^4}m_Q\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}(1+\alpha+\beta)r(m_Q,s)^2$$

$$\rho^{\langle g^2G^2\rangle}(s) = \frac{\langle g^2G^2\rangle}{2^{11}\pi^6}m_Q^2\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)(1+\alpha+\beta)r(m_Q,s)$$

$$\rho^{\langle g\bar{q}\sigma\cdot Gq\rangle}(s) = \frac{3\langle g\bar{q}\sigma\cdot Gq\rangle}{2^8\pi^4}m_Q\int_{\alpha_{min}}^{\alpha_{max}}d\alpha\left\{\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta}r(m_Q,s)-\frac{2}{1-\alpha}[m_Q^2-\alpha(1-\alpha)s]\right\}$$

$$\rho^{\langle\bar{q}q\rangle^2}(s) = \frac{\langle\bar{q}q\rangle^2}{2^4\pi^2}m_Q^2\sqrt{1-\frac{4m_Q^2}{s}}$$

$$\langle q\bar{q}q\bar{q}\rangle = \kappa\langle\bar{q}q\rangle\langle\bar{q}q\rangle \qquad \kappa = 1$$

$$\rho^{\langle g^3G^3\rangle}(s) = \frac{\langle g^3G^3\rangle}{2^{13}\pi^6}\int_{\alpha_{min}}^{\alpha_{max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{min}}^{1-\alpha}d\beta(1-\alpha-\beta)(1+\alpha+\beta)[r(m_Q,s)+2m_Q^2\beta]$$

$$\rho^{\langle\bar{q}q\rangle\langle g^2G^2\rangle}(s) = -\frac{\langle\bar{q}q\rangle\langle g^2G^2\rangle}{2^{11}\pi^4} m_Q \left[\sqrt{1-\frac{4m_Q^2}{s}} + 4\int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2}\int_{\beta_{min}}^{1-\alpha} d\beta\beta(1+\alpha+\beta)\right]$$

$\alpha_{min} = \left(1-\sqrt{1-4m_Q^2/s}\right)/2$ $\alpha_{max} = \left(1+\sqrt{1-4m_Q^2/s}\right)/2$ $\beta_{min} = \alpha m_Q^2/(s\alpha - m_Q^2)$

$$\begin{aligned}
\hat{B}\Pi_1^{\text{cond}} &= \frac{\langle\bar{q}q\rangle\langle g^2G^2\rangle}{3\cdot 2^9\pi^4} m_Q^3 \int_0^1 d\alpha \left[\frac{1}{\alpha^3}\int_0^{1-\alpha} d\beta(\alpha+\beta)(1+\alpha+\beta)e^{-\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}} - \frac{1}{1-\alpha}e^{-\frac{m_Q^2}{\alpha(1-\alpha)M^2}}\right] \\
&\quad -\frac{\langle\bar{q}q\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}{2^5\pi^2} m_Q^2 \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \left[1+\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}\right] e^{-\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}} \\
&\quad +\frac{\langle g^2G^2\rangle^2}{3^2\cdot 2^{15}\pi^6} m_Q^4 \int_0^1 \frac{d\alpha}{\alpha^2}\int_0^{1-\alpha}\frac{d\beta}{\beta^2}(1-\alpha-\beta)(1+\alpha+\beta)\frac{1}{M^2}e^{-\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}} \\
&\quad +\frac{\langle\bar{q}q\rangle\langle g^3G^3\rangle}{3\cdot 2^{11}\pi^4} m_Q \int_0^1 \frac{d\alpha}{\alpha^4}\int_0^{1-\alpha} d\beta(1+\alpha+\beta)\left[\alpha(\alpha+6\beta) - \frac{2(\alpha+\beta)m_Q^2}{M^2}\right]e^{-\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}} \\
&\quad +\frac{\langle g^2G^2\rangle\langle g\bar{q}\sigma\cdot Gq\rangle}{3\cdot 2^{11}\pi^4} m_Q \int_0^1 \frac{d\alpha}{\alpha^3}\left\{2\left[3\alpha(1-\alpha) - \frac{m_Q^2}{M^2}\right]e^{-\frac{m_Q^2}{\alpha(1-\alpha)M^2}}\right. \\
&\quad \left. +\int_0^{1-\alpha} d\beta\left[-3\alpha\beta + (\alpha+\beta)\frac{m_Q^2}{M^2}\right]e^{-\frac{(\alpha+\beta)m_Q^2}{\alpha\beta M^2}}\right\},
\end{aligned}$$

input values

$$m_c = 1.23 \pm 0.05 \text{ GeV}$$

$$m_b = 4.24 \pm 0.06 \text{ GeV}$$

$$\langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3 \text{GeV}^3$$

$$\langle g\bar{q}\sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$$

$$m_0^2 = 0.8 \pm 0.1 \text{GeV}^2$$

$$\langle g^2 G^2 \rangle = 0.88 \text{GeV}^4$$

$$\langle g^3 G^3 \rangle = 0.045 \text{ GeV}^6$$

Choice of work windows

1. OPE convergence
2. pole dominance
3. $\sqrt{s_0} \sim M_H + 0.5 \text{GeV}$

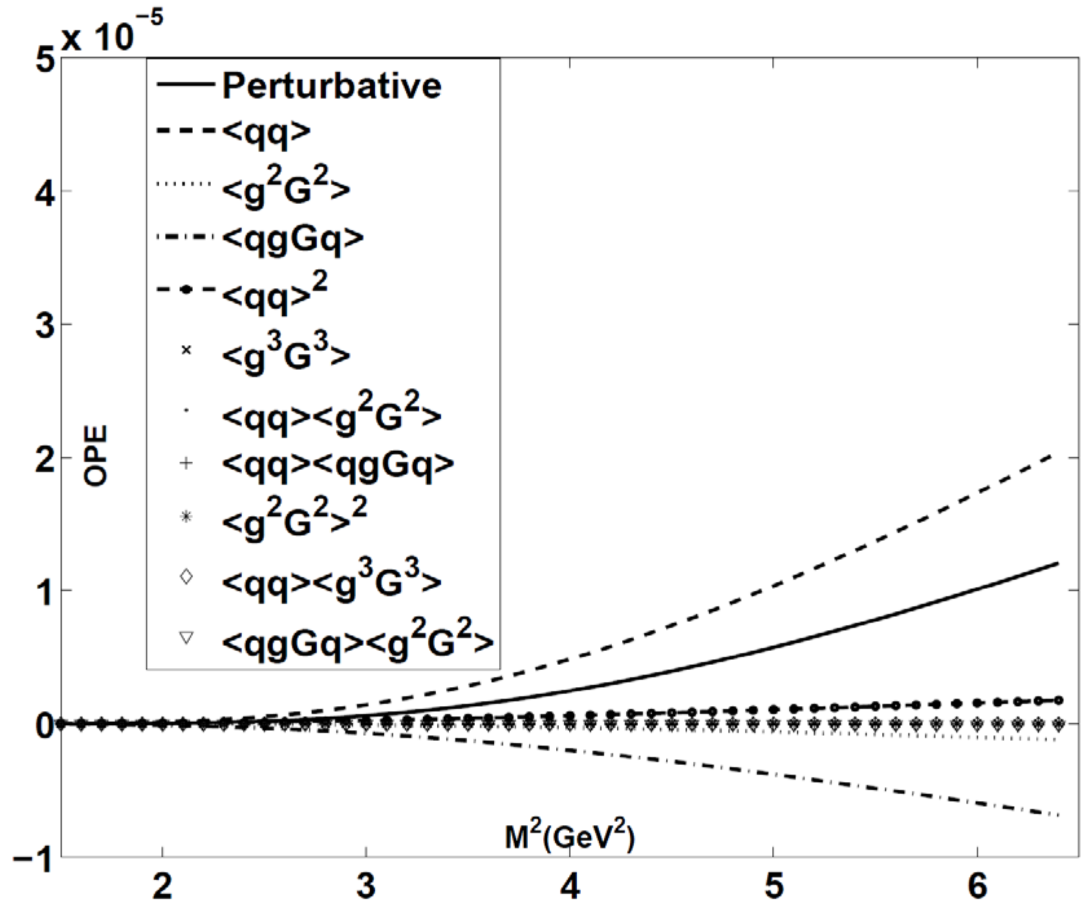


FIG. 1: The OPE contribution in sum rule (7) for $\sqrt{s_0} = 4.4$ GeV. The OPE convergence is shown by comparing the perturbative, two-quark condensate, two-gluon condensate, mixed condensate, four-quark condensate, three-gluon condensate, two-quark multiply two-gluon condensate, two-quark multiply mixed condensate, four-gluon condensate, two-quark multiply three-gluon condensate, mixed multiply two-gluon condensate contributions.

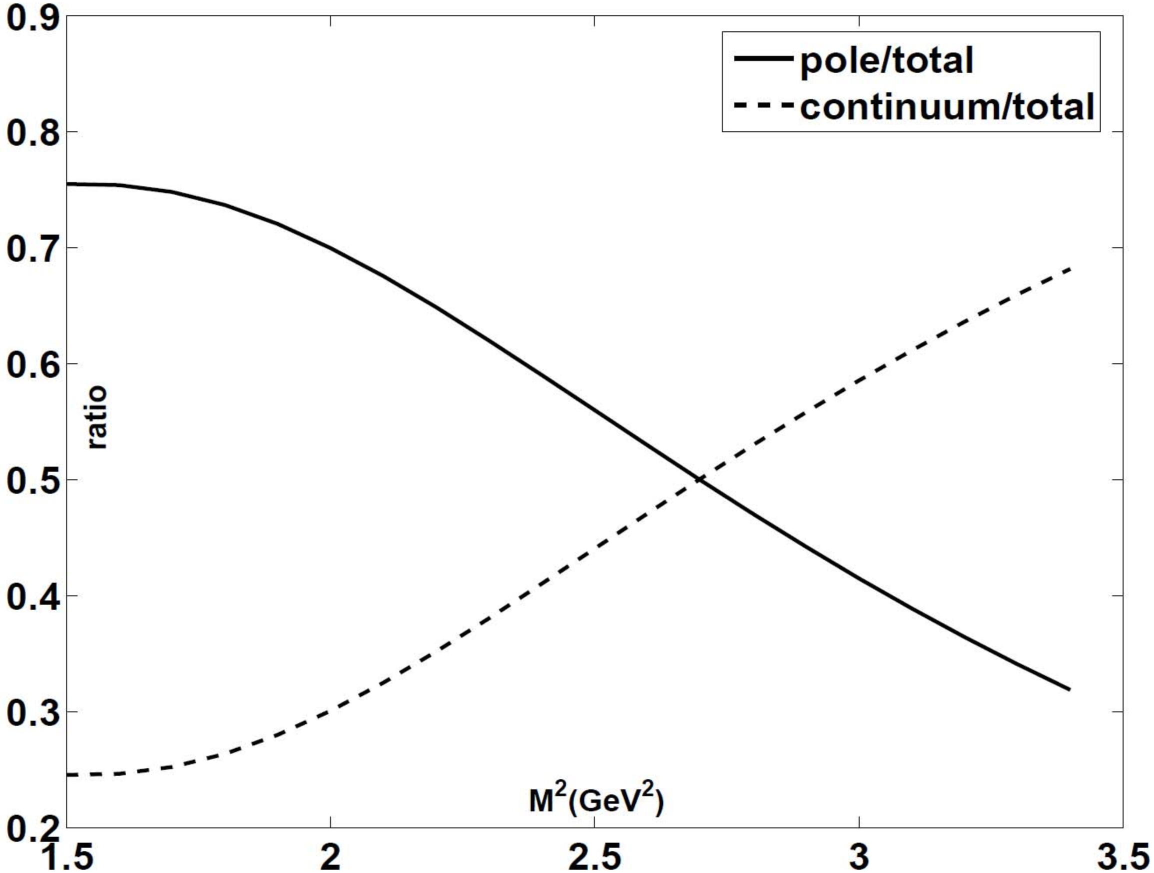


FIG. 2: The phenomenological contribution in sum rule (7) for $\sqrt{s_0} = 4.4$ GeV. The solid line is the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) as a function of M^2 and the dashed line is the relative continuum contribution.

Some discussions

1) **It may be difficult to find a conventional work window rigidly satisfying both of two rules in some cases.**

2) **Some condensates are very large and play an important role in the OPE side, which makes the standard OPE convergence (i.e. the perturbative at least larger than each condensate contribution) happen only at very large values of M^2. The consequence is that it is difficult to find a conventional Borel window where both the OPE converges well (the perturbative at least larger than each condensate contribution) and the pole dominates over the continuum.**

3) **We consider the perturbative dominating over the sum of condensates instead of the perturbative larger than each condensate: there are merely few important condensates and they could cancel out each other to some extent; other condensates are almost negligible.**

Zc(3900) $(3899.0 \pm 3.6 \pm 4.9)$ MeV

Value from QCDSR: $3.86 \pm 0.13 \pm 0.14$ GeV

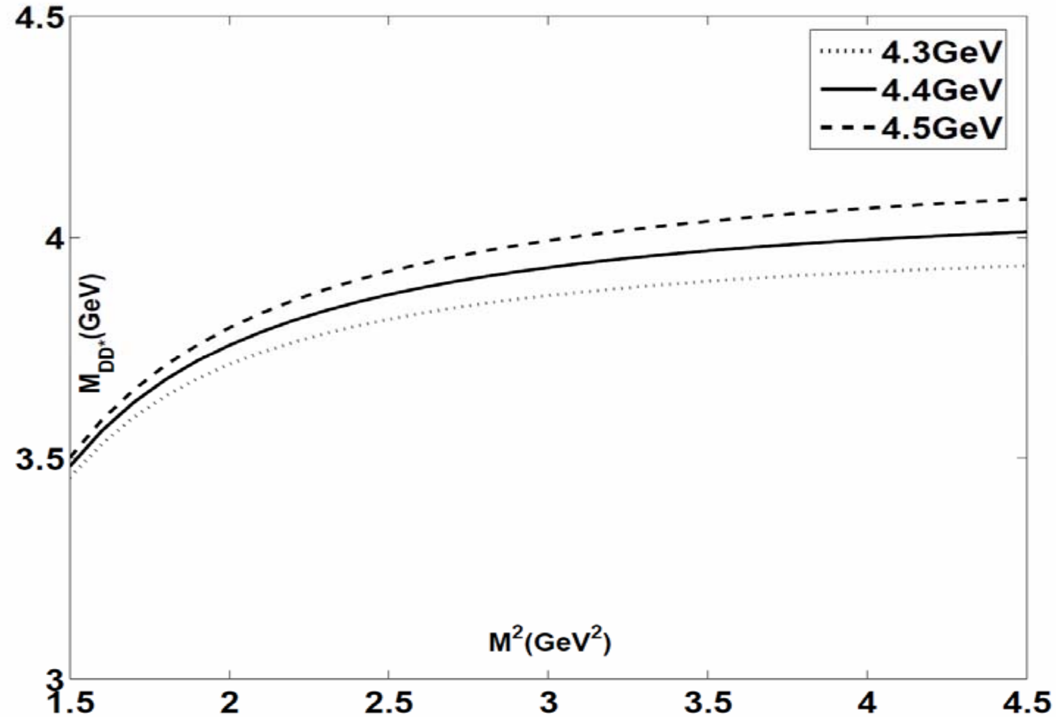


FIG. 3: The mass of the $\bar{D}D^*$ molecular state as a function of M^2 from sum rule (8). The continuum thresholds are taken as $\sqrt{s_0} = 4.3 \sim 4.5$ GeV. The ranges of M^2 is $2.1 \sim 2.6$ GeV^2 for $\sqrt{s_0} = 4.3$ GeV, $2.1 \sim 2.7$ GeV^2 for $\sqrt{s_0} = 4.4$ GeV, and $2.1 \sim 2.9$ GeV^2 for $\sqrt{s_0} = 4.5$ GeV.

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Could $Z_b(10610)$ be a $B^*\bar{B}$ molecular state?

Jian-Rong Zhang[a,*], Ming Zhong[a,b], Ming-Qiu Huang[a]

[a] *Department of Physics, National University of Defense Technology, Hunan 410073, China*
[b] *Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, China*

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Assuming the newly observed structure $Z_b(10610)$ as a bottomonium-like molecular state $B^*\bar{B}$, we calculate its mass in the framework of QCD sum rules. The numerical result is 10.54 ± 0.22 GeV for $B^*\bar{B}$, which coincide with the mass of $Z_b(10610)$. This consolidates the statement made by Belle Collaboration that the $Z_b(10610)$ resonance could be a $B^*\bar{B}$ molecular state.

C.P. Shen, et al., Belle Collaboration, PRL **104**, 112004 (2010)

Y(4140) Zhang & Huang, JPG**37**, 025005 (2010)

X(4350) Zhang & Huang, CTP **54**, 1075 (2010)

Table 1 The mass spectra of molecular states with same heavy quarks.

Hadron	Configuration	Mass/GeV	Hadron	Configuration	Mass/GeV
$D_s\bar{D}_s$	$(c\bar{s})(\bar{c}s)$	3.91 ± 0.10[24]	$B_s\bar{B}_s$	$(b\bar{s})(\bar{b}s)$	10.70 ± 0.10[24]
$D_s^*\bar{D}_s$	$(c\bar{s})^*(\bar{c}s)$	4.01 ± 0.10[24]	$B_s^*\bar{B}_s$	$(b\bar{s})^*(\bar{b}s)$	10.71 ± 0.11[24]
$D_s^*\bar{D}_s^*$	$(c\bar{s})^*(\bar{c}s)^*$	4.13 ± 0.10[24]	$B_s^*\bar{B}_s^*$	$(b\bar{s})^*(\bar{b}s)^*$	10.80 ± 0.10[24]
$D_{s0}^*\bar{D}_{s0}^*$	$(c\bar{s})_0^*(\bar{c}s)_0^*$	4.58 ± 0.10	$B_{s0}^*\bar{B}_{s0}^*$	$(b\bar{s})_0^*(\bar{b}s)_0^*$	11.35 ± 0.09
$D_{s1}\bar{D}_{s0}^*$	$(c\bar{s})_1(\bar{c}s)_0^*$	4.64 ± 0.10	$B_{s1}\bar{B}_{s0}^*$	$(b\bar{s})_1(\bar{b}s)_0^*$	11.38 ± 0.09
$D_{s1}\bar{D}_{s1}$	$(c\bar{s})_1(\bar{c}s)_1$	4.66 ± 0.12	$B_{s1}\bar{B}_{s1}$	$(b\bar{s})_1(\bar{b}s)_1$	11.39 ± 0.13
$D_s\bar{D}_{s0}^*$	$(c\bar{s})(\bar{c}s)_0^*$	4.24 ± 0.08	$B_s\bar{B}_{s0}^*$	$(b\bar{s})(\bar{b}s)_0^*$	11.06 ± 0.10
$D_{s1}\bar{D}_s$	$(c\bar{s})_1(\bar{c}s)$	4.37 ± 0.08	$B_{s1}\bar{B}_s$	$(b\bar{s})_1(\bar{b}s)$	11.10 ± 0.10
$D_s^*\bar{D}_{s0}^*$	$(c\bar{s})^*(\bar{c}s)_0^*$	4.36 ± 0.08	$B_s^*\bar{B}_{s0}^*$	$(b\bar{s})^*(\bar{b}s)_0^*$	11.09 ± 0.10
$D_s^*\bar{D}_{s1}$	$(c\bar{s})^*(\bar{c}s)_1$	4.43 ± 0.09	$B_s^*\bar{B}_{s1}$	$(b\bar{s})^*(\bar{b}s)_1$	11.10 ± 0.10

Table 2 The mass spectra of molecular states with differently heavy quarks.

Hadron	Configuration	Mass/GeV	Hadron	Configuration	Mass/GeV
$B_s\bar{D}_s$	$(b\bar{s})(\bar{c}s)$	7.31 ± 0.09	$B_s^*\bar{D}_{s0}^*$	$(b\bar{s})^*(\bar{c}s)_0^*$	7.71 ± 0.07
$B_s^*\bar{D}_s$	$(b\bar{s})^*(\bar{c}s)$	7.37 ± 0.09	$B_s^*\bar{D}_{s1}$	$(b\bar{s})^*(\bar{c}s)_1$	7.78 ± 0.08
$B_s^*\bar{D}_s^*$	$(b\bar{s})^*(\bar{c}s)^*$	7.46 ± 0.09	$D_s^*\bar{B}_s$	$(c\bar{s})^*(\bar{b}s)$	7.30 ± 0.09
$B_{s0}^*\bar{D}_{s0}^*$	$(b\bar{s})_0^*(\bar{c}s)_0^*$	8.07 ± 0.09	$D_{s1}\bar{B}_{s0}^*$	$(c\bar{s})_1(\bar{b}s)_0^*$	8.07 ± 0.09
$B_{s1}\bar{D}_{s0}^*$	$(b\bar{s})_1(\bar{c}s)_0^*$	8.14 ± 0.09	$D_s\bar{B}_{s0}^*$	$(c\bar{s})(\bar{b}s)_0^*$	7.73 ± 0.07
$B_{s1}\bar{D}_{s1}$	$(b\bar{s})_1(\bar{c}s)_1$	8.17 ± 0.11	$D_{s1}\bar{B}_s$	$(c\bar{s})_1(\bar{b}s)$	7.78 ± 0.08
$B_s\bar{D}_{s0}^*$	$(b\bar{s})(\bar{c}s)_0^*$	7.65 ± 0.07	$D_s^*\bar{B}_{s0}^*$	$(c\bar{s})^*(\bar{b}s)_0^*$	7.79 ± 0.08
$B_{s1}\bar{D}_s$	$(b\bar{s})_1(\bar{c}s)$	7.80 ± 0.08	$D_s^*\bar{B}_{s1}$	$(c\bar{s})^*(\bar{b}s)_1$	7.86 ± 0.08

3. Summary

1) Masses for $\{Q\bar{q}\}\{\bar{Q}^{(\prime)}q\}$ and $\{Q\bar{s}\}\{\bar{Q}^{(\prime)}s\}$ molecular states are systematically calculated in QCD sum rules.

2) Some results could support that some new hadrons' molecular explanations.

3) Besides charmonium-like and bottomnium-like molecular states, Bc-like molecules could be searched in future experiments.

Thanks!