Production of $C = + \ XYZ$ recoiled with γ in e^+e^- experiments

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Outline

Based on : ArXiv: 1310.0374, by Y.J.Li, G.Z.Xu, K.Y.Liu and Y.J.Zhang

- Introduction
- The frame of calculation
- The results for pure charmonium
- The results for $C = +$ XYZ states
  - $X(3872)$
  - $X(3940)$ and $X(4160)$
  - $X(4350)$
- Summary and discussion

Introduction-X(3872)

- Discovered by the Belle collaboration(2003). PRL91,262001
- Confirmed by the CDF 2003 , D0 2004 , BaBar 2004 , LHCb 2011 , and CMS 2013 collaborations.
- Mass: Close to the $D^0\bar{D}^{\star 0}$ threshold within 1MeV, J^{PC}: 1^{++} or 2^{-+}(Excluded by LHCb EPJ C72,1972,2012)
- Theoretical hypothesis: standard charmonium, $D^0\bar{D}^{\star 0}$molecule, tetraquark , quark-gluon mixture state, threshold effect...

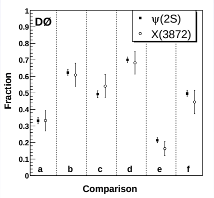
  QWG,2011; N.Drenska, 2010; S.Godfrey, 2008; M.Nielsen, 2010; Eric.S.Swanson, 2006; C.Hambrock, 2013;

$X(3872) \Leftrightarrow \chi_{c1}(2P)$?

- Potential model:
  Mass$[\chi_{c1}(2P)] \approx 3950$MeV, lager than $X(3872)$ about 75MeV.
  If Z(3930)$=\chi_{c2}(2P)$, Mass$[2^3P_2 - X(3872)] = 58$MeV$>$ 50MeV.
  (Screening effects: draw down the mass to 3900MeV PRD79,094004 .)
- If Mass$[\chi_{c1}(2P)] = 3872$MeV, Width$=$ 1.7MeV.
  $B[2^3P_1 \to \gamma\psi(2S)]/B[2^3P_1 \to \gamma J/\psi] \approx 6$.
  CONSISTENT with X(3872)
- D0:no significant differences between the $X(3872)$ and $\psi(2S)$
- Failed to explain the Isospin-violating in the $J/\psi\rho^0$, $J/\psi\omega$ decay patterns.

State		Expt.	Theor.
1P	$\chi_2(1^3\mathrm{P}_2)$	3556.20 ± 0.09	3554
	$\chi_1(1^3\mathrm{P}_1)$	3510.66 ± 0.07	3510
	$\chi_0(1^3\mathrm{P}_0)$	3414.75 ± 0.31	3433
	$h_c(1^1\mathrm{P}_1)$	3525.93 ± 0.27	3519
2P	$\chi_2(2^3\mathrm{P}_2)$	$3929 \pm 5 \pm 2$	3937
	$\chi_1(2^3\mathrm{P}_1)$		3901
	$\chi_0(2^3\mathrm{P}_0)$		3842
	$h_c(2^1\mathrm{P}_1)$		3908

B.Q.Li et al, PRD79,094004,2009



CDF,PRL93,162002,2004

X(3872)$\Leftrightarrow D^0\bar{D}^{\star 0}$ Molecule?

- In 1977, Rugula,Georgi,Glashow and Voloshin,Okun presented molecule conjecture. In 1994, Turnqvist predicted the mass of the ground $D\bar{D}$ molecule state was about 3870MeV.
- Mass and quantum number can be explained naturally.
- Can explain the Isospin-violation in the $J/\psi\rho^0$,$J/\psi\omega$ decay mode E.S.Swanson,PLB598,197,2004 .
- Prediction on charged molecule states(D^+D^{*0},D^0D^{*-}), but no explicit signals in the experimental measurements.
- Puzzle of the production at the hadron colliders: Tevatron and LHC.

X(3872)$\Leftrightarrow$ Mixture with $\chi_{c1}(2P)$ and Molecule?

Others in support of that X(3872) has a $c\bar{c}$ component,

- QCDSR supports $c\bar{c}(97\%)$ mixed with molecule or tetraquark state.
- Screening potential model (calculation on the width) supports the idea of mixture with a primary $c\bar{c}$ component.

X(3872) as mixture with $\chi_{c1}(2P)$ and $D\bar{D}^{\star 0}$ molecule components, (Meng's talk, C.Meng,hep-ph/0506222)

- $Z_{c\bar{c}}$ as the possibility of the $\chi_{c1}(2P)$ component in $X(3872)$. Universal, obtained by fitting to the experimental data.
- Molecule component dominates the decay patterns.
- In the B and hadron production process, $\chi_{c1}(2P)$ dominates. (Predictions of prompt X(3872) hadron-production at NLO in α_s are consistent with the CMS and the CDF data C.Meng,hep-ph/1304.6710 and disfavor the pure $\chi_{c1}(2P)$ view M.Butenschoen,hep-ph/ 1303.6524) Note: LHCb data will also be compatible when taking the relativistic correction contribution into account.

Introduction- Other $C = + XYZ$ states

	State, $m(\Gamma)$ in MeV, J^{PC}		Prod.(Decay)	Ref
$X(3872)$	$3871.68 \pm 0.17 (< 1.2)$	1^{++}	$B \to K\ (\pi\pi J/\psi)$	PRL91,262001
			$B \to K\ (\omega J/\psi)$	(hep-ex/0505037; PRD82,011101)
			$B \to K\ (D^0 \bar{D}^*)$	PRL97,162002; PRD77,011102
			$B \to K\ (\gamma J/\psi)$	PRD74,071101
			$p\bar{p} \to (\pi\pi J/\psi)+$	PRL93,072001; PRL98,132002
			$pp \to (\pi\pi J/\psi)+$	JHEP04(2013)154, 1302.6269
$X(3915)$	$3917.5 \pm 2.7 (27 \pm 10)$	0^{++}	$B \to K(\omega J/\psi)$	PRL94.182002; PRL101,082001
			$e^+e^- \to e^+e^-(\omega J/\psi)$	PRD82,011101; PRD86,072002
$X(3940)$	$3942^{+9}_{-8}(37^{+27}_{-17})$	J^{P+}	$e^+e^- \to J/\psi\ (D\bar{D}^*)$	PRL100,202001
$Y(4140)$	$4143.0 \pm 3.1 (12^{+9}_{-6})$	J^{P+}	$B \to K\ (\phi J/\psi)$	arXiv1101.6058
$X(4160)$	$4156^{+29}_{-25}(139^{+110}_{-60})$	J^{P+}	$e^+e^- \to J/\psi\ (D^*\bar{D}^*)$	PRL100,202001
$Y(4274)$	$4274.4^{+8.4}_{-6.7}(32^{+22}_{-15})$	J^{P+}	$B \to K\ (\phi J/\psi)$	arXiv1101.6058
$X(4350)$	$4350.6^{+4.6}_{-5.1}(13.3^{+18}_{-10})$	$0/2^{++}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	PRL104,112004

C=+ XYZ states through recoiled γ process at B factories

- Motivated by two poionts:
  1)large cross sections for the double charmonium production recoiled by J/ψ
  2)quantum number of photon is same as J/ψ
- Identifying the C = + charmonium states H in the $e^+e^- \to \gamma^\star \to H + \gamma$ at B factories was proposed in the Ref.(D.Li,PRD80,114014,2009 and W.L.Sang,PRD81,034028, 2010).
- The radiative corrections of $e^+e^- \to \gamma^\star \to H + \gamma$ at B factories were calculated.
- The relativistic correction of $e^+e^- \to \gamma^\star \to \eta_c + \gamma$ was also included in the Sang's paper.

$e^+e^- \to X(3872) + \gamma$ at BESIII

- Recently, BesIII reports the cross sections of $e^+e^- \to \gamma X(3872)$ (arxiv/1310.0280,arxiv/1310.4101)

$$\sigma \times \mathrm{Br}[J/\psi\pi\pi] < 0.13\mathrm{pb} \;\; \text{at } 90\% \text{ CL.} \qquad \sqrt{s} = 4.009\mathrm{GeV}$$
$$\sigma \times \mathrm{Br}[J/\psi\pi\pi] = 0.32 \pm 0.15 \pm 0.02\mathrm{pb} \qquad \sqrt{s} = 4.230\mathrm{GeV}$$
$$\sigma \times \mathrm{Br}[J/\psi\pi\pi] = 0.35 \pm 0.12 \pm 0.02\mathrm{pb} \qquad \sqrt{s} = 4.260\mathrm{GeV}$$
$$\sigma \times \mathrm{Br}[J/\psi\pi\pi] < 0.39\mathrm{pb} \;\; \text{at } 90\% \text{ CL.} \qquad \sqrt{s} = 4.360\mathrm{GeV}$$

  Where $\mathrm{Br}[J/\psi\pi\pi]$ means $\mathrm{Br}[X(3872) \to J/\psi\pi\pi]$.

- The studies of $\psi(4160) \to X(3872)\gamma$ (arxiv/1304.8101) and $\psi(4260) \to X(3872)\gamma$ (F.K.Guo's talk, arxiv/1306.3096) are proposed to probe the molecular content of the $X(3872)$.

The frame of Calculation

- Heavy quarkonium is an excellent candidate to probe QCD from the high energy to the low energy regimes.
- In the Nonrelativistic QCD (NRQCD) approach, the production of heavy quarkonium is factored to short distance coefficients and long distance matrix elements(LDMEs).
- The short distance coefficients can be calculated perturbatively with the expansions by α_s.
- The LDMES can be scaled by the relative velocity v between the quark and antiquark. v^2 is about 0.3 for charmonium and about 0.1 for bottomonium.
- $$\begin{aligned} R &= \sum_n F_n < \mathcal{O}(n) > \\ F_n &= F_n^0 (1 + c_1 \alpha_s + c_2 \alpha_s^2 + ...) \\ < \mathcal{O}(n) > &\propto v^{d_n} \end{aligned} \tag{1}$$

The amplitudes

In the NRQCD factorization framework, the amplitude in the rest frame of H as (PRD78,074022; PRD80,114014; PRD81,034028)

$$
\begin{aligned}
&\mathcal{M}(e^-(k_1)e^+(k_2) \to H_{c\bar{c}}(^{2S+1}L_J)(2p_1) + \gamma) \\
= \; &\sum_{L_z S_z} \sum_{s_1 s_2} \sum_{jk} \int \mathrm{d}^3\vec{q}\,\Phi_{c\bar{c}}(\vec{q}) \langle s_1; s_2 \mid S S_z\rangle \langle 3j; \bar{3}k \mid 1\rangle \\
&\times \mathcal{M}\left[e^-(k_1)e^+(k_2) \to c_j^{s_1}(p_1+q) + \bar{c}_k^{s_2}(p_1-q) + \gamma(k)\right] \qquad (2)
\end{aligned}
$$

where $\langle 3j; \bar{3}k \mid 1\rangle = \delta_{jk}/\sqrt{N_c}$, $\langle s_1; s_2 \mid S S_z\rangle$ is the color CG coefficient for $c\bar{c}$ pairs projecting out appropriate bound states, and $\langle s_1; s_2 \mid S S_z\rangle$ is the spin CG coefficient.
$\mathcal{M}\left[e^-(k_1)e^+(k_2) \to c_j^{s_1}(p_1+q) + \bar{c}_k^{s_2}(p_1-q) + \gamma(k)\right]$ is the quark level scattering amplitude.

Expansions of quark-level amplitudes up-to $\mathcal{O}(v^4)$

- S wave

$$\mathcal{M}[(c\bar{c})(^1S_0^{[1]})] = \mathcal{M}_s\Big|_{q=0} + \frac{1}{2}q^\alpha q^\beta \frac{\partial^2(\sqrt{\frac{m_c}{E_q}}\mathcal{M}_s)}{\partial q^\alpha \partial q^\beta}\Big|_{q=0} + \mathcal{O}(q^4). \quad (3)$$

- P wave

$$\mathcal{M}[(c\bar{c})(^3P_J^{[1]})] = \epsilon_\rho(s_z)q_\sigma(L_z)(\frac{\partial \mathcal{M}_t^\rho}{\partial q^\sigma}\Big|_{q=0}$$
$$+\frac{1}{6}q^\alpha q^\beta \frac{\partial^3(\sqrt{\frac{m_c}{E_q}}\mathcal{M}_t^\rho)}{\partial q^\alpha \partial q^\beta \partial q^\sigma}\Big|_{q=0}) + \mathcal{O}(q^5). \quad (4)$$

- D wave

$$\mathcal{M}[(c\bar{c})(^1D_2^{[1]})] = \frac{1}{2}q^\alpha q^\beta \frac{\partial^2(\sqrt{\frac{m_c}{E_q}}\mathcal{M}_s)}{\partial q^\alpha \partial q^\beta}\Big|_{q=0} + \mathcal{O}(q^4). \quad (5)$$

Wave function of $c\bar{c}$

1. Consider the Fourier transform between the momentum space and position space (PRD55,5853; PRD86,094017)

$$\begin{aligned}
\int \mathrm{d}^3\vec{q}\ \ \Phi_{c\bar{c}}(\vec{q}) \quad &\propto \quad \sqrt{Z^H_{c\bar{c}}}R_{c\bar{c}}(0) \\
\int \mathrm{d}^3\vec{q}\ \ \vec{q}^{\alpha}\Phi_{c\bar{c}}(\vec{q}) \quad &\propto \quad \sqrt{Z^H_{c\bar{c}}}R'_{c\bar{c}}(0) \\
\int \mathrm{d}^3\vec{q}\ \ \vec{q}^{\alpha}\vec{q}^{\beta}\Phi_{c\bar{c}}(\vec{q}) \quad &\propto \quad \sqrt{Z^H_{c\bar{c}}}R''_{c\bar{c}}(0) \\
\int \mathrm{d}^3\vec{q}\ \ \vec{q}^{\alpha}\vec{q}^{\beta}\vec{q}^{\delta}\Phi_{c\bar{c}}(\vec{q}) \quad &\propto \quad \sqrt{Z^H_{c\bar{c}}}R'''_{c\bar{c}}(0).
\end{aligned} \tag{6}$$

2. $R_{c\bar{c}}(0)$ is the radial Schrodinger wave function at origin. And $R^l_{c\bar{c}}(0)$ the derivative of the radial Schrodinger wave function at the origin

$$R^l_{c\bar{c}}(0) = \left.\frac{\mathrm{d}^l R_{c\bar{c}}(r)}{\mathrm{d}^l r}\right|_{r=0} \tag{7}$$

Wave function of $c\bar{c}$

- $R_{c\bar{c}}(0)$ is correspond to the $\mathcal{O}(v^0)$ S-wave matrix element.
- $R'_{c\bar{c}}(0)$ is correspond to the $\mathcal{O}(v^0)$ P-wave matrix element.
- $R''_{c\bar{c}}(0)$ is correspond to the $\mathcal{O}(v^2)$ S-wave matrix element or $\mathcal{O}(v^0)$ D-wave matrix element.
- $R'''_{c\bar{c}}(0)$ is correspond to the $\mathcal{O}(v^2)$ P-wave matrix element.

Relativistic correction K factor

$$
\begin{aligned}
K_{v^2}[\eta_c] &= -\frac{5v^2}{6}-\frac{rv^2}{1-r},\\
K_{v^2}[\chi_{c0}] &= -\frac{\left(55r^2-28r+13\right)v^2}{10\left(3r^2-4r+1\right)}-\frac{rv^2}{1-r},\\
K_{v^2}[\chi_{c1}] &= -\frac{\left(21r^2+30r-11\right)v^2}{10\left(r^2-1\right)}-\frac{rv^2}{1-r},\\
K_{v^2}[\chi_{c2}] &= -\frac{\left(90r^3+113r^2+4r-7\right)v^2}{10(r-1)\left(6r^2+3r+1\right)}-\frac{rv^2}{1-r},
\end{aligned}
\tag{8}
$$

where $r=4m_c^2/s$. $-\frac{rv^2}{1-r}$ is the relativistic correction of the phase space. If we select $r\to 0$, the K_{v^2} factor is consistent with the large p_T behavior at hadron colliders xu,PRD86,094017,2012 .

$D\bar{D}$ component contributions in the molecule model

- The parton-level amplitudes may be compared with the $D\bar{D}$ hadron-level amplitudes

$$\begin{aligned} & \mathcal{M}\left[e^-(k_1)e^+(k_2) \to c\bar{c}(2p_1) + \gamma\right] \\ \sim \; & \mathcal{M}\left[e^-(k_1)e^+(k_2) \to D\bar{D}(2p_1) + \gamma\right] \end{aligned} \quad (9)$$

- But the $R^l_{c\bar{c}}(0) \sim v^{2l} R^S_{c\bar{c}}(0) \gg R_{D\bar{D}}(0)$ with the S wave $l = 0$ and P wave $l = 1$.
- For the binding energy of $c\bar{c}$ and $D\bar{D}$ are several hundreds MeV and several MeV, respectively.
- If $Z^H_{c\bar{c}} \sim Z^H_{D\bar{D}}$, we can consider the $c\bar{c}$ contributions only.

Paremeters

Our parameters are selected as

$$m_c = m_H/2, \qquad \alpha_s = 0.23 \pm 0.03, \qquad \alpha = 1/133,$$
$$v^2 = 0.23 \pm 0.03, \qquad R_{1S} = 1.454\mathrm{GeV}^3, \qquad R_{2S} = 0.927\mathrm{GeV}^3,$$
$$R_{3S} = 0.791\mathrm{GeV}^3, \qquad R'_{1P} = 0.131\mathrm{GeV}^5, \qquad R'_{2P} = 0.186\mathrm{GeV}^5,$$
$$R''_{1D} = 0.031\mathrm{GeV}^7. \qquad (10)$$

The wave functions at origin for higher states are estimated as

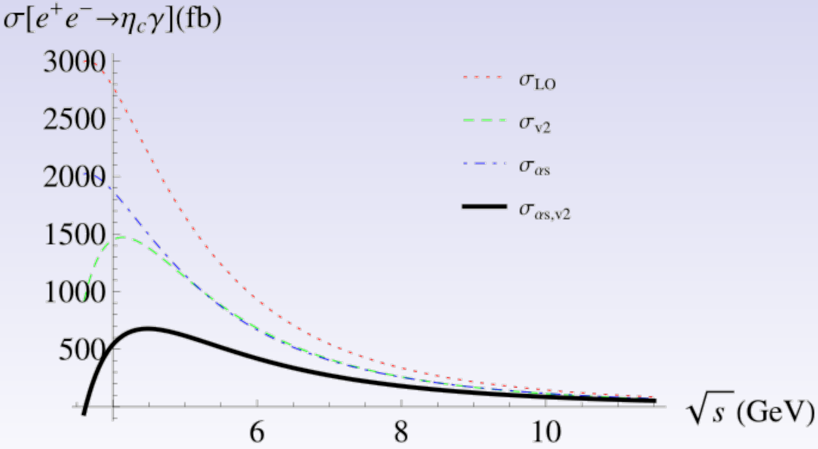
$$\begin{aligned} R_{4S} &= 2 \times R_{3S} - R_{2S} = 0.655\mathrm{GeV}^3, \\ R'_{3P} &= (R'_{1P} + R'_{2P})/2 = 0.159\mathrm{GeV}^5, \\ R''_{2D} &= R''_{1D} = 0.031\mathrm{GeV}^7. \end{aligned} \qquad (11)$$

Numerical results for pure charmonium

$\eta_c(nS)$ and $\eta_{c2}(nD)$

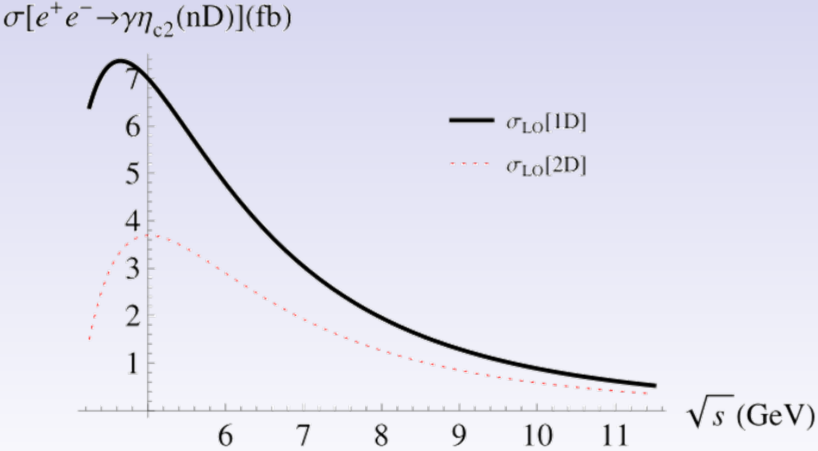
$\sqrt{s}$(GeV)		4.00	4.25	4.50	4.75	5.00	10.6	11.2
$\eta_c(1S)$	LO	2781	2494	2192	1906	1652	117	95
(2981)	RC	-1332	-1033	-814	-650	-526	-25	-20
	QCD	-909	-807	-700	-598	-508	-22	-16
	Total	540±210	653±170	678±140	658±115	617±95	70±4	58±3
$\eta_c(2S)$	LO	563	684	706	679	629	58	48
(3639)	RC	-730	-563	-442	-352	-284	-13	-10
	QCD	-177	-221	-231	-222	-205	-13	-10
	Total	-344±98	-100±79	33±65	105±54	141±46	32±2	27±2
$\eta_c(3S)$	LO		233	337	374	377	44	36
(3994)	RC		-450	-352	-279	-225	-10	-8
	QCD		-72	-107	-121	-123	-10	-8
	Total		-288±59	-122±48	-27±40	29±33	24±2	20±1
$\eta_c(4S)$	LO			133	198	225	34	28
(4250)	RC			-279	-221	-178	-8	-6
	QCD			-41	-63	-73	-8	-7
	Total			-186±37	-86±30	-26±25	17±1	15±1
$\eta_{c2}(1D)$ (3796)	LO	4.0	6.4	7.3	7.3	7.0	0.71	0.58
$\eta_{c2}(2D)$ (4099)	LO		1.5	2.9	3.5	3.7	0.47	0.38

$\eta_c(1S)$



$\sigma[e^+e^- \to \eta_c\gamma]$(fb)
3000
2500
2000
1500
1000
500
6
8
10
$\sqrt{s}$ (GeV)
σ_{LO}
σ_{v2}
$\sigma_{\alpha s}$
$\sigma_{\alpha s, v2}$

$\eta_{c2}(nD)$



The predictions near the threshold may be not reliable

The NRQCD requires that the energy of photon at the center of the mass frame of e^+e^-

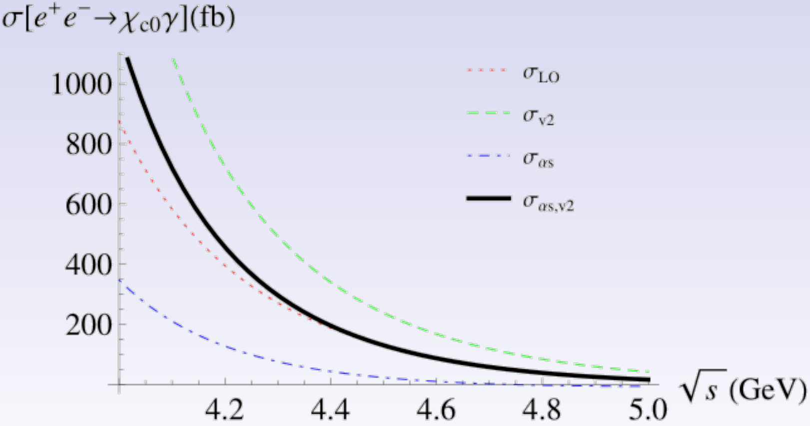
$$E_\gamma = \frac{s - M_H^2}{2\sqrt{s}} \sim \sqrt{s} - M_H + \mathcal{O}\left[(1 - M_H/\sqrt{s})^2\right] \quad (12)$$

be larger than $\Lambda_{QCD} \sim 300 \text{ MeV} \sim m_c v^2$. Although this process is a QED process, the prediction is not reliable and only a reference value if this requirement is not satisfied.

$\chi_{c0}(nP)$

$\sqrt{s}$(GeV)		4.25	4.50	4.75	5.00	10.6	11.2
$\chi_{c0}(1P)$	LO	328	132	53	21	1.81	1.6
(3415)	RC	268	107	48	22	-0.77	-0.63
	QCD	-228	-107	-52	-26	-0.38	-0.29
	Total	368±46	131±20	49±9	17±4	1.42±0.11	1.22±0.09
$\chi_{c0}(2P)$	LO	1991	665	271	119	1.30	1.18
(3918)	RC	3102	680	230	96	-0.64	-0.54
	QCD	-1013	-384	-177	-89	0.39	0.30
	Total	4080±426	962±102	324±38	127±17	1.04±0.10	0.94±0.08
$\chi_{c0}(3P)$	LO		1073	384	164	0.82	0.75
(4131)	RC		1600	391	140	-0.44	-0.38
	QCD		-551	-223	-107	0.29	0.23
	Total		2121±220	554±59	198±23	0.67±0.07	0.61±0.06

$\chi_{c0}(1P)$

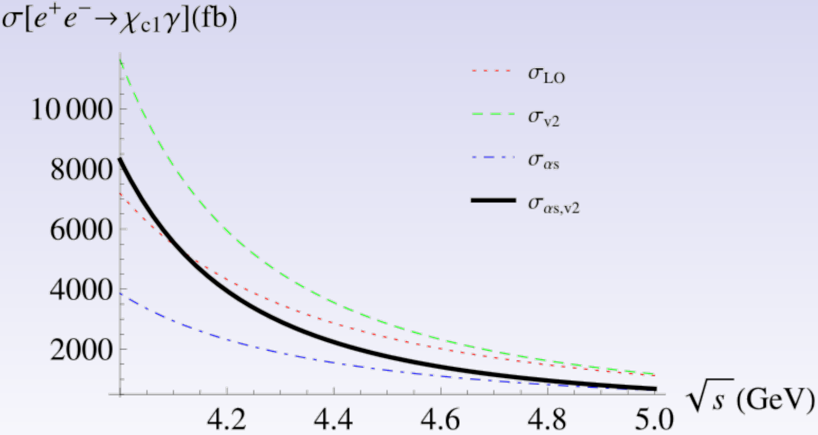


$\sigma[e^+e^- \to \chi_{c0}\gamma]$(fb)
1000
800
600
400
200
4.2
4.4
4.6
4.8
5.0
$\sqrt{s}$ (GeV)
σ_{LO}
σ_{v2}
$\sigma_{\alpha s}$
$\sigma_{\alpha s,v2}$

$\chi_{c1}(nP)$

$\sqrt{s}$(GeV)		4.25	4.50	4.75	5.00	10.6	11.2
$\chi_{c1}(1P)$	LO	3874	2392	1597	1124	23.5	18.5
(3511)	RC	1296	459	168	52	-4.8	-3.8
	QCD	-1791	-1091	-715	-492	-6.5	-4.9
	Total	3379±288	1760±154	1051±96	685±65	12±1	10±1
$\chi_{c1}(2P)$	LO	8854	4244	2495	1624	25.7	20.0
(3901)	RC	9585	2297	789	312	-4.9	-3.9
	QCD	-4041	-1967	-1152	-741	-7.7	-5.7
	Total	14397±1357	4573±394	2131±182	1195±105	13±1	10±1
$\chi_{c1}(3P)$	LO		1073	384	164	0.82	0.75
(4178)	RC		1600	391	140	-0.44	-0.38
	QCD		-551	-223	-107	0.29	0.23
	Total		2121±220	554±59	198±23	0.7±0.1	0.6±0.1

$\chi_{c1}(1P)$

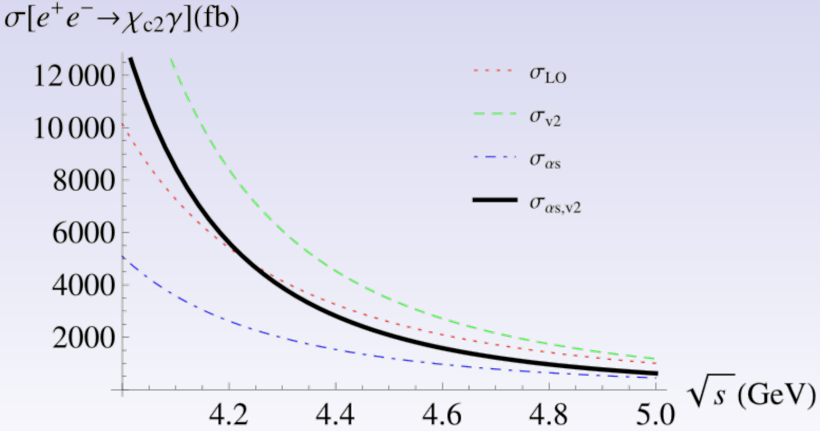


$\sigma[e^+e^- \to \chi_{c1}\gamma]$(fb)
10 000
8000
6000
4000
2000
4.2
4.4
4.6
4.8
5.0
$\sqrt{s}$ (GeV)
σ_{LO}
σ_{v2}
$\sigma_{\alpha s}$
$\sigma_{\alpha s,v2}$

$\chi_{c2}(nP)$

$\sqrt{s}$(GeV)		4.25	4.50	4.75	5.00	10.6	11.2
$\chi_{c2}(1P)$	LO	4724	2590	1562	1004	9.66	7.37
(3556)	RC	2385	880	376	173	-1.16	-0.93
	QCD	-2455	-1384	-851	-557	-6.27	-4.82
	Total	4655±446	2087±213	1086±121	621±76	2±1	2±1
$\chi_{c2}(2P)$	LO	13419	5581	2931	1927	11.29	8.53
(3927)	RC	17835	3965	1355	565	-1.22	-0.99
	QCD	-6423	-2822	-1533	-926	-7.25	-5.52
	Total	24862±2472	6723±635	2754±267	1368±141	3±1	2±1
$\chi_{c2}(3P)$	LO		8938	3607	1886	8.55	6.40
(4208)	RC		14212	2949	995	-0.83	-0.68
	QCD		-4210	-1803	-977	-5.43	-4.10
	Total		18941±1933	4753±451	1904±182	2±1	2±1

$\chi_{c2}(1P)$



$\sigma[e^+e^- \to \chi_{c2}\gamma]$(fb)
12 000
10 000
8000
6000
4000
2000
4.2
4.4
4.6
4.8
5.0
$\sqrt{s}$ (GeV)
σ_{LO}
σ_{v2}
$\sigma_{\alpha s}$
$\sigma_{\alpha s,v2}$

Numerical results for XYZ

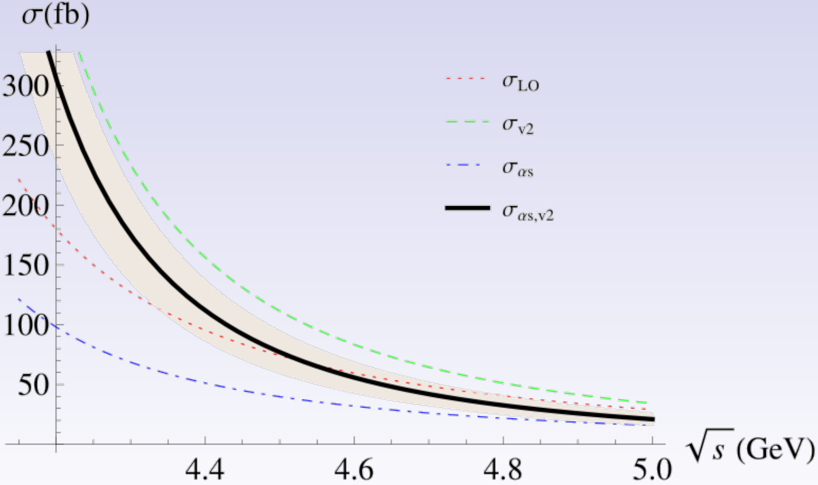
Numerical results for XYZ :$X(3872)$

In the sight of the mixture state of $\chi_{c1}(2P)$ and $D^0\bar{D}^{\star 0}$ molecule, the cross sections of $X(3872)$ production can be expressed as following C.Meng, arXiv:1304.6710

$$d\sigma[X(3872) \to J/\psi\pi^+\pi^-] = d\sigma[\chi_{c1}(2P)]\times k, \qquad (13)$$

where $k = Z_{c\bar{c}}^{X(3875)} \times Br[X(3872) \to J/\psi\pi^+\pi^-]$.
$Br[X(3872) \to J/\psi\pi^+\pi^-]$ is the branching fraction for $X(3872)$ decay to $J/\psi\pi^+\pi^-$. $Z_{c\bar{c}}^{X(3875)}$ is the possibility of the $\chi_{c1}(2P)$ component in $X(3872)$. And $k = 0.018 \pm 0.04$.

$X(3872)$



$X(3872)$-continuum contributions

$\sqrt{s}$(GeV)	4.15	4.2	4.25	4.3	4.35	4.45	4.55
LO	221±49	180±40	150±33	127±28	110±24	84±19	66±15
RC	310±69	208±46	146±32	106±24	80±18	47±10	30±7
QCD	-100±22	-82±18	-69±15	-59±13	-51±11	-39±9	-31±7
Total	431±96	306±68	227±51	175±39	138±31	92±20	65±14

$\sqrt{s}$(GeV)	NRQCD prediction for continue	BESIII [46, 47]
4.009		<130 at 90% CL.
4.160	401 ± 89	
4.230	255 ± 57	$320 \pm 150 \pm 20$
4.260	215 ± 48	$350 \pm 120 \pm 20$
4.360	133 ± 29	<130 at 90% CL.
4.415	105 ± 23	
4.660	47 ± 10	

$$E_\gamma[4.009] = 134\text{MeV}, \;\; E_\gamma[4.160] = 270\text{MeV}. \tag{14}$$

$X(3872)$-resonance contributions

The resonance contributions can be estimated as:

$$\sigma_{Res}[s] = \frac{12\pi\Gamma[Res \to e^+e^-]\Gamma[Res \to \gamma X]}{(s-M^2)^2 + (M\Gamma_{tot}[Res])^2}. \quad (15)$$

With $X(3872)$ considered as $2P$ states, the largest decay widths are $\psi(4040)$ and $\psi(4160)$, which are considered as the mixing of $\psi(3S)$ and $\psi(2D)$.

The parameters for $\psi(4040)$ and $\psi(4160)$

$$\Gamma[\psi(4040) \to e^+e^-] = 0.87 \text{ keV}, \ \Gamma[\psi(4040) \to \gamma X] = 40 \text{ keV}$$
$$\Gamma[\psi(4160) \to e^+e^-] = 0.83 \text{ keV}, \ \Gamma[\psi(4160) \to \gamma X] = 140 \text{ keV}$$
$$\Gamma_{tot}[\psi(4040)] = 80 \text{ MeV}, \ \Gamma_{tot}[\psi(4160)] = 103 \text{ MeV}$$

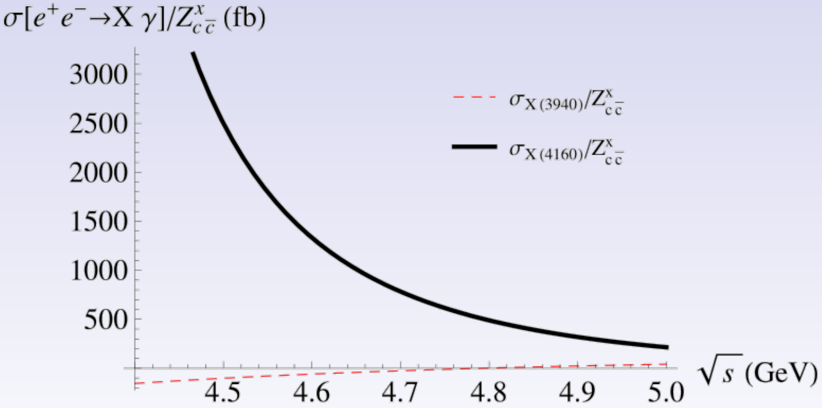
Hence, we can determine the contributions from these resonances

$$(\sigma_{\psi(4040)}[4.23] + \sigma_{\psi(4160)}[4.23]) \times k = (62 \pm 14)\text{fb}$$
$$(\sigma_{\psi(4040)}[4.26] + \sigma_{\psi(4160)}[4.26]) \times k = (37 \pm 8)\text{fb}. \quad (16)$$

Numerical results for XYZ :$X(3940)$ and $X(4160)$

- $X(3940)$ and $X(4160)$ are found in $e^+e^- \to J/\psi\,(D\bar{D})$ at B factories (PRL100,202001) .
- η_c and χ_{c0} are recoiled with J/ψ, but χ_{c1} and χ_{c2} are missed (PRL100,202001) . The theoretical predictions are consistent with the experimental data (hep-ph/0211181, PRD77,014002; PRD84,034022; JHEP02(2013)089) .
- The mass of $\eta_c(3S)$ and $\chi_{c0}(3P)$ are predicted as 3994 MeV and 4130 MeV, respectively (PRD79,094004) .
- So there should be large $\eta_c(3S)$ or $\chi_{c0}(3P)$ component in $X(3940)$ or $X(4160)$ V.V.Braguta,PRD74,094004,2006; K.T.Chao,PLB661,348,2008 .

$X(3940)$ or $X(4160)$



Numerical results for XYZ :$X(4350)$

$X(4350)$ are found in $\gamma\gamma \to H \to \phi J/\psi$ at B factories . J^{PC} is 0^{++} or 2^{++}. PRL104,112004,2010

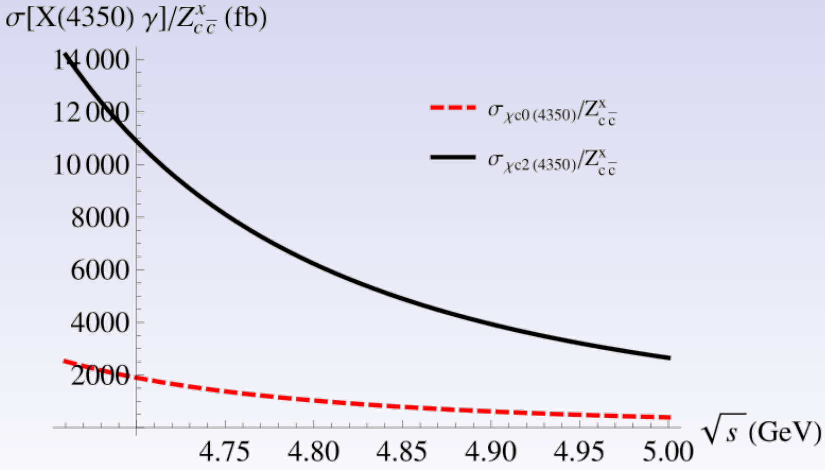
The mass of $\chi_{c2}(3P)$ is 4208 MeV PRD79,094004 .

Ignore more detail of the mass, we considered it as $\chi_{c0}(3P)$ or $\chi_{c2}(3P)$, the wave function at origin are estimated as

$$R' = R'_{3P} \;\; = \;\; (R'_{1P} + R'_{2P})/2 = 0.159\text{GeV}^5, \qquad (17)$$

So there should be large $\chi_{c0}(3P)$ or $\chi_{c2}(3P)$ component in $X(4350)$ X.Liu,PRL104,122001,2009; Z.G.Wang,PLB690,403,2010 .

$X(4350)$



σ[X(4350) γ]/Z^x_{c c̄} (fb)
14 000
12 000
10 000
8000
6000
4000
2000
σ_{χc0 (4350)}/Z^x_{c c̄}
σ_{χc2 (4350)}/Z^x_{c c̄}
4.75
4.80
4.85
4.90
4.95
5.00
√s (GeV)

Summary and discussion

Summary and discussion

1. We can estimate the possible event number at BESIII and Belle. The possible event number is

$$N = \sigma[e^+e^- \to \gamma + c\bar{c}[n]] \times Z^H_{c\bar{c}} \times Br \times \mathcal{L} \times \epsilon, \qquad (18)$$

2. where ϵ is the efficiency of detectors are selected as 20%, Br is the branch ratio of H to the decay mode, $\mathcal{L}$ is the luminosity.
3. The integrated luminosity is $1.0fb^{-1}@4.23$ GeV, $1.0fb^{-1}@4.26$ GeV, $0.5fb^{-1}@4.66$ GeV, and $1ab^{-1}@10.6$ GeV.
4. The decay mode of $nKm\pi$ means $D\bar{D}$ decay and the branch ratio is estimated as 1%.

Possible events at BESIII and Belle

H	Decay	Br	$Z^H_{c\bar{c}}$	4.23	4.26	4.66	10.6
η_c	$K\bar{K}\pi$	7.2%	1	9	9	5	1012
χ_{c0}	$2\pi^+2\pi^-$	2.2%	1	2	2		6
χ_{c1}	$\gamma l^+l^-(\gamma J/\psi)$	4.1%	1	29	27	5	101
χ_{c2}	$\gamma l^+l^-(\gamma J/\psi)$	2.3%	1	23	20	3	10
$\eta_{c2}(1D)$	$\gamma\gamma K\bar{K}\pi$	1.5%	1				2
$\eta_c(2S)$	$K\bar{K}\pi$	1.9%	1				123
$X(3872)(\chi_{c1}(2P))$	$\pi^+\pi^-l^+l^-(\pi^+\pi^-J/\psi)$	0.6%	0.36	6	5	1	6
$X(3915)(\chi_{c0}(2P))$	$\pi^+\pi^-\pi^0l^+l^-(\omega J/\psi)$	1%	1	9	8		2
$Z(3930)(\chi_{c2}(2P))$	$nKm\pi(D\bar{D})$	1%	1	57	46	4	6
$X(3940)(\eta_c(3S))$	$nKm\pi(D\bar{D})$	1%	1				48

Summary

1. We study the production of $C = +$ charmonium states H in $e^+e^- \to \gamma\ +\ H$ at BESIII with $H = \eta_c(nS)$ (n=1,2,3,4), $\chi_{cJ}(nP)$ (n=1,2,3), and ${}^1D_2(nD)$ (n=1,2).
2. The radiative and relativistic corrections are calculated to next to leading order for S and P wave states.
3. We then argue that search for the $C = +$ XYZ states $X(3872)$, $X(4160)$, $X(3940)$ and $X(4350)$ in $e^+e^- \to \gamma\ +\ H$ at BESIII and Belle may be helpful to clarify the nature of these states.

Thanks!