Lattice Study on Y(4260) and Glueballs

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Outline

I. Introduction

II. Hybrid-like vector charmonium relevant to Y(4260)

III. Glueballs in J/psi radiative decays

1. Lattice predictions
2. Phenomenological implication

I. Introduction

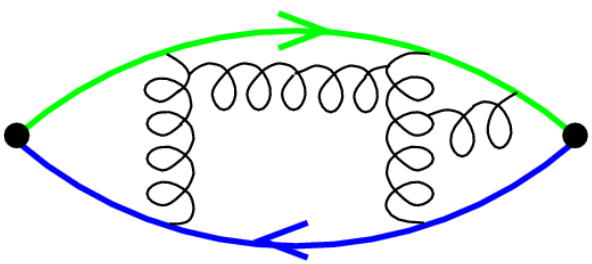
The lattice formulation of QCD---Lattice QCD

$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, e^{-S}$$

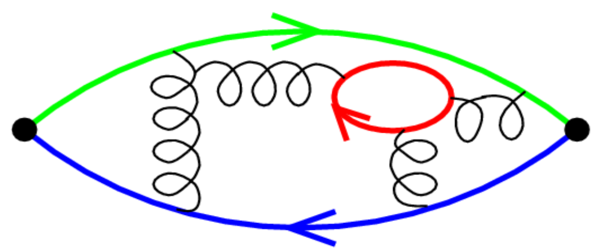
$$S = S_{gauge} + S_{quarks} = \int d^4x \left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) - \sum_i \log(\mathrm{Det}\, M_i)$$

$$Z = \int \mathcal{D}A_\mu \, \det M \; e^{\int d^4x \, \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)}.$$

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \, \mathcal{O} \, e^{-S} \; .$$



(A) Quenched QCD: quark loops neglected



(B) Full QCD

The disadvantage and advantage of the Quenched approximation

Disadvantage:

- not a unitary (physical) theory.
- the systematical uncertainties due to the neglect of sea quarks are not under control

Advantages:

- no sea quarks, so no mixing with q-barq mesons
- pure gauge glueballs are well-defined
- numerically, very large statistics can be obtained for the calculations relevant to glueballs

Anyway,

preliminary full-QCD lattice study shows that the s ystematical uncertainty got by QA may not be that important.

A). Lattice and parameters

Anisotropic lattices

β	ξ	a_s(fm)	La_s(fm)	$L^3 \times T$	N_{conf}
2.4	5	0.222(2)	1.78	$8^3 \times 96$	5000
2.8	5	0.138(1)	1.66	$12^3 \times 144$	5000

The finite volume effects for glueballs have been seriously studied and found to be small (Y.Chen et al, PRD73(2006)).

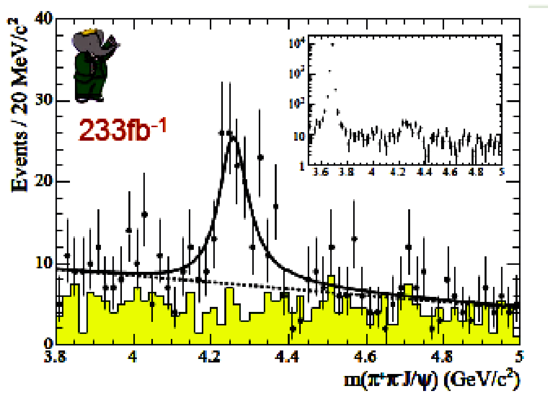
B). Actions

Tadpole improved Symanzik's gauge action

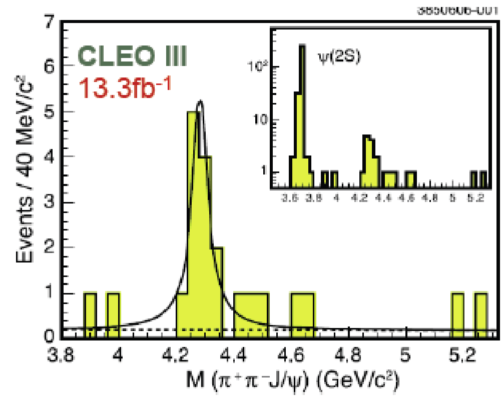
Tadpole improved Clover's fermion action

II. 1- exotic charmonium and X(4260)

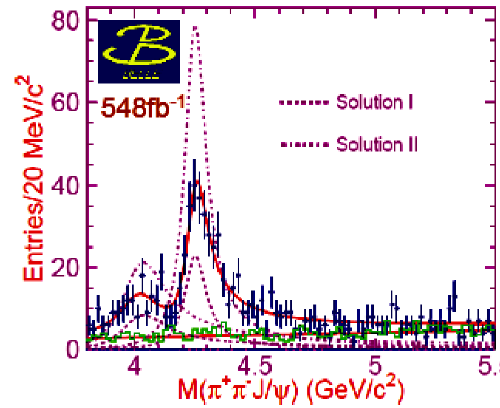
I).Experimental status of Y(4260)



BaBar,PRL95,142001(05)



CLEO-III,PRD74, 091104(R)(2006)



Belle,PRL99,182004(07)

$$e^+e^- \to \gamma_{ISR} X$$
$$X \to J/\psi\pi^+\pi^-$$
$$J/\psi \to l^+l^-$$

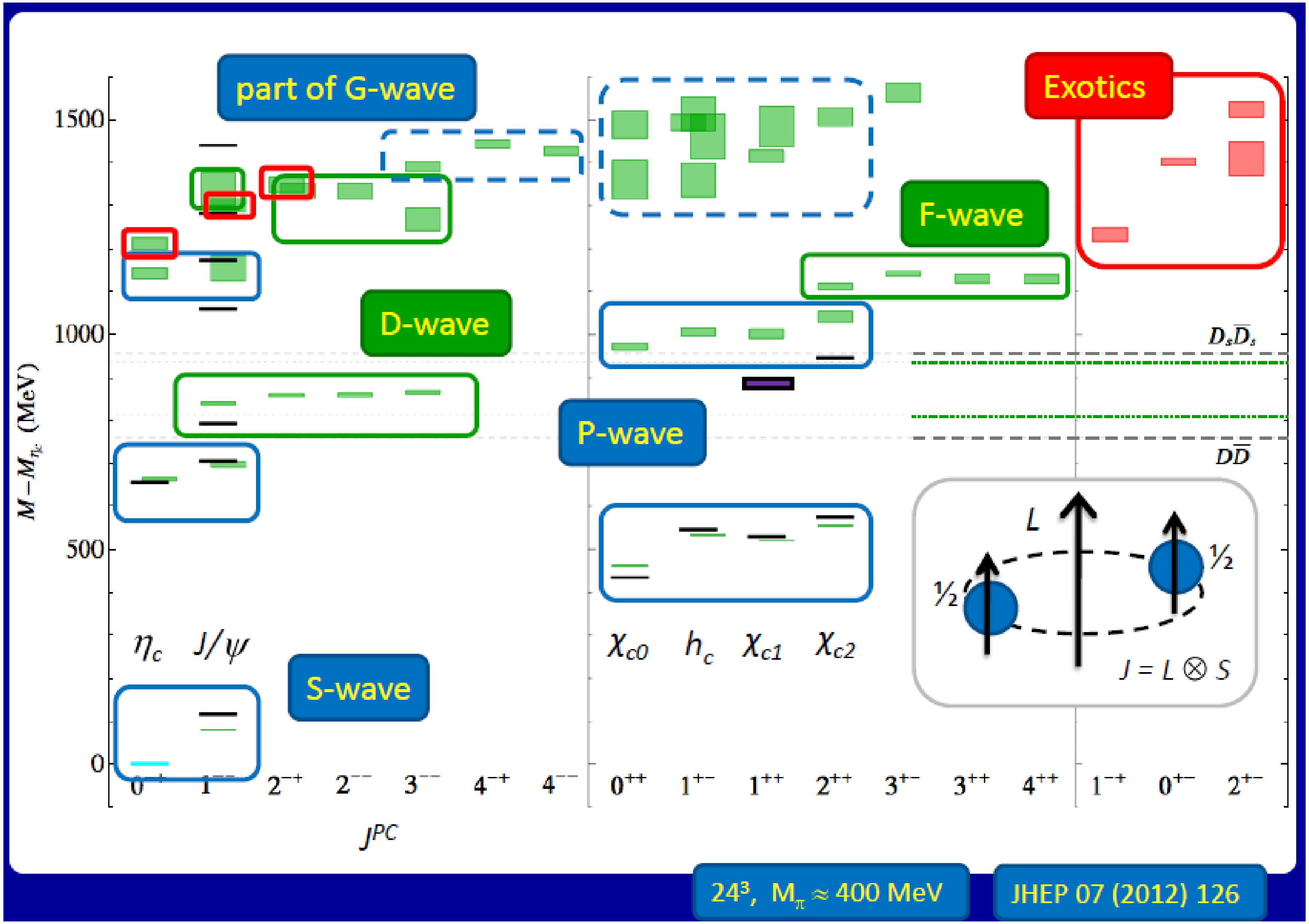
1. Observed in the initial state radiation process
2. The resonance parameter (PDG2012)

$$M_X = 4263(8)MeV$$
$$\Gamma_X = 95(14)MeV$$

3. The leptonic decay width

$$\Gamma(Y(4260)\to e^+e^-)\Gamma(Y(4260)\to J/\psi\pi^+\pi^-)/\Gamma_{tot} = 5.8eV$$

II) Latest charmonium spectrum from lattice QCD



III) Our strategy for the lattice calculation

1. 1^{--} hybrid-like interpolation field operator

$$\overline{\psi}^a \gamma_5 \psi^b B_i^{ab}$$

Nonrelativistic decomposition

$$q = e^{\frac{\gamma\cdot D}{2m}}\begin{pmatrix}\psi\\ \chi\end{pmatrix} = \left[1+\frac{\gamma\cdot D}{2m}+\frac{\gamma\cdot\vec{D}\,\gamma\cdot D}{8m^2}O(1/m^3)\right]\begin{pmatrix}\psi\\ \chi\end{pmatrix}$$

$$= \begin{pmatrix}\psi\\ \chi\end{pmatrix} + \frac{i}{2m}\begin{pmatrix}-\sigma\cdot\vec{D}\chi\\ \sigma\cdot\vec{D}\psi\end{pmatrix} + \frac{(\vec{D}^2+\sigma\cdot B)}{8m^2}\begin{pmatrix}\psi\\ \chi\end{pmatrix} + O(1/m^3),$$

$$\bar{q} = \begin{pmatrix}\psi^\dagger & -\chi^\dagger\end{pmatrix} e^{-\frac{\gamma\cdot\overleftarrow{D}}{2m}} = \begin{pmatrix}\psi^\dagger & -\chi^\dagger\end{pmatrix} + \frac{i}{2m}\begin{pmatrix}\chi^\dagger\sigma\cdot\overleftarrow{D}^\dagger & \psi^\dagger\sigma\cdot\overleftarrow{D}^\dagger\end{pmatrix}$$

$$+\frac{(\overleftarrow{D}^2+\sigma\cdot B)}{8m^2}\begin{pmatrix}\psi^\dagger & -\chi^\dagger\end{pmatrix} + O(1/m^3),$$

0^{-+}	$\overline{\psi}\gamma_5\psi$	$\chi^+\varphi$
1^{--}	$\overline{\psi}\gamma_i\psi$	$\chi^+\sigma_i\varphi$
$1^{--}{}_H$	$\overline{\psi}^a\gamma_5\psi^b B_i^{ab}$	$\chi^{a+}\varphi^b B_i^{ab}$

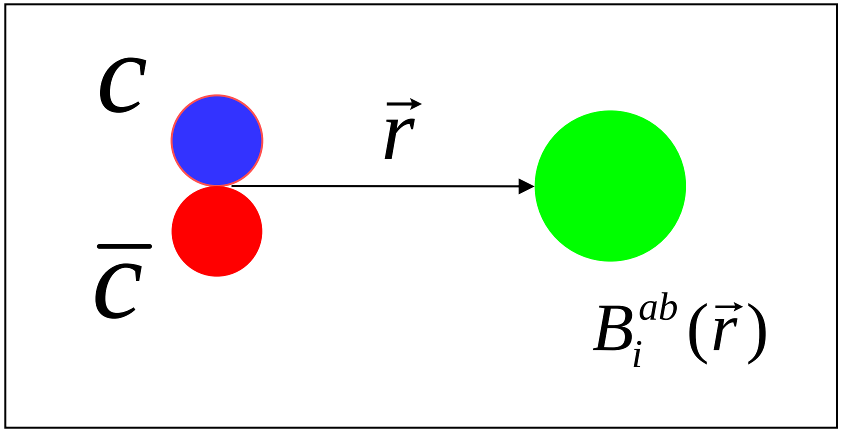
$$O_i^{(H)} \equiv \bar{c}^a\gamma_5 c^b B_i^{ab} \to \chi^{a\dagger}\phi^b B_i^{ab} + O(\frac{1}{m_c}),$$ → c-cbar spin singlet

$$O_i^{(M)} \equiv \bar{c}^a\gamma_i c^a \to \chi^{a\dagger}\sigma_i\phi^a + O(\frac{1}{m_c}).$$ → c-cbar spin triplet

2. Spatially extended interpolation field operator for the vector charmonium-like state

In the Coulomb gauge,

$$O(\vec{r}) = (\bar{c}^a \gamma_5 c^b)(0) B_i^{ab}(\vec{r})$$



This is equivalent to giave a c-cbar center of mass motion, which describes the recoil of the c-cbar against additional degrees of freedom.

Intuitively, the coupling of this kind of operators to conventional vector charmonia can be suppressed from two aspects:

a) spin states of the c-cbar (spin flipping is suppressed by the heavy quark mass.

b) center-of-mass motion (to the leading order of NR, there is no cneter-of-mass motions for conventional charmonia.)

3. The two-point functions calculated on the lattice

We introduce the following wall-source operator

$$O_i^{(W)}(\tau) = \sum_{\mathbf{y},\mathbf{z}} \bar{c}^a(\mathbf{y},\tau)\gamma_5 B_i^{ab}(\mathbf{z},\tau)c^b(\mathbf{z},\tau).$$

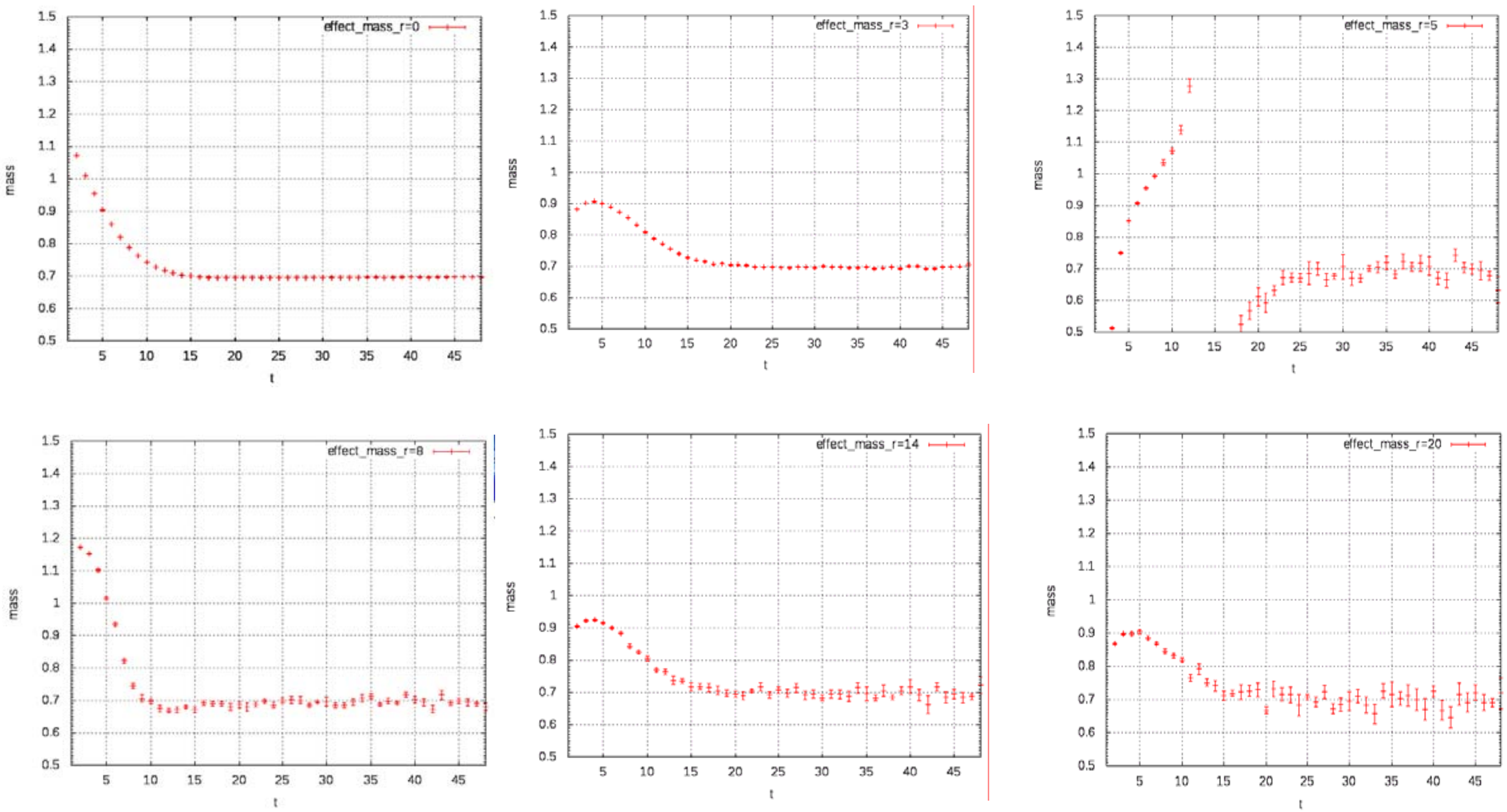
and calculate the following two-point functions

$$C(r,t) = \left\langle O(\vec{r},t) O^{(w)^+}(0) \right\rangle = \sum_i W_i(r) e^{-m_i t}$$

where Wi(r) is the spectral weight of the i-th state, which varies with respect to r, and thereafter can be interpreted as the Coulomb BS wave function versus the distance between the c-cbar and B field.

Effective mass plots for different r

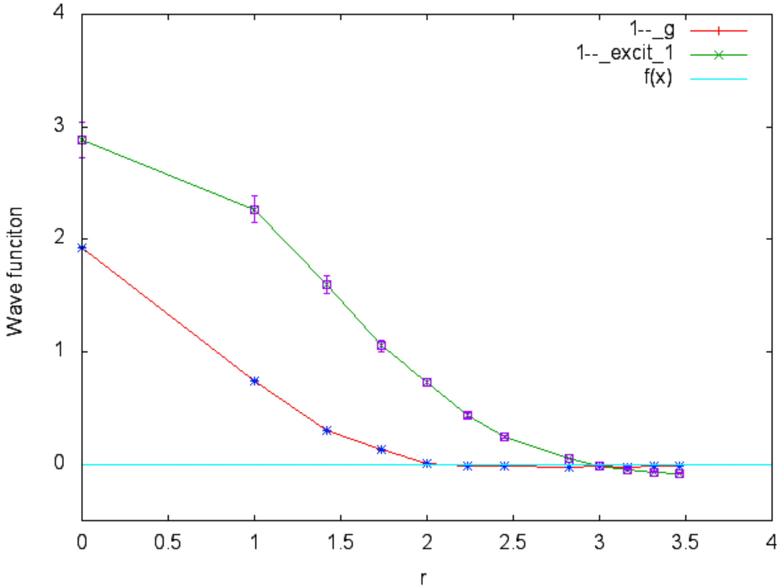
$$m_{eff}(r,t) = \ln \frac{C(r,t)}{C(r,t+1)}$$



The two-point functions with different r can be fit jointly with the same spectrum.

$$C(r,t)=\left\langle O(\vec{r},t)O^{(w)^{+}}(0)\right\rangle=\sum_i W_i(r)e^{-m_i t}$$

2-mass term fit

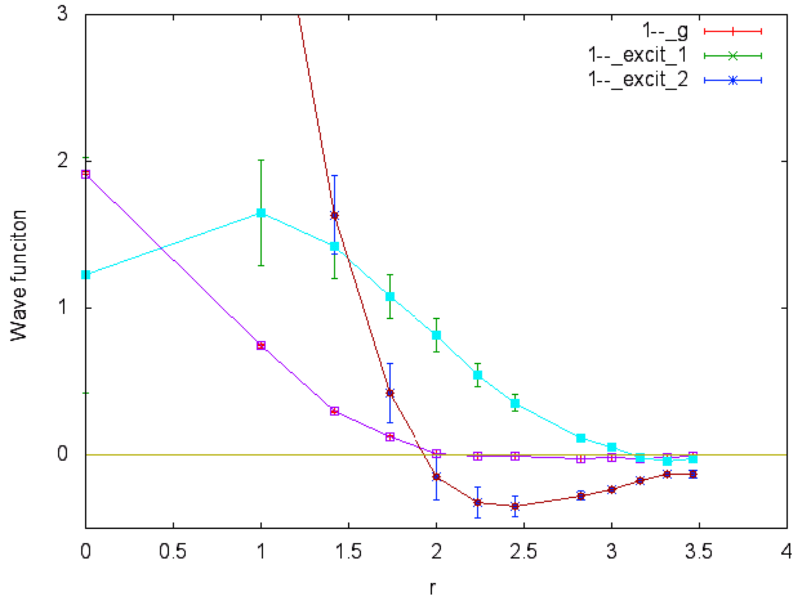


$$m_1=3.076(4)GeV$$

$$m_2=4.275(18)GeV$$

X(4260)???

3-mass term fit



$$m_1=3.072(9)GeV$$

$$m_2=4.311(44)GeV$$

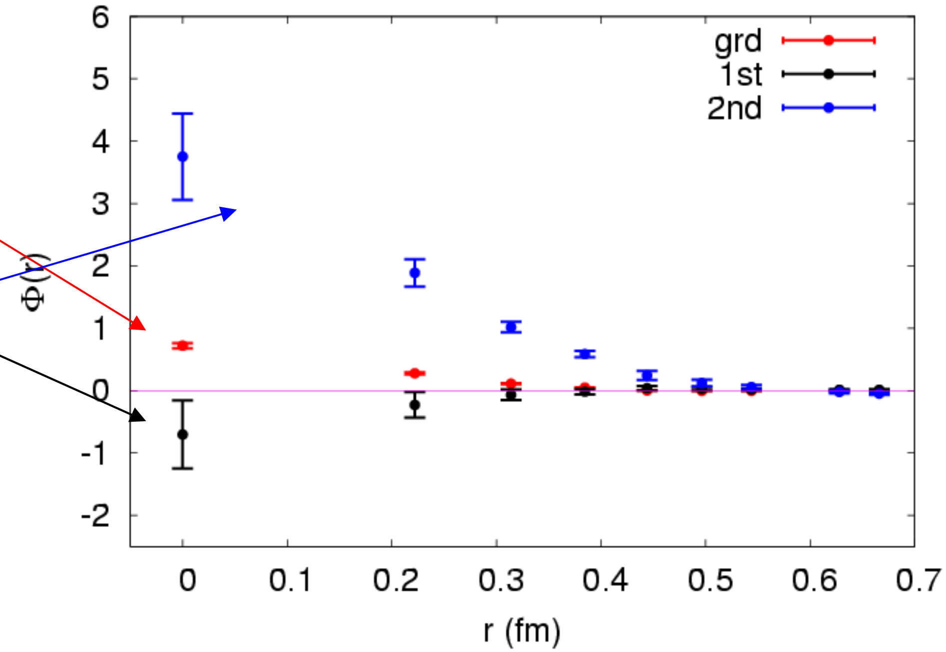
$$m_3=4.932(88)GeV$$

$$grd \Rightarrow J/\psi;$$

$$1st \Rightarrow \psi';$$

$$2nd \Rightarrow 1^{--}_{hyb.}\ ???$$

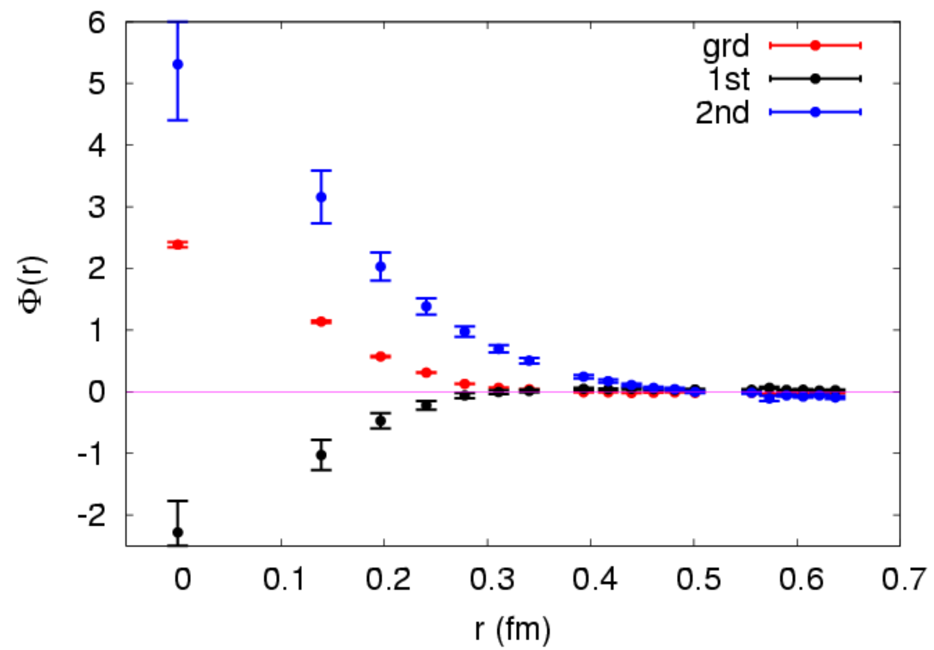
$$C(r,t) = \left\langle O(\vec{r},t) O^{(w)^+}(0) \right\rangle = \sum_i W_i(r) e^{-m_i t}$$



The major observation:

- The hybrid-like operators couples more strongly the 'exotic' state;

- The coupling to the conventional charmonia damps more rapidly as r increases.

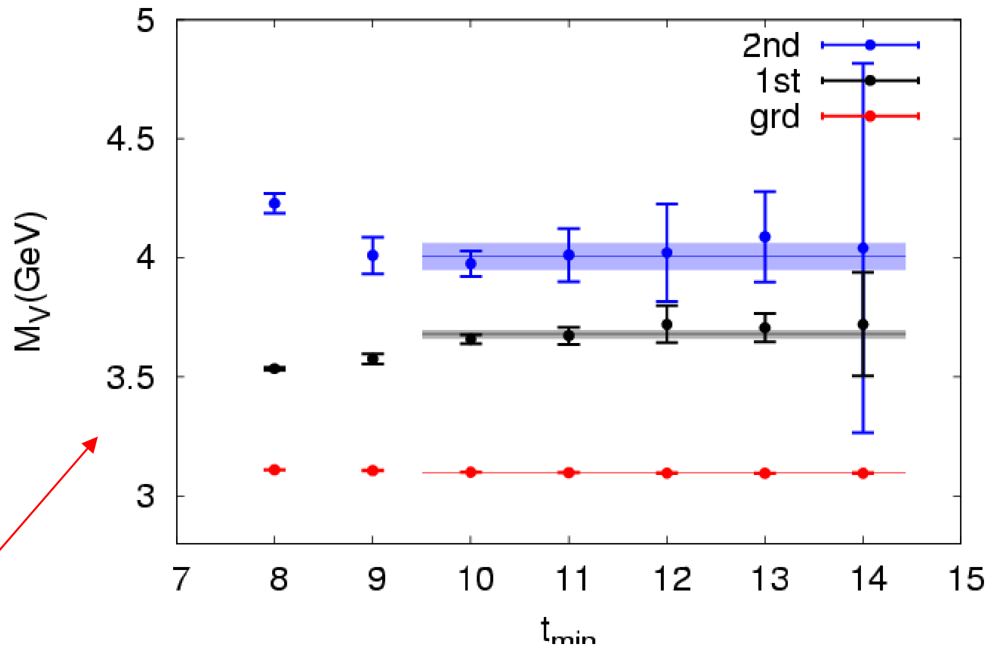


For comparison, we carried out a similar calculation with the conventional c-barc bilinear operators and get the reasonable masses for psi states.

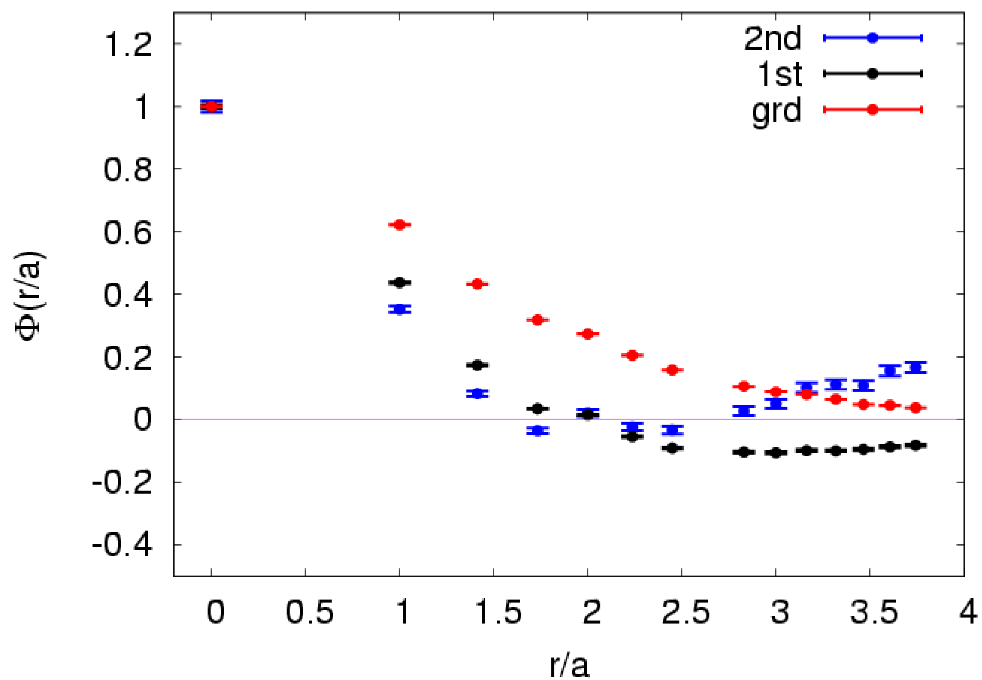
$$M_{grd} = 3.097(1)GeV$$

$$M_{1st} = 3.670(19)GeV$$

$$M_{2nd} = 4.008(43)GeV$$



The figure on the right shows the Coulomb BS wavefunctions for psi states.



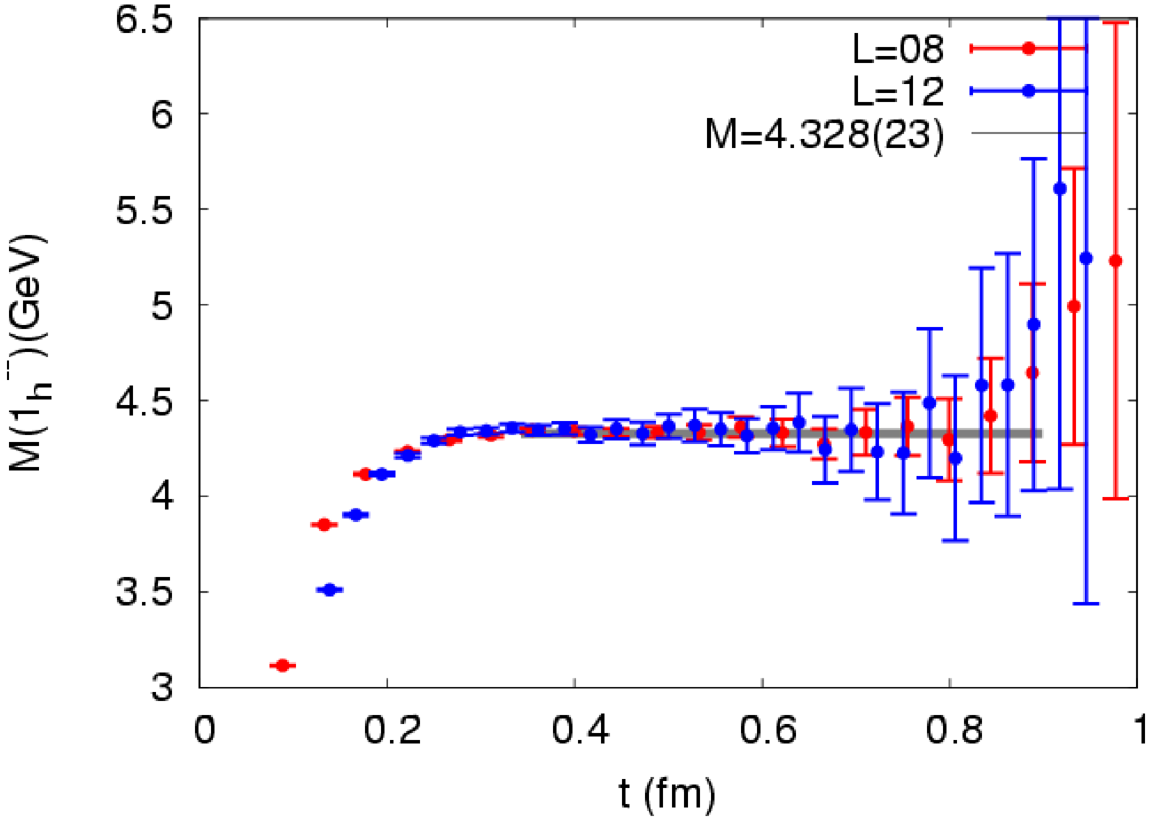
It is obviously seen that

1. The spectral weight of J/psi (and actually other conventional vector charmonia) damps more rapidly with the increment of r;
2. The small deviation of the J/psi mass from the original value 3.097 can be understood as the small contamination of higher conventional charmonia;
3. The different behaviors of the spectral weight can be applied to eliminate the conventional states. In other words, we can linearly combine them to obtain an optimal two-point function

$$C_{optimal}(t) = C(r_1, t) + \omega C(r_2, t)$$

which is dominated by the exotic vector charmonium. The optimal parameter omega can be determined numerically by requesting the plateau lasting as long as enough.

Linearly combine the correlation functions with different r we can eliminate the conventional vector charmonium and get a relatively clean signal of the exotic vector meson.



IV) The leptonic decay width of the exotic vector charnomium

1. The leptonic decay width of this exotic vector chamonium is an important quantity, which can shed light on the nature of Y(4260).

$$\Gamma(Y(4260)\rightarrow e^+e^-)\Gamma(Y(4260)\rightarrow J/\psi\pi^+\pi^-)/\Gamma_{tot}=5.8eV$$

2. The leptonic decay constant of the exotic state can be calculated directly in lattice QCD.

The decay constant of a vector meson is defined as

$$\langle 0|\bar{q}\gamma_\mu q|V(\vec{p},r)\rangle = m_V f_V \varepsilon_\mu(\vec{p},r)$$

where the matrix element on the left can be derived by calculate the two point function

$$\sum_{\vec{x}}\langle 0|\bar{q}\gamma_\mu q(\vec{x},t)O^{(w)}(0)|0\rangle = \sum_{i,r}\frac{1}{2M_i}\langle 0|\bar{q}\gamma_\mu q|V_i,r\rangle\langle V_i,r|O^{(w)}|0\rangle e^{-M_i t}$$

β	$M(J/\psi)$(GeV)	$f_{J/\psi}$ (MeV)	$M(Y)$(GeV)	f_Y (MeV)
2.4	3.076(4)	428(7)	4.43(7)	32(20)
2.8	3.082(1)	378(6)	4.40(7)	31(11)
Exp.	3.097	407(5)	$\cdots$	$\cdots$

Using the formula

$$\Gamma(V_{c\bar{c}} \to e^+e^-) = \frac{16\pi}{27}\alpha_{\rm QED}^2 \frac{f_V^2}{M_V}$$

One can predict the leptonic decay width of the exotic vector charmonium

$$\boxed{\Gamma(Y \to e^+e^-) \approx 25(20)\,{\rm eV}}$$

(*preliminary*)

To summarize,

1. Both quenched and unquenched studies on charmonium spectrum show that there may be an exotic vector charmonium state around M~4.3 GeV.
2. For the first time, the leptonic decay width of this state is predicted to be 23(20) eV, which is much smaller than that of the conventional vector charmonia.

3. If this state corresponds to X(4260) observed in experiments, one can estimate the branch ratio of of it decaying into pi+pi-J/psi

$$\Gamma(V \to e^+e^-) \propto f_V^2$$

$$\langle 0|\bar{q}\gamma_\mu q|V(\vec{p},r)\rangle = m_V f_V \varepsilon_\mu(\vec{p},r)$$

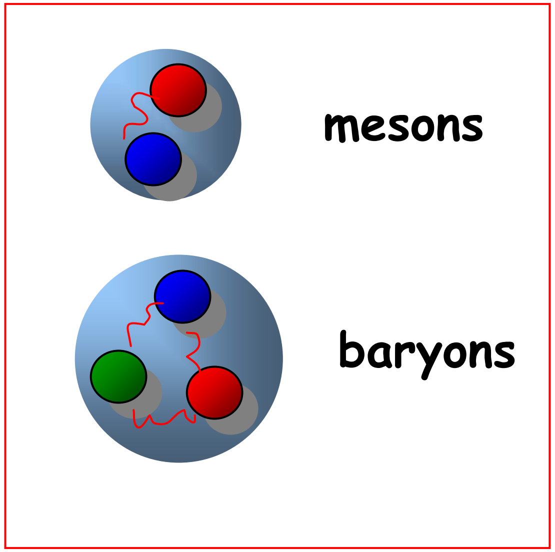
$$\Gamma(Y(4260) \to e^+e^-)\Gamma(Y(4260) \to J/\psi\pi^+\pi^-)/\Gamma_{tot} = 5.8eV$$

$$\sim 20-30\%$$

III. Glueballs in J/psi radiative decays

I). Motivation

- According to QCD, gluons can combined into exotic hadron states, say, glueballs, through their self-interaction.
- Glueballs are distinct from the conventional mesons which are described by the Quark Model.
- The existence or non-existence of glueballs has deep impact to our understanding of the essence of the strong interaction.
- The search for glueballs is a long standing task of high energy experiments.
- **However, the identification of glueballs requires the necessary information of the properties of glueballs from the theoretical study.**



Quark model

- Quenched LQCD predicts glueball spectrum
  - Lowest-lying glueballs have masses in the range 1~3GeV
  - **these are also supported by the preliminary full QCD calculation.**

- Experimentally, f0(1370), f0(1500), f0(1710), ksi(2230),etc., are glueball candidates, but decisive conclusion cannot be drawn.

- J/psi radiative decay can be the best hunting ground.

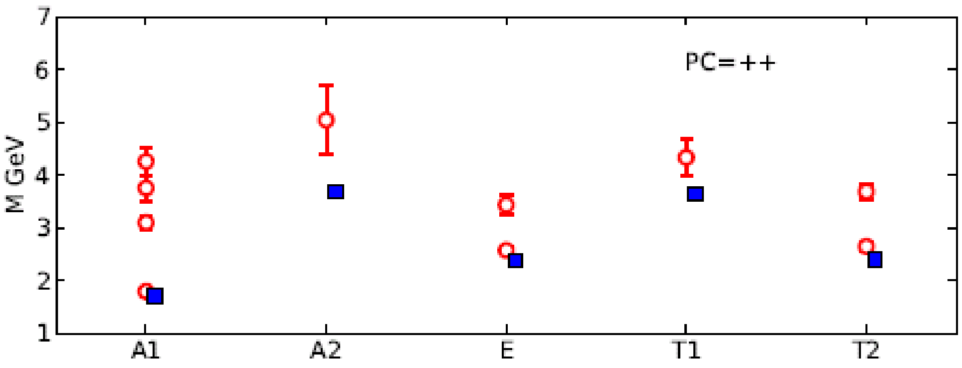
- BESIII in Beijing is producing 10^{10} J/psi events

- The direct calculation of the production rate of glueballs in J/psi radiative decays from lattice QCD

J^{PC}	$r_0 M_G$	M_G (MeV)
0^{++}	4.16(11)(4)	1710(50)(80)
2^{++}	5.83(5)(6)	2390(30)(120)
0^{-+}	6.25(6)(6)	2560(35)(120)
1^{+-}	7.27(4)(7)	2980(30)(140)
2^{-+}	7.42(7)(7)	3040(40)(150)
3^{+-}	8.79(3)(9)	3600(40)(170)
3^{++}	8.94(6)(9)	3670(50)(180)
1^{--}	9.34(4)(9)	3830(40)(190)
2^{--}	9.77(4)(10)	4010(45)(200)
3^{--}	10.25(4))(10)	4200(45)(200)
2^{+-}	10.32(7)(10)	4230(50)(200)
0^{+-}	11.66(7)(12)	4780(60)(230)

Y. Chen et al,
Phys. Rev. D 73, 014516 (2006)

- Latest results of glueball masses from 2+1 flavor dynamical lattice QCD study
[E.Gregory et al, JHEP 10 (2012) 170, arXiv:1208.1858(hep-lat)]



Open circles are full-QCD results, and the filled squares are from quenched lattice QCD studies

II). The direct calculation of the widths of J/psi to glueballs

- Radiative decay width:

$$\Gamma(i \to \gamma f) = \int d\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{M_i^2} \frac{1}{2J_i+1} \times \sum_{r_i, r_j, r_\gamma} \left| M_{r_i, r_j, r_\gamma} \right|^2,$$

- Transition amplitudes: $M_{r_i, r_f, r_\gamma} = \epsilon_\mu^*(\vec{q}, r_\gamma) \langle f(\vec{p}_f, r_f) | j_{\mathrm{em}}^\mu(0) | i(\vec{p}_i, r_i) \rangle$
- Multipole decomposition:

$$\langle f(\vec{p}_f, r_f) | j_{\mathrm{em}}^\mu(0) | i(\vec{p}_i, r_i) \rangle = \sum_k \alpha_k^\mu(p_i, p_f) F_k(Q^2).$$

- Decay width expressed in terms of the form factors

$$\Gamma(i \to \gamma f) \propto \sum_k F_k^2(0).$$

- So the major task is to calculate the matrix elements

III). Our results from lattice QCD calculation

In order to get fair signals of the three point functions, a large enough statistics is required.

- 5000 gauge configurations for each lattice
- Charm quark mass is set by the physical mass of J/psi
- On each configuration,

  N_t charm quark propagators are calculated with point sources on all the N_t time slices.

$$\Gamma^{(3)\mu i}(\vec{p}_f,\vec{q};t_f,t)=\frac{1}{T}\sum_{\tau=0}^{T-1}\sum_{\vec{y}}e^{+i\vec{q}\cdot\vec{y}}\left\langle O_G(\vec{p}_f,t_f+\tau)\,j^{\mu}(\vec{y},t+\tau)O_{J/\psi}^{i,+}(\tau)\right\rangle$$

Equivalent to hundreds of thousands measurements!!!

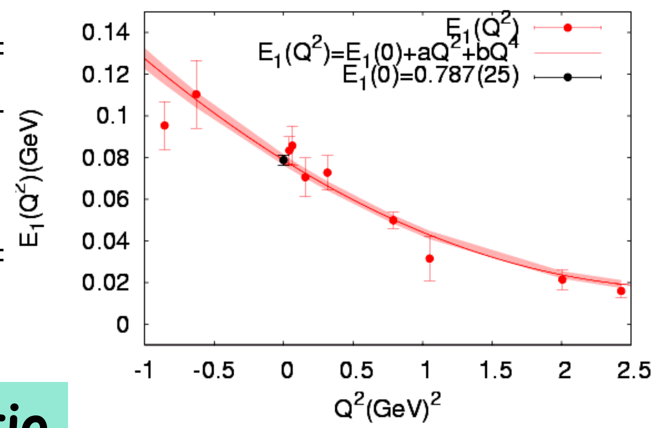
A). J/psi radiatively decaying to the scalar glueball

(L.Gui, et al. (CLQCD Collaboration), Phys. Rev. Lett. 110, 021601 (2013))

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left|E_1(0)\right|^2$$

Interpolated on-shell form factor E1(0) and its continuum limit

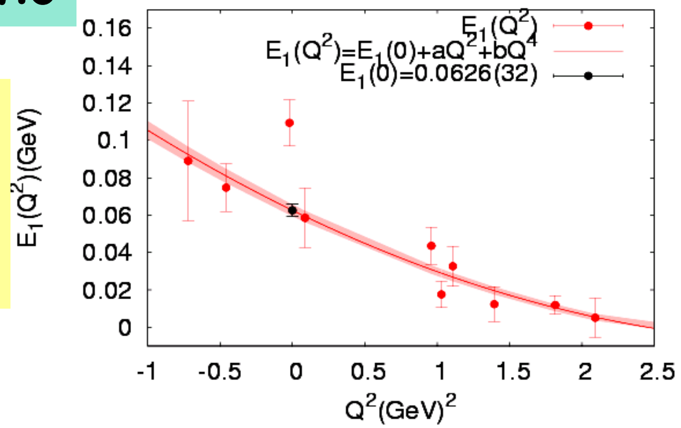
β	M_G(GeV)	$Z_V^{(s)}(a)$	$E_1(0,a)$ (GeV)	Γ(keV)
2.4	1.360(9)	1.39(2)	0.0787(25)	-
2.8	1.537(7)	1.11(1)	0.0626(32)	-
∞	1.710(90) [3]	-	0.0536(57)	0.35(8)



The predicted width and the branch ratio

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left|E_1(0)\right|^2 = 0.35(8)keV$$

$$\Gamma/\Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$



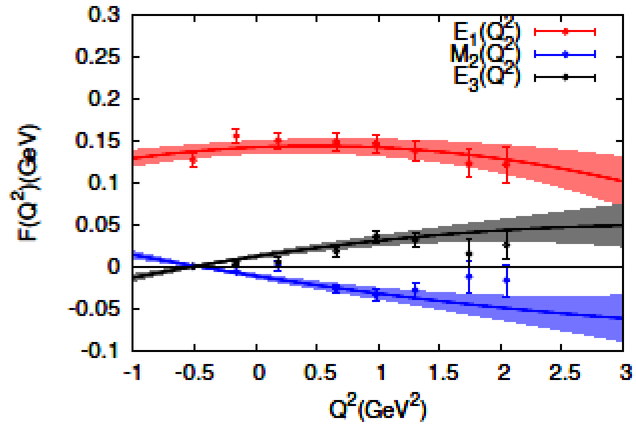
B). J/psi radiatively decaying to the tensor glueball

(Y.B. Yang ,et al .(CLQCD Collaboration), Phys. Rev. Lett. 111, 091601 (2013))

$$\Gamma(J/\psi \to \gamma G_{2^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left[|E_1(0)|^2 + |M_2(0)|^2 + |E_3(0)|^2\right]$$

- The form factors we obtained from the lattice QCD

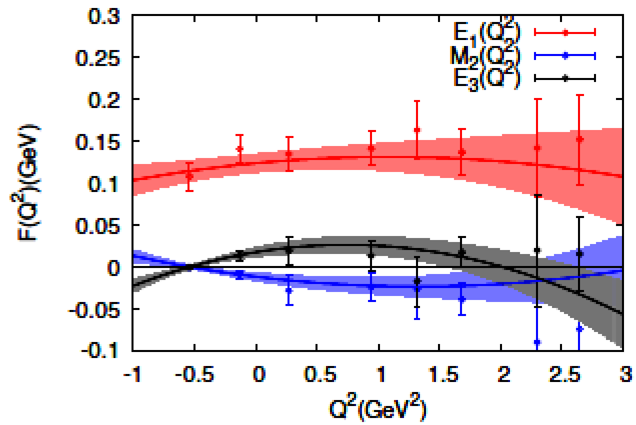
β	M_T (GeV)	E_1 (GeV)	M_2 (GeV)	E_3 (GeV)
2.4	2.360(20)	0.142(07)	−0.012(2)	0.012(2)
2.8	2.367(25)	0.125(10)	−0.011(4)	0.019(6)
∞	2.372(28)	0.114(12)	−0.011(5)	0.023(8)



- We also carry out a similar lattice study on the tensor glueball production rate in J/psi radiative decay.

$$\Gamma(J/\psi \to \gamma G_{2^+}) = 1.01(22)keV$$

$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1(2)\times 10^{-2}$$

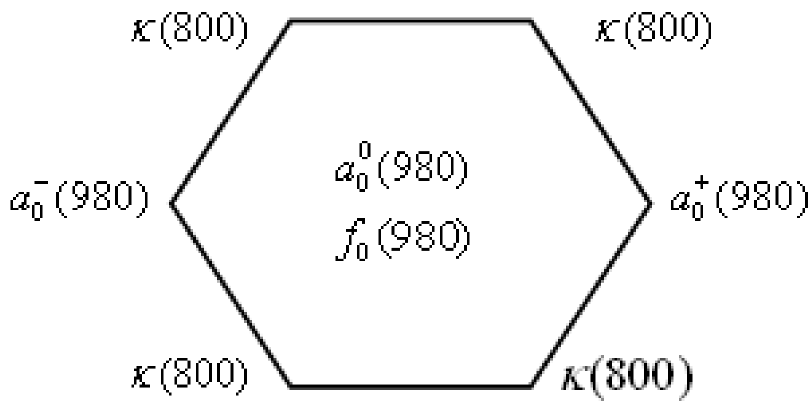


IV). Phenomenological Implications

A) The scalar mesons

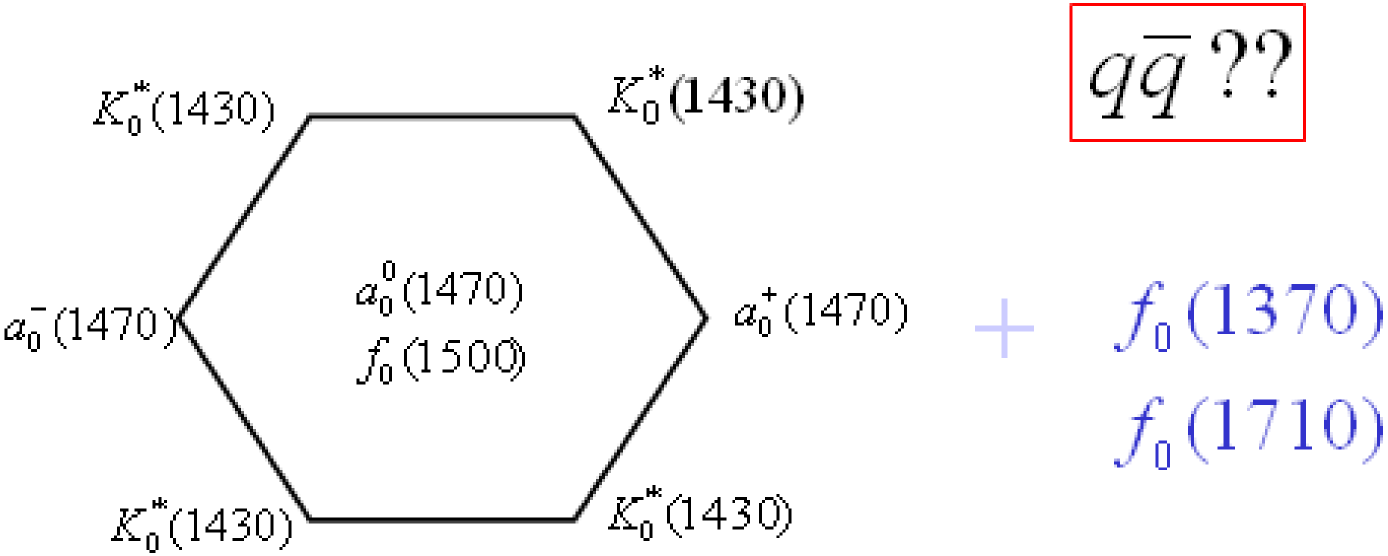
The scalars mesons with masses below 2GeV can be sorted into two multiplets.

Scalar Mesons ($J^{PC}=0^{++}$)



$q^2\bar{q}^2$??

$+ \ \sigma(600)$



These ten scalar mesons can be assigned as a q-barq meson nonet plus a possible scalar glueball which can be either one of the three isoscalars or an admixture of them. There are Many mixing models, the details are out of the scope of this talk.

References for glueball-meson mixing models:

1. C. Amsler and F.E. Close, Phys. Lett. B 353, 385 (1995).
2. C. Amsler and F.E. Close, Phys. Rev. D 53, 295 (1996).
3. F.E. Close and A. Kirk, Eur. Phys. J. C 21, 531 (2001).
4. F.E. Close and Q. Zhao, Phys. Rev. D 71, 094022 (2005)
5. D. Lee and D. Weingarten, Phys. Rev. D 61, 014015 (1999).
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7. F. Giacosa, Th. Gutsche, V.E. Lyubovitskij, and A. Faessler, Phys. Rev. D 72, 094006 (2005).
8. H.Y. Cheng, C.K. Chua, and K.F. Liu, Phys. Rev. D 74, 094005 (2006).

Lattice prediction:

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}\left|E_1(0)\right|^2 = 0.35(8)keV$$

$$\Gamma/\Gamma\_tot = 0.33(7)/93.2 = 3.8(9)\times 10^{-3}$$

Experimental results for J/psi radiatively decaying to scalars

C. Amsler et al. (Particle Data Group), Phy. Rev. D 86, 010001 (2012)

$J/\psi \to \gamma f_0(1500) \to \gamma\pi\pi \qquad (1.01 \pm 0.32)\times 10^{-4}$

$Br(f_0(1500) \to \pi\pi) = (34.9 \pm 2.3)\% \quad \Rightarrow Br(J/\psi \to \gamma f_0(1500)) = 2.9\times 10^{-4}$

$J/\psi \to \gamma f_0(1710) \to \gamma\eta\eta$	$\left(2.35^{+1.27}_{-0.77}\right)\times 10^{-4}$
$J/\psi \to \gamma f_0(1500) \to \gamma\eta\eta$	$\left(1.65^{+0.57}_{-1.50}\right)\times 10^{-4}$

BESIII results (PRD87, 092009)

$J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K}$	$\left(8.5^{+1.2}_{-0.9}\right)\times 10^{-4}$
$J/\psi \to \gamma f_0(1710) \to \gamma\pi\pi$	$(4.0 \pm 1.0)\times 10^{-4}$
$J/\psi \to \gamma f_0(1710) \to \gamma\omega\omega$	$(3.1 \pm 1.0)\times 10^{-4}$
$J/\psi \to \gamma f_0(1710)$	$> (1.5 \pm 0.3)\times 10^{-3}$

Using $Br(f_0(1710) \to K\underline{K}) = 0.36 \Rightarrow Br(J/\psi \to \gamma f_0(1710)) = 2.4\times 10^{-3}$

$Br(f_0(1710) \to \pi\pi) = 0.15 \Rightarrow Br(J/\psi \to \gamma f_0(1710)) = 2.7\times 10^{-3}$

B) The tensor glueball candidate

$$\Gamma(J/\psi \to \gamma G_{2^+}) = 1.01(22)keV$$

$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1(2)\times 10^{-2}$$

- Comparing to the experimental observations, **(PDG2012)**

$$f_J(2220): M = 2231(4)MeV, \Gamma = 23(8)MeV$$

Γ_{172}	$\gamma f_J(2220)$	$> 2.50 \times 10^{-3}$	CL=99.9%
Γ_{173}	$\gamma f_J(2220) \to \gamma\pi\pi$	$(8 \pm 4) \times 10^{-5}$	
Γ_{174}	$\gamma f_J(2220) \to \gamma K\overline{K}$	$(8.1 \pm 3.0) \times 10^{-5}$	
Γ_{175}	$\gamma f_J(2220) \to \gamma p\overline{p}$	$(1.5 \pm 0.8) \times 10^{-5}$	

$$B(f_J \to p\overline{p})B(f_J \to \pi\pi) < 1.8\times 10^{-4}$$

- However, f_J(2220) has not been confirmed by new experiments,
- On the other hand, the tensor glueball can be a broad object.
- With the largest J/psi events sample, our result can provide useful information for BESIII to identify the tensor glueball.

- **BESIII new results for** $J/\psi \to \gamma\eta\eta$

(M. Ablikim et al. (BES Collaboration),
Phys. Rev. D 87, 092009 (2013) (arXiv:1301.0053)

Resonance	Mass(MeV/c^2)	Width(MeV/c^2)	$\mathcal{B}(J/\psi \to \gamma X \to \gamma\eta\eta)$	Significance
$f_0(1500)$	1468^{+14+23}_{-15-74}	$136^{+41+28}_{-26-100}$	$(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$	8.2 σ
$f_0(1710)$	$1759\pm6^{+14}_{-25}$	$172\pm10^{+32}_{-16}$	$(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$	25.0 σ
$f_0(2100)$	$2081\pm13^{+24}_{-36}$	273^{+27+70}_{-24-23}	$(1.13^{+0.09+0.64}_{-0.10-0.28}) \times 10^{-4}$	13.9 σ
$f_2'(1525)$	$1513\pm5^{+4}_{-10}$	75^{+12+16}_{-10-8}	$(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$	11.0 σ
$f_2(1810)$	1822^{+29+66}_{-24-57}	$229^{+52+88}_{-42-155}$	$(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$	6.4 σ
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	$(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$	7.6 σ

In this analysis, the best fit favors the presence of f2(2340) with a mass of 2362(30)MeV and a width of 334(60) MeV. No evident narrow peak around 2.2GeV over the broad bump is observed in the eta-eta mass specturm.

- **Flavor-blindness of glueball decays**

$$\frac{1}{P.S.}\Gamma(G \to \pi\pi : K\bar{K} : \eta\eta : \eta\eta' : \eta'\eta') = 3:4:1:0:1$$

As such, one can estimate,

$$\Gamma(G \to \eta\eta)/\Gamma(G \to PP) \sim O(10\%)$$

which can be compared with that of f0(1710)

- **PP final states in the tensor glueball decays should be in D-wave,** considering the centrifugal barrier effects,

$$\Gamma(G \to M\bar{M}) = \eta\alpha \frac{k^{2L+1}}{m_G^{\ 2L}} = \frac{\eta\alpha}{m_G}\left(\frac{k}{m_G}\right)^{2L+1}$$

$$\frac{k}{m_G} = \frac{1}{2}\sqrt{1-\left(\frac{2m_M}{m_G}\right)^2} \sim 0.5-0.3$$

So the partial width of G to PP can be suppressed by an order of magnitude, so intutitively one has,

$$Br(G \to \eta\eta) \sim O(10\%)$$

- So the production rate of f_2(2340) in the J/psi radiative decay can be estimated as

$$Br(J/\psi \to \gamma f_2(2340) \sim 10^{-2}$$

with the new result of BES

$$Br(J/\psi \to \gamma f_2(2340) \to \gamma\eta\eta) = 5.6(2.3)\times 10^{-5}$$

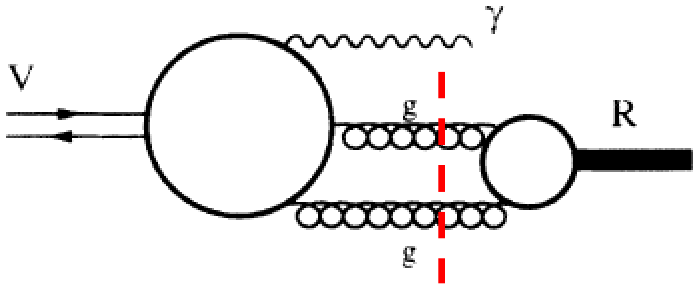
which is consistent with our prediction.

- As for the large width of f_2(2340), we will give a tentative phenomenological discussion.

C).Glueball widths—a phenomenological argument

(M.B. Cakir and G. R. Farrar, Phys. Rev. D 50, 3268 (1994);
F.E. Close, G.R. Farrar, and Z. Li, Phys. Rev. D 55, 5749(1997))

- For a vector heavy quarkonium V, its radiative decay into a resonnance R can be qualitatively treated in the perturbative QCD formalism.



- Intuitively, the dominant contribution can be depicted by the right diagram.

$$\Gamma(V \to \gamma R) = \frac{1}{24\pi}\frac{k}{M_V^2}\sum_{i,f}\left|A^2\right|$$

- To the lowest order of the pQCD, the amplitude A is expressed as

$$A = \frac{1}{2}\sum\int\frac{d^4k}{(2\pi)^4}\frac{1}{k_1^2}\frac{1}{k_2^2}\langle V|\gamma gg\rangle\langle gg|R\rangle$$

- Introducing the width of R decaying into two gluons.

$$\Gamma(R \to gg) = \frac{1}{2J+1}\frac{1}{2m_R\pi}\sum_{\varepsilon_1,\varepsilon_2}\left|\langle gg|R\rangle\right|^2$$

After some calculus, one can get the following formulea, which relate the branch ratios of J/psi radiatively decaying to a resonance, the widths of the resonances, and branch ratios of the resonances decaying into two gluons, **(F.E. Close, G.R. Farrar, and Z. Li, Phys. Rev. D 55, 5749(1997))**

$$Br(J/\psi \to \gamma 0^{++}) = \left(\frac{m_R}{1.5GeV}\right)\left(\frac{\Gamma_R}{96MeV}\right)\left[\frac{x|H_S(x)|^2}{35}\right] Br(R \to gg) \times 10^{-3}$$

$$Br(J/\psi \to \gamma 2^{++}) = \left(\frac{m_R}{1.5GeV}\right)\left(\frac{\Gamma_R}{26MeV}\right)\left[\frac{x|H_T(x)|^2}{34}\right] Br(R \to gg) \times 10^{-3}$$

where the constants are introduced by hand to rescale the expression.

$$\frac{x|H_J(x)|^2}{35} \qquad x = 1 - \frac{m_R^2}{m_V^2}$$

Approximately takes a value close to 1 and insensitive to x for m_R in the range 1.3-2.3GeV

- **Another quntitative criterion for identifying a possible glueball state**

$$Br(R \to gg) \sim \begin{cases} O(1) & glueballs \\ O(\alpha_s{}^2) \sim 0.1 - 0.2 & q\bar{q} \end{cases}$$

It is interesting that most of the well established q-barq states have very small $Br(R \to gg)$

- **Thus, for the scalar glueball, using the branch ratio we get from lattice QCD,**

$$Br(J/\psi \to \gamma 0^{++}) = 3.8(9) \times 10^{-3}$$

$$= \left(\frac{m_R}{1.5GeV}\right)\left(\frac{\Gamma_R}{96MeV}\right)\left[\frac{x|H_S(x)|^2}{35}\right] Br(R \to gg) \times 10^{-3}$$

$$\Rightarrow \Gamma_R \sim O(300MeV)/Br(R \to gg)$$

for f0(1500),

$$Br(J/\psi \to \gamma f_0(1500)) = 2.9 \times 10^{-4}$$

$$= \left(\frac{m_R}{1.5GeV}\right)\left(\frac{\Gamma_R}{96MeV}\right)\left[\frac{x|H_S(x)|^2}{35}\right] Br(R \to gg) \times 10^{-3}$$

$$\Rightarrow Br(f_0(1500) \to gg) \sim 0.2 - 0.3$$

- **While for the tensor glueball,**

$$Br(J/\psi \to \gamma 2^{++}) = 1.1(2)\times 10^{-2}$$

$$= \left(\frac{m_R}{1.5GeV}\right)\left(\frac{\Gamma_R}{26MeV}\right)\left[\frac{x\left|H_S(x)\right|^2}{34}\right] Br(R \to gg)\times 10^{-3}$$

$$\Rightarrow \Gamma_R \sim O(200MeV)/Br(R \to gg)$$

To summarize

- For the first time, the widths of J/psi radiatively decaying to glueballs are calculated directly in lattice QCD (in quenched approximation at present stage).

- The lattice techniques are standard, but with very large statistics

- In the continuum limit, the partial width of $J/\psi \to \gamma G_{0^+}$ is pre-dicted to be 0.35(8) keV, and the branch ratio

$$\Gamma(J/\psi \to \gamma G_{0^+})/\Gamma_{tot} = 3.8(9)\times 10^{-3}$$

  This result supports f0(1710) as the best candidate for the scalar glueball.

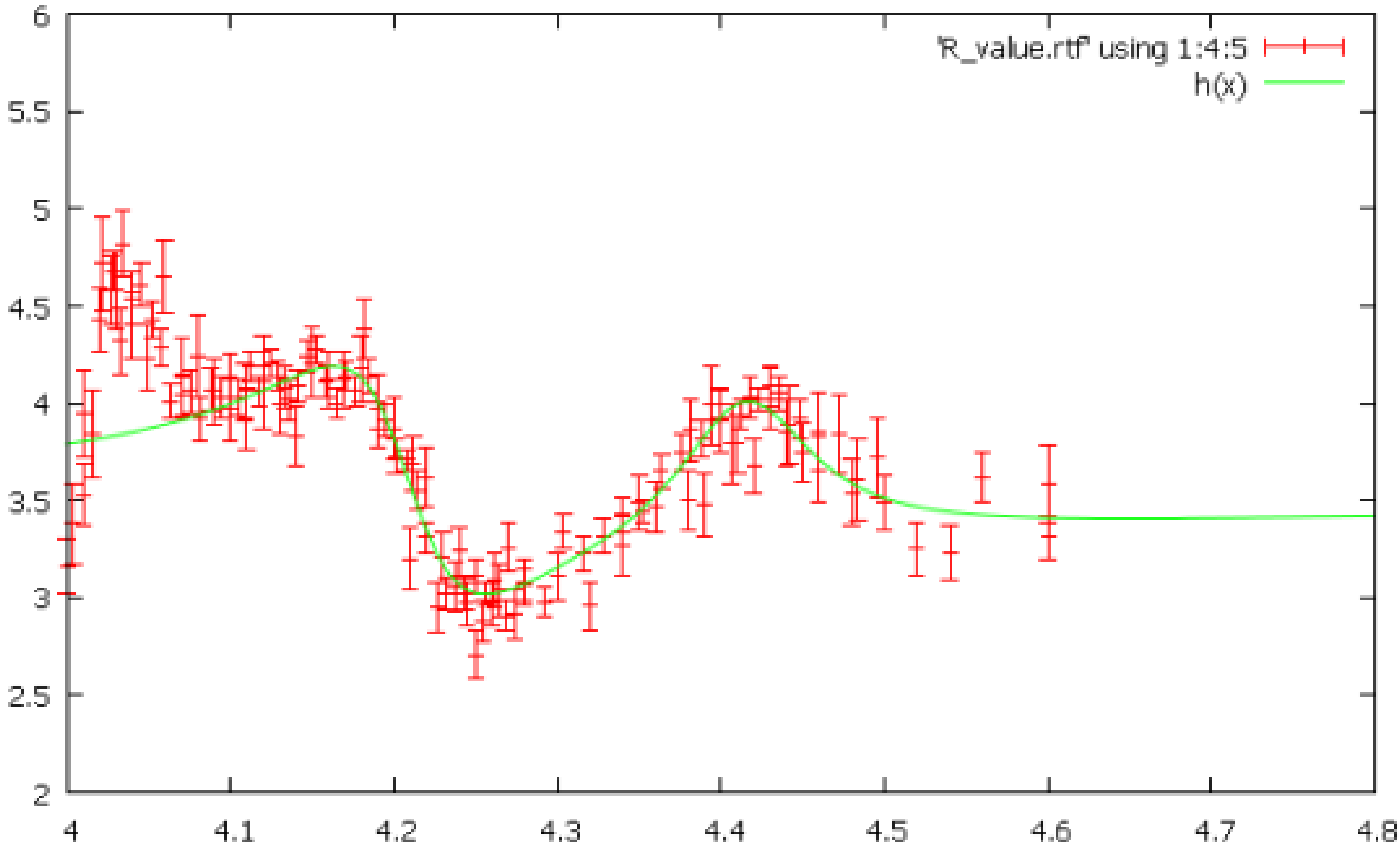
- The production fractrion of the tensor glueball in the J/psi radiative decay to is also presented

$$\Gamma(J/\psi \to \gamma G_{2^+})/\Gamma_{tot} = 1.1(2)\times 10^{-2}$$

- f2(2340) can be tentatively assigned as a possible new candidate for the tensor glueball, based on the new result of the PWA of $J/\psi \to \gamma\eta\eta$ from BESIII Collaboration.

- Anyway, since the production rate of the tensor glueball is predicted to be very large in the J/psi radiative decays, we propose BESIII to do a systematic and scrutinized analysis to the tensor products in the J/psi radiative decays using the world largest J/psi data set they are accumulating.

Thanks!



'R_value.rtf' using 1:4:5
h(x)
6
5.5
5
4.5
4
3.5
3
2.5
2
4
4.1
4.2
4.3
4.4
4.5
4.6
4.7
4.8