Hadronic molecules in effective field theory

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Based on:

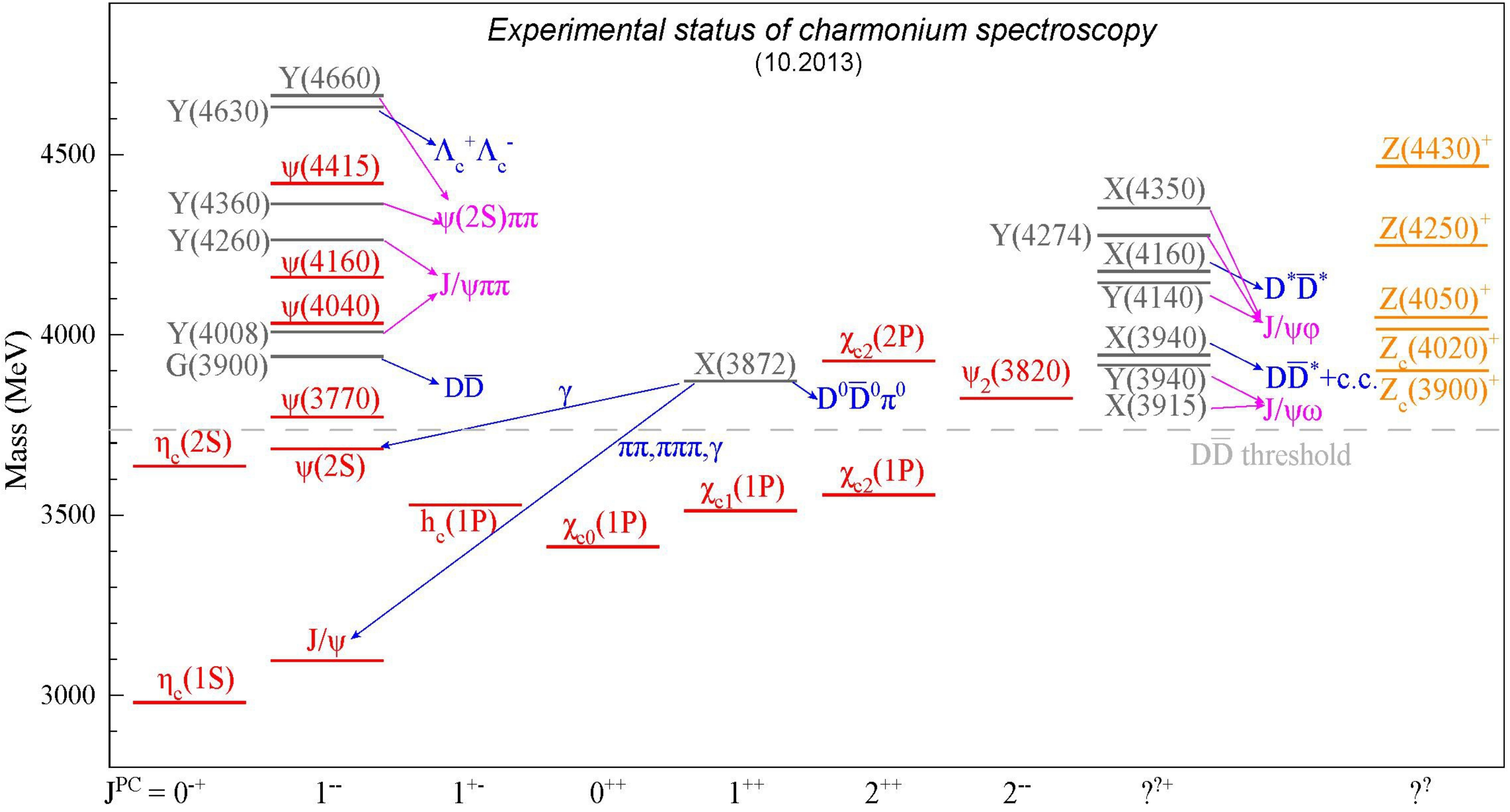
Guo, Hanhart, Meißner, Wang, Zhao, PLB725(2013)127

Guo, Hidalgo-Duque, Nieves, Pavon Valderrama, PRD88(2013)054007

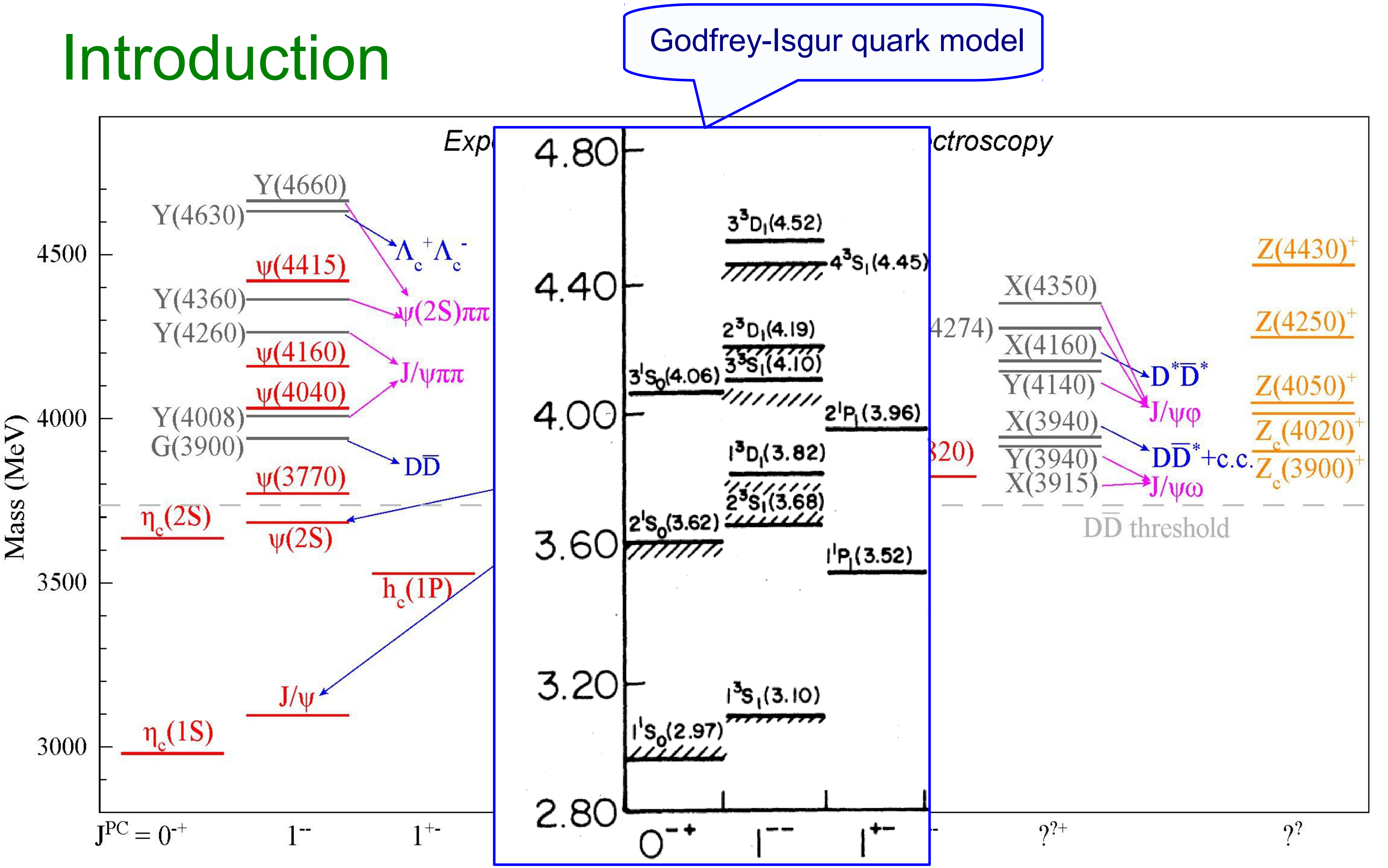
Outline

- Introduction
  - *S*-wave shallow bound states
- Production of $X(3872)$ in radiative charmonium transitions
- $Z_c(3900)$ and $Z_c(4020)$
- Summary

Introduction

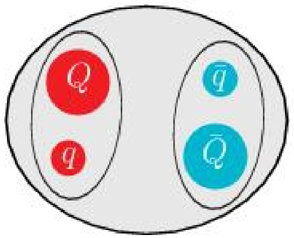


Introduction

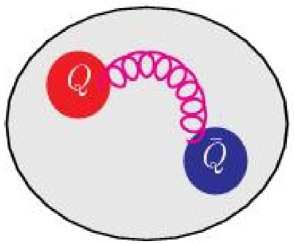


Introduction

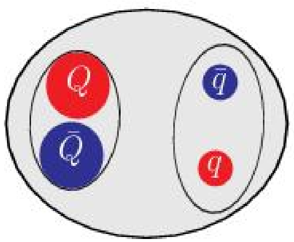
- Many states are beyond naïve quark model



Diquark-diquark tetraquark

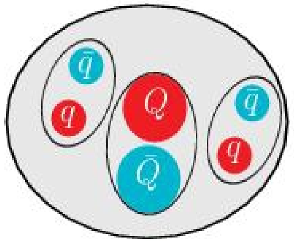


Hybrid



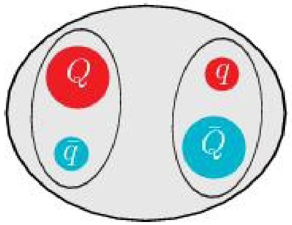
Born-Oppenheimer tetraquark

Braaten 2013



Hadro-charmonium

Voloshin 2007

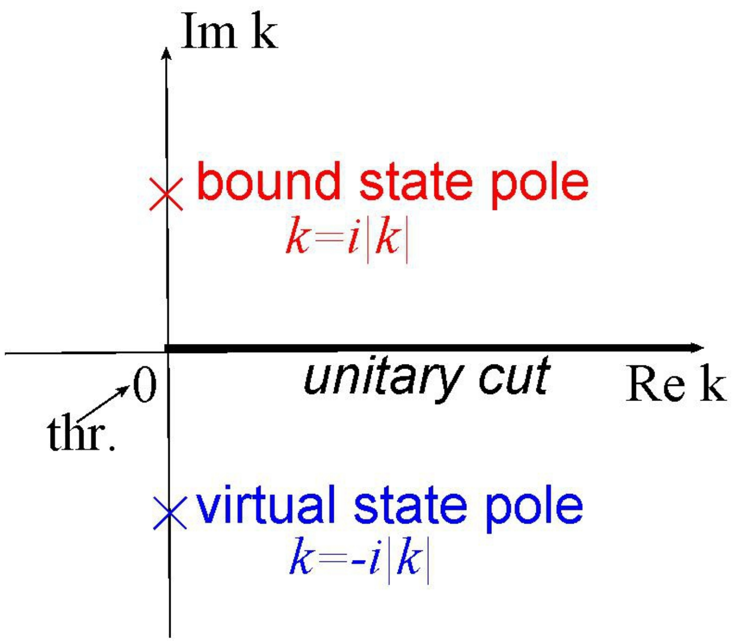


Hadronic molecule, generated from hadron interactions (bound, virtual state or resonance)

Introduction
bound state vs virtual state

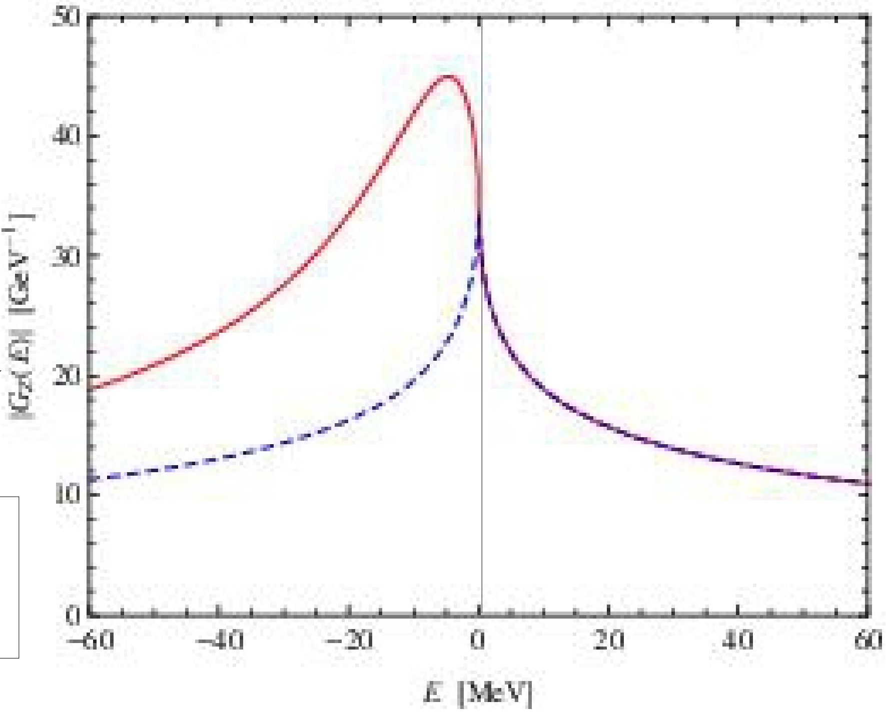
- Both bound state and virtual state are *S*-matrix poles below threshold

$$f(k) \propto \frac{1}{k\cot\delta(k) - ik}$$



- Line shapes of a bound state and a virtual state are same above threshold, difficult to be distinguished it they are close to threshold

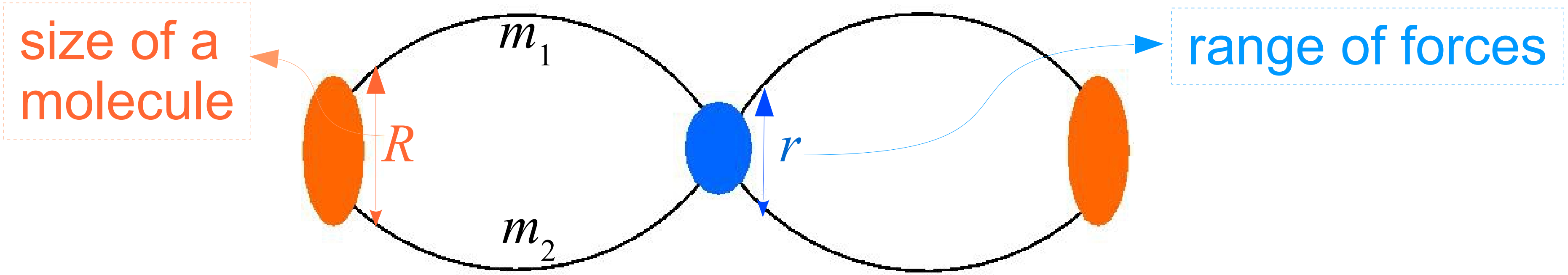
Comparison of the line shapes for a binding energy of ~5 MeV



Introduction

S-wave shallow bound state

- Most important ingredient of an EFT: scale separation



For a molecule with mass $M_{\rm mol}=m_1+m_2-E_B$

energy of the two constituents $m_1+m_2+p^2/(2\mu)$

⟹ momentum $p\sim\sqrt{2\mu E_B}$, $R\sim 1/p\sim 1/\sqrt{2\mu E_B}$

- Shallow bound state:

R>>size of a hadron ⟹ well-defined

$R>>r$ ⟹ scale separation, accessible to EFT

Introduction
S-wave shallow bound state

Suppose the physical state $|\psi\rangle$ contains a two-hadron continuum state $|h_1 h_2\rangle = |\mathbf{q}\rangle$ and something else $|\psi_0\rangle$

The time-independent Schrödinger Equation

$$(\hat{H}_0 + \hat{V})|\psi\rangle = -E_B|\psi\rangle$$

here H_0 is the free Hamiltonian, $\hat{H}_0|\mathbf{q}\rangle = q^2/(2\mu)$, and $E_B > 0$ is the binding energy.

Multiplying by $\langle\mathbf{q}|$, we get

$$\langle\mathbf{q}|\psi\rangle = -\frac{\langle\mathbf{q}|\hat{V}|\psi\rangle}{E_B + q^2/(2\mu)}$$

The probability of finding the physical state in the continuum state is

$$\lambda^2 = \int \frac{d^3\mathbf{q}}{(2\pi)^3} |\langle\mathbf{q}|\psi\rangle|^2 = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{|\langle\mathbf{q}|\hat{V}|\psi\rangle|^2}{[E_B + q^2/(2\mu)]^2}$$

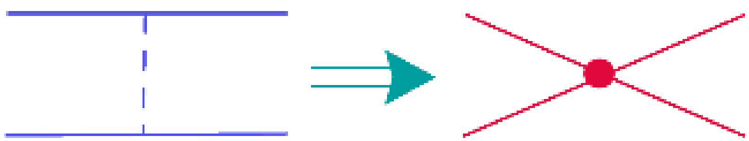
Introduction

S-wave shallow bound state

Denoting $g_{\rm NR}^2(\mathbf{q}) = |\langle \mathbf{q}|\hat{V}|\psi\rangle|^2$, we have

$$\lambda^2 = 4\mu^2 \int \frac{d\Omega_{\mathbf{q}}}{(2\pi)^3} \int_0^\infty dq q^2 \frac{g_{\rm NR}^2(\mathbf{q})}{(q^2+2\mu E_B)^2}$$

If the binding energy is very small, so that the binding momentum $\sqrt{2\mu E_B} \ll 1/r$ with r the range of forces, we have an expansion



$$g_{\rm NR}^2(\mathbf{q}) = q^{2L} g_{\rm NR}^2(0) + \mathcal{O}\left(r\sqrt{2\mu E_B}\right)$$

here L is the orbital angular momentum.

The integral is only convergent for an S-wave interaction with $L=0$. Therefore,

the probability of finding the physical state in the S-wave two-hadron state with a small binding energy is related to the coupling constant $g_{\rm NR}(0)$

Landau (1960), Weinberg (1963,1965), Baru et al (2004),. . .

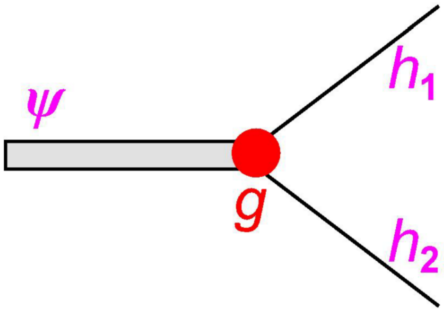
$$\lambda^2 \approx \frac{\mu^2}{2\pi\sqrt{2\mu E_B}} g_{\rm NR}^2(0)$$

Introduction
S-wave shallow bound state

From nonrelativistic quantum mechanics to relativistic QFT:

$$g = \sqrt{2m_1}\sqrt{2m_2}\sqrt{2(m_1+m_2)}\,g_{\rm NR}(0)$$

here g is the coupling constant in the relativistic Lagrangian



$$\mathscr{L} = g\psi^\dagger h_1 h_2 + h.c.$$

Therefore, the coupling constant contains the structure information

$$g^2 \approx 16\pi\lambda^2(m_1+m_2)^2\sqrt{\frac{2E_B}{\mu}} \leq 16\pi(m_1+m_2)^2\sqrt{\frac{2E_B}{\mu}}$$

Bound from above! The maximum corresponds to a pure bound state.
From the coupling constant, one can identify an *S*-wave shallow b.s.

X(3872) in radiative charmonium transitions

Guo, Hanhart, Meißner, Wang, Zhao, PLB725(2013)127

- Assume X(3872) to be a $D\bar{D}^* + c.c.$ bound state

Törnqvist (2003), …

$$\left|X(3872)\right\rangle = \alpha_1 |c\bar{c}\rangle + \frac{\alpha_2}{\sqrt{2}} \left|D\bar{D}^* + c.c.\right\rangle$$

- $|\alpha_2| >> |\alpha_1|$, X(3872) should be produced through meson loops
- Our phase convention under charge conjugation:

$$\mathcal{C} D \mathcal{C}^{-1} = \bar{D}, \qquad \mathcal{C} D^* \mathcal{C}^{-1} = \bar{D}^*$$

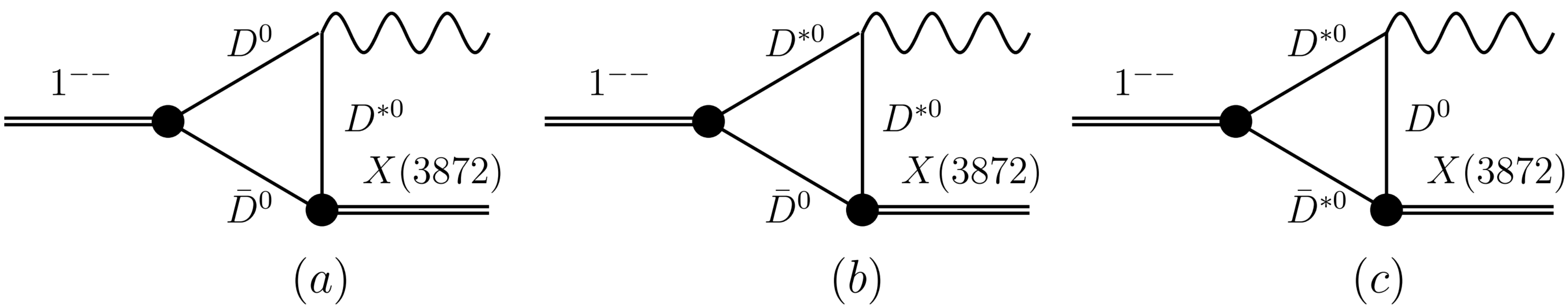
so that $\left|X(3872)\right\rangle = \frac{1}{\sqrt{2}} \left|D\bar{D}^* + D^*\bar{D}\right\rangle$

for quantum numbers 1^{++}

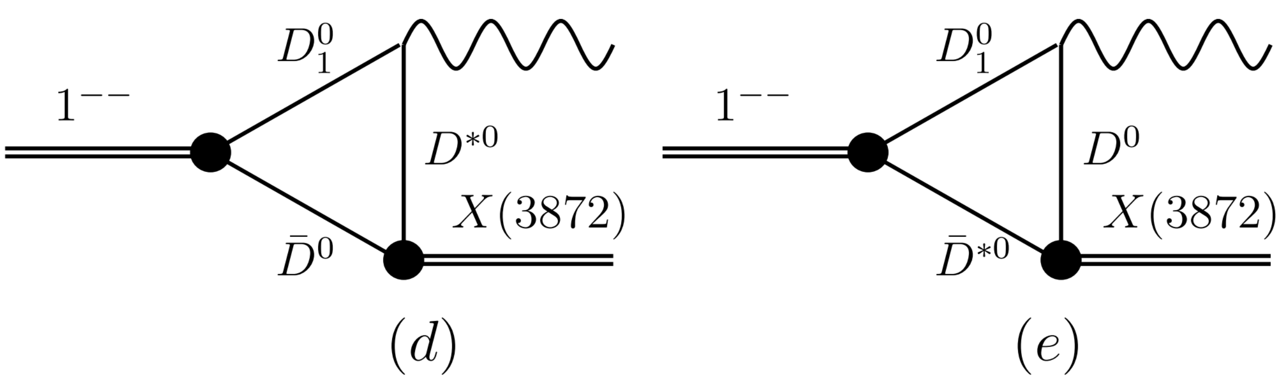
the sign is convention-dependent,
physics is convention-independent

X(3872) in radiative charmonium transitions
Possible loops

- $\Gamma(D^{*0}\to D^0\gamma) >> \Gamma(D^{*+}\to D^+\gamma)$, $\Gamma(D_1^0\to D^0\gamma) >> \Gamma(D_1^+\to D^+\gamma)$
- Type-I: *S*-wave charmed meson + *S*-wave charmed meson



- Type-II: *S*-wave charmed meson + *P*-wave charmed meson



$X(3872)$ in radiative charmonium transitions

- Nonrelativistic effective field theory (NREFT):
  Power counting   Guo et al PRL103(2009)082003; PRD83(2011)034013; …

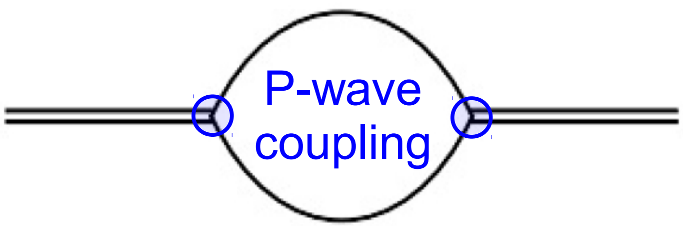
  Nonrelativistic, D-meson velocity $v << 1$

  three-momentum $\boldsymbol{p} \sim O(v)$; energy $E = \frac{\boldsymbol{p}^2}{2m} \sim O(v^2)$

  Propagator $\frac{1}{E - \boldsymbol{p}^2/(2m)} \sim O(v^{-2})$

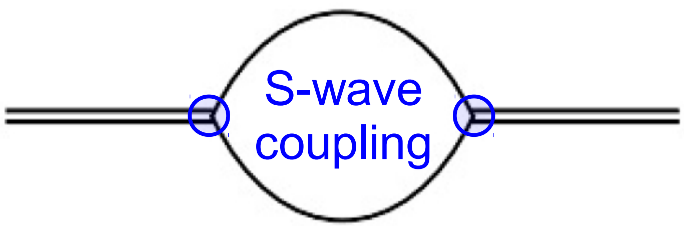
- Examples

- S-wave charmonium



$$O\left(\frac{v^5}{(v^2)^2} v^2\right) = O(v^3)$$

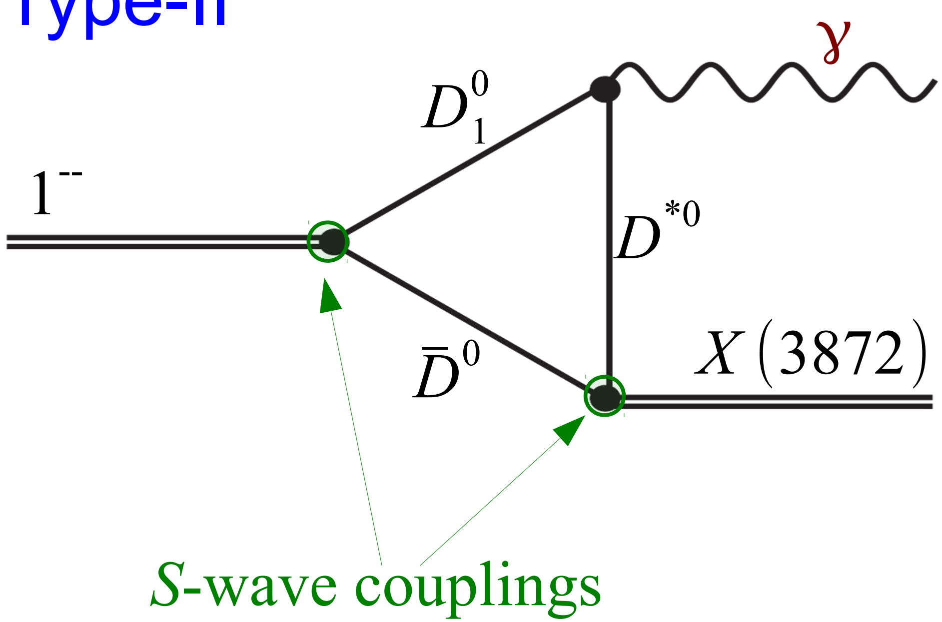
- P-wave charmonium



$$O\left(\frac{v^5}{(v^2)^2}\right) = O(v)$$

$X(3872)$ in radiative charmonium transitions

- Type-II



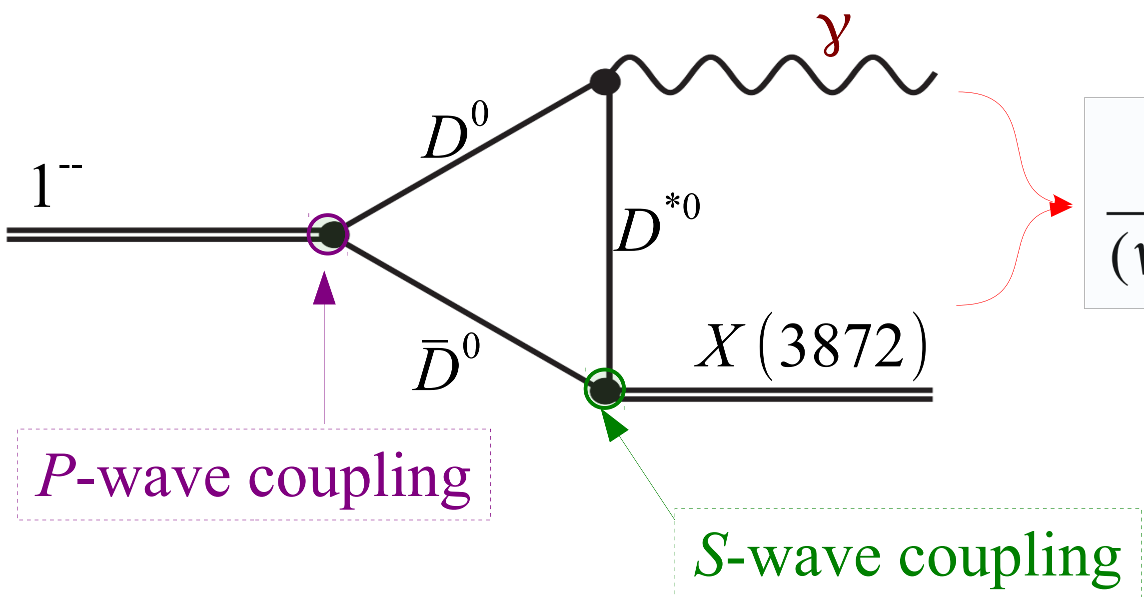
$$\frac{v^5}{(v^2)^3}E_\gamma = \frac{E_\gamma}{v}$$

- The loop integral is convergent

$$I(q) \equiv i\int \frac{d^4 l}{(2\pi)^4}\frac{1}{(l^2 - m_1^2 + i\epsilon)[(P-l)^2 - m_2^2 + i\epsilon][(l-q)^2 - m_3^2 + i\epsilon]}$$

X(3872) in radiative charmonium transitions

- Type-I



$$\frac{v^5}{(v^2)^3}\,\frac{q^2}{m_0} = \frac{E_\gamma^2}{m_0 v}$$

- m_0 is a quantity of the dimension mass.

  For a soft photon, $E_\gamma \ll m_0 \sim 1$ GeV

  ⇨ type-I is suppressed relative to type-II

$X(3872)$ in radiative charmonium transitions

- For $Y(4260)\to\gamma X(3872)$, we assume $Y(4260)$ to be a hadronic molecule with the main component being $D_1(2420)\bar{D}+c.c.$

Ding (2008), Li, Wang, Dong, Zhang (2013), Wang, Hanhart, Zhao (2013)

see also the next talk by Qian Wang

$$M_{D^0}+M_{D^{*0}}-M_X=0.16\pm0.26\ \text{MeV},$$

$$M_D+M_{D_1(2420)}-M_Y=27^{+9}_{-8}\ \text{MeV}.$$

$\mathcal{O}\left(r\sqrt{2\mu E_B}\right)$

- Coupling constants

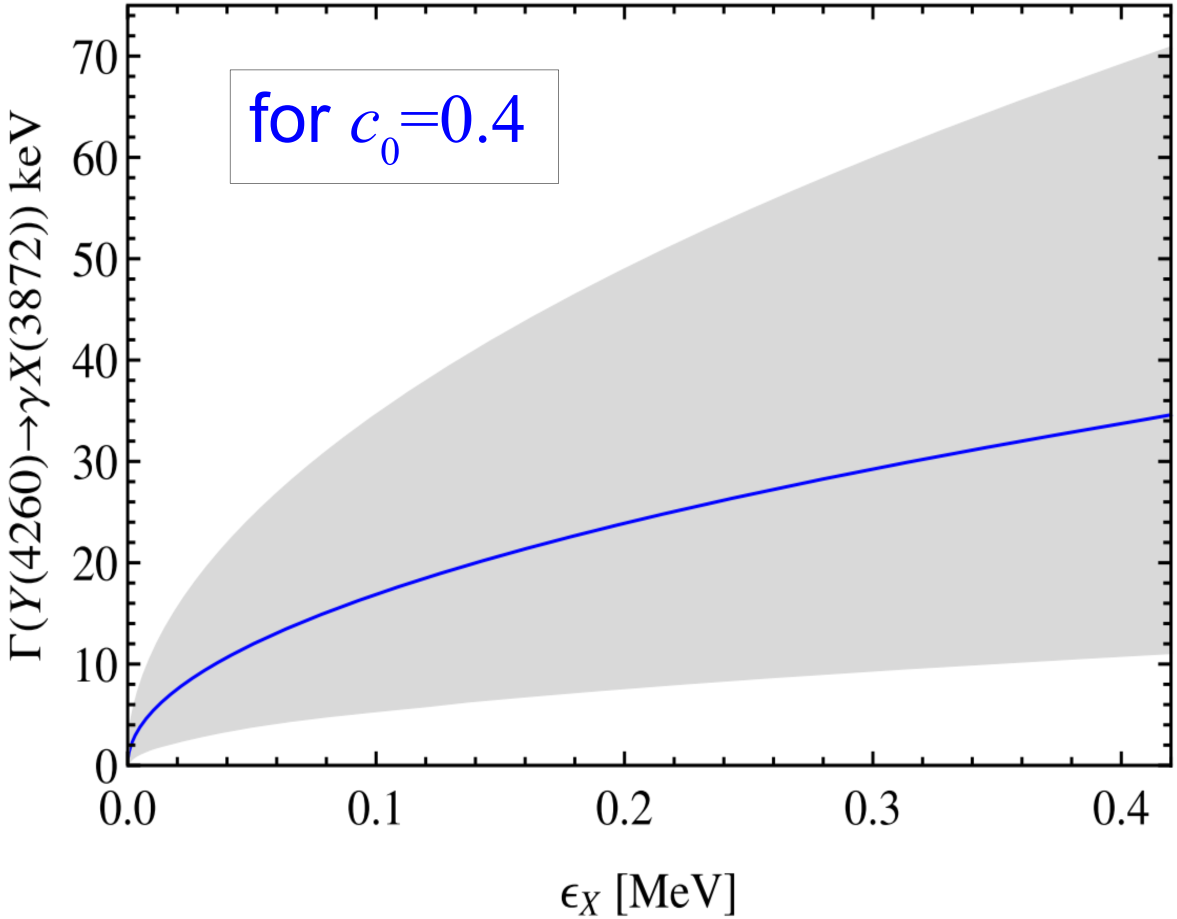
$X(3872)DD^*$: $|x|=0.97^{+0.40}_{-0.97}\pm0.14\ \text{GeV}^{-1/2}$

$Y(4260)DD_1$: $|y|=3.28^{+0.25}_{-0.28}\pm1.39\ \text{GeV}^{-1/2}$

from uncertainties of binding energies

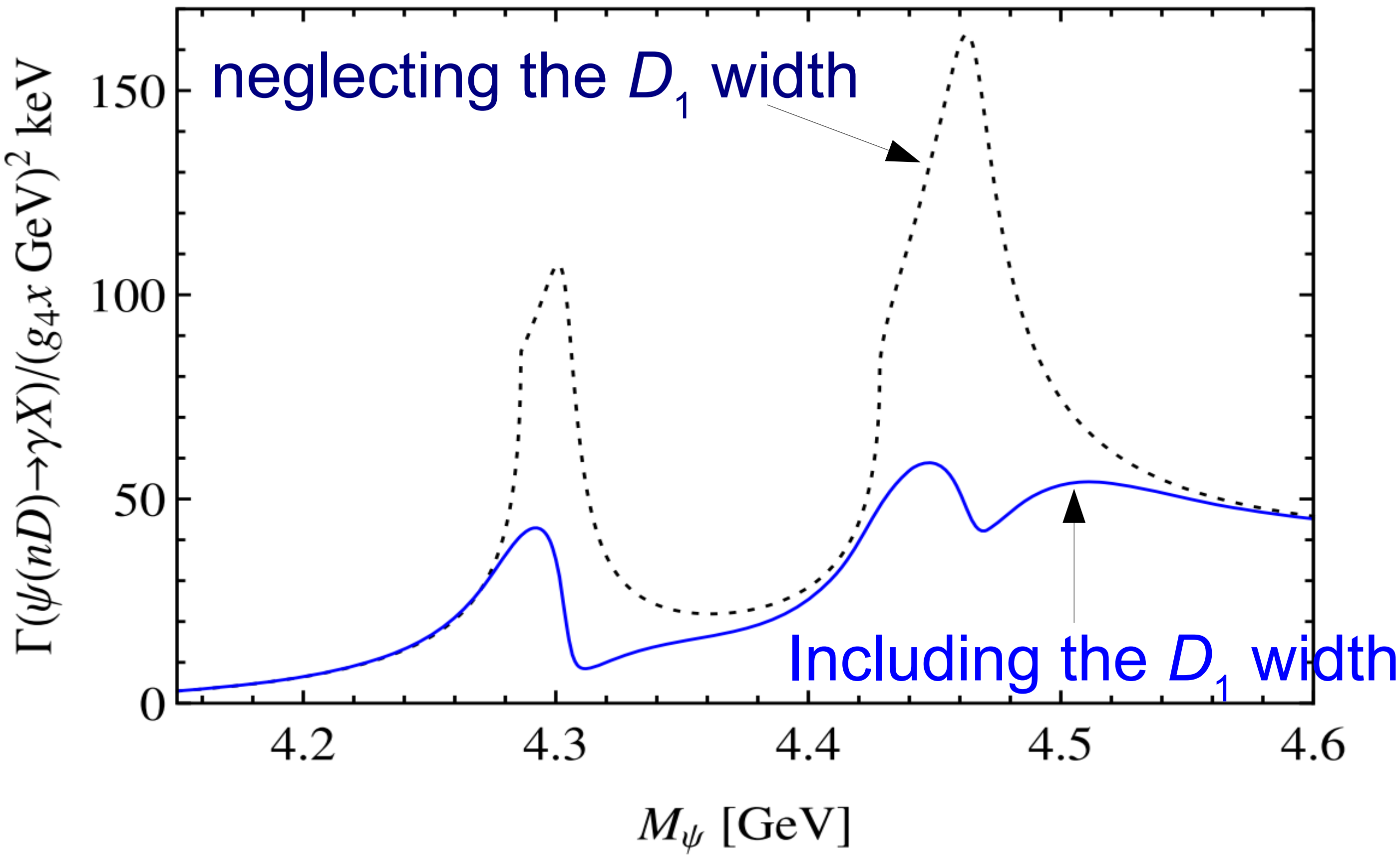
$X(3872)$ in radiative charmonium transitions
Partial width of $Y(4260)\to\gamma X(3872)$

$$\Gamma\left(\psi(4260)\to\gamma X(3872)\right)_{(d)} = 141^{+136}_{-\ 91}\left(x^2\ \text{GeV}\right)c_0^2\ \text{keV}$$



c_0=[0.3, 0.5]
in quark models

$X(3872)$ in radiative charmonium transitions



The best energy regions for producing the X(3872) + photon in e^+e^- are around the Y(4260) and around 4.45 GeV!

Observed by BESIII, see talk by Zhiqing Liu

$Z_c(3900)$ and $Z_c(4020)$

In the heavy quark limit $m_Q \to \infty$

- Spin symmetry: the heavy quark spin decouples

  spin multiplets: (D, D^*), $(J/\psi, \eta_c)$, ...

- Flavor symmetry: charm and bottom not distinguished

- Considering contact interaction between heavy hadron and anti-hadrons Alfiky, Gabbiani, Petrov (2006)

  divergence is regularized with a gaussian cutoff

- HQS $\Rightarrow$ some channels have the same combination of contact terms

$Z_c(3900)$ and $Z_c(4020)$

Consequences of spin symmetry

- X(3872) and its spin partner

$I(J^{PC})$	States	Thresholds	Masses ($\Lambda = 0.5$ GeV)
$0(1^{++})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* - D^*\bar{D})$	3875.87	3871.68 (input)
$0(2^{++})$	$D^*\bar{D}^*$	4017.3	4012^{+4}_{-5}

- $Z_b(10610)$ and $Z_b(10650)$

$1(1^{+-})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	10 604.4	$10\,602.4 \pm 2.0$ (input)
$1(1^{+-})$	$B^*\bar{B}^*$	10 650.2	$10\,648.1 \pm 2.1$

- Effect of OPE is small see Nieves, Valderrama (2011,2012)
- Other isovector states: W_b's Voloshin (2011); Mehen,Powell(2011)

$Z_c(3900)$ and $Z_c(4020)$

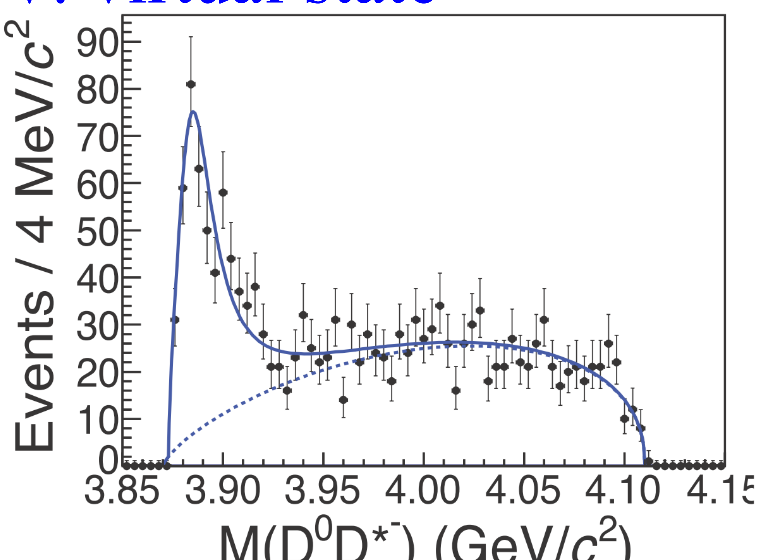
as charm partners of the Z_b *states*

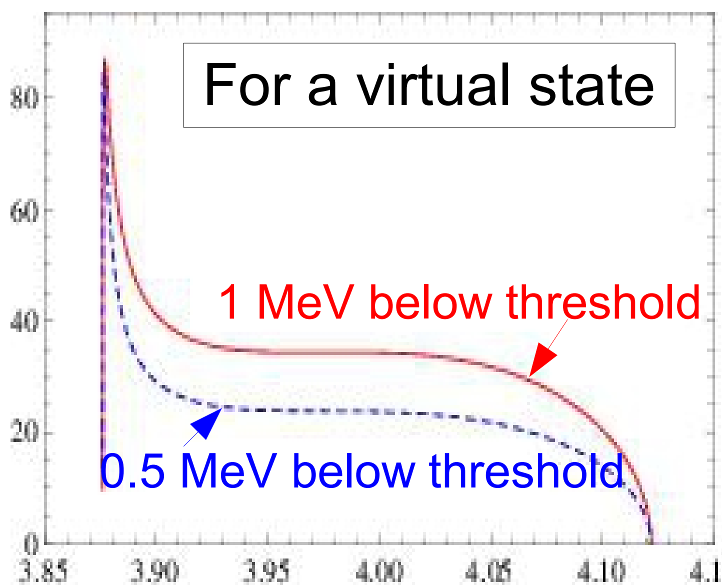
Guo, Hidalgo-Duque, Nieves, Valderrama (2013)

- Heavy flavor partners of the Z_b states

$I(J^{PC})$	States	Thresholds	Masses ($\Lambda = 0.5$ GeV)
$1(1^{+-})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* + D^*\bar{D})$	3875.87	3871^{+4}_{-12} (V)
$1(1^{+-})$	$D^*\bar{D}^*$	4017.3	4013^{+4}_{-11} (V)

- V: virtual state





Summary

- Hadronic molecules close to thresholds can be studied using effective field theory
- Best regions for the production of the *X*(3872)+photon in electron-positron collisions are around the *Y*(4260) and around 4.45 GeV
- The two Z_c states could be flavor partners of the Z_b states. Their poles might be below threshold.

Thank you for your attention