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Neutrino Physics and Astrophysics

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Neutrino Mass Models
part II

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plan of part II

1. Flavor symmetries (II): the lepton mixing puzzle
2. First approximation for a realistic model: TB mixing
3. TB mixing from symmetry breaking of a flavor symmetry
1. A minimal model based on A_4
2. Conclusion

[much more speculative! Only an example out of many existing possibilities, to illustrate current ideas]

based on
AF1 = Guido Altarelli and F. F. hep-ph/0504165
AF2 = Guido Altarelli and F. F. hep-ph/0512103
AFL = Guido Altarelli, F.F. and Yin Lin hep-ph/0610165
FHLM1 = F.F., Claudia Hagedorn, Yin Lin and Luca Merlo hep-ph/0702194
AFH = Guido Altarelli, F.F. and Claudia Hagedorn hep-ph/0702194
FL = F.F. and Yin Lin hep-ph/07121528
L = Yin Lin hep-ph/08042867

Flavor symmetries II (the lepton mixing puzzle)

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu U_{PMNS} \nu_L + h.c.$$

U_{PMNS} depends by three mixing angles ϑ_{12}, ϑ_{23}, ϑ_{13} like V_{CKM}

$$U_{PMNS} = U_e^+ U_\nu$$

ϑ_{ij} have been determined or constrained by neutrino oscillations

	Fogli [NoVe 2008] [0806.2649]	Schwetz et al. [0808.2016]
$\sin^2\vartheta_{12}$	$0.326^{+0.05}_{-0.04}$ $[2\sigma]$	$0.304^{+0.022}_{-0.016}$
$\sin^2\vartheta_{23}$	$0.45^{+0.16}_{-0.09}$ $[2\sigma]$	$0.50^{+0.07}_{-0.06}$
$\sin^2\vartheta_{13}$	0.016 ± 0.010	$0.01^{+0.016}_{-0.011}$
$\Delta m^2_{21}\ (eV^2)$	$(7.66 \pm 0.35) \times 10^{-5}$ $[2\sigma]$	$(7.65^{+0.23}_{-0.20}) \times 10^{-5}$
$\lvert\Delta m^2_{31}\ (eV^2)\rvert$	$(2.38 \pm 0.27) \times 10^{-3}$ $[2\sigma]$	$(2.40^{+0.12}_{-0.11}) \times 10^{-3}$

$\vartheta_{12} = \left(34.8^{+3.0}_{-2.5}\right)^0$ $[2\sigma]$ $\qquad$ $\vartheta_{12} = \left(33.5^{+1.4}_{-1.0}\right)^0$

$\vartheta_{23} = \left(42.1^{+9.2}_{-5.3}\right)^0$ $[2\sigma]$ $\qquad$ $\vartheta_{23} = \left(45.0^{+4.0}_{-3.4}\right)^0$

Tri-Bimaximal Mixing

a good approximation of the data [Harrison, Perkins and Scott; Zhi-Zhong Xing 2002]

$$\sin^2\vartheta_{12}^{TB} = \frac{1}{3} \qquad \sin^2\vartheta_{23}^{TB} = \frac{1}{2} \qquad \sin^2\vartheta_{13}^{TB} = 0$$

quality set by the solar angle

$$\vartheta_{12}^{TB} = 35.3^0 \quad \Longleftrightarrow \quad \vartheta_{12}^{Fogli} = \left(34.8^{+3.0}_{-2.5}\right)^0 \quad [2\sigma]$$

$$\vartheta_{12}^{Schwetz} = \left(33.5^{+1.4}_{-1.0}\right)^0$$

correct within a couple of degrees, about 0.035 rad, less than ϑ_C^2

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-Bimaximal mixing

$$\nu_3 = \frac{-\nu_\mu + \nu_\tau}{\sqrt{2}}$$ maximal

$$\nu_2 = \frac{\nu_e + \nu_\mu + \nu_\tau}{\sqrt{3}}$$ trimaximal

What is the best 1st order approximation to lepton mixing?

in the quark sector

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\vartheta_C)$$

[Wolfenstein 1983; Zhi-Zhong Xing 1994,...]

in the lepton sector

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \ldots$$

agreement of ϑ_{12} suggests that only tiny corrections $[O(\vartheta_C^2)]$ are tolerated. If all corrections are of the same order, then

$\vartheta_{13} \approx O(\vartheta_C^2)$ expected

$$U_{PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} + \ldots$$

can be reconciled with the data through a correction of $O(\vartheta_C)$, for instance a rotation in the 12 sector [from the left side]

$\vartheta_{13} \approx O(\vartheta_C)$ expected

[quark-lepton complementarity ?] [Smirnov; Raidal; Minakata and Smirnov 2004]

$\vartheta_{23} - \pi/4 \approx O(\vartheta_C^2)$

common feature: $\vartheta_{23} \approx \pi/4$ [maximal atm mixing]

... or anarchical U_{PMNS} ? [Hall, Murayama, Weiner 1999]

θ_{23} maximal from some flavour symmetries ?

a no-go theorem
[F. 2004]

$\vartheta_{23} = \pi/4$ can never arise in the limit of an exact realistic symmetry

charged lepton mass matrix:

$$m_l = m_l^0 + \delta m_l^0$$

symmetric limit

symmetry breaking effects: vanishing when flavour symmetry F is exact

realistic symmetry:

(1) $\left|\delta m_l^0\right| < \left|m_l^0\right|$

(2) m_l^0 has rank ≤1

$$m_l^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

ϑ_{12}^e undetermined

$$U_{PMNS} = U_e^+ U_\nu$$

[omitting phases]

$$\tan\vartheta_{23}^0 = \tan\vartheta_{23}^\nu \cos\vartheta_{12}^e + \left(\frac{\tan\vartheta_{13}^\nu}{\cos\vartheta_{23}^\nu}\right)\sin\vartheta_{12}^e$$

undetermined

$$\vartheta_{23} = \frac{\pi}{4}$$

determined entirely by breaking effects
(different, in general, for ν and e sectors)

Lepton mixing from symmetry breaking

Consider a flavor symmetry G_f such that G_f is broken into two different subgroups: G_e in the charged lepton sector, and G_ν in the neutrino sector. $(m_e^+ m_e)$ is invariant under G_e and m_ν is invariant under G_ν. If G_e and G_ν are appropriately chosen, the constraints on m_e and m_ν can give rise to the observed U_{PMNS}.

For instance we can select G_e in such a way that $(m_e^+ m_e)$ is diagonal and G_ν in such a way that m_ν is responsible for the whole lepton mixing.

$$
\begin{tikzcd}
 & & G_f \arrow[dl] \arrow[dr] & & \\
 & G_e \arrow[dl] & & G_\nu \arrow[dr] & \\
\boxed{(m_e^+ m_e) \text{ diagonal}} & & & & \boxed{U_{PMNS}^T\, m_\nu\, U_{PMNS} = (m_\nu)_{diag}}
\end{tikzcd}
$$

[He, Keum, Volkas 0601001
Lam 0708.3665 + 0804.2622]

TB mixing from symmetry breaking

it is easy to find a symmetry that forces ($m_e^+ m_e$) to be diagonal;
a "minimal" example (there are many other possibilities) is

$$G_T=\{1,T,T^2\}$$

$$T=\begin{pmatrix}1 & 0 & 0\\ 0 & \omega^2 & 0\\ 0 & 0 & \omega\end{pmatrix} \qquad \omega=e^{i\frac{2\pi}{3}}$$

[$T^3=1$ and mathematicians call a group with this property Z_3]

$$T^+ (m_e^+ m_e)\, T = (m_e^+ m_e) \quad \Rightarrow \quad \left(m_e^+ m_e\right)=\begin{pmatrix}m_e^2 & 0 & 0\\ 0 & m_\mu^2 & 0\\ 0 & 0 & m_\tau^2\end{pmatrix}$$

in such a framework TB mixing should arise entirely from m_ν

$$m_\nu(TB) \equiv \frac{m_3}{2}\begin{pmatrix}0&0&0\\0&1&-1\\0&-1&1\end{pmatrix}+\frac{m_2}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}+\frac{m_1}{6}\begin{pmatrix}4&-2&-2\\-2&1&1\\-2&1&1\end{pmatrix}$$

most general neutrino mass matrix giving rise to TB mixing

easy to construct from the eigenvectors:

$$m_3 \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\1\end{pmatrix} \qquad m_2 \leftrightarrow \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix} \qquad m_1 \leftrightarrow \frac{1}{\sqrt{6}}\begin{pmatrix}2\\-1\\-1\end{pmatrix}$$

a "minimal" symmetry guaranteeing such a pattern [C.S. Lam 0804.2622]

$$G_S \times G_U \quad G_S=\{1,S\} \quad G_U=\{1,U\}$$

$$S=\frac{1}{3}\begin{pmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{pmatrix} \quad U=\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

[this group corresponds to $Z_2 \times Z_2$ since $S^2=U^2=1$]

$$S^T m_\nu S = m_\nu \qquad U^T m_\nu U = m_\nu \quad \Longrightarrow \quad m_\nu = m_\nu(TB)$$

Algorithm to generate TB mixing

- start from a flavour symmetry group G_f containing G_T, G_S, G_U
- arrange appropriate symmetry breaking

$$\begin{tikzcd}
 & G_f \arrow[dl] \arrow[dr] & \\
\text{charged lepton sector} \quad G_T & & G_S \times G_U \quad \text{neutrino sector}
\end{tikzcd}$$

if the breaking is spontaneous, induced by $\langle\varphi_T\rangle$,$\langle\varphi_S\rangle$,... there is a vacuum alignment problem

Minimal choice

G_f generated by S and T (U can arise as an accidental symmetry) they satisfy

$$S^2 = T^3 = (ST)^3 = 1$$

these are the defining relations of A_4, group of even permutations of 4 objects, subgroup of SO(3) leaving invariant a regular tetrahedron. S and T generate 12 elements

[Ma and Rajasekaran 2001, Ma 2002, Babu, Ma and Valle 2003, ...]

$$A_4 = \{1, S, T, ST, TS, T^2, ST^2, STS, TST, T^2S, TST^2, T^2ST\}$$

there are many many non-minimal possibilities: G_f=S_4, $\Delta(27)$, $\Delta(108)$, ...

[Medeiros Varzielas,
King and Ross 2005 and 2006;
Luhn, Nasri and Ramond 2007,
Blum, Hagedorn and Lindner 2007,...]

A_4 has 4 irreducible representations: 1, 1′, 1″ and 3

$$\omega \equiv e^{i\frac{2\pi}{3}}$$

$$\begin{array}{lll} 1 & S=1 & T=1 \\ 1' & S=1 & T=\omega^2 \\ 1'' & S=1 & T=\omega \end{array}$$

$$3 \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

Building blocks of a minimal model [AF1, AF2]

	l	e^c	μ^c	τ^c	h_u	h_d	φ_T	φ_S	ξ_i
A_4	3	1	1''	1'	1	1	3	3	1

$\underbrace{l \quad e^c \quad \mu^c \quad \tau^c}_{\text{matter fields}} \quad \underbrace{h_u \quad h_d}_{\text{Higgses}} \quad \underbrace{\varphi_T \quad \varphi_S \quad \xi_i}_{A_4 \text{ breaking sector}}$

[change of notation: Higgs doublets are denoted by h_u and h_d]

SU(2)xU(1)xA_4x... invariant Lagrangian:

$$L = \frac{y_e}{\Lambda} e^c h_d (\varphi_T l) + \frac{y_\mu}{\Lambda} \mu^c h_d (\varphi_T l)' + \frac{y_\tau}{\Lambda} \tau^c h_d (\varphi_T l)''$$

$$+ \frac{x_a}{\Lambda^2} h_u h_u \xi (ll) + \frac{x_b}{\Lambda^2} h_u h_u (\varphi_S ll) + V(\xi, \varphi_S, \varphi_T) \ldots$$

[(...) denotes an A_4 singlet,...]

higher dimensional operators in $1/\Lambda$ expansion [Λ = cutoff]

additional symmetry: Z_3, acts as a discrete lepton number; avoids additional invariants

$\varphi_S \leftrightarrow \varphi_T$

$x(ll)$

under appropriate conditions (SUSY,...) a natural minimum of the scalar potential V is

$$\frac{\langle \varphi_T \rangle}{\Lambda} = (u,0,0)$$ ⟺ breaks A_4 down to G_T

$$\frac{\langle \varphi_S \rangle}{\Lambda} = y_b(u,u,u)$$ ⟺ breaks A_4 down to G_S

$$\frac{\langle \xi \rangle}{\Lambda} = y_a u$$

[y_a and y_b are numbers of order one]

then:

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d u$$

⇒ charged fermion masses

$$m_f = y_f v_d u$$

free parameters as in the SM at this level

$$m_\nu = \begin{pmatrix} a+\frac{2}{3}b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3}b & a-\frac{b}{3} \\ -\frac{b}{3} & a-\frac{b}{3} & \frac{2}{3}b \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$a \equiv 2x_a y_a u$$
$$b \equiv 2x_b y_b u$$

2 complex parameters in ν sector (overall phase unphysical)

is also invariant under G_U (accidental symmetry)

TB mixing automatically guaranteed by pattern of symmetry breaking

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

independent from
$|a|$, $|b|$, $\Delta \equiv \arg(a)-\arg(b)$!!

ν spectrum

$$r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{35}$$

requires a (moderate) tuning

- in this minimal model the mass spectrum is always of normal hierarchy type
the model predicts

$$m_1 \geq 0.017 \ \text{eV} \qquad \sum_i m_i \geq 0.09 \ \text{eV} \qquad |m_3|^2 = |m_{ee}|^2 + \frac{10}{9} \Delta m^2_{atm} \left(1 - \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$$

- in a see-saw realization both normal and inverted hierarchies can be accommodated

Sub-leading corrections

arising from higher dimensional operators, depleted by additional powers of $1/\Lambda$. $\Rightarrow$ they affect m_l, m_ν and they can deform the VEVs.

results

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + O(u)$$

TB pattern is preserved if corrections are $\leq \vartheta_C^2 \approx 0.04$

generic prediction for ϑ_{13}
$\vartheta_{13}=O(u)$

range of VEVs:

$$m_\tau = y_\tau v_d u$$

$$y_\tau < 4\pi$$

$\Rightarrow$

$$u > 0.002(0.02)$$

$$\tan\beta = 2.5(30)$$

$$\tan\beta = \frac{v_u}{v_d}$$

$$0.002 \leq u \leq 0.04$$

the range expected for ϑ_{13} is similar additional tests are possible if there is new physics at a scale M close to TeV

$$L_{eff} = i\frac{e}{M^2} l^c h_d \left(\sigma^{\mu\nu} F_{\mu\nu}\right) \mathcal{M}(\langle\varphi\rangle) l + [4\text{ - fermion}] + h.c. + \ldots$$

dominant 4-fermion LFV operators

$$\frac{1}{M^2}\bar{e}^c \bar{\tau}^c \mu^c \mu^c \qquad\qquad \frac{1}{M^2}(\bar{l}\,\bar{l}\,l\,l)$$

selection rule $\Delta L_e \Delta L_\mu \Delta L_\tau = \pm 2$

$$\tau^- \to \mu^+ e^- e^- \qquad\qquad \tau^- \to e^+ \mu^- \mu^-$$

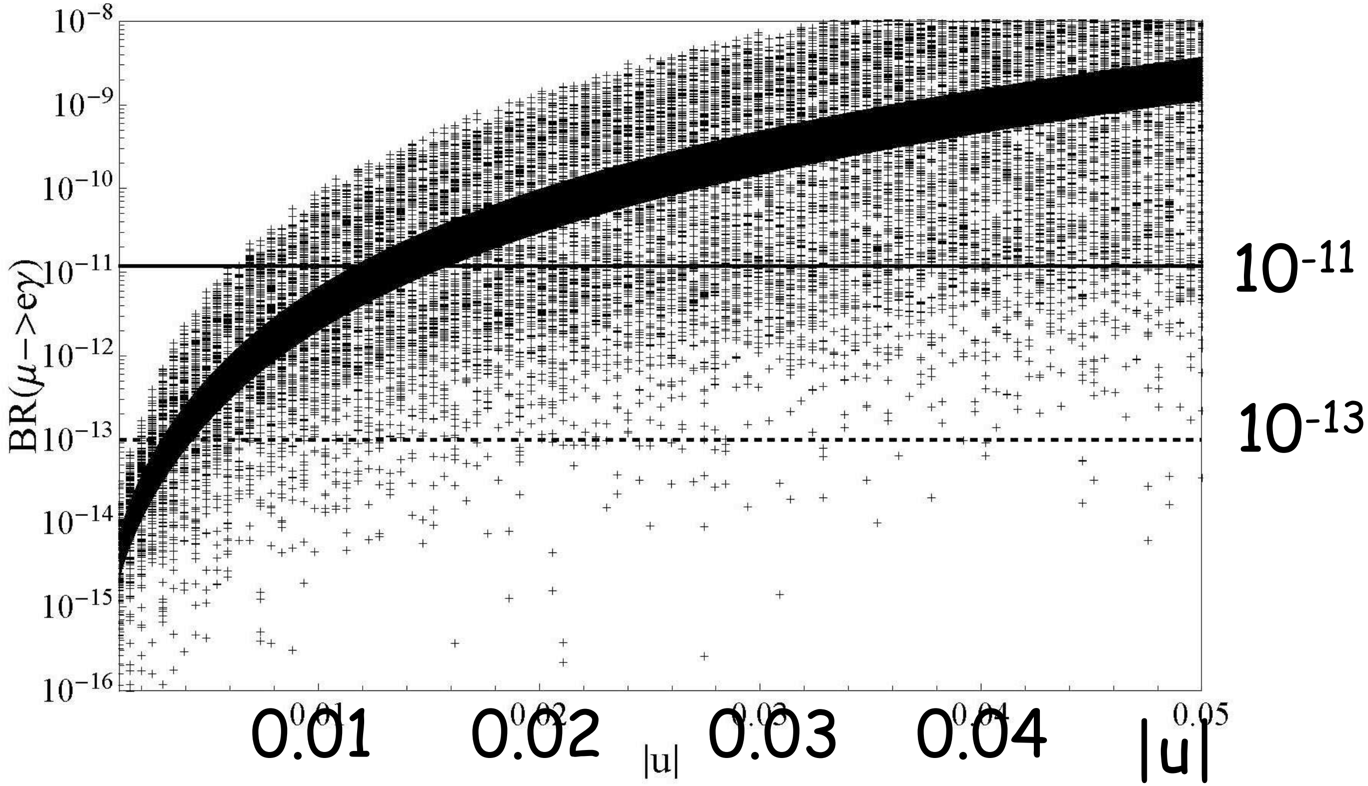
this term contributes to magnetic dipole moments and to LFV transitions such as $\mu \to e\gamma \quad \tau \to \mu\gamma \quad \tau \to e\gamma$ usually discussed in terms of

$$R_{ij} = \frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)}$$

up to O(1) coefficients $R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$ independently from u

$\tau \to \mu\gamma \quad \tau \to e\gamma$ below expected future sensitivity

In a SUSY realization of this model



$$BR(\mu \to e\gamma) = \underbrace{\frac{12\pi^3 \alpha_{em}}{G_F^2 m_\mu^4}\left(\delta a_\mu\right)^2}_{0.0014\times\left(\frac{\delta a_\mu}{30\times 10^{-10}}\right)^2}\left[\gamma u\right]^4$$

O(1) coefficient

Conclusion

theory of neutrino masses

it does not exist! Neither for neutrinos nor for charged fermions. We lack a unifying principle.

like weak interactions before the electroweak theory

$SU(2)_L \otimes U(1)_Y$

gauge invariance

all fermion-gauge boson interactions in terms of 2 parameters: g and g'

?

Yukawa interactions between fermions and spin 0 particles: many free parameters (up to 22 in the SM!)

only few ideas and prejudices about neutrino masses and mixing angles

caveat: several prejudices turned out to be wrong in the past!

- $m_\nu \approx 10$ eV because is the cosmologically relevant range
- solution to solar is MSW Small Angle
- atmospheric neutrino problem will disappear because it implies a large angle

[other slides]

many models predicts a **large** but **not necessarily maximal** θ_{23}

an example: abelian flavour symmetry group $U(1)_F$

$$F(l) = (\times,0,0) \qquad [\times \neq 0]$$

$$F(e^c) = (\times,\times,0)$$

$$m_e = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & O(1) & O(1) \end{pmatrix} v_d \qquad m_\nu = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & O(1) & O(1) \\ \cdot & O(1) & O(1) \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$\vartheta_{23} \approx O(1)$$

maximal only by a fine-tuning!

similarly for all other abelian charge assignements

$$F(l) = (1,-1,-1)$$

$$m_\nu = \begin{pmatrix} \cdot & O(1) & O(1) \\ O(1) & \cdot & \cdot \\ O(1) & \cdot & \cdot \end{pmatrix} \frac{v_u^2}{\Lambda}$$

$$\vartheta_{23} \approx O(1) + \text{charged lepton contribution}$$

no help from the see-saw mechanism within abelian symmetries…

θ_{23} maximal by RGE effects?

[Ellis, Lola 1999
Casas, Espinoza, Ibarra, Navarro 1999-2003
Broncano, Gavela, Jenkins 0406019]

running effects important only for quasi-degenerate neutrinos

2 flavour case

boundary conditions at Λ>> e.w. scale $\quad m_2, m_3, \vartheta_{23}$

at $Q < \Lambda$

$$\vartheta_{23}(Q) \approx \frac{\pi}{4} \quad \Leftrightarrow \quad \varepsilon \approx -\frac{\delta m}{m}\cos 2\vartheta_{23}$$

[possible only if $\quad \delta m \equiv m_2 - m_3 << m_2 + m_3 \approx 2m$]

$$\varepsilon \approx \frac{1}{16\pi^2} y_\tau^2 \log\frac{\Lambda}{Q}$$

gives the scale Q at which
θ_{23}(Q) becomes maximal

$\Rightarrow$

m_2, m_3, ϑ_{23} fine tuned
to obtain Q at the e.w. scale

a similar conclusion also for the 3 flavour case:

$$\sin^2 2\vartheta_{12} = \frac{\sin^2\vartheta_{13}\sin^2 2\vartheta_{23}}{(\sin^2\vartheta_{23}\cos^2\vartheta_{13} + \sin^2\vartheta_{13})^2}$$

infrared stable fixed point

[Chankowski, Pokorski 2002]

if $\vartheta_{23} = \dfrac{\pi}{4}$

wrong!

$$\sin^2 2\vartheta_{12} = \frac{4\sin^2\vartheta_{13}}{(1+\sin^2\vartheta_{13})^2} < 0.2 \ \ \text{(Chooz)}$$

Alignment and mass hierarchies

$$m_l = \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d \left(\frac{v_T}{\Lambda} \right)$$

charged fermion masses
are already diagonal

$$\boxed{m_e << m_\mu << m_\tau}$$

can be reproduced by
U(1) flavour symmetry

$$\left.\begin{array}{l} Q(e^c) = 4 \quad Q(\mu^c) = 2 \quad Q(\tau^c) = 0 \\ Q(l) = 0 \\ Q(\vartheta) = -1 \qquad \langle \vartheta \rangle \neq 0 \end{array}\right\}$$

compatible with A_4

$$y_e \approx \frac{\langle \vartheta \rangle^4}{\Lambda^4} \qquad y_\mu \approx \frac{\langle \vartheta \rangle^2}{\Lambda^2} \qquad y_\tau \approx 1$$

[see also Lin hep-ph/08042867 for a realization without an additional U(1)]

Quark masses – grand unification

quarks assigned to the same A_4 representations used for leptons?

	q	u^c	c^c	t^c	d^c	s^c	b^c
A_4	3	1	1''	1'	1	1''	1'

- fermion masses from dim ≥ 5 operators, e.g. $\dfrac{\tau^c \varphi_T l H_d}{\Lambda}$ good for leptons, but not for the top quark
- naïve extension to quarks leads diagonal quark mass matrices and to $V_{CKM}=1$ departure from this approximation is problematic [expansion parameter (VEV/Λ) too small]

possible solution within T', the double covering of A_4

[FHLM1]

$$S^2 = R \quad R^2 = 1 \quad (ST)^3 = T^3 = 1$$

24 elements

representations: 1 1′ 1″ 3 2 2′ 2″

	$\begin{pmatrix} u & d \\ c & s \end{pmatrix}$	$\begin{pmatrix} u^c \\ c^c \end{pmatrix}$	$\begin{pmatrix} d^c \\ s^c \end{pmatrix}$	$(t \quad b)$	t^c	b^c	η	ξ''
T'	2''	2''	2''	1	1	1	2'	1''

[older T′ models by
Frampton, Kephard 1994
Aranda, Carone, Lebed 1999, 2000
Carr, Frampton 2007
similar U(2) constructions by
Barbieri, Dvali, Hall 1996
Barbieri, Hall, Raby, Romanino 1997
Barbieri, Hall, Romanino 1997]

- lepton sector as in the A_4 model
- t and b masses at the renormalizable level (τ mass from higher dim operators) at the leading order

$$m_{u,d} \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

33>>22,23,32

$$\langle \eta \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$$

$$m_t, m_b > m_c, m_s \neq 0$$

$$V_{cb}$$

- masses and mixing angles of 1st generation from higher-order effects
- despite the large number of parameters two relations are predicted

$$\sqrt{\frac{m_d}{m_s}} = |V_{us}| + O(\lambda^2)$$

$0.213 \div 0.243 \qquad 0.2257 \pm 0.0021$

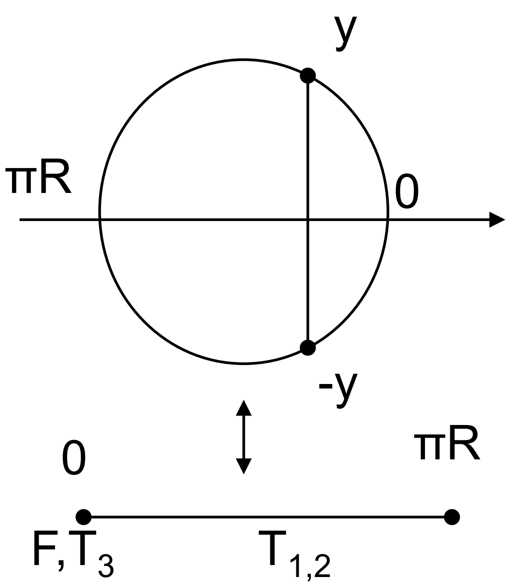
$$\sqrt{\frac{m_d}{m_s}} = \left|\frac{V_{td}}{V_{ts}}\right| + O(\lambda^2)$$

$0.208^{+0.008}_{-0.006}$

- vacuum alignment explicitly solved
- lepton sector not spoiled by the corrections coming from the quark sector

other option: SUSY SU(5) in $5D=M_4 x(S^1 x Z_2)$ + flavour symmetry $A_4 x U(1)$

[AFH]



DT splitting problem solved
via SU(5) breaking induced by compactification

dim 5 B-violating operators forbidden!
p-decay dominated by gauge boson exchange (dim 6)

unwanted minimal SU(5) mass relation $m_e=m_d^T$ avoided by assigning $T_{1,2}$ to the bulk

the construction is compatible with A_4!

	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$
$SU(5)$	1	$\bar{5}$	10	10	10	5	$\bar{5}$
A_4	3	3	1''	1'	1	1	1'

reshuffling of singlet reps.

unsuppressed top Yukawa coupling $T_3 T_3$

realistic quark mass matrices
by an additional U(1) acting on $T_{1,2}$

neutrino masses from see-saw
compatible with both normal and
inverted hierarchy

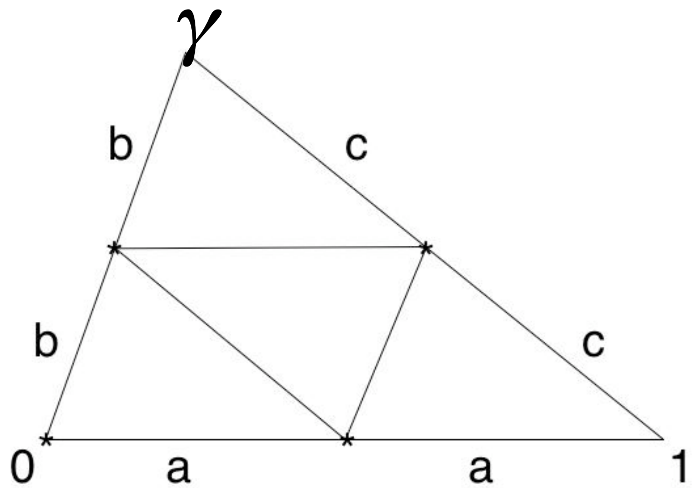
TB mixing + small corrections

A_4 as a leftover of Poincare symmetry in D>4 [AFL]

D dimensional
Poincare symmetry:
D-translations x SO(1,D-1)

usually broken by
compactification down to 4 dimensions:
4-translations x SO(1,3) x …

a discrete subgroup of the (D-4) euclidean group = translations x rotations
can survive in specific geometries



Example: D=6

2 dimensions
compactified on T^2/Z_2

$$z \to z+1$$
$$z \to z+\gamma$$
$$z \to -z$$

four fixed points

$$\text{if} \quad \gamma = e^{i\frac{\pi}{3}}$$

compact space is a regular tetrahedron
invariant under

$$S: \quad z \to z+\frac{1}{2}$$ [translation]

$$T: \quad z \to \gamma^2 z$$ [rotation by 120^0]

[subgroup of 2 dim Euclidean group = 2-translations x SO(2)] the four fixed points (z_1, z_2, z_3, z_4) are permuted under the action of S and T

$$S: \qquad (z_1, z_2, z_3, z_4) \to (z_4, z_3, z_2, z_1)$$
$$T: \qquad (z_1, z_2, z_3, z_4) \to (z_2, z_3, z_1, z_4)$$

S and T satisfy

$$\boxed{S^2 = T^3 = (ST)^3 = 1}$$

the compact space is invariant under a remnant of 2-translations x SO(2) isomorphic to the A_4 group

Field Theory

brane fields $\varphi_1(x)$, $\varphi_2(x)$, $\varphi_3(x)$, $\varphi_4(x)$ transform as 3 + (a singlet) under A_4

The previous model can be reproduced by choosing l, e^c, μ^c, τ^c, $H_{u,d}$ as brane fields and φ_T, φ_S and ξ as bulk fields.

String Theory [heterotic string compactified on orbifolds]

in string theory the discrete flavour symmetry is in general bigger than the isometry of the compact space. [Kobayashi, Nilles, Ploger, Raby, Ratz 2006]

orbifolds are defined by the identification

$$(\vartheta\, x) \approx x + l \qquad \begin{cases} l = n_a e_a & \text{translation in a lattice} \\ \vartheta & \text{twist} \end{cases}$$

group generated by (ϑ, l) is called space group

fixed points: special points x_F satisfying

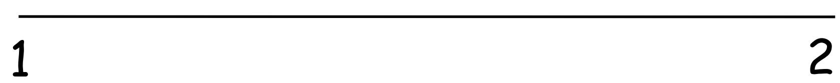
$$x_F \equiv (\vartheta_F^K\, x_F) + l_F \qquad \text{for some} \quad (\vartheta_F^K, l_F)$$

twisted states living at the fixed point $x_F=(\vartheta_F^K, l_F)$ have couplings satisfying space group selection rules [SGSR]. Non-vanishing couplings allowed for

$$\prod_F (\vartheta_F^K, l_F) \equiv (1,0)$$

G_f is the group generated by the orbifold isometry and the SGSR

Example: S^1/Z_2



Isometry group = S_2 generated by σ^1 in the basis {|1>,|2>}

SGSR = $Z_2 \times Z_2$ generated by $(\sigma^3,-1)$

[allowed couplings when number n_1 of twisted states at |1> and the number n_2 of twisted states at |2> are even]

G_f = semidirect product of S_2 and $(Z_2 \times Z_2) \equiv D_4$

group leaving invariant a square

relation between A_4 and the modular group [AF2]

modular group PSL(2,Z): linear fractional transformation

complex variable

$$z \to \frac{az+b}{cz+d} \qquad a,b,c,d \in Z \qquad ad-bc=1$$

discrete, infinite group generated by two elements

$$\underbrace{z \to -\frac{1}{z}}_{S} \qquad \underbrace{z \to z+1}_{T}$$

obeying

$$S^2 = (ST)^3 = 1$$

the modular group is present everywhere in string theory

[any relation to string theory approaches to fermion masses?]

Ibanez; Hamidi, Vafa; Dixon, Friedan, Martinec, Shenker; Casas, Munoz; Cremades, Ibanez, Marchesano; Abel, Owen

A_4 is a finite subgroup of the modular group and

$$A_4 = \frac{PSL(2,Z)}{H} \quad \Rightarrow \quad \boxed{\text{representations of } A_4 \text{ are representations of } PSL(2,Z)}$$

infinite discrete normal subgroup of PSL(2,Z)

future improvements
on
atmospheric and reactor angles

$\sin^2\theta_{23}$

$\delta(\sin^2\theta_{23})$ reduced by future LBL experiments from $\nu_\mu \to \nu_\mu$ disappearance channel

$$P_{\mu\mu} \approx 1 - \sin^2 2\vartheta_{23} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$\vartheta_{23} \approx \frac{\pi}{4}$$

$$\delta\vartheta_{23} \approx \frac{\sqrt{\delta P_{\mu\mu}}}{2}$$

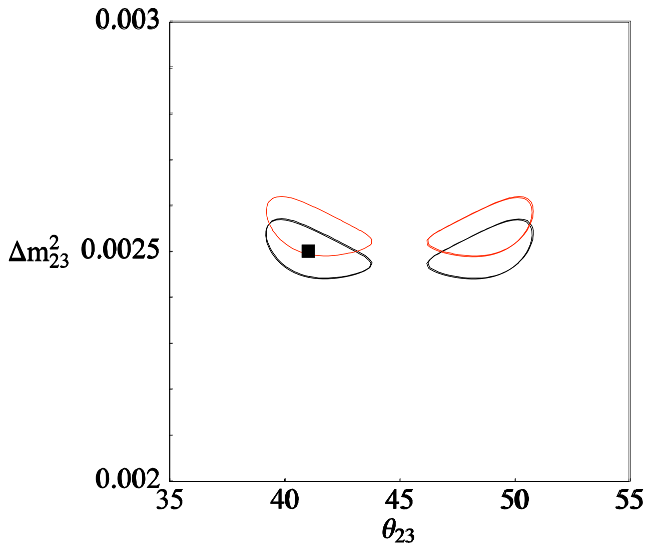
i.e. a small uncertainty on $P_{\mu\mu}$ leads to a large uncertainty on θ_{23}

- no substantial improvements from conventional beams
- superbeams (e.g. T2K in 5 yr of run)

$$\delta P_{\mu\mu} \approx 0.01$$

$$\delta\vartheta_{23} \approx 0.05\ \text{rad} \leftrightarrow 2.9^0$$

improvement by about a factor 2



T2K-1
90% CL
black = normal hierarchy
red = inverted hierarchy
true value 41^0
[courtesy by Enrique Fernandez]

maximal mixing from renormalization group running?

θ_{23} maximal by RGE effects?

[Ellis, Lola 1999
Casas, Espinoza, Ibarra, Navarro 1999-2003
Broncano, Gavela, Jenkins 0406019]

running effects important only for quasi-degenerate neutrinos

2 flavour case

boundary conditions at Λ>> e.w. scale m_2, m_3, ϑ_{23}

at $Q < \Lambda$

$$\vartheta_{23}(Q) \approx \frac{\pi}{4} \quad \Leftrightarrow \quad \varepsilon \approx -\frac{\delta m}{m} \cos 2\vartheta_{23}$$

[possible only if $\delta m \equiv m_2 - m_3 << m_2 + m_3 \approx 2m$]

$$\varepsilon \approx \frac{1}{16\pi^2} y_\tau^2 \log \frac{\Lambda}{Q}$$

gives the scale Q at which $\theta_{23}(Q)$ becomes maximal

$\Rightarrow$

m_2, m_3, ϑ_{23} fine tuned to obtain Q at the e.w. scale

a similar conclusion also for the 3 flavour case:

$$\sin^2 2\vartheta_{12} = \frac{\sin^2 \vartheta_{13} \sin^2 2\vartheta_{23}}{(\sin^2 \vartheta_{23} \cos^2 \vartheta_{13} + \sin^2 \vartheta_{13})^2}$$

infrared stable fixed point

[Chankowski, Pokorski 2002]

if $\vartheta_{23} = \dfrac{\pi}{4}$

$$\sin^2 2\vartheta_{12} = \frac{4\sin^2 \vartheta_{13}}{(1 + \sin^2 \vartheta_{13})^2} < 0.2 \quad \text{(Chooz)}$$

wrong!

vacuum alignment from minimization of the scalar potential

(1) natural vacuum alignment

$$\langle \varphi_T \rangle = (v_T, 0, 0)$$
$$\langle \varphi_S \rangle = (v_S, v_S, v_S)$$
$$\langle \xi \rangle = u$$

$$v_T \approx v_S \approx u$$

it is not a local minimum of the most general renormalizable scalar potential V depending on φ_S , φ_T , ξ and invariant under A_4

a simple solution in 1 extra dimension ≡ ED

[Altarelli, F. 0504165]

$\langle \varphi_T \rangle = (v_T, 0, 0)$

local minimum of V_0

e^c, μ^c, τ^c $\qquad$ $l, h_{u,d}$

0 $\qquad$ y $\qquad$ L

$\langle \varphi_S \rangle = (v_S, v_S, v_S)$

$\langle \xi \rangle = u$

local minimum of V_L

ν masses arise from local operators at y=L

$$\frac{(\varphi_S ll) h_u h_u}{\Lambda^2} \qquad \frac{\xi (ll) h_u h_u}{\Lambda^2}$$

this explains also the absence of the terms with $\varphi_S \leftrightarrow \varphi_T$

charged lepton masses from non-local operators

bulk fermionY=-1

$$\left.\begin{aligned} &\frac{(f^c \varphi_T F)\delta(y)}{\sqrt{\Lambda}} \\ &- MFF^c \\ &\frac{(F^c l) h_d}{\sqrt{\Lambda}}\delta(y-L) \end{aligned}\right\} \quad E << M$$

$$\frac{(f^c \varphi_T l) h_d}{\Lambda} e^{-ML}$$

a 4D supersymmetric solution ≡ SUSY [Altarelli,F. hep-ph/0512103]

L is identified with the superpotential w_{lepton} in the lepton sector

w_{lepton} is invariant under $A_4 \times Z_3 \times U(1)_R$

	l	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	1	1''	1'	1	3	3	1	1	3	3	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$U(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2

matter fields (l, e^c, μ^c, τ^c) — Higgses ($h_{u,d}$) — A_4 breaking sector (φ_T, φ_S, ξ, $\tilde{\xi}$) — ``driving fields'' (φ_0^T, φ_0^S, ξ_0)

absence of $\varphi_S \leftrightarrow \varphi_T$ $\quad x(ll)$ automatic

$$w = w_{lepton} + w_d + \ldots$$

$$w_d = M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) + g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0 (\varphi_S \varphi_S) + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2$$

minimum of the scalar potential at:

$$\begin{aligned} \langle \varphi_T \rangle &= (v_T, 0, 0) \\ \langle \varphi_S \rangle &= (v_S, v_S, v_S) \\ \langle \xi \rangle &= u \\ \langle \tilde{\xi} \rangle &= 0 \end{aligned}$$

$$v_T = -\frac{3M}{2g} \qquad v_S^2 = -\frac{g_4}{3g_3} u^2$$

u undetermined